RESEMBLANCE-BASED LOGIC: A MORPHOLOGICAL MODEL

1. 1. Introduction

This document presents a reformulation of logic grounded in **morphological relationships**—specifically, the distinction between **resemblance** (sameness) and **diversity** (difference).

We show that all binary logical operators can be understood as functions that respond to whether inputs are the same or different, and that the complete space of such operators can be counted systematically.

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2. 2. Basic Definitions

2.1. Definition 2.1 (Alphabet)

Let L be a finite set of symbols with $|L| = n$.

2.2. Definition 2.2 (Input Pairs)

For any binary operator $f : L \times L \to L$, we consider all ordered input pairs (a, b) where $a, b \in L$.

These pairs fall into two categories:

- **Resemblance pairs**: (a, a) where inputs are identical – n such pairs.
- **Diversity pairs**: (a, b) where $a \neq b$ – $n(n-1)$ such pairs.

Total: n^2 ordered pairs.

2.3. Definition 2.3 (Output Rules)

For each input pair, the output is chosen according to morphological relationships.

When inputs are the same (a, a):

1. **Self**: output $= a$ (return the input)
2. **Diverse**: output $\in L \setminus \{a\}$ (return something different)

Thus there are n **possible outputs**: the input itself, or any of the $n-1$ other symbols.

When inputs are different (a, b) with $a \neq b$:

1. **First**: output $= a$
2. **Second**: output $= b$
3. **Other**: output $\in L \setminus \{a, b\}$ (return something different from both)

Again there are n **possible outputs**: first input, second input, or any of the $n-2$ other symbols.

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3. 3. Counting Operators

3.1. Theorem 3.1 (Total Operator Count)

The total number of binary operators that can be constructed using resemblance-based rules is

$$n^{n^2}.$$

Proof.
We have n^2 total input pairs.
For each pair, we may choose independently among n possible outputs.
Hence the total number of distinct operators is

$$\underbrace{n \times n \times \cdots \times n}_{n^2 \text{ times}} = n^{n^2}.$$

This equals the cardinality of the function space $L^{L \times L}$, confirming that the resemblance-based framework is **complete**. ∎

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4. 4. Boolean Case ($n = 2$)

For the Boolean alphabet $L = \{0, 1\}$:

$$2^{2^2} = 2^4 = 16 \text{ operators}.$$

This matches exactly the 16 binary Boolean connectives (AND, OR, NAND, NOR, XOR, …).

Input pairs:

- Resemblance: $(0, 0)$ and $(1, 1)$
- Diversity: $(0, 1)$ and $(1, 0)$

Output choices per pair:

- $(0, 0)$: output 0 (self) or 1 (diverse) — 2 choices
- $(1, 1)$: output 1 (self) or 0 (diverse) — 2 choices
- $(0, 1)$: output 0 (first) or 1 (second) — 2 choices
- $(1, 0)$: output 1 (first) or 0 (second) — 2 choices

Total: $2 \times 2 \times 2 \times 2 = 16$ ✓

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5. 5. Example: NAND Operator

The NAND operator can be expressed using resemblance-based rules:

Inputs	Type	Rule	Output
$(0,0)$	Resemblance	Diverse	1
$(1,1)$	Resemblance	Diverse	0
$(0,1)$	Diversity	Second	1
$(1,0)$	Diversity	First	1

Pattern:

- For resemblance pairs: always choose "diverse" (output ≠ input)
- For diversity pairs: always choose the output that equals 1

Thus NAND emerges naturally from morphological choices without invoking the NOT operator as a primitive.

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6. 6. Key Insights

6.1. 6.1 Pure Morphological Foundation

Logic can be completely grounded in two levels of morphological analysis:

1. Are the inputs the same or different? (Resemblance vs. Diversity)
2. Should the output be the same as or different from the input(s)? (Self/First/Second vs. Diverse/Other)

6.2. 6.2 Complete Expressiveness

The resemblance-based model generates **all** possible binary operators:

- n^{n^2} total operators
- Equals the complete function space $L^{L \times L}$
- No restrictions on expressiveness

6.3. 6.3 Independence of Pairs

Each ordered input pair is treated as an independent decision point.
Hence:

- $(0,1)$ and $(1,0)$ receive separate rules
- Order matters for diversity pairs
- Total count grows as n^{n^2}

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7. 7. General Formula

For an alphabet of size n:

Property	Value
Total input pairs	n^2
Resemblance pairs	n
Diversity pairs	$n(n-1)$
Output choices per pair	n
$\boxed{\text{Total operators}}$	$\boxed{n^{n^2}}$

Examples:

- $n = 2$: $2^4 = 16$ (Boolean logic)
- $n = 3$: $3^9 = 19{,}683$ (Ternary logic)
- $n = 4$: $4^{16} = 4{,}294{,}967{,}296$ (Quaternary logic)

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8. 8. Conclusion

The resemblance-based model demonstrates that:

1. **Logic is morphology** - All logical operations can be understood as patterns of sameness and difference.
2. **No primitives needed** - Traditional operators (AND, OR, NOT) emerge from morphological choices rather than being assumed as primitives.
3. **Complete framework** - The model captures the entire space of binary operators.

This reformulation suggests that logical reasoning is fundamentally about recognizing and preserving (or transforming) patterns of resemblance and diversity.