Università degli Studi di Milano

Data Science for Economics

Dynamic Economic Modeling: Problem Set 1

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1 Consumption with different generations

An economy is composed of identical individuals. Each individual lives for 2 periods (you may imagine them as adulthood and old age). Individuals may work during the first period of their life for a proportion L of the day, for an income equal to wL. In the second period they retire and consume their remaining lifetime savings. Their lifetime utility is given by:

$$U = \log(C_1) + \log(C_2) + \log(1 - L), \tag{1}$$

where C_i is consumption in period i.

(a)

If the rate of interest on savings is R, write down the individual's budget constraints for both periods, and then combine them in a lifetime (inter-temporal) budget constraint.

Solution

For a detailed process of obtaining the solutions, I suggest checking out the PDF file on my GitHub, which I uploaded as *proof of work*.
The budget constraints for the two periods are:

$$\begin{cases} C_1 + S_1 \leq wL \\ C_2 + S_2 \leq (1+R)S_1, \end{cases}$$

where $S_2 = 0$ since agents will not "die" neither in debt nor credit. However, since the agents are assumed not to waste resources, the budget constraints become:

$$\begin{cases} C_1 + S_1 = wL \\ C_2 + S_2 = (1+R)S_1, \end{cases}$$

Solving for S_1, the inter-temporal budget constraint (IBC) is given by

$$IBC : C_1 + \frac{C_2}{(1+R)} = wL. \tag{2}$$

(b)

Solve for optimal consumption each period and the optimal work effort. Comment on what you find.

Solution

We solve the IBC for C_2:

$$C_2 = (wL - C_1)(1 + R)$$

We then plug C_2 in U:

$$U = ln(C_1) + ln(wL - C_1 + wLR - C_1R) + ln(1 - L)$$

We now solved the *unconstrained* optimization problem, by setting the FOC:

$$\frac{\partial U}{\partial C_1} = 0 \quad \text{and} \quad \frac{\partial U}{\partial L} = 0$$

After some algebra, the first FOC yields:

$$C_1^* = \frac{wL}{2}$$

The second FOC yields:

$$L^* = \frac{w + C_1}{2w}$$

If we substitute C_1^* in L^* we get the values of:

$$L^* = \frac{2}{3} \quad \text{and} \quad C_1^* = \frac{w}{3}$$

We can now derive C_2^* , by substituting C_1^* into $C_2 = (wL - C_1)(1 + R)$, and we get:

$$C_2^* = \frac{w}{3}\Big(1 + R\Big).$$

Comment

L^* does not depend on wages. Even if the salary increases, the actor does not seize such an increase by working more. The interpretation can be twofold:

1. It could be argued that the actor is keen on preserving their leisure time, given that they are not willing to sacrifice it for an overall higher salary

2. The individual is still working for 2/3 of their total time endowment, which could be regarded as a high fraction of it. Furthermore, even a decrease in wages would not "convince" the individual to work less.

Overall, we may consider them as an actor interested in maintaining their 1/3 - 2/3 "work-life balance".
C_1^* is directly proportional to wage, being 1/3 of it. If the wage doubles, C_1 doubles as well. The relationship between C_2^* and w is still directly proportional, but it also considers the interest rate: if R increases, so does C_2^* .

(c)

The government introduces a fixed pension paid to individuals in the second period of their lives, funded by a "lump sum tax" paid by those who work. Re-write the inter-temporal budget constraint. What will be the impact of the pension on consumption and labour supply decisions of the young? What about the old when the pension is introduced? Give intuition for your answers.

Solution

Let t be the amount of the lump sum tax. We assume the fixed pension has the same value of the taxation. The IBC is found by

$$\begin{cases} C_1 + S_1 = wL - t \\ C_2 + S_2 = (1+R)S_1 + t \end{cases}$$

where $S_2 = 0$. Solving for S_1 we find:

$$S_1 = \frac{C_2 - t}{1+R}. \tag{3}$$

and by substituting S_1 in the budget constraint of the first period, we get the IBC:

$$C_1 + \frac{C_2 - t}{1+R} = wL - t \tag{4}$$

Deriving C_2 we get:

$$C_2 = (wL - t - C_1)(1+R) + t$$

By substituting C_2 in U we get:

$$U = ln(C_1) + ln((wL - t - C_1)(1+R) + t) + ln(1-L)$$

Once again, we set the following FOC:

$$\frac{\partial U}{\partial C_1} = 0 \quad \text{and} \quad \frac{\partial U}{\partial L} = 0$$

From the first one we get:

$$C_1 = \frac{wL}{2} - \frac{tR}{2(1+R)}$$

From the second one we get:

$$L = \frac{1}{2} + \frac{C_1}{2w} + \frac{tR}{2w(1+R)}$$

We find C_1^* and L^* by plugging C_1 into L or vice-versa; we then do the same for C_2^*. After some algebra we get:

$$L^* = \frac{2}{3} + \frac{1}{3}\frac{tR}{w(1+R)}$$

$$C_1^* = \frac{1}{3}w - \frac{tR}{3(1+R)}$$

$$C_2^* = \frac{1}{3}w(1+R) - \frac{1}{3}tR$$

Comment

The labour supply is positively proportional to t: as t increases, the individual works more to compensate for the loss of income, on top of the 2/3 of the time observed in the scenario without taxation.
Additionally, the optimal level of labour *increases less* as w increases, differently from the scenario without taxation (where L was fixed at 2/3). I use the term *increases less*, because while 2/3 of the time endowment is still fixed, there is an increase of L that is a positive function of t, but a negative one of w. At a given level of taxation, if the wages are higher the actor will work less.
Consumption in period 1 is negatively affected by t, and so is consumption in period 2. The negative relation between C_2 and t is due to consumption smoothing.

(d)

The pension is now funded by an income tax, i.e., a tax equal to twL, where t is the tax rate. Will the behaviour of the young change in the case where $R = 0$? Interpret this result.

Solution

New budget constraints

$$\begin{cases} C_1 + S_1 = wL(1-t) \\ C_2 + S_2 = (1+R)S_1 + wLt \end{cases}$$

By assumption $S_2 = 0$:

$$S_1 = \frac{C_2 - twL}{(1+R)}$$

substituting in $t = 1$

$$C_1 + \frac{C_2 - twL}{1+R} = (1-t)wL$$

with $R = 0$, we get the new IBC:

$$C_1 + C_2 = wL$$

By following the previous steps, we first get:

$$C_1 = \frac{wL}{2} \quad \textit{and} \quad L = \frac{1}{2} + \frac{C_1}{2w}$$

from which we get the optimal values of:

$$\begin{cases} C_1^* = \frac{w}{3} \\ L^* = \frac{2}{3} \end{cases}$$

These solutions are the exact same of point (b).
We conclude that behaviour does not change with an income tax if $R = 0$.

As an exercise, I tried to calculate what happens when $R \neq 0$.
The complete process is in the handwritten PDF on GitHub, but the final results should be:

$$\begin{cases} C_1^* = \frac{w(1-2t+R-tR)}{3(1+R)} \\ L^* = \frac{2}{3} \end{cases}$$

2 Stylised facts of the business cycle

A business cycle is made of an expansion (boom) and a contraction (recession). During the expansion all good things (GDP, employment, productivity, and so on) tend to go up, or grow faster than "normal", and bad things (e.g. unemployment) tend to fall. During the contraction good things go down and bad things go up.

In order to reproduce the results I used the R statistical software. The code was developed by myself and can be found on my GitHub .

Most of the data used comes from the Fred API, fetched through the *fredr* package. The variables obtained through this source are the following:

- GNP | Gross National Product | (GNPC96)
- CD | Durables consumption | (PCEDG)
- CND | Nondurable consumption | (PCEND)
- AveH | Average Weekly Hours | (AWHNONAG)
- Y | Real Output | (A939RX0Q048SBEA)
- C | Real Consumption |(A794RX0Q048SBEA)
- I | Real Investment |(GPDIC1)
- r | Interest rate and discount factor | INTDSRUSM193N

Other variables were downloaded from the U.S. Bureau of labour statistics, such as:

- N | Total work hours
- L | Total employment
- w | Average wage

These variables can be found in my GitHub repository in the "additional variables" xlsx file.
Since quarterly data for A (total factor productivity) could not be found, I retrieved the quarterly percentage change in total factor productivity from the

website of the *Federal Reserve Bank of San Francisco*; however, as Prof. Massaro explained the class of the 14th of June, this is not a good proxy and to retrieve quarterly data a specific approach is necessary.
However, the corresponding variable in my data is called dA in the tables (or dtfp in the R code).
The remaining variables, which are combinations of the aforementioned ones, have been computed through the R software.
The computation of *real* variables from *nominal* variables has been carried out through the Consumer Price Index found on FRED (CPIAUCSL).

(a)

Replicate Table 1 and 2 for the US economy from - ideally - 1950 Q1 to the newest data you can find (either 2020 or 2023). You can use real GDP instead of GNP if you want.

Solution

Table 1: Cyclical Behaviour of the US Economy (1964Q1-2021Q2)

Variables	sd%	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
GNP	1.58	0.15	0.34	0.56	0.76	1.00	0.76	0.56	0.34	0.15
CND	1.91	0.21	0.25	0.32	0.34	0.39	0.16	-0.01	-0.19	-0.33
CD	4.00	-0.17	-0.03	0.18	0.36	0.67	0.69	0.61	0.50	0.38
H	1.67	0.34	0.50	0.64	0.77	0.88	0.58	0.38	0.18	0.02
AveH	0.44	-0.14	0.03	0.21	0.42	0.61	0.64	0.55	0.41	0.26
L	1.39	0.44	0.57	0.68	0.76	0.82	0.49	0.28	0.08	-0.06
GNP/L	0.91	-0.41	-0.28	-0.07	0.14	0.48	0.56	0.54	0.47	0.36
w	0.85	0.06	0.16	0.28	0.35	0.37	0.49	0.46	0.37	0.26

Table 2: Business Cycle Statistics for the US Economy (1964Q1-2021Q2)

Variable	SD	Relative_SD	First Order Auto-Correlation	Contemporaneous Corr with Y
Y	1.57	1.00	0.76	1.00
C	1.39	0.89	0.68	0.89
I	6.55	4.17	0.82	0.88
N	0.45	0.29	0.71	-0.73
Y/N	1.34	0.85	0.71	0.97
w	0.85	0.54	0.86	0.37
r	0.34	0.22	0.87	0.46
dA	313.32	199.57	0.03	0.07

(b)

Verify whether or not the following business cycle facts from Cooley and Prescott (1995) still hold today:

1. Consumption is smoother than output.
2. Volatility in GNP is similar in magnitude to volatility in total hours.
3. Volatility in employment is greater than volatility in average hours. Therefore most labour market adjustments operate on the extensive rather than intensive margin.
4. Productivity is slightly pro-cyclical.
5. Wages are less variable than productivity.
6. There is no correlation between wages and output (nor with employment for that matter).

Answers

1. By looking at the variables Y and C in table 2. it appears that consumption is still smoother than output, having a relative standard deviation of 0.89 (where the standard deviation of output is 1). Comparing Cooley and Prescott's table with mine, however, it appears that non-durables' consumption became much more volatile. Indeed, while durable consumption has remained well above GNP's volatility (4.00 standard deviation in our data, compared to Cooley and Prescott's value of 4.96), non-durable consumption's standard deviation spiked from 0.86 to 1.91, becoming thus more volatile than GNP.
2. This statement appears to remain true, given that the value of the standard deviation for GNP is 1.58, while the standard deviation for H (total hours) is 1.67
3. Volatility in employment is greater than volatility in average hours: the standard deviation of L is 1.39, compared to a 0.44 standard deviation of AveH.
4. This seems to remain true. The cross-correlation of productivity with output at zero lag is positive and similar in magnitude to Cooley and Prescott's data (their value was 0.41, compared to my 0.48).
5. While this remains the case, the gap between the two's standard deviations reduced; in particular, the standard deviation of wage (w) increased to 0.85, compared to a value of 0.91 for productivity (GNP/L).
6. This does not seem to be true anymore: according to my data, wages appear to have a correlation with output of 0.37; thus identifying a relevant positive correlation, making the variable wage pro-cyclical. The correlation between output and employment appears to remain stable, with a value that moved from 0.85 to 0.82

(c)

Verify whether or not the following business cycle facts from King and Rebelo (1999) still hold today:

1. Consumption of non-durables is less volatile than output.
2. Consumer durables are more volatile than output.
3. Investment is three times more volatile than output.
4. Government expenditures are less volatile than output.
5. Total hours worked are about the same volatility as output.
6. Capital is much less volatile than output.
7. Employment is as volatile as output, while hours per worker are much less volatile than output.
8. Labour productivity is less volatile than output.
9. The real wage is much less volatile than output.

Answers

1. As explained in the previous answer, the volatility of non-durables consumption appears to have increased dramatically, from 0.86 to 1.91. Given that the current volatility of GNP is now 1.58, the statement appears to be false according to my data.
2. This appears to remain the case. As noted in a previous answer, the standard deviation of durables consumption is 4.00, well above the 1.58 of GNP.
3. While investment's volatility still appears to be higher than output, its relative standard deviation increased to 4.17, suggesting that it is now fourfold higher than output's.
4. Although government expenditure is not present in the table, I downloaded the relevant data through FRED (A955RX1Q020SBEA). After de-trending with an HP filter, the value of the standard deviation of G appears to be 0.01040649. Thus being less volatile than output.
5. This appears to not be the case anymore, since the relative standard deviation of total hours worked with respect to output is 0.29, thus one-third of the volatility of output.
6. While we do not have actual data for 'capital' we do have data for capital productivity, which appears to be the variable in King and Rebelo's table. However, given that quarterly data for TFP is not available, I retrieved the values of quarterly percentage change in factor productivity, which appear to be not adequate for this task (given the extremely high volatility). However, we could approximate $K_{t+1} = I + (1-\delta)K_t$, highlighting lower volatility than output.

7. This appears to remain the case since the relative standard deviation of employment compared to GNP is 0.88, whereas the average hours per worker is 0.28.

8. This statement remains true according to our data, since the percentage standard deviation of GNP is 1.58, while labour productivity's (GNP/L) is 0.91.

9. This last statement appears to be true as well since the real wage's relative standard deviation is 0.22 with respect to the output.