UNIVERSITÀ
DEGLI STUDI
DI TRIESTE

Statistics

Descriptive Data Analysis

Luca Pennella

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Statistics

Statistics is the science that deals with collecting data and extracting information/knowledge from them.

Data can help to understand the phenomena, but it is necessary to collect the necessary one and do it well; data must then be examined to isolate and highlight the information.

Starting from a question about a phenomenon, statistics is involved in determining which data can be used to answer that question, and, if the data is not already available, how it should be collected.

Then follows the phase in which the data is analyzed to extract the information.

Statistics I

Briefly, the steps of a statistical analysis are

- *formulate a question*, translate a cognitive need so that it is susceptible of an answer in statistical terms;
- *identifying or collecting data*, is a broad topic, which goes by the name of experimental design and sampling; it is based on the probability theory;
- *organize and look at the data*, from the collected data it is not immediate to extract the information you need, but you can either synthesize it appropriately and/or represent it graphically, according to the information sought;
- *formulate a model* to explain the observed data, based on the assumptions made about the phenomenon. The model is then estimated using the available data. This model can be utilized to either confirm or reject hypotheses about the phenomenon or to make predictions for future occurrences.

Object of the analysis: the population

The final aim of the statistical analysis is to know some aspects of interest of the population

A population is an entire group, a complete set of units.

- each component of the whole population is known as statistical unit, such as:
  - the population of Italian men aged 18 years old on 01/01/2012;
  - Italian families at 01/01/2012;
  - 50 major urban centres spread across Europe.
- The population can be finite (i.e. Italian population) or infinite (i.e. all the people affected by a disease, in the past and the future).

To conduct a statistical analysis we collect variables on the statistical units.

Statistical unit

A student of a scientific lyceum can be a component of several populations, due to the context in which he/she is observed:

- the student is an element of the population of pupils attending a given school
- the student is part of the population of people living in the same municipality on a given date
- he/she can belong to the population of young people in the given age range
- if we refer to the context of Italian secondary schools, he can no longer be considered a statistical unit. In this case, the statistical units are the individual schools (school population), where the variable can be the number of pupils enrolled in a given year.

Sampling

When we cannot (want to) observe the entire population we can use a sampling procedure.

Sampling and Inference

We observe just a part of the population, the sample, and we generalize to the whole population what was observed on the sample.

- Sampling is natural: it is also done in the kitchen and our daily life.
- Let's imagine that we are cooking a soup: to get an idea of the possible success, we take a taste.
- Tasting a spoon of soup and deciding if it is enough salted, you are doing exploratory-descriptive analysis.
- If you conclude that all the soup is tasteless, you do inference.

Sampling and representativeness

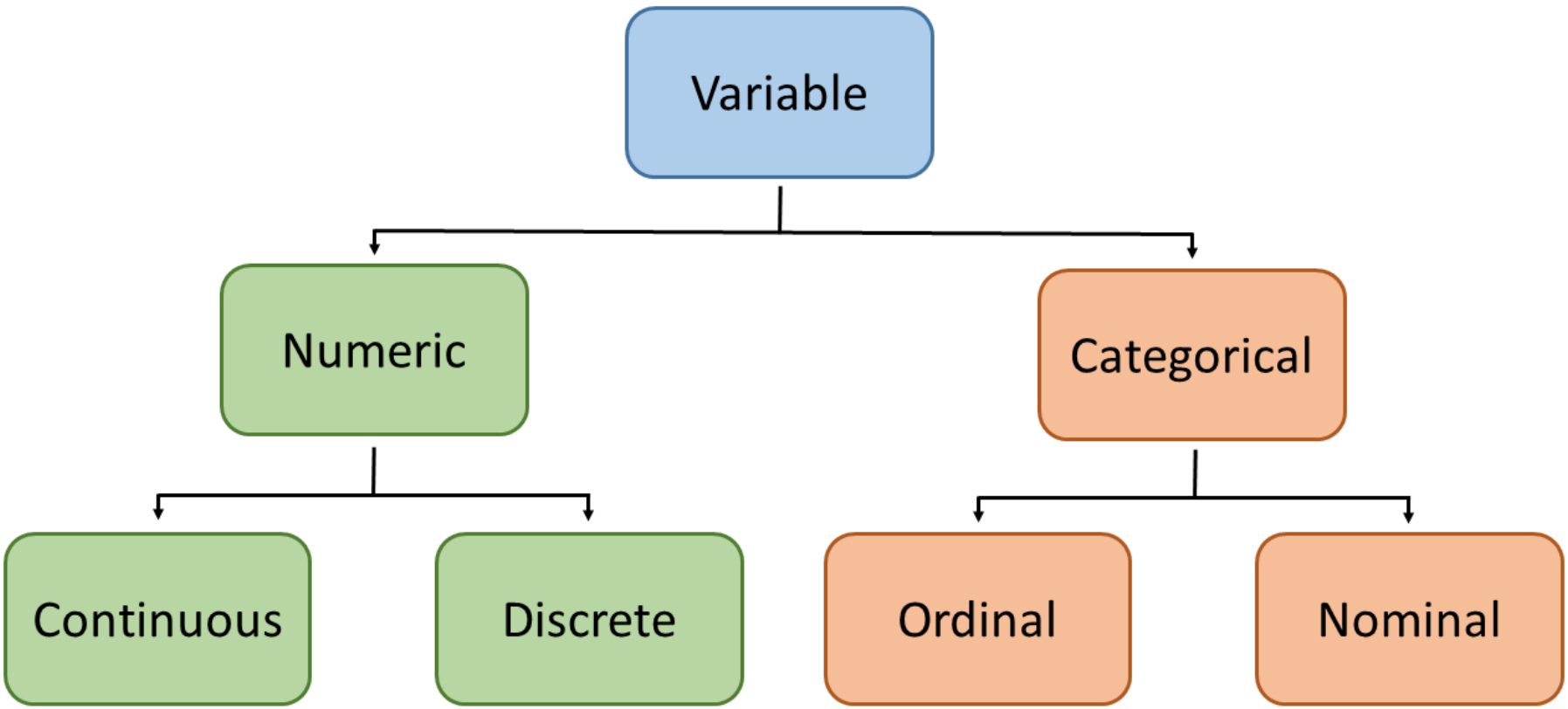
- For the inference to be valid, the spoon we taste must be representative of the entire preparation (soup).
- If we add the salt first, then all the ingredients, never mix and taste the soup on the surface, we probably don't have a "representative" sample.
- If we throw in the salt first, then all the ingredients, and mix all the ingredients well before tasting, probably the representativeness of the sample is better.

Basic terminology

A statistic data is the result of the survey (measurement/observation) of some characteristic (variables) on a statistical unit belonging to a population.

- statistical unit: the individual component of the population;
- variable: the elementary aspect surveyed on the statistical units of the population;
- modalities/categories of a variable: the different ways in which the variable presents itself in the statistical units
- support: the (theoretical) set of modalities of a variable.

Types of variables



Categorical/qualitative variables

- A variable is categorical if modalities are expressed in verbal form;
  - a qualitative variable is nominal if its modalities do not imply an order;
  - a qualitative variable is ordinal if its modalities imply an order;

Example:

- Categorical nominal
  Did you like the last movie directed by Q. Tarantino?
  - I watched it and I liked it
  - I watched it but I didn't like it
  - I haven't watched it
- Categorical ordinal
  How frequently did you drink beer?
  - Never
  - Once a week
  - More than once a week
  - Everyday
  - More than once per day

Numerical/quantitative variables

A variable is numerical if modalities are expressed in numerical form. We distinguish

- with respect to the value that it can assume
  - a numerical variable is discrete if the set of its modalities is finite or countable (in other words, if the quantity that it represents varies with "jumps");
  - a numerical variable is continuous variable if the set of its modalities is a range, limited or unlimited.;
- with respect to the operations that it is reasonable to do: interval or ratio.

Examples

- Discrete ratio
  How often have you been to the cinema in the last three months?
- Continuous ratio
  What is your height (cm)?
- Discrete interval
  In what year were you born?
- Continuous interval
  External temperature

Data matrix

The data is organized in a matrix, for example, the data to construct the gap-minder graph looks like this:

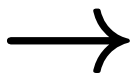
variable ↓

	Country	**time**	**gdppc**	**lifexp**	**Region**
1	Albania	1980	1061	70	Europe
2	Albania	1990	978	74	Europe
3	Albania	1991	688	73	Europe
4	Albania	1992	643	73	Europe
5	Albania	1993	714	73	Europe
6	Albania	1994	785	74	Europe
⋮	⋮	⋮	⋮	⋮	⋮
N-1	Zimbabwe	2010	323	50	Africa
N	Zimbabwe	2011	348	52	Africa

← statistical units

lifexp	Region
70	Europe
74	Europe
73	Europe
73	Europe
73	Europe
74	Europe
⋮	⋮
50	Africa
52	Africa

Sometimes, it is convenient to code categorical variables, such as Region, using 0/1 variables. Here we have 4 possible categories: The Americas, Europe, Africa and Asia.



Thus, we replace the Region variable with 4 variables: Am (yes/no), Eu (yes/no), Af (yes/no) and As (yes/no).

lifexp	Am	Eu	Af	As
70	0	1	0	0
74	0	1	0	0
73	0	1	0	0
73	0	1	0	0
73	0	1	0	0
74	0	1	0	0
⋮	⋮	⋮	⋮	⋮
50	0	0	1	0
52	0	0	1	0

0/1 variables are usually called indicator variables (because they indicate the presence/absence of a mode), binary variables, or dummy variables (they are not truly variables).

- Looking closer, we note that our last coding is redundant because we know that, in our study, if a country is not in Americas (Am), Europe (Eu) and Africa (Af), can only be in Asia (As).
- Then only 3 of the 4 variables are enough to encode the region.
- Here we code the modality "yes" as 1.

lifexp	Am	Eu	Af
70	0	1	0
74	0	1	0
73	0	1	0
73	0	1	0
73	0	1	0
74	0	1	0
⋮	⋮	⋮	⋮
50	0	0	1
52	0	0	1

To encode a categorical variable with k categories we need $k-1$ indicator variables. The choice of using 0 and 1 is conventional.

Statistical distribution

Consider a set of N statistical units on which we observe the variable Y. We call distribution of the variable Y the set of observations (represented by numbers or verbal expressions) relating to the N units of the set of data. In symbols, the distribution will be denoted as:

$$y_1, y_2, \ldots, y_N$$

where y_1 is the observation related to the unit identified by the number 1, y_2 is the one related to the unit identified by the number 2, and so on.

Sometimes this is referred to as a raw distribution, which does not offer a clear overview.

Distributions

For example, for the variable **Gender** the distribution is:

*Male Male Male Male Male Female Female Female Male Male Female Female Male Male Female Female
Female Male Male Female Female Male Female Female Male Female Female Female Female Male Male Male
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Female Male Male Female Female Female Male Female Female Male Male Male Male Male Female Female
Female Male Male Female Male Male Male Female Male Female Male Female Female Male Male Female*

Distributions

For the variable **Height** the distribution is:

178 180 175 186 170 164 158 165 182 172 158 158 185 194 170 162 162 169 193 NA 173 186 162 165 175
163 160 170 170 173 186 188 192 165 191 187 162 181 193 173 169 185 170 181 165 181 178 165 178 175
184 192 180 180 188 168 185 181 160 181 160 174 173 168 175 170 179 165 160 158 176 170 158 180 197
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Absolute frequency distribution

Let Y be a variable, we call absolute frequency distribution the list of observed modalities associated with the number of times they are observed, that is, associated with the respective absolute frequencies.

Most of the time, it is easy to obtain absolute frequency distributions for discrete qualitative and quantitative variables.

On the other hand, handling continuous variables (and discrete ones, if they have many categories) requires some preliminary operations.

Example: frequency distribution of **Gender**

Modality	Absolute frequency
Female	232
Male	261

Example: frequency distribution of **favorite literary genre**

Modality	Absolute frequency
Other	117
Comedy	9
Science fiction	17
Fantasy	42
Noir/Thriller	103
Horror	12
Psychological	68
Romantic	78
History	37

Example: frequency distribution of **sleep hours**

Modality	Absolute frequency
5	9
6	49
7	198
8	203
9	23
10	3
12	1

Example: frequency distribution of **height**

For the height, it is advisable to define classes.

Modality	Absolute frequency
147	1
155	1
157	3
158	11
159	3
160	28
161	7
162	12
163	9
164	8
165	32
166	4
167	10
168	18
169	7
170	39
171	4
172	18
173	17
174	6
175	37
176	8

Modality	Absolute frequency
177	6
178	22
179	8
180	39
181	13
182	17
183	9
184	10
185	23
186	10
187	12
188	10
189	3
190	10
191	1
192	4
193	2
194	1
195	2
196	1
197	2

Example: frequency distribution of **height**

Interval	Absolute frequency
[145,160]	47
(160,165]	68
(165,170]	78
(170,175]	82
(175,180]	83
(180,185]	72
(185,190]	45
(190,195]	10

Interval	Absolute frequency
[145,185]	430
(185,195]	55

The choice of classes is up to the researcher (no fixed rules), it must be done reasonably.

Intervals of different width

By choice (if you wish to provide more detailed information about a portion of the distribution) or by necessity (if the data has already been grouped into classes by someone else), the intervals can be of varying lengths.

In such cases, it is crucial to introduce the frequency density, which is defined as:

$$\left(\begin{array}{c}\text{density}\\ \text{of an interval}\end{array}\right) = \frac{\text{absolute frequency of } Y \text{ on the interval}}{\text{length of the interval}}$$

Example: Instagram followers

Modality	Frequency
0	1
8	1
10	1
15	1
22	1
30	1
40	1
41	1
50	3
80	3
99	1
100	2
120	2
123	1
124	1
130	1
150	4
173	1
183	1
186	1
192	1
200	5
209	1
215	1
229	1
236	1
240	1
242	1
247	1
250	6
252	1
270	1

Modality	Frequency
527	1
531	1
548	1
550	1
558	1
561	1
573	1
576	1
580	1
581	1
584	1
586	2
587	1
590	1
598	1
600	17
605	1
615	1
617	1
622	2
626	1
630	1
635	1
639	1
644	1
645	1
650	4
652	1
654	1
655	1
658	2
660	1

Modality	Frequency
970	1
976	1
987	1
1000	21
1003	1
1017	1
1018	1
1019	1
1031	1
1032	1
1040	1
1042	1
1043	1
1047	1
1050	1
1067	1
1068	1
1070	1
1075	1
1082	1
1100	4
1110	1
1113	1
1136	1
1164	1
1173	1
1174	1
1200	8
1205	1
1235	1
1259	1

Example: Instagram followers

Interval	Frequency
[0,500]	169
(500,1000]	150
(1000,1500]	59
(1500,2000]	17
(2000,2500]	10
(2500,3000]	6
(3000,100000]	5

Frequency Density

Interval	Frequency	Interval width	Density
[0,100]	17	100	17/100=0.17
(100,200]	18	100	18/100=0.18
(200,300]	43	100	43/100=0.43
(300,400]	48	100	48/100=0.48
(400,500]	43	100	43/100=0.43
(500,1000]	150	500	150/500=0.3
(1000,10000]	96	9000	96/9000=0.01067

The density tells us the expected number of statistical units for each unit of measurement of the variable. In the first class, for example, we expect to see 17 people in a range of 100 units, in the second last class we expect to see 150 units for every 500.

Conditional frequency distributions

- Let us call
  - Y the variable we are studying (the hours of sleep, for example)
  - X the variable through which we extract the statistical units to be considered in the analysis (gender, in our case)

  we define the variable Y conditional on $X = x$ and we write $Y|X = x$ to express the restriction of Y to the modality $X = x$.
- The distribution of the variable $Y|X = x$ is usually called the distribution of Y conditional on $X = x$ or, equivalently, the distribution of Y given $X = x$.
- Note that there is a conditional distribution (of Y given X) for each modality of X.
- The distribution of the variable Y, without conditioning for an X modality, is called the marginal distribution.

Example: height, males and females

In the case of continuous quantitative variables, which can be divided into intervals, things work in the same way.

Females

Interval	Frequency
[145,160]	47
(160,165]	67
(165,170]	65
(170,175]	40
(175,180]	9
(180,185]	0
(185,190]	0
(190,195]	0

Males

Interval	Frequency
[145,160]	0
(160,165]	1
(165,170]	13
(170,175]	42
(175,180]	74
(180,185]	72
(185,190]	45
(190,195]	10

Relative Frequencies

By dividing an absolute frequency by the total number of statistical units (N in our case) we obtain the so-called relative frequencies, that is

$$\left(\begin{array}{c}\text{relative}\\ \text{frequencies}\end{array}\right) = \frac{\left(\begin{array}{c}\text{absolute}\\ \text{frequencies}\end{array}\right)}{\left(\begin{array}{c}\text{total number of}\\ \text{observations}\end{array}\right)}$$

They allow comparisons of distributions based on different numbers of statistical units.

Relative frequencies: hours of sleep

Modality	Freq. F	Freq. M	Freq. Tot
5	4	5	9
6	24	25	49
7	95	103	198
8	98	105	203
9	6	17	23
10	1	2	3
12	0	1	1

Modality	Rel. F. F	Rel. F. M	Rel. F. Tot
5	0.02	0.02	0.02
6	0.11	0.10	0.10
7	0.42	0.40	0.41
8	0.43	0.41	0.42
9	0.03	0.07	0.05
10	0.00	0.01	0.01
12	0.00	0.00	0.00

Example: Instagram followers

Interval	Freq	Rel. Freq.
[0,100]	17	17/416=0.041
(100,200]	18	18/416=0.043
(200,300]	43	43/416=0.103
(300,400]	48	48/416=0.115
(400,500]	43	43/416=0.103
(500,750]	85	85/416=0.204
(750,1000]	65	65/416=0.156
(1000,2000]	76	76/416=0.183
(2000,5000]	19	19/416=0.046
(5000,100000]	2	2/416=0.005

Frequency distribution: notation

- y_i i-th modality / interval $(c_{i-1}, c_i]$ of the variable Y, for $i = 1, 2, \ldots, k$ (k modalities / intervals);
- n_i absolute frequency: number of statistical units that present the modality/interval y_i;
- N total number of observations ($N = n_1 + n_2 + \cdots + n_k$);
- f_i relative frequency ($f_i = n_i/N$).

modality/interval	Freq.	Rel.F.
y_1	n_1	$f_1 = n_1/N$
y_2	n_2	$f_2 = n_2/N$
$\vdots$	$\vdots$	$\vdots$
y_k	n_k	$f_k = n_k/N$
Total	N	1

The symbol $\sum$ (summation)

What do we mean for:

$$N = \sum_{i=1}^{k} n_i$$

that is for 'Sum for i ranging from 1 to k'?

$$N = n_1 + n_2 + \cdots + n_k$$

Some properties:

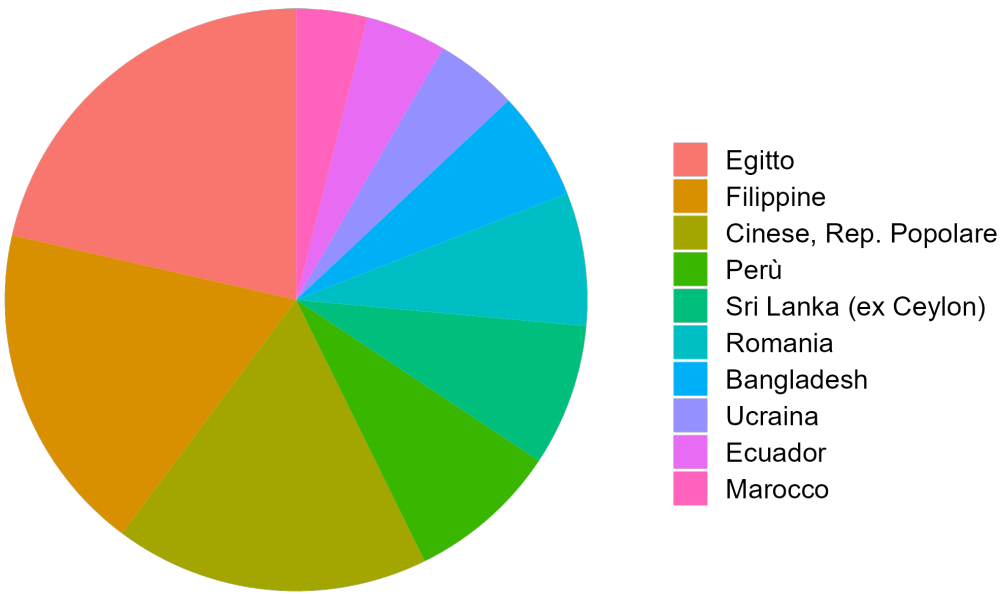
1. $\sum_{i=1}^{k}(y_i + x_i) = \sum_{i=1}^{k} y_i + \sum_{i=1}^{k} x_i$
2. $\sum_{i=1}^{k} a y_i = a \sum_{i=1}^{k} y_i$
3. Be careful: $\sum_{i=1}^{k} a = ak$

Finally a plot!

We can visualize the frequency distributions of a variable representing each modality with the relative frequency.

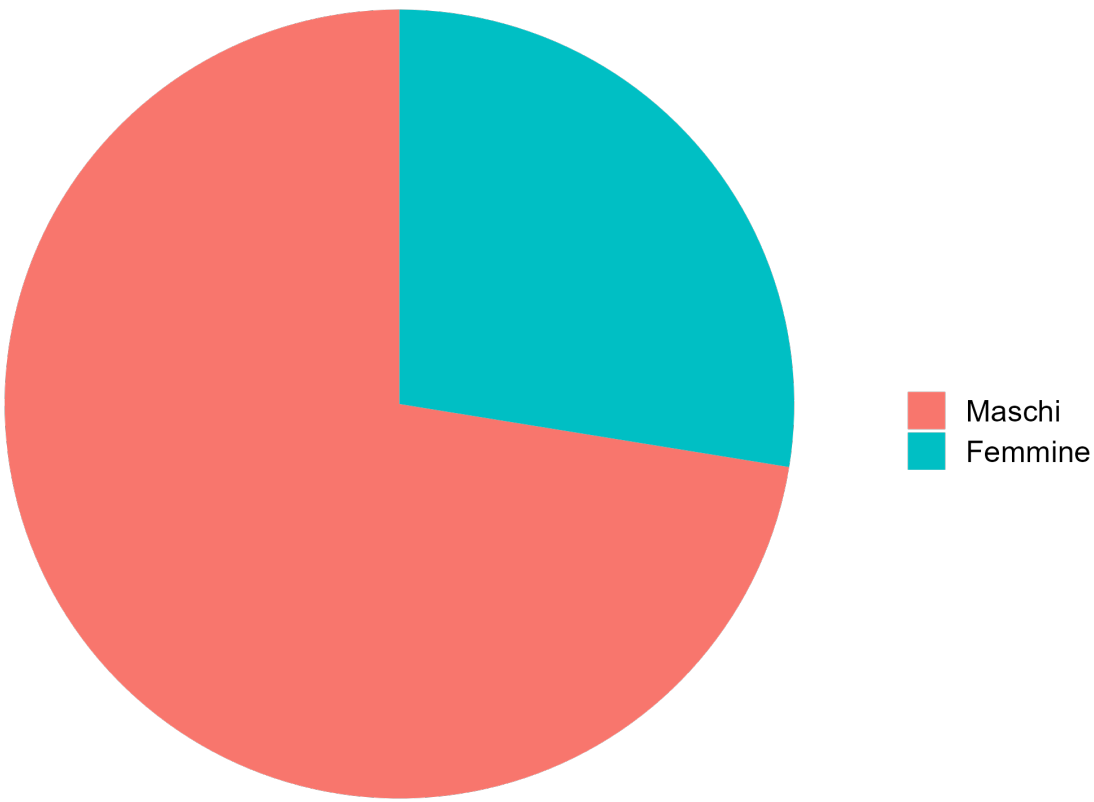
Example: Relative frequency distribution of the top 10 nationalities in Milan in 2023

Nationality	Inhabitants
Egitto	45457
Filippine	38942
Cinese, Rep. Popolare	37041
Perù	17799
Sri Lanka (ex Ceylon)	16724
Romania	15673
Bangladesh	12802
Ucraina	9704
Ecuador	9513
Marocco	8351



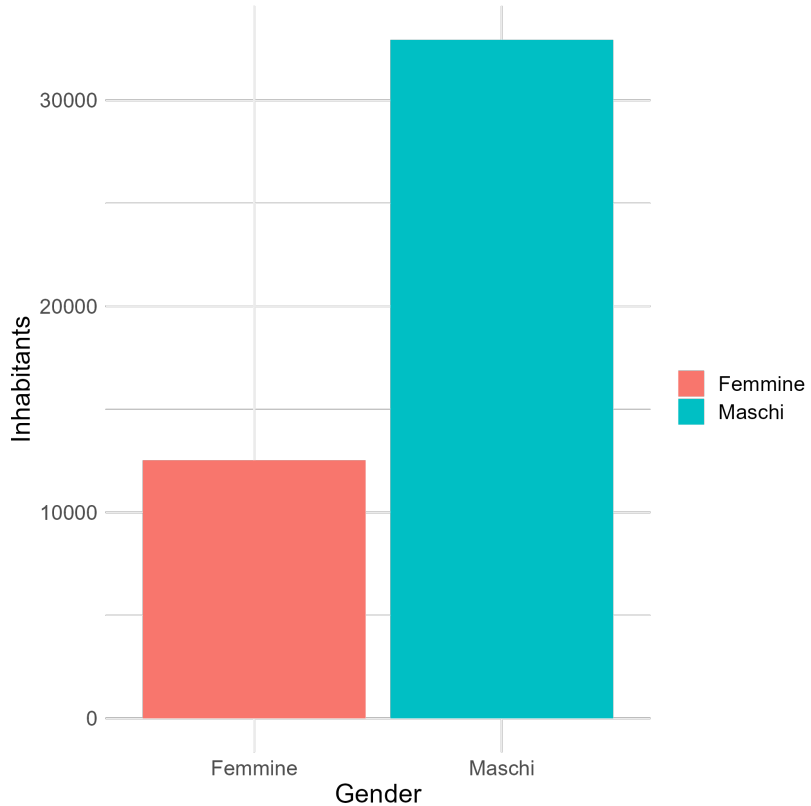
Another plot...

Example: Relative frequency distribution of Egyptians by gender in Milan in 2023



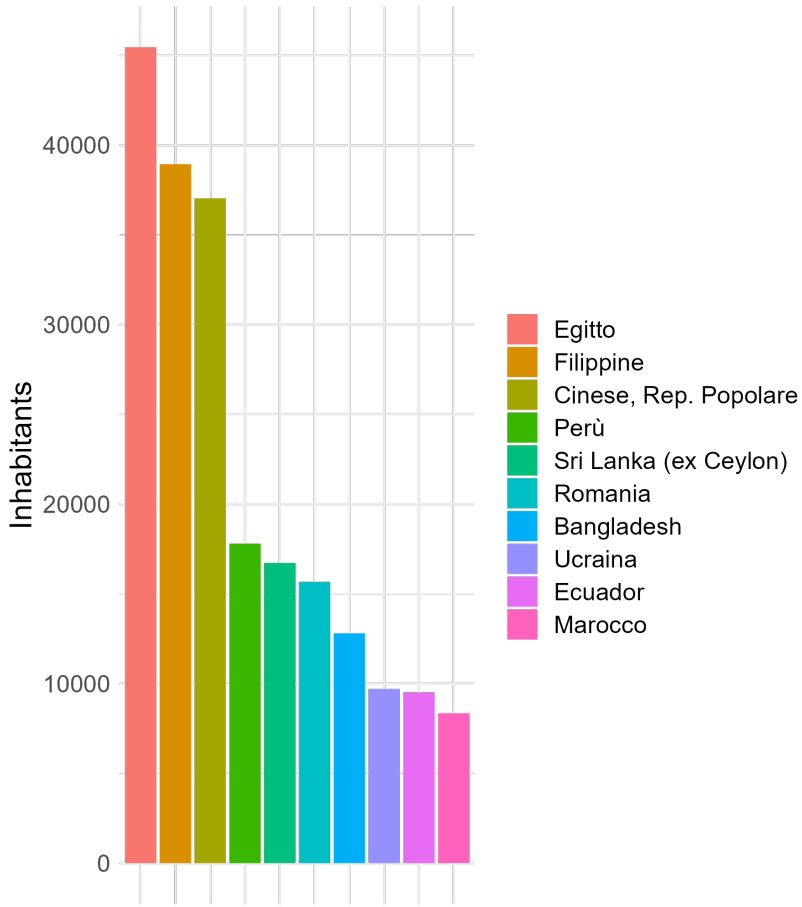
Another plot...

Example: Absolute frequency distribution of Egyptians by gender in Milan in 2023



When the variable is nominal (as in this case), the position of the modalities is arbitrary

Barplot

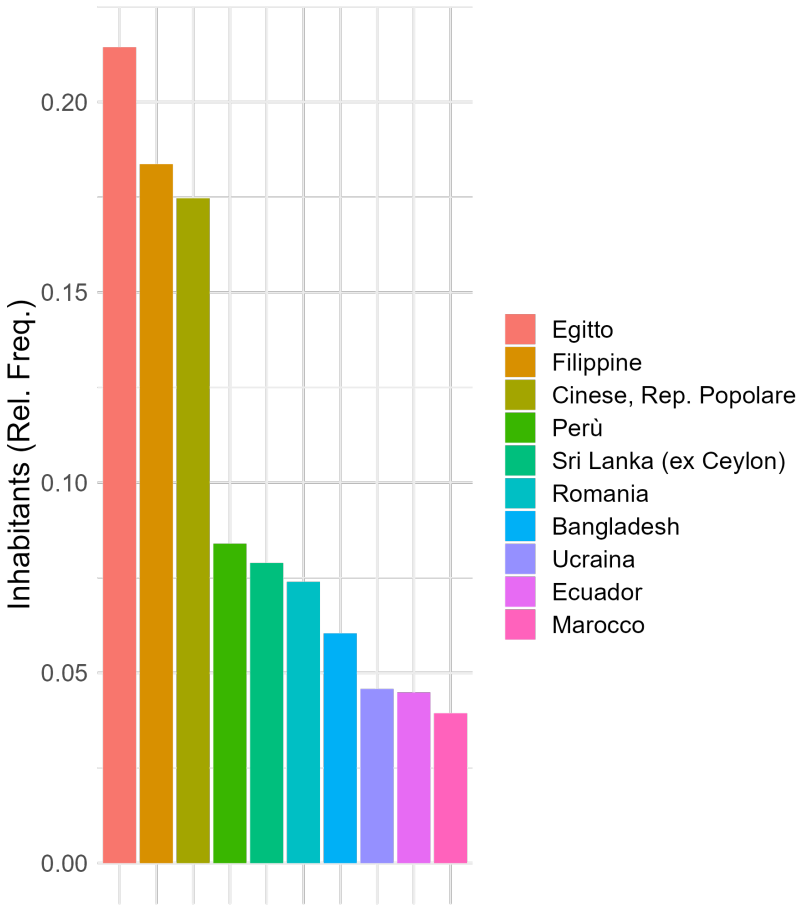


The plot shows:

$$x\text{-axis} = \begin{pmatrix} \text{modalities} \\ \text{reported in the} \\ \text{frequency} \\ \text{distribution} \end{pmatrix}$$

(bars height) = (absolute frequencies)

Barplot



The plot shows:

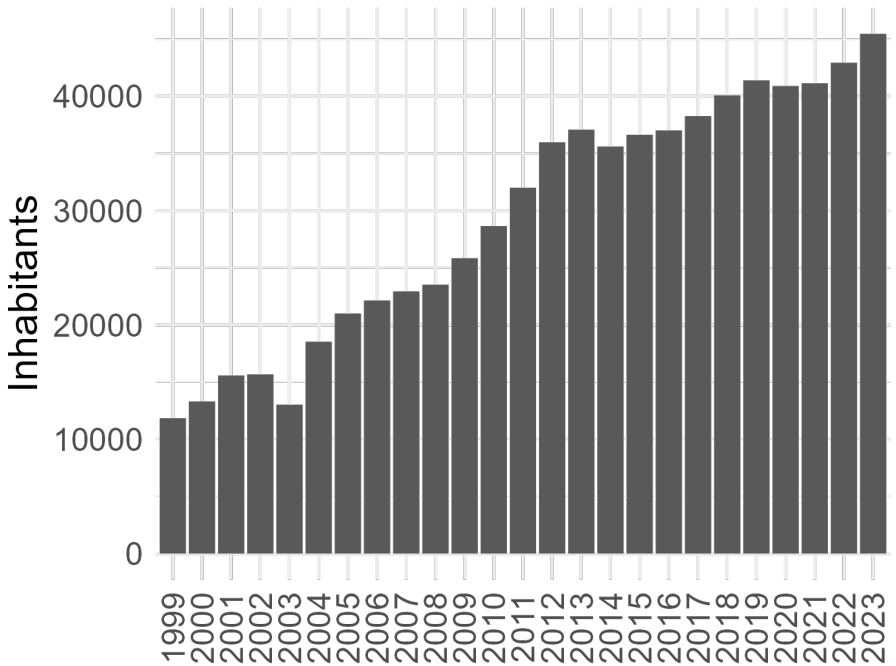
$$x\text{-axis} = \begin{pmatrix} \text{modalities} \\ \text{reported in the} \\ \text{frequency} \\ \text{distribution} \end{pmatrix}$$

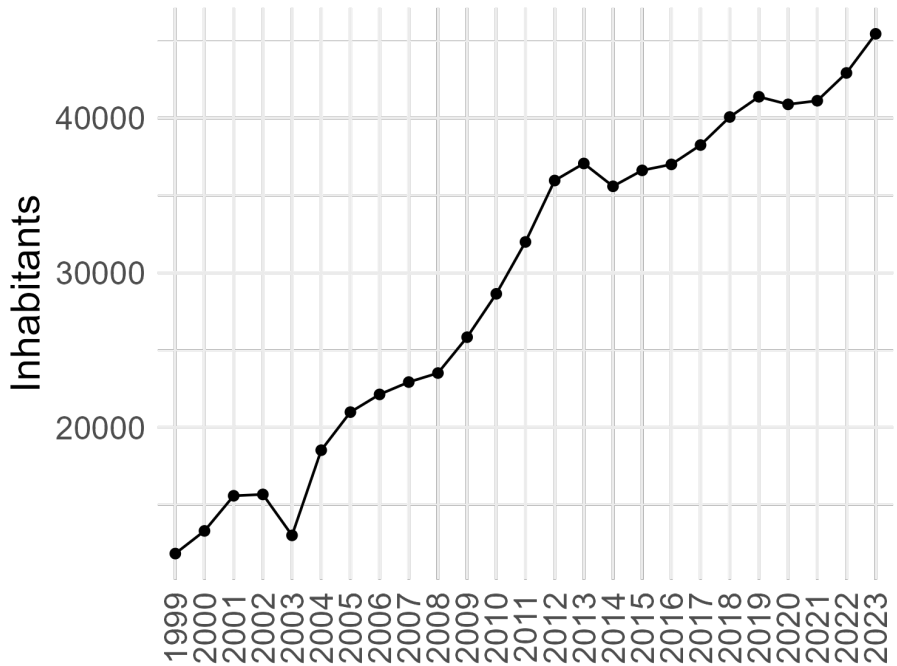
(bars height) = (absolute frequencies)

Using the relative frequencies
we obtain the same plot

(bars height) = (relative frequencies)

Barplot: Frequency by year

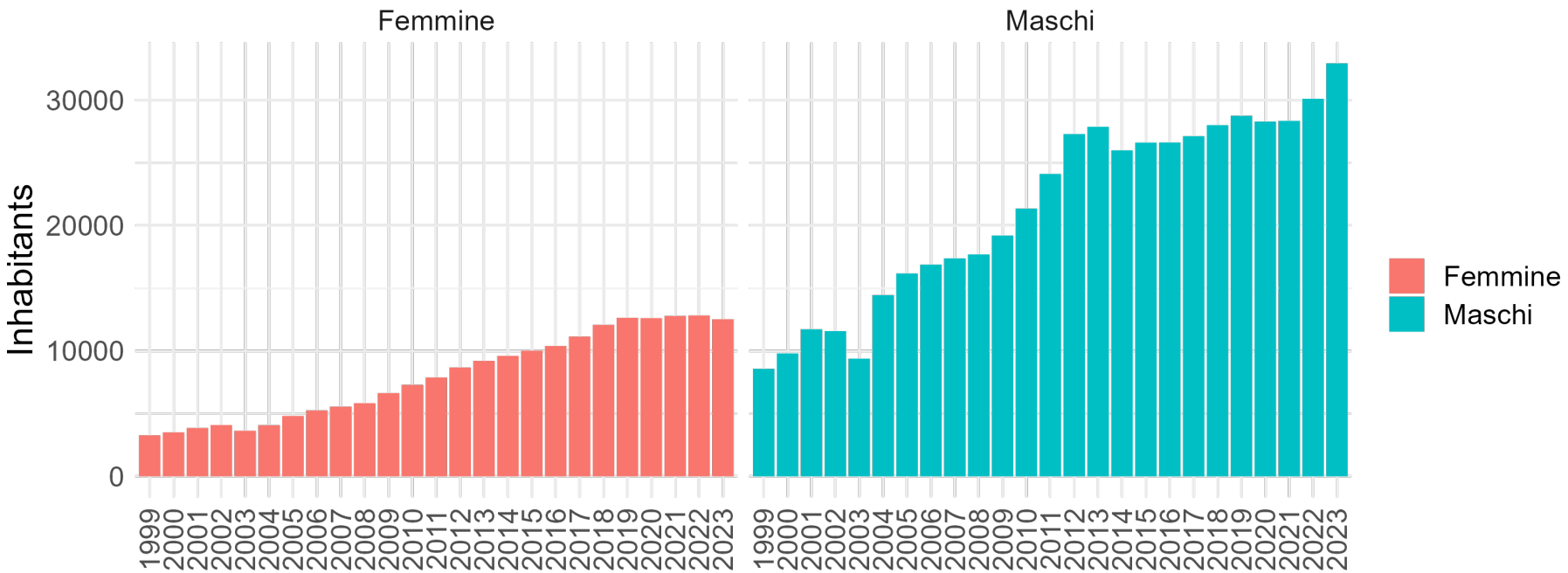




When the variable is ordinal or quantitative discrete, the position of the modalities is meaningful and follows a specific order.

Barplot and comparisons: year and gender

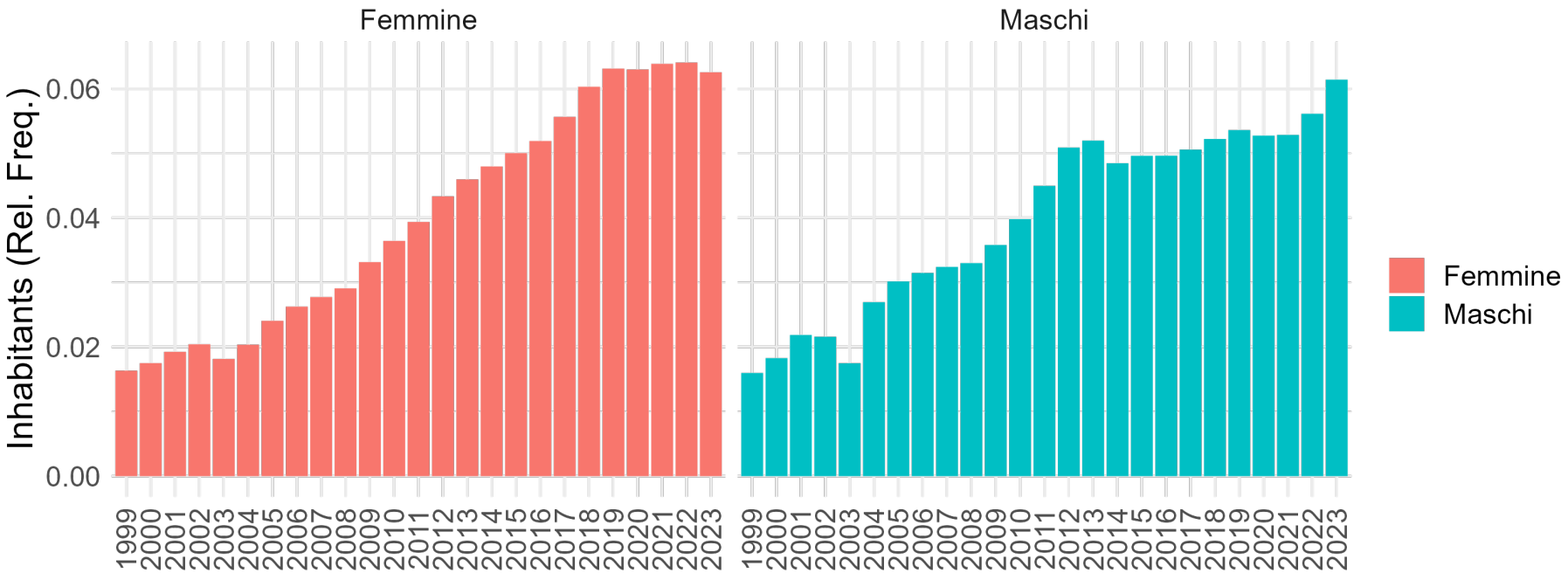
We can compare the population distribution in Milan in the last years by gender.



What type of frequencies are these? Are they suitable for this task?

Barplot and comparisons: year and gender

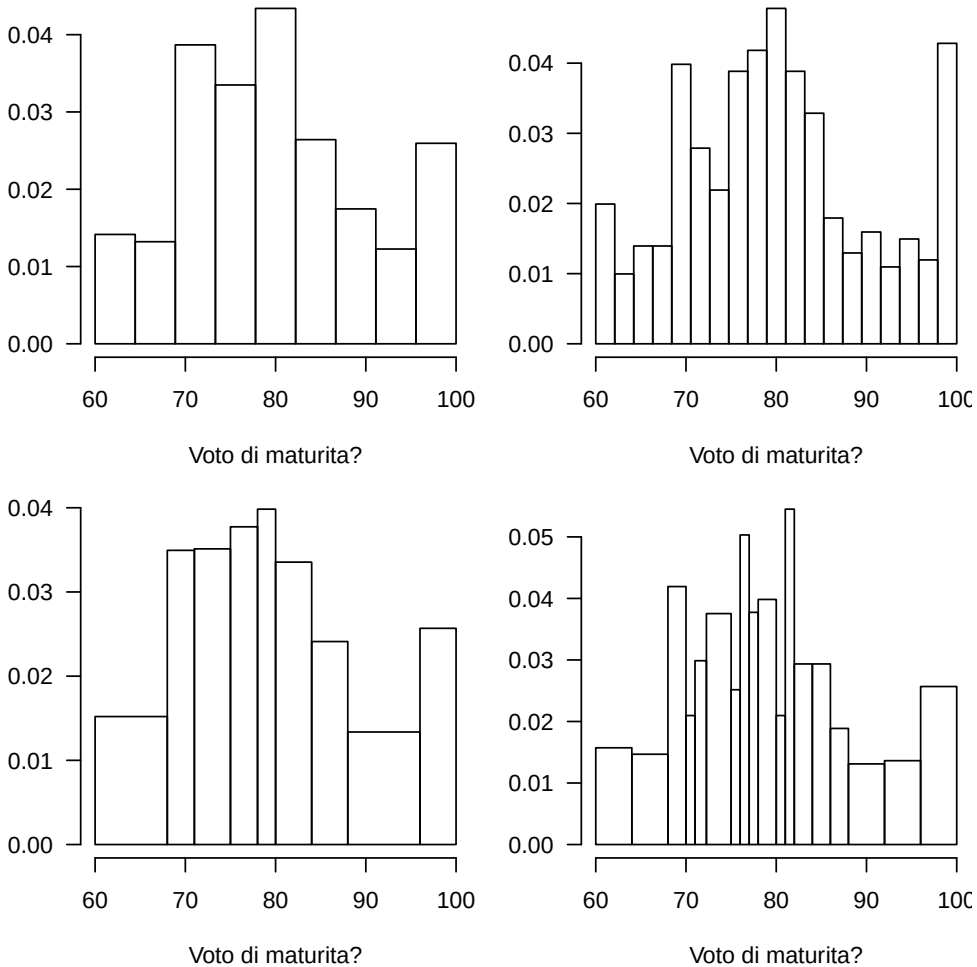
No! This is exactly the reason why we need to compute relative frequencies.



"There are 3 kinds of lies: lies, damned lies, and statistics."

High school grade

Which one of these histograms is useful? Which one gives too many details? Which one hides too much of the data?



Histograms and intervals

- The fewer the number of intervals, the fewer the details,
- however, too many intervals means too many details. This might lead to sample bias: we depict characteristics that are related to our specific sample.
- A good strategy is to make several graphs from the same data, changing the interval widths, and then choose the best one
- The interval number is related to the data size!