

# Chapter 1

## Analysis

In the first part of this chapter we introduce some some theoretical arguments useful in the analysis. ~~Then~~<sup>e</sup> we start considering a basic version of the model, trying to understand the basic concepts and ensure that everything ~~work~~<sup>e</sup> works fine. ~~Then~~<sup>e</sup> we try to explain what's are the consequences of a specific agent's behavior and finally ~~analyze~~<sup>e</sup> some aspect of the ~~complete~~<sup>e</sup> model.  
<sub>analyzing                      entire</sub>

### 1.0.1 Update Evaluation

Analyzing how ~~person~~<sup>people</sup> update their knowledge database, we can find an exponential law.

Let  $B$  be the **agent's boldness**,  $Q$  the **real quality** of a restaurant and  $V$  the **internal valuation** about that restaurant. thought by the agent on his own

~~Persons~~<sup>people</sup> update  $V$  ~~for the next turn~~<sup>step by step</sup> with the following recursion rule

$$\begin{aligned} V(t+1) &= V(t) + B(Q - V(t)) \\ V(t+1) - V(t) &= B(Q - V(t)) \end{aligned}$$

~~we can interpretate that as a discrete derivate~~ by considering an infinitesimal time step  $dt$  we can look at this relation such as a partial derivative tha we could integrate

$$\frac{1}{B} \frac{dV}{dt} = Q - V(t)$$

~~Homogeneous solution:~~ In light of this we obtain both an homogeneous solution:

$$\begin{aligned} \frac{1}{B} \frac{dV}{dt} &= -V(t) \\ V(t) &= V(0) e^{-Bt} \end{aligned}$$

~~Particular solution~~

And a particular one:

$$V(t) = Q$$

Let's now analyse the model on what concerns the global knowledge related to all agents about restaurant's quality. As mentioned before each restaurant is characterised by a particular ranking value which outlines its quality. Moreover, the ranking is actually a dual variable: there's the Real ranking  $Q$  unknown to customers and the personal thinking, for each person, of restaurant's quality,  $V$ ; this ranking is, thus, strictly related to agent's knowledge database.

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## CHAPTER 1. ANALYSIS

Thus, the complete solution of this partial differential equation will be the following

So

$$V(t) = V_0 e^{-Bt} + Q$$

### 1.0.2 Analyzing the global knowledge

~~We want to test the model observing the global knowledge about the restaurants quality. So we introduce the *uncertainty* factor for a specific restaurant  $K_{re}$  as the variance of valuations of all person at time  $t$~~

Since that, we introduce for, each restaurant, an uncertainty factor ... which is defined as the variance of the evaluation of all people at the specific time step  $t$ :

$$K_{re}(t) = \sum_{per} (\mu(t) - V_{re,per}(t))^2 \quad (1.1)$$

~~The *global uncertainty* about all the world is  $K$ , the mean of  $K_{re}$  over all restaurants~~

As a natural consequence, the global uncertainty  $K$  is simply the mean value of  $K_{re}$  estimated over all restaurants:

$$K(t) = \mathbb{E}[K_{re}(t)] \quad (1.2)$$

### 1.0.3 Analyzing satisfaction

da fare...  
da fare...  
da fare...

As mathematically proved before, we can easily verify that every person updates his knowledge database following an exponential law

#### FIRST SIMULATION: ONE RESTAURANT

\*SOTTOLINEARE CHE IL RANK DEL RISTORANTE E' FISSO A 2.5 MENTRE IL THINKING INIZIALE DEI PERSON E' CASUALE

### 1.1 First Tests

in order to study how people update their thinking of each restaurant

~~In order to test the valuation updating we start with a very simple situation: 1 restaurant, 20 persons and no friends. \* As predict every person update the knowledge database about a restaurant, in exponential way. Notice that different persons with different boldness coefficient, converge at the true restaurant quality with different velocities.~~

Computing the average valuation and the standard deviation over time

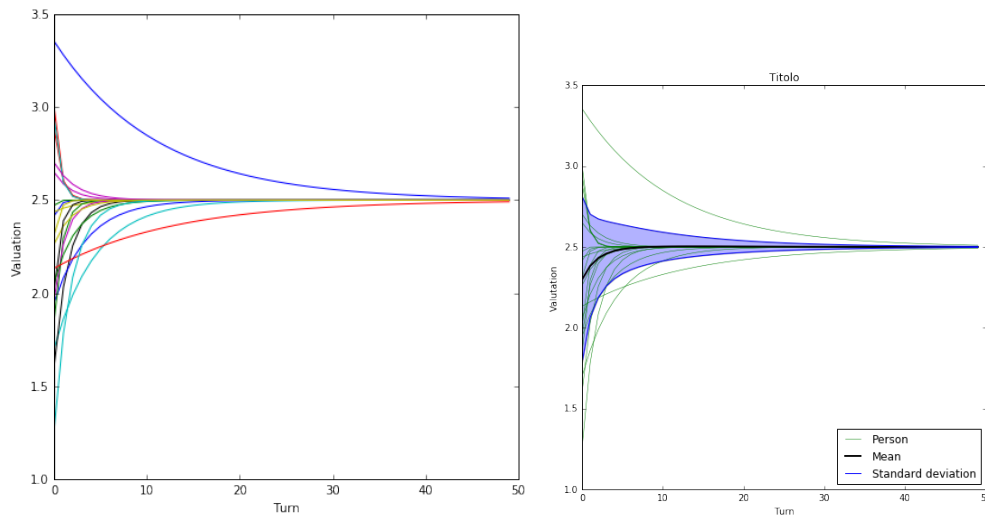
~~We can also compute the  $K_{re}$  coefficient.~~

GRAFICOOOO!!!!

As we see in plot 1.1(?), the  $K_{re}$  coefficient after a sufficient number of turns

~~It's important to see that  $K_{re}$  converge to 0, this means that at  $t \rightarrow \infty$ , all persons will know the real quality of the restaurant.~~

It is important to outline that different people, with different boldness coefficient, will converge to the true restaurant quality in different ways. As we could expect people who are more obstinated (higher boldness) will be slower in converging to the true value of the restaurant; people more open-minded, instead, will be more incline in updating their knowledge database faster.



Personal thinking of a person

Figure 1.1: Each color represents the internal valuation for an agent. Simulation with: 1 restaurant, 20 persons, no friends and restaurant is big enough for all persons.

For the simulation we set 1 restaurant, 20 persons and no friends; the restaurant capacity should be set high enough to host all people.

## 1.2 With multiple restaurants

### SECOND SIMULATION: MANY RESTAURANTS

Now we set up a test with different restaurants. So it's important the rule used by person to choose where to eat.

#### Randomly choose

People select randomly the place where to eat.

In this case, Here the process of updating the valuation is a bit different, in fact a person goes to different restaurants and updates the valuation only for that place; so focusing on a single restaurant we see that the curve is not an exponential but an exponential with some constant zones. The process on discovering the real valuation is obviously slower than before.

Actually

thus, since a person focuses only on...

#### Person that choose the best

All person try to eat at the best restaurant they believe. We see that selecting a restaurant, some person never updating their valuation about it. This happen because persons start with a random valuation about places and if they think that a specific one is disgusting they never go there. This seem

In this second section we set up simulations by considering a more realistic situation with many restaurants. Since people now have to choose where to go among many places becomes important the way how they take this choice.

Every person tries to eat at the restaurant he thinks is the best. Thus, we observe that there are some restaurants that a person will never go to, and consequently will never updates his thought about it. This happens because each customer starts the simulation with a random valuation of every single restaurant. Since that, maybe there are people thinking from the beginning that the best restaurant in town is, instead, the worst one and they will never try it.

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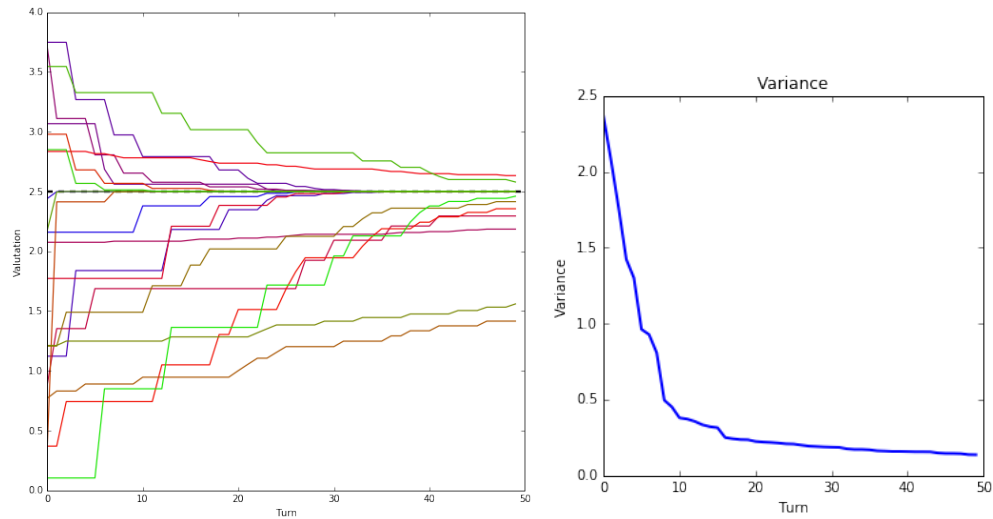


Figure 1.2: Simulation with: 4 restaurant, 20 persons, no friend, 50 turns, random choose.

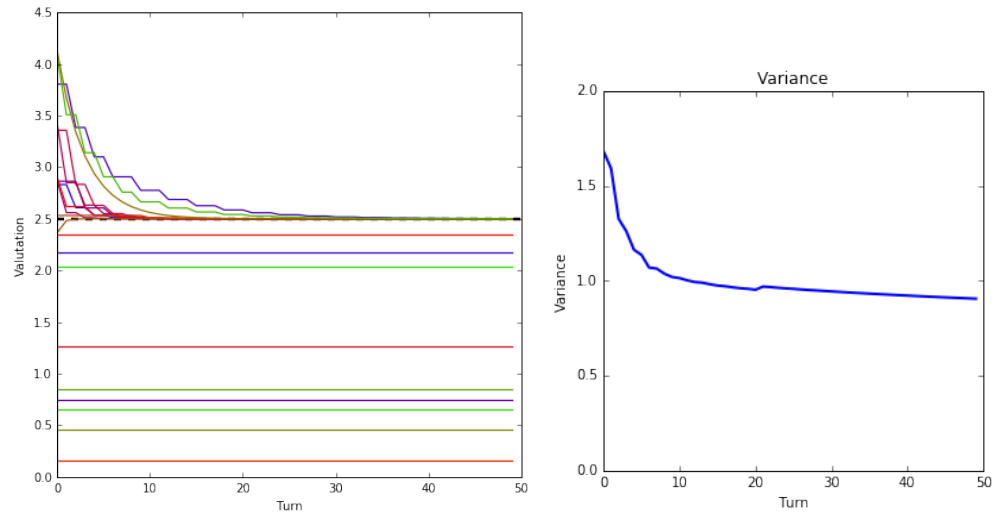


Figure 1.3: Simulation with: restaurant, persons, no friend, turns, best thinking choosing.

natural if the restaurant is really a bad one, but can happen that a restaurant with a good valuation is never discovered by persons.  
We can see that  $K_{re}$  not converge to zero.

This situation may seems to be sensible if the restaurant is a really bad one, but would be ungodly if the restaurant is indeed a good one; in light of this, could happen that a good restaurant would be never discovered by someone just because the agent is convinced since the beginning (in real life, maybe because he heard about that) that the quality of it is really low.

The simulations we considered until now are indeed too simple to reproduce a satisfying situation describing what happens in a real world; as a matter of fact, we could assume that people usually choose what they think is the best but we reasonably suppose that they will even explore the world.

In this sense we introduce a bit more complex situation where we organize the probability of adopting a strategy instead of another in a different way: 50% of time people goes to a restaurant randomly without considering its rank (random choose) and remaining 50% of time they go to the one they think is the best choice.

### 1.3. COMPLETE BEHAVIOR

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~~OSS: Notice that this issue can also be resolved later with introduction of communication between persons.~~

However, this issue could be resolved through the introduction of the communication between friends

### 1.3 Complete Behavior

#### THIRD SIMULATION: A BIT MORE COMPLICATED

Previous behavior are too simple to model the real world, we want that persons choose the best they think, but also explore the world. So we a more complex behavior in which 50% of time person use *random choose* and 50% *best choose*. Observing the  $K_{re}$  we are sure that people explore the world

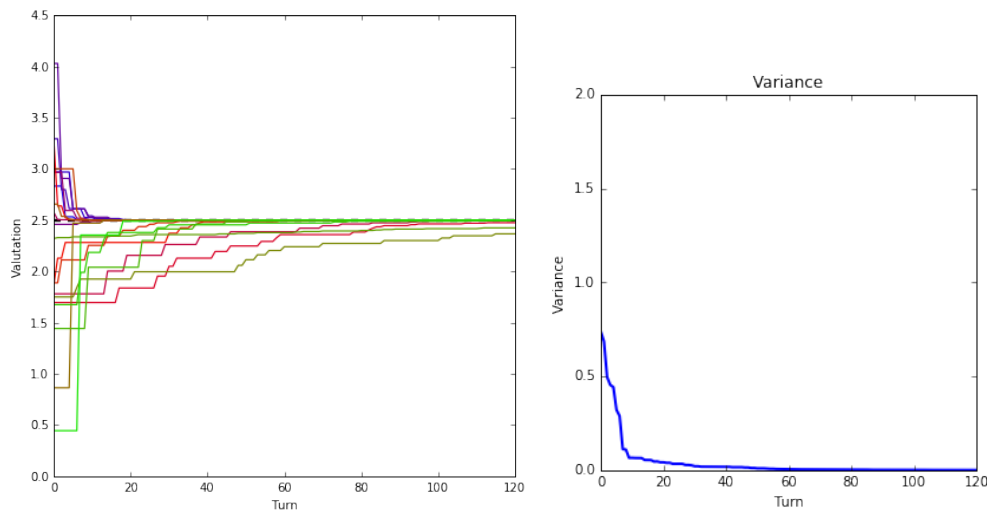


Figure 1.4: Particular situation for a fixed restaurant. Simulation with: 4 restaurant, 20 persons, no friend, 300 turns, fifty-fifty.

although occasionally they choose the best.

OSS: notice that in the graph Valuation vs. Turn under the quality of restaurant the convergence is slower than above

### 1.4 Complete Model

#### FOURTH SIMULATION: COMMUNICATION AMONG FRIENDS

~~For all the following experiment we use a model in which persons can communicate each other and use 50% of time random choose behavior and 50% of time the best choose.~~

Looking at the uncertainty factor  $K_{re}$ , introduced before, we observe that people are more incline in discovering new places even if they think, at first, they are not so good.

Moreover, in this case, we notice that people thought will converge to the real best restaurant, which rank is fixed to 2.5, faster than before and, furthermore, every person will convince itself that the best restaurant is indeed that one.

This, simply means that more open-minded people, who are incline to try even places they don't think are so good at first, will soon recognize which restaurant is the best one.

In this section we start to introduce the communication among people. Thus, a person can communicate its thought to another. People base the choice of the restaurant again on both two strategies: the 50% of turns they will adopt random choose, remaining ones they will go for best choose.

Due to how we set the parameters in this simulation we obtain a more stochastic situation cause to the random choice (the 50% of the time) and even produced by the fact that people could easily find full restaurant; as a matter of fact the capacity of each place is finite.

In this sense, we need to make many different simulations that represent a single realization of the model (NON HO BEN CAPITO COSA SI VOLEVA DIRE IN QUESTO PARAGRAFO, AL MASSIMO APPROFONDISCI UN ATTIMINO)

This model introduce stochastic elements, not only in the random choose, but also due to the fact that person can find full restaurants. So we have to do multiple experiments that represent only a single realization of the process (model).

#### MANY FRIENDS MEANS BETTER CHOICE

##### 1.4.1 ~~Difference between number of friends~~

We want to analyze how  $K(t)$  varied by varying the number of person's friends. We set up four different experiments with 0,1,3 and 5 friends, for each we consider 5 different realizations.

Start considering the case with 20 persons 4 restaurants and no friends. We plot  $K(t)$  for all the 15 realizations. than we compute the mean and

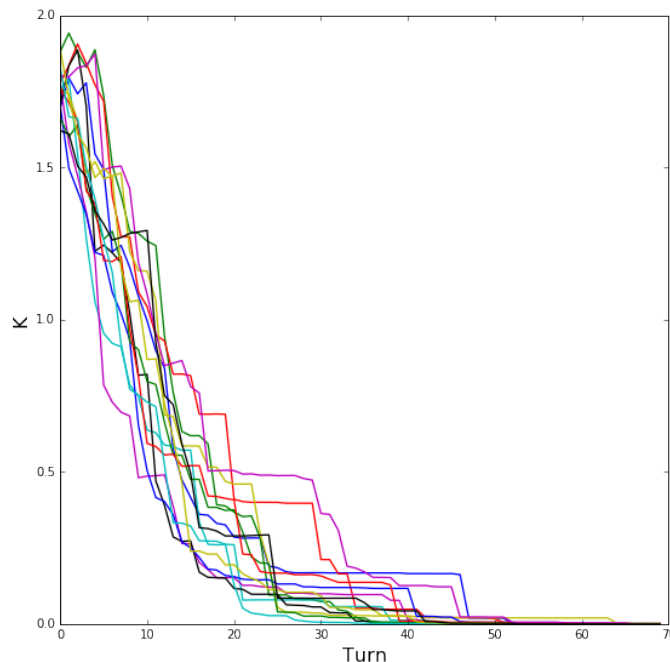


Figure 1.5:

the variance of  $K(t)$  and try to approximate it with an exponential. The approximation is good as confirmed with a chi-squared test.  $\chi^2 = 43.4$  with  $DF = 47$  The exponential fit:

$$K(t) = ae^{-bt+c}$$

IN QUEST'ULTIMA PARTE NON MI E' MOLTO CHIARO IL PROCEDIMENTO, SE RIUSCISSI FRA DOVRESTI DILUNGARTI UN PO' DI PIÙ TU SPIEGANDO PER BENE CHE COSA HAI FATTO IN OGNI PASSAGGIO E COSA SONO LE QUANTITA' CHE TIRI IN BALLO. ALTRIMENTI NON SI CAPISCE MOLTO

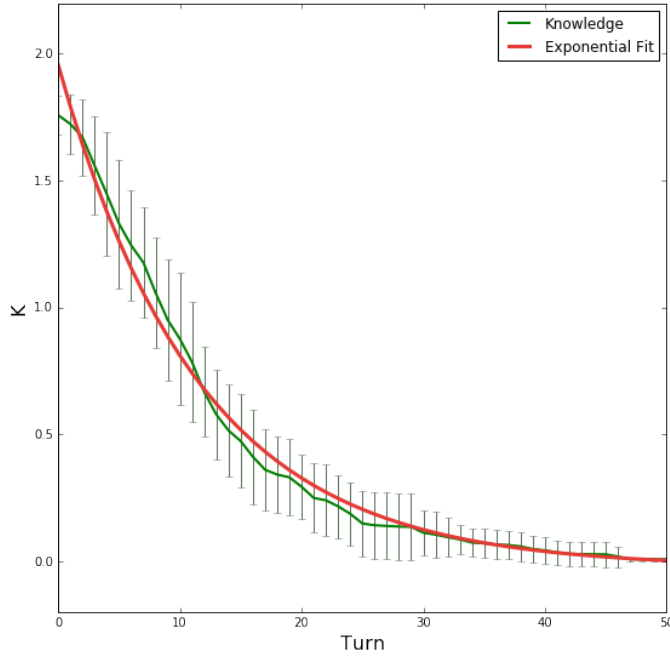


Figure 1.6:

with

$$\begin{aligned} a &= 1.97 \\ b &= 0.086 \\ c &= -0.02 \end{aligned}$$

We can consider exponential time constant  $\tau = \frac{1}{b} = 11.5$  that measures the number of turns needed to reduce  $K$  by 63%, so we can think approximately that in  $5\tau = 57$  turns uncertainty is 0.

Then we reproduce the same experiment with friends and find: As we expect model with higher number of friends have  $K$  that go to zero faster than other, in fact persons inform each other about the quality of restaurants and in less time they discover the real rating. In particular

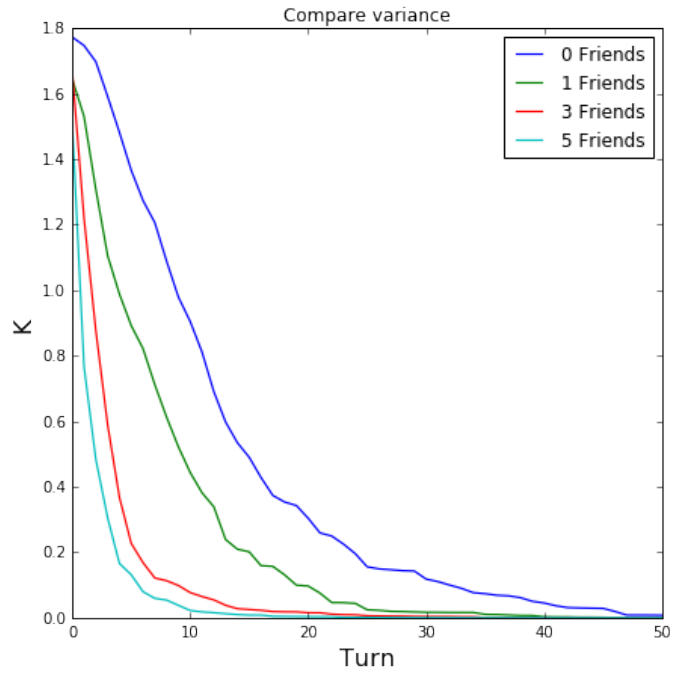


Figure 1.7:

Table 1.1:

n friends	$b$ coefficient	$\tau$	n turn to zero
0	- 0.086	11.5	58
1	- 0.134	7.4	37
3	- 0.359	2.7	14
5	- 0.560	1.8	9