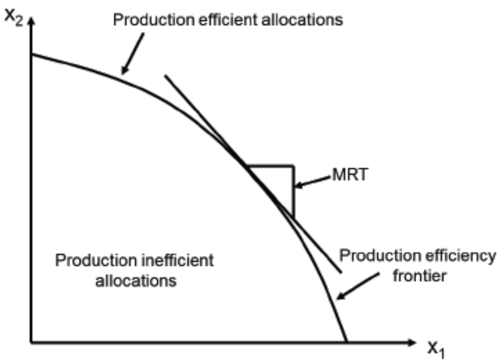
SECOND BEST THEORY

1 BASIC MODEL

1.1 FUNDAMENTAL COMPONENTS

- Two Prices: producer prices p and consumer prices q
- Production:
  - Area under curve shows production set (i.e. feasible points)
  - Curve is production-frontier (i.e. efficient points; see later)
    - If concave DRTS; linear CRTS; convex IRTS
    - If CRTS $\pi = 0$; if DRTS then $\pi > 0$ in SR or if imperfect competition
    - (Absolute) slope of production frontier is MRT: $-\frac{\delta x_2}{\delta x_1} = MRT_{x_1,x_2}$

Figure 3: The Production Fronter



- Firms:
  - Generally, $\pi^j(p) = \sum p_i y_i^j - p_0 l_0^j - p_k k^j$ (note use producer prices!)
  - Supply of firm: $\frac{\delta \pi^j(p)}{\delta p_i} = y_i^j(p)$ why FOC?
  - Demand for labour/capital: $-\frac{\delta \pi^j(p)}{\delta p_0} = l_0^j(p)$ and $-\frac{\delta \pi^j(p)}{\delta p_k} = k^j(p)$
  - Assume that consumers get a share of profits from firm as income
- Government:
  - If producer prices are fixed, gov. can buy the bundle of inputs: $R = \sum (q_i - p_i) x_i(p)$
  - If producer prices are not fixed, specify revenue vector in terms of commodities
  - Two types of taxes:
    - Unit tax: $t_i = q_i - p_i$
    - Ad-valorem tax: $q_i = (1 + t_i) p_i$

2 FUNDAMENTAL WELFARE THEOREMS

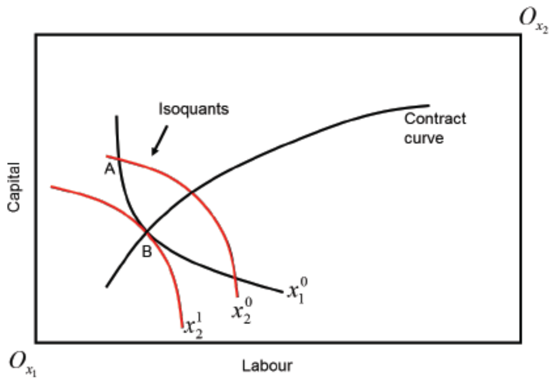
2.1 PARETO CONDITIONS

2.1.1 PRODUCTION EFFICIENCY

- Formally, marginal rates of technical substitution (i.e. slopes of isoquants) between any two factors of production is equal in production of all commodities $MRTS_{k,l_0}^{x_1} = MRTS_{k,l_0}^{x_2}$
  - If not, can transfer factors of production and produce same with spare left over
- Graphically, plot these in (x_1, x_2) space to get production efficiency frontier. Thus economy is necessarily producing at the boundary of the production set

- Intuitively, any given amount of output is produced using the inputs in such a way that the output of any other commodity is maximized

Figure 2: Production Efficiency



2.1.2 *Consumption Efficiency*

- Formally, marginal rate of substitution between any two commodities must be the same for all consumers: $MRS^A_{x_1,x_2} = MRS^B_{x_1,x_2}$
- Graphically, the available amounts of the two commodities are divided between the two consumers along the contract curve

2.1.3 *Product Mix Efficiency*

- Formally, the (equal) marginal rate of substitution in consumption must be equal to the marginal rate of transformation (MRT) in production: $MRS^A_{x_1,x_2} = MRS^B_{x_1,x_2} = MRT_{x_1,x_2}$
- Graphically, point on the contract curve where the marginal rates of substitution for the two consumers are equal to the marginal rate of transformation
    - At point D the MRS (say 1) is smaller than MRT (say 3) at point E
    - Can produce one unit less of $x_1$in return for 3 units of x_2
    - Take the one unit of $x_1$away from person A and give him one unit of x_2
    - Their utility is unchanged and give extra two units of x_2 to person B. Thus not efficient!

Figure 5: Production Mix Efficiency

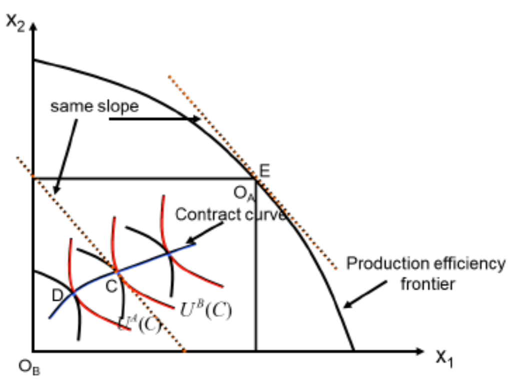
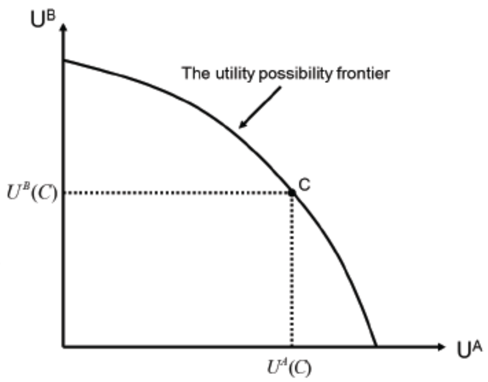


Figure 6: The Utility possibility frontier



2.1.4 *Utility Possibility Frontier*

- Summarize "efficient" points. Look at lecture notes for how to derive

2.2 Welfare Theorems

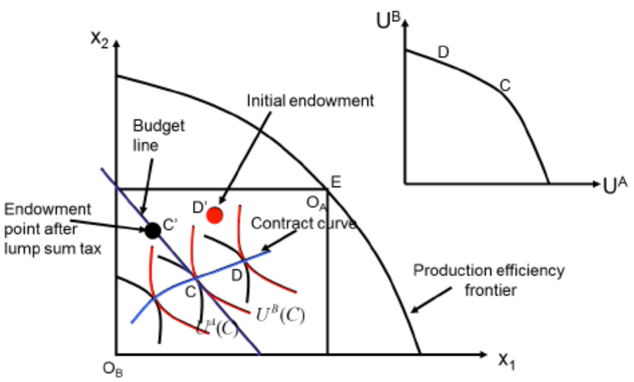
2.2.1 *First Welfare Theorem*

- "Competitive equilibrium is necessarily efficient because consumers and producers face same relative prices and optimize by equating these to the relevant marginal rates"
- Can derive this as follows
  - Firms minimize costs by employing two factors of production until MRTS is equal to the relative wage (p_0) rental rate (p_k): $MRTS^{x_1}_{l_0,k} = MRTS^{x_2}_{l_0,k} = \frac{p_0}{p_k}$
  - Firms maximize profits by equating the marginal costs of production to the relevant producer prices: $MC_{x_1} = p_1$ and $MC_{x_2} = p_2$
  - Thus $C^{x_1} = p_0 l_0^1 + p_k k^1$ and $C^{x_2} = p_0 l_0^2 + p_k k^2$ linked as: $\delta C^{x_1} = p_0 \delta l_0^1 + p_k \delta k = -\delta C^{x_2}$
    - This is because an increase in the use of a factor in the production of x_1 means an equal reduction in the use of that factor n the production of x_2 ($\delta l_0^1 = -\delta l_0^2$ etc.)
  - Thus the marginal cost of x_1 can be written: $\frac{MC_{x_1}}{MC_{x_2}} = \frac{\delta C^{x_1}}{\delta C^{x_2}} \frac{\delta x_2}{\delta x_1} = (-)\frac{\delta x_2}{\delta x_1}$
  - And $(-)\frac{\delta x_2}{\delta x_1} = MRT_{x_1,x_2} = \frac{p_1}{p_2}$ since firms produce until MC equals producer price
  - Consumers max utility when MRS equals relative consumer price: $MRS^A_{x_1,x_2} = MRS^B_{x_1,x_2} = \frac{q_1}{q_2}$
  - Since in competitive equilibrium $\frac{q_1}{q_2} = \frac{p_1}{p_2}$ we get $MRS^A_{x_1,x_2} = MRS^B_{x_1,x_2} = MRT_{x_1,x_2}$

2.2.2 *Second Welfare Theorem*

- "Any feasible Pareto efficient allocation on the utility possibility frontier can be attained as a competitive equilibrium by suitable lump sum redistribution of the endowments"
- Consider incomes of agents as: $m^A = q_k k_A + q_0 l_A$ and $m^B = q_k(\bar{k} - k_A) + q_0(\bar{l} - l_A)$
- Redistribution of the endowments is redistributing of income in a way that they cannot avoid their tax liabilities. Let the market do the rest

Figure 7: The Second Welfare Theorem



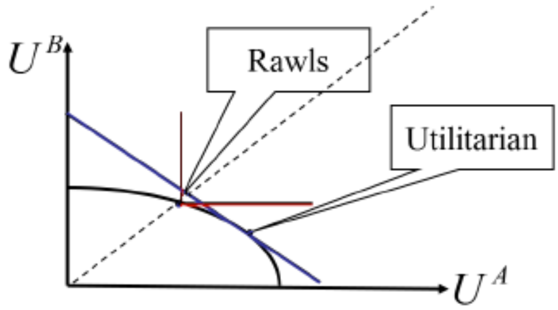
2.3 Social Welfare Functions

- Pareto criterion alone is insufficient to allow the government to decide between different allocations on the utility possibility frontier
- Use Bergson-Samuelson approach: $SWF = W(U^1, \ldots U^H)$ where $\frac{\delta W}{\delta U^h} \geq 0$
- Can only produce a unique social choice if (1) individual utility functions are cardinal and (2) can make interpersonal comparisons
  - Suppose that we allocate 4 beers amongst two people and {2,2} is the best outcome:

        - $U^A(2) + U^B(2) > U^A(1) + U^B(3)$
        - $U^A(2) - U^A(1) > U^B(3) - U^B(2)$
        - $U^A(2) - U^A(1) > \lambda[U^B(3) - U^B(2)]$
        - But since $U^B(3) - U^B(2) > 0$ we know this cannot hold for large λ

- Two extremes of SWF:
    - <u>Utilitarian</u>: $SWF = \sum \beta^h U^h$ with special case $\beta^h = 1$ perfect substitutes
    - <u>Rawls</u>: $SWF = min\{U^1, \ \dots, U^h\}$ where we have perfect complements

Figure 8: Social choices with Utilitarian and Rawlsian SWFs



2.4 Classifying Problems

2.4.1 *Types of Worlds*

- <u>Perfect world:</u> First best (i.e. pareto efficient; only constraints irreducible associates with tech and endowments) attained without government. Have perfect separation of efficiency and equity
    - This relies on unrealistic assumptions:
        - Perfect competition and price taking behaviour in all markets
        - Complete set of markets (or the absence of externalities and public goods
        - Complete and symmetric information.
- <u>Imperfect world:</u> Any redistribution of resources will have to trade-off the benefits in terms of satisfying equity objectives with the efficiency cost

- <u>First best world:</u> Government has the capacity to correct all market failures and therefore to ensure that all the Pareto conditions are satisfied
- <u>Second best world:</u> Government only has the capacity to correct some market failures and, therefore, cannot ensure that all the Pareto conditions are satisfied

2.4.2 *Types of distortion*

- <u>Endogenous distortions:</u> arise from market imperfections (e.g. monopoly, externalities)
    - Product mix inefficiencies: presence of a production externality, such as CO2
    - Production inefficiencies: sectoral wage differentials for factor of production
    - Consumption inefficiencies: consumption externality, such a envy
- <u>Policy-imposed distortions:</u> originating from policy choices
    - Product mix efficiency: Taxes cause producer and consumer prices to diverge
    - Production inefficiencies: Sector-specific factor taxes or subsidies
    - Consumption inefficiencies: Consumers face different relative prices e.g. after-tax

3 Tax Incidence

3.1 ADD-ONS

3.1.1 PRINCIPLE OF TARGETING

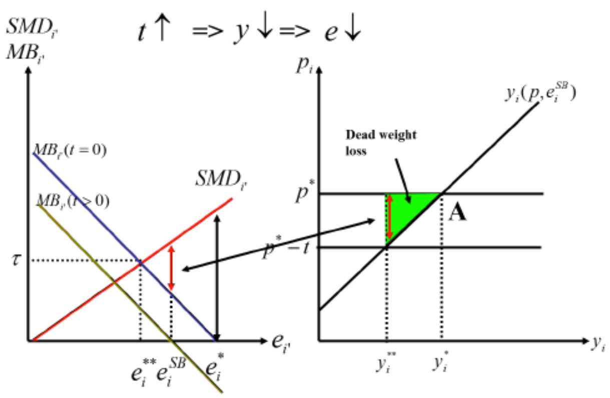
Bhagwati (1971): To correct distortion in first best world use instrument that most directly offsets source

E.g. Pigouvian tax on CO2 (externality, where $\tau = SMD$) better than green tax on SUVs (output of the good that causes the externality)

Green tax can be used to fix pollution problem but in doing so distorts the choice between y_i and other goods thus imposing a new inefficiency on the economy

Look at doc for specific example shown in Figure 9

Figure 9: The Principle of Targeting



3.1.2 LUMP SUM TAXES

Lump Sum Tax: Levied on the unchangeable endowments of individuals. Thus incidence is trivial as by definition falls on taxpayer

Optimal: Redistributes income from individuals with "good" endowment to "bad" and thus varies with "ability" (i.e. characteristics of endowments).

But… "ability" is private information and no incentive to reveal "good" endowment

Universal lump-sum (i.e. poll tax) is not optimal but instead regressive

3.2 PRINCIPLES OF TAX INCIDENCE

3.2.1 SET-UP

One commodity x traded in *competitive* market

Producer price p, consumer price q, unit tax $q = p + t$, ad-valorem tax $q = (1+t)p$

No income effects so $D'(q) \leq 0$ and decreasing returns so $S'(p) \geq 0$

3.2.2 INCIDENCE OF DISTORTIONARY TAX IS SHIFTED

Tax Liability Side Equivalence: Does not matter which side levy tax in a competitive market. This is because taxes changes market prices, which shifts incidence forwards/backwards

Also does not matter if tax is levied as a unit or as an ad valorem tax if raise same revenue

Figure 1: Unit tax collected from the producers

$$D(q) = S(q - t)$$

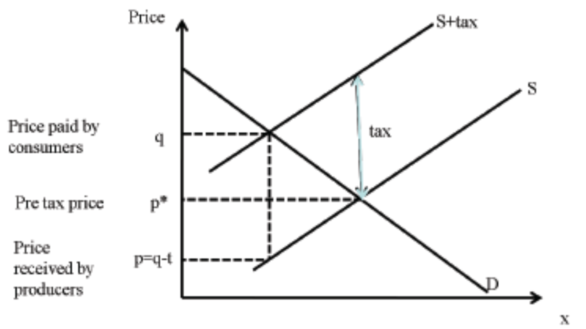
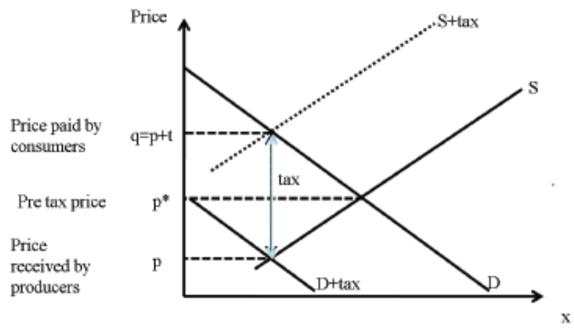


Figure 2: Unit tax collected from the consumers

$$D(p + t) = S(p)$$



E.g. social security: depends on whether employers can pass tax on to consumers as higher prices of outputs (forward shifting) or lowers wages of employees (backward shifting)

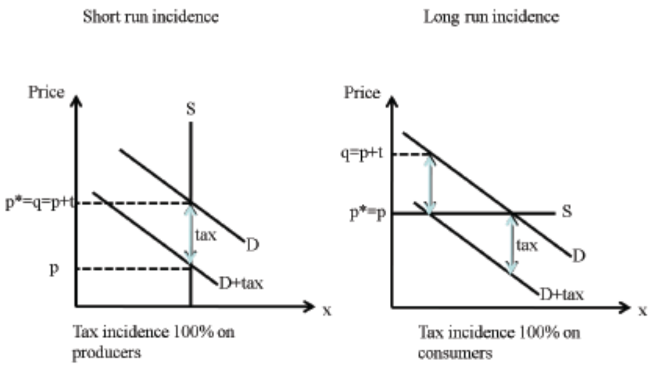
3.2.3 Borne by those who cannot easily change their behavior

Burden of a tax depends on elasticity of supply or demand:
Note in equilibrium, markets must clear. That is, $D(p+t) = S(p)$
Taking total derivative: $\frac{\delta D}{\delta q}(\delta p + \delta t) = \frac{\delta S}{\delta p}\ \delta p$
Rearrange: $\frac{\delta p}{\delta t} = \frac{\frac{\delta D}{\delta q}}{\frac{\delta S}{\delta p} - \frac{\delta D}{\delta q}}$
At equilibrium $x^D = x^S$ and $q \cong p$ for a small tax. Thus multiply with $\frac{q}{x^D} = \frac{p}{x^S}$

$$\frac{\delta p}{\delta t} = \frac{\frac{\delta D}{\delta q}\frac{q}{x^D}}{\frac{\delta S}{\delta p}\frac{p}{x^S} - \frac{\delta D}{\delta q}\frac{q}{x^D}} = \frac{\epsilon^D}{\epsilon^S - \epsilon^D} = \frac{1}{\frac{\epsilon^S}{\epsilon^D} - 1} < 0$$

Thus see:
If $\epsilon^D \rightarrow -\infty$ incidence falls entirely on producer $\frac{\delta p}{\delta t} = -1$.
If $\epsilon^S \rightarrow \infty$ or $\epsilon^D = 0$ incidence falls entirely on consumer $\frac{\delta q}{\delta t} = -1$
Intuitively:
If demand is very elastic any change in the consumer price will induce large reduction in demand as consumers substitute away
If supply is very inelastic suppliers will supply to the market whatever the price is and so they will bear the incidence of the tax
Implies short run and long run incidence of a tax can differ since firm/supply substitution possibilities are larger in the long run

Figure 3: Unit tax collected from the consumers



3.3 General and Specific Tax equivalence (optional, see Lecture Notes)

3.4 Experimental Evidence (Sansgruber & Tyran, 2005)

Market constructed so demand is inelastic and tax incidence always fall on buyers
Propose two taxes where revenue equally divided amongst buyers and sellers
Transparent Tax: collected directly from buyers
In-Transparent Tax: collected from sellers
Buyers should not accept it in either case. This is not the case:
In first referendum 9/10 times buyers accept IT and 9/10 times reject TT
Buyer inconsistency due to "fiscal illusion"
But… second referendum 5/10 times buyers accept IT and 8/10 times reject TT

4 Deadweight Loss

4.1.1 *Definitions*

Deadweight Loss:
(i) Amount in excess of tax revenue collected that taxpayer can pay the government in exchange for removal of the tax while maintaining the post-tax utility level
(ii) How much more could be collected from taxpayer than collected, with no extra loss in utility, if the collection method were lump sum (based on EV not CV)
Consult lectures on duality

<u>Indirect utility function:</u> Links maximized utility to consumer prices and income i.e. $V(q,m) = \max_{x_1,x_2} U(x_1,x_2;l_0)\ \text{s.t.}\ q_1x_1 + q_2x_2 = m = q_0l_0$
Solution gives us the <u>Marshallian/uncompensated demand function</u>, recording demand for the good at given consumer prices and income $x_i(q,m)$
<u>Expenditure function:</u> Minimum expenditure at given consumer prices needed to obtain a given level of utility i.e. $E(q,\bar{U}) = \min_{x_1,x_2} \sum q_i x_i\ \ s.t.\ U(x_1,x_2;l_0) = \bar{U}$
Solution gives <u>Hicksian/compensated demand function</u>. It varies consumer and prices and income so as to keep utility level fixed $x_i^c(q,\bar{U})$

4.2 Fixed Producer Prices: DWL as Substitution Effect

4.2.1 *Set Up*

Tax is shifted completely on to consumers ($q_1 = p_1 + t_1$). Thus post-tax utility $V(p_1 + t_1, m)$

Revenue generated is $R(t_1) = x_1(p_1 + t_1, m)t_1$

Note $x_1(p_1 + t_1, m) = x_1^c(p_1 + t_1, U_1)$ [Hicksian demand why? check this- depends on income effect?]

<u>Equivalent variation:</u> Consumer wtp to avoid tax (i.e. post-tax utility at pre-tax prices):

$$EV(t_1) = E\big(p_1 + t_1, V(p_1 + t_1, m)\big) - E\big(p_1, V(p_1 + t_1, m)\big)$$
$$= m - E\big(p_1, V(p_1 + t_1, m)\big)$$

Define $DWL(t_1) = EV(t_1) - R(t_1)$

4.2.2 Calculating DWL via decomposition

Graphically, see that DWL is purely caused by the substitution effect!

In face of a change in relative prices, consumer reduce tax liabilities by substituting away from taxed good into sub-optimal arrangement

Hence lump sum tax is necessarily efficient as it only causes income effect (i.e. shifts budget line in without changing relative prices)

Figure 1: Illustrating the DWL calculation

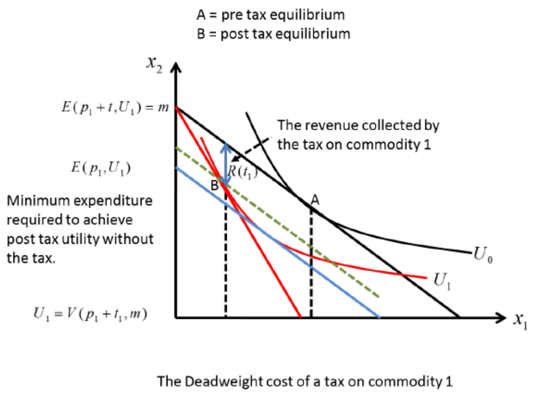


Figure 3: Illustrating the DWL calculation

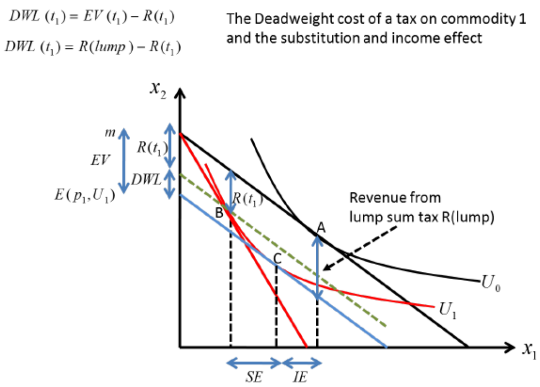
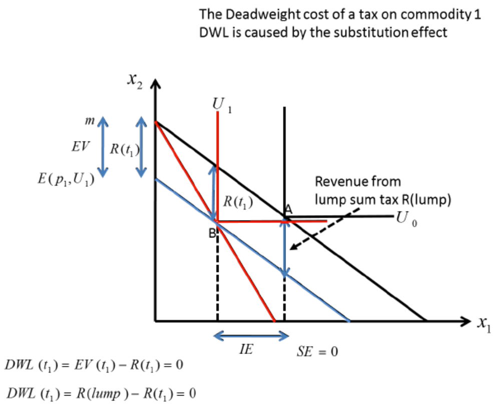
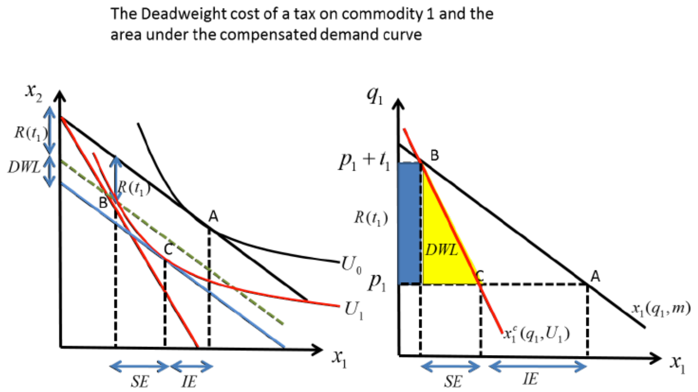


Figure 4: No substitution effect = no DWL



4.2.3 Calculating DWL via Hicksian Demand Function

Figure 5: DWL and the Area under the Compensated Demand Curve



Hicksian/compensated demand function isolates substitution effect

DWL is equal to area under Hicksian demand curve $\int_{p_1}^{p_1+t_1} x_1^c(q_1,U_1)dq_1 - t_1x_1(p_1+t_1,m)$

$\int_{p_1}^{p_1+t_1} x_1^c(q_1,U_1)dq_1 = \int_{p_1}^{p_1+t_1} \frac{\delta E(q_1,U_1)}{\delta q_1}dq_1$ [using Sheperd's Lemma $\frac{\delta E}{\delta q_1} = x_1^c$]

$= E(p_1+t_1,U_1) - E(p_1,U_1) = m - E(p_1,U_1) = EV(t_1)$ by definition

DWL is directly related to elasticity of compensated demand curve: More elastic compensated demand; tax causes bigger substitution effect; DWL is bigger

4.2.4 Harberger Approximation Formula

Use local estimates of compensated demand slopes evaluated at post-tax values

$$DWL(t_1,t_2) - \frac{1}{2}\sum\sum t_i t_j \frac{\delta x_i^c(.)}{\delta q_j} = -\frac{1}{2}t_1^2\frac{\delta x_1^c(.)}{\delta q_1} - \frac{1}{2}t_2^2\frac{\delta x_2^c(.)}{\delta q_2} - \frac{1}{2}t_1t_2\frac{\delta x_2^c(.)}{\delta q_1} - \frac{1}{2}t_2t_1\frac{\delta x_1^c(.)}{\delta q_2}$$

Since $\frac{\delta x_2^c(.)}{\delta q_1} = \frac{\delta x_1^c(.)}{\delta q_2}$: $DWL(t_1,t_2) = -\frac{1}{2}t_1^2\frac{\delta x_1^c(.)}{\delta q_1} - \frac{1}{2}t_2^2\frac{\delta x_2^c(.)}{\delta q_2} - t_1t_2\frac{\delta x_2^c(.)}{\delta q_1}$

First and second terms are DWL in the two markets created by respective tax

Third term is the tax interaction effect

Note special case where do not tax good 2: $DWL = -\frac{1}{2}\sum t^2|\epsilon_{11}^c| \frac{\delta x_i^c(.)}{\delta q_j}\frac{x_1^c}{q_1}$

Directly see that elasticity (of compensate demand) causes bigger DWL.

DWL function is a convex function of the tax rate

Low tax on a lot of different tax bases tend to reduce the overall DWL.

- Observe tax interaction effects:; If complements tax causes bigger DWL
  - If subsites, tax causes smaller DWL
    - Rise in price of good 1 shifts out demand curve for good 2
    - Since there is a tax levied on good 2 more revenue is collected, reducing DWL
  - If complements, tax causes bigger DWL
    - Rise in price of good 1 shifts in demand curve for good 2
    - Since there is a tax levied on good 1 less revenue is collected, raising DWL

4.3 Fixed Consumer Prices: DWL as loss of Profit

Tax entirely shifted onto producers.

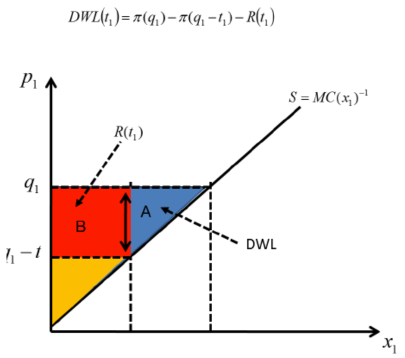
Assume in short run supply curve is upwards sloping and representative firm earns profit

Define DWL is related to loss of profits: $DWL = \pi(q_1) - \pi(q_1 - t_1) - R(t_1)$;

Loss in π is larger than R because firm reduces production in the presence of the tax

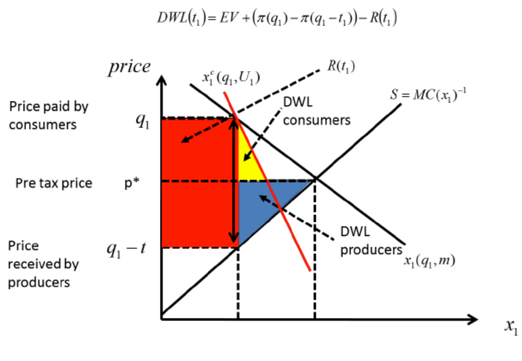
For units not produced, (marginal) cost of production is below the consumer price

Figure 7: The DWL on the supply side



The excess burden of a tax borne by producers

Figure 8: The overall DWL when the incidence of the tax is shared



The excess burden of a tax shared between producers and consumers

4.4 SHARED INCIDENCE

Just combine above to define $DWL = EV(t_1) + \pi(q_1) - \pi(q_1 - t_1) - R(t_1)$

4.5 EMPIRICAL ESTIMATES OF DWL

4.5.1 METHODOLOGY

Need data on hourly wages (q_0), non-labour income (I), hours of work (L_0) and all kinds of relevant background characteristics ($\boldsymbol{X}$) of representative sample of taxpayers
Estimate labour supply $\ln l_{0t}^h = \alpha_h + \beta_1 \ln q_{0t}^h + \beta_2 \ln I_t^h + \gamma X_t^h + \epsilon_t^h$
Derive parameters β_1 (uncompensated wage elasticity) and β_2 (income elasticity)
As compensation is never paid, compensation wage elasticity is counterfactual. Instead estimate by back it out by using Slutsky decomposition and β_1, β_2

4.5.2 LITERATURE REVIEW

- See Tax Review for Literature Summary and much more detail

Harberger (1964) estimates DWL at 2.5% of the revenue raised. But…

- Early literature focused on U.S. 1986 tax cut, finding such large elasticity implying not only high marginal efficiency cost but also US is on wrong side of Laffer curve
  - Feldstein (1999) says much higher, at 32%, since incorporates taxpayer liability by substituting into an untaxed good (leisure), exclusions (E) and deductions (D)
    - $TI = q_0 l_0 - E - D$ so $DWL = \frac{1}{2} TI \frac{t_0^2}{1-t_0} \epsilon_{TI}^C$
    - A 10% increase in all personal tax rates creates a marginal deadweight loss in the order of $2 per $1 revenue collected
  - Auten and Carroll (1999) estimate elasticity of 0.55
- Subsequent research revised this down due to better data, improved methodology, and that tax changes after 1986 separated out changes from non-tax-related changes in inequality
  - Moffitt and Wilhelm (2000): AGI tax elasticity 1.76-1.99 (like Feldstein) but once apply 2SLS falls to 0.35-0.97
  - Giertz (2007) seems good example too
  - In range 0.12 to 0.40. So marginal excess burden per dollar revenue is $ 0.195 for an across-the-board proportional tax increase, and $ 0.339 for top 1% of earners
- Recent evidence emphasizes elasticity can be very different for demographics. Taking account of how different population groups respond to incentives allows any particular level of redistribution to be achieved at a minimum efficiency cost!

- Institute for Fiscal Studies investigated tax returns of UK self-employed pre- and post tax changes, finding elasticity of taxable is well above 1
- Mirrless Review: Largest responses in reducing labour supply due to taxes are by women with school age children and for those aged over 50
- Blundell and Shephard (2011): labour supply responses by low-earning parents matter a great deal
- Saez et al. (2012): ETI is higher for high income individuals who have more access to avoidance opportunities, especially deductible expenses.

Negative Income Tax

- Higher guaranteed income will reduce the incentive to take a job at all (extensive); higher marginal tax rate reduces incentive to work if substitution > income effect (intensive)
- To provide non-workers with one-third of average income would require a tax rate of 33% in addition to other purposes. Total is thus 67%!

Odd Bits

- Maximization of 'social welfare' is not same crude measure of aggregate income. It allows for work effort, needs, and inequality.
- Substitution effects are generally larger than income effects so taxes reduce labour supply. Especially for low earners, responses are larger at the extensive margin—employment—than at the intensive margin—hours of work.
- Good quotes in Mirless: "A tax change that would have been revenue neutral…" "When personal tax rates on ordinary income rise…"

5 General Theory of Second Best

5.1 Principles

Demand side interdependency: If the price of one good goes up they may increase (substitutes) or decrease (complements) their demand for another
Supply side interdependency: Via cost of production being pushed up for other goods (draws factors of production)

Second best world with interdependencies:
Do not attempt to restore Pareto conditions in some markets if cannot do so in all
Distortions can offset/magnify each other and new distortions can counteract/extenuate existing ones
Optimal policy interventions balance MC of new against MB of alleviating existing
Moving the economy towards more efficient allocation is not same as moving prices towards marginal cost / removing inefficiencies for some markets
Second best world with no interdependencies:
Pareto conditions should be satisfied in independent markets
Policy interventions governed by same principles as in First Best (i.e. targeting)

5.2 Guide For Analysis

1. Is it a second best problem (i.e. irremovable distortion)? If so are there interdependencies? Are interdependences supply or demand?
2. Set up a framework. What markets do you need to consider? Make it simple and clear

3. What is the primary welfare effect of the policy? What distortion is created at the margin (do so graphically)
4. What is the secondary welfare effect of the policy? What is the irremovable constraint and what are the initial (pre-policy) consequences? What is it at the margin (MB/C of policy)
5. What is the optimal direction of travel? Consider the effects of a marginal change to establish whether policy intervention is optimal
6. What is the optimal policy? Find point where marginal cost and benefit balance

Commonly make two assumptions:
No income effects in demand for the relevant goods:
Thus Hicksian and Marshallian demand are same
Calculate DWL under Marshallian demand curve (easier)
Relevant sectors are small
Resources drawn/released to rest of economy without affecting factor prices
Any interdependences between markets will be on demand side only

6 James Meade 'Trade and Welfare' (1955)

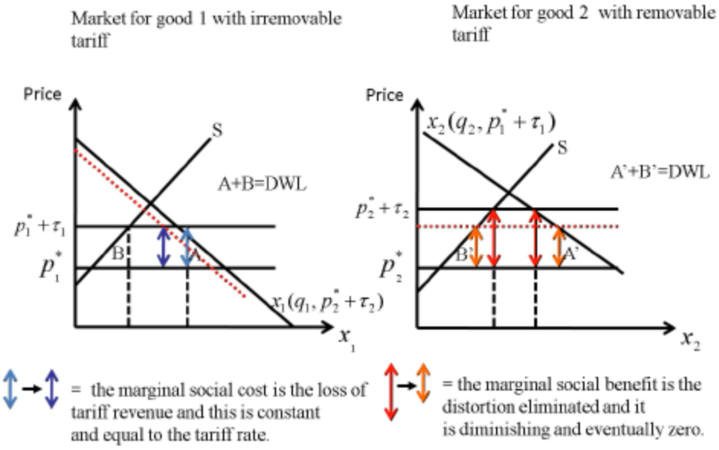
6.1 Set Up

Small economy producing two goods: x_1 irremovable tariff τ_1; x_2 removeable tariff τ_2
No income effect: DWL can be measured under Marshallian demand curve
Demand side interdependency: Two goods are substitutes in consumption
No supply side interdependency: Two sectors are “small”

6.2 Equilibrium

Two inefficiencies pre-policy:
B/B’ Product mix inefficiencies: consumers face domestic consumer price while world market produces face world market price
A/A’ production inefficiencies: Domestic producers face domestic producer price while world market producers face world market price
Note trade-off:
Two red arrows for x_2 indicate initial marginal DWL of tariff. Lowering t_2 stimulates domestic consumption and expensive domestic production replaced
Blue arrow in x_1 indicates MSB/C of an expansion/contraction of demand. Fall in consumer price of good x_2 shifts in demand curve x_1 (sub.) and reduces revenue
As progress marginal benefit from cutting τ_2 falls by less (and approaches zero). Marginal cost does not fall. Thus reach an optimum with *some* reduction (but not all the way to zero)

Figure 1: Tariffs in small open economy



6.3 LIPSEY & LANCASTER (1956-57)

Pricing of *one* public service in presence of private sector market failures. Given one irremovable price distortion should other prices bet set at marginal cost?

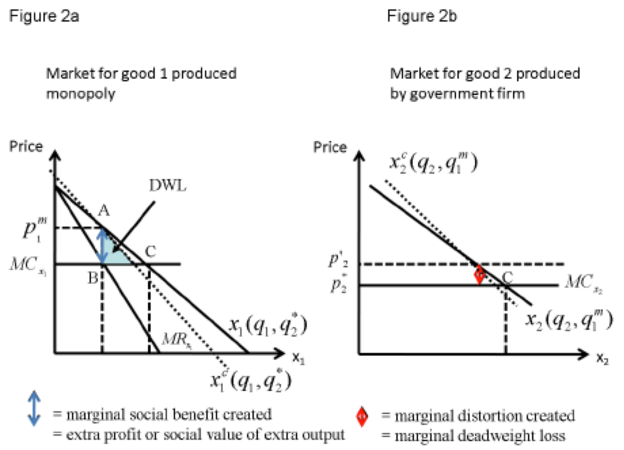
6.3.1 *SET UP*

Two goods: x_1 produced by unregulated monopoly; x_2 by SOE. Both have constant MC
Demand side interdependence: two goods are substitutes

6.3.2 *EQUILIBRIUM*

Inefficiency pre-policy: Monopoly will produce x_1^M so $MC_1 = MR_1$ resulting in $p > MC_1$
First Best: The simplest is to regulate the price of the monopoly and demand that it sets $p_1 = MC_1$ and supplies x_1^* units to the market. Require same for state owned firm
Second Best with independent (cannot change monopoly): optimal to seek to satisfy Pareto condition in market for x_2 in isolation and hence demand SOE sets $p_2 = MC_2$
Second Best with interdependent (cannot change monopoly): optimal to create a distortion in the market for x_2 in order to alleviated unremovable distortion in market for x_1
Note trade-off:
Primary welfare effect: Deviating from marginal cost pricing creates loss in market for x_1. This is increasing at an increasing rate
Secondary welfare effect: Rise in p_2 shifts out x_1 demand, creating marginal social gain as strictly positive gap between monopoly p (A) and MC (C) falls
Monopolist produces at A but MC=p is at C. Difference is DWL
Note optimal direction of travel: Loss is small at start ($p_2 = MC_2$) so optimal price is to have a balancing distortion: $p_2 = MC_2 - [q_1^m - MC_1]\frac{dx_1}{dx_2}$
$\frac{dx_1}{dx_2}$ is negative if subsites and positive if complements
$q_1^m - MC_1$ is social value per unit of good 1 supplied
Look at back of DWL triangles: primary is very small (0) at first but increasing with gap between price and MC
Note we don't minimize DWL but maximize social welfare. Triangle might not get smaller but monopoly revenue increases

Figure 2: Regulation of state-owned firm in the presence of a private monopoly



It might be helpful to do go through a concrete example. Suppose that demand is linear in both sectors and that preferences are quasilinear preferences

(i.e., $\frac{\partial v}{\partial I} = 1$ because there is a third good that enters the utility function in a linear way). With these assumptions, we can get closed form solutions for everything.

Specifically, the demand curve is $x_1 = \frac{a}{b} - \frac{p_1}{b} + \frac{cp_2}{b}$ and the inverse demand curve is $p_1 = a - bx_1 + cp_2$. The demand for good 2 is $x_2 = d - ep_2 + fp_1$. The parameter c controls whether the two goods are substitutes or complements. We assume that $c > 0$ (substitutes). All the other parameters are positive. The monopolist solves

$$\pi(p_2) = \max x_1(a - bx_1 + cp_2) - MC_{x1}x_1$$

which gives the first order condition

$$a + cp_2 - 2bx_1 - MC_{x1} = 0$$

and $x_1^m(p_2) = \frac{a+cp_2-MC_{x1}}{2b}$ and $p_1^m = \frac{a+cp_2+MC_{x1}}{2}$. The profit is

$$\pi_1(p_2) = \frac{(a + cp_2 - MC_{x1})^2}{4b}.$$

Social welfare is

$$SW = v(p_1(x_1^m(p_2), p_2), p_2) + \pi_1(p_2) + (p_2 - MC_{x2})\,x_2$$

We can now, using equation (6), evaluate the cost and benefit of increasing the price in market 2 above marginal cost:

$$\frac{\partial SW}{\partial p_2} = \frac{c}{2b}\left(\frac{a + cp_2 - MC_{x1}}{2}\right) - (p_2 - MC_{x_2})\,e.$$

from which we can find the optimal price policy by solving $\frac{\partial SW}{\partial p_2} = 0$. The result is

$$p_2^* = \frac{1}{4eb - c^2}\left(c(a - MC_{x1}) + 4ebMC_{x2}\right).$$

The second order condition requires that $4eb - c^2 > 0$. We see that $p_2^* > MC_{x2}$ because $c\,(a - MC_{x1} + cMC_{x2}) > 0$ as $(a - MC_{x1} + cMC_{x2}) > 0$ is required for production under competition or monopoly to be positive if p_2 is priced at marginal cost. So, the state owned firm should price above marginal cost: doing so increases social welfare. If $c = 0$, then $p_2^* = MC_{x2}$. What if $c < 0$, so that the two goods are complements? In this case, we see that the optimal price of good 2 is below marginal cost because this is what is needed to stimulate the demand for good 1.

6.4 Ramsey Pricing

- In contrast to L&L this acknowledges Fixed Cost is what causes natural monopoly to arise in first place and gives independent reason for above-MC-pricing.
- Considers natural monopoly with large fixed costs and multiple outputs

6.4.1 *Set Up*

x_1 (competitive private, CRTS, time endowment), x_2 (public, IRTS), and x_3 (public IRTS)
One factor of production l_0
Constant MC and large fixed cost that justifies IRTS so $C = F + c_2y_2 + c_3y_3$
Supply interdependencies: x_1 can be consumed or used as input for x_2/x_3 (s)

No demand interdependencies: Preferences are quasi-linear in x_1 and separable in x_2/x_3 so $U = x_1 + v(x_2) + w(x_3)$

Market clearing conditions:
Optimal demand for public goods: $p_2 = v'(x_2)$ and $p_3 = w'(x_3)$
By making utility function additive we have no demand interdependencies
Demand equals supply: $x_2 = y_2, x_3 = y_3$ thus $x_1 = m - (F + c_2 y_2 + c_3 y_3)$
Fix quantities and let market choose price. Other way is identical

6.4.2 *FIRST BEST*

- Assume can levy lump-sum tax to cover fixed cost
- IRTS means x_2/x_3 would be a natural monopoly if produced by private sector. Thus solve $\max_{y_2, y_3} m - (F + c_2 y_2 + c_3 y_3) + v(y_2) + w(y_3)$
- From FOC see two goods should be priced at MC: $MRS_{x_2,x_3} = \frac{p_2}{p_3} = MRT_{x_2,x_3} = \frac{c_2}{c_3}$ and this holds if $p_3 = c_3$ and $p_3 = c_3$

6.4.3 *SECOND BEST*

Suppose government cannot levy lump sum tax to cover fixed cost of two public goods
Budget constraint of SOE is now $p_2 y_2 + p_3 y_3 = F + c_2 y_2 + c_3 y_3$. Thus Lagrange…

$$\mathcal{L} = m - (F + c_2 y_2 + c_3 y_3) + v(y_2) + w(y_3) + \lambda[(p_2 - c_2)y_2 + (p_3 - c_3)y_3 - F]$$

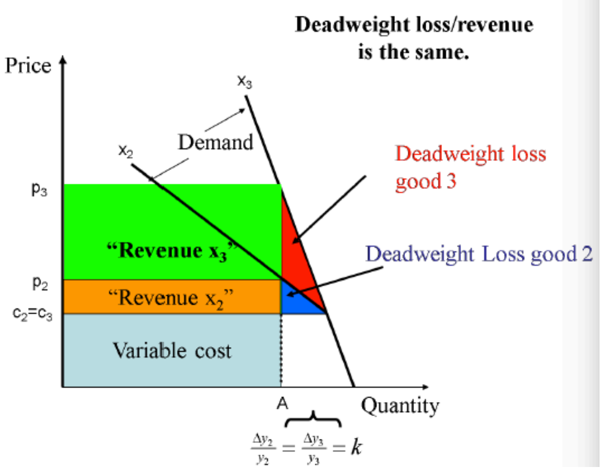
Solving:

$$\frac{d\mathcal{L}}{dy_i} = -c_i + \frac{dv}{dy_2} y_i + \lambda\left(p_i - c_i + y_i \frac{dp_i}{dy_i}\right) = 0$$

$-c_{\text{i}} + \frac{dv}{dy_i}(y_i)$ is direct benefit of an extra unit y_i net of the utility cost generated by need of input c_i
$p_i - c_i + y_i \frac{dp_i}{dy_i}$ is marginal net revenue effect
Channel 1: more y_i, more c_i, less revenue
Channel 2: if sold raise p_i but to sell need to lower prices $\frac{dp_i}{dy_i}$, thus also lose revenue on infra-marginal units
Since $MU_2 = p_2$, can rewrite FOC as e.g. $\frac{d\mathcal{L}}{dy_2} = (1+\lambda)(p_2 - c_2) + \lambda \frac{dp_2}{dy_2} = 0$
Multiplying last term by $\frac{p_3}{p_2}$ get $\epsilon_{ii} = \frac{dy_i}{dp_i}\frac{p_i}{y_i}$
Thus $\frac{p_2 - c_2}{p_2} = -\frac{\lambda}{1+\lambda}\frac{1}{\epsilon_{22}} = \frac{k}{\epsilon_{22}}$. Likewise for x_3
Get multiple insights from this:
In general, optimal to price both goods above marginal cost. “Do not satisfy Pareto ins some markets if you cannot do it everywhere”
Better to have small distortion on two goods than large on one
Distortion is (inversely) proportional to (compensated) elasticity of demand (ϵ_{ii})
Substitution effect (i.e. DWL) generated by a small price increase is large for goods with a numerically large compensated own price elasticity.
Total distortion is minimized, not by increasing all prices by the same percentage, but by reducing the supplies of all publicly produced goods by the same percentage
Interpret now as SOE fixing supply, pushing p_i above MC_i as per formula
$\frac{p_2 - c_2}{p_2} = \frac{k}{\epsilon_{22}}$ approximated by $\frac{\Delta p_i}{p_i} \approx k \frac{y_i}{p_i}\frac{\Delta p_i}{\Delta y_i}$ and thus $\frac{\Delta y_i}{y_i} \approx k < 0$

Total distortion is minimized by equalizing (marginal) DWL per unit of revenue generated not costs per se
Demonstrated in Figure 3 where for simplification assume MC of both goods are same and if priced at MC both quantities same as well
Justifies difference in DWL triangles: revenue from large increase in price of inelastic good is also bigger than modest increase in price of elastic good.
Thus alternative interpretation that prices should increase above MC so to equalize the DWL cost per unit of revenue at the margin.

Figure 3: Illustration of Ramsey Price Rule



7 Diamond-Mirrlees Production Efficiency Result

7.1 Assumptions

1. Competitive markets
2. CRTS
3. Full instrument set
4. Revenue requirement
5. No lump sum taxes
6. SWF is individualistic
7. Non-satiation: At least one good

Assumptions 1&2:
Producers do not earn profits and hence income of consumers does not depend on producer prices
Supply curves are horizontal/perfectly-elastic, including labour. Thus tax incidence falls entirely on consumers
Assumption 3:
Taxes puts a wedge between p_i and q_i
Gov. has enough flexibility to decide if it wants to distort relative consumer prices, relative factor (input) prices or any mixture of the two
Assumptions 4&5:
Implies that gov. is operating in a second best world; revenue requirement is an irremovable distortion/constraint

7.2 Set-Up

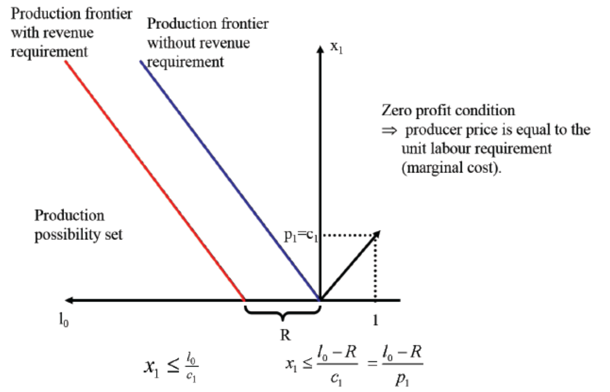


Figure 1: The production frontier and production choices

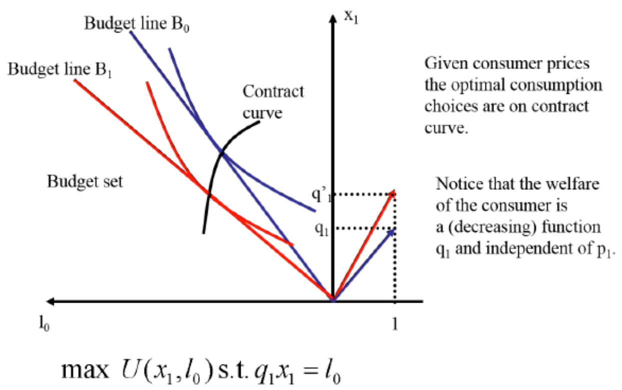


Figure 2: The optimal consumption-labour choice

Production frontier shows maximum amount of output that can be produced with the resources available: $x_1 = \frac{l_0 - R}{c_1}$
Since CRTS this is a straight line
Shifts according to government revenue requirement
Consumption-labour supply choice of the consumer: $\max U(x_1; l_0)\ \text{s.t.}\ q_1x_1 = l_0$
Optimal choice is where $MRS_{l_0;x_1} = \frac{1}{q_1}$
Contract curve: Varying q_1 trace out optimal choices as function of consumer prices
Note Pareto Efficiency conditions:
Production Efficiency: $x_1 = \frac{l_0 - R}{c_1}$
Product Mix Efficiency: $MRS_{l_0,x_1} = MRT_{l_0,x_1} = \frac{1}{c_1}$
Consumption Efficiency: Trivially satisfied as only one consumer

7.3 Equilibrium

7.3.1 *First Best Benchmark (with Lump-Sum taxes)*

Lump Sum so $p_1x_1 = l_0 - R$. Hence set so budget line and the production frontier coincide
This satisfies PE and PME as $MRS_{l_0,x_1} = MRT_{l_0,x_1} = \frac{1}{c_1}$ as $p_1 = q_1 = c_1$

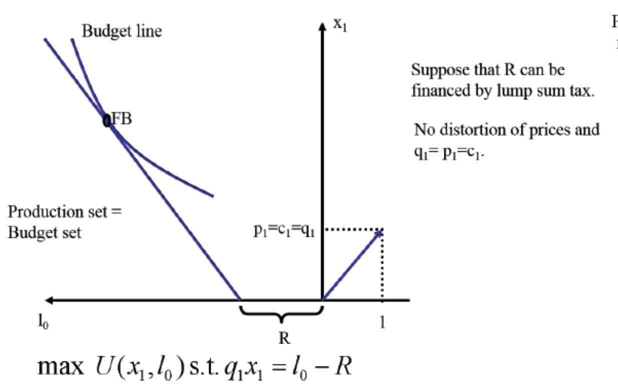


Figure 3: The first-best allocation with lump sum taxation

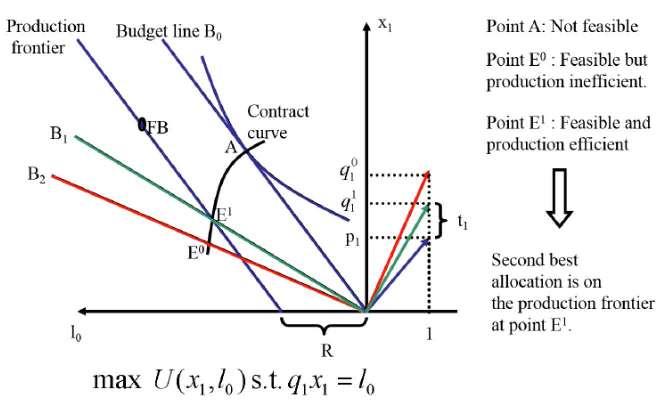


Figure 4: The second best tax and production efficiency

7.3.2 *Second Best (Distortionary Commodity Taxes)*

No Lump Sum Tax so need Commodity Tax $R = (q_1 - p_1)x_1(q_1)$. This implies $q_1 > p_1$
Any combination of (x_1, l_0) that is on the budget line defined by relative price q_1 and on the production frontier will raise exactly the right amount of revenue

Optimal choice is point E^0. This is below production frontier and raises more revenue than required
Since consumer dislikes supplying labour, utility can be increased by reducing labour supply to minimum required to produce x_1 at E_0. Hence cannot be optimal
Government should lower tax and present consumer with steeper budget line B_1
Hence choose E_1. On production frontier at point where it incepts budget line and so the consumption-labour choice raises exactly right amount of revenue
Cannot lower further else too little revenue collected
Satisfies PE but not PME, as $MRS_{l_0,x_1} = \frac{1}{q_1^1} < MRT_{l_0,x_1} = \frac{1}{p_1} = \frac{1}{c_1}$
The second best optimal tax system maintains economy on production possibility frontier

7.4 Interpretation

As government can adjust taxes on all (but one) goods/factors of production, it can bring any configuration of relative consumer prices consistent with the revenue requirement
Any utility level attained at relative consumer price that involves production inefficiency can be obtained with one without
Thus production inefficiency only adds distortions without correcting others or generating benefits to consumers through profits

Does this contradict Second Best? No!
Diamond-Mirrlees assumes consumer and producer prices are independent
Incidence of any taxes falls consumers so there no reason to impose taxes that distort production efficiency
Hence an example that Pareto conditions should be satisfied in independent markets
Independence between consumers and producers is consequence of full set of tax instruments and that there is no profit income.
Hence can we just tax profit 100% and return to model? No! Hard to tax pure profit because it is hard to observe what is pure profit and what is just reward to entrepreneurship

Preserving production efficiency has many tax implications:

<u>Input Taxes:</u> Taxes on production factors should not vary across firms (or sectors), else MRTS are not equal and can increase production by reallocating inputs
<u>Public and private sector production same relative prices:</u> E.g teachers should be paid same in public and private sector (assuming same skill level)
<u>MRTS within public sector should be same</u> – i.e. public enterprises should be exposed to the same shadow prices.
<u>Intermediate goods should not be taxed</u> – Else can reallocate from firm producing intermediate to firm using intermediate. Important for VAT!
<u>Small open economy should not use tariffs</u>: Tariff on a commodity is equivalent to production subsidy and consumption tax levied at same rate
Implies can expose producers to world market prices by removing subsidy component, restoring production efficiency
I.e. if purpose is to raise money, there is not point in distorting production with a subsidy, just levy the consumption tax.

8 Ramsey Taxes

In DM, optimal tax structure preserves production efficiency. So tax commodities not production. But how do we tax commodities, differentiated or uniform taxes?
Ramsey taxes follow general principle of equalizing marginal DWL per unit of revenue and thus justifies different tax rates on different goods

8.1 Assumptions

Two commodities x_1 and x_2 and one factor of production l_0
x_1 and x_2 are produced with l_0 via CRTS (hence fixed producer prices and no profit)
Fixed producer prices p_0, p_1, p_2 (equal to MC) and consumer prices q_0, q_1, q_2
Normalise so $p_0 = 1$
Representative consumer has direct $U(x_1, x_2, l_0)$ and time budget $T_0 = \ell_0 + l_0$
As only one agent we only consider efficiency and ignore distribution for now
Government collects R and has full set of taxes $q_i = (1 + t_i)p_i$ (choose any relative prices)
Since government chooses $t_{1,2}$ and $p_{1,2}$ are fixed, government picks $q_{1,2}$ directly

8.2 First Best

Can tax without distortions: uniform tax t on all consumption goods <u>including value of leisure</u>!
Equivalent to a lump sum tax on (exogenous) full income:

$$p_1x_1 + p_2x_2 + p_0L_0 = \frac{p_0T_0}{1 + t}$$

Only feasible if leisure can be taxed directly but irl government cannot value this and only observes market transactions
Indirectly taxing leisure by taxing labour causes problems as subsidy of labour just cancels out consumer good taxes

$$(1 + t)p_1x_1 + (1 + t)p_2x_2 = (1 + t)p_0l_0$$

8.3 Second Best: Ramsey tax solution

Now consider we cannot tax leisure as per above: $t_0 = 0$ [or is this just normalizing? If so where is the second best limitation?]

8.3.1 *Solving*

First, derive the basics:
Indirect $V(q_0, q_1, q_2, m) = \max U(x_1, x_2, l_0)$ s.t. $q_1x_1 + q_2x_2 = q_0l_0 + m$
Revenue requirement $R = x_1(q_1, q_2, m)(q_1 - p_1) + x_2(q_1, q_2, m)(q_2 - p_2)$
Lagrange $\mathcal{L} = V + \mu[x_1(q_1 - p_1) + x_2(q_2 - p_2) - R]$
μ is marginal cost of public funds (of one unit R via distorting taxes)
$\frac{dL}{dq_1} = \frac{dV}{dq_1} + \mu\left[x_1 + \frac{dx_1}{dq_1}(q_1 - p_1) + \frac{dx_2}{dq_1}(q_2 - p_2)\right] = 0$ and likewise for good 2
First term is direct welfare loss associated with an increase in q_1
Bracket term contains two components of revenue effect
mechanical revenue effect (increase tax a bit when tax base does not change)
behavioural effect (demand changes for x_1 AND x_2 changes)
Thus matter is x_1 is ordinary (so demand falls) and x_2 is substitute/complements (demand rises/falls so tax revenue rises/falls)
Optimal tax on each good balances direct welfare cost of the tax with the revenue effect.

Recall that:

Roy's Identity: Links Marshallian (uncompensated) demand function to indirect $x_i(q,m) = -\frac{\frac{dV(q,m)}{dq_i}}{\lambda}$ where $\lambda = \frac{dV(q;m)}{dm}$ is marginal utility of income

Slutsky Equation: total change in (Marshallian) uncompensated demand for commodity i decomposed into (Hicksian) compensated change (i.e. substitution effect) and income effect $\frac{dx_i(q;m)}{dq_j} = \frac{dx_i^c(q;V(q,m))}{dq_j} - \frac{dx_i(q;m)}{dm} x_j$

Slutsky Symmetry: $\frac{dx_1^c}{dq_2} = \frac{dx_2^c}{dq_1}$

Use this to rewrite:

Roy's $\frac{dV}{dq_1} = -\lambda x_1$ rewrite $-\lambda x_1 + \mu\left[x_1 + \frac{dx_1}{dq_1}(q_1 - p_1) + \frac{dx_2}{dq_1}(q_2 - p_2)\right] = 0$

Slutsky $\frac{dx_1}{dq_1} = \frac{dx_1^c}{dq_1} - \frac{dx_1}{dm}x_1$ rewrite $-\lambda x_1 + \mu\left[x_1 + \left(\frac{dx_1^c}{dq_1} - \frac{dx_1}{dm}x_1\right)(q_1 - p_1) + \left(\frac{dx_2^c}{dq_1} - \frac{dx_2}{dm}x_1\right)(q_2 - p_2)\right] = 0$

Rearrange as $\mu\frac{dx_1^c}{dq_1}(q_1 - p_1) + \mu\frac{dx_2^c}{dq_1}(q_2 - p_2) + \mu x_1 = \alpha x_1$

Where $\alpha = \lambda + \mu\left[\frac{dx_1}{\text{dm}}(q_1 - p_1) + \frac{dx_2}{\text{dm}}(q_2 - p_2)\right]$ i.e. private marginal value of income plus marginal revenue effect due to income effect

Rearrange as $\frac{\frac{dx_1^c}{dq_1}(q_1-p_1)+\frac{dx_2^c}{dq_1}(q_2-p_2)}{x_1} = \frac{\alpha-\mu}{\mu}$

Sub in $q_i - p_i = p_i t_i$ so $-\frac{\frac{dx_1^c}{dq_1}p_1^2 t_1+\frac{dx_2^c}{dq_1}p_1 p_2 t_1}{p_1 x_1} = \frac{\frac{dx_1^c}{dq_1}p_1 t_1+\frac{dx_2^c}{dq_1}p_2 t_1}{x_1} = \frac{\mu-\alpha}{\mu}$

8.3.2 *Interpretation*

LHS is marginal DWL per marginal unit of revenue collected from tax on good $\frac{\frac{dDWL_1}{dt_1}+\frac{dDWL_2}{dt_1}}{\left.\frac{dR}{dt_1}\right|_{x_1}}$

$\frac{dx_1^c}{dq_1}p_1 t_1 + \frac{dx_2^c}{dq_1}p_2 t_1$ is the portion of behavioural effect due to compensated demand, where second term is tax interaction effect (tax on x_1 has on DWL in x_2)

Denominator x_1 is mechanical revenue effect i.e., $\left.\frac{dR}{dt_1}\right|_{x_1} = p_1 x_1$ (where p_1 cancels)

For $t_2 = 0$, we can approximate DWL by Harberger formula using area under compensated demand curve: $DWL = -\frac{1}{2}p_1 t_1^2 \frac{dx_1^c}{dt_1}$ where $\frac{dx_1^c}{dt_1} = \frac{dx_1^c}{dq_1}p_1$ thus $\frac{1}{p_1}\frac{dDWL}{dt_1} = -p_1 t_1 \frac{dx_1^c}{dt_1}$

RHS is marginal DWL per marginal unit of revenue and same for all goods

μ is marginal cost of public funds from distortionary commodity taxation

α tells us what happens if we take unit of income away from consumers lump sum

$\mu - \alpha$ thus difference between raising a unit of revenue through distortionary commodity taxation versus lump sum. This reveals DWL!

Key is that RHS is same for all goods. Thus optimal to distort all relative prices in order to counter-act this one unavoidable distortion

Applies back to General Theory of Second Best

Intuitively, DWL is a convex function so best approach is low taxes on all goods.

Suppose not all equal. Then could lower tax with high marginal DWL and raise it in another to reduce total DWL

8.4 Special Case of Independent Demands

Assume compensated cross-price effects between two taxed goods are zero $\frac{dx_1^c}{dq_2} = \frac{dx_2^c}{dq_1} = 0$

Our equations simplify to $-\frac{\frac{dx_1^c}{dq_1}p_1 t_1}{x_1} = \frac{\mu-\alpha}{\mu}$ and $\frac{\frac{dx_1^c}{dq_2}p_2 t_2}{x_1} = \frac{\mu-\alpha}{\mu}$

Since $\epsilon_{ii} = \frac{dx_1^c}{dq_1}\frac{q_1}{x_i^C} < 0$ can rewrite as $\frac{t_1}{1+t_1} = -\frac{(\mu+\alpha)}{\mu}\frac{1}{\epsilon_{11}^c}$ and $\frac{t_2}{1+t_2} = -\frac{(\mu+\alpha)}{\mu}\frac{1}{\epsilon_{22}^c}$

How can there be elasticity with independent markets? Have free good, leisure!

Tax rate should be inversely proportional to good's (compensated) -price elasticity

Intuitively, substitution effect and thus DWL generated by such a tax is smaller

8.5 Equity Considerations

Set Up

Ramsey tax rule considered above is purely based on efficiency. Now consider two types of consumers: rich (R) and poor (P)

$V_R(q; m_R)$ and $V_P(q; m_P)$ where $m_R > m_P$ (note we allow for different preferences)

SWF $= \theta^R V^R(q; m^R) + \theta^P V^P(q; m^P)$ where $\theta^P > \theta^R$

For simplicity assume independent demands

Interpretation

Now $\frac{t_i}{1+t_i} = -\frac{(\mu+\alpha_i)}{\mu}\frac{1}{\epsilon_{ii}^c}$ where $\alpha_i = s_i^P\alpha^P + s_i^R\alpha^R$

where $s_i^{\mathrm{h}} = \frac{x_i^{\mathrm{h}}}{x_i^P + x_i^R}$ [i.e. share of consumption by consumer types NOT budget share]

where $\alpha^{\mathrm{h}} = \theta^{\mathrm{h}}\lambda^{\mathrm{h}} + \mu\left(\frac{dx_1^{\mathrm{h}}}{dm^{\mathrm{h}}}(q_1 - p_1) + \frac{dx_2^{\mathrm{h}}}{dm^{\mathrm{h}}}(q_2 - p_2)\right)$ [i.e. m. social value of income]

Implies low tax on goods with high α_i since:

Marginal social value of income is higher for poor ($\alpha^P > \alpha^R$)

Society places more value on a unit of income in hands of poor ($\theta^P > \theta^R$)

Poor have higher private marginal value of income ($\lambda^P > \lambda^R$)

Revenue effect set in motion by the income effect of the tax might differ

Poor consume more necessities ($s_i^P = 0, s_i^P = 1$) than luxuries ($s_i^P > 0, s_i^P < 1$)

- Often paraphrased as "goods that constitutes a large share of the budget of the poor should everything else being equal be taxed at a lower rate"
  - Or "inferior goods which are consumed mostly by individuals with low incomes should tax more leniently"
- To show this define budget share as $\sigma_i^h = \frac{q_i x_i^h}{m^h + l_0^h} = \frac{q_i x_i^h}{\overline{m}^h}$ thus $\sigma_i^h \overline{m}^h = q_i x_i^h$
- Substituting in we get $s_i^h = \frac{\sigma_i^{\mathrm{h}}\overline{m}^{\mathrm{h}}}{\sigma_i^P\overline{m}^P + \sigma_i^R\overline{m}^R}$ thus $\frac{ds_i^{\mathrm{P}}}{d\sigma_i^p} = \frac{\sigma_i^P\overline{m}^P\overline{m}^R}{(\sigma_i^P\overline{m}^P + \sigma_i^R\overline{m}^R)^2} > 0$
  - s_i^p and thus α_i increases for a larger poor's budget share σ_i^p

Efficiency vs. Equity

Equality aspect of optimal Ramsey taxes represented by $\frac{\mu-\alpha^i}{\mu}$; efficiency aspect by inverse elasticity term as before

<u>Trade-off occurs</u> if goods mostly consumed by individuals for whom the social marginal value of income is large (poor) are also commodities which are price inelastic.

Poor spend more of their income on necessities (food and housing) and little on luxuries (sports cars and caviar). Fo r equity necessity needs low tax!
Necessity goods are consumed by everyone but relatively price inelastic. Luxury goods are relatively price elastic. For efficiency necessity needs high tax!

For practical application of Ramsey taxes see Lecture Slides 8