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words, diagrams, equ.

01. The Market

Model

Model: Simplified representation of reality, eliminating irrelevant details and allow economics to focus on essential features.
Exogenous variable: taken as determined by factors not discussed in a model
Endogenous variable: determined by forces described in the model

Benefits

- Clearly state hypothesis and assumptions (e.g. First Theorem of Welfare Economics states competitive market are Pareto efficient if we assume perfectly competitive markets)
- Expand set of plausible explanations (e.g. Game Theory and Public Choice give different perspectives by including math)
- Allow arguments to be evaluated (e.g. Phillips Curve has been discredited recently due to recent data)

Drawbacks

- Comprehensive barrier (e.g. 2008 Financial Crisis blamed on silos; "All models are wrong" but math makes it easy to forget one's limitations)
- Rely on good available date, which is rare (e.g. Argentinian economic figures were ignored until recently; can't predict "Black Swans" from past e.g. Brexit effect)

Optimisation

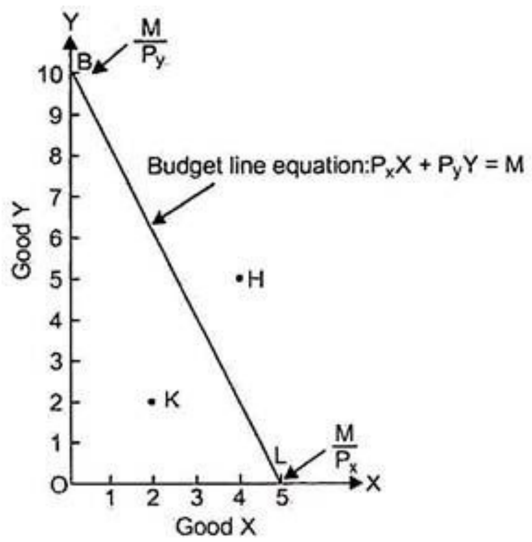
Optimisation principle: people try to choose the best patterns of consumption affordable
Equilibrium principle: prices adjust until demand is equal to supply (i.e. markets clear)
Comparative Statics: comparing two 'static' equilibria without worrying about how the market moves from one to another
Pareto Efficiency: Cannot make someone better off without making someone else worse off. Description of efficiency not distribution. Theoretically competitive market and discriminating monopoly are, ordinary monopoly and price controls aren't.

02. Budget Constraint

Budget Set: $= \{(x_1, x_2) \in \mathbb{R}^2_+ \mid p_1x_1 + p_2x_2 \leq m\}$
Consumption Bundle: $X = (x_1, x_2)$
Budget Constraint: $p_1x_1 + p_2x_2 \leq m$
Budget slope $= -\frac{p_1}{p_2}$

Increase in m: parallel shift outward
change in p_1: increase makes line steeper/flatter

Often let x_2 be a composite good (income spent on everything but x_1)
Numeraire price: Expresses p_1 in terms of p_2 by setting peg $p_2 = 1$ (i.e. divide by p_2: $\frac{p_1}{p_2}x_1 + x_2 \leq \frac{m}{p_2}$)



Taxes (& Subsides)

Quantity Tax: $p'_1 = p_1 + t$ (i.e. budget line gets steeper)
Value Tax: $p'_1 = (1+t)\,p_1$ (i.e. budget line gets steeper)
Lump Sum Tax: $m' = m - l$ (i.e budget line has inward shift)
Rationing: remove all $x_1 > \bar{x}_1$ (i.e. budget line is no existent for some values)
[vice versa for subsidies]

03. Preferences

Axioms of Rationality

Complete: can compare any two bundles (e.g. X and Y)
Reflexive: any good is at least as good as itself (e.g. $X \succcurlyeq X$)
Transitive: if $X \succcurlyeq Y$ and $Y \succcurlyeq Z$ then $X \succcurlyeq Z$ (i.e. ICs never cross)

If well behaved additional assumptions/qualities apply…
Monotonicity of Preferences: non-satiation (e.g. if $X > Y$ then $X \succ Y$); IC always has ve- slope
Convex Preferences: averages are better than extremes; IC is convex. This can be mathematically shown as…
if $(x_1, x_2) \sim (y_1, y_2)$ then $(\theta x_1 + (1-\theta)y_1, \theta x_2 + (1-\theta)y_2) \succcurlyeq (x_1, x_2)$ for all $0 \le \theta \le 1$

Indifference Curves (IC)

Indifference Curves: all consumption bundles for which a decision maker is indifferent between. To remain indifferent a change in x_2 must be offset by a change in x_1, hence a curve forms. Small arrow indicates 'direction' of preference

Strictly Prefers: $X \succ Y$ | Weakly Prefers: $X \succcurlyeq Y$ | Indifferent: $X \sim Y$

Perfect Substitutes	A $+\Delta x_1$ results in an equal $-\Delta x_2$ e.g. different coloured pencils	
Perfect Complements	Consumed in a fixed proportion e.g. left and right shoes	
Bads	$+\Delta x_2$ is compensated with $+\Delta x_1$ e.g. chocolate (☺ and raisin (☹) cookies	
Neutrals	$+\Delta x_2$ has an effect but $+\Delta x_1$ doesn't e.g. strawberry (☺) and cream (☹)	
Satiation	Perfect bundle (bliss point); closer is better e.g. apple juice and water	*Note: slope is… ve+ if too much/little of one good ve- if too much/little of both goods
Discrete Goods	Set of line segments due to small quantity e.g. household cars	
Quasi-linear	IC just shifts vertically; general equation $x_2 = k - f(x_1)$. Not very realistic.	

Marginal Rate of Substitution

MRS is how much Δx_2 a consumer must be compensated for a Δx_1. Slope of an IC $\left(\frac{\Delta x_2}{\Delta x_1}\right)$

- Can also be expressed as $-E$ (exchange rate) that must be tangent to IC. Else higher IC is affordable.
- $MRS = k$ for perfect substitutes, $= 0$ or ∞ for perfect complements, $= \infty$ for neutrals.
- Strictly diminishing as x_1 increases for convex preferences

04. Utility

Utility Function: Assigns a number to a consumer preference (i.e. IC), the higher the more satisfaction is gained (i.e. $X \succ Y$ if and only if $u(X) > u(Y)$)
Ordinal Utility: only order of numbers matters | **Cardinal Utility**: magnitude of number matters as well
Monotonic Transformation: Changes number but preserves order (e.g. $v(X) = k\, u(X)$). Graph always ve⁺
Lexicographic Preferences: $X \succ Y$ with $x_1 >> x_2$

If alternatives are finite, preferences need to be complete, reflexive, and transitive
If alternatives are infinite, preferences also need to be continuous (if $X \succ Y$ then $X + h \succ Y - h$ for small h)

> This allows us to take limits as the bundles approach infinity without special cases. E.g. ...
> Assume lexicographic preferences $x_1 \gg x_2$
> Let $X_n = \left(\frac{1}{n}, 0\right)$ and $Y_n = (0,1)$.
> For all finite n, $X \succ Y$ but... $\lim_{n\to\infty} X = (0,0) \prec \lim_{n\to\infty} Y = (0,1)$

Utility Function Equations

Perfect Substitutes: $u(x_1, x_2) = ax_1 + bx_2$
Perfect Complements: $u(x_1, x_2) = min\{ax_1, bx_2\}$
Quasilinear Preferences: $u(x_1, x_2) = k = f(x_1) + x_2$
Cobb-Douglas Preferences: $u(x_1, x_2) = {x_1}^c {x_2}^d$

> All utility max. DMs have rational pref.
> Not all DMs with rational pref. are utility max.
> Yes if finite-; no if infinite options

Marginal Utility

$MU_i = \frac{\Delta U}{\Delta x_i}$ keeping all other x_j constant (i.e. partial derivative $\frac{du(x_1,x_2)}{dx_1}$)
Magnitude is irrelevant on its own but consider utility-neutral change ($MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$) ...
$-\frac{MU_1}{MU_2} = \frac{\Delta x_2}{\Delta x_1} = MRS.$
MU_i is abstract and depends on $u(\quad)$ but MRS is observable and constant under monotonic trans.

05. Choice

At **optimal choice** (x^*, y^*) BL is just touching the highest IC possible (tangent or intersects at corner).

For monotonic preferences...
If **interior solution** it is a necessary but insufficient condition for $MRS = -\frac{p_x}{p_y}$; occurs if **convex** preferences.

> Can solve via Lagrange...
> $\mathcal{L} = u(x, y) - \lambda(p_x x + p_y y - m)$
> 1. $\frac{\partial \mathcal{L}}{\partial x} = MU_x - \lambda p_x = 0$ 2. $\frac{\partial \mathcal{L}}{\partial y} = MU_y - \lambda p_y = 0$ 3. $\frac{\partial \mathcal{L}}{\partial \lambda} = p_x x + p_y y - m = 0$ 4. Compare sol.
> Obtain $MRS = -\frac{p_1}{p_2}$ form 1. and 2. then sub into 3. to find x^* and y^* in terms of p_x, p_y, m

If **corner solution** then it is always the case that $MRS\ [\leq \text{ or } \geq]\ -\frac{p_x}{p_y}$; occurs if **concave** preferences.

> Can solve via intuition... do Lagrange above. If $x^* < 0$ then corner solution @x-axis [same for y^*]
> Verify by knowing that...
> If $@x-\text{axis}\ \left|\frac{MU_x}{MU_y}\right| \geq \left|\frac{p_x}{p_y}\right|$ then x has more bang-for-buck so consumer wants more/only x
> If $@y-\text{axis}\ \left|\frac{MU_x}{MU_y}\right| \leq \left|\frac{p_x}{p_y}\right|$ then y has more bang-for-buck so consumer wants more/only y

If strictly convex there is only one solution:

> Suppose two opt. solutions/bundles, A and B
> Strict convexity implies $u\left(\frac{1}{2}A + \frac{1}{2}B\right) > \min\{u(A), u(B)\} = u(A) = u(B)$. Hence can't be opt.

06. Demand

Demand Function: relates optimal choice to different prices and income i.e. $x_i(p_i, p_j, m)$

Comparative Statics

Income (m)

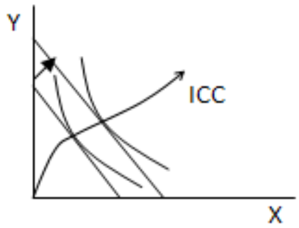
Normal Good: $\frac{\Delta x_i}{\Delta m} > 0$ | **Inferior Good**: $\frac{\Delta x_i}{\Delta m} < 0$

Necessary Good: $\frac{\%\Delta x_i}{\%\Delta m} < 1$ | **Luxury Good**: $\frac{\%\Delta x_i}{\%\Delta m} > 1$

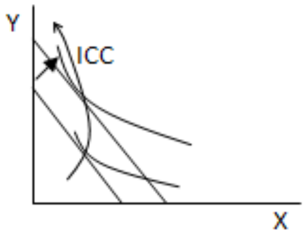
Prices (p_i, p_j)

Ordinary Good: $\frac{\Delta x_i}{\Delta p_i} < 0$ | **Giffen Good**: $\frac{\Delta x_i}{\Delta p_i} > 0$ [Note: All Giffen goods have to be inferior as well]

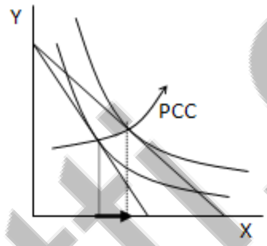
Substitute: $\frac{\Delta x_i}{\Delta p_j} > 0$ | **Complement**: $\frac{\Delta x_i}{\Delta p_j} < 0$ [In two-good world, tells us if $\Delta {x_i}^s$ <> $|\Delta {x_i}^m|$ as $+\Delta p_1 = -\Delta p_2$]



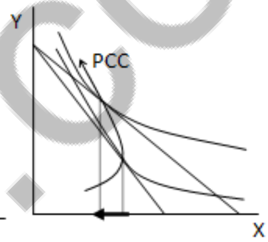
X is a normal good



X is inferior good

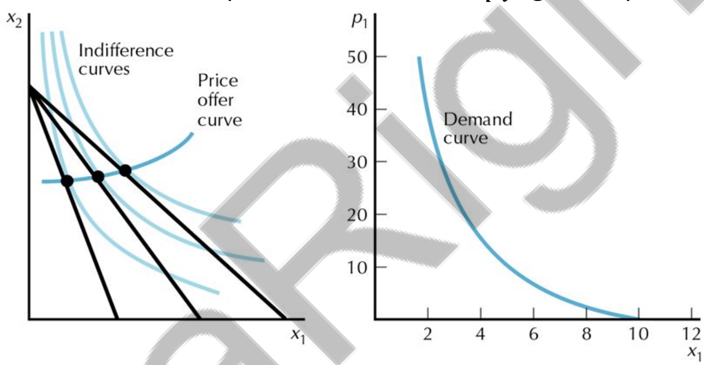


X is an ordinary good



X is a Giffen good

Income Offer Curve: connects optimal demand bundles as m increases
Price Offer Curve: connects optimal demand bundles as p_i increases
Engel Curve: plots the optimal consumption of x_i against different values m (all else fixed)
Demand Curve: plots the optimal consumption of x_i against different values p_i (all else fixed)
Inverse Demand Curve: plots different values p_i against optimal consumption of x_i (all else fixed)



A Price offer curve B Demand curve

	Demand Function	IOC and POC	Engel Curve	Demand Curve
Perfect Substitutes (i.e. spend all income on cheapest good)*	$x_1 \begin{cases} \frac{m}{ap_1} \text{when } p_1 < p_2 \\ 0 \text{ when } p_1 > p_2 \end{cases}$			
Perfect Complements (i.e. spend income in fixed proportion)	$x_1 = kx_2 = \frac{m}{ap_1 + bp_2}$			
Cobb-Douglas (i.e. spend income in fixed proportion)	$x_1 = \frac{c}{c+d} \times \frac{m}{p_1}$			

*If preferences are homothetic [u is homogenous to degree one e.g. $u(ax, ay) = a \times u(x, y)$] then IOC is a linear through origin and Engels Curve is linear

Quasi-linear functions have no income effect

Need to solve $\max[f(x) + y]\, s.t. p_x x + p_y y = m$

Can rearrange budget constraint: $y = \frac{m - p_x x}{p_y}$

Substitute this into utility function: $f(x) + \frac{m - p_x x}{p_y}$

Differentiate w.r.t. x to get FOC: $f'(x^*) = \frac{p_x}{p_y}$

This demand function shows x^* is independent of m

Reservation prices: Price at which consumer is indifferent between consuming another unit or not

Hence the equation $u(0, m) = u(1, m - r_1)$ must hold

If quasilinear $(x, m - p_x)$ we can deduct some simple equations and infer:

$v(0) + m = m = v(1) + m - r_1$	$r_0 = \text{n/a}$
$v(1) + m - r_2 = v(2) + m - 2r_2$	$r_1 = v(1) - v(0)$
…	$r_2 = v(2) - v(1)$
$v(n-1) + m - (n-1)r_n = v(n) + m - nr_n$	…
	$r_n = v(n) - v(n-1)$

In this special case r =marginal-u necessary to buy an additional unit x; y is irrelevant in this.

07. Revealed Preference

Directly Revealed Preference: chosen bundle reveals preference over all other alternatives

Indirectly Revealed Preference: same but does so via transitivity (if $X \succcurlyeq Y$ and $Y \succcurlyeq Z$ then $X \succcurlyeq Z$)

Further assumptions can 'recover' further preferences:

- Preferences are strictly convex: unique demand bundle for each budget (e.g. X)
- Monotonic: all bundles that are more than X are more preferred (e.g. Y and Z)
- Convex: all weighted averages of X and preferred bundles are preferred also
- We can hence trap an IC

Weak Axiom of Revealed Preference: If X is DRP-ed to Y and different, Y cannot be DRP-ed to X

i.e. If X is purchased when Y is affordable $[(\boldsymbol{p} \cdot \boldsymbol{x}) \geq (\boldsymbol{p} \cdot \boldsymbol{y})]$ then when Y is purchased X must be unaffordable $[(\boldsymbol{p}' \cdot \boldsymbol{y}) \ngeq (\boldsymbol{p}' \cdot \boldsymbol{x})]$

Necessary but insufficient for rational & maximising behaviour (tests reflexivity)

Test for WARP:

- Use IRL observations to construct table (A, B, C) working out how much each cost given observed prices (a, b, c)
- Underline bundles (s, s) [these were chosen and hence optimal]; put a * if $(s, t) < (s, s)$ [these have to be affordable]
- If $(s, t) < (s, s)$ and $(t, s) < (s, s)$ then WARP is broken [e.g. @a A chosen when B affordable @b B chosen when A affordable]

	p_1	p_2	x_1	x_2
A	1	2	1	2
B	2	1	2	1
C	1	1	2	2

	A	B	C
a	<u>5</u>	4*	6
b	4*	<u>5</u>	6
c	3*	3*	<u>4</u>

Strong Axiom of Revealed Preference: If X is DRP-ed to Y and different, Y cannot be DRP-ed/inDRP-ed to Y

Necessary and sufficient test for rational & maximising behaviour (tests reflexivity and transitivity)

["If observed choices satisfy SARP, there exist well-behaved preferences that could have generated them"]

Test for SARP):

- Repeat same steps as with WARP. Infer indirectly revealed preferences [e.g. $A \succ B, B \succ C \therefore A \succ C$]. Mark these with (*).
- If $(s, t))$ and (t, s) are * or (*), SARP is broken

	A	B	C
a	<u>20</u>	10*	22(*)
b	21	<u>20</u>	15*
c	12	15	<u>10</u>

08. Slutsky Equation

$\underset{(?)}{\Delta x_i} = \underset{(-)}{\Delta x_i^s} - \underset{(?)}{\Delta x_i^m}$ or $\underset{(?)}{\frac{\Delta x_i}{\Delta p_i}} = \underset{(-)}{\frac{\Delta x_i^s}{\Delta p_i}} - \underset{(?)}{\frac{\Delta x_i^m}{\Delta m}} x_i$ (derived by dividing by Δp_i and using identiy $\Delta m = x_i \Delta p_i$)

Total: $\Delta x_i = x_i(p'_i, m) - x_i(p_i, m)$
Sub: $\Delta x_i^s = x_i(p'_i, m') - x_i(p'_i, m)$ [where m' is abstract compensated income]
Income: $-\Delta x_i^m = x_i(p'_i, m) - x_i(p'_i, m')$

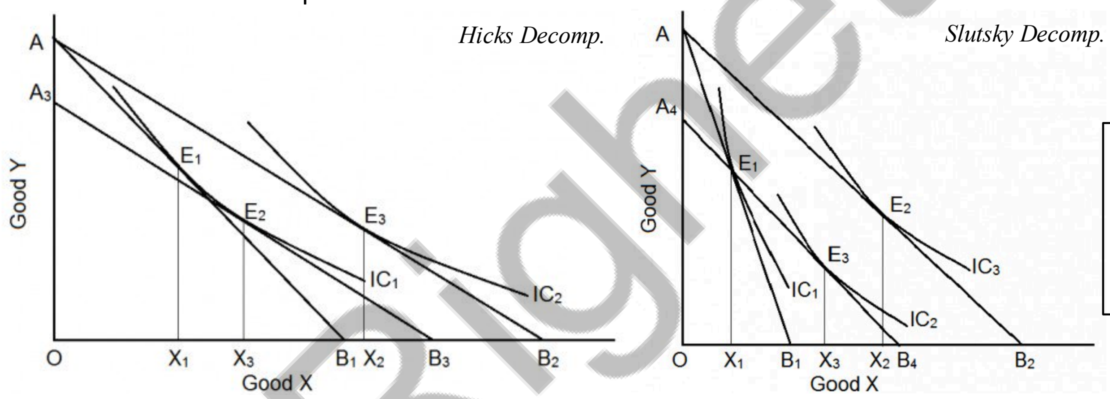
Sign of effects and properties of goods can be crucial for insights e.g. ordinary/giffen shows if x_i^s or x_i^m bigger
Special cases: Perfect Comp.: 0 sub-effect | Perfect Sub.: 0 income effect | Quasi-linear: 0 income-effect

Slutsky Decomposition (purchasing power)

Substitution Effect: holds purchasing power constant and changes relative prices i.e. pivots around X so it has same gradient as the new budget constraint
Income Effect: changes purchasing power and holds new relative prices constant i.e. new budget line shifts so it intersects with the new optimal choice Y

Hicks Decomposition (utility)

Substitution Effect: holds utility constant and changes relative prices i.e. pivots around IC so it has the same gradient as the new budget constraint
Income Effect: changes purchasing power and holds new relative prices constant i.e. budget line shifts so it intersects with the new optimal choice Y



Confusingly, Slutsky equation corresponds to Hicks decomp.

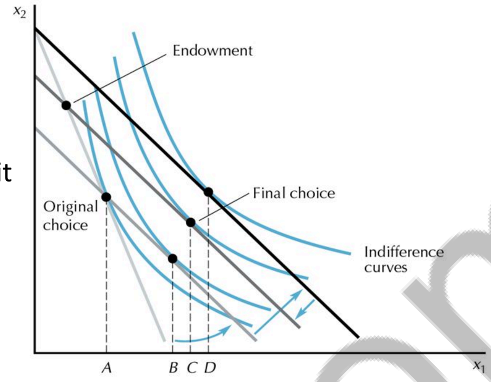
Duality [Non-Varian]

Marshallian Demand: $x(\boldsymbol{p}, m)$: observable demand. Keeps m fixed and changes $\boldsymbol{p}$
$\boldsymbol{x}^*$ comes from problem "$\max u^*(\boldsymbol{x}, \boldsymbol{p}, m)\ s.t.\ \boldsymbol{p} \cdot \boldsymbol{x} = m$"
Hicksian Demand: $h(p, \bar{u})$: abstract demand. Keeps $\bar{u}$ fixed [DM is compensated via Δm)] and changes $\boldsymbol{p}$
$\boldsymbol{h}$ comes from problem "$\min e(\boldsymbol{p}, u)\ s.t.\ u(\boldsymbol{h}) = \bar{u}$"

- If we sub in $\boldsymbol{h}$ into budget constraint [$\boldsymbol{p} \cdot \boldsymbol{x} = m$] we get $\boldsymbol{p} \cdot \boldsymbol{h}(\boldsymbol{p}, \bar{u}) = e(\boldsymbol{p}, \bar{u})$ [Sheppard's Lemma]
- If we let $u^*(\boldsymbol{x}) = \bar{u}$ then $e(\boldsymbol{p}, u^*) = m$. If less than m not u^*; if more then consumer can't afford it.
- Hence $\boldsymbol{h}(\boldsymbol{p}, u^*) = \boldsymbol{x}^*(\boldsymbol{p}, e(\boldsymbol{p}, u^*))$ which can be differentiated

$\frac{\partial h_i}{\partial p_i} = \frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial e} \times \frac{\partial e}{\partial p_i}$ [chain rule as p_i occurs in two places: $x_i(p_i, e(p_i, u))$]
$\frac{\partial h_i}{\partial p_i} = \frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial e} h_i$ [via Shephard's lemma]
$\frac{\partial h_i}{\partial p_i} = \frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial e} x_i$ [due to duality]
$\frac{\partial x_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial x_i}{\partial e} x_i$ [which is the Slutsky Equation]

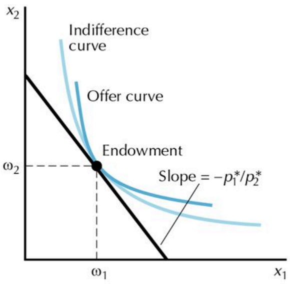
09. Endowment



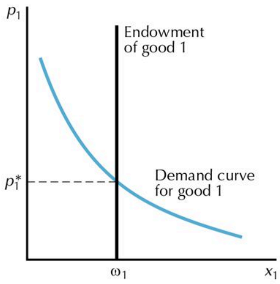
Endowment: (ω_1, ω_2): what consumer starts with
Gross Demand: (x_1, x_2): what consumer ends with
Net Demand: $(x_1 - \omega_1, x_2 - \omega_2)$: what consumer buys/sells
$p_1x_1 + p_2x_2 \equiv p_1\omega_1 + p_2\omega_2$; $\boldsymbol{\omega}$ always on BL; Δp causes pivot on it

$$\underset{(?)}{\frac{\Delta x_i}{\Delta p_i}} = \underset{(-)}{\frac{\Delta x_i^s}{\Delta p_i}} + (\omega_i - x_i)\underset{(?)}{\frac{\Delta x_i^m}{\Delta m}}$$

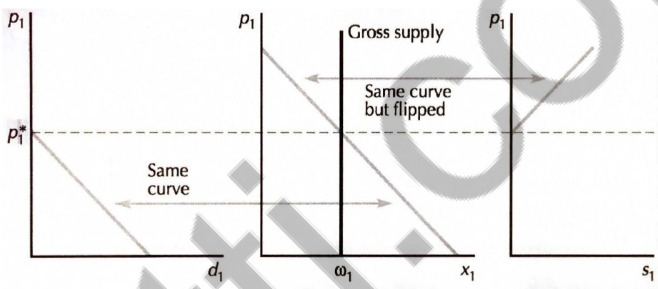
[EE is another shift in new BL so it intersects with $\boldsymbol{\omega}$]



A Offer curve



B Demand curve



A Net demand | B Gross demand | C Net supply

$d_1(p_1, p_2) = \begin{cases} x_1 - \omega_1 \ if \ ve^+ \\ 0 \ if \ otherwise \end{cases}$ | $s_1(p_1, p_2) = \begin{cases} \omega_1 - x_1 \ if \ ve^+ \\ 0 \ if \ otherwise \end{cases}$

- Offer curve will always intersect with (ω_1, ω_2) [there must exist $\boldsymbol{p}$ so consumer chooses not to trade]
- More valuable bundle is always preferred (as bundle doesn't have to be consumed, can be traded)
- Use DRP graphs to infer if consumer is better/worse off and if behaviour changes; often indeterminate

Labour Supply Model

M: non-labour income | C: consumption | p: price of consumption | w: wage rate | L: labour supplied
$pC = M + wL$
$pC - wL = M$
$pC + w(\bar{L} - L) = M + wL$ [Assume max $L = \bar{L}$ (e.g. 24h a day) and add $w\bar{L}$ to both sides]
$pC + wR = M + w\bar{R}$ [Define Recreation as $R = \bar{L} - L$ and by def. $\bar{R} = \bar{L}$]
$pC + wR = p\bar{C} + w\bar{R}$ [Define endowment C as $\bar{C} = \frac{M}{p}$ and rearrange; w is opportunity cost (i.e. price) of R]
i.e. value of consumption plus leisure= value of endowment of consumption and time
[Note: w is opportunity cost (and hence price) of leisure]

- Assuming R is normal we get an ambiguous w-effect: $\underset{(?)}{\frac{\Delta R}{\Delta w}} = \underset{(-)}{\frac{\Delta R^s}{\Delta w}} - \underset{(+)}{(\bar{R} - R)}\underset{(+)}{\frac{\Delta R}{\Delta m}}$
- Backwards-bending labour supply assumes bigger SE for smaller R (so L increases) and vice versa
- By contrast, just increasing overtime pay has unambiguous decrease in R (so L increases) [DRP!]

Index Numbers and Price Indices

Paashe = Present, Laspeyres = Last

Index Numbers (q)

Passhe $q = \frac{p_1^t x_1^t + p_1^t x_1^t}{p_1^t x_1^b + p_1^t x_1^b}$
If P$q > 1$ DM better off under t. Else indet.

Laspeyres $q = \frac{p_1^b x_1^t + p_1^b x_1^t}{p_1^b x_1^b + p_1^b x_1^b}$
If L$q < 1$ DM worse off under t. Else indet.

[DRP: Could have consumed b-bundle at t but didn't]

Price Indices (p)

Passhe $p = \frac{p_1^t x_1^t + p_1^t x_1^t}{p_1^b x_1^t + p_1^b x_1^t}$
If P$p > M$ DM worse off under t. Else indet.

Laspeyres $p = \frac{p_1^t x_1^b + p_1^t x_1^b}{p_1^b x_1^b + p_1^b x_1^b}$
L$p < M$ DM better off under t. Else indet

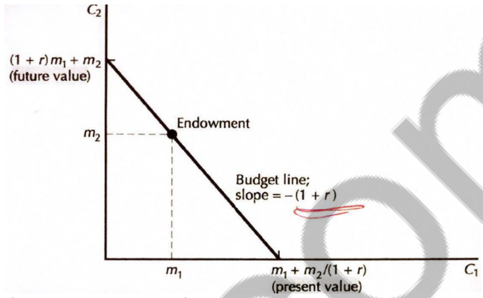
$M = \frac{p_1^t x_1^t + p_1^t x_1^t}{p_1^b x_1^b + p_1^b x_1^b}$ [Can't use 1 as p is different for numerator and denominator

10. Intertemporal

$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$ [Consumption: (c_1, c_2) | Income: (m_1, m_2) | Interest Rate: r]

Lender if $c_1 < m_1$, Borrower if $c_1 > m_1$, Polonius Point if $c_1 = m_1$ and $c_2 = m_2$

- Always Monotonic (i.e. prefers endowment with higher PV) as DM doesn't have to consume
- Convexity is natural (likely to want an "average" amount of c)
- Well-behaved pref. is reasonable (forgo some c_1 for more c_2)



Values

Real Interest Rate: $1 + \rho = \frac{1+r}{1+\pi}$ so $\rho \approx r - \pi$ for small π

Present Value $= \frac{\text{Future Value}}{1+r}$

P£ $[p_1 = 1, p_2 = \frac{1}{1+r}]$: $c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$

F£ $[p_1 = 1+r, p_2 = 1+\pi]$: $(1+r)c_1 + \frac{c_2}{1+\pi} = (1+r)m_1 + \frac{m_2}{1+\pi} \rightarrow \frac{1+r}{1+\pi}c_1 + c_2 = \frac{1+r}{1+\pi}m_1 + m_2$

If multiple time periods: PV: $c_1 + \frac{c_2}{(1+r_1)} + \frac{c_3}{(1+r_1)(1+r_2)} \ldots = m_1 + \frac{m_2}{(1+r_1)} + \frac{m_3}{(1+r_1)(1+r_2)} \ldots$

Net Present Value $= M_1 - P_1 + \frac{M_2 - P_2}{1+r}$ [income stream$=(M_1, M_2)$ and payments stream$=(P_1, P_2)$]

Comparative Statics

Use DRP, strict convexity (i.e. one solution) and Slutsky to see if welfare or behaviour changes

SLUTSKY EQUATION: raising r is like raising p_1, assume consumption is a normal good

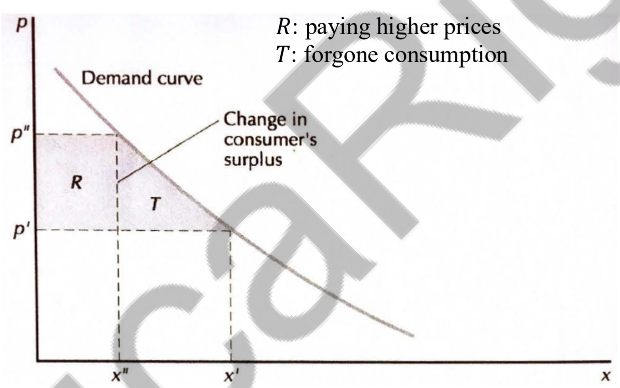
$$\underset{(?)}{\frac{\Delta c_1}{\Delta p_1}} = \underset{(-)}{\frac{\Delta {c_1}^s}{\Delta p_1}} - \underset{(?)}{(m_1 - c_1)}\underset{(+)}{\frac{\Delta c_1^m}{\Delta m}}$$

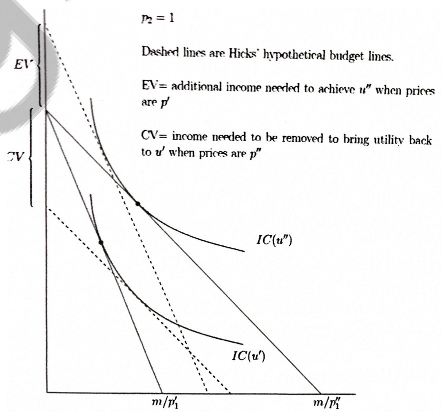
14. Consumer's Surplus

Consumer Surplus: max p willing to buy x^* units $-$ actual p [i.e. area under demand curve]

$\approx \int_0^x P(q)\partial q - xp \approx \int_0^p x(\boldsymbol{p}, m)\partial p - xp$ [approx. if divisible; exact if quasi-linear (no income effect)]

$\Delta CS = \int_{p''}^{p'} x^*(\boldsymbol{p}, m)\partial p$





Compensating Variation: Δm for consumer after Δp to make them have u' e.g. CO_2 tax offset by tax credit

Graphically: Shift new BL to make it tangent to old consumption point's IC

Algebraically: $e(p'', u') - e(p', u')$ [i.e. how much DM has to spend at new prices for old utility$-\, m$]

$CV = \int_{p''}^{p'} h(\boldsymbol{p}, u')\partial p$ [via Sheppard's Lema $\frac{\partial e}{\partial p_i} = h_i$]

Equivalent Variation: Δm for consumer before Δp to make them have u'' e.g. tax credit instead of subsidy

Graphically: Shift old BL to make it tangent to new consumption point's IC

Algebraically: $e(p'', u'') - e(p', u'')$ [i.e. $m -$ how much DM has to spend at old prices for new utility]

$EV = \int_{p''}^{p'} h(\boldsymbol{p}, u'')\partial p$ [via Sheppard's Lema $\frac{\partial e}{\partial p_i} = h_i$]

Use Slutsky Equation to see how CV, EV, and CS compare via $h()$ and $x()$ gradients.

15. Market Demand

Market Demand = Sum of Individual Demand: $X_i(\boldsymbol{p}, \boldsymbol{m}) = \sum_{a \in A} x_i^a(\boldsymbol{p}, m^a)$

- Assume market has well-behaved pref. Hence we can use demand function $X(\boldsymbol{p}, M)$ for entire market
- If all consumers are facing same prices for same goods, MRS must be the same at their optimal choices

Elasticity

Price Elasticity: $\frac{\left(\frac{\Delta x_i^*}{x_i^*}\right)}{\left(\frac{\Delta p_i}{p_i}\right)} = \frac{p_i}{x_i^*} \frac{\partial x_i^*}{\partial p_i} = \epsilon(p)$ [standardises units]

Elastic: $|\epsilon(p)| > 1$ | **Inelastic**: $|\epsilon(p)| < 1$ | **Unit Elastic**: $\epsilon(p) = -1$ [Unless luxury, $\epsilon \leq 0$]

Other: **Income Elasticity**: $\epsilon_m = \frac{m_i}{x_i^*} \frac{\partial x_i^*}{\partial m_i}$ | **Cross-Price Elasticity**: $\epsilon_{ij} = \frac{p_j}{x_i^*} \frac{\partial x_i^*}{\partial p_j}$

- **Constant Elasticity**: $x = Ap^{\epsilon}$ will have elasticity ϵ
- **Linear Elasticity**: $x = a - bp$ will have elasticity $\frac{-bp}{a-bp}$. Changes depending on where it is!

Marginal Revenue

Revenue: $R = pq$

Profit-max firm never produces on inelastic part of Demand Curve

Price	Quantity
$\frac{\partial R}{\partial p} = q + p\frac{\partial q}{\partial p} = q\left(1 + \frac{p}{q}\frac{\partial q}{\partial p}\right) = q(1 + \epsilon)$	$\frac{\partial R}{\partial q} = p + q\frac{\partial p}{\partial q} = p\left(1 + \frac{q}{p}\frac{\partial p}{\partial q}\right) = q\left(1 + \frac{1}{\epsilon}\right)$
=0 @max R. If inelastic, raising p raises R	=0 @max R. If inelastic, raising q (+costs) lowers R

➔ Raising p and lowering q would raise R and lowers costs

Marginal Revenue of x (if necessity) will always be less than Demand Curve

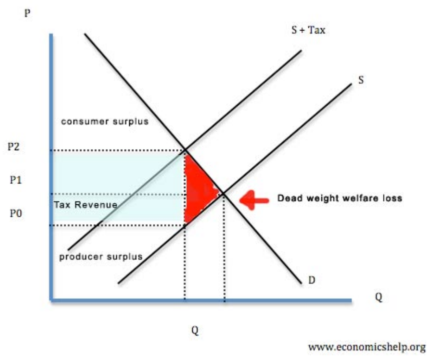
- $MR = \frac{\partial R}{\partial q} = p + q\frac{\partial p}{\partial q}$
- For any $q > 0, MR < p$ (as $\frac{\partial p}{\partial q}$ is ve⁻ unless a luxury good)
- i.e. to sell another unit good you have to lower the price

16. Equilibrium

Equilibrium $D(p^*) = S(p^*)$ or, the inverse, $P_S(q^*) = P_D(q^*)$

The more elastic (flatter) the supply curve, the more of a tax gets passed along to the consumer

Dead Weight Loss intuition: you can't tax what isn't there



19. Technology

Production Function: $f(x_1, x_2) = \bar{q}$

Marginal Product: $MP_1 = \frac{\partial f}{\partial x_1}$

Diminishing MP: $\frac{\partial^2 f}{\partial x_1{}^2} < 0$ [given one factor variable]

Returns to scale: $f(tx_1, tx_2) <=> tf(x_1, x_2)$ [given all factors variable]

Isoquants: ICs for production firms: $\{(x_1, x_2) \mid f(x_1, x_2) = q\}$

Technical-RS: MRS for production firms: $TRS_{1,2} = \frac{\Delta x_2}{\Delta x_1} - \frac{MP_1}{MP_2}$; $TRS = -\frac{w_1}{w_2}$ [Nec. but insuf. for non-corner]

Long Run: all factors/inputs are variables. Short Run: one or more factors/inputs may be fixed e.g. $\bar{x}_1$

Assumptions

- Monotonic: if you increase the amount of an inputs you will produce at least as much output as before
- Convex: if (x_1, x_2) and (y_1, y_2) produce q any weighted avg. will produce at least q as well. This is natural when the output has separate production processes

20. Profit Maximising & 21. Cost Minimisation

To maximise profits a firm must solve two problems…

1. $c(q) = \min \bar{w}L + \bar{r}K \; s.t. \, f(L, K) = \bar{q}$ i.e. Find cheapest input bundle for any level of output
2. $\pi^* = \max pq - c(q)$ i.e. Choose the optimum level of output q^*

- 1. can be done via Lagrange
- 2. can be solved in the long run as follows, assuming non-corner solution:

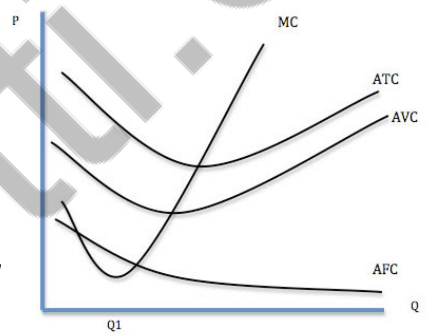
$$\frac{\partial \pi^*}{\partial q^*} = p - c'(q^*) = 0$$
$$p[MR] = c'(q^*)[MC]$$

- 2. can be solved in the short run as follows, assuming non-corner solution:

$$\frac{\partial \pi^*}{\partial x_i} = pq_i^* - c_i(q^*) = pMP_i - w_i$$
$$pMP_i[MR] = w_i[MC]$$
$MP_i = \frac{w_i}{p}$ [i.e. production function and isoprofit line are tangent]

[**Isoprofit** line: $\pi = pq - (w_1x_1 + w_2\bar{x}_2)$ can be rearranged as $q = \frac{w_1}{p}x_1 + \frac{w_2}{p}\bar{x}_2 + \frac{\pi}{p}$]

22. Cost Curves



Total Cost: $TC = c(q)$ [RTS: $c(tq) <=> tc(q)$ for all $t > 1$]

Average Cost: $AC = \frac{c(q)}{q} = \frac{c_v(q)+F}{q}$ [RTS: $\frac{c(q)}{q} <=> \frac{c(q+h)}{q+h}$]

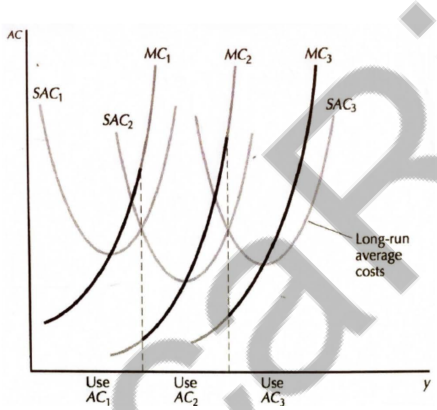
Marginal Cost: $MC = c'(q) = c'_v(q)$ [i.e. $\int MC = c_v(q)$]

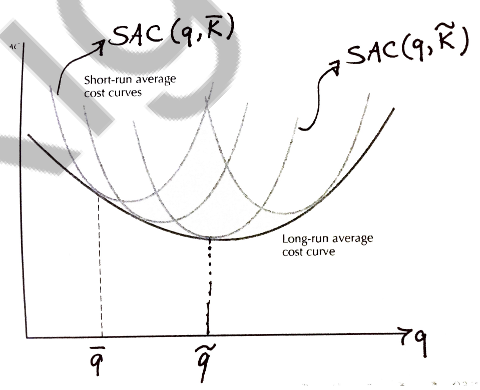
- AC or AVC attain min when intersecting with MC
- Multiple tech: $c(q) = cf(m) + cg(n) \; s.t. \, m + n = q \; and \; c'f = c'g$

SAC and LAC

$SAC(y, k = \bar{k}) \geq LAC(y)$. There likely exists some $SAC(\bar{y}, k = \bar{k}) = LAC(\bar{y})$

If there are semi-flexible options, AC = lower envelopes of SACs





23. Firm Supply [LR]

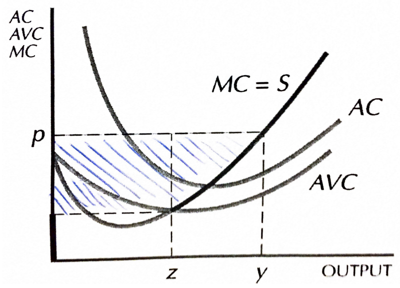
Competitive Firm: market price is independent of its own level of output i.e. price taker

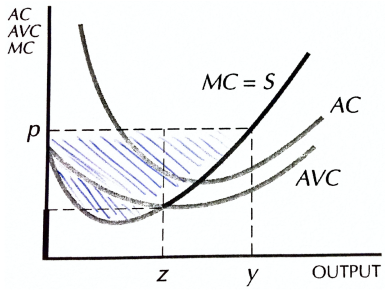
<u>Competitive firm produces along MC that has ve$^+$ slope and is above AVC line. Else produces nothing</u>.

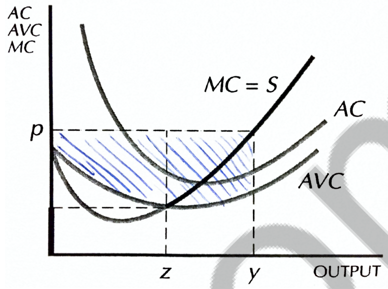
- NBNS for firm to supply when $\frac{\partial \pi}{\partial q} = 0$ (i.e. $MC = MR = p$) [Note: implies all active firms have same MC!]
- NBNS for firm to supply when $\frac{\partial^2 \pi}{\partial q^2} > 0$ (if MC ve$^-$ slope increasing q decreases AC and so increases π)
- **Shutdown Cost:** If $AVC > p$ then firm can't make profit in long run. Hence, for such values $q = 0$
- Optimum q^* or y^* is where area

Producer Surplus: actual $p - \min p$ willing to sell x^* units ; aka rent. Can be calculated in several ways…

$$y^*p^* - \int_0^{y^*} p(q)\partial q \text{ [Area left } SC] = y^*p^* - \int_0^{y^*} MC(q)\partial q \text{ [Area above } MC]$$
$$= y^*p^* - y^*AVC(y) \text{ [Revenue} - c_v]$$







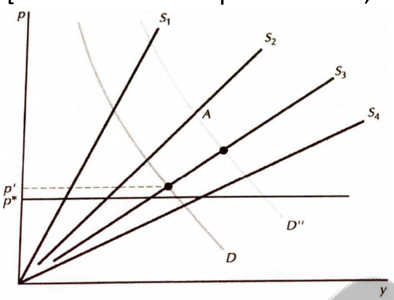
24. Industry Supply [LR]

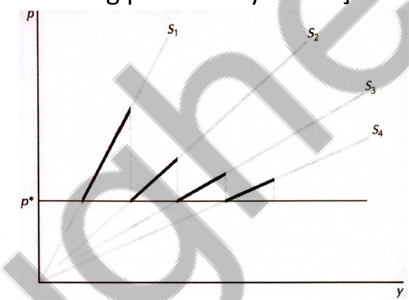
Market Supply = Sum of Individual Supply: $S_i(p) = \sum_{a \in A} x_i^a(p)$

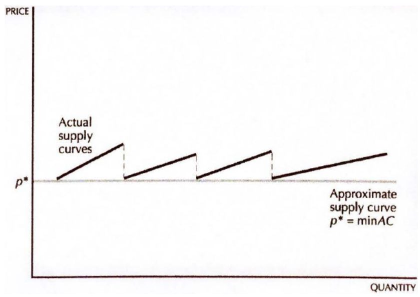
Competitive industry with free entry sees LR profits close to 0

- Firm exits if no LR profit. Hence (assuming no barriers to entry) p' is lowest intersection above p^*
- Firm enters if LR profit. Hence (assuming no barriers to entry) market will 'move along' S_i curves
- If n firms in market and then Δp causes $n\Delta y$ [i.e. S_i curve gets steeper as $i \to n$]
- Hence, S is approximately a straight line $= p^* = \min AC$
- **Mature industries:** $\pi = 0$; all factors paid their market price (i.e. opportunity cost). None exit/enter.

[NOTE: Economic profits are 0, but accounting profits may be ve+]

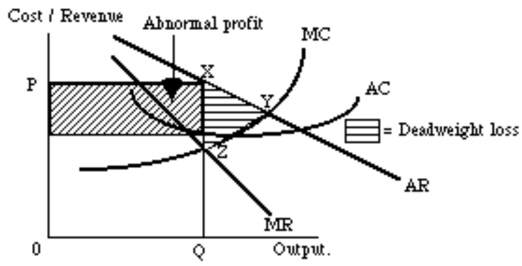






0-Profits may exist even if there is a Fixed-Factor/Barrier-To-Entry

Potential firms (PF) can enter market by "buying out" active firms (AF). AF's option of not "selling out" is their opportunity cost which must be the value of their rent!



25. Monopoly

Price Maker so $p \Rightarrow P(q)$ [i.e. inverse demand]. $P(q)q = R(q)$

q^* so $\max P(q)q - c(q) = \max R(q) - c(q)$

$R'(q^*)[MR] = c'(q^*)[MC]$

$MR(q) = R'(q) = \left(qP(q)\right)' = P(q) + qP'(q) = P(q)\left[1 + \frac{1}{\epsilon(q)}\right] = P(q)\left[1 - \frac{1}{|\epsilon(q)|}\right]$ (assuming necessity)

$|\epsilon(q)| > 1$ so $MR(q) < P(q)$ (see p10 for why firm will never produce where p is inelastic)

$MC(q^*) = P(q^*)\left[1 - \frac{1}{|\epsilon(q^*)|}\right] \therefore P(q^*) = \frac{MC(q^*)}{\left[1 - \frac{1}{|\epsilon(q^*)|}\right]}$ [i.e. mark-up> 1 as $|\epsilon(q)| > 1$]

Dead Weight Loss: potential surplus lost due to inefficiencies. For non-price discriminating monopoly, this is because $MC(q^*) = MR(q^*) < P(q^*)$ [i.e. some are willing to pay more than what it costs to produce]

TAXES

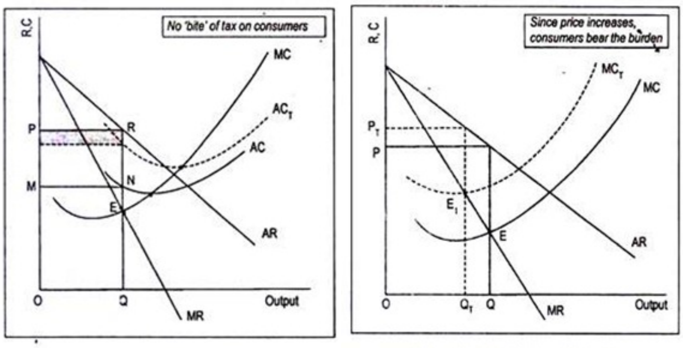
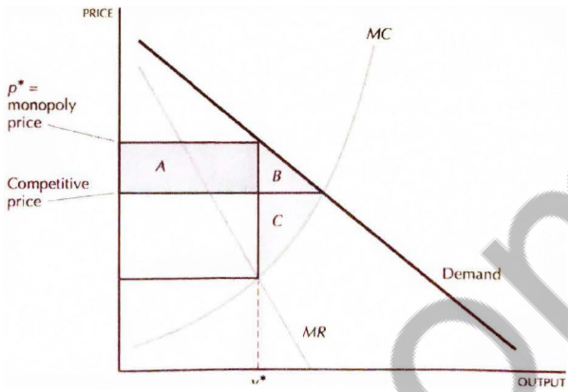


Fig. 5.8: Lump sum Tax and Monopoly

Fig. 5.9: Specific Sales Tax and Monopoly

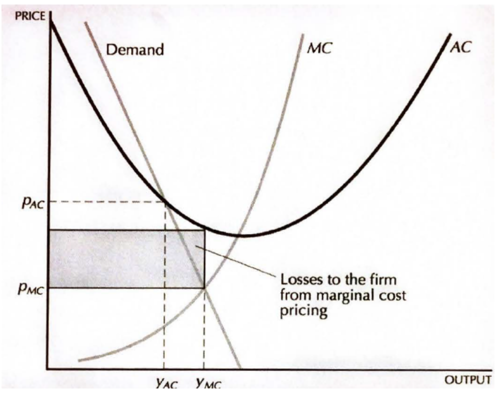
Unit tax increases DWL and worse for consumer;
Lump tax lowers profit but no behaviour change

N-PC Monopoly vs. Comp. Market



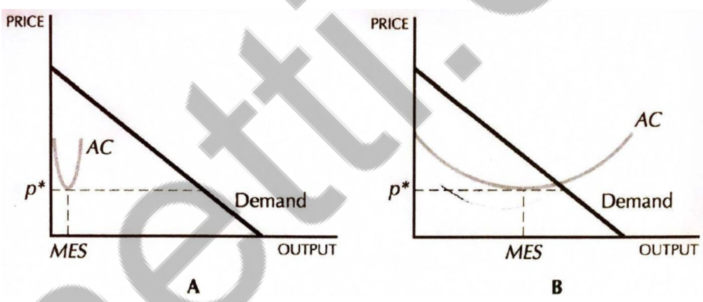
A=PS gained; C=PS lostl; A+B=CS Lost

Natural Monopolies



Producing at 'efficient outcome' is unprofitable as high fixed costs mean $D(\hat{p}) = MC(\hat{q}) < AC(\hat{q})$ (e.g. L-shaped LAC)

Causes of Monopolies



1. **Minimum-efficient-scale** (q *where* $AC(q)' = 0$) is large relevant to $D(q)$ i.e. pays to be big (see Natural Monopolies)
2. 'First mover' advantage means firm can price-out competitors
3. Firms collude

26. Monopoly Behaviour

Price Discrimination

First Degree Price Discrimination: sells each unit on a 'take it or leave it' p i.e. at reservation price (e.g. unrealistic). Pareto efficient and $TS = PS$

Third Degree Price Discrimination: different prices for different types of consumers aka non-linear pricing (e.g. Student Discount)

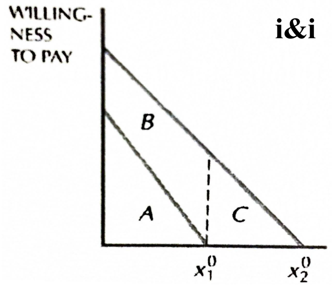
- For $\max_q P_1(q_1)q_1 + \dots + P_n(q_n)q_n$, by def. $MC = MR_i(q_i^*) = P_i(q_i^*)q_i^* = P_i(q_i^*)\left[1 - \frac{1}{|\varepsilon_i(q_i)|}\right]$
- Hence, the less elastic the demand in market i, the higher the price

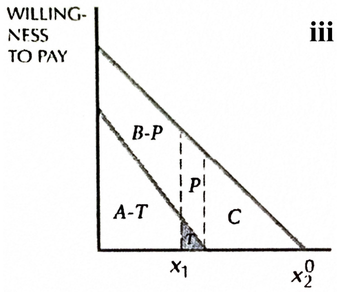
Finding Optimal PD

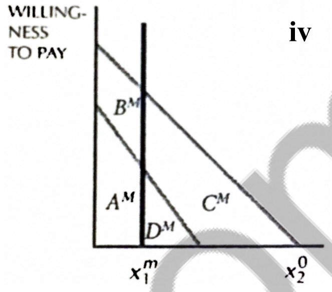
1. Construct inverse demand function $D_i(p_i) \Rightarrow P_i(q_i) = f(Dorq)$
2. Multiply by q to get revenue function $R_i(q_i) = f(Dorq)q$
3. Solve simultaneous equations $R_i(q_i^*) = MC$ to get $q_i{}^*$
4. Substitute back into $P_i(q_i^*)$ to get $p_i{}^*$
5. Calculate Profit
6. Check this isn't higher than if simple p^*, q^* were chosen
   i. $D(p) = D_1(p_1) + \dots + D_n(p_n) \Rightarrow P(q) \Rightarrow R(q)$
   ii. Repeat steps 3., 4., and 5.

Second Degree Price Discrimination: different prices for different units of output (e.g. Buy 1 Get 1 Free)

- IRL High-willingness-to-pay can pretend to be low-willingness-to-pay so FDPC doesn't work
- So, monopoly creates price-quantity packages incentivising consumers to self-select… (assume $MC = 0$)





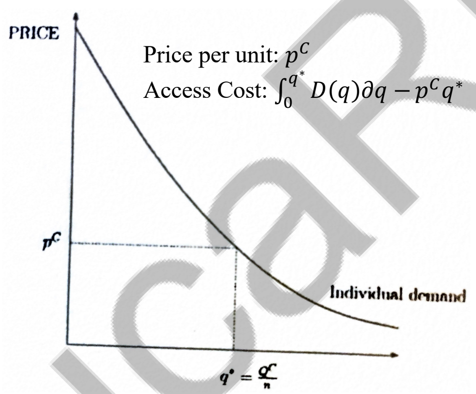


i. FDPC: Monopolist offers x_1^0 at price A and x_2^0 at price $A + B + C$.
- Customer I_1 chooses x_1^0 (gets CS$=0$)
- Customer I_2 chooses x_1^0 (gets CS$=B$ instead of CS$=0$)
- IRL Monopolist sells $2x_1^0$ so gets PS$=2A$. Inefficient.

ii. SDPC: Monopolist offers x_1^0 at price A and x_2^0 at a price $A + C$ (i.e. gets I_2 to switch by offering 'cheaper' bundle x_1)
- Customer I_1 chooses x_1^0 (gets CS$=0$)
- Customer I_2 chooses x_2^0 (gets CS$=B$)
- IRL Monopolist sells $x_1^0 + x_2^0$ so gets PS$=2A + C$. Less inefficient.

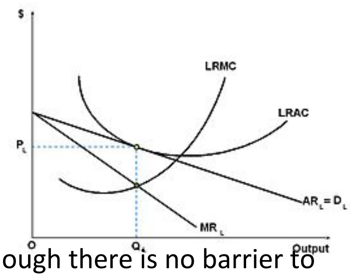
iii. SDPC: Monopolist offers $x_1 = x_1^0 - h$ at price $A - T$ and x_2^0 at a price $A + P + T + C$ (i.e. gets I_2 to switch by offering smaller bundle x_1 and hence being able to offer 'pricier' bundle x_2)
- Customer I_1 chooses x_1 (gets CS$=0$)
- Customer I_2 chooses x_2^0 (gets CS$=B - P$)
- IRL Monopolist sells $x_1 + x_2^0$ so gets PS$=2A - T + P + C$. Even less inefficient (assuming $P > T$).

iv. SDPC: Monopolist optimises so x_1^m at price A^M (so to max $P - T$) and x_2^0 at a price $A^M + D^M + C^M$
- Customer I_1 chooses x_1^m (gets CS$=0$)
- Customer I_2 chooses x_2^0 (gets CS$=B^M$)
- IRL Monopolist $x_1^m + x_2^0$gets PS$=2A^M + D^M + C^M$. Efficient.



Quality-differentiation: akin to SDPC
Two-part tariffs: akin to FDPC; sets a fixed 'access cost' then a 'price per unit' to extract all surplus
Bundling: selling products together to extract profit e.g. \$200 seperate but \$220 bundle

	Word	Excel
I_1	\$120	\$100
I_2	\$100	\$120



Monopolistic Competition

Monopolistic Competition: Each firm has some degree of market power as though there is no barrier to entry firms can differentiate themselves through different brands (substitutes to some degree of each other)

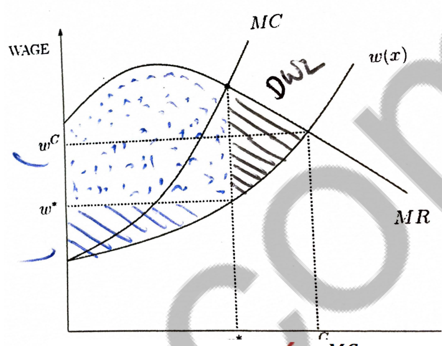
- Each firm faces a downward-sloping demand curve (has some market power)
- Profit in SR and so new competitors enter market. Demand curve falls until tangent to LRAC
- No-profit in LR but still not Pareto-efficient

27. Factor Markets

Monopsony

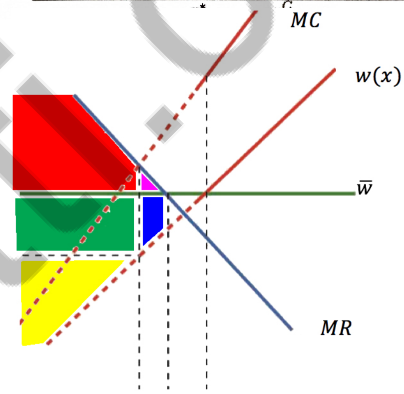
Monopsony: firm is the sole buyer of a factor supplied by many sellers in a market and hence it affects the price of its inputs by choosing demand [i.e. supply curve: $w = w(x)$]

- $\max_x pf(x) - w(x)x$ so $f'(x^*)[MR] = w'(x^*)x + w(x^*)[MC]$
- assume $w(x)$ ve⁺ slope [$w'(x) > 0$] so $MC = w'(x)x + w(x) > w(x)$ [i.e. MC above supply curve]
- Production functions $f(x)$ are concave for large x [i.e. $f''(x) < 0$ diminishing marginal return to x] so…
- $MR = pf'(x)$ is eventually decreasing
- Equilibrium sees underemployment and discounted wages



Minimum wage ($\overline{w}$)

- New MC is $\overline{w}$ until it intersects with $w^C(x)$ then same as before
- If $\overline{w} < MR(x^*)$ it increases wage and employment (less DWL)
- Need to work out new surpluses (TS↑, WS↑, PS↓)
- If $\overline{w} > MR(x^*)$ it increases wage and <u>un</u>employment (more DWL)



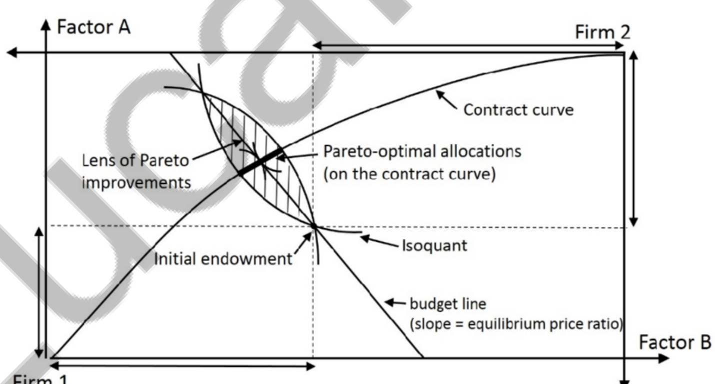
Segment	Change
	Existing PS
	Increase in PS (producing more goods)
	Decrease in PS (higher cost of wages)
	Increase in WS (employing more)
	Existing WS

When comparing utilities we must now assume they are cardinal!

32. Exchange

α is a **Pareto-improvement** over β if: Nobody strictly prefers β to α but at least one agent strictly prefers α
α is **Pareto-efficient** if: There is no feasible state of affairs which is a Pareto-improvement over α

- If an interior solution, Pareto-efficiency requires $MRS_A = MRS_B$, else a Pareto-improvement can be made via trade. Graphically this means that A's and B's indifference curves are tangent to each other



Edgeworth Box

- The set of all Pareto-efficient points form the contract curve
- A point off the contract curve will have two indifference curves forming an enclosed lens
- Any point in this lens is a Pareto improvement
- In bargaining model, A and B will negotiate to a point on the contract curve in the lens
-At equilibrium, each agent chooses the same point

Central Planner in Exchange Economy

- Central planner would simply choose an allocation along contract curve, such that $MRS_A = MRS_B$

Partial Equilibrium

1. Demand = Supply
2. $x^1{}_A(\boldsymbol{p}^*) + x^1{}_B(\boldsymbol{p}^*) = \omega^1{}_A + \omega^1{}_B$
3. $x^1{}_A(\boldsymbol{p}^*) - \omega^1{}_A + x^1{}_B(\boldsymbol{p}^*) - \omega^1{}_B = 0$
4. $e^1{}_A(\boldsymbol{p}^*) + e^1{}_B(\boldsymbol{p}^*) = 0$
5. $z^1(\boldsymbol{p}^*) = 0$ (Repeat for each market)
6. This is an unnecessarily strong condition…

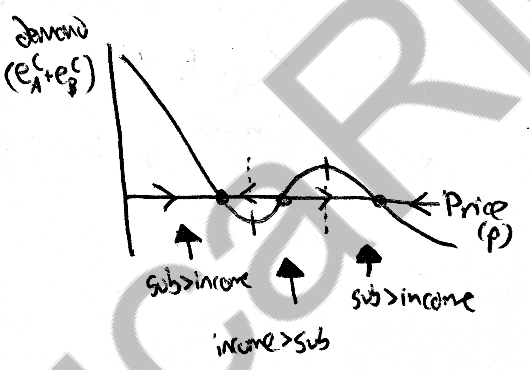
Walras' Law

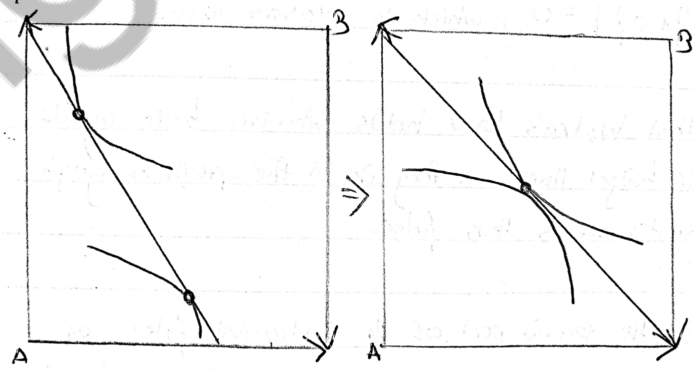
1. Recall budget constraint: $p^1x^1{}_A(\boldsymbol{p}^*) + p^2x^2{}_A(\boldsymbol{p}^*) \equiv p^1\omega^1{}_A + p^2\omega^2{}_A$
2. Rearrange to: $p^1e^1{}_A(\boldsymbol{p}^*) + p^2e^2{}_B(\boldsymbol{p}^*) \equiv 0$
3. Sum to: $p^1\left(e^1{}_A(\boldsymbol{p}^*) + e^1{}_B(\boldsymbol{p}^*)\right) + p^2\left(e^2{}_A(\boldsymbol{p}^*) + e^2{}_B(\boldsymbol{p}^*)\right) \equiv 0$
4. Hence: $p^1z^1(\boldsymbol{p}^*) + p^2z^2(\boldsymbol{p}^*) \equiv 0$
5. If $p^1z^1(\boldsymbol{p}^*) = 0$, then $p^2z^2(\boldsymbol{p}^*) = 0$ by identity!

- Generally, "if there are k markets and $k-1$ clear, then the last markets must also be in equilibrium"
- Hence, if we have $k-1$ independent equations in a k-good general equilibrium model, we can net one price equal to a constant and then solve. The intuition behind this is that only relative prices matter

Competitive Equilibrium in Exchange Economy

- Assume A and B are price takers, with a Walrasian auctioneer announcing a pair of prices (mimics perfect competition whilst maintaining a two-agent model)
- Given a price pair (p^1, p^2), an agent chooses their most preferred bundle on budget line e.g. $(x^1{}_A, x^2{}_A)$.
- There is no guarantee that the market will clear for any good (i.e. $e^1{}_A + e^1{}_B = z^1 \not\equiv 0$)
- But prices will adjust, via the auctioneer, until they do.
  - *Guaranteed Existence*: If aggregate excess demand (z^1) changes continuously with price (p^1). If $p^1$is high $z^1 < 0$, if p^1 is low $z^1 > 0$, hence there must exist p^1 such that $z^1 = 0$.
    - This in turn either requires continuous individual demand functions (i.e. convex preferences) or that discontinuous demand behaviour of the consumer is small relative to the market
  - *No Uniqueness:* There can be multiple equilibria but (unless just touching case) it must be odd
- At equilibrium, A and B select a bundle at which their IC is tangent to budget line. As they face the same budget line (i.e. same relative prices), they must also be tangent to each other!
- <u>This gives us equation</u> $MRS_A = MRS_B = \frac{p^1}{p^2}$, akin to central planner!





Exam Question (narrate your workings!)

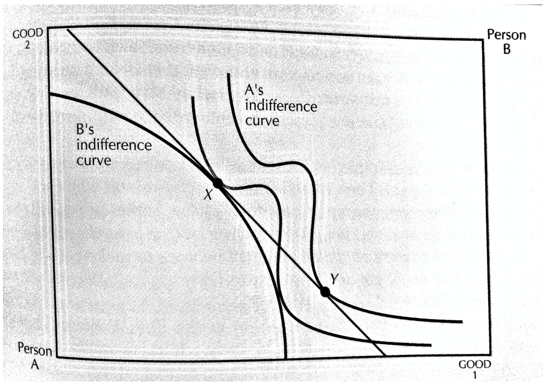
1. $\max U_A$ $s.t.$ budget constraint via Lagrange derive demand function (e.g. $x^1{}_A$). Can use CD shortcut $(x^1 = \frac{am}{p^1})$ after solving generally once.
2. Derive excess demand function $(e^1{}_A = x^1{}_A - \omega^1{}_A)$
3. Solve via market clearing condition $(e^1{}_A + e^1{}_B = z^1 = 0)$

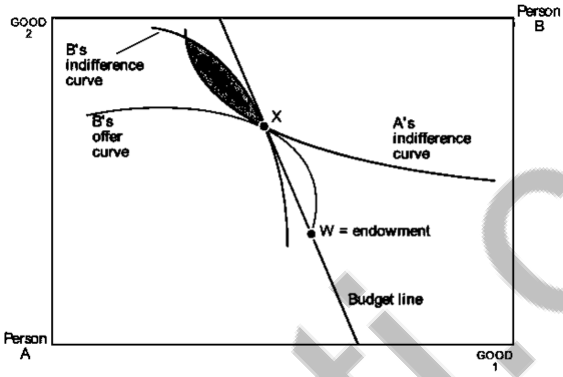
First Welfare Theorem

"A competitive equilibrium is Pareto efficient assuming complete markets (no externalities), price-taking behaviour (face same relative prices), non-satiation of preferences, and that CE exists (continuous z^1)"

The Monopoly Case

- If A is a (non-price discriminating) monopolist it will set their IC tangent to B's offer curve. We then just draw a price line connecting this equilibrium point (X) to the endowment (ω).
- This is not Pareto Efficient. Intuition: A would like to sell more but can only do so by lowering the price at which they are currently sold, making her overall worse off
- If A could perfectly price discriminate, then the allocation would move along the IC, not the price line, and hence this wouldn't be a problem .





Second Welfare Theorem

"Any Pareto efficient allocation can be attained via competitive equilibrium given a corresponding initial endowment, assuming preferences and production sets are convex and the FWT holds".

Criticisms

- Non-convexity of preferences: Especially relevant when concerning non-divisible and small units (e.g. a zoo may prefer a lion or a zebra instead of half a lion and half a zebra)
- Non convexity of production sets: If there are economies of scale or increasing returns to scale, a firm will always find it profitable to expand production and so equilibrium is never attained.
- Infeasible lump sum taxation: A government has to tax endowments so it doesn't distort prices but no such tax has ever been discovered. Requires an omnipotent state, and if it were so it may as well be a central planner.

33a. Production: Crusoe

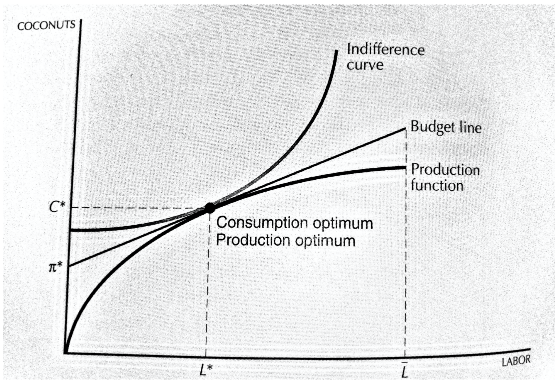
- Crusoe is the sole owner of a firm, looking to maximize profits, and its sole employee, looking to maximize utility. We must now solve two problems.

Central Planner in Crusoe Economy

- Solve $\max u \; s.t. \, c = f(L) \; [\mathcal{L}]$
- Rearrange to attain $MRS = MRT (= MPL)$
- Intuition: A central planner chooses the point where the indifference curve and production function are tangent i.e. $MRS = MRT$.

Competitive Equilibrium in Crusoe Economy

- Firm is bounded by equation $\pi = pc - wL$, which gives us isoprofit line $c = \frac{\pi}{p} + \frac{w}{p} L$
- A firm wants to get onto the highest possible isoprofit line within feasible production set.
- Firm will hence choose point where isoprofit line is tangent to production function i.e. $\frac{w}{p} = MRT = MPL$
- Note that the intercept of isoprofit line is $\frac{\pi}{p}$ (i.e. profit measured in units of product)
- Consumer is bounded by equation $pc \leq m = \pi + wL$, which gives use the same isoprofit line!
- Consumer will hence choose point where isoprofit line is tangent to indifference curve i.e. $\frac{w}{p} = MRS$
- Together, this gives us the equation $MRS = \frac{w}{p} = MRT (= MPL)$, akin to central planner!



[Note that Walras' Law still holds]
Via budget constraints: $\pi = pc^s - wL^d$; $pc^d = \pi + wL^s$
Combine and rearrange: $p(c^d - c^s) + w(L^d - L^s) \equiv 0$

Exam Question

Firm's Problem

1. Solve $\max \pi = pc - wL \; s.t. c \leq f(L) \; [\mathcal{L}]$ or $\max \pi = pc - wL \; substituting \; c = f(L) \left[\frac{d\pi}{dL} = 0\right]$
   (Remember that a firm can always make 0 profit. This feed through to L choices!)
2. Use this to derive optimal (L^d, c^s) and π^*

Consumer's Problem

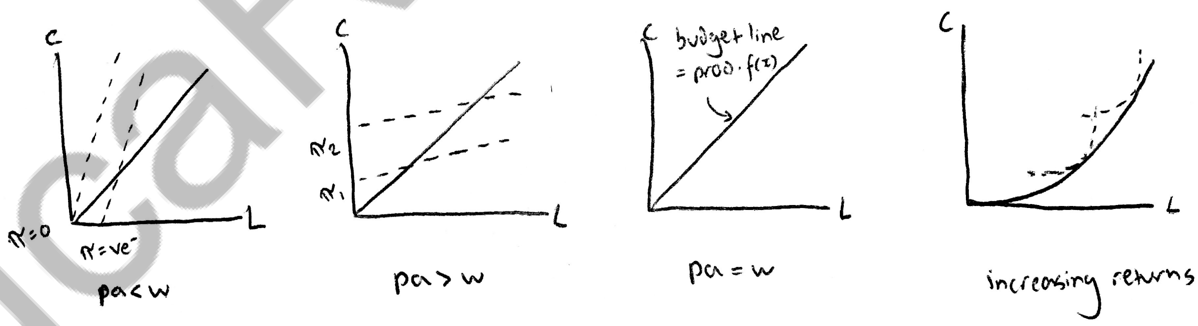
1. $\max u(c, T - L) \; s.t. c \leq \frac{\pi^*}{p} + \frac{w}{p} L \; [\mathcal{L}]$
2. Use this to derive optimal (L^s, c^d)

Putting it together

- Use market clearing conditions to find values $(L^s = L^d; c^s = c^d)$
- $|MRT| = |MPL| = f'(L^*) = \frac{w}{p}$

Different Technologies

- Decreasing returns to labour: Everything works
- Constant returns to Labour: $f(L) = \alpha L$:
  - If $p\alpha < w$, $L = 0$ is optimal but this contradicts well-behaved-preferences (i.e. starve to death)
  - If $p\alpha > w$, increasing L always increases π. Hence no equilibrium
  - Thus must be $p\alpha = w$, giving **zero-profit-condition**. Hence know $\frac{w}{p} = \alpha$, w/out u-function
- Increasing returns to labour: Increasing L always increases π. Hence no equilibrium

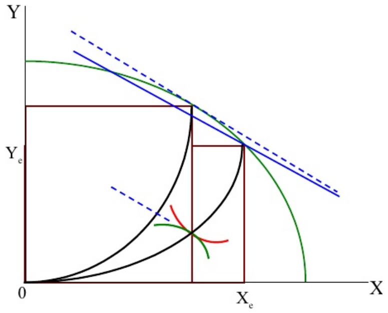


33b. Production: Castaways Inc.

Central Planner in Castaways Inc. Economy

- If we talk about more than one agent working for the firm, Pareto-efficiency becomes more complex and can be broken down into three components. For simplicity we assume labour supply is inelastic
  - Productive efficiency: $MRT_i = k$ (firms produce on the PPF)
  - Efficiency in exchange: $MRS_i = k$ (goods go to the consumer who most value them)

  - Allocative efficiency: $MRS_i = MRT$ (firms produce the goods that consumers most want)
- Even if $MRS_A = MRS_B$ and firms produce on the PPF, unless $MRS_i = MRT$, the outcome is not Pareto efficient. This is because we can also "exchange" goods via production, a Pareto improvement.



Competitive Equilibrium in Castaways Inc. Economy

- Firm is bounded by equation $\pi = p_c c + p_y y - wL$, which gives us isoprofit line $c = \frac{\pi}{p_c} + \frac{w}{p_c} L - \frac{p_y}{p_c} Y$
- A firm wants to get onto the highest possible isoprofit line that touches the PPF
- Firm will hence choose point where isoprofit line is tangent to PPF i.e. $MRT = -\frac{p_y}{p_c}$
- Consumers will choose consumption such that $MRS = -\frac{p_y}{p_c}$
- Together, this gives us the equation $MRS = -\frac{p_y}{p_c} = MRT$, akin to central planner!
- The intuition is the same as before: Firms look to price signals as they profit maximize, consumer as they utility maximize. If everyone faces the same price ratio we get the above equation.

The Multiple Firms Case

- There must be a single wage, else one firm would get all L, leaving the other with excess demand for L
  - A yam firm solves $\max \pi = p_y f_y(L_y) - wL_y \Rightarrow f_y'(L_y) = \frac{w}{p_y}$
  - A coconut firm solves $\max \pi = p_c f_c(L_c) - wL_c \Rightarrow f_c'(L_c) = \frac{w}{p_c}$
- Compare this to a single firm that produces both: $\max \pi = p_y f_y(L_y) + p_c f_c(L_c) - wL_y - wL_c$
  - $\pi_y = p_y f_y'(L_y) - w \Rightarrow f_y'(L_y) = \frac{w}{p_y}$
  - $\pi_c = p_c f_c'(L_c) - w \Rightarrow f_c'(L_c) = \frac{w}{p_c}$
- The equations, and hence the outcome, is identical to above!

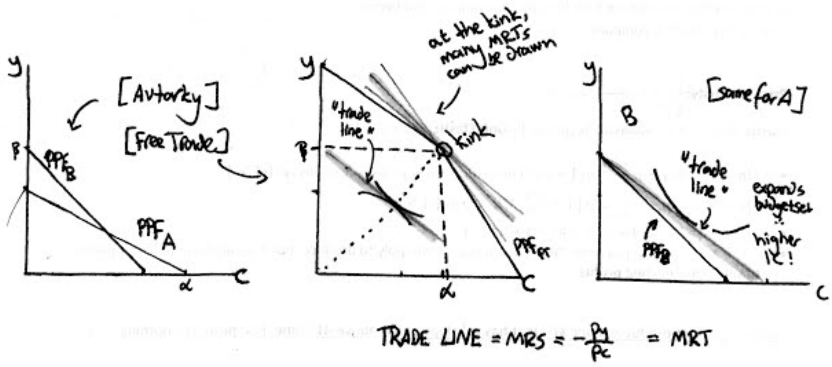
33c. Production: Ricardian Trade

- If we talk about more than one agent working for different firms, we get a basic model for trade. Again, for simplicity, we assume labour supply is inelastic. Also assume constant returns (i.e. $\boldsymbol{f_c{}^A = \alpha_c{}^A L_c}$) so zero-profit-condition!
- **Comparative advantage:** If an agent can produce a good at a lower opportunity cost compared to another country/agent (e.g. $\frac{f_c{}^A}{f_y{}^A} > \frac{f_c{}^B}{f_y{}^B}$ hence $\frac{\alpha_c{}^A}{\alpha_y{}^A} > \frac{\alpha_c{}^B}{\alpha_y{}^B}$). Irrespective of absolute advantage.

Central Planner in Ricardian Trade

- Under autarky we get two PPFs with one slope each. Under trade we get a one PPF with two slopes
- The kink is the point of complete specialisation (each agent is only producing where they have a comparative advantage).
- <u>Intuition</u>: Trade introduces an extra production possibility. It leads to at least the same amount of both goods, expanding the budget set of both agents. Hence it must be a Pareto efficient.

- Graphically: An allocation is efficient if $MRS_i = MRT$. Note that at the kink we can draw many different MRTs. The price line is now called the trade line. (Note also that we can get a Pareto efficient point with only partial specialisation)



Competitive Equilibrium in Ricardian Trade

- Note that the price for goods must be the same in both countries, else no producer would sell them in the lower-priced market. But, assuming barriers to migration, nothing says wages must be equal.
- Let A have a comparative advantage in c, that is $\frac{\alpha_c{}^A}{\alpha_y{}^A} > \frac{\alpha_c{}^B}{\alpha_y{}^B}$. There are two possibilities for the price ratio

1. $\frac{\alpha_c{}^A}{\alpha_y{}^A} > \frac{\alpha_c{}^B}{\alpha_y{}^B} > \frac{p_y}{p_c}$ or $\frac{p_y}{p_c} > \frac{\alpha_c{}^A}{\alpha_y{}^A} > \frac{\alpha_c{}^B}{\alpha_y{}^B}$ [can treat these as the same]
   - Can rewrite as $p_c\boldsymbol{\alpha}_c{}^A > p_y\boldsymbol{\alpha}_y{}^A$ and $p_c\boldsymbol{\alpha}_c{}^B > p_y\boldsymbol{\alpha}_y{}^B$. In both countries the revenue from 1 unit of labour in y is strictly less than in c. Hence no y produced.
   - This is incompatible with equilibrium (i.e. well behaved preferences)
2. $\frac{\alpha_c{}^A}{\alpha_y{}^A} > \frac{p_y}{p_c} > \frac{\alpha_c{}^B}{\alpha_y{}^B}$
   - Can rewrite as $p_c\boldsymbol{\alpha}_c{}^A - w^A > p_y\boldsymbol{\alpha}_y{}^A - w^A$. As max profit in CR is 0, left side must be 0 and right side is negative. Hence A will produce only c (with $p_c\boldsymbol{\alpha}_c{}^A = w^A$).
   - Recall budget constraint $p_c c + p_y y = w^A = p_c\boldsymbol{\alpha}_c{}^A$. Hence get trade line $c = \frac{p_y}{p_c}(\boldsymbol{\alpha}_c{}^A - y)$. Doing the same for B we get $c = \frac{p_y}{p_c}(\boldsymbol{\alpha}_y{}^A - y)$. Both have MRS of $-\frac{p_y}{p_c}$, which is also MRT! Same as Central Planner solving The Multiple Firms Case.
   - Note that wage in A adjusts so that profits in c are zero and in y are negative, similarly for B. Hence there will be wage inequality: If $\boldsymbol{\alpha}_y{}^B < \boldsymbol{\alpha}_y{}^A$ then $w^B < w^A$

Exam Question

1. Use zero-profit condition to find wage in terms of prices.
2. Firms $\max \pi$: Recall zero-profit-condition under constant returns. Substitute into budget constraint.
3. Consumer $\max u$: Solve subject to budgets constraint [$\mathcal{L}$] (normalise a price if need to).
4. Markets Clear: Derive demand functions for both agents and use these to find market clearing values.

35a. Externalities: Basics

Consumption externality: If one agent's utility function depends on another agent's consumption
Production externality: If one firm's cost/production function depends on that of another firm
Worked Example

- A chooses a and x to $\max p_a a - c_a(a,x)$; B chooses b to $\max p_b b - c_b(b,x)$
- FOC (assuming interior sol.) tells us $MR = MC$ so $p_a = \frac{\partial c_a}{\partial a}$ and $p_b = \frac{\partial c_b}{\partial b}$.
- Note $0 = \frac{\partial c_a}{\partial x}$, that is price of polluting for A is zero. This is not Pareto efficient as a DWL exists!

Central Planner

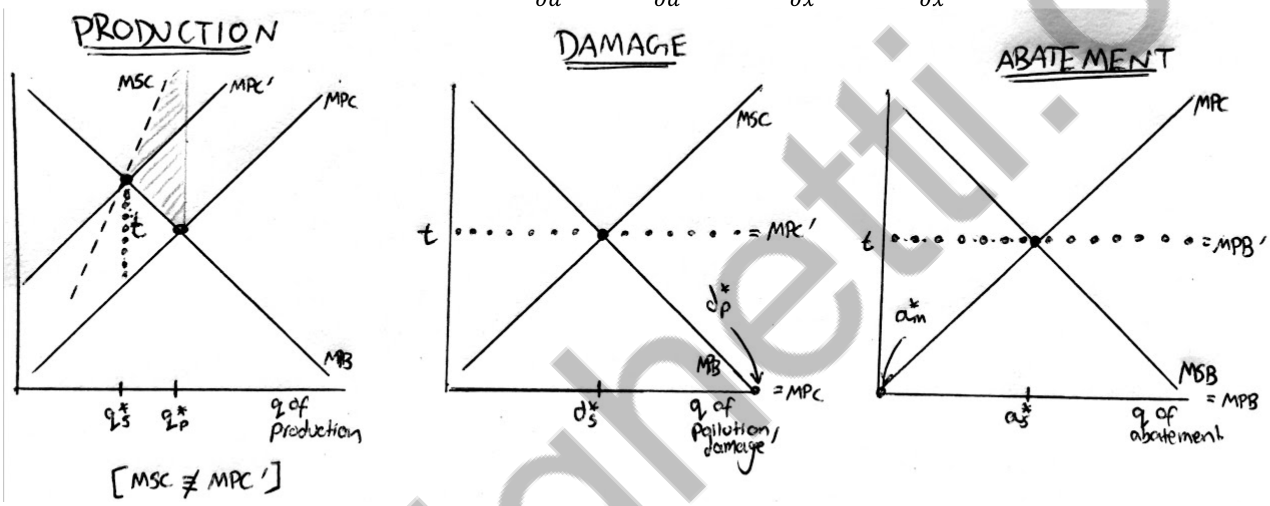
- Would merge the two firms so $\max p_a a + p_b b - c_a(a,x) - c_b(b,x)$
- FOC (assuming interior sol.) tells us $MR = MC$ so $p_a = \frac{\partial c_a}{\partial a}$ and $p_b = \frac{\partial c_b}{\partial b}$.
- Note $0 = \frac{\partial c_a}{\partial x} + \frac{\partial c_b}{\partial x}$, that is the firm balances the cost and benefit of extra pollution. Pareto efficient!

Pigou Tax

- Would implement tax on externality so $A \max p_a a - c_a(a,x) - tx$
- Note $0 = \frac{\partial c_a}{\partial x} + t$. If $t = \frac{\partial c_b}{\partial x}$ we get Pareto efficiency!

Missing Markets

- Would distribute property rights and then allow trade. If B gets property rights and can sell x'…
- $A \max p_a a - qx - c_a(a,x)$; $B \max p_b b + qx' - c_b(b,x')$. Markets clear when $x = x'$
- FOC (assuming interior sol.) tell us $p_a = \frac{\partial c_a}{\partial a}$, $p_b = \frac{\partial c_b}{\partial a}$, and $-\frac{\partial c_a}{\partial x} = q = \frac{\partial c_b}{\partial x}$. Pareto efficient!



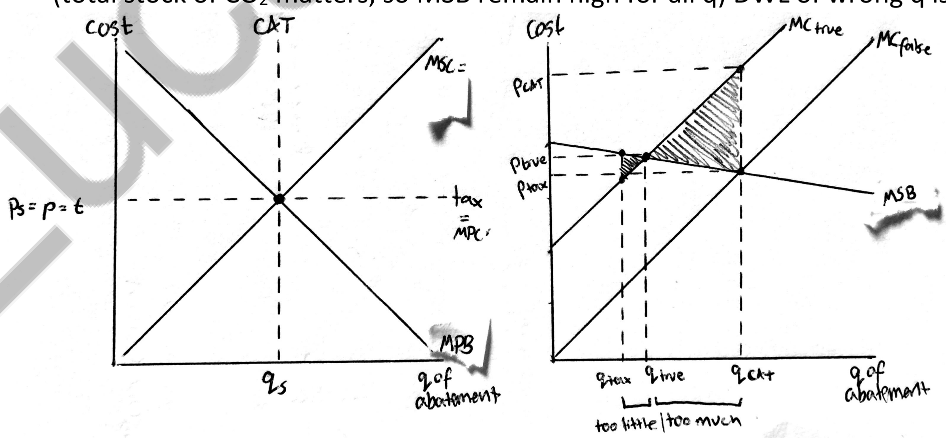
Case Study Comparison: Climate Change

In theory the should be identical

- Carbon Tax (Pigou) fixes the price of abatement and ensures a socially optimal quantity. Cap & Trade (Missing Markets) fixes the quantity and ensures the price is cost effective.
- If C&T sees gov. auctions off permits then same revenue and distributional impact.
- Intuition: shifts up MPC so intersects at same point MSC does (or in extreme case maps onto entire curve). Neither requires knowing the whole MC of each individual firm, unlike command & control.

In practice there C&T has some major disadvantages

- <u>Government is not omniscient</u>: Must estimate MC (Nordhaus \$30/ton; Stern \$300/ton). If MSB is flat (total stock of CO_2 matters, so MSB remain high for all q) DWL of wrong q is greater than wrong p

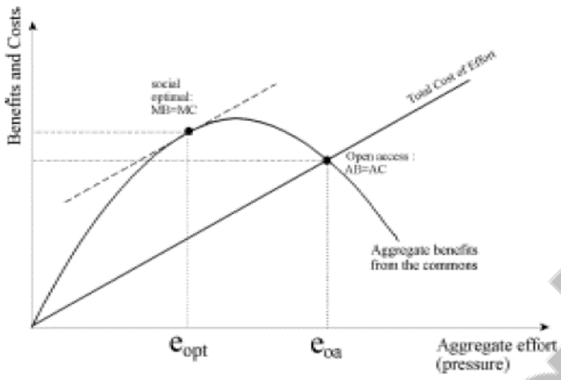


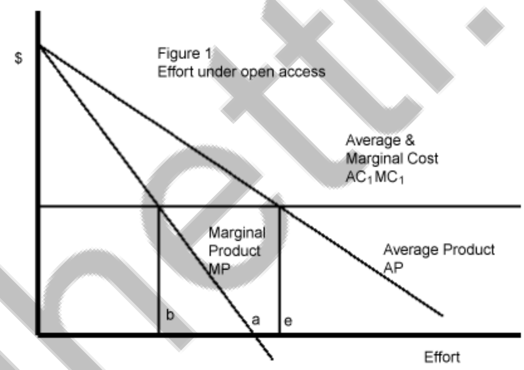
- Doesn't't' auction permits:
    - Prevents "double dividend" as firms get rents instead of gov., which could use money to offset distortions elsewhere (Parry and William: 50% more costly without auctioning)
    - Grandfarthering: firms raise pollution just before permits are handed out on polluting basis
    - Issues of equity: Disadvantage for new firms
- Difficult to get international agreement: Money crosses borders and requires global governance (hence Kyoto failed). Likely gov.s allowed to issue unlimited permits, defeating point (EU's ETS fell to 0.06€/ton)
- Leakage: C&T isn't compatible with 'extra' policies by lesser authorities as permits get traded elsewhere
- Volatility: Fluctuating price (RECLAIM \$400 to \$70,000/ton) means investment is more difficult. But… can argue that it helps play a countercyclical role

35b. Externalities: Coase Theorem

Tragedy of the Commons

- Without well-defined property rights, farmers will until TC=TR (i.e. $f(c) = ac$)
- With well-defined property rights, a farmer will graze cows until MC=MR (i.e. $f'(c) = a$





Coase Theorem

Coase Theorem (McCloskey): If no transaction costs, bargaining leads to efficient solution of externality. But
- Transaction Cost: Include technical limitations, (smoker doesn't have time to negotiate with each restaurant goer), information asymmetry (don't reveal true utility function), and free rider problem (C is incentivised to let B pay the full cost of abating A's externality)
- Doesn't account for 'equity': Unless quasi-linear (see below), outcome depends on initial endowment and how property rights are defined (i.e. different starting point in Edgeworth Box)
- Practical limitations: future generations and animals can't "bargain efficiently"

Coase Theorem (textbook): If preferences are quasilinear (so ICs are all horizontal translations of each other), the efficient amount of the good involved in the externality is independent of property rights.

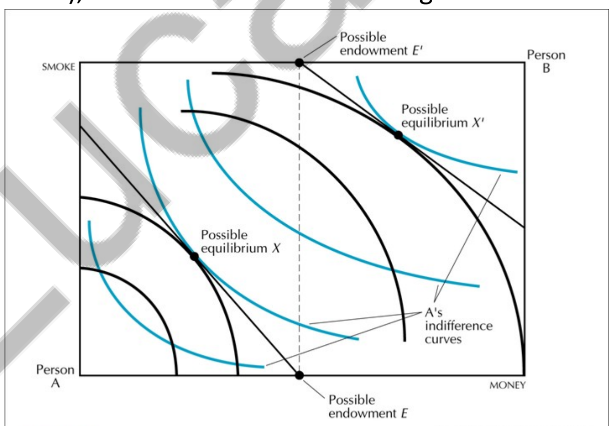


FIGURE 34.1 Preferences for money and smoke

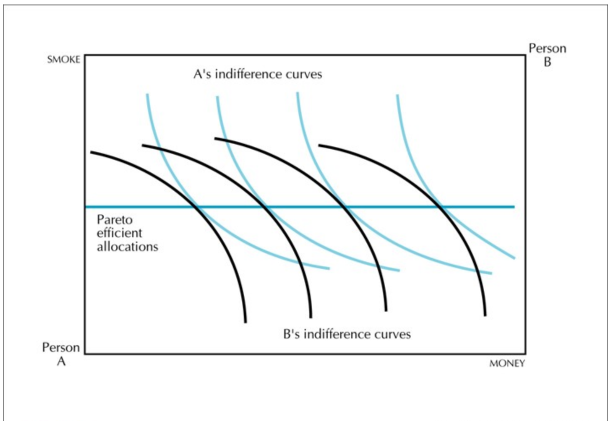


FIGURE 34.2 Quasilinear preferences and the Coase theorem

37. Public Goods

- **Public Good**: a special case of a consumption externality, where a good is non-rival and non-excludable
- Market provision likely Pareto-inefficient (under-supplied) as is collective provision (free-rider problem)

The Discrete Case

"It is Pareto-efficient to supply a discrete public good if the sum of the consumers' reservation values is at least as high as the cost of supplying it. Otherwise it is Pareto-inefficient to supply it."

$$\sum u_i = \max\begin{cases} v_a + v_b - (p_a + p_b \text{ i.e. cost})[\text{supplied}] \\ 0\ [\text{not supplied}] \end{cases}$$

- A and B decide whether to buy a WiFi router or not. This is a binary choice.

Example I: $v_a = 8;\ v_b = 3, \text{cost} = 10$
  - If $p_a = 7.5, u_a = 0.5$ and $p_b = 2.5, u_b = 0.5$
  - $0.5 > 0$ so buying is a Pareto improvement

Example II: $v_a = 8;\ v_b = 3, \text{cost} = 12$
  - Assume they buy a TV in whatever proportions ($p_a + p_b = 12$).
  - $u_a = 8 - p_a$ and $u_b = 3 - p_b = p_a - 9$
  - Now assume A just paid B $p_a - 8$ [may be negative]
  - $u_a = 8 - p_a$ and $u_b = p_a - 8$
  - $p_a - 8 > p_a - 9$ so buying is a Pareto improvement

Exam Question

	A	B	Sum
v_i	8	3	11
p_i	7	5	12
$v_i - p_i$	1	-2	1

Free Rider Problem: *Example III:* $v_a = 8;\ v_b = 3, \text{cost} = 7$
  - B may refuse to pay anything ($p_b = 0$), A to pay all of it. Hiding true values prevents Pareto efficient

The Continuous Case

Samuelsson Rule: "The level of supply of a public good is Pareto efficient if, and only if, the sum of the consumer's MRS is equal to the MRT between the public good and any other good (i.e. wealth m)"

$$MRS_a + MRS_b = MRT \text{ where } MRS_i = \frac{MU_G}{MU_{mi}} \text{ and } MRT = c'(G)$$

[Stems from non-rivalry, with on unit enjoyed by all; Note difference to market $MRS_a = MRS_b = MRT$]

- A and B decide how much WiFi allowance to buy. This is a continuous choice.

Proof

- $\max u_a(m_a, G)$ s.t. $u_b(m_b, G) = \bar{u}_b$ [utility constraint] and $m_a + m_b = \bar{m}_a + \bar{m}_b - c(G)$ [feasibile]
- $\mathcal{L} = u_a - \lambda_1(u_b - \bar{u}_b) - \lambda_2(m_a + m_b - \bar{m}_a - \bar{m}_b + c(G))$
  - $\frac{\partial u_a}{\partial m_a} - \lambda_2 = 0\ ;\ 0 - \lambda_1 \frac{\partial u_b}{\partial m_b} - \lambda_2 = 0\ ;\ \frac{\partial u_a}{\partial G} - \lambda_1 \frac{\partial u_B}{\partial G} - \lambda_2 c'(G) = 0$
  - Rearrange $\frac{\partial u_a}{\partial G} / \frac{\partial u_a}{\partial m_a} + \frac{\partial u_B}{\partial G} / \frac{\partial u_b}{\partial m_b} = c'(G) \Rightarrow \frac{MU_G}{MU_{m_a}} + \frac{MU_G}{MU_{m_b}} = c'(G) \Rightarrow MRS_a + MRS_b = MRT$

Intuition

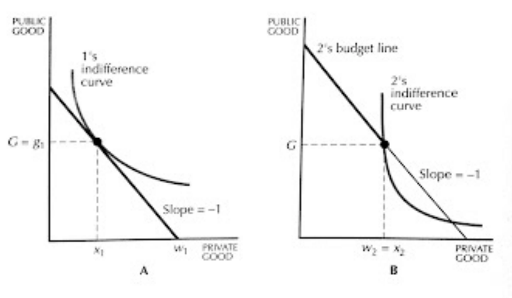
- Suppose $MRS_a = 0.5;\ MRS_b = 0.4, MRT = 1$
- A and B can both be made strictly better off by reducing G
- Same logic holds true for opposite case

The market will undersupply

- A assumes B contributes $\bar{g}_b$ so chooses g_a to $\max u_a(m_a, G) = \max u_a(M_a - g_a, g_a + \bar{g}_b)$ via FOC…
- $\frac{\partial u_a}{\partial m_a}\frac{\partial (M_a - g_a)}{\partial g_a} + \frac{\partial u_a}{\partial G}\frac{\partial (g_a + \bar{g}_b)}{\partial g_a} = \frac{\partial u_a}{\partial m_a} \times -1 + \frac{\partial u_a}{\partial G} \times 1 = 0$
- Rearrange to $\frac{\partial u_a}{\partial G} / \frac{\partial u_a}{\partial m_a} = MRS_a = 1$. Get same for B
- $MRS_a + MRS_b = 1 + 1 > MRT = 1$
- Intuition: each ignores positive externality, hence too low

Free Riding

- Only increase, not decreases, the provision public good
- A already contributed $g_{a=1}$ so B endowment $(w_{=2}, g_{a=1})$
- Given the shape of B's IC it is optimal to free ride so $g_{b=2} = 0$



The free rider problem. Person 1 contributes while person 2 free rides.

Quasi Linear Preferences: Amount of public goods stays fixed at efficient level

- Algebra: $u_i = x_i + v_i(G)$. Using $\mathcal{L}$ (see proof) we get $\frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial G} / 1 = MRS_i$. Note the absence of x_i
- Intuition: Slope is same when shifting curve so redistributing private good (x_i) means $\sum MRS_i$ is constant
- Trick: If quasilinear problem can just solve for $\max u_a + u_b$

Groves-Clarke Scheme

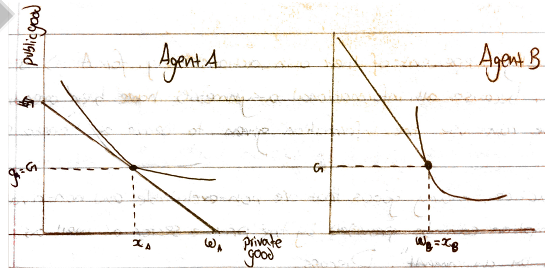
- If the TV is bought, each pays C/i. Let $n_i = v_i - C/i$
- Each actor announces their net reservation value (n_i). Buy the TV if $\sum n_i \geq 0$
- If i is pivotal also pays a tax equal to externality ($\sum n_{\text{all but } i}$)

Example: The TV costs 300, A's true $v_a = 70$ so $n_a = -30$. Truth is weakly dominant strategy in all cases

1. $n_b + n_c \geq 30$ (if A tells truth, decision is buy and A is not pivotal)
   - If A tells the truth the TV is still bought so $u_a = -30$
   - Same is true for any over-/understatement resulting in TV still being bought
   - If A understates and the TV is not bought $u_a = -(n_b + n_c) < -30$ so worse off
2. $0 \leq n_b + n_c < 30$ (if A tells truth, decision is not buy and A is pivotal)
   - If A tells the truth the TV will not be bought so $u_a = -(n_b + n_c)$
   - Same is true for any over-/understatement resulting in TV not being bought
   - If A overstates and the TV is bought $u_a = -30 < -(n_b + n_c)$ so worse off
3. $n_b + n_c < 0$ (if A tells truth, decision is not buy and A is not pivotal)
   - If A tells the truth the TV is not bought and no externality payment so $u_a = 0$
   - Same is true for any over-/understatement resulting in TV not being bought
   - If A overstates and the TV is bought $u_a = -30 + n_b + n_c < 0$
   - If i is pivotal also pays a tax equal to externality ($\sum n_{\text{all but } i}$)

- Note we assume quasilinear pref (n_i does not depend on tax) and constrained PE (PE use of tax money)

Essay Plan



Private Sector cannot supply Public Goods

- Non-rivalry: FWT assumes no externalities and hence does not hold. Samuelson Rule ($\sum MRS_i = MRT$) shows market will tend to undersupply.
- Hinders even when excludable (e.g. lighthouse)
- Non-excludable: Price system cannot work when agents are incentivised to lie (overstate if payment doesn't depend, understate if it does). May also be impossible to get payment. Free rider problem shown below (Agent B free rides and not tangent!). Hinders even when rivalrous (e.g. FT paywall has Pareto improvement of giving everyone access)

Private Sector can supply Public Goods

- Tie to private good: e.g. WiFi password printed on receipt, toilets for customers only
- Profit indirectly: advertisement (public radio stations), rents (Montgomery & Bean: Climate-Controlled-Walkways best provided where market was most free and contracts long term)
- Altruism and Game Theory: may seek reputation (Wikipedia) or seek long run corporation (chores)
- Coase-style bargains: requires private property rights (British bodies of water) or contracts (R&D projects). But… both enforced by government.

Government cannot supply Public Goods: Public Choice Theory states politicians seek votes and civil servants power for the department (Pork Barrelling), electorate may be misinformed, not

Bridging the divide:

- Zandt: all lighthouses , even when owned privately , where linked to government via fixed price, granted monopoly, or enforcement of user levies.
- Question is not if government should provide, but how much government should help.
- Note this depends on context and can change with tech (scramble cable TV, detect Ad-Blockers)