Macro Policy

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2 MONETARY POLICY

2.1 BASELINE MONETARY POLICY MODEL

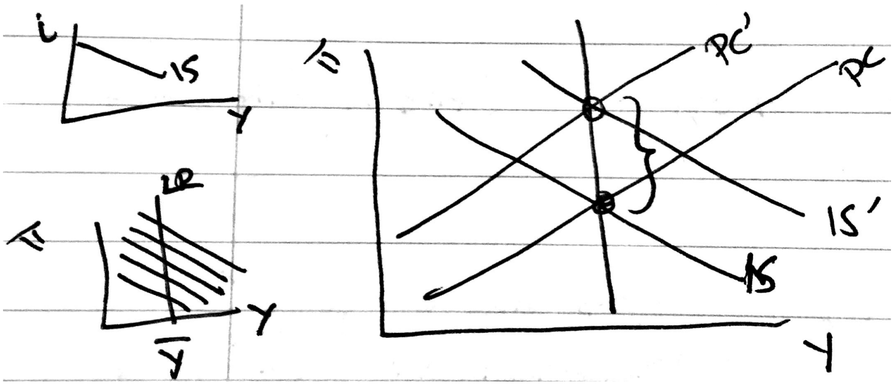
2.1.1 *SET UP*

- Timing is such that: π_t^e set using rational expectations; η_t, ϵ_t observed; r_t set; y_t, π_t realised
- [IS] describes good market equilibrium: $Y_t = \bar{Y} - \alpha(r_t - \bar{r}) + \eta_t$
  - Where η_t is real-demand shock: iid white noise with $E[\eta_t] = 0$ and $Var[\eta_t] = \sigma_\eta^2$
  - Lower r lowers borrowing cost, raising I, raising Y
- Phillips Curve [PC] describes Inflation-Output trade-off: $\pi_t = \pi_t^e + \theta(Y_t - \bar{Y}) + \epsilon_t$
  - Where ϵ_t is cost-push shock: iid white noise with $E[\epsilon_t] = 0$ $Var[\epsilon_t] = \sigma_\epsilon^2 > 0$
  - Higher prices, result in lower wages $\frac{\bar{W}}{P}$ due to fixed contracts, raising L, raising Y
- CB sets r_t to minimizes expected value of loss function: $L_t = \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}\lambda(Y_t - \bar{Y})^2$
  - λ is inverse of CB 'conservativeness' (trade-off between output and inflation target)
  - Losses are quadratic and thus greater deviations have greater marginal costs

2.1.2 *EQUILIBRIUM*

- CB observes shocks perfectly and hence effectively controls y through r:
  - $\min_{r_t} L_t = \min_{y_t} L_t = \frac{1}{2}(\pi^e + \theta(Y_t - \bar{Y}) + \epsilon_t)^2 + \frac{1}{2}\lambda(Y_t - \bar{Y})^2$
- FOC wrt y_t gives Targeting Rule [TR]:
  - $\theta(\pi_t^e + \theta(Y_t - \bar{Y}) + \epsilon_t - \pi^*) + \lambda(Y_t - \bar{Y}) = 0$
  - $\theta(\pi_t - \pi^*) + \lambda(Y_t - \bar{Y}) = 0$
  - $(Y_t - \bar{Y}) = -\frac{\theta}{\lambda}(\pi_t - \pi^*)$
- Take expectations through [PC]:
  - Ratex thus $\pi_t^e = E[\pi_t]$
  - Sub in[PC] gives $E[Y_t] = \bar{Y}$
  - Sub in [TR] gives $E[\pi_t] = \pi^*$
- Sub back into FOC to get y_t (easiest since only one unknown)
  - $Y_t = \bar{Y} - \frac{\theta}{\lambda+\theta^2}\epsilon_t$ thus $E[Y_t] = \bar{Y}$ [natural rate hypothesis]
- Sub into [IS] to get r_t, that is the reaction function
  - $r_t = \bar{r} - \frac{1}{\alpha}(Y_t - \bar{Y}) + \frac{1}{\alpha}\eta_t = \bar{r} + \frac{1}{\alpha}\frac{\theta}{\lambda+\theta^2}\epsilon_t + \frac{1}{\alpha}\eta_t$
- Sub into [PC] to get π_t
  - $\pi_t = \pi^* + \frac{\lambda}{\lambda+\theta^2}\epsilon_t$ thus $E[\pi_t] = \pi^*$

- η_t is fully offset (i.e. not in Y_t or π_t) but ϵ_t is not due to transmission mechanism:
  - $\uparrow \eta_t;\ \uparrow y_t, \uparrow \pi_t;\ \uparrow i_t\ s.t. \Delta y_t = \Delta\pi_t = 0$ Divine coincidence!
  - $\uparrow \epsilon_t;\ - y_t, \uparrow \pi_t;\ \uparrow i_t\ s.t. \downarrow y_t\ \uparrow \pi_t$ Forces trade-off
- Always at LRAS as π^e is rational. If $\uparrow \epsilon$, IS shifts up vertical line, so $\uparrow \pi$ by full amount



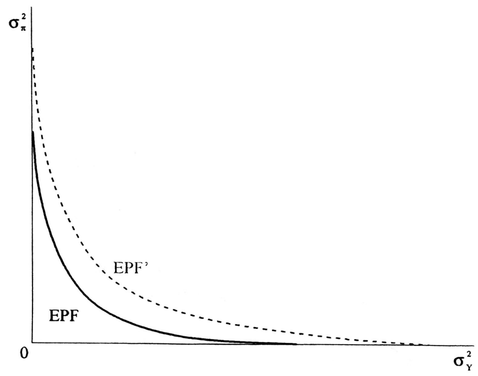
- No inflation bias (i.e. $\pi^e = \pi^*$). People know CB will perfectly offset demand shock (i.e. $Y^* = \bar{Y}$) and cost push is equally likely to be positive/negative
- Note extreme cases: If $\lambda = \infty$, Y_t is fixed; if $\lambda = 0$, π_t is fixed

2.1.3 *Solving for Volatility*

- Recall $Var(X) = E(X^2) - E(X)^2$
- Thus $\sigma_\epsilon^2 = Var[\epsilon_t] = E[(\epsilon_t - E[\epsilon_t])^2] = E[(\epsilon_t - 0)^2] = E[\epsilon_t^2]$
- Thus
  - $\sigma_Y^2 = Var[Y_t] = E[(Y_t - \bar{Y})^2] = \left(\frac{\theta}{\lambda+\theta^2}\right)^2 E[\epsilon_t^2] = \frac{\theta^2}{(\lambda+\theta^2)^2}\sigma_\epsilon^2$
  - $\sigma_\pi^2 = Var[\pi_t] = E[(\pi_t - \pi^*)^2] = \left(\frac{\lambda}{\lambda+\theta^2}\right)^2 E[\epsilon_t^2] = \frac{1}{(1+\theta^2/\lambda)^2}\sigma_\epsilon^2$

2.1.4 *Efficient Policy Frontier (Taylor, 1979)*

- In LR no trade-off between levels of inflation and output (since LRAS is vertical) but there is a trade-off between volatilities:
  - Note $\sigma_\pi + \theta\sigma_Y = \frac{\lambda+\theta^2}{\lambda+\theta^2}\sigma_\epsilon = \sigma_\epsilon$
  - Rearrange to $\sigma_\pi^2 = \left(\sqrt{\sigma_\epsilon^2} - \theta\sqrt{\sigma_Y^2}\right)^2$ [EPF]
    - As $\theta\sigma_Y = \frac{\theta^2}{\lambda+\theta^2}\sigma_\epsilon \le \sigma_\epsilon$ thus $\sqrt{\sigma_\epsilon^2} \ge \theta\sqrt{\sigma_Y^2}$ thus $\sigma_\pi^2 \ge 0$
- Can plot this in $[\sigma_Y^2, \sigma_\pi^2]$ space by examining properties:
  - Limits:
    - If $\sigma_Y^2 = 0, \sigma_\pi^2 = \sigma_\epsilon^2$ ($\lambda \to \infty$, hyper 'liberal')
    - If $\sigma_\pi^2 = 0, \sigma_Y^2 = \frac{\sigma_\epsilon^2}{\theta^2}$ ($\lambda = 0$, hyper 'conservative')
  - Slope:
    - $\frac{d\sigma_\pi^2}{d\sigma_Y^2} = -\frac{\theta}{\sqrt{\sigma_Y^2}}\left(\sqrt{\sigma_\epsilon^2} - \theta\sqrt{\sigma_Y^2}\right) = -\lambda < 0$ so downward sloping
    - $\frac{d^2\sigma_\pi^2}{d(\sigma_Y^2)^2} = \frac{d}{d\sigma_Y^2}\left[-\theta\frac{\sqrt{\sigma_\epsilon^2}}{\sqrt{\sigma_Y^2}} + \theta^2\right] = \frac{1}{2}\theta\frac{\sqrt{\sigma_\epsilon^2}}{\sigma_Y^2/\sqrt{\sigma_Y^2}} > 0$ so convex
- A rise in σ_η^2 has no effect since demand shocks are perfectly offset due to divine coincidence. A rise in σ_ϵ^2 causes a worse trade-off everywhere
- Whilst response to a single given shock $r_t = \bar{r} + \frac{1}{\alpha}\frac{\theta}{\lambda+\theta^2}\epsilon_t + \frac{1}{\alpha}\eta_t$ does not change, the second moment does $Var(r_t) = \left(\frac{1}{\alpha}\frac{\theta}{\lambda+\theta^2}\right)^2\sigma_\epsilon^2 + \left(\frac{1}{\alpha}\right)^2\sigma_\eta^2$



2.2 OPTIMAL FORECAST ADD-ONS

2.2.1 SIGNAL EXTRACTION

- Optimal Forecast must be [i] unbiased $E(f) = \eta$ and [ii] minimize variance $\min V(y - f)$
- Use guess and verification where $f^* = \phi + \phi_A \tilde{\eta}_A + \phi_B \tilde{\eta}_B$
- To be unbiased:
  - $E(\phi + \phi_A \tilde{\eta}_A + \phi_B \tilde{\eta}_B) = \phi + \phi_A(\eta_A + E[v_A]) + \phi_B(\eta_B + E[v_B])= \phi + \phi_A \eta_A + \phi_B \eta_B$
  - (If use full linear equation all terms but $\phi_A \eta_A + \phi_B \eta_B$ will cancel)
  - Hence know $\phi = 0$ and $\phi_A + \phi_B = 1$ hence $f = \phi_A \eta_A + (1 - \phi_A)\eta_B$
- To min variance:
  - $V(\phi_A \eta_A + (1 - \phi_A)\eta_B) = \phi_A^2 V(v_A) + (1 - \phi_B^2)V(v_B) = \phi_A^2 \sigma_A^2 + (1 - \phi_A)^2 \sigma_B^2$
  - FOC wrt ϕ_A: $2\phi_A \sigma_A^2 - 2(1 - \phi_A)\sigma_B^2 = 0$ hence $\phi_A^* = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}$
  - Checking SOC: $2\sigma_A^2 + 2\sigma_B^2 > 0$ hence minimum
- Optimal forecast $f^* = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2} \tilde{\eta}_A + \frac{\sigma_A^2}{\sigma_A^2 + \sigma_B^2} \tilde{\eta}_B$
  - Use both because independent so contain separate information. If interdependent still use both also have covariance term against it
  - Noisier signal gets assigned less weight

2.2.2 RATIONAL EXPECTATIONS

- Optimal forecast f^* of y given $\tilde{y}$ minimizes conditional mean squared error: $E[(y - f^*)^2 | \Omega]$ where MSE of estimator $\hat{y}$ for y: $E[(y - \hat{y})^2]$
- Note Conditional expectation $E[y|\Omega]$ minimizes conditional MSE:
  - $\min_{\hat{y}} E[(y - \hat{y})^2 | \Omega]$
  - FOC wrt $\hat{y}$: $-2E[y - \hat{y}|\Omega] = 0$ Thus $\hat{y} = E[y|\Omega]$
- Thus rational expectations is optimal forecast $f^* = y^e = E[y|\Omega]$
  - Unbiased: $E[y - f^*] = E\big[E[y - f^*|\Omega]\big] = E[E[y|\Omega] - f^*] = 0$
    - Law of iterated expectations
  - Min Variance: $E[(y - f^*)^2] = E[(y - f^* - E[y - f^*])^2] = Var[y - f^*]$

2.3 UNCERTAINTY

2.3.1 TYPES OF UNCERTAINTY IRL

- Structure of economy: Relationship between different economic variables (pass through of exchange rate on inflation; short term i/r and inflation etc.)

- State of economy: GDP data takes a quarter to collect and frequently refined thereafter; do not know true values of $\bar{Y}, \bar{r}$ as they are concepts
- Uncertainty about future events ϵ, η
- Akin to Brainard principle, Frank Semts notes that as the accurate of estimates deteriorates, it is optimal for policymakers to put less weight on these estimates when making decisions. Central Banks thus instead "feel their way"

2.3.2 *SET-UP*

- Set-up same as Baseline Monetary Model:
  - [IS]: $Y_t = \bar{Y} - \alpha(r_t - \bar{r}) + \eta_t$
  - [PC]: $\pi_t = \pi_t^e + \theta(Y_t - \bar{Y}) + \epsilon_t$
  - Loss function: $L_t = \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}\lambda(Y_t - \bar{Y})^2$
- But now CB does not have complete information but faces uncertainty:
  - Additive Uncertainty: CB does not observe real demand shock η_t but signal $\widetilde{\eta_t}$ where $\hat{\eta}^*$ is optimal forecast $\hat{\eta}^* = E(\eta|\tilde{\eta})$
  - Multiplicative Uncertainty: CB is uncertain slope of IS equation (or 'true' effectives) but instead faces $\alpha = \bar{\alpha} + v$

2.3.3 *EQUILIBRIUM: ADDITIVE*

- CB minimizes expected loss function $E[L_t|\Omega_t^{CB}]$. For simplicity assume CB inflation-nutter
- Substitute in [PC] then sub Y via [IS]
  - $E[(\pi_t - \pi^*)^2|\Omega^{CB}] = \frac{1}{2}E[(\pi_t^e + \theta[Y_t - \bar{Y}] + \epsilon_t - \pi^*)^2|\Omega_t^{CB}]$
  - $= \frac{1}{2}E[(\pi_t^e + \theta[-\alpha(r_t - \bar{r}) + \eta_t] + \epsilon_t - \pi^*)^2|\Omega_t^{CB}]$
- Take FOC wrt r_t. Note, cannot use y_t since CB no longer has perfect control over it
  - $\frac{1}{2}E[-\alpha\theta\{\pi_t^e + \theta[-\alpha(r_t - \bar{r}) + \eta_t] + \epsilon_t - \pi^*\}|\Omega_t^{CB}]$
- Expand and simplify
  - $\pi_t^e - \theta\alpha(r_t - \bar{r}) + \theta E[\eta_t|\Omega_t^{CB}] + \epsilon_t - \pi^* = 0$
  - [**] $\pi_t^e + \theta E[\eta_t|\Omega_t^{CB}] + \epsilon_t - \pi^* = \theta\alpha(r_t - \bar{r})$
- Take expectations
  - Note $E[\eta_t|\Omega_t^{CB}] = \hat{\eta}_t^*$; $E\left[\pi_t\middle|\Omega_t^{prv})\right] = \pi_t^e$
  - [PC]: $E[\pi_t|\Omega_t^{prv}] = \pi_t^e + \theta\left(E[Y_t|\Omega_t^{prv}]_t - \bar{Y}\right) + E[\epsilon_t|\Omega_t^{prv}]$ thus $E\left[Y_t\middle|\Omega_t^{prv}\right] = \bar{Y}$
  - [IS]: $E\left[Y_t\middle|\Omega_t^{prv}\right] = \bar{Y} - \alpha\left(E\left[r_t\middle|\Omega_t^{prv}\right] - \bar{r}\right) + E\left[\eta_t\middle|\Omega_t^{prv}\right]$ thus $E\left[r_t\middle|\Omega_t^{prv}\right] = \bar{r}$
  - [**]: $\pi_t^e + \theta E[\eta_t|\Omega_t^{CB}] + E[\epsilon_t] - \pi^* = \theta\alpha(E[r_t] - \bar{r})$ thus $\pi_t^e = \pi^*$
- Substitute in for macroeconomic variables.
  - Let $v = \eta_t - \hat{\eta}_t^*$ i.e. forecast error
  - [**] $\pi^* + \theta\hat{\eta}_t^* - \pi^* = \theta\alpha(r_t - \bar{r})$ thus $r_t = \bar{r} + \frac{1}{\alpha}\hat{\eta}_t^* + \frac{1}{\alpha\theta}\epsilon_t$
  - [IS]: $Y_t = \bar{Y} - \alpha\left[\frac{1}{\alpha}\hat{\eta}_t^* + \frac{1}{\alpha\theta}\epsilon_t\right] + \eta_t = \bar{Y} + (\eta_t - \hat{\eta}_t^*) - \frac{1}{\theta}\epsilon_t$ thus $Y = \bar{Y} + v - \frac{1}{\theta}\epsilon_t$
  - [PC]: $\pi_t = \pi_t^e + \theta\left(\eta_t - \hat{\eta}_t^* - \frac{1}{\theta}\epsilon_t\right) + \epsilon_t = \pi^* + \theta(\eta_t - \hat{\eta}_t^*)$ thus $\pi = \pi^* + \theta v$
- Interpret results
  - Coefficient of optimal policy reaction function is unaffected due to Certainty Equivalence (i.e. equally likely that shock is positive or negative so do nothing)
  - Output and inflation increasing in unanticipated real demand shock v
  - Volatility increases with uncertainty: $Var[Y_t] = \frac{1}{\theta^2}\sigma_\epsilon^2 + \sigma_v^2$ and $Var[\pi_t] = \theta\sigma_v^2$

2.3.4 *EQUILIBRIUM: MULTIPLICATIVE*

- CB minimizes expected loss function $E[L_t|\Omega_t^{CB}]$. For simplicity assume CB hyper-liberal
- Substitute in [PC] then sub Y via [IS]
  - $E[(Y_t - \bar{Y})^2|\Omega^{CB}] = E[(-\alpha(r_t - \bar{r}) + \eta_t)^2|\Omega_t^{CB}]$
- Take FOC wrt r_t. Note, cannot use y_t since CB no longer has perfect control over it
  - $2E[-\alpha(-\alpha(r_t - \bar{r}) + \eta_t)|\Omega_t^{CB}] = 0$
- Expand and simplify
  - $E[\alpha^2|\Omega_t^{CB}](r_t - \bar{r}) - E[\alpha|\Omega_t^{CB}]\eta_t = 0$
  - $r_t = \bar{r} + \frac{E[\alpha|\Omega_t^{CB}]}{E[\alpha^2|\Omega_t^{CB}]}\eta_t$
- Substitute in Multiplicative Uncertainty
  - $r_t = \bar{r} + \frac{E[\bar{\alpha}+v|\Omega_t^{CB}]}{E[(\bar{\alpha}+v)^2|\Omega_t^{CB}]}\eta_t$
- Take expectations
  - Note $E[v|\Omega_t^{CB}] = 0$; $E[\alpha|\Omega_t^{CB}] = \bar{\alpha}$; $E[v^2|\Omega_t^{CB}] = \sigma_v^2$; $E[\alpha^2|\Omega_t^{CB}] = \bar{\alpha}^2 + \sigma_v^2$; v_t, η_t i.i.d.
  - $r_t = \bar{r} + \frac{\bar{\alpha}}{\bar{\alpha}^2+\sigma_v^2}\eta_t = \bar{r} + \frac{1}{1+\left(\frac{\sigma_v}{\bar{\alpha}}\right)^2}\frac{1}{\bar{\alpha}}\eta_t$
- Substitute in for macroeconomic variables:
  - $Y_t = \bar{Y} + \left[1 - \frac{1}{1+(\sigma_v/\bar{\alpha})^2}\frac{\alpha}{\bar{\alpha}}\right]\eta_t$
  - If $\alpha = \bar{\alpha}$ (ex-post)
    - $Y_t = \bar{Y} + \left(1 - \frac{1}{1+\left(\frac{\sigma_v}{\bar{\alpha}}\right)^2}\right)\eta_t$
    - $Var[Y_t] = \left(\frac{\sigma_v^2}{\bar{\alpha}^2+\sigma_v^2}\right)^2 \sigma_\eta^2 = \left(\frac{1}{\bar{\alpha}^2/\sigma_v^2+1}\right)\sigma_\eta^2$
- Interpret result
  - Coefficient of optimal policy reaction function is affected due to Brainard Principle: CB operates cautiously and responds less under uncertainty
    - Attenuation of shock η_t increasing in coeff. of variation of α: $\frac{\sigma_v}{\bar{\alpha}} = \frac{sd(\alpha)}{|E(\alpha)|}$. As σ_v^2 increases, MP creates more additional uncertainty so CB turns it off
    - Thus limits: $\lim_{\sigma_v^2\to\infty} r_t \to \bar{r}$ (off) and $\lim_{\sigma_v^2\to 0} r_t \to \bar{r} + \frac{1}{\bar{\alpha}}\eta_t$ (on)
    - This is a feature of quadratic loss function

2.4 Effective Lower Bound

- ZLB has been empirically refuted, so instead believe in 'Effective Lower Bound'
  - ECB -0.5% deposit rate; BoJ -0.1% main policy rate; Germany -0.63% short-term bond yields (private sector!)
  - Because taking money out as cash has associated expenses (storage, security etc.)

2.4.1 Set Up

- Rewrite MP reaction function (i.e. Taylor rule) for nominal interest rate:
  - Recall $r_t = \bar{r} + \phi_\pi(\pi_t - \pi^*) + \phi_Y(Y_t - \bar{Y})$
  - Assume Fisher equation with perfect foresight ($\pi_t^e = \pi_t$): $i_t = r_t + \pi_t$
  - Thus $i_t^* = \bar{r} + \pi^* + (1+\phi_\pi)(\pi_t - \pi^*) + \phi_Y(Y_t - \bar{Y})$
  - Taylor Principle: $\frac{di_t^*}{d\pi_t} > 1$. Slope is trade-off between y-gap and π-stabilisation
- ELB exists and may prevent full response to shocks
  - $i_t = \max\{i_t^*, i_{ELB}\}$

- Neutral nominal interest rate is counterfactual that would keep real interest rate at natural level: $\tilde{\imath}_t = \bar{r} + \pi_t$
    - So $r_t > \bar{r}$ corresponds to $i_t > \tilde{\imath}_t$ (contractionary monetary policy stance)

2.4.2 *EQUILIBRIUM*

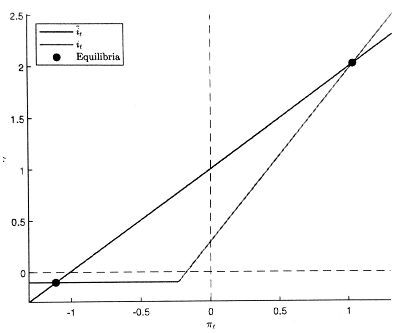
- System is in equilibrium when $r_t = \bar{r}$, that is $i_t = \tilde{\imath}_t$. This arises when $\pi_t = \pi^*$ and $Y_t = \bar{Y}$
    - "Whenever the nominal interest rate chosen by policymakers i_t coincides with natural level of nominal interest rate"
    - Two such states: "high" and "low"
- So real interest rate r_t increasing in inflation π_t, ensuring stability
    - If $\pi_t > \pi^*$, then $r_t > \bar{r}$, which will reduce output (via IS) and inflation (via PC) bringing inflation back to π^* over time

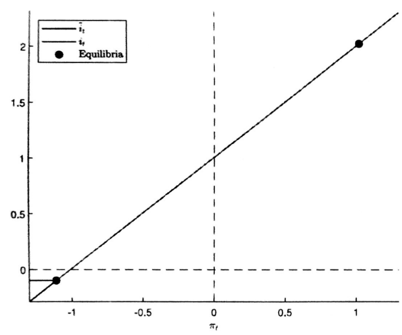
2.4.3 *DYNAMICS*

- Sufficient to consider only inflationary shocks. Output shocks should be translated first

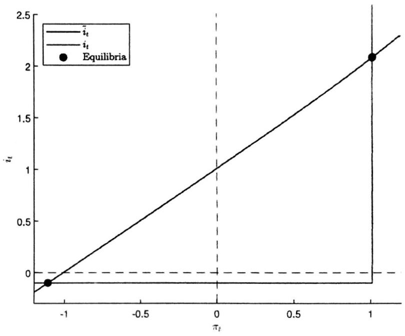
- Positive (negative) perturbation to inflation around high steady state shows it is stable
    - CB responds to shock according to policy rule: $i_t > \tilde{\imath}_t$ ($i_t < \tilde{\imath}_t$)
    - Moves real interest rate above (below) natural level
    - Via IS curve output y_t responds negatively (positively) via investment channel
    - Via PC curve inflation π_t responds negative (positively) to this change in output
    - Economy moves back towards original stead state. It is hence stable
- Negative perturbation to inflation around lower steady state shows it is unstable
    - CB responds to shock according to policy rule: $i_t > \tilde{\imath}_t$
    - Moves real interest rate above natural level. ELB constrains CB so not "far enough"
    - Via IS curve output y_t responds negatively via investment channel
    - Via PC curve inflation π_t responds negatively to this change in output
    - Economy moves further from steady state. Deflationary spiral if $\pi_t < i_{ELB} - \bar{r}$

- Consider extreme cases:
    - CB only cares about output gap ($\phi_\pi \to 0;\ \phi_Y \to \infty$): Policy keep $i_t = \tilde{\imath}_t$ so two lines collapse into each other
    - CB only care about inflation ($\phi_\pi \to \infty;\ \phi_Y \to 0$): Nominal interest rate will always attempt to adjust inflation back to target





Notes: No weight on inflation, all weight on ouptut gap.



Notes: All weight on inflation, no weight on ouptut gap.

2.4.4 *MODEL AVERAGING*

- Suppose CB faces uncertainty about economic model: A with prob μ; B with prob $1 - \mu$
- Optimal to minimize weighted average of expected loses: $\mathcal{L} = \mu E_A[L_t] + (1-\mu) E_B[L_t]$

- Take special case $L_t = \frac{1}{2}(Y_t - \bar{Y})^2$
    - If $\alpha_A = \alpha_B = \alpha$ (i.e. no uncertainty) then $r_t = \bar{r} + \frac{1}{\alpha}\eta_t$
    - If $\alpha = \alpha_A$ and $\alpha = \alpha_B$ and equally likely $\mu = \frac{1}{2}$ then $r_t = \bar{r} + \frac{\alpha_A + \alpha_B}{\alpha_A^2 + \alpha_B^2}\eta_t$
        - Sub in IS: $\mathcal{L} = \frac{1}{2}\frac{1}{2}(-\alpha_A[r_t - \bar{r}] + \eta_t)^2 + \frac{1}{2}\frac{1}{2}(-\alpha_B[r_t - \bar{r}] + \eta_t)^2$
        - FOC wrt r_t: $-\alpha_A \frac{1}{2}(-\alpha_A[r_t - \bar{r}] + \eta_t) - \alpha_B \frac{1}{2}(-\alpha_B[r_t - \bar{r}] + \eta_t) = 0$
        - Rearrange: $(\alpha_A^2 + \alpha_B^2)\frac{1}{2}[r_t - \bar{r}] = (\alpha_A + \alpha_B)\frac{1}{2}\eta_t$
        - Rearrange: $r_t = \bar{r} + \frac{\alpha_A + \alpha_B}{\alpha_A^2 + \alpha_B^2}\eta_t$
    - If $\alpha_A = \bar{\alpha} + \delta$ and $\alpha_B = \bar{\alpha} - \delta$ where $\bar{\alpha} = \frac{1}{2}(\alpha_A + \alpha_B)$ then $r_t = \bar{r} + \frac{1}{1 + \frac{\delta^2}{\bar{\alpha}^2}}\frac{1}{\bar{\alpha}}\eta_t$
        - Sub in an simplify: $r - \bar{r} = \frac{(\bar{\alpha}+\delta)+(\bar{\alpha}-\delta)}{(\bar{\alpha}+\delta)^2+(\bar{\alpha}-\delta)^2}\eta_t = \frac{1}{1+\delta^2/\bar{\alpha}^2}\frac{1}{\bar{\alpha}}\eta_t$
        - Contrast to multiplicative uncertainty about α: $r = \bar{r} + \frac{1}{1+\sigma_v^2/\bar{\alpha}^2}\frac{1}{\bar{\alpha}}\eta_t$

- If there is deep model uncertainty (i.e. CB faces uncertainty but cannot assign probabilities), common robust control method is minimax strategy:
    - "Prepare for the worst" by minimizing maximum loss
    - "Do whatever it takes" to prevent highly detrimental possible outcomes

2.5 Central Bank Tools

2.5.1 *Tinbergen Rule*

- To achieve N independent policy objectives need N effective policy instruments
    - Financial crisis shows price stability does not equal financial stability
    - Using MP for price and financial stability could lead to trade-off: tightening to reduce asset price bubbles by 'leaning against the wind'
    - Need macroprudential policy tools: leverage ratio, countercyclical capital buffer etc.

2.5.2 *Forward Guidance*

- Communication by CB of future monetary policy actions. Can be qualitative (i.e. code words) or quantitative (i.e. time-dependent or state-contingent)
- Enables CB to 'manage expectations' of future policy rates and thereby influence long-term interest rates, improving effectiveness:
    - Expectations Theory of Term Structure: $i_{n,t} = \frac{1}{n}(i_t + i_{t+1}^e + \cdots + i_{t+n-1}^e)$
    - FG that policy rate likely to remain lower for longer reduces i_{t+s}^e and thereby $i_{n,t}$

2.5.3 *Large Scale Asset Purchases*

- Usually of medium/long-term government bonds via creation of reserve balances (QE) Acts through various channels:
    - Bernanke-Blinder: Expanding monetary base reduces lending rates and stimulates lending
    - Bond Market Model: Rise in bond demand by CB reduces liquidity premium and 'signals' lower expectations of future interest rates
        - Liquidity Premium Theory: $i_{n,t} = \frac{1}{n}(i_t + i_{t+1}^e + \cdots + i_{t+n-1}^e) + \theta_{n,t}$
    - 'Portfolio rebalancing' by private sector leads to greater demand for other assets (e.g. corporate bonds, equity) increasing other asset prices as well

- Empirically, significant reduction in bond yields and increase asset prices boosting output growth and inflation (Joyce, Tong and Woods, 2011)

3 FISCAL POLICY

3.1 DEFINITIONS

- Government budget deficit: $D_t = G_t + rB_t - T_t = \Delta B_t$
- Primary budget deficit: $D_t^p = D_t - rB_t = G_t - T_t$ (i.e. excludes interest payments)
    - If $D_t^p = -rB_t$ then $D_t = \Delta B_t = 0$ (no net debt issue and B_t remains constant)
- Derive ratios as follows:
    - Let $b_t = \frac{B_t}{Y_t}$ (ratio of government debt to national income). Both are in real terms
    - Let B_t^n denote stock of government debt in nominal terms so $B_t = \frac{B_t^n}{P_t}$
    - Hence $b_t = \frac{B_t}{Y_t} = \frac{B_t^n}{P_t Y_t}$. Likewise $d_t = \frac{D_t}{Y_t} = \frac{D_t^n}{P_t Y_t}$
- Note $D_t = D_t^p + rB_t$ implies $d_t = d_t^p + rb_t$

3.2 TAX SMOOTHING

3.2.1 *GOVERNMENT BUDGET CONSTRAINT*

- $G_t^C + G_t^I + rB_{t-1} = T_t + S_t + R_t^I + D_T$ i.e. Consumption + Investment + Interests on Debt = Tax Revenue (minus Transfers) + Seignorage + Investment Revenue + Deficit
- Note that $D_t = B_t - B_{t-1}$ and $R_t^I = (1 + r_G)G_{t-1}^I$
- Hence rewrite: $G_t^C + G_t^I + (1 + r)B_{t-1} = T_t + S_t + (1 + r_G)G_{t-1}^I + B_t$

- Two periods with no prior investment and constraint $B_2 = 0$ (i.e. need to balance books)
    - $D_1 + D_2 = (B_1 - B_0) + (B_2 - B_1) = -B_0$
    - So initial debt $B_0 > 0$ requires budget surplus $D_1 + D_2 < 0$
- Combine two periods to get intertemporal budget constraint (via substituting B_1)
    - $G_1^C + G_1^I + (1 + r)B_0 = T_1 + S_1 + B_1$
    - $G_1^C + (1 + r)B_1 = T_2 + S_2 + (1 + r_G)G_1^I$
    - $(1 + r)B_0 + G_1^C + \frac{G_2^C}{1+r} + G_1^I = T_1 + \frac{T_2}{1+r} + S_1 + \frac{S_2}{1+r} + \frac{1+r_G}{1+r} G_1^I$
- Gives <u>Golden Rule of Public Finance</u>: $(1 + r)B_0 + G_1^C + \frac{G_2^C}{1+r} = T_1 + \frac{T_2}{1+r} + S_1 + \frac{S_2}{1+r} + \frac{r_G - r}{1+r} G_1^I$
    - If $r_G = r$ then G_1^I drops out (has no effect on net liability of government). So debt financing of public investment is fine!
    - Invest up to the point where $r_G = r$ (and r is negative at the moment!)

3.2.2 *TAX SMOOTHING MODEL*

Assumptions

- Government sets distortionary tax τ_t to minimize DWL: $L_G = \frac{1}{2}\kappa\tau_1^2 Y_1 + \frac{1}{1+\rho_G}\frac{1}{2}\kappa\tau_2^2 Y_2$
    - Discount rate is given by $\rho_G \geq 0$ and $\kappa > 0$
- Subject to constraint $(1 + r)B_0 + G_1^C + \frac{G_2^C}{1+r} + \frac{r - r_G}{1+r} G_1^I = \tau_1 Y_1 + \frac{\tau_2 Y_2}{1+r} + S_1 + \frac{S_2}{1+r}$
    - B_0 predetermined and $r, r_G, Y_t, G_t^C, G_t^I, S_t$ exogenous

Solving

- Note BC is necessarily binding. Hence set up lagrangian $\mathcal{L} = DWL - BC$

- FOC wrt τ_1: $\kappa\tau_1 Y_1 - \lambda Y_1 = 0$ | wrt τ_2: $\frac{1}{1+\rho_G}\kappa\tau_2 Y_2 - \lambda\frac{Y_2}{1+r} = 0$
- Hence $\tau_1 = \frac{1+r}{1+\rho_G}\tau_2$

Key Results

- Note there is tax-smoothing: $\frac{d\tau_1}{d\tau_2} = \frac{1+r}{1+\rho_G} > 0$ so there is co-movement
    - This minimizes DWL as loss, $\frac{1}{2}\kappa\tau_t^2 Y_t$, is increasing and convex in tax rate τ_t
    - Impatient government with $\rho_G > r$ sets lower current tax rate $\tau_1 < \tau_2$

- Perfect tax smoothing when $\rho_G = r$ so $\tau_1 = \tau_2 = \bar{\tau}$
    - Intuitively, discount rate of market equals discount rate of government OR borrow at $1 + r$ to have lower taxes today and discount this at $\delta = 1 + r$ as well
    - Can now rearrange and simplify
        - $(1+r)B_0 + G_1^C + \frac{G_2^C}{1+r} = \tau_1 Y_1 + \frac{\tau_2 Y_2}{1+r} + S_1 + \frac{S_2}{1+r}$
        - $\mathbb{L} = \tau_1 Y_1 + \frac{\tau_2 Y_2}{1+r} + \mathbb{S} = \bar{\tau}\left[Y_1 + \frac{Y_2}{1+r}\right] + \mathbb{S}$
- Optimal income tax rate $\bar{\tau}$ equals ratio of present value of net government liabilities $\mathbb{L}$ adjusted for present value of seignorage $\mathbb{S}$, and present value of income [intuition?]

3.2.3 Budget Deficit

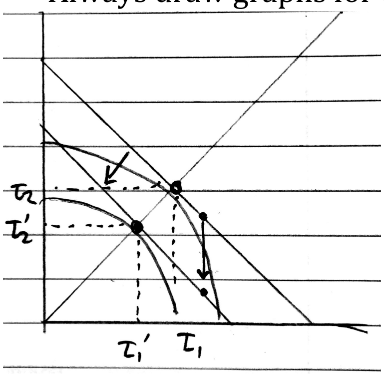
Optimal Tax Rate

- Assume $B_0 = 0, S_1 = S_2, r_G = r = \rho_G$ so have perfect tax smoothing as above
    - Simplifies as $\mathbb{L} = G_1^C + \frac{G_2^C}{1+r}$ and $\mathbb{S} = 0$
    - Thus $\bar{\tau} = \frac{G_1^C + \frac{G_2^C}{1+r}}{Y_1 + \frac{Y_2}{1+r}} = \frac{\tilde{G}^C + \frac{\tilde{G}^C}{1+r}}{\tilde{Y} + \frac{\tilde{Y}}{1+r}} = \frac{\tilde{G}^C}{\tilde{Y}}$ where $\tilde{X}$ denotes permanent level

Key Results

- Consider comparative statics:
    - ΔG_1^C: Rise in both τ_i to pay off increased spending. D_1^P rises due to more spending and D_2^P falls to compensate
        - $\frac{d\tau_1}{dG_1^C} = \frac{d\tau_2}{dG_1^C} = \frac{1}{Y_1 + \frac{Y_2}{1+r}} > 0$ | $\frac{dD_1^P}{dG_1^C} = 1 - \frac{d\tau_1}{dG_1^C} > 0$ | $\frac{dD_2^P}{dG_1^C} = -(1+r)\frac{dD_1^P}{dG_1^C} < 0$
    - ΔY_2: Rise in tax base so can afford to lower both τ_i. As only happens in second period, D_1^P rises as effect not yet realised and D_2^P falls to compensate
        - $\frac{d\tau_1}{dY_2} = \frac{d\tau_2}{dY_2} = -\frac{1}{1+r}\frac{G_1 + \frac{G_2}{1+r}}{\left(Y_1 + \frac{Y_2}{1+r}\right)^2} < 0 | \frac{dD_1^P}{dY_2} = 0 - \frac{d\tau_1}{dY_2} > 0 | \frac{dD_2^P}{dG_1^C} = -(1+r)\frac{dD_1^P}{dY_2} < 0$

- Always draw graphs for these!



- Now assume $G_1^C = G_2^C = \tilde{G}^C = \bar{G}^c$ and $Y_1 = Y_2 = \tilde{Y} = \bar{Y}$. Now get $D_1 = G_1^I$. Government should only borrow to invest (can invest as golden rule applies)
    - Define $d_t = \frac{D_t}{Y_t}$. Then optimal deficit ratio: $d_1 = \left(\frac{G_1^C}{Y_1} - \frac{\tilde{G}^C}{\tilde{Y}}\right) + \frac{G_1^I}{Y_1}$
    - Ignoring government investment, should borrow iff $\frac{G_1^C}{Y_1} > \frac{\tilde{G}^C}{\tilde{Y}}$ (i.e. government consumption temporarily high or output temporarily low)
        - Tax rate smoothing leads to countercyclical government deficit acting as 'automatic stabilizer'. Tax revenues decline/rise in recessions/booms
    - Consider effect of anticipated drop in future output Y_2 (e.g. Brexit)
        - Drop in future output $Y_2' < Y_2$ increases optimal tax rate $\bar{\tau}' > \bar{\tau}$
        - Requires immediate increase in τ_1 to smooth tax rates, and reduction in budget deficit D_1, with $D_1' - D_1 = -(\bar{\tau}' - \bar{\tau})Y_1 < 0$

3.3 GOVERNMENT DEBT DYNAMICS

- For simplicity assume $G_t^I = 0, R_t^I = 0, G_t = G_t^C, S_t = 0$. Thus constraint $G_t + rB_t = T_t + \Delta B_t$
- Typically worry about b not B because it better describes governments ability to repay
- Reinhart-Rogoff noted that $b = 0.9$ is a worrying cut-off point where debt becomes unsustainable. But in 2013 methodology became under serious scrutiny

3.3.1 MODEL

- Steady state b^* is level of b s.t. $\Delta b = 0$. Note when $b = 0$ we are at $\Delta b = d^p$

Solving for Steady State

- $\Delta B = G - T + rB$ and $b = \frac{B}{Y}$. Hence
    - $\hat{b} = \hat{B} - \hat{Y} = \frac{G-T+rB}{B} - g = \frac{G-T}{Y}\frac{Y}{B} + r - g = d^p\frac{1}{b} + r - g$
    - $\Delta b = \hat{b}b = d^p + (r-g)b$ and at steady state $\Delta b = 0$ have $b^* = \frac{d^P}{g-r}$
- Let us interpret each term of $\Delta b = d^p + (r-g)b$
    - d^p: Fundamental component. Increase in primary deficit increases government deficit 1:1 (i.e. rate of debt accumulation)
    - rb: d^p excluded interest payments and this adds it back in
    - $-gb$: Adds the denominator effect as the economy grows
- In $(b, \Delta b)$ space: x-intercept is b^*, y-intercept is d^p, and slope is $(r-g)$

Checking for stability

- Do so graphically noting following definitions:
    - Stable: Converge back to ss for shocks in both directions
    - Semi-stable: Converge back to ss for shocks in one directions
    - Unstable: Diverge from ss for shocks in both directions
    - Globally Stable: Converge to a ss from any starting point
    - Locally Stable: Converge to a ss from starting points in some area
    - Explosive: Never converge to a ss from any starting point

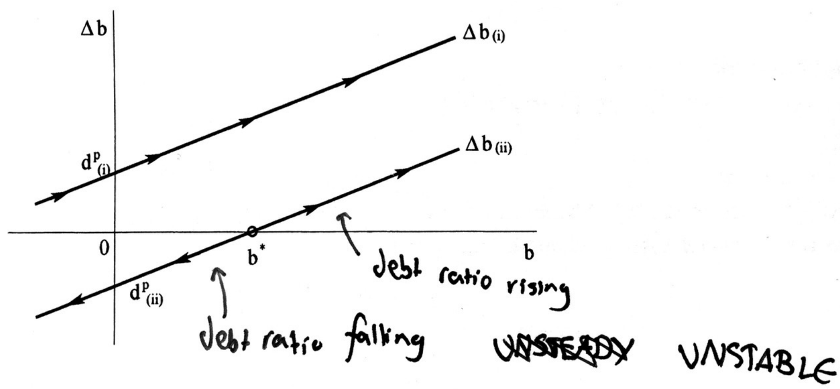
Case $r > g$

- Note line is upwards sloping. Debt-ratio has a natural tendency to increase over time as rate at which debt accumulates due to interest outpaces rate of growth
- Total volume effect depends on rate $(r-g)$ and stock b

- For large values $b > b^*$ this is strong enough to overpower surplus ($d^p < 0$) and thus increase b. Diverge from left to right: $(r-g)b > d^p$. Alarming!!
- For small values of $b < b^*$ this is offsetting but not sufficient to overturn surplus (d^p>0). b decreases but at a decreasing rate. Converge right to left: $(r-g)b < d^p$
- Note $b^* = \frac{d^p}{g-r} > 0$ under primary surplus $d^p < 0$. So even if have primary surplus once $b > b^*$ we are unstable

Phase line for $\Delta b = d^p + (r-g)\,b$ in case A: $r > g$

(i) Primary deficit $d^p_{(i)} > 0$ $\rightarrow$ explosive debt ratio b (debt crisis)



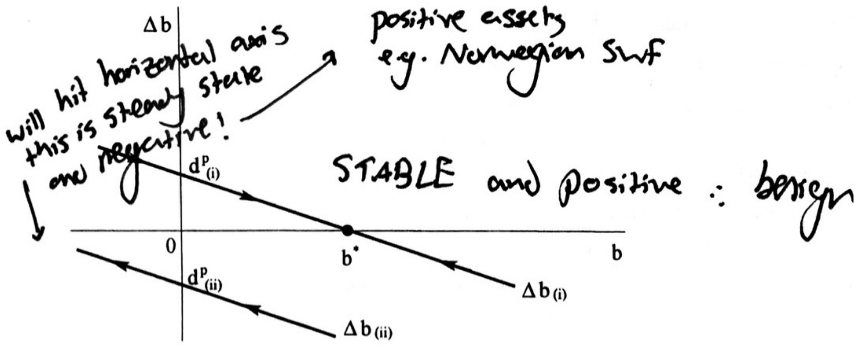
(ii) Primary surplus $d^p_{(ii)} < 0$ $\rightarrow$ unstable steady state $b^* > 0$

Case $r < g$

- Note line is downwards sloping. Debt-ratio has a natural tendency to decline over time as national income outgrows rate at which debt accumulates due to interest.
- Total volume effect depends on rate $(r-g)$ and stock b
  - For large values $b > b^*$ this is strong enough to overpower deficit ($d^p > 0$) and thus decrease b. Convergence from right to left: $(r-g)b > d^p$. Benign!!
  - For small values of $b < b^*$ this is offsetting but not sufficient to overturn deficit ($d^p > 0$). b increases but at a decreasing rate. converge left to right: $(r-g)b < d^p$
  - Note $b^* = \frac{d^p}{g-r} > 0$ under primary deficit $d^p < 0$. So even if have primary deficit there is a self-correcting mechanism

Phase line for $\Delta b = d^p + (r-g)\, b$ in case B: $r < g$

(i) Primary deficit $d^p_{(i)} > 0 \quad \rightarrow$ stable steady state $b^* > 0$



(ii) Primary surplus $d^p_{(ii)} < 0 \quad \rightarrow$ decreasing debt ratio b (to $b^* < 0$)

Case $r = g$

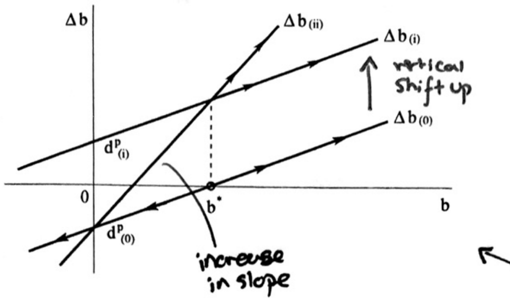
- If $r = g$ then $\Delta b = d^p$
    - b rising if $d^p > 0$; b falling if $d^p < 0$;
    - $d^p = 0$ implies $\Delta b = 0$ so infinitely many steady states b^*
- On knife edge but… unlikely to happen as both are continuous variables

3.3.2 Improving Government Debt Dynamics

♠ Understanding government debt crises - case A: $r > g$

Recall: $\Delta b = d^p + (r - g)\, b$

For country with $r > g$ and primary surplus $d^p < 0$
at (unstable) steady state $b^* = \frac{-d^p}{r-g} > 0$:
(i) Increase in d^p (even if temporary) results in explosive b.
(ii) Rise in r or decline in g (even if temporary) results in explosive b.



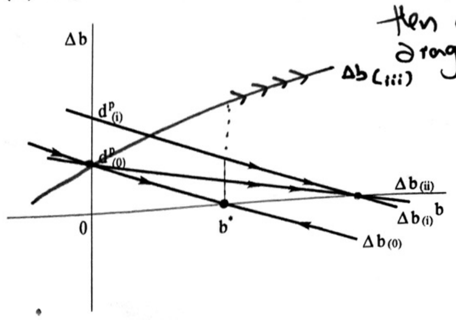
even if temporary!
(shift back but from $b > b^*$ so spiral

♠ Understanding government debt crises - case B: $r < g$

Recall: $\Delta b = d^p + (r - g)\, b$

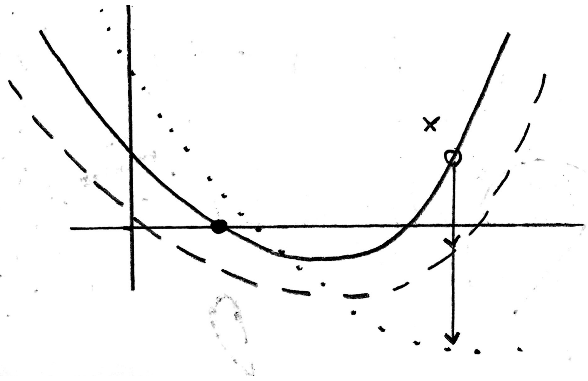
For country with $r < g$ and primary deficit $d^p > 0$
at (stable) steady state $b^* = \frac{d^p}{g-r} > 0$:
(i) Increase in d^p results in increase in b to higher b^*.
(ii) Small rise in r or decline in g results in increase in b to higher b^*. } higher b^* but stay
(iii) Larger rise in r or decline in g such that $r' > g'$ results in explosive b.

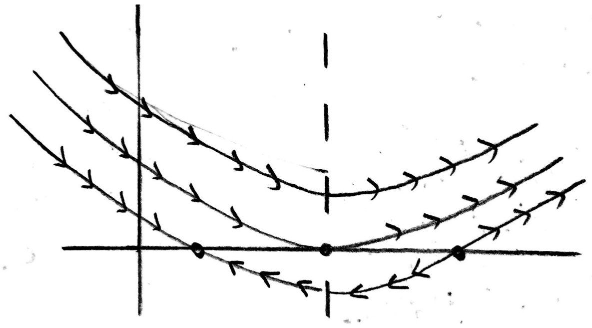
then will flip to above diagram



Nominal vs. Real Debt

- Recall dynamics $\Delta b = d^p + (r - g)b$.
- If debt issued in nominal terms then $\Delta b = d^p + (i - \pi - g)b$
- Thus…
    1. Lower primary deficit d^p (fiscal consolidation, austerity)
    2. Lower nominal interest rate i (e.g. LSAPs) or higher inflation π (inflating away debt; expansionary MP). Both lower r
    3. Higher output growth (outgrowing debt; e.g. structural reform)
    4. Lower debt ratio b (debt restructuring)
- 1. Can be seen via dashed line; 2.-3. Can be seen via dotted line. 4. is a movement along





Risk Premium

- Assume country with initially $r < g, d^p > 0, b_0 = b^* = \frac{d^p}{g-r} > 0$ (stable)
- Suppose rise in sovereign risk premium ρ so $r_1 = r + \Delta\rho > g$. Creates quadratic!!

- E.g. European Sovereign Debt Crisis 2010-12
    - Financial crisis increased deficit d^p and reduced output g
    - Bank bailouts increased public debt b (especially Ireland, Spain)
    - Worries about sovereign debt increased risk premium, raising r
    - Policy responses: austerity (decreased d^p), ECB announced OMT (reduce r), sovereign debt restructuring in Greece (decreased b)

3.4 Political Approach

Alesnia and Tabellini (1990)
Deficit bias in fiscal policy due to strategic use of government debt by political parties

Political business cycle: Macroeconomic fluctuations induced by political considerations and electoral cycle

Opportunistic policymakers try to boost economy before election to improve re-election chances (e.g. Nordhaus (1975) model has short-sighted voters with adaptive expectations re-elect incumbent if economy is doing well, resulting in boom-bust around election)

Partisan policymakers cause business cycles around elections with liberal/conservative government yielding increase/decrease in output and inflation (Alesnia, 1987)

Also see Pol notes!!

3.4.1 Deficit Bias (Alesnia & Tabellini, 1990)

Set Up

- Two political parties (L, R) with preferences $U^i(G^i, G^j) = \ln G^i$ over public goods G^L, G^R
    - i.e. only care about spending on own interest
- Two periods. Incumbent in period 1 maximizes: $V^I = \ln G_1^I + \beta\rho \ln G_2^I$
    - β is intertemporal discount factor; ρ probability of re-election in period 2
- Budget constraint $G_1 = \tau Y_1 + D$ and $G_2 + D = \tau Y_2$ (where $G_t = G_t^L + G_t^R$; $Y_2 \geq Y_1$)

Solving

- $\max V^I = \ln G_1^I + \beta\rho \ln G_2^I == \ln(\tau Y_1 + D - G_1^O) + \beta\rho \ln(\tau Y_2 - D - G_2^O)$
- As $\frac{dV^I}{dG_t^O} < 0$, know incumbent I sets $G_t^O = 0$ and $G_t = G_t^I$
- Thus solve $V^I = \ln(\tau Y_1 + D) + \beta\rho \ln(\tau Y_2 - D)$...
    - FOC wrt D: $\frac{1}{\tau Y_1 + D} = \beta\rho \frac{1}{\tau Y_2 - D}$ rearranging to $D = \frac{\tau(Y_2 - \beta\rho Y_1)}{1+\beta\rho} > 0$

Key-Results

- $D = \frac{\tau(Y_2 - \beta\rho Y_1)}{1+\beta\rho} > 0$. This has two components
    - Consumption Smoothing: Even if $\rho = 1$ have $D = \frac{\tau(Y_2 - Y_1)}{1+\beta} \equiv D_{\min}$. Since $Y_2 > \beta Y_1$ and convex preferences there is an innate deficit to allow for smoothing
    - Deficit Bias: Explains remaining $D - D_{\min}$. Incumbent strategically overspends on desired public good to constrain opposing party of not re-elected
        - Heterogenous preferences means care about being re-elected
        - $\rho < 1$ means not sure of being re-elected with bias decreasing in ρ
- Model doesn't just describe politics but also private sector budget spending!

3.4.2 Political Business Cycle: Partisan Model (Alesnia, 1987)

Assumptions

- Objective Function: Government of party $P \in \{L, R\}$ sets Y_1, Y_2 to max $V^P = U_1^P + \beta U_2^P$
    - $U_t^P = -\frac{1}{2}(\pi_t - \pi^*)^2 + \lambda^P(Y_t - \bar{Y})$
    - $\lambda^L > \lambda^R \geq 0$
- Phillips Curve $\pi_t = \pi_t^e + \theta(Y_t - \bar{Y})$
    - Assume no shocks, $\theta > 0$, private sector ratex formed at end $t-1$

Solving

- Sub PC into OF:

$$U_t^P = -\frac{1}{2}(\pi_1^e + \theta(Y_1 - \bar{Y}) - \pi^*)^2 + \lambda^P(Y_t - \bar{Y}) - \beta\frac{1}{2}(\pi_2^e + \theta(Y_2 - \bar{Y}) - \pi^*)^2 + \beta\lambda^P(Y_2 - \bar{Y})$$

- FOC wrt Y_2 (given π_t^e): $= -\theta\beta(\pi_2^e + \theta(Y_2 - \bar{Y}) - \pi^*) + \beta\lambda^P = 0$
    - Simplify $\theta(\pi_2 - \pi^*) = \lambda^P$ and rearrange $\pi_2^P > \pi^* + \frac{\lambda^P}{\theta} > \pi^*$ to show π-bias!!
    - Using ratex $\pi_2^e = E[\pi_2^P|P] = \pi^* + \frac{\lambda^P}{\theta}$ (no uncertainty so perfect foresight)
    - Substitute into PC: $Y_2^P = \bar{Y}$
- FOC wrt Y_1(given π_t^e): $= -\theta(\pi_1^e + \theta(Y_1 - \bar{Y}) - \pi^*) + \lambda^P = 0$
    - Simplify $\theta(\pi_1 - \pi^*) = \lambda^P$ and rearrange $\pi_1^P > \pi^* + \frac{\lambda^P}{\theta} > \pi^*$ to show π-bias!!
    - Using ratex $\pi_1^e = E[\pi_1^P] = p\pi_1^L + (1-p)\pi_1^R = \pi^* + \frac{[p\lambda^L + (1-p)\lambda^R]}{\theta}$
    - Using PC: $Y_1^P = \bar{Y} + \frac{\{\pi_1^P - \pi_1^e\}}{\theta} = \bar{Y} + \frac{\{\lambda^P - [p\lambda^L + (1-p)\lambda^R]\}}{\theta^2}$
        - Note $Y_1^R < \bar{Y}; Y_1^L > \bar{Y}$

Key Results

- Electoral uncertainty leads to political business cycle
    - $Y_1^L > \bar{Y} > Y_1^R; Y_2^L = \bar{Y} = Y_2^R$
    - $\pi_1^L = \pi_2^L = \pi^L; \pi_1^R = \pi_2^R = \pi^R$ where $\pi^L = \pi^* + \frac{\lambda^L}{\theta} > \pi^* + \frac{\lambda^R}{\theta} = \pi^R$
- Election of L leads to temporary boom, R recession. Party L leads to worse inflation bias

- Evidence from Alesnia, Roubini, and Cohen (1997):
    - Opportunistic models: MP not more expansionary during election years, although opportunistic cycle in fiscal policy with pre-election boosts
    - Partisan models: Systematic differences in growth, inflation and unemployment between Democratic and Republican administrations. Size of political cycle increasing in degree of election surprise

[Add to this from Pol]

3.5 Fiscal Policy Rules and Frameworks

3.5.1 Prudent Fiscal Policy

- Using debt dynamic $\Delta b = d^p + (r - g)b$ and primary deficit $D_t^p = G_t - T_t$ note that $\Delta b = \left(\frac{G-T}{Y}\right) + (r - g)b$ [1]
- Using prudent fiscal policy rule $\Delta b < 0$ in normal times note that $\left(\frac{\tilde{G}-\tilde{T}}{\tilde{Y}}\right) + (r^n - g^n)b \leq 0$ [2]
- Using tax smoothing result $\frac{T}{Y} = \bar{\tau} = \frac{\tilde{T}}{\tilde{Y}}$ note [1]-[2]: $\Delta b \leq \left(\frac{G}{Y} - \frac{\tilde{G}}{\tilde{Y}}\right) + [(r - r^n) - (g - g^n)]b$
    - $\Delta b \leq 0$ required if $\frac{G}{Y} \leq \frac{\tilde{G}}{\tilde{Y}}$, $r \leq r^n$, $g \geq g^n$
    - Limited $\Delta b > 0$ allowed if $\frac{G}{Y} > \frac{\tilde{G}}{\tilde{Y}}$, $r > r^n$, or $g < g^n$

FOR REGIMES SEE LECTURE NOTE 8. LEARN OFF BY HEART!