SOCIAL COST BENEFIT ANALYSIS

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BASIC MODEL

CONTEXT

- Social cost benefit analysis is about selection of a policy from a set of possible alternatives to be implemented (i.e. ex ante)
    - Contrast to policy evaluation, which is about evaluating the consequences of a policy (i.e. ex post)
    - Consistent procedure when market prices need not fully reflect social costs and benefit
    - Applying different weights to the gains and loses can make a big difference! Thus important to scrutinise assumptions
    - IRL: Green Book in UK, Little-Mirrlees manual in developing world
- Fundamental Principle: undertake a project iff $\Delta SWF > 0$ but we do not observe individual utility functions so cannot calculate this directly. Can we instead use market outcomes?

ASSUMPTIONS

1. Many identical consumers, $h = 1, \dots, H$
2. Two private goods $x = (x_1; x_2)$ sold in markets with consumer prices $q = (q_1; q_2)$ and producer prices $p = (p_1; p_2)$
3. Direct utility function of a representative consumer $U(x_1; x_2)$ subject to budget constraint $q_1 x_1 + q_2 x_2 = m$ (m includes any distributed profit or reimbursed tax payments)
4. Maximization (solving via Lagrange) gives optimal condition of consuming until $\frac{\delta U}{\delta x_j} = \lambda q_j$ where λ is private marginal utility of income and thus same for both equations
5. Two goods are produced with labour l_0 and capital k at prices p_0 and p_k: $x_j = F^i(l_0^j, k^j)$. Total supplies of inputs are fixed and amounts used respectively given by l_0^j, k^j
6. Social welfare is utilitarian: $SWF(U_1, \dots, U_H) = \sum_{h=1} U_h$

- Model a small [government] project as increases supply of x_1 by $\Delta x_1 > 0$ per capita via reallocating resources from production of x_2 that thus decreases by $\Delta x_2 > 0$ per capita
    - This is the opportunity cost: by using resources to the project the option to use those resources in the next-best employment is foregone

RESULT 1: CORE EVALUATION

Utility Calculation

- We do project iff $\Delta SWF > 0$ and decompose this by defining shadow price: marginal social value $v_j = \frac{\delta SWF}{\delta x_j} = H \frac{\delta SWF}{\delta U} \frac{\delta U}{\delta x_j}$
    - Formally, "product of the marginal utility of the extra output and the increase in social welfare per unit of utility summed over the H individuals"
- Hence $\Delta SWF > 0 \Leftrightarrow v_1 \Delta x_1 - v_2 \Delta x_2 = H \frac{\delta SWF}{\delta U} \frac{\delta U}{\delta x_1} \Delta x_1 - H \frac{\delta SWF}{\delta U} \frac{\delta U}{\delta x_2} \Delta x_2 > 0$
- But… do not observe individual utility functions so cannot calculate this directly
- Can we instead use market outcomes?

Market Calculation

- But we do observe market outcomes so instead consider using $q_1 \Delta x_1 - q_2 \Delta x_2 > 0$
- Substitute in optimality condition from (4.) $q_j = \frac{1}{\lambda} \frac{\delta U}{\delta x_j}$

- Now $q_1\Delta x_1 - q_2\Delta x_2 = \frac{1}{\lambda}\frac{dU}{dx_1}\Delta x_1 - \frac{1}{\lambda}\frac{dU}{dx_2}\Delta x_2 > 0$

- Now note $\boldsymbol{\frac{1}{\lambda}\frac{\delta U}{\delta x_1}\Delta x_1 - \frac{1}{\lambda}\frac{\delta U}{\delta x_2}\Delta x_2 > 0}$ **iff** $\boldsymbol{\frac{1}{\lambda}\left(H\frac{\delta SWF}{\delta U}\frac{\delta U}{\delta x_1}\Delta x_1 - H\frac{\delta SWF}{\delta U}\frac{\delta U}{\delta x_2}\Delta x_2\right) > 0}$
  - As all individuals are identical $U_i = U$ and we consider individualistic SW, an increase in individual U corresponds to an increase in SW
  - If we are utilitarian then also know that $\frac{\delta SWF}{\delta U} = 1$
- **Market value reveals social value. SW increasing if difference between market value of project output and goods forgone is positive:** $\boldsymbol{q_1\Delta x_1 - q_2\Delta x_2 > 0 \Leftrightarrow \Delta \mathrm{SWF} > 0}$

- Shadow prices are proportional to market prices: $v_j = H\frac{\delta SWF}{\delta U}\lambda p_j$ where $H\frac{\delta SWF}{\delta U}\lambda$ is the same for all goods. But implies market prices need adjustment before used as shadow
  - If identical consumers and utilitarian SWF this simplifies to $v_j = \lambda p_j$

Note on the Numeraire:

- Numeraire is unit of account (good relative to which other's value is measured). Choice is arbitrary but must be applied systematically
- If shadow price is defined in monetary terms $q_1^{sp} = \frac{v_1}{\lambda} = \frac{1}{\lambda}\frac{dU}{dx_1}$ (as per Boardman), the numeraire is $\frac{1}{\lambda}$, private income valued at marginal private value of income

Result 2: Overcoming issue with Δqx_2

- IRL may be impossible to work out how much (consumption) goods must be sacrificed. Instead consider using market values of inputs
- Value of foregone production of x_2 is diverged labour + capital:
  - $q_2\Delta x_2 = -q_2\left(\frac{\delta F^2}{\delta l_0^2}\Delta l_0^2 + \frac{\delta F^2}{\delta k^2}\Delta k^2\right) = q_2\left(\frac{\delta F^2}{\delta l_0^2}\Delta l_0^1 + \frac{\delta F^2}{\delta k^2}\Delta k^1\right)$
    - Note $\frac{\delta F^2}{\delta X^2}$ is MP product of X
    - As $\Delta l_0^1 = -\Delta l_0^2; \Delta k^1 = -\Delta k^2$
  - $q_2\Delta x_2 = \frac{q_2}{p_2}(p_0\Delta l_0^1 + p_k\Delta k^1)$ as
- Profit maximization by price taking firms in competitive factor markets states FoP employed until value MP equals producer price: $p_2\frac{\delta F^2}{\delta l_0^2} = p_0$ and $p_2\frac{\delta F^2}{\delta k^2} = p_k$
  - $q_1\Delta x_2 = \frac{q_2}{p_2}(p_0\Delta l_0^1 + p_k\Delta k^1)$
- If producer and consumer prices coincide this simplifies $\boldsymbol{q_1\Delta x_2 = p_0\Delta l_0^1 + p_k\Delta k^1}$**. Market value of foregone consumption equals market value of inputs used**

Fundamental Principle

- Combining above we obtain **if** $\boldsymbol{q_1\Delta x_1 - \left(p_0\Delta l_0^1 + p_k\Delta k^1\right) > 0}$ **then** $\boldsymbol{\Delta SWF > 0}$ assuming (i) price taking behaviour, (ii) no taxes or subsidies, and (iii) project is small
- But… any three of those could go wrong

Short Essay Question: Can we use market prices as shadow prices

No. Unless very specific circumstances.

- For this we require to write Net Benefit (NB) $= kq_1\Delta x_1 - kq_2\Delta x_2$
  - And we then simply set k as the numeraire

- We note we have big SWF formal $\left(\sum \frac{\delta SWF}{\delta U^h}\frac{\delta U^h}{\delta x_1^h}\Delta x_1^h\right) - \left(\sum \frac{\delta SWF}{\delta U^h}\frac{\delta U^h}{\delta x_2^h}\Delta x_2^h\right)$
- Project impacts everyone the same $\Delta x_1^h = \frac{1}{H}\Delta x_1$
    - Sufficient as now $NB = \left(\sum \frac{dSWF}{U_h}\lambda^h\right) q_1 \Delta x_1 - \left(\sum \frac{dSWF}{U_h}\lambda^h\right) q_2 \Delta x_2$ where () is k
    - Intuitively, don't know if people (thus society) like more money but constant for all goods so just see if more
- OR $\lambda^h = \lambda \forall h$ AND utilitarian SWF with equal weights $\frac{dSWF}{U_h} = 1$
    - Sufficient as now $NB = (\sum \Delta x_1^h)\lambda q_1 - (\sum \Delta x_2^h)\lambda q_2 = \lambda q_1 \Delta x_1 - \lambda q_2 \Delta x_2$
    - Intuitively, people are affected differently but same value of money, hence same benefit of extra GP and society values this same way regardless who gets it

Limitations: What might go wrong?

- May need to adjust observed market price to get an accurate estimate of the shadow price:
    - Could be that government project is not small so affects prices. If large, which market price should we use? Pre- or post-project or combination? If post how to estimate it?
    - Could be that input markets are not competitive. If not then market prices do not reflect social value and cannot be used to as shadow prices
    - Inputs and outputs could be subject to taxes and subsidies or financed through distortionary taxation. Need to adjust for this
    - Could be that uncorrected externalities are not reflected in market prices. Need to adjust for this
- Also fundamental assumptions:
    - Output could not be traded in a market. Must either use survey data on willingness to pay or try deduce from other existing market prices
    - Impact of projects and policies may be different for different groups of individuals may want to take such distributional considerations into account
    - Costs and benefits accrue over time and this must be factored into the analysis. In particular, one must decide on a social discount rate

Dealing with Distortions

"Boardman Method" for Price Effects

- Assumes throughout:
    - Private income as numeraire
    - Shadow price is GSB/unit, so now average not marginal unit
    - Demand and supply curves are (locally) linear (thus simply calculate areas of squares and triangles)
    - Utility structure is quasi-linear (does not need to distinguish compensated and uncompensated demand)
- $\Delta SS = \Delta SS_{\text{output}} + \Delta SS_{\text{input}} + \Delta SS_{\text{secondary}}$
    - $\Delta SS_{\text{output}} = \gamma_g \Delta R + \Delta CS + \Delta PS$ [revenue]
    - $\Delta SS_{\text{input}} = \gamma_g \Delta E + \Delta CS + \Delta PS$ [expenditure]
    - For simplicity assume no marginal excess tax burden so $\gamma_g = 1$ and no secondary market effects so $\Delta SS_s = 0$. Both are notable limitations though!

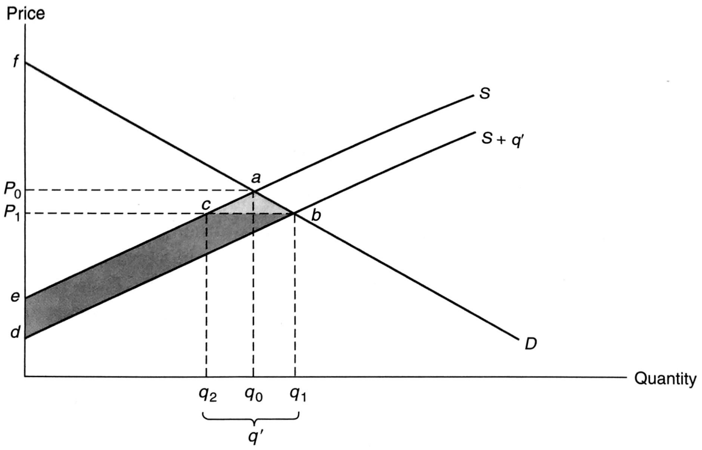
IMPORTANT NOTE: Only consider some distortions (taxes and externalities). SCBA Textbook Chp 5 and 6 give a rich categorization.

- Reason for discussing market failures is that their presence provides prima farce rationale for most programs in the first place!

IMPORTANT NOTE: below is simple graph analysis. Combine with tax revenue discussion (ignoring tax terms) to see critical interaction between the two

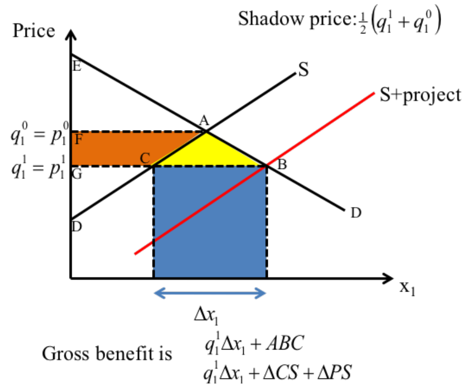
OUTPUT MARKETS

- No price effect assumption is only sensible at small quantities for homogenous goods traded in large, national markets
- In undistorted market shadow price is average of pre- and post-project: $\frac{1}{2}(q_1^1 + q_1^0)$
  IRL do not know q_1^1 ex-ante so need knowledge of supply and demand elasticities



Social surplus change (ignoring costs of project inputs to the government):
Project (a): Direct increase in supply of q'—gain of triangle *abc* plus project revenue equal to area of rectangle q_2cbq_1
Project (b): Supply schedule shift through cost reductions for producers—gain of trapezoid *abde*

Figure 5.2 Measuring impacts in an efficient market with price effects.



INPUT MARKETS

- Look at drastic differences between perfectly elastic and special case inelastic supply curve
  - Latter seeks market prices underestimate because do not consider opportunity cost of removing land from private sector (i.e. destroying CS)

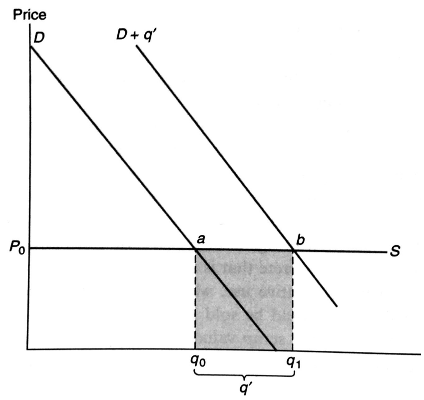


Figure 6.1 Opportunity costs with no price effects.

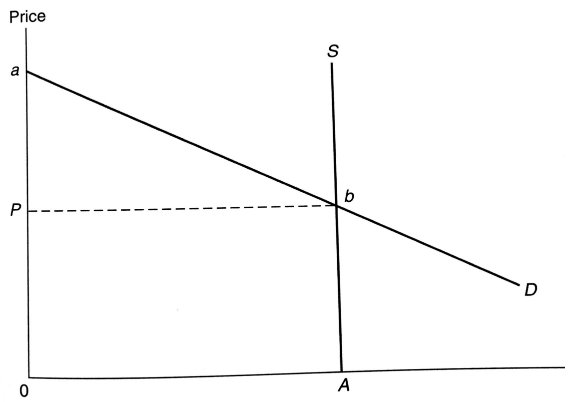
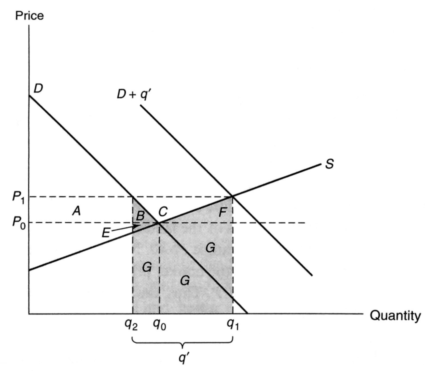


Figure 6.2 Opportunity costs with inelastic supply curve.

- Now look at what happens with some price effect:



	Gains	Losses	Net gains
Consumers		$A + B$	$-\{A + B\}$
Producers	$A + B + C$		$A + B + C$
Government		$B + C + G + E + F$	$-\{B + C + G + E + F\}$
Society			$-\{B + G + E + F\}$

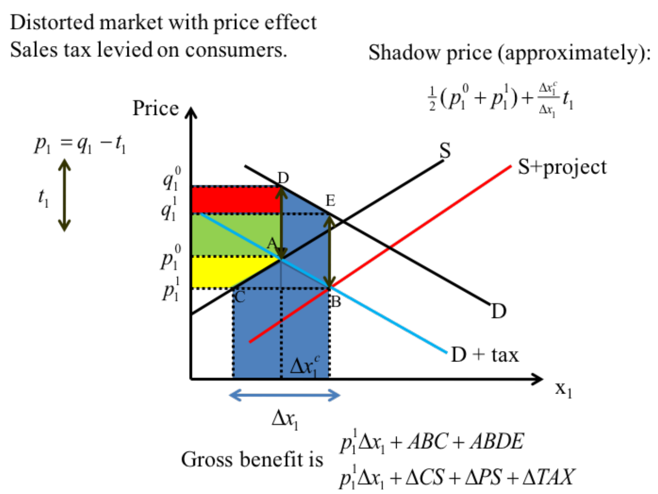
TAXES AND SUBSIDIES

- Extend framework with following assumptions:
    - Indirect utility function of representative consumer : $V(q_1, q_2, m) = \max_{x_1, x_2} U(x_1, x_2)$ s.t. $q_1x_1 + q_2x_2 = m$
    - Individual accrues income through firm profits (due to DRTS) and sales tax ($q_1 = p_1 + t_1$). Thus $m = \pi(q_1 - t_1,\ p_2, p_0) + t_1x_1(q_1, q_2, m) + \cdots$
    - Assume one input l_0 at price p_0 to produce two outputs

- Consider project increases x_1 by Δx_1 and causes consumer price q_1 to be pushed down a little. This has welfare effect: $\frac{\Delta V}{\Delta q_1} = \frac{\delta V}{\delta q_1} + \frac{\delta V}{\delta m}\frac{\delta m}{\delta q_1} = \frac{\delta V}{\delta q_1} + \lambda\left[\frac{\delta \pi}{\delta p_1} + t_1\frac{\delta x_1}{\delta q_1}\right]$
- Use (1) Roy's identity and (2) Hotelling's Lemma to rewrite (see Part I for derivation)

    - Roy's identity says $\frac{\delta V}{\delta q_1} = -\lambda x_1$
    - Hotelling's Lemma says $\frac{\delta \pi}{\delta p_1} = y_1$ where y_1 is supply function of x_1
    - Thus $\frac{\Delta V}{\Delta q_1} = -\lambda x_1 + \lambda\left[y_1 + t_1\frac{\delta x_1}{\delta q_1}\right] = \lambda(y_1 - x_1) + \lambda t_1\frac{\delta x_1}{\delta q_1} = 0$
        - **Welfare effect of small price change is extra tax revenue generated**
        - No change in social surplus as supply = demand in equ. so $\Delta\text{CS} = \Delta\text{PS}$

- Thus $GSB = p_1\Delta x_1 + \Delta CS + \Delta PS + \Delta\text{tax} = p_1\Delta x_1 + \frac{\Delta V}{\lambda} \approx \boldsymbol{p_1\Delta x_1 + t_1\Delta x_1}$ and shadow price is given by $v_1 = \frac{GSB}{\Delta x_1} = p_1 + t_1$



- Gives us following interpretation:
  - If no tax $t_1 = 0$ effect of (small) price reduction is zero ($\frac{\Delta V}{\Delta q_1} = 0$) as S=D ($y_1 = x_1$)
    - Intuitively, consumer's surplus gain netted by producer's loss. Can use the pre-project price in calculation even if there is a (small) price effect.
  - If there is tax $t_1 > 0$ we have effect even for small changes: $\frac{\Delta V}{\Delta q_1} = \lambda t_1 \frac{\delta x_1}{\delta q_1}$
    - Intuitively, now have extra tax revenue collected when tax base expands as q_1 falls. Have to adjust the pre-project producer price for shadow price

EXTERNALITIES

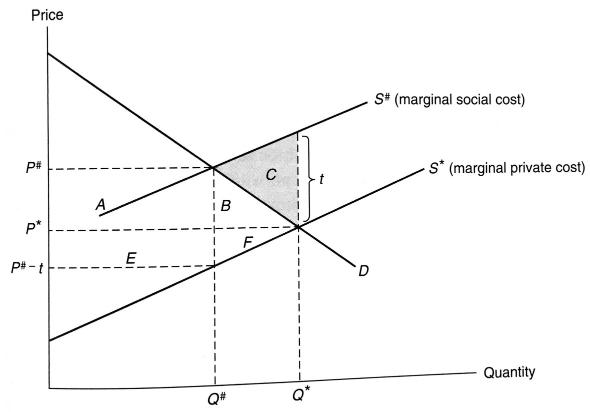
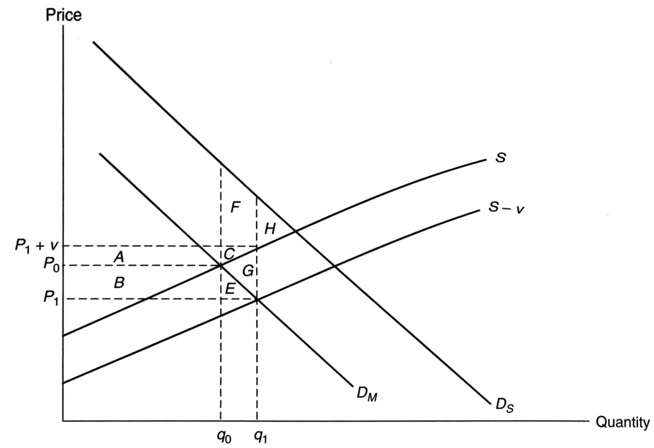


Figure 5.6 Negative externality.

	Gains	Losses	Change
Consumers of good		A + B	–(A + B)
Producers of good		E + F	–(E + F)
Third-party consumers	B + C + F		B + C + F
Government revenue	A + E		A + E
Society			C



Gain to consumers in target neighborhood:	$B + E$
Gain to persons in nearby neighborhood:	$C + G + F$
Gain to producers:	$A + C$
Program costs:	$A + B + C + G + E$
Net benefits:	$C + F$

Figure 5.7 Social benefits for direct supply of a good with a positive externality.

CONTROL AREA MATTERS

- Note scope of policy-maker varies and takes place in the context of the second best: tax structure, market imperfections, externalities etc. may be controlled by higher government
- Extend framework with following assumptions:
  - Production of the good generates externality with constant social marginal damage d per x_1 produced (measured in money)
  - Assume demand curve for x_1 is horizontal at q_1

- We see this matters. In narrow analyst need to adjust market price for the externality; in broad can use market price because the externality is already reflected in the market price

- Narrow control gross benefit is $p_1\Delta x_1 - d\Delta x_1$ so shadow price $p_1 - d = q_1 - d$
- Broad control optimal externality tax is to set $t_1 = d$. Thus gross benefit is just $p_1\Delta x_1$ and the shadow price is $p_1 = q_1 - t_1 = q_1 - d$

Figure 1: Narrow control area

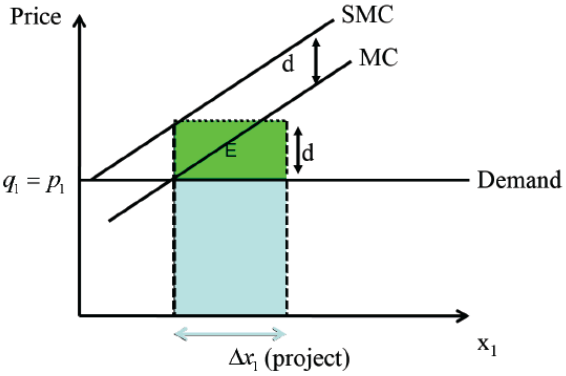
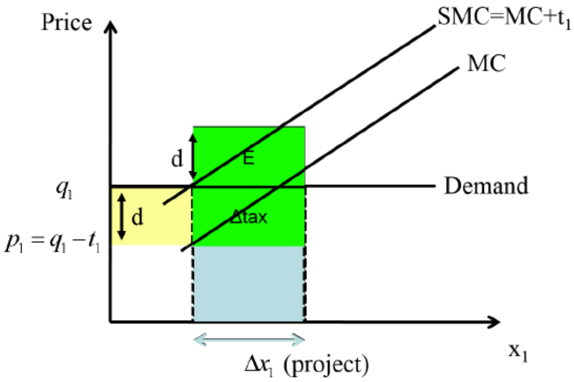


Figure 2: Narrow control area



Type of intervention	Efficient markets	Inefficient markets
Change in input markets (Concept: value costs as the opportunity cost of the purchased resources.)	If supply schedule is flat, value cost as direct budgetary expenditure. (Example: purchase of materials from a competitive national market.) If supply schedule is not flat, value cost as direct budgetary expenditure less (plus) any increase (decrease) in social surplus in market. (Example: purchases of materials from a competitive local market.)	Value costs as direct budgetary expenditure less (plus) any increase (decrease) in social surplus in market. (Examples: hiring unemployed labor; purchases of materials from a monopoly.)
Changes in output markets (Concept: value benefits as WTP for the change and costs as WTP to avoid the change.)	Value change as net change in social (i.e., consumer and producer) surplus plus (less) any increase (decrease) in government revenues. (Example: government provision of goods and services to consumers or producers.)	Value change as net change in social (i.e., consumer, producer, and third party) surplus plus (less) any increase (decrease) in government revenues. (Example: tax or subsidy in market with externality.)

IMPORTANT NOTE: For Small Open Economy and HS2 application see Lecture 3

Distribution

Motivation

The Problem

- So far, in assuming agents are identical, we have implicitly assumed Kaldon-Hicks potential compensation test: If total benefits are greater than total costs, program is justified
- Consider now following changes in assumptions:
  - 1. Economy is populated by different consumers
  - 3. Directly utility function $U_h(x_1^h, x_2^h)$ and budget constraint $q_1x_1^h + q_2x_2^h = m^h$
    - In practice may also face different prices!

- 6. SWF is individualistic but no longer necessarily equal: $\frac{dSWF}{dU_h} \geq 0$

- Now we see that just using market prices (and ignoring distributive analysis) cannot generally be true
  - $\Delta SWF = \sum^H \frac{dSWF}{dU_h}\frac{dU_h}{dx_1^h}\Delta x_1^h + \sum^H \frac{dSWF}{dU_h}\frac{dU_h}{dx_2^h}\Delta x_2^h$
  - $\Delta SWF = \sum^H \frac{dSWF}{dU_h}\lambda^h q_1 \Delta x_1^h + \sum^H \frac{dSWF}{dU_h}\lambda^h q_2 \Delta x_2^h$
  - $\Delta SWF = \sum^H \beta_h^b \Delta x_1^h + \sum^H \beta_h^c \Delta x_2^h$
- When can we ignore distributive effects? Consider three conditions:
  1. Projects impact same $\Delta x_i^h = \Delta x_i \ \forall h$
  2. Consumers same $\lambda_h = \lambda \ \forall h$
  3. SWF values increments equally $\frac{dSWF}{dU_h} = \frac{dSWF}{dU} \ \forall h$
  - If all hold get $\Delta SWF = H\frac{dSWF}{dU}\lambda(q_1\Delta x_1 - q_2\Delta x_2)$, sufficient but not necessary
  - If 1. or 2.&3. hold also can ignore these (see Lecture note 2)

Justifying Distributive Analysis

1. Diminishing marginal utility of income $\frac{\Delta U_l}{\Delta y_l} > \frac{\Delta U_h}{\Delta y_h}$
   - Still treat people equally, just noting helping poor gives more bang for our buck
2. Inequality aversion because of riots, human dignity, relative utility etc. $\frac{\Delta SWF}{\Delta y_l} > \frac{\Delta SWF}{\Delta y_h}$ even if $\frac{\Delta U_l}{\Delta y_l} \ngtr \frac{\Delta U_h}{\Delta y_h}$. This contradicts KH directly as should not do project even if efficient
   - See Lecture note 2 for discussion of inequality aversion equation $g(y) = \frac{y^{1-a}}{1-a}$
3. One person one vote: Rich have more income -> Spend more -> more weight given by CS

- Distributive analysis can have important implications
  - Azar (1999): WTP for statistical life is lower in poor countries. Fixing this has dramatic consequences for climate change. Likewise Nurmi and Ahtiainen (2017)
  - Ross and Winker (2004): Can mergers causing price hikes be okay? US only considers CS as PS depends to benefit wealthier. Canada gives equal weight

One Pound Method

- We note the following problem:
  - $\Delta SWF = \sum^H \frac{dSWF}{dU_h}\frac{dU_h}{dx_1^h}\Delta x_1^h + \sum^H \frac{dSWF}{dU_h}\frac{dU_h}{dx_2^h}\Delta x_2^h$
  - $\Delta SWF = \sum^H \frac{dSWF}{dU_h}\lambda^h q_1 \Delta x_1^h + \sum^H \frac{dSWF}{dU_h}\lambda^h q_2 \Delta x_2^h$
  - $\Delta SWF = \sum^H \beta_h^b \Delta x_1^h + \sum^H \beta_h^c \Delta x_2^h$
    - Policy affects people differently -> define groups with standing -> report CBA
  - $\Delta SWF = \Delta SWF^+ - C\text{term} = \sum \hat{\beta}_h \Delta x_1^h - \frac{1}{\mathrm{H}}\sum \hat{\beta}_h C$
    - Project will have some benefit ΔSWF^+ (utility) and cost $C > 0$ (money)
- To make terms comparable need consistent numeraire i.e. define $\hat{\beta}_h$ wrt to reference policy
- UK government uses benchmark of "uniform £1 lump sum transfer" i.e. project needs to be better than opportunity cost of spending money on universal benefit / transferring

- $\Delta SWF = \Delta SWF^{+} - C = \sum \hat{\beta}_h \Delta x_1^h - C$
- Define $\hat{\beta}_h = \hat{c}\beta$ s.t. $\frac{1}{\mathrm{H}}\sum\hat{\beta}_h = 1$ (utility per pound) i.e. $\sum\hat{\beta}_h = \hat{c}\sum\beta_h = H$ so $\hat{c} = \frac{H}{\sum\beta_h}$
- $\frac{\Delta SWF}{\frac{1}{\mathrm{H}}\sum\hat{\beta}_h} = \frac{1}{\frac{1}{\mathrm{H}}\sum\hat{\beta}_h}\sum\hat{\beta}_h\Delta x_1^h - C = \sum\hat{\beta}_h\Delta x_h - C$

- Inspired by Harberger Principle (1978) decomposes efficient and distribution $NB = NB^E + NB^D$. Do not accept project if $NB > 0$ but $NB^E < 0$ as simply redistributing is even better
  - See lecture slides 4, page 3
- Note IRL benchmark is hard: Gramlich: costs tax payer approximately \$1.50-\$2.00 to redistribute \$1.00 via transfer. This should be upper bound (less leaky bucket)

- If don't have market prices, use hedonic pricing, contingent valuation surveys etc.
- Tax example: DWL plus admin to implement costs, and fall in L causes fall in Y
- Clearly distribution argument has force because many government still implement these e.g. SNAP

Social Discounting

Marginal rate of time preference δ MRTransformation ρ Consumer/producer interest rate i/ r Market interest rate MIR

Basic

- Impact of project can be spread out over time

Model Set Up

Make following assumptions to analyse:

1. Time runs from $t = 0,1,2\ldots,T$
2. One all-purpose good so $y_t = c_t + I_t$
3. One representative consumer with instantaneous utility $u(c_t)$ where it's increasing at a decreasing rate $\frac{\partial u_t}{\partial c_t} > 0$; $\frac{\partial^2 u_t}{\partial c_t^2} < 0$
4. Budget constraint $c_t + s_t = \pi_t + (1+i)s_{t-1}$
   - s_t is saving, π_t real profit and i real consumer interest rate
5. Allow for weights on utility experienced at different points in time $SWF(u_0(c_0),\ldots,u_T(c_T))$
6. Output in period $t+1$ is function of investments I_t made in period t so $y_{t+1} = G(I_t)$ given real producer interest rate r where $\frac{\partial G}{\partial I_t} > 0$ and $\frac{\partial^2 G}{\partial I_t^2} < 0$ and y0 is given. Take y_0 as given
7. Market for loanable funds clears each period such that $s_t = I_t$

- Can model project as affecting c_t by Δc_t
  - Many, but not all, projects require reduction in c at start and pay off later
- Support project iff $\Delta SWF = \sum_{t=0}\frac{\partial SWF}{\partial u_t}\frac{\partial u_t}{\partial c_t}\Delta c_t > 0$
  - Can decompose to $\Delta SWF = \sum_{t\in T^+}\frac{\partial SWF}{\partial u_t}\frac{\partial u_t}{\partial c_t}\Delta c_t^+ + \sum_{t\in T^-}\frac{\partial SWF}{\partial u_t}\frac{\partial u_t}{\partial c_t}\Delta c_t^-$

Defining Terms

- **Social Discount Factor** are weights attached to consumption incremental: $\beta_t = \frac{\partial SWF}{\partial u_t}\frac{\partial u_t}{\partial c_t}$
  - Captures that small increase in c_t has on u_t and how this in turn increases SWF (but in supo say SDR discounts consumption not utility!)

- **Social Discount Rate** relates SDF at two successive periods $\beta_t = \frac{\beta_{t-1}}{1+SDR_t}$ where $SDR_0 = 0$
- Substituting back in: $\Delta SWF = \sum_{t=0} \beta_t \Delta c_t = \sum_{t=0} \frac{\beta_0}{\Pi_{k=0}(1+SDR_k)} \Delta c_t$
  - where $\Pi_{k=0}^{t}$ is the product of terms $(1 + SDR_k)$ between $k = 0$ and t
  - If assume SDR is constant then $\Delta SWF = \sum_{t=0} \frac{\beta_0}{(1+SDR_k)^t} \Delta c_t$

- Some things to note
  - Constant discount rate means discount factor is falling over time: social marginal value of consumption must be falling over time
  - β_0 is essentially our numeraire
  - Each t may represent the life span of one generation

IMPORTANT NOTES ON APPLICATION:

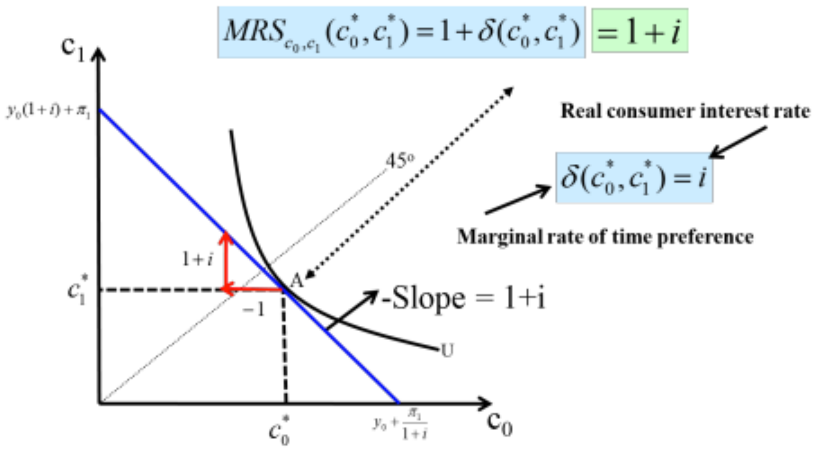
- Projects must be considered over time period. If not (25y vs. 75y project) need to align:
  - Roll-Over Method: consider 3x25y and 1x75y project
  - Equivalent Annual Net Benefit Method: $\frac{NPV}{\alpha_i^N}$ where α_i^N is annuity factor i.e. amount which, if received each year for duration of project, has same NPV as project
  - But… there might also be "quasi-option value" in having flexibility after short project versus lock in of long project.
- Also need to account for inflation, with there being many problems with CPI versus RPI etc.

Social Discount Rate and Market real Interest Rate

- We see that they are interchangeable in a first best world (without market/policy distortions, perfect capital market) but else must use with great care
- Overview:
  - Project displaces private consumption and private investment
  - Thus need two shadow prices δ (for c_1 vs. c_2) and ρ (for I_0 vs. I_1)
  - Thus get $d = \delta \frac{\Delta c_0}{\Delta I^P} + \rho \frac{\Delta c_0}{\Delta I^P}$
  - In first best $\delta = \rho$ so social and consumption discount rate are interchangeable $i = r$
    - In first best SDR=CDR because doesn't matter if CDR or firm rate
  - In second best take weighted average of these. Supo question 1 is good overview.

Consumer Choice (i.e. Supply of Loanable Funds)

- Solving consumer problem $[\max u_0(c_0) + u_1(c_1)]$ get opt. condition $MRS = \frac{\left(\frac{du_0}{dc_0}\right)}{\left(\frac{du_0}{dc_1}\right)} = 1 + i$
  - Will assume consumer is net saver (as must be in market equilibrium)
- Define **marginal rate of time preference** such that $MRS(c_0, c_1) \equiv 1 + \delta(c_0, c_1)$
  - Intuitively, if $\delta > 0$ then only willing to give up one unit of consumption today if compensated by more than one unit of consumption tomorrow, and vice versa
  - Implies MU of consumption decreasing over time so SDF is decreasing
  - Marginal rate of **pure** time preference as $\delta^*(c_0, c_1)$ evaluated at $c_0 = c_1$ (when I consume as much today as tomorrow – so ignore changes to income)
- Thus restate opt. condition as $\delta(c_0, c_1) = i$ or tangency between intertemporal budget line and indifference curve [δ cost of saving – loss of value due to discounting; i benefit of saving – more money in future than today]
  - If assume substitution effect dominates income effect, then increasing function of i



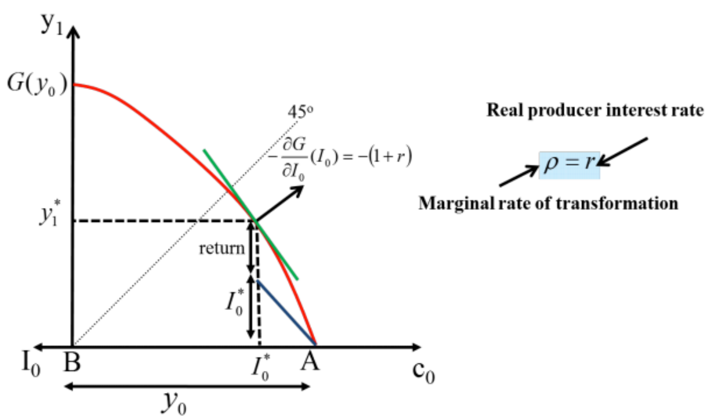
Back To Model

- Recall two-period model rule: $\Delta SWF = \frac{\partial SWF}{\partial u_0}\frac{\partial u_0}{\partial c_0}\Delta c_0 + \frac{\partial SWF}{\partial u_1}\frac{\partial u_1}{\partial c_1}\Delta c_1$
- Assumed that $\frac{\partial SWF}{\partial u_0} = \frac{\partial SWF}{\partial u_1} = 1$ so rewrite as $\frac{\Delta SWF}{\frac{\partial u_0}{\partial c_0}} = \Delta c_0 + \frac{\left(\frac{\partial u_1}{\partial c_1}\right)}{\left(\frac{\partial u_0}{\partial c_0}\right)}\Delta c_1 = \Delta c_0 + \frac{1}{1+\delta(c_0^*,c_1^*)}\Delta c_1$
- Marginal rate of time preference is right choice for SDF when we use consumption in first period as numeraire!
- In perfect capital market (i=r) the marginal rate of time preference (already shown \delta = 1) is also equal to r and thus a justified proxy for SDR

PRODUCER CHOICE (I.E. DEMAND OF LOANABLE FUNDS)

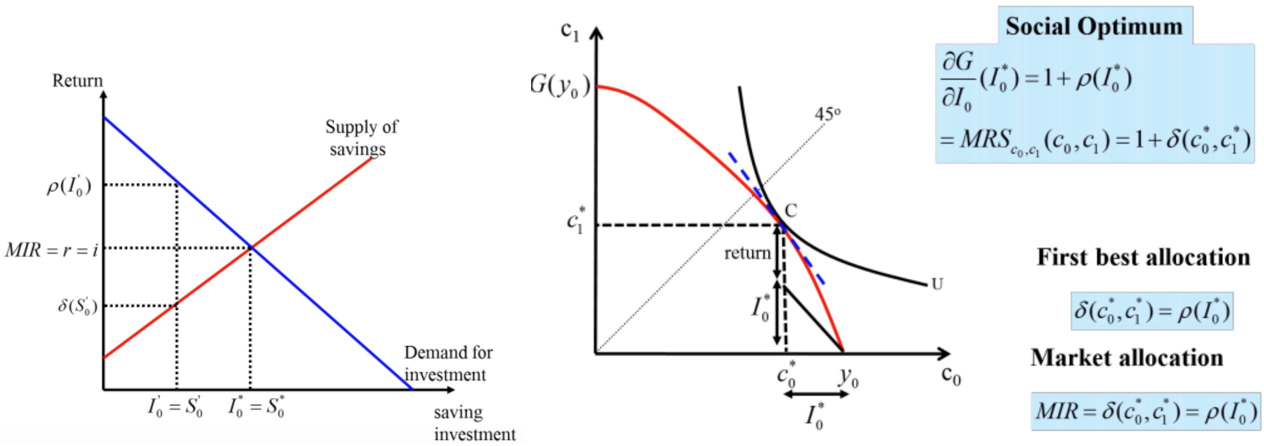
Study demand for investment. Derive demand for loanable funds

- Observe the production frontier $G(\quad)$:
    - Savings used by producers to invest/increase capital stock and thus future output
    - Distance A-B represents real income of period 0 allocated between I_0 and c_0
    - Slope at each investment level is MRT between income today and tomorrow
        - $\frac{dG}{dI_0} = 1 + \rho(I_0)$. Decreasing in investment, thus convex curvature
- \rho is rate at which you turn investment today into output tomorrow
- Firms solve profit maximization: $\pi_1 = \max G(I_0) - (1+r)I_0$ thus…
    - FOC given by $\frac{dG}{dI_0} = 1 + r$ i.e. where the iso-cost line is tangent to production frontier
    - Thus $\rho(I_0^*) = r$ (cost of investment = benefit)



MARKET EQUILIBRIUM

- Given by where saving is equal to investment: $S_0(MIR) = I_0(MIR)$ thus $MIR - r - i$
- Implies market = first-best allocation as $\delta(c_0^*, c_1^*) \;=\; \rho(I_0^*) = MIR$
    - Production efficient, i.e., on the frontier
    - Utility maximizing allocation of resources over the two periods is where the slope of an intertemporal indifference curve is tangency to the frontier



Choosing Weights

- Consider project causing $\Delta I^P > 0$ (period 0) and $\Delta c_1^P > 0$ (period 1) with small price effect
- Displacement is thus $\Delta I^P = \Delta I_0 + \Delta c_0$
- Thus SCBA equation: $\Delta SWF = \frac{du_1}{dc_1}\left(\Delta c_1^p - \frac{dG}{dI_0}\Delta I_0\right) - \frac{du_0}{dc_0}\Delta c_0 = \frac{du_1}{dc_1}\Delta c_1^p - \frac{du_1}{dc_1}\frac{dG}{dI_0}\Delta I_0 - \frac{du_0}{dc_0}\Delta c_0$
    - i.e. project benefit t=1, foregone t=1 C due to I-displacement, forgone t=0 C
- Using $\frac{du_0}{dc_0}$ as numeraire: $\frac{\Delta SWF}{\left(\frac{du_0}{dc_0}\right)} = \frac{\left(\frac{du_1}{dc_1}\right)}{\left(\frac{du_0}{dc_0}\right)}\Delta c_1^p - \frac{\left(\frac{du_1}{dc_1}\right)}{\left(\frac{du_0}{dc_0}\right)}\frac{dG}{dI_0}\Delta I_0 - \Delta c_0 = \frac{1}{1+\delta}\Delta c_1^p - \frac{1+\rho}{1+\delta}\Delta I_0 - \Delta c_0$

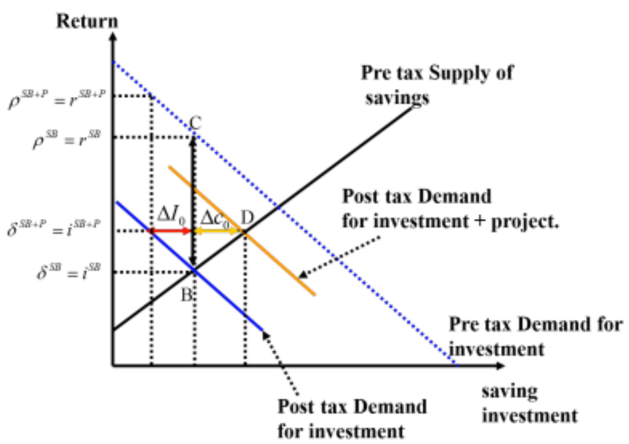
First Best

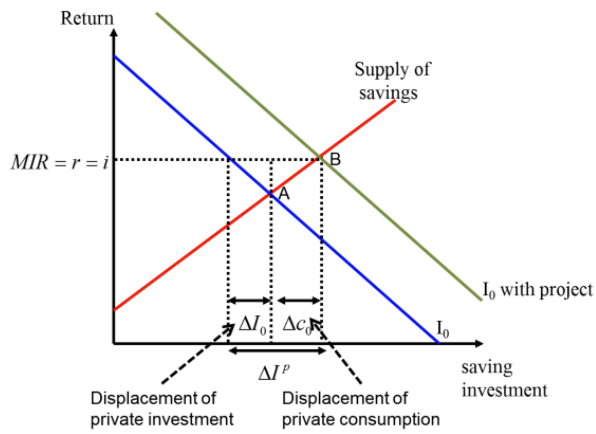
- In an undistorted economy we have $r \;=\; i \;=\; MIR \;=\; \delta \;=\; \rho$
- Thus simplifies to $\frac{\Delta SWF}{\left(\frac{du_0}{dc_0}\right)} = \frac{1}{1+\delta}\Delta c_1^p - (\Delta I_0 + \Delta c_0) = \frac{1}{1+\delta}\Delta c_1^p - \Delta I^p$
    - Opportunity cost of resources used by the project is the same whether private sector investment ΔI_0 or private consumption Δc_0 is foregone
- Can thus use real consumer interest rate (marginal rate of time preference) or producer interest rate (marginal rate of transformation) as the social discount rate

Second Best

- Suppose government levies corporate income tax (i.e. tax on investment demand)
- We note the following set up:
    - Consumers optimize by setting marginal rate of time preference equal to real consumer interest rate
    - Producers optimize by setting the marginal rate of transformation equal to real producer interest rate
    - Thus marginal rate of transformation is larger than the marginal rate of time preference at the initial distorted equilibrium!
- We note the following project effects:

- Project increases demand for investment shifting the demand curve out and increasing the market interest rate. Displaces ΔI_0 and Δc_0
- Which of the two rates –marginal rate of time preference or marginal rate of transformation – should we use as social discount rate?





Post tax not project is done

- We derive the Harberger rule to solve this:
  - $\frac{\Delta SWF}{\left(\frac{du_0}{dc_0}\right)} = \frac{1}{1+\delta^{SB}}\Delta c_1^p - \frac{1+\rho^{SB}}{1+\delta^{SB}}\Delta I_0 - \Delta c_0 = \frac{1}{1+\delta^{SB}}\Delta c_1^p - \frac{\Delta I^P}{1+\delta^{SB}}\left[(1+\rho^{SB})\frac{\Delta I_0}{\Delta I^P} + (1+\delta^{SB})\frac{\Delta c_0}{\Delta I^P}\right] = \frac{1}{1+\delta^{SB}}\Delta c_1^p - \frac{\Delta I^P}{1+\delta^{SB}}\left(1 + \rho^{SB}\frac{\Delta I_0}{\Delta I^P} + \delta^{SB}\frac{\Delta c_0}{\Delta I^P}\right)$
  - Define weighted average of the marginal rate of time preference and the marginal rate of transformation as $d = \rho^{SB}\frac{\Delta I_0}{\Delta I^P} + \delta^{SB}\frac{\Delta c_0}{\Delta I^P}$
  - Thus $\frac{\Delta SWF}{\left(\frac{du_0}{dc_0}\right)} = \frac{1}{1+\delta^{SB}}\left\{\Delta c_1^p - (1+d)\Delta I^P\right\} = \frac{1+d}{1+\delta^{SB}}\left\{\frac{\Delta c_1^p}{1+d} - \Delta I^P\right\}$
  - Thus $\frac{\Delta SWF}{\left(\frac{du_0}{dc_0}\right)} > 0$ iff $\frac{\Delta c_1^p}{1+d} - \Delta I^P > 0$. Should a convex combination of the real consumer and real producer interest rate for the social discount rate

(Important) Discussion

- In practice do not know exactly how much project displaces investment or consumption. General rule is to use producer real interest rate:
  - Presence of tax distortions on real producer interest rate (marginal rate of transformation) is higher than consumer (marginal rate of time preference)
  - Thus if choose consumer real interest as social discount rate, willing to accept public projects with lower marginal return than next-best private opportunity
    - Case for MRT: If government wants to take resources away from private sector, it should be able to demonstrate that society will get better return
- Case against MRT: Suspicion that private investment is associated with negative externalities or default risk is factored into market rates
- So far assumed closed domestic market (thus investment demand curve given by ROI). But Lind et al. note foreign borrowing is possibility to fund project without crowding out
  - $SDR = a * CRI + b * ROI + c * CFF$
  - Burgess and Zerbe estimate $SDR = 0.1 * 3.5\% + 0.54 * 8.5\% + 0.36 * 5.5\% = 7\%$

- IRL very hard to calculate weights and they change over time. Also projects are typically funded in different ways so very ad hoc
  - See textbook p248 for all criticisms

RAMSEY FORMULA

- (back to first best)
- **Social/C Time Preference $\boldsymbol{\delta = \delta_p + \gamma g}$**: value society attaches to present, as opposed to future, consumption. Based on comparisons of utility across different time/generations
  - **Time preference $\boldsymbol{\delta_p}$:** Rate at which consumption and public spending are discounted over time <u>assuming no change in per capita consumption</u> (perfect smoothing)
    - i.e. preference for value now rather than later
    - Consists of pure time preference and a measure of catastrophic risk
  - **Wealth effect $\boldsymbol{\gamma g}$**: Reflects expected growth in per capita consumption, where future consumption will be relatively higher and expected to have lower [marginal] utility
    - γ multiplied by expected growth rate of future real per capita consumption g
      - γ equation is elasticity of MU of consumption and interpretation is degree of inequality aversion
- Note per-period utility function exhibits constant relative consumption inequality aversion: $-\frac{\left(\frac{d^2y}{dc^2}\right)}{\left(\frac{du}{dc}\right)}c = \gamma$. Thus γ controls degree of inequality aversion (how much care about smoothing)
  - Higher γ means society is more adverse to large differences in consumption across time because it induces large variations in marginal utility
  - Higher growth means consumption will be higher in future and so a more equal allocation requires reallocating consumption from the future to the present

ASSUMPTIONS

1. Infinite time periods $t = 0,1,2\ldots,\infty$
2. One all-purpose good c_t. Define vector $c_t = \{c_0, c_1, \ldots c_\infty\}$
3. Two ways to think of individuals and SWF
   a) **Representative Consumer**: One representative consumer who lives forever with per-period utility function $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$ and intertemporal $V(c) = \sum^\infty \frac{1}{(1+\delta_p)^t} u(c_t)$. Thus simply $SWF = V(c)$
      - Utility discount rate δ_p represents marginal rate of pure time preference i.e. $MRS_{c_t,c_{t-1}}|_{c_t=c_{t-1}} = \frac{c_{t-1}^{-\gamma}}{\frac{1}{1+\delta}c_t^{-\gamma}} = 1 + \delta_p$
      - SWF put different weight on <u>utility</u> at different points in time because the representative consumer is <u>impatient</u>

b) **Generations**: Society is made up of a sequence of unrelated generations with life spans represented by t. Per-generation utility function is same $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$ and $SWF(c) = \sum^{\infty} \frac{1}{(1+\delta_p)^t} u(c_t)$
   - Discount rate δ_p now represents different weights put on generations and is an ethical choice

4. Society experiences constant consumption growth $c_t = c_0(1+g)^t$ where $c_0 > 0$

Derivation of SDR Formula

- Consider project induce change in consumption profile Δc
- Choose consumption at time 0 as numeraire. Workout social consumption discount rate:
    - $\Delta SWF = \sum^{\infty} \frac{1}{(1+\delta_p)^t} \frac{du}{dc_t} \Delta c_t$ thus $\frac{\Delta SWF}{\left(\frac{du}{dc_t}\right)} = \sum^{\infty} \frac{1}{(1+\delta_p)^t} \Delta c_t = \sum^{\infty} \beta_t \Delta c_t$ [check this]
- Thus $\beta_t = \frac{(c_0(1+g)^t)^{-\gamma}}{(1+\delta_p)^t} = \frac{c_0^{-\gamma}}{(1+\delta_p)^t (1+g)^{t\gamma}} = \frac{\beta_0}{(1+\delta_p)^t (1+g)^{t\gamma}}$
    - Where $c_0^{-\gamma} = \beta_0$ is social discount factor at time 0

- Thus $SDR = \frac{\beta_{t-1}-\beta_t}{\beta_t} = -\frac{\beta_t - \beta_{t-1}}{\beta_t} = (1+\delta_p)(1+g)^{\gamma} - 1$
- Transform as $\ln(1+SDR) = \ln(1+\delta_p) + \gamma \ln(1+g)$ so $SDR \approx \delta_p + \gamma g$
- OR… derive via $SDR_{\Delta t \to 0} = -\frac{\Delta \ln \beta_t}{\Delta t} = -\frac{\Delta \ln\left(\frac{\beta_0}{(1+\delta_p)^t (1+g)^{t\gamma}}\right)}{\Delta t} = -\frac{\Delta\left(\ln \beta_0 - t\left(\ln(1+\delta_p) + \gamma \ln(1+g)\right)\right)}{\Delta t} = \ln(1+\delta_p) + \gamma \ln(1+g) \approx \delta_p + \gamma g$

Estimating SDR

- Two approaches: Market data or ethical judgement
    - Market Data: Nordhaus observes MIR of 5.5% and sets this as consumption discount rate δ_p. Combines with best forecast γg (2*2%). Thus $\delta = 1.5\%$
        - But… $MIR \neq \delta$ due to market imperfections, only reflects those alive now, and social≠private preferences
    - Ethical Judgement:
        - Stern picks $\delta_p = 0$ and observes $\gamma g = 1 * 1.3\%$. Thus implied consumption rate is 1.4%. Much lower than Nordhaus!
        - Greenbook estimates pure time preference as 0.5% but adds 1% catastrophic risk so $\delta_p = 1.5\%$. Combines with best forecast γg (1*2%). Thus $\delta = 3.5\%$
        - Ramsey (1928) warned against discounting future utilities without reason
        - Consistent with explicit welfare criterion but inconsistent with data

Is Constant SDR Valid?

- When projects become inter-generational (already arbitrary) there are four reasons to have SDR be time declining:
    - Individuals are time inconsistent, applying lower rates over time. Laibson "Golden Eggs and hyperbolic discounting" demonstrates this
    - Even using modest constant SDR has seemingly unethical implications due to geometrics where small "world saving" sacrifices today are rejected

    - People today fail to account properly for future generations. See Nicholas Stern critique of using market rates
    - The farther we look in the future, the greater uncertainty we face. SDR have less influence as time horizon increases, resulting in time-declining discount rates
- Results in different schedules:
    - Greenbook hence settles on schedule that starts at 3.5% and falls by 0.5%-points after 30 then 45, 50, 75, 100 years
    - Newell and Prizer: 0-50 3.5%, 2.5% 50-100, 1.5% 100-200, 0.5% 200-300, after 0%

DEALING WITH UNCERTAINTY?

- See above for why this contributes to time-declining discount rates
- Do we add risk premium to SDR? Broadman says treat separately. Translate net benefits into certainty equivalents then discount those at risk-free SDR
- Uncertainty about marginal value of consumption? in future is important and possible to mitigate. Hence might save for precautionary reasons if risk-averse.

- Need to factor in catastrophic risk: Makes all public investment worthless, aking to death probability in consumer problem
    - This justifies applying less weight on future
    - Green Book gives probability of 1%. See Toby Ord for higher estimate of 1/6
    - Relate to climate change – can do something based on our actions

If I am poor in future want to save
If I am uncertain about poor in future want to save more
If I am dead in future I don't want to save