1. Social networks: social signatures and weight-topology correlations (17 pts)

In this exercise, we will do some weighted network analysis using a social network dataset describing private messaging in a Facebook-like webpage[1]. In the network, each node corresponds to a user of the website and link weights describe the total number of messages exchanged between users.

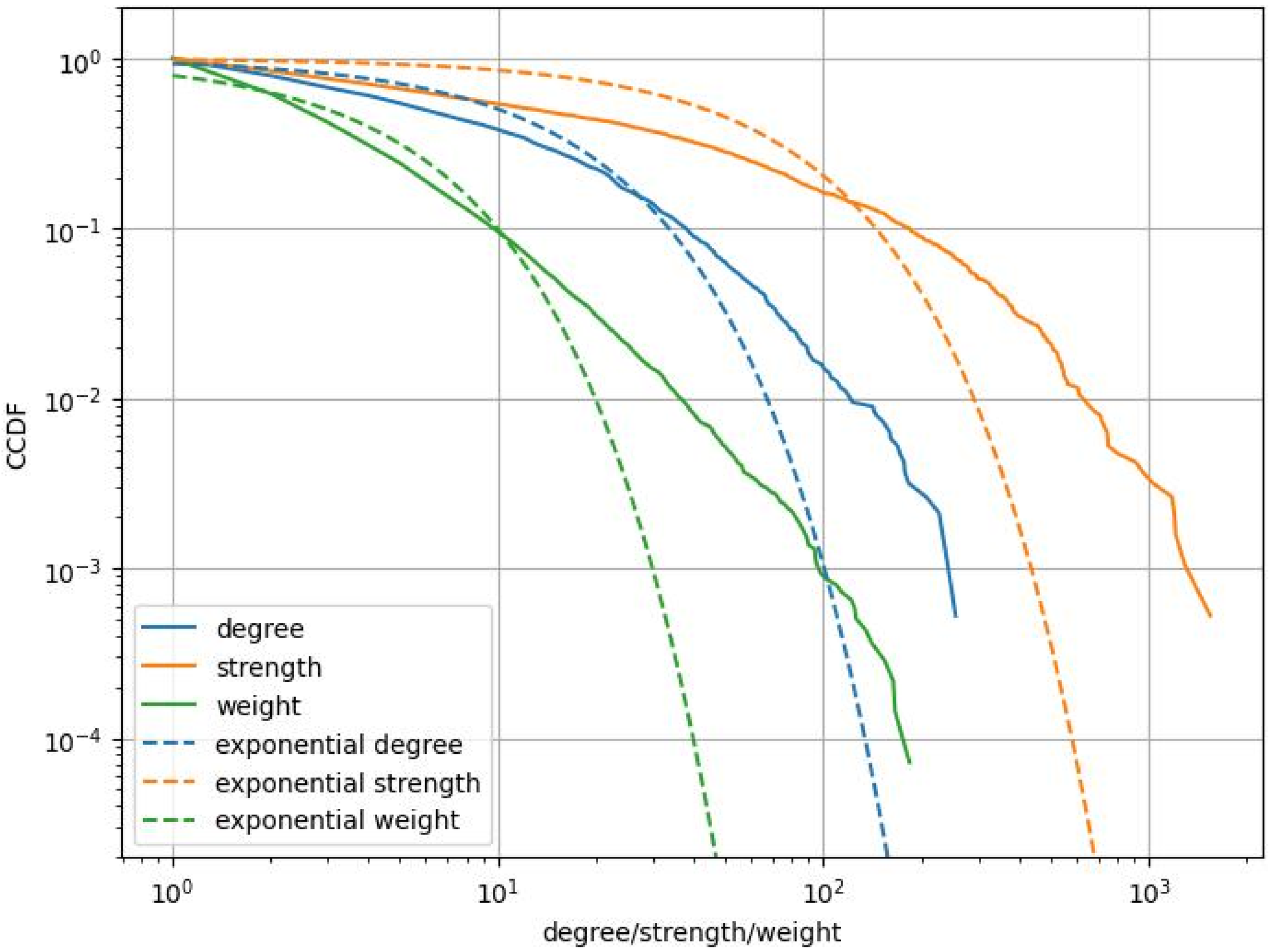
In the file `OClinks_w_undir.edg`, the three entries of each row describe one link:
`(node_i node_j w_ij)`,
where the last entry `w_ij` is the weight of the link between nodes `node_i` and `node_j`.

You can use the accompanying code template to get started.
The function `scipy.stats.binned_statistic` is especially useful throughout this exercise.

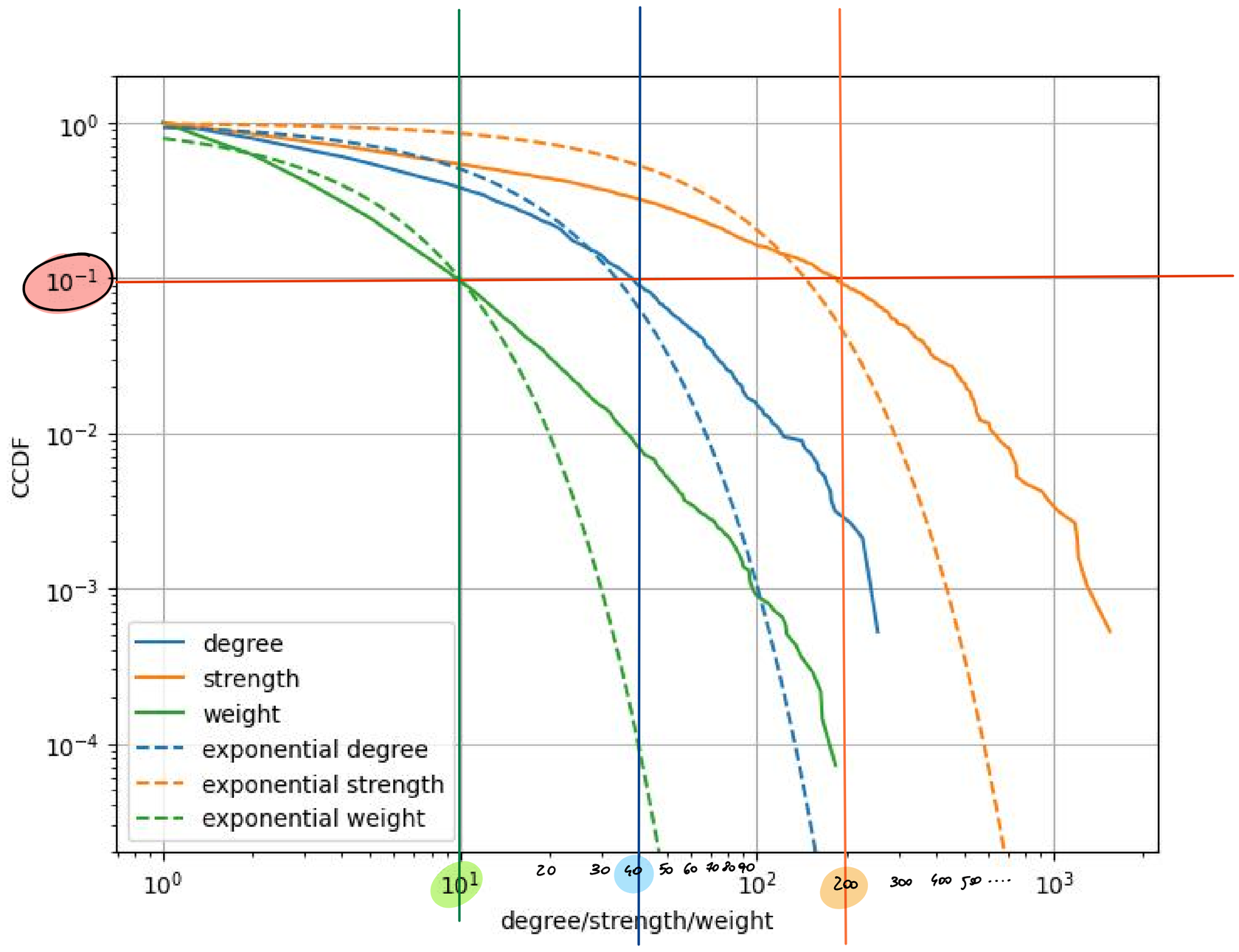
Submit all your solutions to MyCourses as a single pdf report. Remember to include in your report all the visualizations as well as all the numerical and verbal answers.

a) (4 pts) Before performing a more sophisticated analysis, let us begin with taking a look at some basic network statistics to get a rough idea of what the network is like. To this end, **plot** the complementary cumulative distribution function (CCDF) for node degree k, node strength s and link weight w.

   - Show all three distributions in one plot using log-log scale.
   - The resulting distributions have *heavy tails*, meaning that the right part of the distributions decays slower than exponentially. To see this, **plot** in the same figure the CCDFs of exponential distributions[2] having the same mean as each empirical distribution.
   - Based visually on the empirical CCDF plots, roughly **estimate** the 90th percentiles of the degree, strength, and weight distributions. **Explain** also how can they be read off from the plots.



The 90th percentile represents the value below which 90% of the data in the distribution lies. To locate it on the CCDF plot, we have to look for the point where the CCDF curves cross the value $0.10 = 10^{-1}$ on the ordinate axis.



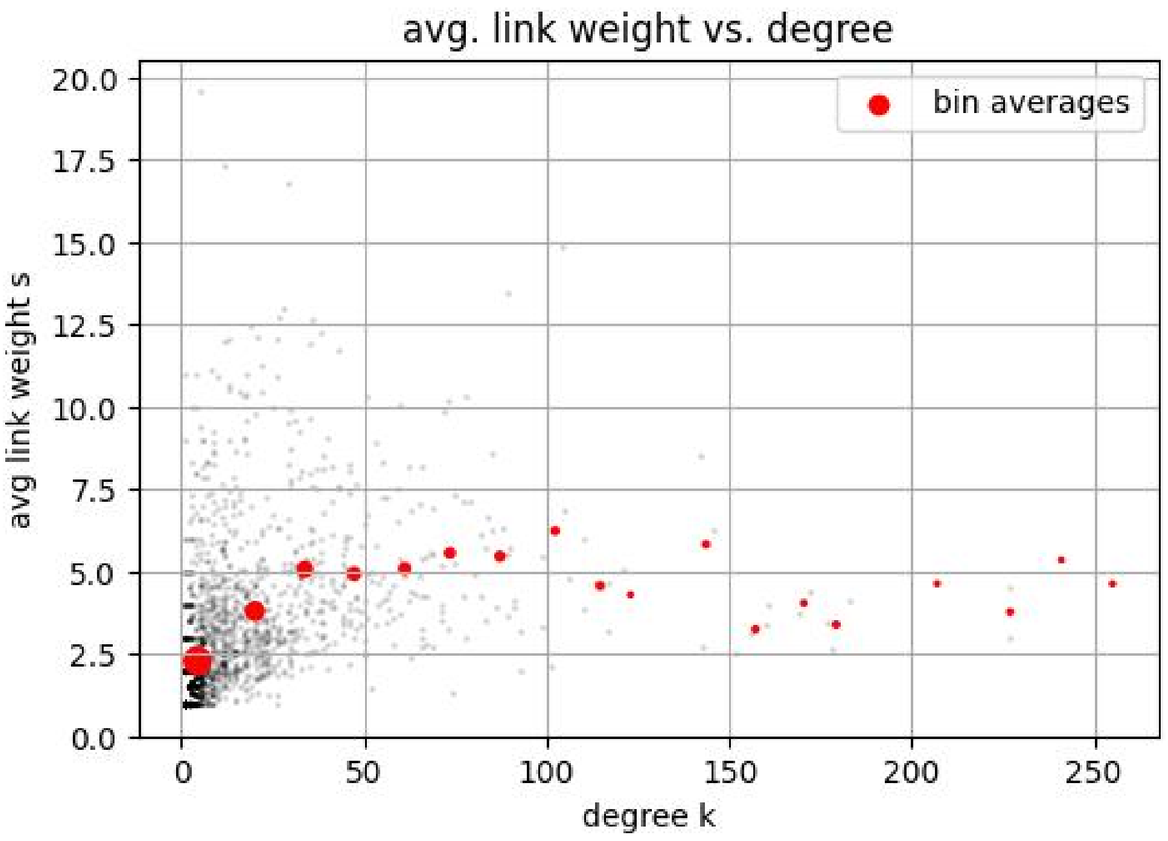
So, based on the plot, we can say that:

- 90th percentile of degree distribution $\simeq 40$
- 90th percentile of strength distribution $\simeq 200$
- 90th percentile of weight distribution $\simeq 10$

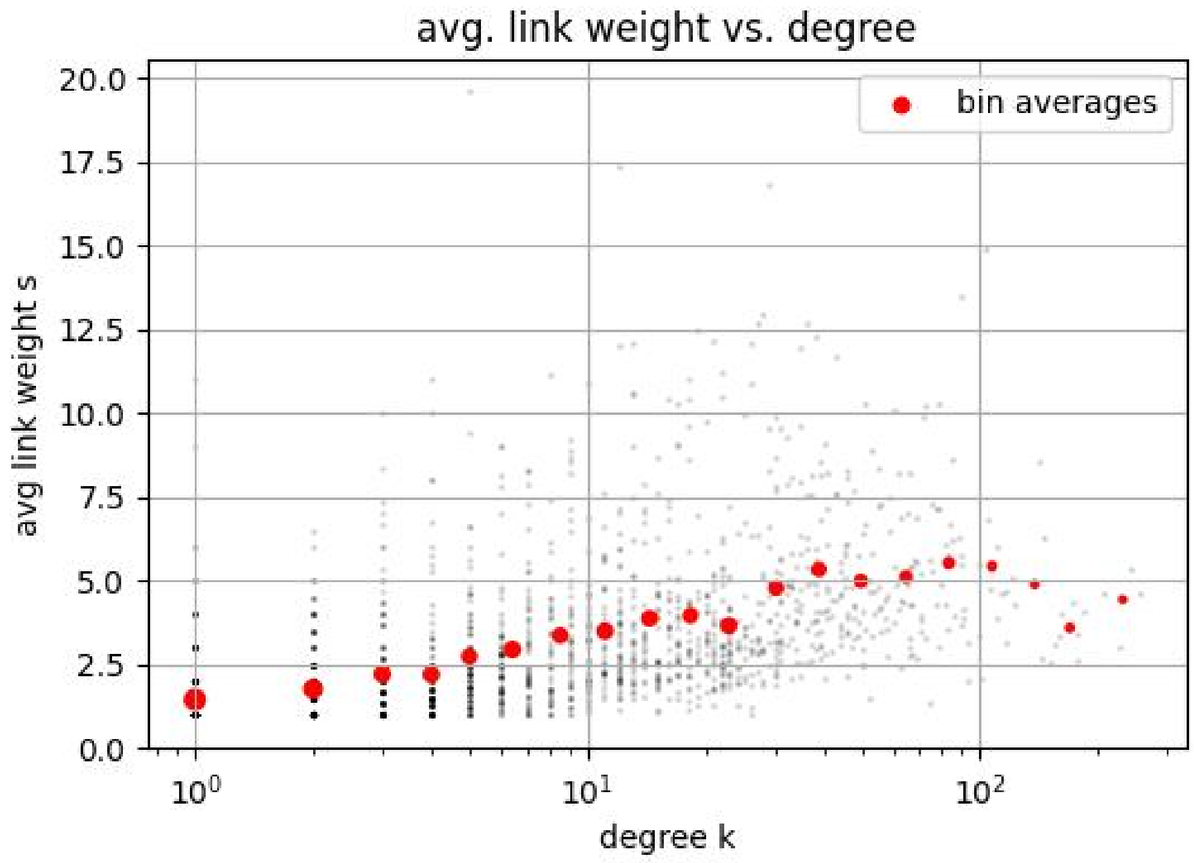
b) (5 pts) Next, we will study how the average link weight per node $v = \frac{s}{k}$ behaves as a function of the node degree k.

- **Compute** s, k, and v for each node.
- Make **a scatter plot** of all the data points of v as a function of k. Create two versions of the plot: one with linear axes and one with logarithmic horizontal axes.
- The large variance of the data can make the scatter plots a bit messy. To make the relationship between v and k more visible, **create** bin-averaged versions of the plots, i.e. divide nodes into bins based on their degree and calculate the average v in each bin. Plot the bin-averaged versions on top of the scatter plots.
- Based on the plots, **which** of the two approaches (linear or logarithmic horizontal axes) suits better for presenting v as a function of k? **Why?**

LINEAR AXES



LOGARITHMIC HORIZONTAL AXES



In linear scales the data points are more scattered, and it is challenging to highlight a relationship between the average link weight per node and the node degree. In the logarithmic case, it is easier to interpret a relatively linear trend.

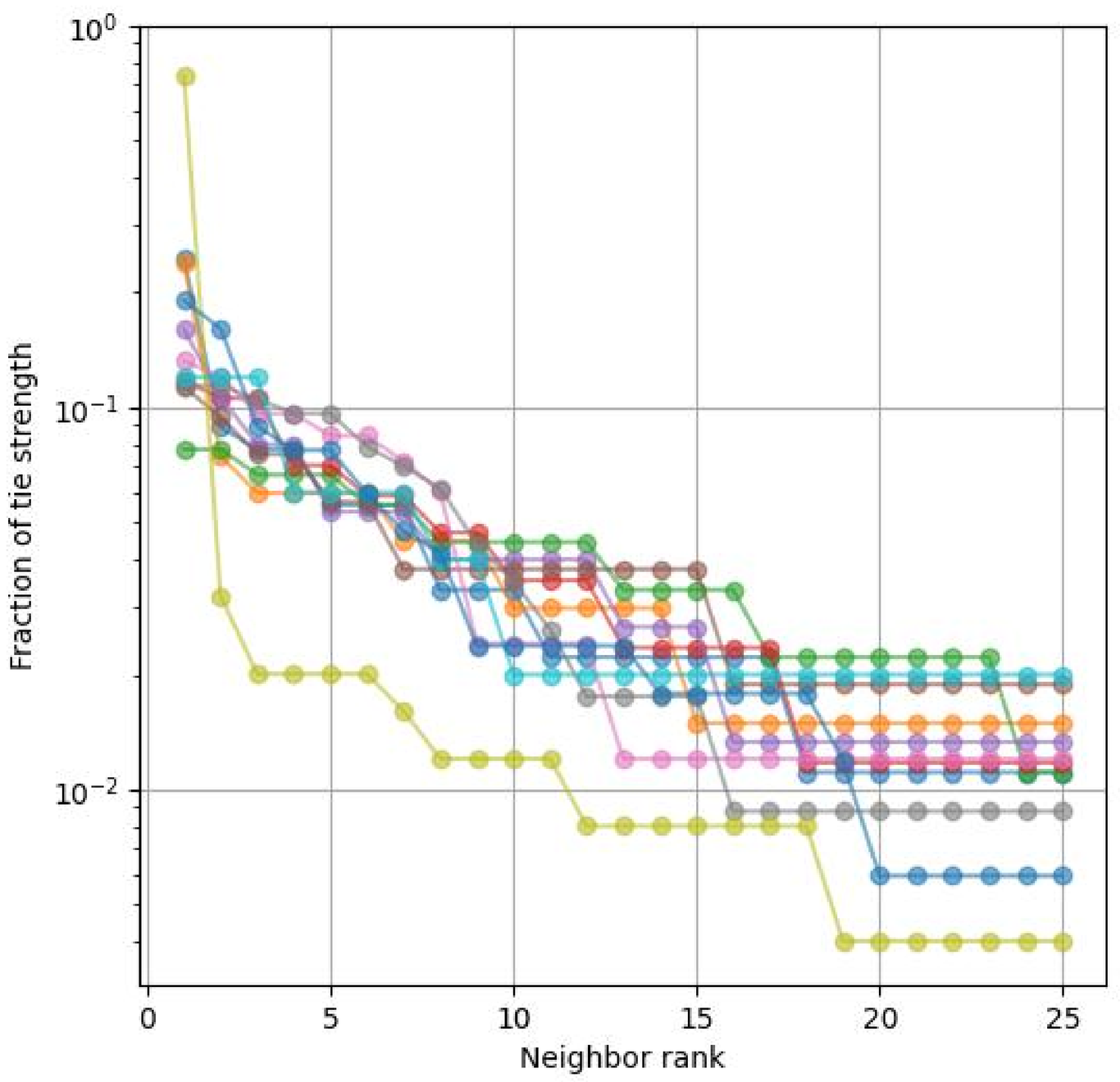
c) (4 pts) Next, we will study the social signature of egocentric networks. The social signature of a node is a vector that describes the distribution of link weights in the egocentric network of the node. We define the social signature of node i with degree k_i as

$$\mathbf{p}_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,k_i}), \quad p_{i,1} \geq p_{i,2} \geq \ldots \geq p_{i,k_i}, \tag{1}$$

where $p_{i,j}$ is the weight of j-th link (sorted in the descending order) incident to node i normalized by the sum of the weights of all links incident to i (i.e., strength of node i) :

$$p_{i,j} = \frac{w_{i,j}}{\sum_{j=1}^{k_i} w_{i,j}}. \tag{2}$$

Compute the social signatures of the nodes in the network that have degree $k_i = 25$ and **plot** the signatures $p_{i,j}$ as a function of rank j.



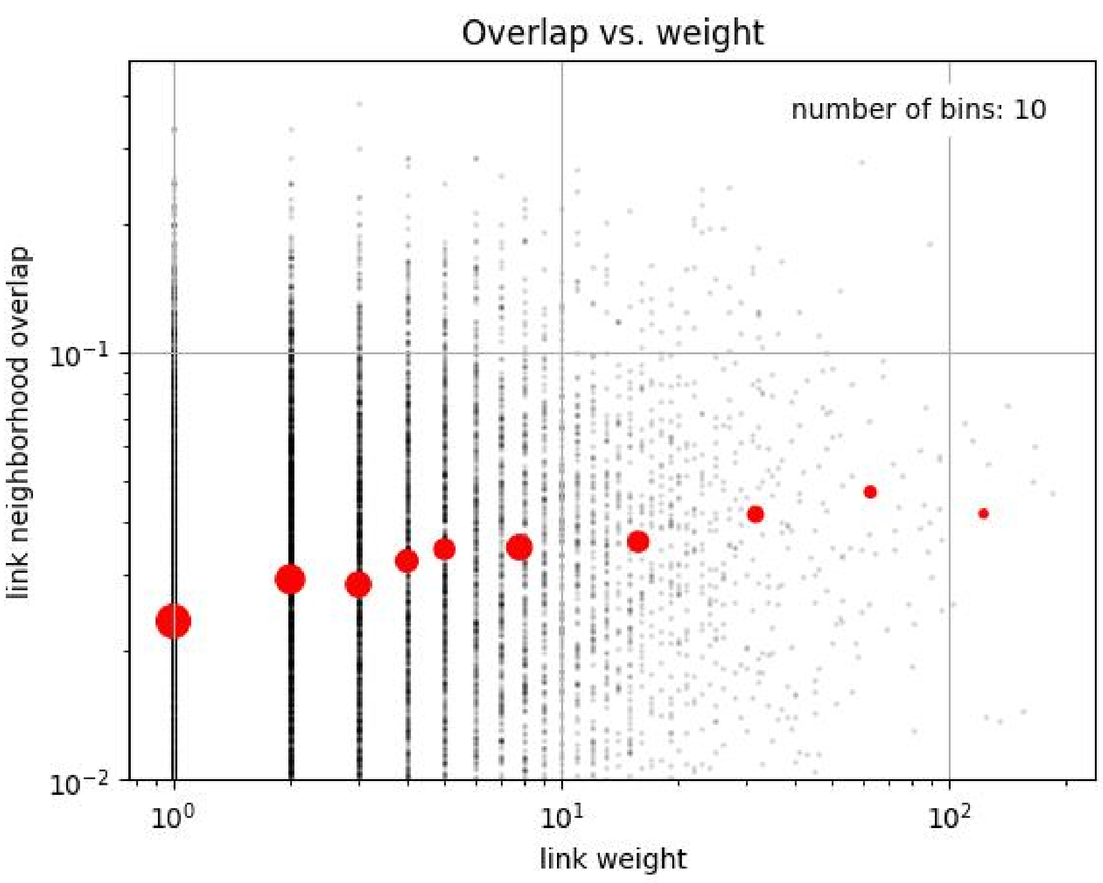
d) (4 pts) Let's consider a link between nodes i and j. For this link, *link neighborhood overlap* O_{ij} is defined as the fraction of common neighbors of i and j out of all their neighbors:

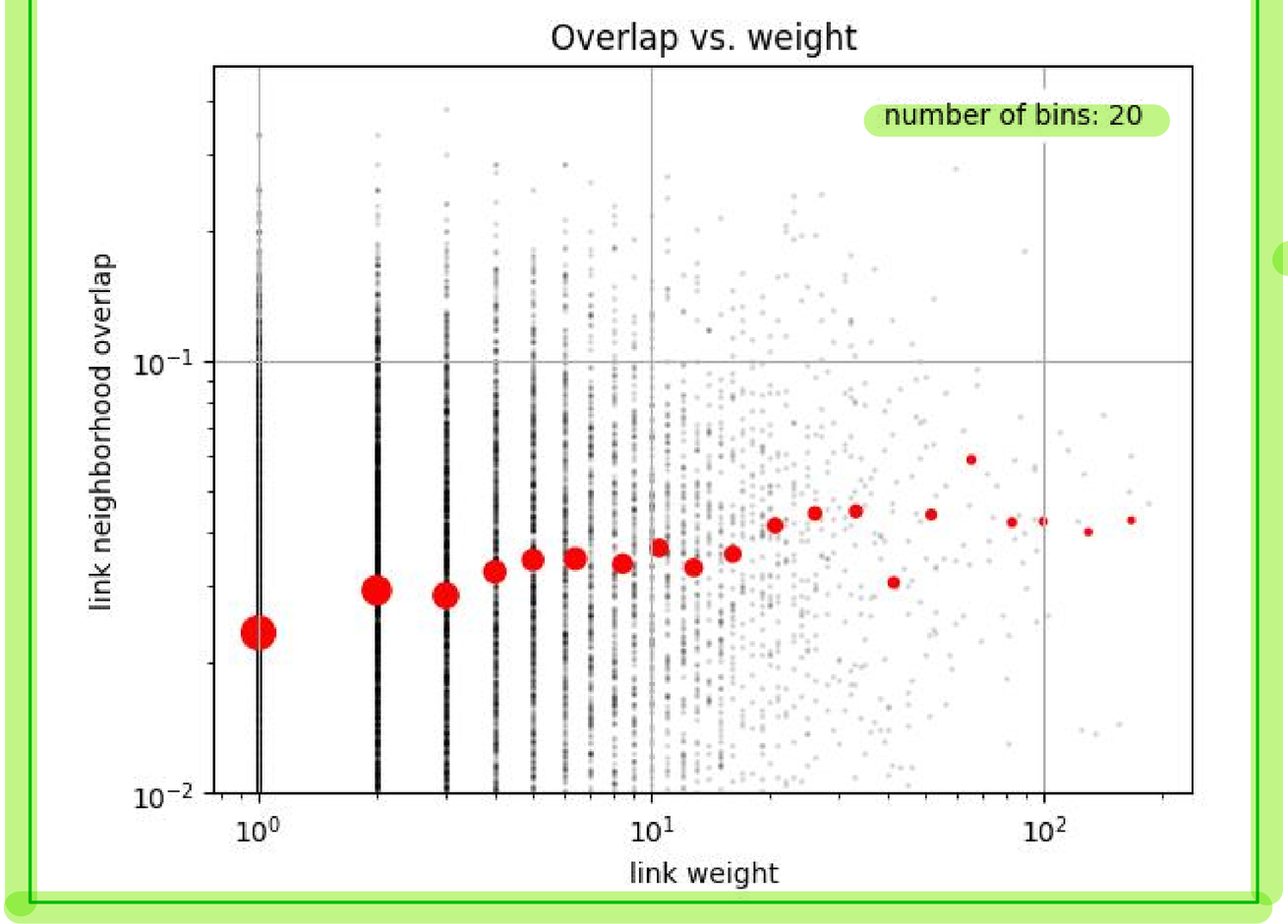
$$O_{ij} = \frac{n_{ij}}{(k_i - 1) + (k_j - 1) - n_{ij}}, \tag{3}$$

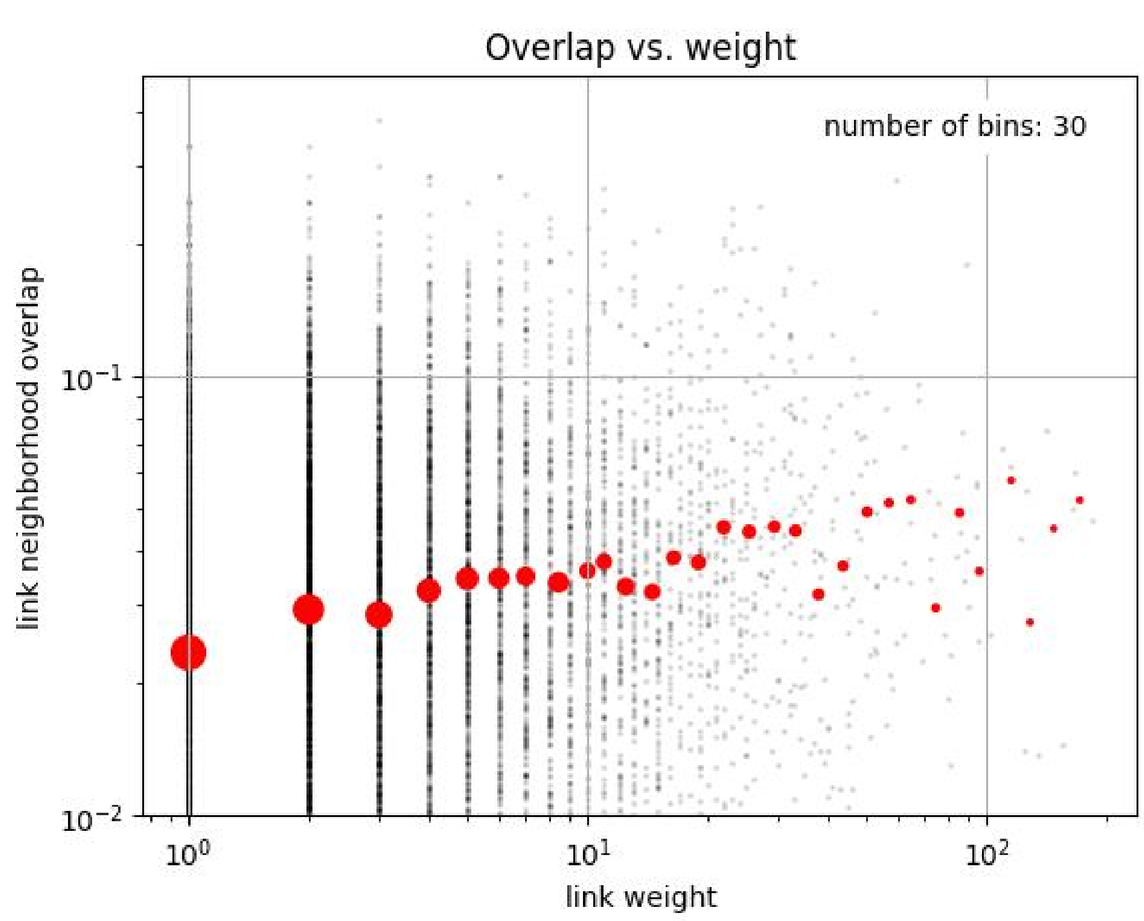
where n_{ij} denotes the number of common neighbors.

According to the Granovetter hypothesis, link neighborhood overlap is an increasing function of link weight in social networks. Your task is to find out whether this is the case also for the present dataset by visualizing it in an appropriate way. Use the binning strategy (linear or logarithmic) that is most suitable for this case:

- **Calculate** the link neighborhood overlap for each link.
- Create a **scatter plot** showing the overlap for every link as a function of link weight.
- As in b), **produce** a bin-averaged version of the plot on top of the scatter plot. In doing this, choose a reasonable number of bins (which, in this case, will be between 10 and 30). **Specify** how many bins you used.
- In the end, you should be able to spot a subtle trend in the data. Based on your plot, **comment** on if the trend is in accordance with the Granovetter hypothesis.







It is noticeable that with 30 bins the scatterplot is much more scattered, and it is not easy to observe a linear relationship, especially for higher link weights. On the other hand, 10 bins are too few, while 20 bins are sufficient to observe a linear trend, that is in line with the Granovetter hypothesis.