Bipedal Locomotion Control based on Divergent Component of Motion

First Mini-Project of Legged Robots

Introduction

In this project you will work on optimization-based control of bipedal robots by using the Divergent Component of Motion (DCM) concept. The code structure will include three parts: the DCM planning and optimization, Foot Trajectory Planner, and Inverse Kinematics. The only block that you need to implement is the DCM planning and optimization. In other words, you only need to open and edit the `DCMTrajectoryGenerator.py` class and implement the quadratic programming optimization in the Jupyter Notebook and follow the comments that have been written in the codes (it is specified by #todo). You can find the code by the following link:
https://github.com/MiladShafiee/lr-biped-start

1 Method

1.1 Linear Inverted Pendulum Model (LIPM)

The LIPM has been widely utilized to describe the dynamics of the CoM for bipedal locomotion [3]. The LIPM assumes a constant rate of change of centroidal angular momentum and movement of the CoM height within a plane. These assumptions are suitable for planning humanoid robot locomotion, as research on human walking indicates minimal variations in centroidal angular momentum and CoM height [1]. Based on the assumptions, the equations of motion for the LIPM can be derived as follows:

$$\ddot{\mathbf{x}} = \omega^2(\mathbf{x} - \mathbf{cop}) \tag{1}$$

in which $\mathbf{x} = [x_{com}, y_{com}]^T$ is the horizontal position of the CoM, $\omega_0 = \sqrt{\frac{g}{\Delta z}}$ is the natural frequency of the LIPM, and $\mathbf{cop} = [cop_x, cop_y]^T$ is the horizontal position of the center of pressure (CoP).

1.2 Divergent Component of Motion

In this section, we provide an overview of the DCM concept's background. The dynamics of the CoM, as modeled by the LIPM, can be split into stable and unstable components. The unstable component is referred to as the DCM and is defined as follows:

$$\textbf{DCM} \quad \boldsymbol{\xi} = \mathbf{x} + \frac{\dot{\mathbf{x}}}{\omega} \tag{2}$$

Throughout this study, DCM is represented as ξ as the notation in [6]. From (2), the CoM dynamics is given by:

$$\textbf{COM dynamics} \qquad \dot{\mathbf{x}} = \omega(\boldsymbol{\xi} - \mathbf{x}) \tag{3}$$

By differentiating (2) and substituting (1), the DCM dynamics is expressed as :

$$\textbf{DCM dynamics} \qquad \dot{\boldsymbol{\xi}} = \omega(\boldsymbol{\xi} - \mathbf{cop}) \tag{4}$$

Fig. (1) illustrates the relationship between DCM dynamics, CoM, and the CoP. By re-arranging DCM dynamics (4), the following ordinary differential equation (ODE) holds:

$$\dot{\boldsymbol{\xi}} - \omega\boldsymbol{\xi} = -\omega\,\mathbf{cop}_0 \tag{5}$$

The solution to (5) writes:

$$\boldsymbol{\xi}(t) = e^{\int \omega dt}\left[\int (-\mathbf{cop}_0\,\omega)\,e^{\int -\omega dt}dt + \mathbf{C}\right], \tag{6}$$

where $C \in \mathbb{R}^2$ is the vector of unknown coefficients that can be found by imposing the boundary conditions. Therefore, we can find these coefficients by solving the problem (6) either as an initial value problem, namely

$$\boldsymbol{\xi}(0) = \boldsymbol{\xi}_0 = \mathbf{cop}_0 + \mathbf{C}_0, \tag{7}$$

or as a final value problem:

$$\boldsymbol{\xi}(T) = \boldsymbol{\xi}_T = \mathbf{cop_0} + \mathbf{C_f}\,e^{\omega T}. \tag{8}$$

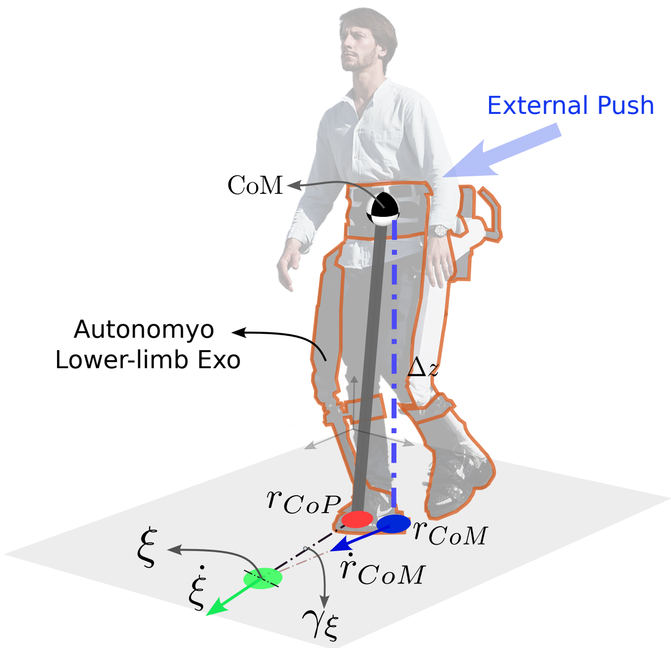

Figure 1: DCM, CoM and CoP Points correlations for Centroidal Dynamics [4]

Therefore, by solving the equation (4) as an initial value problem, we arrive at the following equation that represents the time evolution of the DCM:

$$\boldsymbol{\xi} = (\boldsymbol{\xi}_0 - \mathbf{cop_0})\exp(\omega t) + \mathbf{cop_0} \tag{9}$$

We also can solve the CoM dynamics (3) by treating it as an initial value problem:

$$\mathbf{x} = (\mathbf{x_0} - \boldsymbol{\xi}_0)\exp(-\omega t) + \boldsymbol{\xi}_0 \tag{10}$$

As evident from the above equation, the CoM exhibits stable dynamics, with the exponential term being negative. However, the DCM exhibits unstable dynamics, characterized by a positive exponential term. This indicates that the difference between ξ_0 and cop_0 increases exponentially over time. Therefore, a prerequisite for ensuring the stability of the CoM trajectory is that the DCM trajectory remains stable. As stated in [2], the concept of DCM provides a relatively straightforward approach for formulating stability requirements in walking. A simple rule proves effective in ensuring walking stability: when placing the foot, position the Center of Pressure (CoP) at a certain distance behind and outward from the DCM at the moment of foot contact. This distance between the CoP and the DCM during foot placement is referred to as the DCM offset, and minimizing this distance is crucial for maintaining viable states.

1.3 Push Recovery Controller

This section presents the step adaption mechanism based on DCM dynamics [5]. More precisely, we present below a step adjustment strategy that optimizes the next step position and timing based on the measured DCM.

To find a DCM trajectory that satisfies both the initial and the final condition problems, the coefficient C_0 must equal C_f. Thus, by combining (7) and (8), one has:

$$\boldsymbol{\xi}_0 - \mathbf{cop}_0 = (\boldsymbol{\xi}_T - \mathbf{cop}_0)\, e^{-\omega T}. \tag{11}$$

Now by defining $\sigma = e^{\omega T}$ we obtain :

$$\boldsymbol{\xi}_T + \mathbf{cop}_0(-1+\sigma) - \boldsymbol{\xi}_0\sigma = 0. \tag{12}$$

Let $\mathbf{cop}_T$ represent the CoP position at the start of the next step, and $\boldsymbol{\gamma_T} = \boldsymbol{\xi_T} - \mathbf{cop_T}$ denote the DCM offset for the next step (i.e, the end of this step). Therefore, straightforward calculations lead to:

$$\boldsymbol{\gamma_T} + \mathbf{cop}_T + (\mathbf{cop}_0 - \boldsymbol{\xi}_0)\,\sigma = \mathbf{cop}_0. \tag{13}$$

The step adjustment problem can be formalized as a constrained optimization problem, wherein the search variables consist of γ_T, $\mathbf{cop}_T$, and σ, and the cost function is appropriately defined in a quadratic manner. It's worth noting that the desired final DCM position and step timing are dependent on γ_T and σ, respectively. Additionally, $\mathbf{cop}_T$ is assumed to be located at the center of the foot at the start of the next step. Therefore, we can treat this position as the target for the upcoming footstep placement. The selected cost function aims to minimize the deviation of the desired gait values from the nominal ones:

$$J = \alpha_1|cop_{xT} - cop_{xT,nom}|^2 + \alpha_2|cop_{yT} - cop_{yT,nom}|^2 + \alpha_3|\gamma_{T,x} - \gamma_{nom,x}|^2 + \alpha_4|\gamma_{T,y} - \gamma_{nom,y}|^2 + \alpha_5|\sigma - e^{\omega T_{nom}}|^2, \tag{14}$$

where α_1, α_2, α_3 are positive numbers and the next ZMP position $\mathbf{cop}_{T,nom}$, step duration T_{nom} and next DCM offset γ_{nom} are the desired values.

We also present the following set of inequality constraints:

$$\begin{bmatrix} I_2 & 0_{2\times1} & 0_2 \\ -I_2 & 0_{2\times1} & 0_2 \\ 0_{1\times2} & I_1 & 0_{1\times2} \\ 0_{1\times2} & -I_1 & 0_{1\times2} \end{bmatrix} \begin{bmatrix} \mathbf{cop}_T \\ \sigma \\ \gamma_T \end{bmatrix} \leq \begin{bmatrix} \mathbf{cop}_{T,max} \\ -\mathbf{cop}_{T,min} \\ \sigma_{max} \\ -\sigma_{min} \end{bmatrix}, \tag{15}$$

Here, $\mathbf{cop}_{T,max}$ and $\mathbf{cop}_{T,min}$ are in $\mathbb{R}^2$, while σ_{max} and σ_{min} belong to $\mathbb{R}$. These inequality constraints are established considering the constraints imposed by leg kinematics on the maximum step length and by the maximum achievable velocity on the minimum step duration. Lastly, the relationship described in (13) is considered as an equality constraint. It is worth noting that during push recovery, the controller attempts to minimize the DCM offset to have a zero dcm offset. Due to the quadratic and linear dependence of the cost function and constraints on the unknown variables, the entire framework can be formulated as a Quadratic Programming (QP) problem:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \tfrac{1}{2}x^T P x + q^T x \\ \text{subject to} \quad & Gx \leq h \\ & Ax = b \end{aligned} \tag{16}$$

Thus, the equations 14, 15, 13 should be written in the form of equation 16 and QP library can be used to solve this problem.

Report

Please upload your code, a video of your results, and create a short report (less than 5 pages) including plots and explanation, and answers to the following questions. You do not need to repeat the methodology that already exists in this file. Only include the results that you generated.

Questions

1. Based on equation (9), which physical parameters will affect the rate of divergence of the DCM dynamics?
2. In the optimization-based DCM motion planning, how do we consider the stability of DCM movement?
3. You may notice in the code that we have a threshold for activating the use of the real DCM in optimization. If the error between the real DCM and the desired DCM is below a certain value, we ignore the error and use the desired DCM. But why does an error still exist between the real and desired DCM, even when there is no external push?
4. What other balance recovery strategies can be used in addition to the stepping strategy? (Think about your balance recovery strategies, how do you recover from external disturbances in different situations?)
5. What's the reason that we use the DCM equations in the optimization for balance recovery? Why we do not use the whole dynamic equation of the robot to solve the control problem?
6. Adjust various parameters such as nominal parameters and the boundaries of the optimization variables, then report the maximum and minimum speeds at which the robot can achieve stable push recovery. During this process, you may need to modify the timing or magnitude of the external push to maintain stability. If any changes to the push are made, please provide detailed information on the push parameters, including its timing, magnitude, and direction.
7. (**Bonus**) Adjust the parameters and increase the magnitude of the applied push to trigger a sequence of lateral recovery steps. The goal is to simulate a stronger disturbance that requires the system to take multiple corrective steps sideways to regain balance.)

References

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