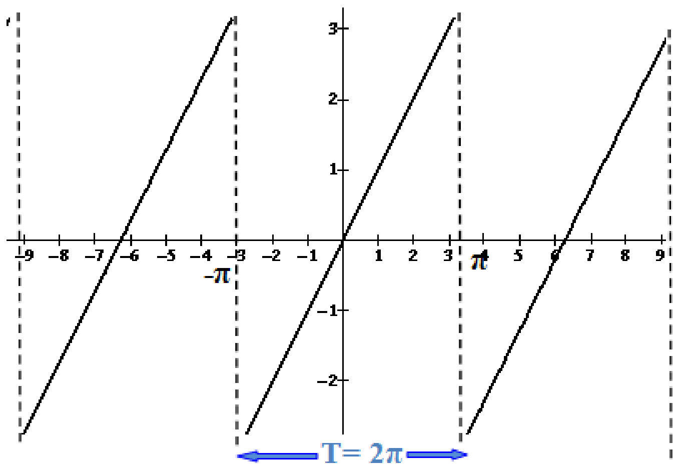
Desarrollo en serie de Fourier de la función definida por:

f(x) = x para $-\pi < x < \pi$ T = 2π



Cálculo de los coeficientes:

$$\boxed{f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(nx) + b_n \operatorname{sen}(nx)\right]}$$

$$a_0 = \int_{-\pi}^{\pi} x.\,dx = \left.\frac{x^2}{2}\right|_{-\pi}^{\pi} = 0$$

Cálculo de a_n :

$$a_n = \int_{-\pi}^{\pi} x.\cos(nx)\,dx = x.\frac{1}{n}\operatorname{sen}(nx)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{x}{n}.\operatorname{sen}(nx)\,dx$$

$$a_n = \pi.\frac{1}{n}\operatorname{sen}(n\pi) + \frac{1}{n^2}\left[\cos(n\pi) - \cos(-n\pi)\right] = 0$$

<u>Cálculo de b_n :</u>

$$b_n = \int_{-\pi}^{\pi} x.\,\mathrm{sen}\,(nx)\,dx = -x\,.\frac{1}{n}\cos(nx)\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \frac{x}{n}.\cos(nx)\,dx$$

$$b_n = -\left[\pi\frac{1}{n}\cos(n\,\pi) + \pi\cos(-\,n\,\pi)\right] + \frac{1}{n^2}\left[\mathrm{sen}\,(\,n\,\pi\,) - \mathrm{sen}\,(\,-n\pi\,)\right]$$

$$b_n = -\left[\pi\frac{1}{n}\cos(n\,\pi) + \frac{\pi}{n}\cos(-\,n\,\pi)\right] = -\frac{2}{n}\cos(n\,\pi)$$

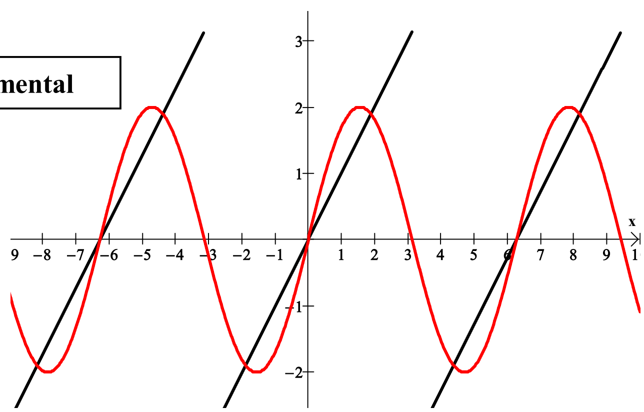
La sumatoria nos queda:

$$f(x) = \sum_{n=1}^{\infty} -\frac{2}{n}\cos(\,n\,\pi\,)\;.\mathrm{sen}\,(n\,x)$$

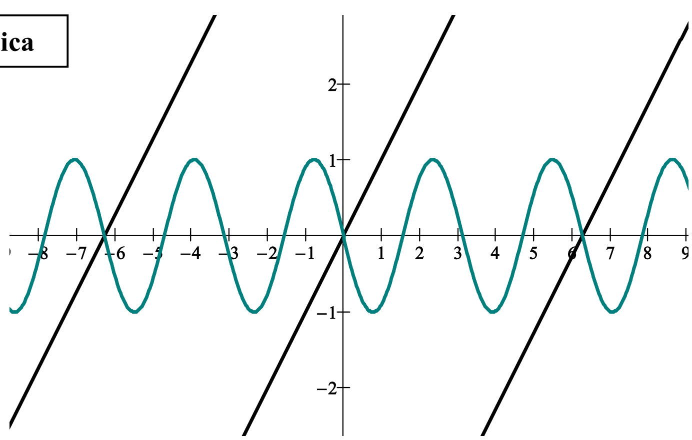
$$f(x) = 2\,\mathrm{sen}(x) - \mathrm{sen}\,(2x) + \frac{2}{3}\mathrm{sen}(3x) - \frac{2}{4}\mathrm{sen}(4x) + \frac{2}{5}\mathrm{sen}(5x) - \frac{2}{6}\mathrm{sen}(6x) + \frac{2}{7}\mathrm{sen}(7x) - ...$$

Graficamos los cuatro primeros términos de la suma

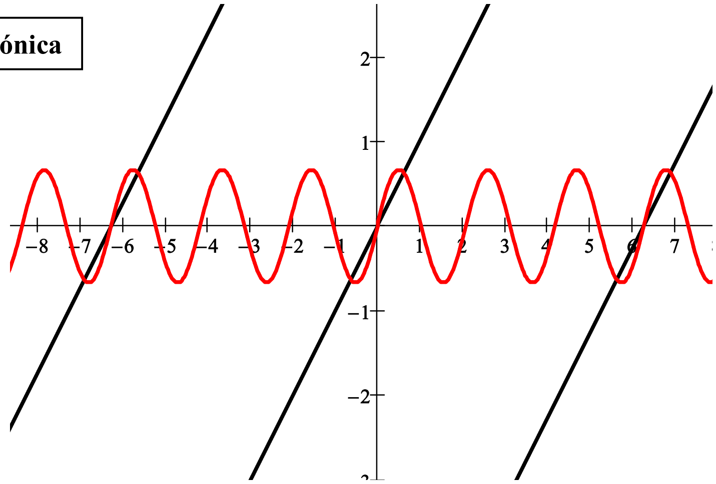
Primer armónica o fundamental



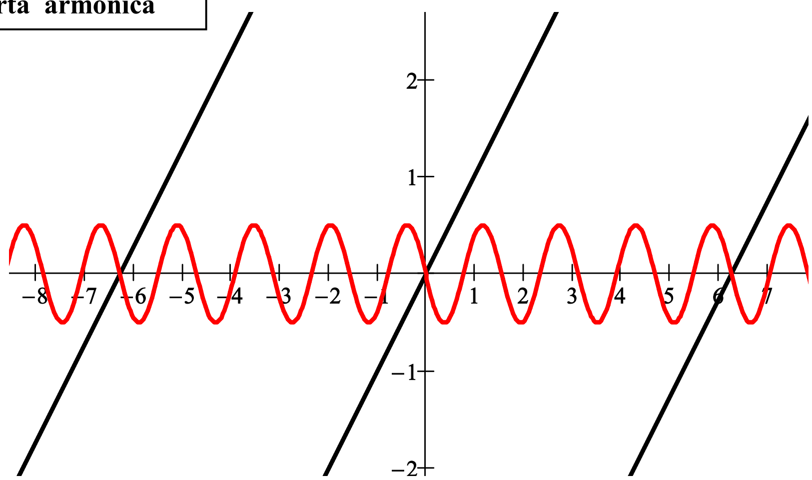
Segunda armónica



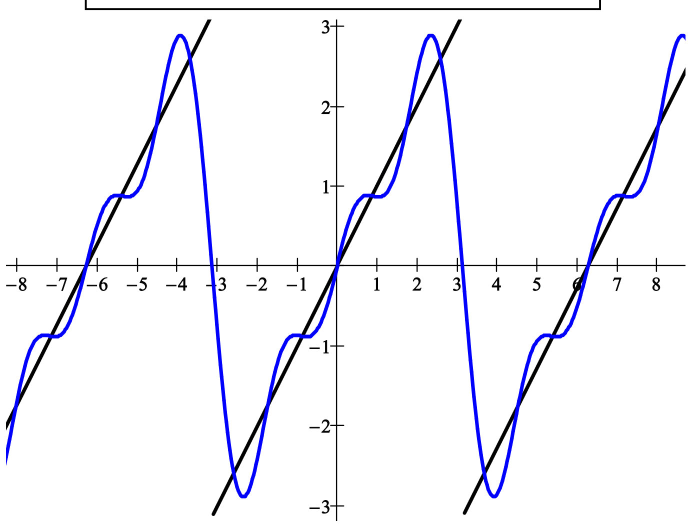
Tercer armónica



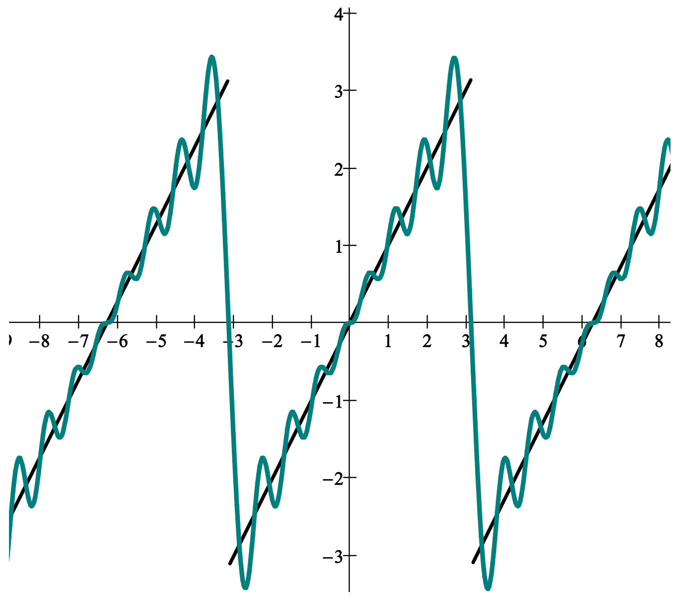
Cuarta armónica



Suma de las tres primeras armónicas



Suma de las cinco primeras armónicas



Suma de las trece primeras armónicas