Piracy in Commercial vs. Open-Source Software Competition*

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March 7, 2016

Abstract

This paper studies pricing strategies for a commercial software provider who faces competition from both a free-of-charge open-source software substitute and illegal copies of its own product. When network externalities are present, the commercial software provider may choose to set a price sufficiently low such that a critical mass of users is not attainable for the open-source product, which consequently reduces the value of open-source software. I coin this pricing strategy “network limit pricing”. Due to lack of network benefits, network limit pricing leads to increased quality differences between the commercial and open-source products, which consequently leads to a market dominated by the commercial software product. Software piracy strengthens the incentives of imposing network limit pricing. This is because piracy takes over parts of the open-source product’s market share, and because the price decrease to network limit pricing is lower, which reduces the opportunity cost of imposing this pricing strategy.

JEL classification: L13; L17; O34

Keywords: Piracy, open-source software, pricing strategies, network externalities

*This paper has benefited from helpful comments and suggestions from Torben Mideksa, Tore Nilssen and Espen Stokkereit.

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1 Introduction

Developers of commercial and proprietary software (PS) products often face competition from both free-of-charge open-source software (OSS) and unauthorized copies of their own products (piracy). While consumers make their adoption decisions based on relative prices and attributes of the available products, their choices may also depend on what *other* consumers do, because of network externalities. In this complex environment the developers of commercial software faces a difficult task in optimally pricing their products.

In this paper, I develop a model that incorporates these aspects to analyze optimal pricing strategies for a seller of a commercial software product who faces competition from both a free-of-charge open-source software product and pirated copies of its own product.[1] The primary objective is to understand how incentives for strategically taking advantage of network externalities in the pricing decision are affected by piracy, and as a consequence, how equilibrium market shares are affected by such pricing behavior.

Open-source software, a class of software released with a publicly available source code and public copyright license,[2] is in some markets competing head-to-head with proprietary software products. The Linux operating system for servers, the Firefox web browser, and Apache web servers are some examples of open-source success stories. Nontheless, the market penetration of OSS products is generally varying. Major product markets like desktop operating systems, office software, and e-mail clients, are still dominated by proprietary Microsoft products, despite the existence of free-of-charge OSS substitutes.

These product markets are also associated with strong network effects, which likely is an important explanation for the relatively unsuccessful entry of OSS in some markets. When a software product suffers from low market penetration, consumers may face costs associated with lack of compatible applications and lack of compatibility with proprietary file standards.[3] Thus, despite the unbeatable price of OSS, consumers may instead adopt the proprietary industry standard due to lack of network benefits of OSS.

This paper argues that software piracy is important in this context. Business Software Alliance (2014) estimates that 43% of all commercial software used in 2013 globally were illegal copies. This is a substantial number, and is likely to increase market shares of commercial software directly, through illegal downloads at the expense of legal commercial software and OSS, and indirectly, through lower prices of legal copies. These effect are amplified if network externalities are strong, as they contribute to increased differences in network benefits between

[1]With some abuse of language the terms "commercial" and "proprietary" are used interchangeably.

[2]The two most used classes of OSS licenses are the GNU General Public License (GPL) (`http://www.gnu.org/licenses/licenses.html`) and the Berkeley Software Distribution (BSD) license (`https://opensource.org/licenses/BSD-3-Clause`).

[3]See e.g. Katz and Shapiro (1985) for a general treatment of network externalities. Products where the quantity of compatible application increases in the installed base are typically "two-sided" in the sense that there exist positive externalities between consumers and developers of compatible applications, see e.g. Rochet and Tirole (2006).

commercial software and OSS.

The channel I emphasize in this paper is how piracy affects the way commercial software providers choose to face competition from OSS. When network effects are present and the commercial software product is endowed with a large installed base of users, the commercial software provider may impose a pricing strategy in which it is ensured that substantial network benefits are *not* attained for the OSS product. I coin this strategy as "network limit pricing". The consequence of imposing the strategy is increased differences in quality between the two products. Since it can involve a substantial price reduction, network limit pricing may not be profitable. However, software piracy affects the incentives of commercial software providers. Since software piracy both reduces OSS adoption and reduces prices of legal copies of commercial software, the cost of imposing network limit pricing is reduced as well. Hence, software piracy can provide incentives for aggressive pricing behavior, which again leads to market dominance at the expense of OSS.

The analysis provides a rationale for pricing behavior of some major software firms. As an example, in the United States v. Microsoft Corp. (2001) antitrust case, expert witnesses argued that the pricing of Microsoft Windows was "too low (...) to be legally classified as a monopoly."[4] Although Microsoft exercises price discrimination of their products, low prices, especially to the non-business segment, remain today. For instance, the Windows 8 license to original equipment manufacturers (OEMs) was priced at just \$15 in 2014,[5] and upgrades to Windows 10 were in 2015 made free-of-charge for a limited time for existing users of Windows 7 and 8 home and pro editions.[6] The future threat of successful adoption of Linux in the non-enterprise segment may have contributed to this pricing strategy.

This paper relates to previous research on pricing strategies with network effects as well as on the impact of open-source software and software piracy on competition.

Pricing strategies with network effects are studied in dynamic models by among others Fudenberg and Tirole (2000) and Cabral (2011). Fudenberg and Tirole (2000) consider a market with room for only one firm at the time, and an incumbent firm who may use a limit-pricing strategy to discourage entry of firms selling incompatible network goods. Through a different mechanism Cabral (2011) finds that network effects may lead to reduced prices because of the concern for future profits in dynamic price competition. The mechanism in the present paper shares some similarities of that of Fudenberg and Tirole (2000). However, an important

[4] "Pricing at Issue As U.S. Finishes Microsoft Case". New York Times, January 6, 1999, `http://www.nytimes.com/1999/01/06/business/pricing-at-issue-as-us-finishes-microsoft-case.html` (accessed April 29, 2014).

[5] "Microsoft Said to Cut Windows Price 70% to Counter Rivals". Bloomberg, February 22, 2014, `http://www.bloomberg.com/news/2014-02-22/microsoft-said-to-cut-windows-price-70-to-counter-rivals.html` (accessed February 27, 2014).

[6] "What Windows as a Service and a "free upgrade" mean at home and at work". Ars Technica, July 29, 2015, `http://arstechnica.com/information-technology/2015/07/what-windows-as-a-service-and-a-free-upgrade-mean-at-home-and-at-work/` (accessed February 25, 2016).

distinction is that the limit-pricing strategy in the present paper is not a strategy of entry deterrence. Rather, sufficiently low prices affect relative qualities between the available software products due to differences in network sizes.

To my knowledge, Casadesus-Masanell and Ghemawat (2006) and Le Texier and Zeroukhi (2015) are the only ones who previously have analyzed the interaction between open-source software and piracy. In a dynamic Windows vs. Linux duopoly model with network externalities, Casadesus-Masanell and Ghemawat (2006) show that Windows may use a forward-looking pricing strategy ensuring that the market does not tip to Linux dominance. By extending the model to include Windows piracy they find larger steady-state differences in user shares between Windows and Linux. However, the scope of this extension is limited as they model the piracy rate as exogenous. In Le Texier and Zeroukhi (2015) a proprietary software (PS) firm decides on price, compatibility with OSS, and copyright protection. Their model shows that piracy affects the PS firm's compatibility choice towards OSS. In particular, if the potential piracy rate is high, the PS firm chooses low compatibility with OSS to undermine network benefits of OSS and therefore soften competition. On the other hand, if the potential piracy rate is low or non-existent, the PS firm sets high compatibility with OSS which allows for higher prices from indirect network benefits from a large OSS network. A limitation to their model is that there is no substitution between PS piracy and OSS adoption. In addition to theoretical studies, cross-country empirial investigations by Casadesus-Masanell and Ghemawat (2006) and Gramstad (2016) have established a negative relationship between software piracy and Linux adoption for server and desktop versions of Linux, respectively.

The literature on proprietary and open-source software competition focuses on various aspects such as product heterogeneity (Bitzer, 2004), compatibility in two-sided platforms (Economides and Katsamakas, 2006), and competition for attracting programmers (Mustonen, 2003). Moreover, competition where PS firms face competition from OSS firms with non-profit objectives like user shares and value to contributors has been analyzed in among others Lee and Mendelson (2008) and Lanzi (2009).[7]

In the theoretical literature on software piracy it is argued that software piracy can be used as a commitment to intertemporal price discrimination (Takeyama, 1997), as a device for signaling of quality (Takeyama, 2003), and as a method of by-passing antitrust regulation on predatory pricing (Ben-Shahar and Jacob, 2004). Moreover, the impact from piracy on price and quality may differ substantially between short run and long run equilibria (Bae and Choi, 2006).

Surveys suggest that, among students, price is the most important motivation for obtaining pirated products over legal copies (Cheng et al., 1997), while norms and fear of punishment are of equal importance among working adults (Peace et al., 2003). Such heterogeneity in preferences towards piracy is also incorporated in the model of the present paper.

[7] This paper abstracts from development in open-source projects. Lerner and Tirole (2002), Lakhani and Wolf (2005), and Lerner and Schankerman (2010) analyze motivations for OSS development in firms and among individual voluntary contributors.

Piracy and competition is adressed by among others Shy and Thisse (1999), Jain (2008), and Rasch and Wenzel (2013). This work suggests that limited copy protection can be optimal for the software industry if network externalities are strong (Shy and Thisse, 1999), and may be beneficial to firms even in the absence of network externalities since piracy might reduce price competition (Jain, 2008). Rasch and Wenzel (2013) address the trade-off between consumers' surplus and software developers' surplus of copy protection in two-sided platform competition.[8]

The rest of the paper is structured as follows: Part 2 provides the benchmark model without software piracy and the main model with piracy. Numerical analysis and simulation results are presented in Part 3. Part 4 concludes. A minor extension, welfare analysis, and proofs are provided in the Appendix.

2 Model

2.1 Preliminaries

There are three types of software products available, legal and illegal copies of W (as in *Windows*) and L (as in *Linux*). As the choice of notation suggests, W is the commercial type, and the legal version is sold at a price p, while L is the open-source type which is free of charge. W and L are imperfect substitutes and incompatible to each other. Consumers choose obtain at most one of the three products.

We consider a population of software users of size normalized to one. There are two types of software users: *new users*, who make up a share σ of the population, and *old users* with a share of $1-\sigma$. The new users choose between purchasing W legally, obtaining an illegal copy of W, and downloading L for free. The old users can be thought of as consumers who have made their adoption decisions at an earlier stage, and who will not revise their decision. Hence, it is not possible to extract further profits from the old user population.[9]

Old users

Among the old user population a share $(1+\lambda)/2$ belongs to the W network, while the rest $(1-\lambda)/2$ belongs to the L network, where $\lambda \in [-1, 1]$.

The measures of old users of W and L are therefore given by

$$W^{old} = (1-\sigma)(1+\lambda)/2$$

and

$$L^{old} = (1-\sigma)(1-\lambda)/2,$$

respectively. Clearly, if $\lambda = 1$ the entire old-user population is W users. Likewise, $\lambda = -1$ means that all of the old users are L users.

[8]For a general treatment of the economics of digital piracy, excellent literature reviews are provided by Peitz and Waelbroeck (2006) and Belleflamme and Peitz (2012).

[9]Hence, whether the old users are pirates or legitimate customers of W is not relevant in this model.

W^{old} and L^{old} can be interpreted as an exogenous *installed base* of each software product. For instance, as of February 2016 approximately 11% and 52% of all desktop computer run generations old Windows XP (released in 2001) and Windows 7 (released in 2009), respectively. Before support of Windows XP ended in April 2014, the share of Windows XP was as high as 30%, despite being a more than 13 year old operating system at the time.[10] These numbers suggest there is a population of substantial size contributing to the network size of all Windows versions, while having a negligible direct effect on Microsoft's revenue. Whenever network externalities are assumed away, the old users do not affect prices and can therefore be disregarded.

New users

New users, the section of consumers whose choice of software product is not predetermined, make one of the following three choices: buy W, obtain a pirated version of W, or download L for free.

Legal versions of W are sold at a price p. The payoff of obtaining a given piece of software is split into a network dependent part $\hat{v}_i, i \in \{W, L\}$, which depends positively on size of the total installed base of the software product in question (to be addressed shortly), and a network independent part ω, which is heterogeneous over the population and uniformly distributed over $[-t/2, t/2]$. The parameter t is a measure of differentiation, which is explained in detail in Part 2.2. In addition, the network dependent value of pirated versions of W are degraded in quality by a rate $\alpha \in [0, 1]$. New users also incur a "cost-of-copying", η, uniformly distributed over $[0, z]$ by obtaining a pirated copy. This cost captures the technical and ethical costs associated with obtaining illegally copies of commercial software (details in Part 2.3). The payoffs from obtaining one of the three software products are summarized as follows:

$$\text{Legal version of } W\text{:} \quad U_W = \hat{v}_W - p - \frac{1}{2}\omega$$

$$\text{Pirated version of } W\text{:} \quad U_W^p = (1-\alpha)\hat{v}_W - \eta - \frac{1}{2}\omega$$

$$\text{Open-source product } L\text{:} \quad U_L = \hat{v}_L + \frac{1}{2}\omega.$$

The total size of the installed bases, i.e., the sum of old and new users for each software product of L and W are denoted M_L and M_W, respectively. Note that both legal and pirated copies of W belong to M_W. The network dependent value of software i, $\hat{v}_i$, is a weakly increasing function of the total market size M_W. In this paper, this function takes a simplified form by assuming the existence of a so-called *critical mass* $N \in [0, 1/2]$, defined as follows:

Definition 1. *The critical mass, $N + \epsilon, \epsilon \to 0$, is the lowest size of the installed base in which the network dependent value of product i, $\hat{v}_i$, takes a positive value: $M_i > N \Rightarrow \hat{v}_i > 0$.*

[10] Net Market Share. Desktop Operating System Version Market Share. `http://www.netmarketshare.com/operating-system-market-share.aspx?qprid=10&qpcustomd=0` (accessed February 3, 2014 and February 25, 2016).

Following the above definition, the network dependent value $\hat{v}_i$ is a function that returns binary values:

$$\hat{v}_W = \begin{cases} v_W & \text{if } M_W > N \\ 0 & \text{if } M_W \leq N \end{cases}$$

$$\hat{v}_L = \begin{cases} v_L & \text{if } M_L > N \\ 0 & \text{if } M_L \leq N. \end{cases}$$

Hence, if the total user share of product i falls below the critical mass, N, then the value from using the product falls by v_i. The size of N is critical for the optimal price set by firm W, since network externalities can be used strategically in the pricing decision. Appendix A provides a more generalized model for the existence of a critical mass.

2.2 Benchmark: No piracy

As a benchmark, competition is analyzed without the availability of illegal copies, i.e., new users only have the choice between purchasing W at price p and downloading L free-of-charge. Optimal pricing is first identified when network externalities are non-existent, i.e. $N = 0$ (or sufficiently small to affect the pricing decision). By extending the benchmark model to allow for a positive value of N, the incentives for setting the price strategically by accounting for network externalities are analyzed.

No network externalities: $N = 0$

When the critical mass is zero, $N = 0$, the network dependent value $\hat{v}_i$ always exceeds the critical mass. Thus $\hat{v}_i = v_i$ for all market shares, M_i. A new user prefers to purchase W rather than download L if $U_W \geq U_L$, i.e., whenever ω satisfies the following inequality:

$$\omega \leq (v_W - p) - v_L,$$

which implies that L is preferred to W when the opposite is true:

$$\omega > (v_W - p) - v_L.$$

The consumer who is indifferent between the two products is given by the taste $\bar{\omega} = v_W - p - v_L$. Demand for W as a function of the price is therefore the integral of all consumers with taste $\omega \in [-t/2, \bar{\omega}]$. Likewise, demand for L as a function of the price of W is the integral of consumers with taste $\omega \in (\bar{\omega}, t/2]$.

The parameter t is a measure of product heterogeneity. An increase in t increases (decreases) the benefit of using W (L) for any consumer with taste $\omega < 0$ (> 0), and equivalently, reduce (increase) the benefit of using W (L) for consumers with taste $\omega > 0$ (< 0). The benefit of modeling differentiation this way, as opposed to the pure "transportation cost" interpretation in the standard Hotelling framework, is that this model captures that some consumers will enjoy increased product differentiation.[11]

[11] E.g., the payoff of obtaining W can be rewritten to $U_W = v_W + \frac{t}{4} - \frac{1}{2}(\frac{t}{2} + \omega)$, where $\frac{t}{2} + \omega$ is the distance to W.

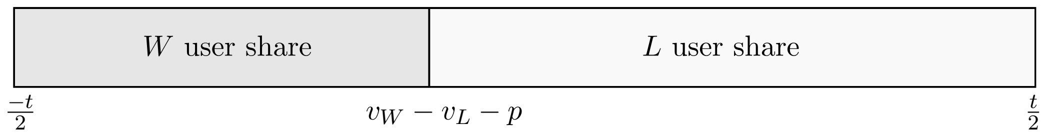


Figure 1: User shares among new users for a given level of quality and price

To take a concrete example this parameter can measure the differentiation between the products with regard to simplicity and customizability. Consumers with preferences leaning towards open-source software might prefer a simple system with little room for customizing, while consumers leaning towards open-source software will increase their benefit when the system becomes more customizable.[12]

The subscript "0" in the demand functions for new users below denotes absence of network externalities, i.e., for $N = 0$:

$$W_0^{new}(p) = \begin{cases} 0 & \text{if } v_W - p - v_L \leq -\frac{t}{2} \\ \sigma(\frac{1}{2} + \frac{v_W - v_L - p}{t}) & \text{if } -\frac{t}{2} \leq v_W - p - v_L \leq \frac{t}{2} \\ \sigma & \text{if } v_W - p - v_L \geq \frac{t}{2} \end{cases} \tag{1}$$

$$L_0^{new}(p) = \begin{cases} \sigma & \text{if } v_W - p - v_L \leq -\frac{t}{2} \\ \sigma(\frac{1}{2} - \frac{v_W - v_L - p}{t}) & \text{if } -\frac{t}{2} \leq v_W - p - v_L \leq \frac{t}{2} \\ 0 & \text{if } v_W - p - v_L \geq \frac{t}{2}. \end{cases} \tag{2}$$

Interior solutions are assumed throughout the analysis without network externalities, i.e. t is sufficiently large in order for the restriction $-\frac{t}{2} \leq v_W - p - v_L \leq \frac{t}{2}$ to be true for any profit maximizing price, p.[13] Figure 1 provides a graphical representation of the market shares at a given price.

Maintaining the assumption on interior solution, the profit maximizing price for the developer of W is straightforward:

$$p_0 = \arg\max_p W_0^{new}(p)p.$$

The first order condition solves the optimal price:

$$p_0 = \frac{t}{4} + \frac{1}{2}(v_W - v_L). \tag{3}$$

The above solution presumes that that the market is covered, i.e., that all new users have their participation constraints satisfied. The following assumption ensures that the worst-off consumer located at $\bar{\omega}$ has her participation constraint satisfied:

Assumption 1. *The marginal consumer with taste $\bar{\omega} = v_W - v_L - p_0 = (v_W - v_L)/2 - t/4$ receives a non-negative payoff, i.e., parameters are such that $U_W(\bar{\omega}, p_0) = U_L(\bar{\omega}, p_0) \geq 0$, translating into*

[12] Open-source operating systems like Android and Linux desktop distributions are known to be more customazible than proprietary substitutes like Windows and iOS. Increased customizability may to some degree be at the cost of simplicity for the end-user.

[13] It will be clear shortly that this restriction specifically requires $\frac{v_W - v_L}{t} \in [-\frac{1}{2}, \frac{3}{2}]$.

the inequality:

$$t \leq 2v_W + 6v_L.$$

By inserting equation (3) into equations (1) and (2), equilibrium user shares for the new user population are found:

$$W_0^{new}(p_0) = \sigma(\frac{1}{4} + \frac{v_W - v_L}{2t})$$
$$L_0^{new}(p_0) = \sigma(\frac{3}{4} - \frac{v_W - v_L}{2t}).$$

The profits to the developer of W at optimal price p_0 is:

$$\pi(p_0) = \sigma[\frac{1}{4} + \frac{v_W - v_L}{2t}][\frac{t}{4} + \frac{1}{2}(v_W - v_L)]. \tag{4}$$

The *total* user shares between the two software products as functions of price p, i.e. the sum of old and new users, are given by the following equations when network externalities are absent:

$$M_0^W(p) = \sigma(\frac{1}{2} + \frac{v_W - v_L - p}{t}) + (1-\sigma)\frac{1+\lambda}{2} \tag{5}$$
$$M_0^L(p) = \sigma(\frac{1}{2} - \frac{v_W - v_L - p}{t}) + (1-\sigma)\frac{1-\lambda}{2}. \tag{6}$$

The total installed base for each software type is not of importance when network externalitites are absent. In the next subsection equations (5) and (6) are important when the developer of W considers the optimal pricing strategy.

Network externalities: $N > 0$

Network externalities are modeled by $N > 0$. The choice of assuming a *critical mass* as a method of modeling network externalities deserves further elaboration. Loosely defined, a critical mass is a minimum number of adopters of a technology sufficient for self-sustained further adoption, and can therefore be viewed as a *tipping point* in which adoption converges to small or a larger number depending on the initial number of adopters being below or above some given tipping point. Following the definition of Economides and Himmelberg (1995) a critical mass is the smallest network supported in an equilibrium.

My notion of a critical mass is somewhat weaker than that of Economides and Himmelberg (1995), since the model allows for having a non-zero network size even if adoption is below the threshold N. However, products with a network size below the critical mass is perceived as less valuable to consumers. A more detailed description of the critical mass is provided in Appendix A.

Another issue, pointed out by among others Allen (1988) and Economides (1996), is that the outcome of such markets is difficult to predict due to a multiplicity of equilbria. Since a prospective member of a network may decide to adopt a product only if it is expected that at least a critical mass (here N) of others also adopt, the market outcome depends on whether consumers are able to successfully coordinate their actions.

Throughout this paper, I assume that the maximal Nash equilibrium is attained. That is, adoption above the critical mass is attained if feasible. Expectations of the network dependent value $\hat{v}_i$ are therefore formed as follows:

$$E(\hat{v}_i) = \begin{cases} v_i & \text{if } M_0^i > N \\ 0 & \text{if } M_0^i \leq N. \end{cases}$$

Hence, if the total installed base exceeds the critical mass in the hypothetical situation without network externalities, the total installed base exceeds the critical mass when network externalities are present.

Thus, demand among *new users* at a given price depends on whether the total installed base exceeds the critical mass or not.

$$W_N^{new}(p) = \begin{cases} W_0^{new}(p) & \text{if } M_0^W(p) > N \text{ and } M_0^L(p) > N \\ \min\{\sigma, \sigma(\frac{1}{2} + \frac{v_W - p}{t})\} & \text{if } M_0^W(p) > N \text{ and } M_0^L(p) \leq N \\ \max\{0, \sigma(\frac{1}{2} - \frac{v_L + p}{t})\} & \text{if } M_0^W(p) \leq N \text{ and } M_0^L(p) > N \end{cases} \quad (7)$$

$$L_N^{new}(p) = \begin{cases} L_0^{new}(p) & \text{if } M_0^W(p) > N \text{ and } M_0^L(p) > N \\ \max\{0, \sigma(\frac{1}{2} - \frac{v_W - p}{t})\} & \text{if } M_0^W(p) > N \text{ and } M_0^L(p) \leq N \\ \min\{\sigma, \sigma(\frac{1}{2} + \frac{v_L}{t})\} & \text{if } M_0^W(p) \leq N \text{ and } M_0^L(p) > N. \end{cases} \quad (8)$$

The subscript "N" in the demand functions denotes the presence of network externalities.[14] M_0^W and M_0^L are given by equations (5) and (6). Hence, if demand for both software types exceeds the critical mass, then the demand functions are identical to those in the benchmark where network externalities are unimportant.

Provided that $L^{old} < N$, the developer of the commercial software type has an additional pricing strategy: to set the price such that $M_0^L(p) \leq 0$. From equations (7) and (8) it is clear that this pricing strategy leads to a positive shift in demand for W, while demand for L falls by v_L/t, or to zero demand if $t \leq 2(v_W - p)$.

The effect on demand for W from network limit pricing is illustrated in Figure 2. For prices less than or equal to p_N, the user share of L is below critical mass and is regarded as less valuable. Therefore, there is a jump in demand for W at this exact price due to lack of valuable substitutes. In effect, W is a monopolist when $L_0(p) \leq N$ and $t \leq 2(v_W - p)$. However, the monopoly power is constrained since W is limited in the price setting since any price above the threshold such that $L_0(p) > N$ would cause W to lose its monopoly position.

For the remainder of the paper, I impose the following assumption on parameter values:

Assumption 2. *Parameter values are such that $M_0^W(p_0) > N$ and $M_0^L(p_0) > N$, which are equivalent to the equalities:*

$$N - L^{old} < \sigma(\frac{3}{4} - \frac{v_W - v_L}{2t}),$$

$$N - W^{old} < \sigma(\frac{1}{4} + \frac{v_W - v_L}{2t}).$$

[14] The subscript "0" can therefore be interpreted as a special case where $N = 0$.

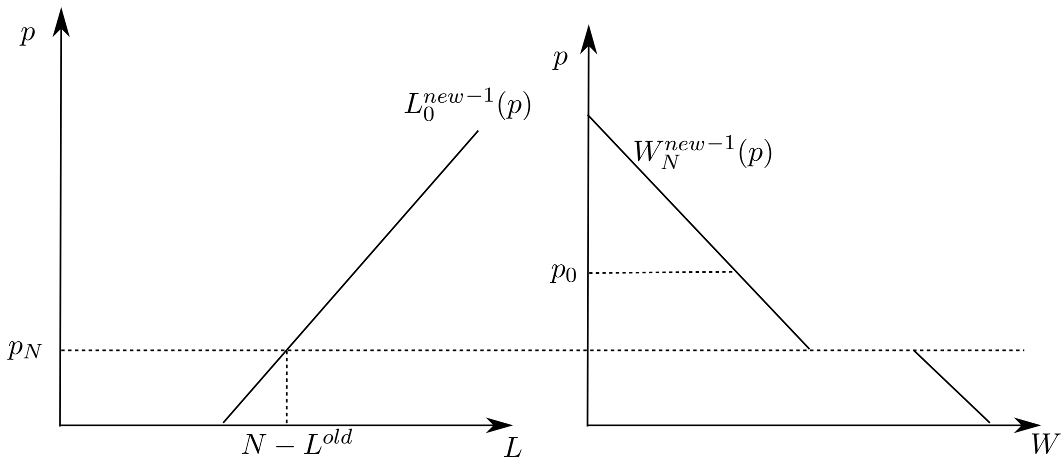


Figure 2: Network limit pricing leads to increased demand for W following reduced quality of open-source substitutes.

That is, at the profit maximizing prices absent network externalities, both software types have user shares above critical mass. Hence, the developer of W does not need to lower the price to ensure network benefits, and the user share of L does not fall below critical mass unless the seller of W makes a strategic pricing decision with the attempt to pull demand for L below critical mass.

The network limit price, p_N, is given by the equality $L_0^{new}(p_N) + L^{old} = N$, which has the following closed-from solution:

$$p_N = (\frac{N - L^{old}}{\sigma} - \frac{1}{2})t + v_W - v_L. \tag{9}$$

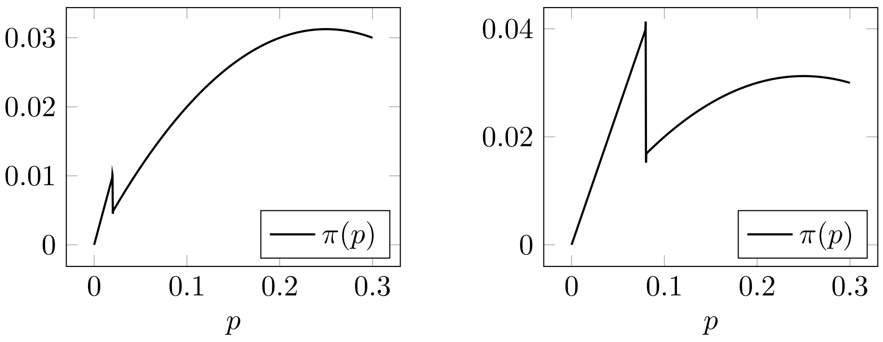
As before, $L^{old} = (1-\sigma)(1-\lambda)/2$. Conditional on the network limit price being imposed, the profits for the seller of W is given by:

$$\pi(p_N) = \min\Big\{\sigma, \sigma(1 + \frac{v_L}{t}) - N + L^{old}\Big\}\Big((\frac{N - L^{old}}{\sigma} - \frac{1}{2})t + v_W - v_L\Big). \tag{10}$$

The positive output effect by imposing network limit pricing is large. However network limit pricing may also imply a large price reduction. As is clear from equation (9), p_N is increasing in N. Intuitively, the larger the critical mass is, the less the price must be reduced in order to force demand for L below critical mass.

As Figure 3 illustrates, the size of N is crucial for the optimal pricing strategy. Profits have a kink at the network limit price, p_N. For $p \leq p_N$, demand for L among new users is lower (in fact zero at given parameter values) due to reduced network benefits, while for $p > p_N$, both software types exist in the market for new users, and demand for W drops once the price exceed the threshold p_N. The larger N is, the smaller is the price differential between p_N and p_0, and p_N is more likely to be the profit maximizing price.

Among other parameters of interest it is easy to check that $\frac{\partial \pi(p_N)}{\partial \lambda} > \frac{\partial \pi(p_0)}{\partial \lambda} = 0$, and



(a) $N = .26$ **(b)** $N = .29$

Figure 3: Profits at different prices. Profits under network limit pricing is at the kinks, and is more likely to be profit-maximizing for larger N.
Parameter values: $\sigma = 0.5, v_W = v_L = t = \lambda = 1$.

$\frac{\partial \pi(p_N)}{\partial t} < \frac{\partial \pi(p_0)}{\partial t}$ for sufficiently small N.[15] Hence, in the absence of piracy, network limit pricing is more likely to occur whenever W has the benefit of a large installed base (λ close to one) and when the degree of differentiation is small (small t). The intuitive reason is that differentiation is a source of market power, and thus the opportunity cost (higher prices) of imposing network limit pricing is higher.

2.3 Piracy

The model is extended by allowing new users to obtain a pirated version of W instead of the legal version of W or the open-source product L. Optimal pricing strategies are once again identified, with and without network externalities, and the network limit pricing strategy where demand for L falls below the critical mass is shown to be more likely to be an optimal strategy with piracy present in the market.

No network externalities: $N = 0$

Piracy is added to the model by allowing users to obtain a free-of-charge version of W. Despite W being available for free, there may be reasons for users rather to purchase W legally. First, a pirated version may have some degree of quality degradation. This may for instance be due to lack of customer support or incompatibility with patches.[16] Second, users may face constraints, for both technical and ethical reasons, of obtaining illegal copies of a piece of software. Moreover, such non-monetary costs may vary between individuals.

The timing of the choices of the new users is illustrated in Figure 4 and is as follows: (i) users choose between joining the W network and joining the L network; (ii) users in the W network

[15] If $N - L^{old} \leq \frac{\sigma v_L}{t}$, then the restriction is $N - L^{old} < \sigma[\frac{9}{16} - \frac{(v_W - v_L)^2}{4t^2}]$. If $N - L^{old} > \frac{\sigma v_L}{t}$, then the restriction is $N - L^{old} < \sigma[\frac{3}{4} + \frac{1}{2t}\sqrt{(v_L - v_W)(5v_L - v_W)}]$. Since the inequality $M_0^L(p_0) > N$ is assumed throughout the paper, the latter restriction is satisfied for the equality $v_W = v_L$.

[16] A patch is a software update designed to fix bugs and security vulnerabilities.

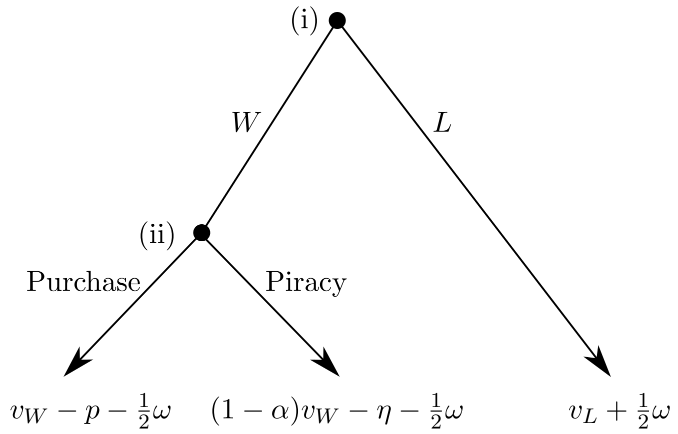


Figure 4: Pay-offs for choice of software product.

choose whether to obtain a pirated or a legal version. In addition it is assumed that new users are unaware of their personal costs of obtaining pirated software before choosing network in stage (i). Instead, they make their choices based on the expected value of choosing a software type at given price.[17]

Legal and illegal versions of W differ among new users in two ways. The pirated version is degraded in quality by a parameter $\alpha \in [0, 1]$, where $\alpha = 0$ means that there is no difference in quality between legal and illegal copies, and $\alpha = 1$ means that the pirated version is of no value. Also, new users differ by the personal cost, η, of obtaining pirated copies which is uniformly distributed over the interval $[0, z]$. This parameter is meant to capture the above mentioned observation that individuals differ in their technical abilities and ethical constraints in obtaining pirated software.

Demand functions are found by solving the model backwards. Given that a user has chosen to join the W network, her value of η is revealed. A new user chooses to purchase rather than pirate whenever the payoff is higher for purchasing:

$$v_W - p \geq (1-\alpha)v_W - \eta.$$

Otherwise, piracy is chosen. From the uniform distribution of η it is straightforward to calculate the share (or probability) of new users purchasing W:

$$\Pr(\text{buy}) = \phi(p) = \min\{1 - \frac{p - \alpha v_W}{z}, 1\}. \tag{11}$$

Hence, the piracy rate among new users of W, i.e. the share who do not purchase is:

$$\Pr(\text{pirate}) = 1 - \phi(p) = \max\{\frac{p - \alpha v_W}{z}, 0\} \tag{12}$$

Intuitively, the piracy rate increases in the purchase price. Moreover, z is a measure of the price-sensitivity of the piracy rate: A unit increase in the puchasing price increases the piracy rate by $1/z$ percentage points whenever $p > \alpha v_W$.

[17]Solving the model with full initial information on personal "cost of copying" is feasible, but will in addition to the distributions of ω and η, depend on the distribution of $\omega + \eta$ which greatly complicates the analysis since this distribution generally is not continuous over the area of interest.

By backward induction, a new user joins the W network if the expected value of choosing W is larger than the value of downloading L. In the first stage, a new user either purchases or pirates W rather than downloading L whenever:

$$\omega \leq \phi(p)[v_W - p] + (1 - \phi(p))[(1 - \alpha)v_W - (p - \alpha v_W)/2] - v_L,$$

where $\phi(p)$ is given in (11), and $(p - \alpha v_W)/2$ is the expected value of η conditional on the consumer being a pirate.[18] This implies that a new user downloads L when the opposite is true, i.e., when:

$$\omega > \phi(p)[v_W - p] + (1 - \phi(p))[(1 - \alpha)v_W - (p - \alpha v_W)/2] - v_L.$$

The demand functions for the two software products among new users, independent on pirating or purchasing, are found by integrating over all consumers who satisfy the above inequalities.[19]

$$\tilde{W}_0^{new}(p) = \sigma\Big[\frac{1}{2} + \frac{\phi(p)[v_W - p] + (1 - \phi(p))[(1 - \alpha)v_W - (p - \alpha v_W)/2] - v_L}{t}\Big] \tag{13}$$

$$\tilde{L}_0^{new}(p) = \sigma\Big[\frac{1}{2} - \frac{\phi(p)[v_W - p] + (1 - \phi(p))[(1 - \alpha)v_W - (p - \alpha v_W)/2] - v_L}{t}\Big] \tag{14}$$

From (13) and (14), and their equivalents in (1) and (2), the following result is of interest:

Proposition 1. *At a given price, total demand for W is larger with piracy than without piracy. Equivalently, total demand for L is smaller with piracy than without piracy: $\tilde{W}_0^{new}(p) > W_0^{new}(p)$ and $\tilde{L}_0^{new}(p) < L_0^{new}(p)$ if $\phi(p) < 1$.*

Proof. The inequalities $\tilde{W}_0^{new}(p) > W_0^{new}(p)$ and $\tilde{L}_0^{new}(p) < L_0^{new}(p)$ are equivalent to:

$$\frac{1}{2}[p(1 + \phi(p)) + \alpha v_W(1 - \phi(p))] < p$$

$$\Leftrightarrow \frac{-(\max\{p - \alpha v_W, 0\})^2}{2z} < 0$$

$$\Leftrightarrow p > \alpha v_W.$$

This is also the condition for a positive piracy rate, i.e. $\phi(p) < 1$. □

The demand function for purchased (pirated) versions of W among new users is equivalent to the probability of both being a W user and a legal user (pirate). Hence, by rearranging (13) and multiplying with the probabilities of purchasing and pirating, respectively, the demand functions become:

$$\tilde{W}_{buy,0}^{new}(p) = \phi(p)\sigma\Big[\frac{1}{2} + \frac{v_W - \frac{1}{2}[p(1 + \phi(p)) + \alpha v_W(1 - \phi(p))] - v_L}{t}\Big] \tag{15}$$

$$\tilde{W}_{piracy,0}^{new}(p) = (1 - \phi(p))\sigma\Big[\frac{1}{2} + \frac{v_W - \frac{1}{2}[p(1 + \phi(p)) + \alpha v_W(1 - \phi(p))] - v_L}{t}\Big]. \tag{16}$$

[18] $E[\eta | \eta \leq p - \alpha v_W] = \dfrac{p - \alpha v_W}{2}$.

[19] A diacritical tilde is included on relevant functions and results to distinguish the piracy case from the no-piracy benchmark.

The profit maximizing price for the developer of W is then:

$$\tilde{p}_0 = \arg\max_p W^{new}_{buy,0}(p)p. \tag{17}$$

Hence, $\tilde{p}_0 = p^*$ is determined by the first order condition of the above maximization problem:

$$\tilde{W}^{new}_{buy,0}(p^*) + p^*[\phi'(p^*)\tilde{W}^{new}_0(p^*) + \phi(p^*)\frac{\partial \tilde{W}^{new}_0(p)}{\partial p}] = 0. \tag{18}$$

Here, $\tilde{W}^{new}_0(p)$ is given by equation (13). From equation (18), it can be seen that the negative output effect from a marginal price increase can be broken down into two parts: $\phi(p)(\partial \tilde{W}^{new}_0(p)/\partial p)$ is the decrease in demand for purchased W in favor of downloading L, while $\phi'(p)\tilde{W}^{new}_0(p)$ represents the loss in revenue from an increase in the piracy rate.[20]

Proposition 2. *The price with piracy can be both higher and lower than without piracy, i.e., both $\tilde{p}_0 > p_0$ and $\tilde{p}_0 \leq p_0$ is feasible.*

Proof. Appendix C. □

It is not difficult to find instances where the price with piracy, $\tilde{p}_0$, is lower than the price without, p_0. The opposite case is true only for a small subset of parameters, which is proven in Appendix C. Nevertheless, the result may seem like a paradox as one would normally expect competition from piracy to always lead to a price reduction. $\tilde{p}_0$ exceeding p_0 can only be true if the degree of degradation of illegal copies, α, is close to or equals zero, i.e., that legal and illegal copies are almost perfect substitutes. For $\alpha = 0$, the *expected* cost of obtaining any copy of W is $p(1 + \phi(p))/2 < p$ for $\phi(p) < 1$. Thus setting an ex-post price $\tilde{p}_0$ slightly above p_0 may be optimal, since the ex-ante cost of W may still be below p_0. Note, however, that the mark-up above p_0, $(\tilde{p}_0 - p_0)/p_0$, is small,[21] and it can hardly be argued that $\tilde{p}_0 > \tilde{p}$ is a likely outcome within the model setup. Nonetheless, the result gives a rationale for why $\tilde{p}_0$ may not necessarily be monotonically increasing in α.

Another observation of the optimal price with piracy, which is important when we consider network externalities, is summarized by the following proposition:

Proposition 3. *The price eventually seizes to increase in the degree of differentiation:*

$$\frac{d\tilde{p}_0}{dt} > 0, \quad \lim_{t\to\infty} \frac{d\tilde{p}_0}{dt} = 0.$$

Proof. Write the first-order condition for profit maximization as:

$$[\phi(p^*) + p^*\phi'(p^*)]f(p^*, t) + p^*\phi(p^*)f'_p(p^*, t) = 0,$$

where $f(p^*, t) = \dfrac{1}{2} + \dfrac{v_W - \frac{1}{2}[p(1 + \phi(p)) + \alpha v_W(1 - \phi(p))] - v_L}{t}$

By applying implicit differentiation w.r.t. p^* and t, the following expression is found:

$$\frac{dp^*}{dt} = \frac{-[\phi(p^*) + p^*\phi'(p^*)]f'_t + p^*\phi(p^*)f''_{pt}}{2\phi'(p^*)f + 2[\phi(p^*) + p^*\phi'(p^*)]f'_p + p^*\phi(p^*)f''_{pp}}$$

[20] The closed-form solution to (18) is the only non-complex solution of a cubic equation, and is omitted.

[21] In numerical computations I did not encounter a markup exceeding 2%.

		No piracy	Piracy
$\sigma = 1$	price	0.25	0.197
$v_W = v_L = 1$	W^{new} network	0.25	0.312
$t = 1$	L^{new} network	0.75	0.688
$\alpha = 0.1$	piracy rate	0%	19.5%
$z = 0.5$	profits	0.0625	0.049
$\sigma = 1$	price	0.5	0.26
$v_W = v_L = 1$	W^{new} network	0.25	0.38
$\boldsymbol{t = 2}$	L^{new} network	0.75	0.62
$\alpha = 0.1$	piracy rate	0%	32%
$z = 0.5$	profits	0.125	0.068
$\sigma = 1$	price	0.25	0.2
$v_W = v_L = 1$	W^{new} network	0.25	0.32
$t = 1$	L^{new} network	0.75	0.68
$\boldsymbol{\alpha = 0.05}$	piracy rate	0%	29.8%
$z = 0.5$	profits	0.0625	0.045
$\sigma = 1$	price	0.25	0.155
$v_W = v_L = 1$	W^{new} network	0.25	0.35
$t = 1$	L^{new} network	0.75	0.65
$\alpha = 0.1$	piracy rate	0%	18.5%
$\boldsymbol{z = 0.3}$	profits	0.0625	0.044

Table 1: Price, network size, piracy rate and profits for various parameter values without network externalitites. Values in bold are changes relative to the first row.

It is easy to check that $\lim_{t\to\infty} f'_t(p^*, t) = 0, \lim_{t\to\infty} f'_p(p^*, t) = 0, \lim_{t\to\infty} f''_{pp}(p^*, t) = 0, f''_{pt}(p^*, t) = 0$ and $\lim_{t\to\infty} f(p^*, t) = 1/2$. Hence,

$$\lim_{t\to\infty} \frac{dp^*}{dt} = \frac{0}{\phi'(p^*)} = 0.$$

□

The intuition behind Proposition 3 is as follows: As t increases, a larger share of the population has strong preferences for W which leads to an increase in price. On the other hand, as their preferences towards piracy are unchanged, there is a limit to how high the price can be set without the W developer losing all the customers to pirated versions of the product.

Table 1 provides a few examples on how prices, and therefore other endogenous variables, are affected by changes in parameter values. In all the examples the quality of the commercial and open-source products are equal. The price increases in the degree of product differentiation, t, when piracy is present, but at a smaller scale relative to the benchmark case without piracy, which is consistent with Proposition 3.

Higher degradation in the quality of pirated versions, i.e., an increase in α, has an ambiguous effect on the price. On one hand, piracy is now a less valuable option since the competitive pressure from piracy is weaker, which allows for higher prices. On the other hand, a higher degree of degradation may lower the *ex-ante* value of obtaining W. Hence, for low values of α the price may decrease in α since the piracy rate initially is non-negligible, and the value of obtaining any version of W may fall. In a small subset of parameter values the price may even exceed p_0 when α is decreasing (from Proposition 2). In the numerical examples in Table 1 the latter effect dominates, which explains why the price increases when the quality of illegal copies is improved.

Lastly, a decrease in z lowers the expected cost of copying, resulting in a more price sensitive piracy rate. When individuals are more likely to acquire illegal copies at a given price, the seller of W responds by cutting the price, which ultimately leads to a larger W network and a smaller L network. However, due to the result in Proposition 2 there may exist a small interval where p is decreasing in z by a small amount for low values of α.

Network externalities: $N > 0$

With the existence of a critical mass of users, i.e., $N > 0$, the commercial software provider has the option to impose network limit pricing. Piracy affects incentives for imposing such pricing for three reasons. First, as stated by Proposition 1, the market share of L is lower at a given price when piracy is present, suggesting that the price reduction from $\tilde{p}_0$ to $\tilde{p}_N$ is lower relative to the no-piracy benchmark. Second, $\tilde{p}_0$ is likely to be lower than p_0, meaning that the negative price effect on profits by imposing network limit pricing is weaker. Third, the price reduction from network limit pricing reduces demand for piracy in favor of legal versions.

As before, I assume that users' expectations on market shares at given prices are based on the hypothetical market shares in the absence of network externalities. Furthermore, I impose an assumption equivalent to Assumption 2 from the case without piracy.

Assumption 3. *At the price $\tilde{p}_0$ defined by equation* (17)*, parameter values are such that the following two inequalities are satisfied:*

$$\tilde{M}_0^W(\tilde{p}_0) := \tilde{W}_0^{new}(\tilde{p}_0) + W^{old} > N$$

$$\tilde{M}_0^L(\tilde{p}_0) := \tilde{L}_0^{new}(\tilde{p}_0) + L^{old} > N.$$

I.e., for the profit maximizing price of W without network externalities, user shares of both software types exceed the critical mass, meaning that the network dependent value is positive for both software products when network limit pricing is not imposed.

The network limit price with piracy is equivalent to its counterpart in the no-piracy benchmark. The network limit price, $\tilde{p}_N$, is the highest price of W in which demand for L is below the critical mass, i.e., the price is set such that $\tilde{L}_0^{new}(p_N) + L^{old} = N$. In other words, $\tilde{p}_N$ solves:

$$\sigma[\frac{1}{2} - \frac{v_W - \frac{1}{2}[\tilde{p}_N(1+\phi(\tilde{p}_N)) + \alpha v_W(1-\phi(\tilde{p}_N))] - v_L}{t}] + L_{old} = N. \tag{19}$$

If $\phi(\tilde{p}_N) = 1$, i.e., $\tilde{p}_N < \alpha v_W$, then the solution to the above equation equals the network limit price, p_N, in equation (9) from the no-piracy case. For $\phi(\tilde{p}_N) < 1$, i.e. $\tilde{p}_N > \alpha v_W$, the solution to the above expression is:

$$\tilde{p}_N = \alpha v_W + z - \sqrt{z\Big[2t\big([\frac{1}{2} - \frac{N - L^{old}}{\sigma}] + v_L - (1-\alpha)v_W\big) + z\Big]}.$$

Or, inserting p_N from equation (9):

$$\tilde{p}_N = \alpha v_W + z - \sqrt{z\Big[z - 2(p_N - \alpha v_W)\Big]}. \tag{20}$$

Hence, there is a non-linear relationship between the network limit prices when piracy is present and in the no-piracy benchmark. From (20) we find that the network limit price in the no-piracy benchmark can never exceed its equivalence in the presence of piracy.

Proposition 4. *The network limit price with piracy is higher than or equal to the one without piracy:* $\tilde{p}_N \geq p_N$.

Proof. For $\tilde{p}_N \leq \alpha v_W$ the piracy rate is zero, i.e. $\phi(\tilde{p}_N) = 1 \Rightarrow \tilde{L}_0^{new}(\tilde{p}_N) = L_0^{new}(\tilde{p}_N)$, which in turn implies that $\tilde{p}_N = p_N$.

For $\tilde{p}_N > \alpha v_W$, the piracy rate is positive. Solving equation (20) for p_N yields the following expression:

$$p_N = \tilde{p}_N - \frac{1}{2z}(\tilde{p}_N - \alpha v_W)^2,$$

which clearly implies that $\tilde{p}_N > p_N$ for $\tilde{p}_N > \alpha v_W$. □

This result can be intuitively explained by the fact that $\tilde{L}_0^{new}(p) \leq L_0^{new}(p)$, by Proposition 1. By definition, $M_0^L(p_N) = N$, which implies the inequality $\tilde{M}_0^L(p_N) \leq N$. Hence, for the same N, the price can be increased from p_N while still maintaining demand for L below critical mass.

The network limit price, $\tilde{p}_N$, reacts similarly to changes in parameter values as its counterpart in the no-piracy benchmark. Obviously, stronger network externalities, interpreted as a larger critical mass N, and a larger installed base of W users (λ) lead to an increase in $\tilde{p}_N$. The response from increased degree of differentiation, t, depends on the strength of network externalities, as summarized by the following proposition:

Proposition 5. *If* $N - L^{old} > \sigma/2$, *then* $\tilde{p}_N$ *increases in* t *at an increasing rate for a positive piracy rate:* $\dfrac{d\tilde{p}_N}{dt} > 0$ *and* $\dfrac{d^2\tilde{p}_N}{dt^2} > 0$.

Proof. The proof follows from applying implicit differentiation w.r.t. t in equation (19):

$$\frac{dp}{dt} = \frac{\frac{N-L^{old}}{\sigma} - \frac{1}{2}}{\phi(p)} > 0 \text{ if } N - L^{old} > \sigma/2 \ ,$$

$$\frac{d^2p}{dt^2} = \frac{\frac{N-L^{old}}{\sigma} - \frac{1}{2}}{z\phi(p)^2}\frac{dp}{dt} > 0 \text{ if } \phi(p) < 1 \ .$$

□

The network limit price $\tilde{p}_N$ is increasing in t for sufficiently strong network externalities. The intuition behind this result is that a higher degree of differentiation yields lower demand for L at any price $p > \bar{p}$, where $\bar{p}$ is the price that solves $L_0^{new}(\bar{p}) = \sigma/2$. Hence, the price reduction from $\tilde{p}_0$ that network limit pricing implies is lower for large t. Moreover, following the concavity of $L_0^{new}(p)$ in p, $\partial^2\tilde{p}_N/\partial t^2 > 0$ whenever a solution exists. A graphical illustration of the relationship between $\tilde{p}_N$ and t is provided in Figure 5.

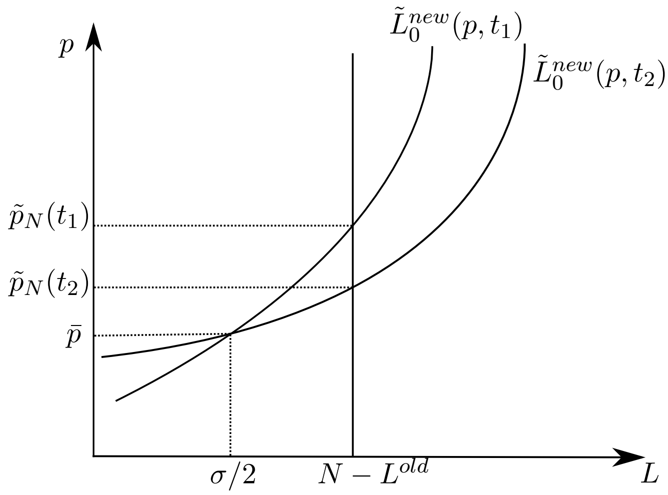


Figure 5: $\tilde{p}_N$ increasing in t for $N - L^{old} > \tilde{L}_0^{new}(\tilde{p})$, $t_1 > t_2$.

Comparative statics differs from the no-piracy benchmark. While $\tilde{p}_0$ converges to a constant in t, $\tilde{p}_N$ increases convexly in t for $\tilde{p}_N < \tilde{p}_0$, which is always true given the assumptions in the model.[22] Some implications of this result are illustrated numerically in the next section.

3 Numerical analysis and simulation results

Network limit pricing may result in corner solutions with respect to both the piracy rate and the size of the L user population, and the model outcomes depend on the combination of several parameters, which in addition must satisfy the restriction of $\phi(\tilde{p}_0) < 1$. While some of the results from the preceding section help understand the main mechanisms, important questions with regard to incentives for imposing network limit pricing and to profits are difficult to answer formally. I therefore rely on numerical tools for the rest of the analysis.

Three results from using numerical tools are emphasized in this section: (i) software piracy strengthens incentives of the commercial software provider to implement network limit pricing, (ii) high degree of product differentiation strengthens incentives to impose network limit pricing with piracy, which is the opposite of what is the case without piracy, and (iii) so-called "profitable piracy" does not occur in this model.

3.1 Incentives for imposing network limit pricing

Figure 6 compares profits under the two pricing strategies in the cases with and without piracy for different values of N, and other parameters taken as given. The solid lines represent profits under network limit pricing, while profits with the price p_0 where demand for L is above critical mass among new users is represented by the dashed lines. As the figure illustrates, network limit pricing is optimal under a larger set of values of N when piracy is present. If the critical mass is interpreted as the strength of network externalities, this means that network limit pricing is optimal for weaker degrees of network externalitites in the presence of piracy relative to the benchmark case.

[22] $\tilde{p}_N < \tilde{p}_0$ is true whenever $\tilde{M}_0^L(\tilde{p}_0) > N$ which holds by assumption.

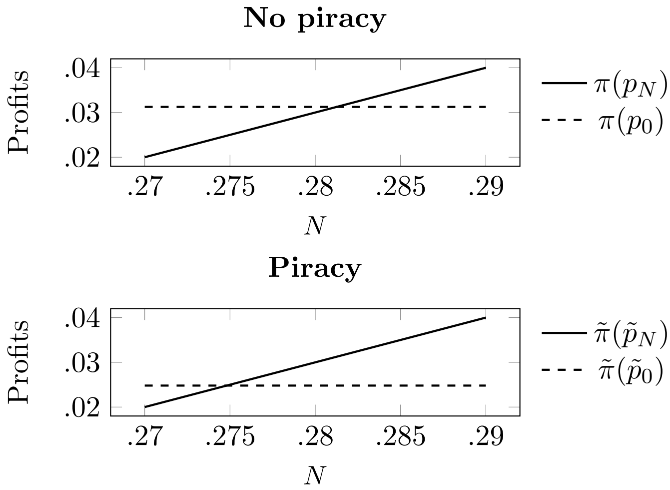


Figure 6: Network limit pricing optimal for lower values of N in the presence of piracy. Parameters: $\sigma = 0.5, z = 0.5, v = t = \lambda = 1, \alpha = 0.1$

If the share of new users willing to obtain pirated copies falls, i.e., an increase in z, or the quality of pirated products is decreased, i.e. α increases, then the critical value of N for network limit pricing to be optimal rises, and this critical value is for large values of z and α equal to that of the no-piracy benchmark.[23] That is, the two lines in the bottom panel of Figure 6 never cross at a larger N than what is the case for the no-piracy benchmark in the top panel. I conjecture that for *any* set of parameters it cannot be the case that network limit pricing is optimal in the absence of piracy, while not being optimal when piracy is present. The first simulation result supports this conjecture:

Simulation result 1. *Consider parameter values such that total network sizes exceed the critical mass whenever network limit pricing is not imposed:* $M_0^W(p_0)$, $\tilde{M}_0^W(\tilde{p}_0)$, $M_0^L(p_0)$, $\tilde{M}_0^L(\tilde{p}_0) > N$. *Then no set of parameters where network limit pricing is optimal without piracy while not being optimal with piracy are found:* $\pi(p_N) > \pi(p_0)$ *and* $\tilde{\pi}(\tilde{p}_N) < \tilde{\pi}(\tilde{p}_0)$ *do not occur for any of the simulated model outcomes.*

By repeated draws of random parameter values from large variance uniform distributions no observations found the inequalities $\pi(p_N) > \pi(p_0)$ and $\tilde{\pi}(\tilde{p}_N) < \tilde{\pi}(\tilde{p}_0)$ to hold for the same values of parameters. On the other hand, for a non-negligible share of the draws the opposite was the case, namely that network limit pricing was not optimal in the absence of piracy, but optimal in the presence of piracy. I.e., network limit pricing is imposed as a reaction to piracy. Figure 7 shows a scatter plot of 1297 simulated realizations of the model, and there is not one single observation in the fourth quadrant representing the two inequalities $\pi(p_N) > \pi(p_0)$ and $\tilde{\pi}(\tilde{p}_N) < \tilde{\pi}(\tilde{p}_0)$. Details on the simulation exercise are provided in Appendix C.

Although difficult to formally prove, the result is intuitive. Except for a very limited set of parameters, p_0 exceeds $\tilde{p}_0$,[24] and from Proposition 5 we know that $\tilde{p}_N \geq p_N$. Hence, the

[23]Notice that $\lim_{z\to\infty} \phi(p^*) = 1$ and $\phi(p^*) = 1$ if $p^* < \alpha v_W$ for $p^* \in \{p_N, p_0\}$.

[24]By running simulations restricting parameters such that the inequality $\tilde{p}_0 > p_0$ holds, the hypothesis from the conjecture was still valid.

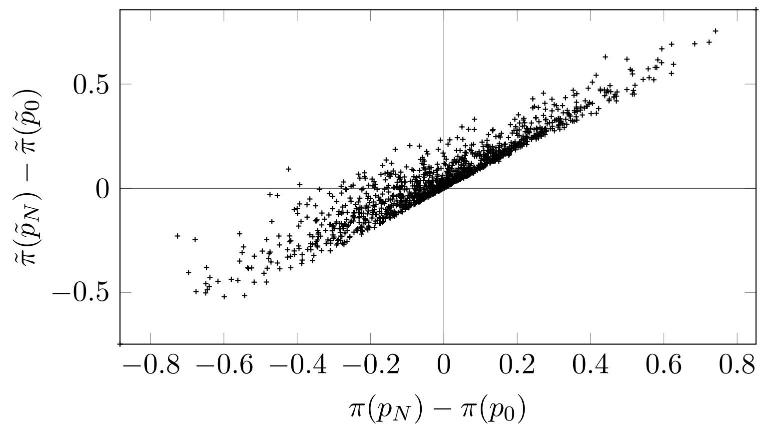


Figure 7: Scatter plot of 1297 simulated model outcomes. $\pi(p_N) > \pi(p_0)$ and $\tilde{\pi}(\tilde{p}_N) < \tilde{\pi}(\tilde{p}_0)$ does not occur for the same parameter values.

price reduction by imposing network limit pricing, $\tilde{p}_0 - \tilde{p}_N$, is reduced, which increases the likelihood of $\tilde{p}_N$ being the profit maximizing price. In addition, besides making the open-source competitor less valuable to consumers, the price p_N also has a positive effect on sales through a lower piracy rate, which further incentivizes the commercial software developer to impose network limit pricing.

3.2 Product differentiation

The importance of the degree of differentiation between W and L, measured by the parameter t, was discussed in Section 2.3. Figure 8 illustrates how the size of t matters numerically. As t becomes large, a larger share of the new users values W (and L) highly, meaning that the commercial developer can set a high price of the commercial software product in the case without piracy. Hence, the cost of imposing network limit pricing is high, as the price differential between p_0 and p_N is large. Therefore, in the absence of piracy, setting the price at p_0 is likely to be the profit maximizing strategy for large t.

However, if an illegal copy of W is available, the commercial developer is limited in the price-setting. For a given z, there is a limit to how high the price can be without the firm losing a substantial share of the sales to piracy. For the numerical example in the bottom panel of Figure 8, $\tilde{p}_0$ seizes to increase as t becomes large, consistent with Proposition 3. On the other hand, the network limit price $\tilde{p}_N$ grows at an increasing rate in t for $\tilde{p}_N < \tilde{p}_0$, meaning that the cost of imposing network limit pricing is low due to the small price differential between the two pricing strategies. Hence, $\tilde{p}_N$ is likely to be the optimal price for large t (and from Proposition 5, for a sufficiently large N) in the presence of piracy.

3.3 Piracy and profits

A recurring topic in the literature on digital piracy is whether so-called "profitable piracy" is feasible, i.e., that the owner of the intellectual property can increase profits as a consequence

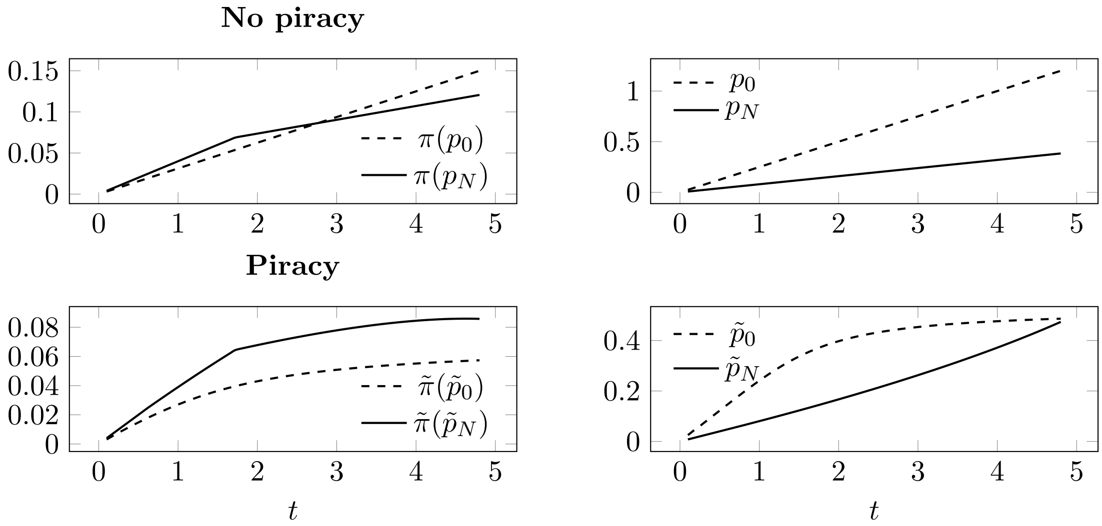


Figure 8: Network limit pricing not optimal for large degree of differentiation in no-piracy benchmark, but optimal in the presence of piracy
Parameters: $\sigma = 0.5, v = \lambda = z = 1, \alpha = 0.05, N = 0.29$.

of piracy.[25]

In the present model, I investigate this possibility in a simulation exercise with 1267 random draws of parameter values, similar to the one in Part 3.1, where I also take into account that the commercial software provider may choose different pricing strategies with and without piracy, and that $\tilde{p}_N > p_N$ for a positive piracy rate (Proposition 2). Figure 9 shows profits with and without piracy for each draw of parameters, and no observation lies above the 45 degree line. Thus, in this model "profitable piracy" does not occur.

Simulation result 2. *No combination of parameters where piracy increases profits for the commercial software provider are found.*

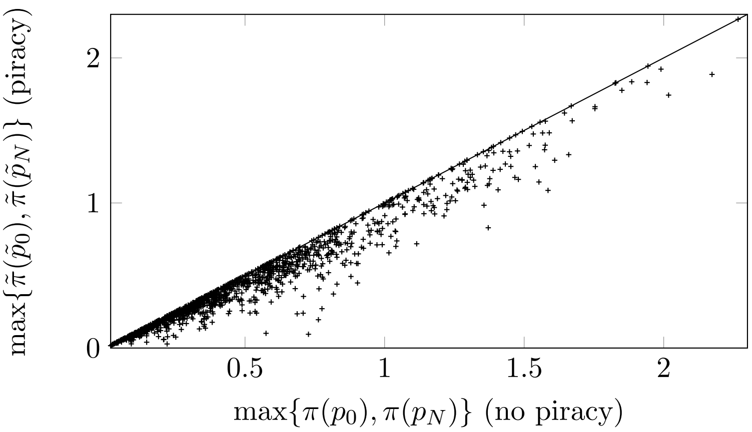


Figure 9: Scatter plot of profits of 1267 random draws of parameter values. No instances of profitable piracy were identified.

[25]Takeyama (1997), Shy and Thisse (1999), and Jain (2008) are some examples of theoretical models of "profitable piracy" in software industries. With endogenous compatibility Le Texier and Zeroukhi (2015) identify profitable piracy in an environment with open-source competition.

Piracy facilitates network limit pricing, which by itself suggests higher profits from piracy since $\tilde{p}_N > p_N$, i.e., W could reap a substantial market share, potentially all of the new users, at a higher price than without piracy. Nonetheless, a higher network limit price relative to the no-piracy case is not sufficient to compensate for lost sales from illegal copying.

4 Conclusion

This paper has analyzed optimal pricing strategies when a proprietary software developer faces competition from both piracy and free-of-charge open-source software in a market with network externalitities. Two pricing strategies for the commercial software provider are identified: (i) "standard" pricing where the network size of the OSS product is not strategically accounted for in the pricing decision, and (ii) "network limit pricing" where the price of the commercial software product is deliberately set with the intention of a critical mass of users not being attained for the OSS type. The consequence of network limit pricing is a concentrated market dominated by commercial software at the expense of open-source software adoption.

The main finding from the theoretical model is that the conditions for network limit pricing are more easily satisfied in the presence of piracy relative to the no-piracy benchmark. The explanation is threefold: First, piracy of commercial software adds to the network of the commercial software type at the expense of open-source software. When the installed base of OSS users is initially smaller, the cost of imposing network limit pricing is reduced, and therefore it is more likely to be optimal. Second, competition from piracy typically forces the developer of commercial software to cut the price, which increases the number of purchasing customers, partly at the expense of the installed base of OSS. Through this channel, the cost of introducing network limit pricing is reduced further. Third, network limit pricing leads consumers to substitute away from piracy towards legal copies, which strengthens the positive output effect a lower price compared to the case without piracy.

Network limit pricing is more likely to be optimal when network externalities are strong, independently of the existence of illegal copies of commercial software. In addition, a large degree of differentiation between open-source and commercial software makes network limit pricing unlikely when piracy is absent, but very likely in the presence of piracy. In a world without piracy, product differentiation creates market power translating into high prices. Piracy reduces the market power from product differentiation since high prices translates to a high piracy rate. Hence, with piracy the price differential between the two pricing strategies is small when product differentiation is large, and the commercial software provider can force open-source software out of the market with a relatively small price reduction.

Since software goods consist of information only, unauthorized replication typically leads to little loss in quality. In addition, price is arguably the most important factor explaining the extent of software piracy. Although piracy rates vary substantially between countries, the piracy rate is high for commercial software products in general, which suggests that the average cost of copying is fairly low for a substantial share of software users. Hence, network limit pricing is likely to be an optimal pricing strategy for commercial software products benefiting from a large installed base relative to OSS substitutes.

The results might provide an explanation for Microsoft's pricing of products like Windows and Office. Despite their dominant market position and a huge army of faithful users, their products are not priced nearly as high as some software products provided by other dominant developers of proprietary software who face similar threats from piracy (e.g. Adobe). Microsoft products are associated with strong network externalities, easily accessible illegal copies, and large installed bases. It can therefore be argued that it is a strategic decision by Microsoft not to price their products too high in order to prevent open-source competition from increasing its installed base, thus preventing network benefits being obtained.

The results also give an additional rationale for the existence of student discounts on software. Being both experimental and price-sensitive, students might be more likely to try out free alternatives to proprietary software products. Hence, proprietary software developers might be incentivized to lower the price in order to avoid open-source competition gaining network benefits. Arguably, young people have lower technical costs and perhaps also ethical costs in obtaining pirated products. Hence, the existence of high-quality illegal copies incentivizes proprietary developers further to impose network limit prices to students.

The analysis in this paper suggest that piracy is hurtful to both proprietary and open-source software developers. While piracy increases usage of commercial software, unauthorized copying is likely to affect profits negatively through lower prices and possibly reduced sales. Piracy reduces usage of open-source software both through the direct effect of individuals leaving open-source software products in favor of free-of-charge illegal copies, as well as the indirect effect of lower prices on legal proprietary alternatives. In addition, the presence of piracy may more easily provide incentives for proprietary software developers to lower prices substantially in order to avoid free-of-charge competitors to generate the network benefits needed to create demand. It should therefore be in the interest of open-source enthusiasts to advocate for increased copyright protection.

A Network externalities and critical mass

The assumption on binary values of the network dependent value for determining of a critical mass is clearly an over-simplification made for the sake of tractability. In this section, I relax this assumption by allowing the network dependent value $v_i, i \in \{W, L\}$ to be a continuously increasing function of the respective installed bases. For simplicity, I only consider the case without software piracy. The results show that under certain conditions, the binary assumption of $\hat{v}_i$ is a close approximation to the continuous case.

Throughout this analysis I assume the market is covered, i.e., $W^{new}(p) + L^{new}(p) = \sigma$ for all p. For the sake of simpler notation, I denote $x_L = L^{new}(p)$, implying $\sigma - x_L = W^{new}(p)$. The network dependent value of the two software products is non-decreasing in the expected sizes of the respective installed bases:

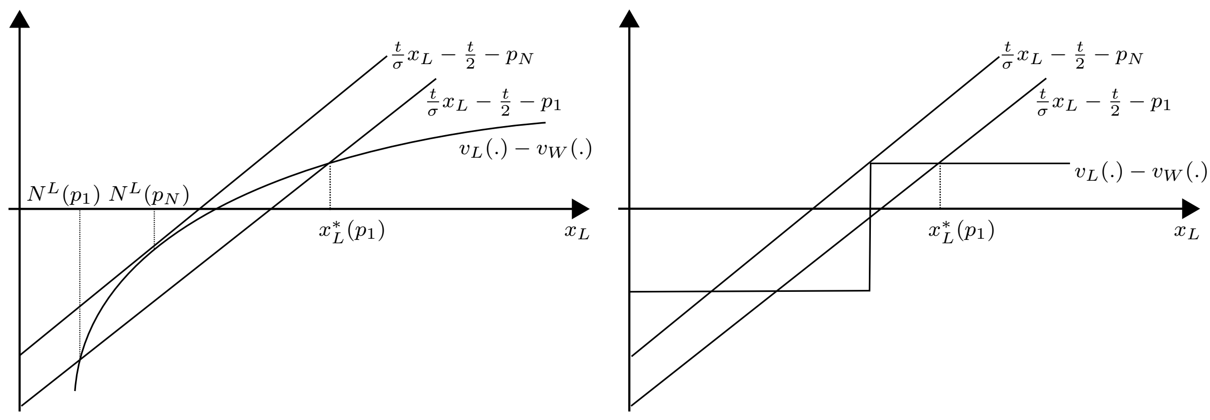
$$v_L = v_L(x_L + L^{old}), v_W(\sigma - x_L + W^{old}), v_i' \geq 0.$$

Similarly to the demand in equation (2), demand for L among new users is given by:

$$x_L = \sigma[\frac{1}{2} + \frac{v_L(x_L + L^{old}) - v_W(\sigma - x_L + W^{old}) + p}{t}]$$

Rearranging, an interior solution for L with fulfilled expectations on the network sizes satisfies the following equality:

$$\frac{t}{\sigma}x_L - \frac{t}{2} - p = v_L(x_L + L^{old}) - v_W(\sigma - x_L + W^{old}) \tag{A1}$$



(a) Critical mass as a function of price. $N^L(p) = N(p) - L^{new}$

(b) Critical mass independent of price.

Figure A1: Critical mass and network limit pricing for different functional forms of v_i.

Figure A1 shows various equilibrium outcomes of L for different functional forms of $v_L(x_L + L^{old}) - v_W(\sigma - x_L + W^{old})$, and shows that there may be multiple equilibria. In Figure A1 (a) there are three equilibrium outcomes of x_L for a price $p_1 > p_N$: two stable equilibria at $x_L = 0$ and $x_L = x_L^*(p_1)$, as well as an unstable equilibrium which is interpreted as the *critical mass* $N(p_1) - L^{old}$. Since $v_L(.) - v_W(.)$ is continuously increasing in x_L, the critical mass is endogenous as it depends on p. *Network limit pricing* is in this setting the price which ensures that there is there is only a minimal equilibrium, i.e., p_N is such that:

$$\frac{t}{\sigma} = v_L'(x_L^*(p_N) + L^{old}) - v_W'(\sigma - x_L^*(p_N) + W^{old}).$$

I.e, the price that satisfies the tangent condition in Figure A1 (a).

Figure A1 (b) corresponds to the model in Part 2 of the paper. Once x_L exceeds some threshold value, v_L jumps up. As a consequence of the discontinuity, the critical mass is here independent of the price, and the price p_N corresponds to the price in which the function $\frac{t}{\sigma}x_L - \frac{t}{2} - p_N$ merely touches the kink at $v_L(.) - v_W(.)$. Also note that the functional form of $v_L(.) - v_W(.)$ as well as the size of t determines whether there will be a positive adoption level of L among new users when the critical mass is not attained.

B Welfare

Since dynamic effects from piracy and pricing strategies is not considered in the model, a welfare analysis is somewhat limited. Within the model one unambiguous welfare result is derived: the social optimum will not be realized in the market solution, irrespective on which pricing strategy is imposed and whether piracy is present in the market.

Since the social optimum is unlikely to be achieved it is more fruitful to compare social welfare under different pricing strategies and for the no-piracy benchmark vs. presence of piracy. It turns out that welfare effects are generally ambiguous and depend on parameter values. When network externalities are not present or not important for the pricing decision, too few will use the commercial software type and too many will use open-source software. When network limit pricing is imposed the opposite is the case: too many will use commercial software and too few (potentially none) will use open-source software.

Piracy also has an ambiguous effect on welfare. When network externalities is not important for the pricing decision user shares will be closer to the social optimum. On the other hand, there will be an efficiency loss from potential quality degradation and from the cost of obtaining illegal copies. It is unclear whether the increased likelihood of network limit pricing is welfare enhancing, since it is not generally clear whether network limit pricing is welfare enhancing.

B.1 First best

Since the marginal cost is zero and investment costs are sunk, the the social surplus is equivalent to the gross consumer surplus. The payoff of the old users is not defined, thus the welfare analysis will only apply to the gross welfare of the new users. Therefore, the welfare results will be normalized to σ, the new-user population size. The gross value of using W and L for a consumer at location ω is $v_W - \frac{1}{2}\omega$ and $v_L + \frac{1}{2}\omega$, respectively. First best allocations are given by the optimization problem:

$$\max_m \int_{-t/2}^{m} (v_W - \frac{1}{2}\omega)d\omega + \int_{m}^{t/2} (v_L + \frac{1}{2}\omega)d\omega,$$

where m is the marginal consumer who is indifferent between the products. The maximization problem has the solution

$$m^* = v_W - v_L.$$

The first best user share among new users are then given by:

$$W^* = \frac{1}{2} + \frac{v_W - v_L}{t},$$

$$L^* = \frac{1}{2} - \frac{v_W - v_L}{t}.$$

The maximized social surplus becomes:

$$WF^{FB} = \frac{t^2}{8} + \frac{t}{2}(v_W + v_L) + \frac{1}{2}(v_W - v_L)^2.$$

First best is implemented if the commercial software provider gives away the product for free. Arguably, the first best allocation is unlikely to be achieved. The rest of the welfare analysis therefore focuses on welfare in the market solution.

B.2 Benchmark: No piracy

No network externalities

When network externalities are non-existent or are not accounted for in the pricing decision, the profit maximizing price is p_0 given by equation (3). It can be shown that, under this price, the consumer indifferent between W and L will be located at:

$$m_0 = \frac{1}{2}(v_W - v_L) - \frac{t}{4} = \frac{1}{2}m^* - \frac{t}{4}$$

The social surplus under price p_0 is then:

$$WF_0 = WF^{FB} - \frac{t}{4}(v_W - v_L) - \frac{1}{8}(v_W - v_L - t/2)^2$$

Welfare result 1. *The user share of commercial software is too small, and user share of open-source software is too large, in the market solution with price p_0.*

The reason for the welfare maximum not being attained is due to the market power the commercial software provider obtains from product differentiation. Marginal cost is zero, and the price is higher than the marginal cost at the profit maximizing price. Hence, some consumers will choose to download the open-source software product for free, even if the gross benefit from the commercial software type is higher.

Network limit pricing

When network limit pricing is imposed there are two opposing effects in play. On one hand, open-source software will not be available, which will reduce the payoff of individuals with high values of ω, and the market will not be covered for high values of t. On the other hand, the commercial software product will be available at a lower price, which will increase the payoff of former L users with $\omega < 0$, who consequently switch to W. The location of the worst-off consumers who will obtain W is:

$$m_N = t/2 - \min\{0, v_L - t(N - L^{old})/\sigma\}.$$

Note that, if $(N - L^{old})t < \sigma v_L$ there will be no effects on aggregate consumption as the market is covered in both regimes. The social surplus with network limit pricing is given by:

$$WF_N = \int_{-t/2}^{m_N} (v_W - \frac{1}{2}\omega) d\omega$$

$$= \begin{cases} tv_W & \text{if } (N - L^{old})t/\sigma \leq v_L \\ tv_W - \frac{t}{2}(v_W - \frac{t}{8}) - \left(\frac{(N-L^{old})t}{\sigma} - v_L\right)\left(v_W - \frac{1}{4}\frac{(N-L^{old})t}{\sigma}\right) & \text{if } (N - L^{old})t/\sigma > v_L \end{cases}$$

Intuitively, the degree of differentiation matters, even if there are no aggregate effects on quantity. If the degree of differentiation is small, the negative welfare effects are not substantial since W is a close substitute to L. If the two products are more differentiated, welfare is more likely to be reduced. The following welfare result summarizes the welfare effects relative to the price where network externalities are not directly accounted for in the price.

Welfare result 2. *Network limit pricing increases welfare relative to p_0 if and only if $v_W > v_L$ and:*

$$t < 6(v_W - v_L).$$

This welfare result implies that network limit pricing always reduces welfare if there are no quality differences between the two products for the median user, $v_W = v_L$, since t is strictly positive. When there are no quality differences, the price p_0 yields user shares of 1/4 and 3/4 between W and L. When network limit pricing is imposed a share 1/4 will increase their gross surplus (user with $\omega < 0$ who download L at price p_0), while a share 1/2 will be worse off (all users with $\omega > 0$).

Relative to the social optimum there will be over-consumption of commercial software and under-consumption (zero) of open-source software.

Welfare result 3. *The user share of commercial software is too large, and user share of open-source software is too small in the market solution with price p_N.*

B.3 Welfare with piracy

No network externalities

Relative to the no-piracy benchmark, the profit maximizing price $\tilde{p}_0$, given by equation (17), has two opposing effects on welfare. From Proposition 1 the total user share of W is larger relative to the no-piracy benchmark. Hence, the location of the consumer indifferent between W (any type) and L, $\tilde{m}_0$, is closer to the social optimum: $m_0 \leq \tilde{m}_0 < m^*$. However, piracy also entails a welfare loss through a possible degradation of quality, α, and the cost of obtaining working illegal copies, η. In relation to the social surplus in the no-piracy benchmark, welfare with piracy and price $\tilde{p}_0$ is:

$$\widetilde{WF}_0 = WF_0 + \int_{m_0}^{\tilde{m}_0} (v_W - v_L - \omega) d\omega + (1 - \phi(\tilde{p}_0))\Big[\int_{-t/2}^{\tilde{m}_0} (-\alpha v_W - \frac{1}{2}\omega) d\omega - \int_0^{\tilde{p}_0 - \alpha v_W} \eta d\eta\Big]$$

Whether welfare increases or decreases relative to the no-piracy benchmark is generally ambiguous and depends on a combination of parameters.

Network externalities

The effects on welfare by going from the price $\tilde{p}_0$ to $\tilde{p}_N$ are similar to the no-piracy benchmark. However, since $\tilde{p}_N < \tilde{p}_0$, the piracy rate will be lower, meaning the welfare loss from quality degradation and copying cost is reduced.

Taking pricing incentives into account, it is unclear whether the finding that network limit pricing is more likely to occur in the presence of piracy is good for welfare. However, if network limit pricing is imposed due to large product differentiation, welfare is most likely reduced due to the reduction in the value of users located at large ω.

From simulations with randomly generated parameter values there is no significant correlation between differences in profits under different pricing strategies, $\pi(p_N) - \pi(p_0)$, and differences in welfare, $WF_N - WF_0$, in both the case with piracy and in the no-piracy benchmark. This suggests that the incentives for engaging in network limit pricing do not have a clear effect on total welfare. By only inspecting parameter values where network limit pricing is imposed with piracy, but not without it, welfare increased relative to the no-piracy benchmark in approximately 61%, and reduced in 39%, of a total of 1614 simulated model realizations.

C Proofs

Proof of Proposition 2: By inserting p_0 from equation (3) into the LHS of equation (18), i.e. the marginal profit function $\tilde{\pi}'(p)$, the result is proven if there exist parameter values such that $\tilde{\pi}'(p_0) > 0$.

Note that if $\phi(p_0) = 1$, then $p_0 = \tilde{p}_0$ since this implies a zero piracy rate. If $\phi(p_0) = 0$, then $p_0 > \tilde{p}_0$ since this price yields a piracy rate of 100% and zero profits, thus it must be optimal to set a price lower than p_0.

Hence, we must restrict the parameters such that $0 < \phi(p_0) < 1$, which is satisfied if we set $p_0 = \alpha v_W + hz$, where $h \in (0, 1)$. By inserting (3) for p_0 we restrict t to be of the form:

$$t = 4(\alpha v_W - \frac{v_W - v_L}{2} + hz)$$

It can be shown that $\tilde{\pi}'(p_0)$ now can be written as:

$$\tilde{\pi}'(p_0) = -\frac{\sigma}{2tz}(2\alpha^2 v_W^2 + 3\alpha v_W h^2 z + 2\alpha v_W hz + 4h^3 z^2 - h^2 z^2).$$

Hence, $\tilde{\pi}'(p_0) > 0$ if

$$2\alpha^2 v_W^2 + 3\alpha v_W h^2 z + 2\alpha v_W hz + 4h^3 z^2 - h^2 z^2 < 0,$$

then there exists parameter values where $\tilde{p}_0 > p_0$ if the set $(0, -2\alpha^2 v_W^2 - \alpha v_W hz\,(3h + 2) + h^2 z^2)$ is non-empty. In order to show that a non-empty set may exist within the model's assumptions, we can set $\alpha = 0$. Then $\tilde{\pi}'(p_0) > 0$ if $0 < h < \frac{1}{4}$.

$\square$

Details on Simulation result 1: Parameter values where randomly drawn from the following uniform distributions: $\sigma \in [0.1, 0.7], v_W \in [0.1, 10], v_L \in [0.1, 10], t \in [0.01, 10], \lambda \in [0.5, 1], N \in$

$[0.2, 0.35], \alpha \in [0, 0.5], z \in [0, 10]$. The model was simulated 20 000 times, and the restrictions $M_0^W(p_0)$, $\tilde{M}_0^W(\tilde{p}_0)$, $M_0^L(p_0)$, $\tilde{M}_0^L(\tilde{p}_0) > N$ as well as $\phi(\tilde{p}_0) < 1$ were true 1297 times of these draws.[26]

Figure 7 in the text plots the 1297 random draws with $\pi(p_N) - \pi(p_0)$ on the horizontal axis and $\tilde{\pi}(\tilde{p}_N) - \tilde{\pi}(\tilde{p}_0)$ on the vertical axis. From the figure it is clear that there is not one single observation where $\pi(p_N) > \pi(p_0)$ and $\tilde{\pi}(\tilde{p}_N) < \tilde{\pi}(\tilde{p}_0)$ is true for the same parameter values, while for a non-trivial share (exactly 178 observations) of the simulated data $\pi(p_N) < \pi(p_0)$ and $\tilde{\pi}(\tilde{p}_N) > \tilde{\pi}(\tilde{p}_0)$. i.e., the commercial software provider imposes network limit pricing as a response to piracy being present.

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[26] The restriction $\phi(\tilde{p}_0) < 1$ was included to ensure that the piracy and no-piracy cases did not give identical results.

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