$f(x) := x \cdot e^{-x}$

$f1(x) := 1 \qquad f2(x) := x \qquad f3(x) := x^2 \qquad w(x) := e^x$

PARABOLA

$$A := \begin{pmatrix} \int_0^1 f1(x)\cdot f1(x)\cdot w(x)\,dx & \int_0^1 f2(x)\cdot f1(x)\cdot w(x)\,dx & \int_0^1 f3(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_0^1 f1(x)\cdot f2(x)\cdot w(x)\,dx & \int_0^1 f2(x)\cdot f2(x)\cdot w(x)\,dx & \int_0^1 f3(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_0^1 f1(x)\cdot f3(x)\cdot w(x)\,dx & \int_0^1 f2(x)\cdot f3(x)\cdot w(x)\,dx & \int_0^1 f3(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix}$$

$$B := \begin{pmatrix} \int_0^1 f(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_0^1 f(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_0^1 f(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix}$$

$$A = \begin{pmatrix} 1.71828 & 1 & 0.71828 \\ 1 & 0.71828 & 0.56344 \\ 0.71828 & 0.56344 & 0.46454 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.5 \\ 0.33333 \\ 0.25 \end{pmatrix}$$

$X := A^{-1} \cdot B$

$$X = \begin{pmatrix} 0.01737 \\ 0.79882 \\ -0.45758 \end{pmatrix}$$

$g(x) := X_0 \cdot 1 + X_1 \cdot x + X_2 \cdot x^2$

$$E := \sqrt{\int_0^1 f(x)\cdot f(x)\cdot w(x)\,dx - \left(X_0 \cdot B_0 + X_1 \cdot B_1 + X_2 \cdot B_2\right)}$$

$E = 6.172 \times 10^{-3}$

RECTA

$$M := \begin{pmatrix} \int_0^1 f1(x)\cdot f1(x)\cdot w(x)\,dx & \int_0^1 f2(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_0^1 f2(x)\cdot f1(x)\cdot w(x)\,dx & \int_0^1 f2(x)\cdot f2(x)\cdot w(x)\,dx \end{pmatrix} \qquad N := \begin{pmatrix} \int_0^1 f(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_0^1 f(x)\cdot f2(x)\cdot w(x)\,dx \end{pmatrix}$$

$$M = \begin{pmatrix} 1.71828 & 1 \\ 1 & 0.71828 \end{pmatrix}$$

$$H := M^{-1}\cdot N$$

$$N = \begin{pmatrix} 0.5 \\ 0.33333 \end{pmatrix}$$

$$H = \begin{pmatrix} 0.11019 \\ 0.31066 \end{pmatrix}$$

$$h(x) := H_0\cdot 1 + H_1\cdot x$$

$$E1 := \sqrt{\int_0^1 f(x)\cdot f(x)\cdot w(x)\,dx - \left(H_0\cdot N_0 + H_1\cdot N_1\right)}$$

$$E1 = 0.044$$