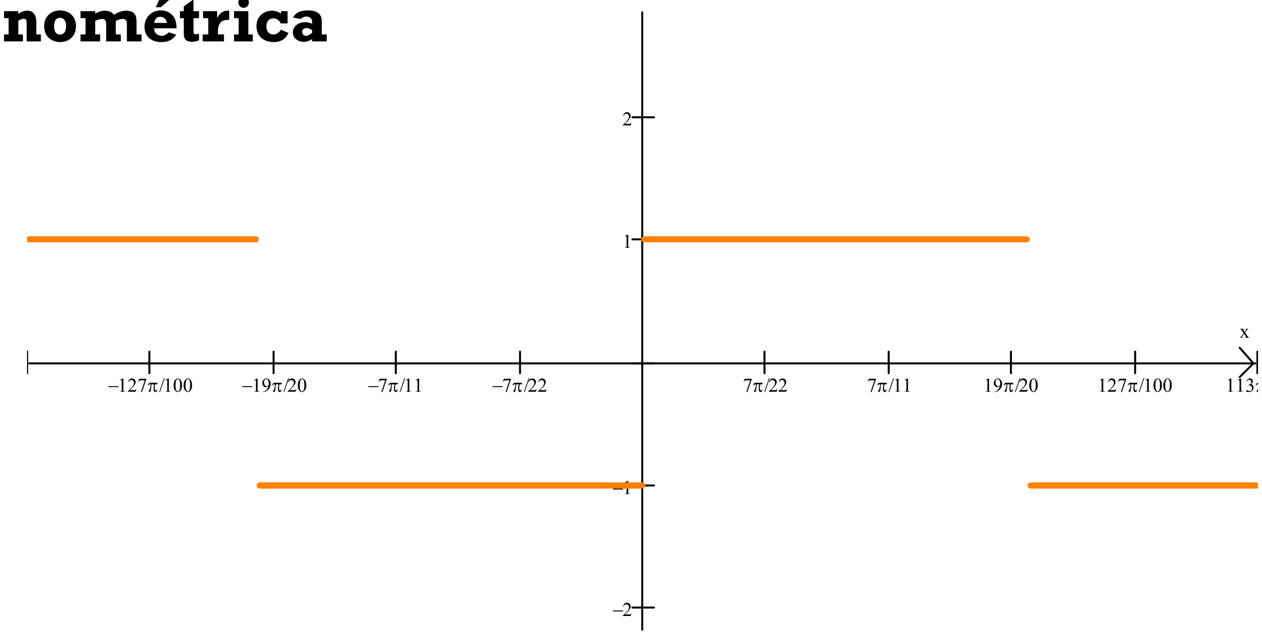
DESARROLLO EN SERIE DE FOURIER

- **Forma trigonométrica**
- **Forma exponencial**
- **Interpretación de gráficos**

Desarrollo en forma trigonométrica

$$f(x) = \begin{cases} 1 & \text{Para} \quad 0 < x < \pi \\ -1 & \text{Para} \quad -\pi < x < 0 \end{cases}$$

Ej.1



$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)sen(nx)dx$$

$$b_n = \frac{2}{\pi}\int_{-\pi}^{0} - sen(nx)dx = \frac{2}{\pi}\left[\frac{1}{n}\cos(nx) \quad \Big|_{-\pi}^{0} \right] = \frac{2}{n\pi}[1-\cos(n\pi)]$$

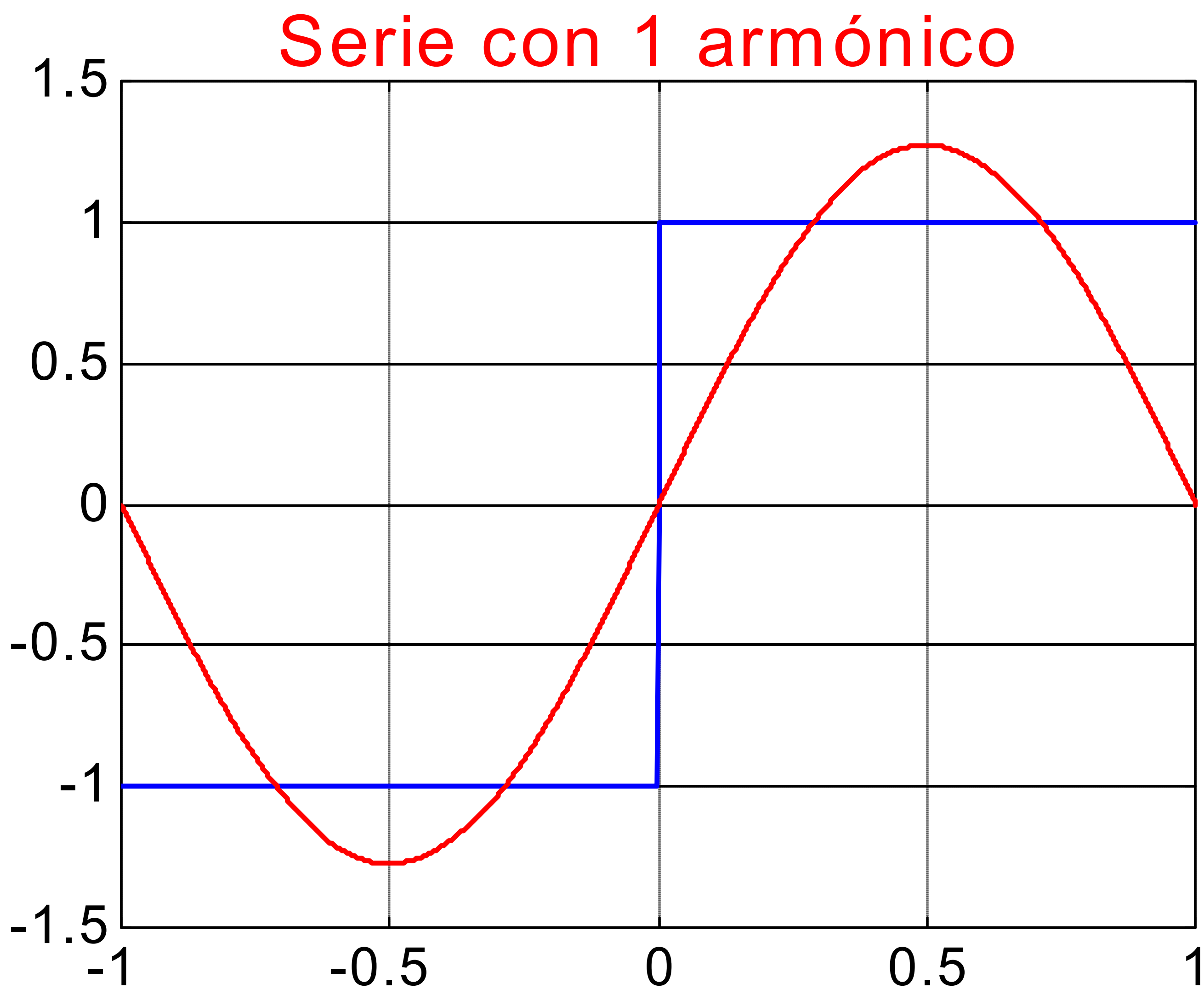
$$b_n = \frac{2}{n\pi}[1-\cos(n\pi)] \qquad n : par \rightarrow b_n = 0 \qquad n : impar \rightarrow b_n = \frac{4}{n\pi}$$

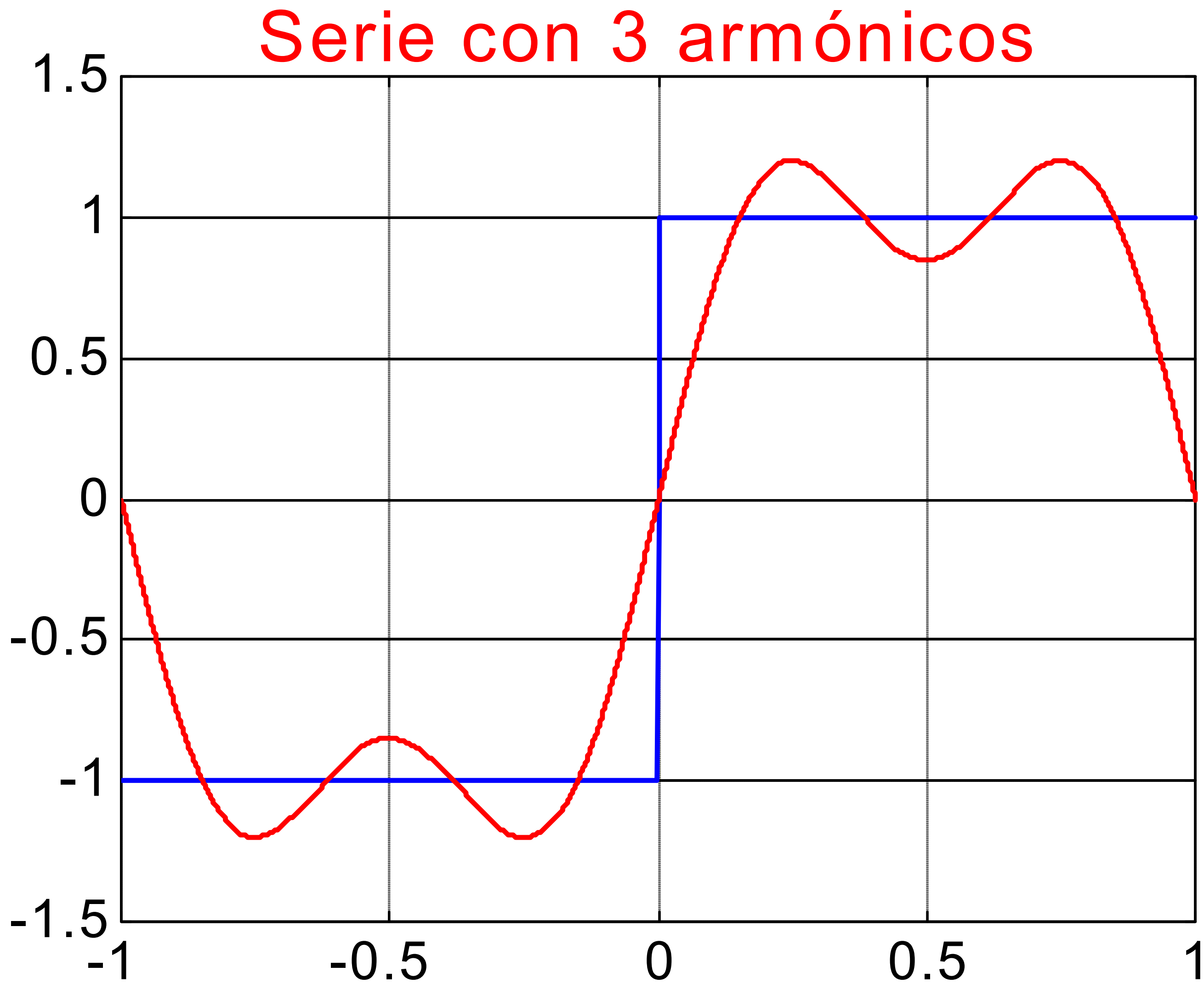
$$f(x) = \frac{4}{\pi}\left[senx + \frac{1}{3}sen3x + \frac{1}{5}sen5x + \frac{1}{7}sen7x + \ldots \right]$$

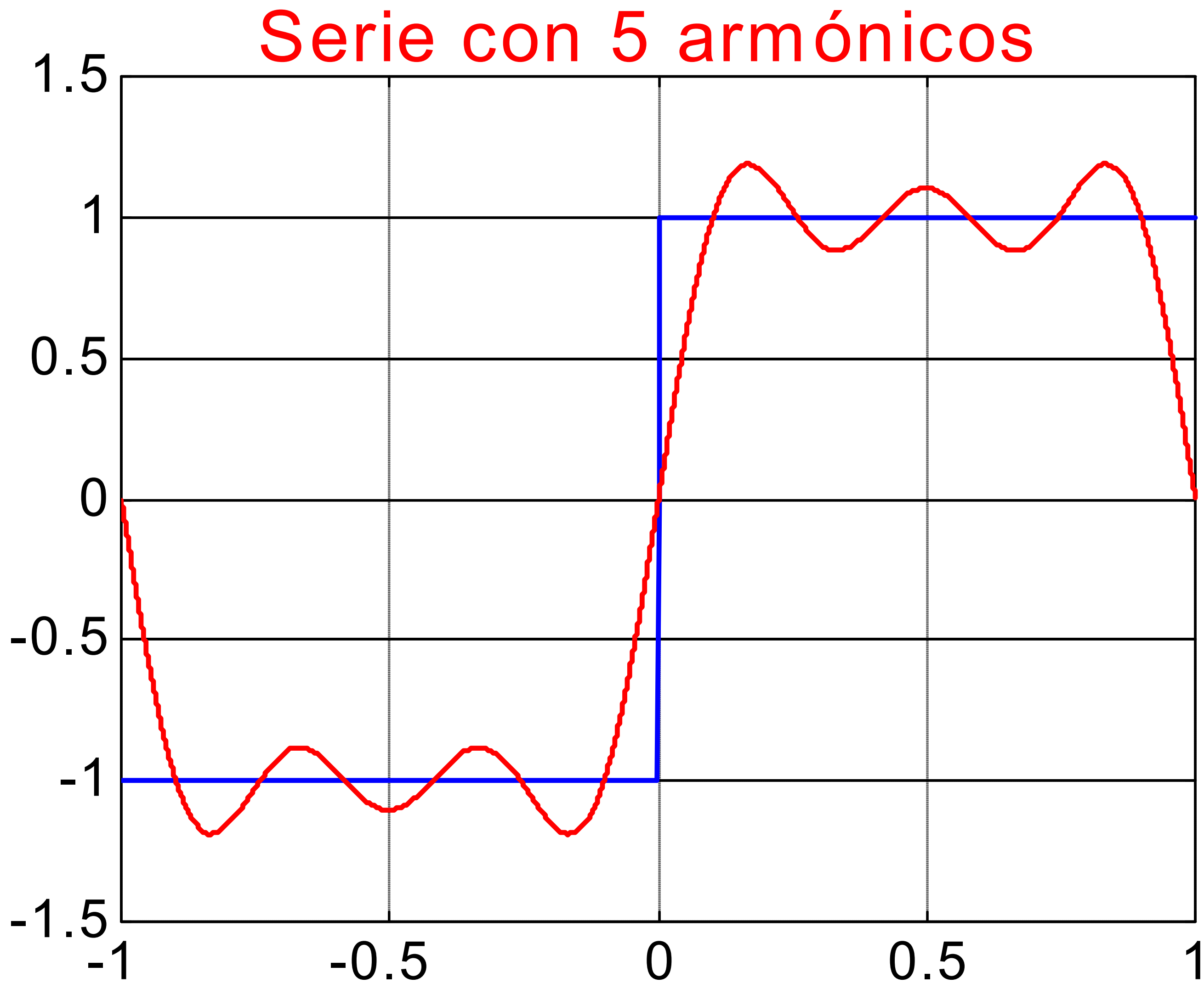
Fenómeno de Gibbs

Si la serie de Fourier para una función f(t) se trunca para lograr una *aproximación en suma finita de senos y cosenos*, es natural pensar que a medida que agreguemos más armónicos, la sumatoria se aproximará más a f(t).

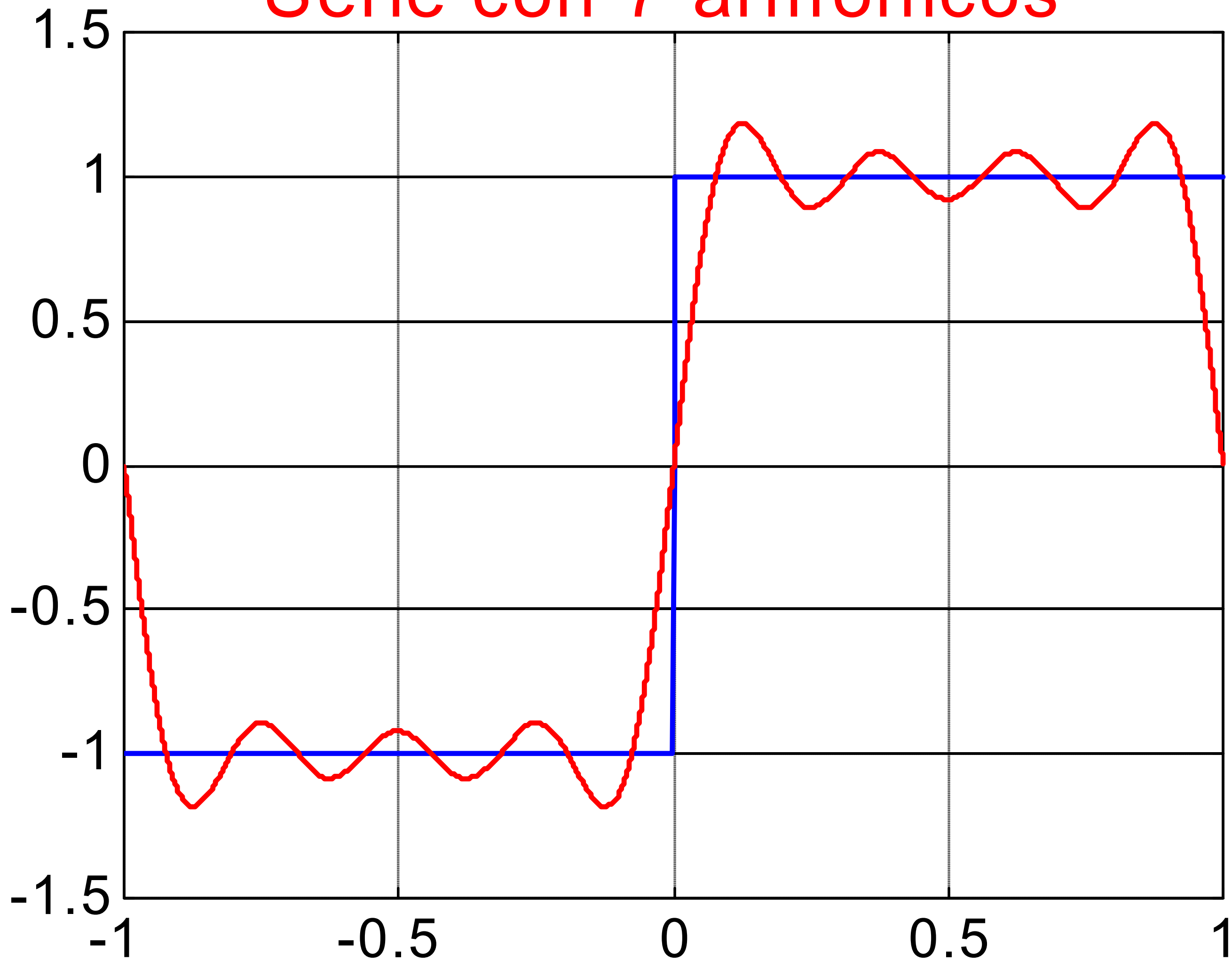
Esto se cumple excepto en las discontinuidades de f(t), en donde el error de la suma finita no tiende a cero a medida que agregamos armónicos.

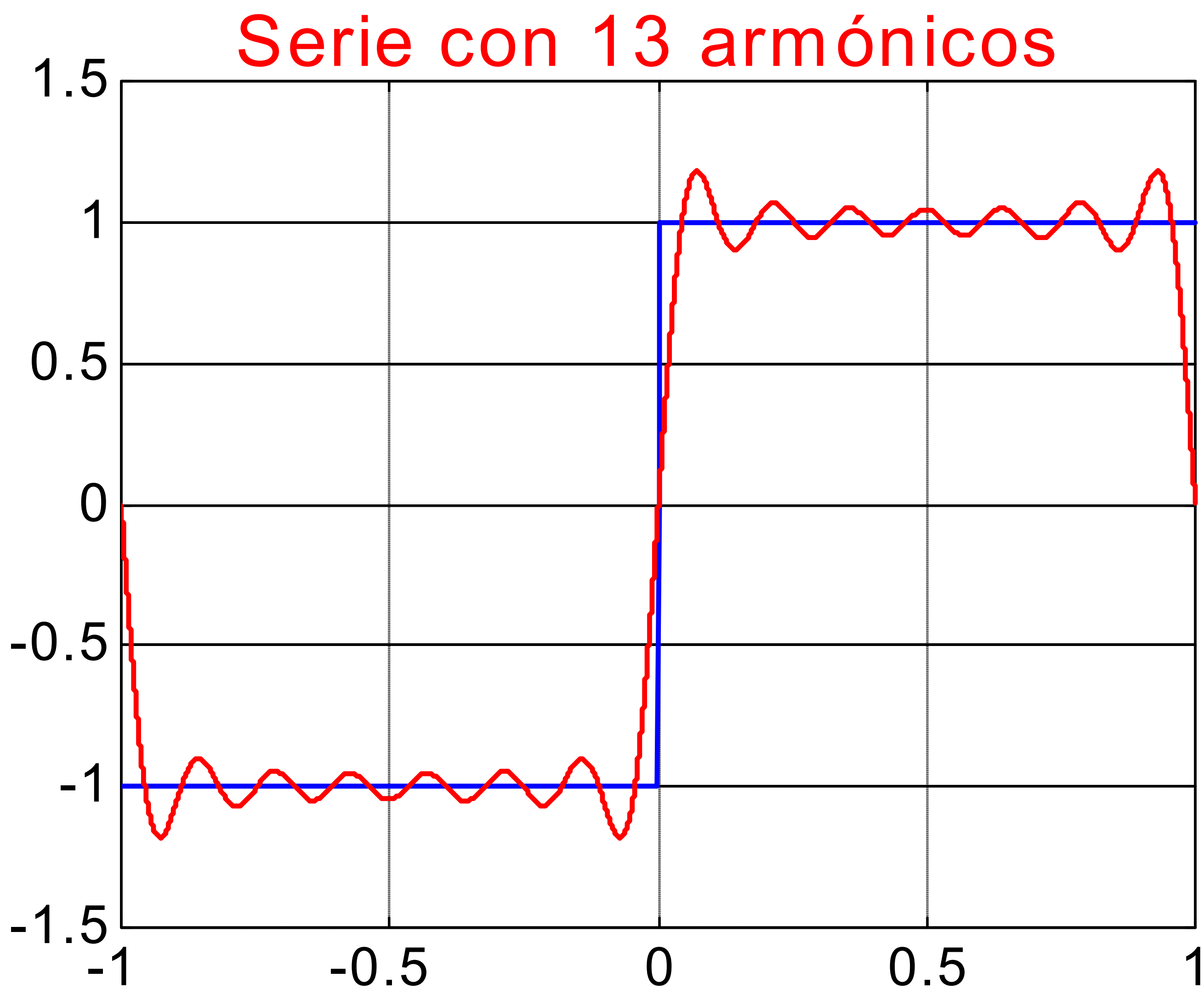

Serie con 1 armónico
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1

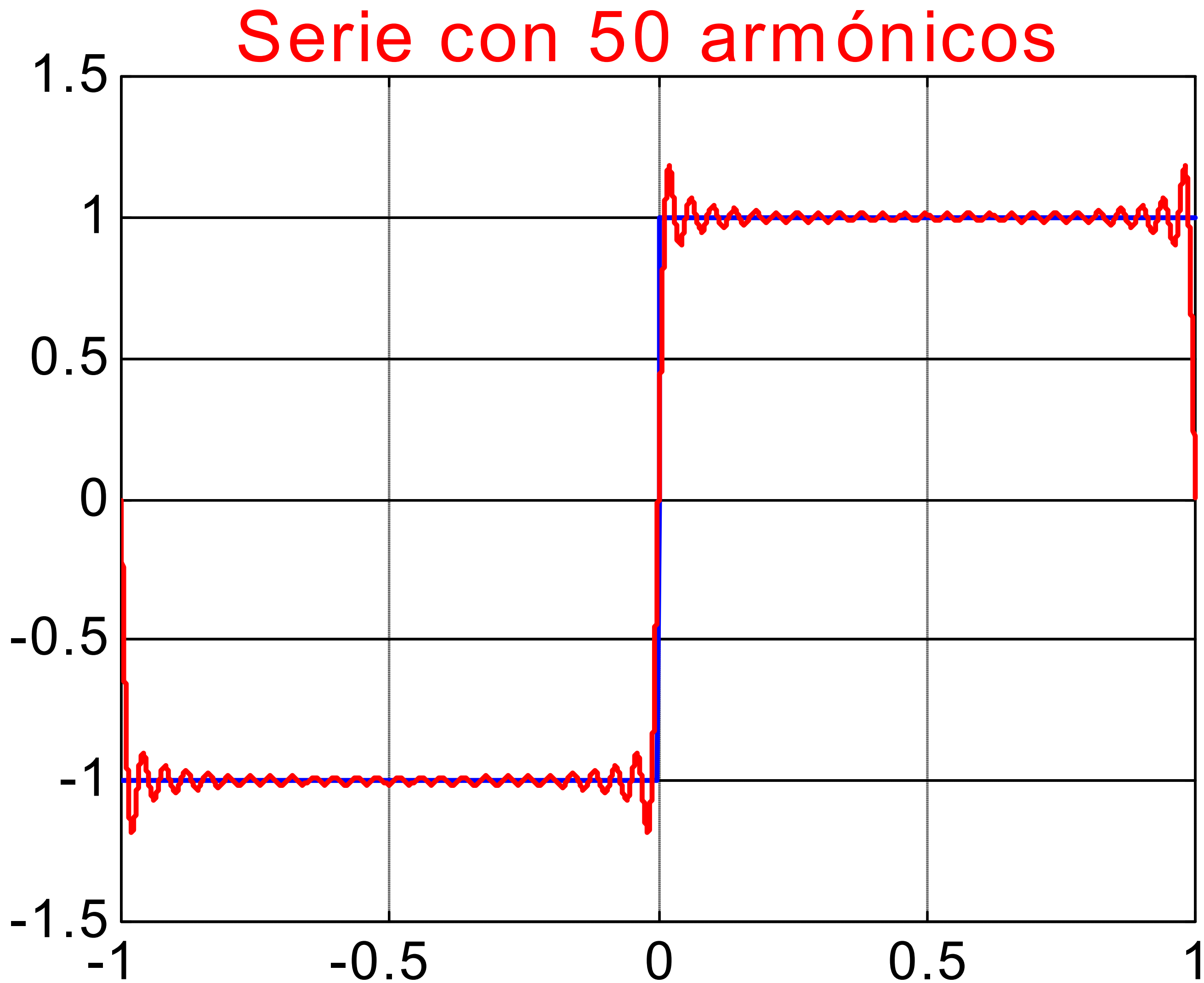

Serie con 3 armónicos
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1


Serie con 5 armónicos
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1

Serie con 7 armónicos



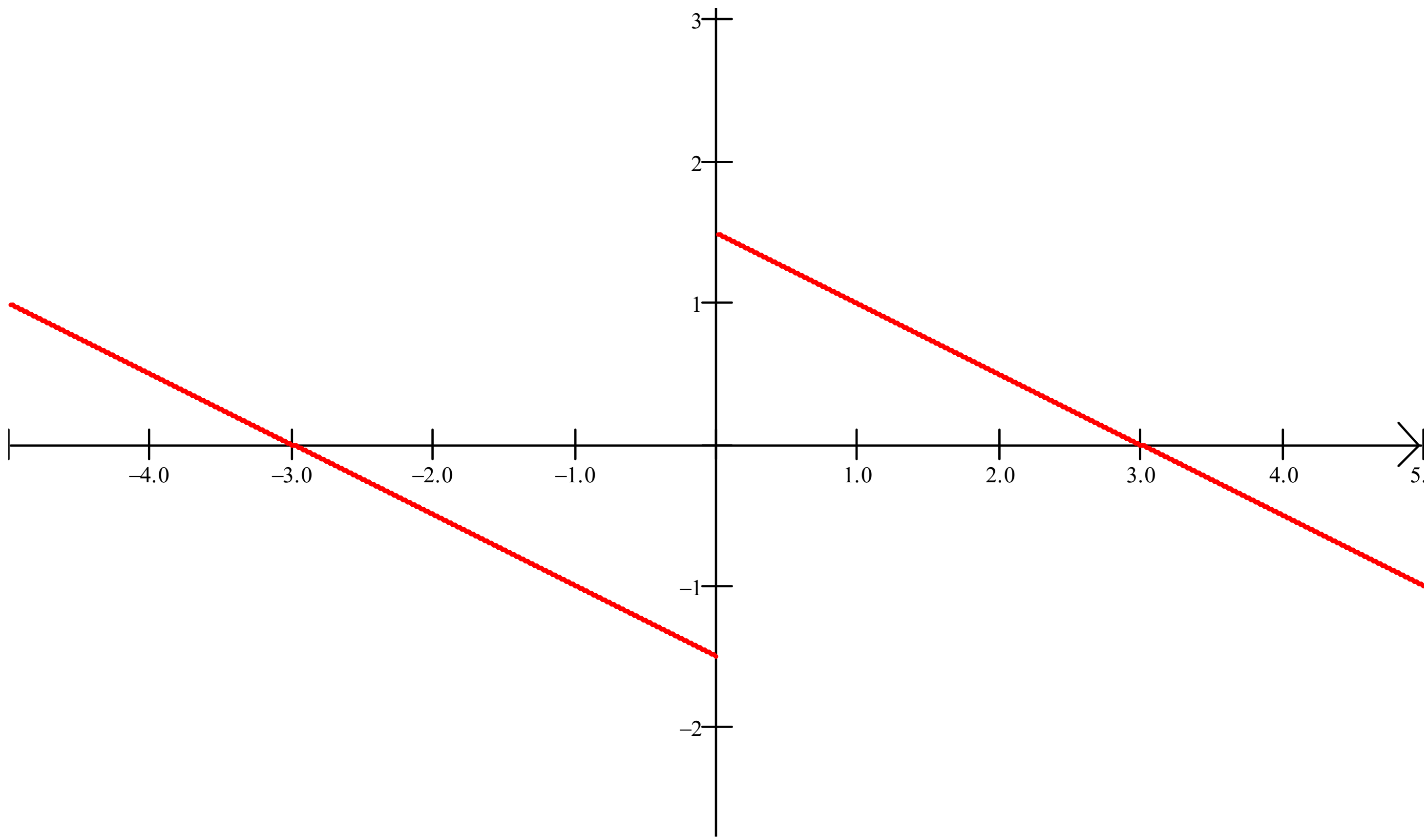

Serie con 13 armónicos
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1


Serie con 50 armónicos
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1

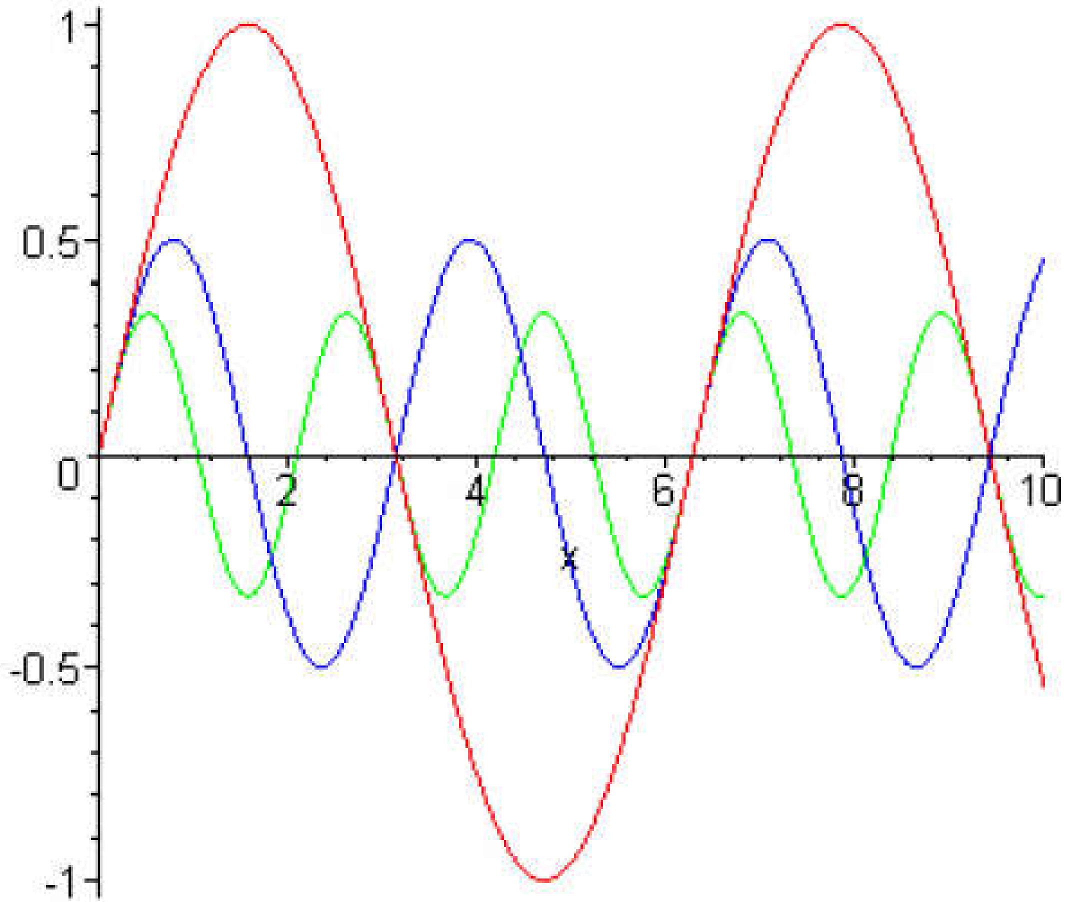
Ej.2

f(t) = -0.5t +1.5   T = 6

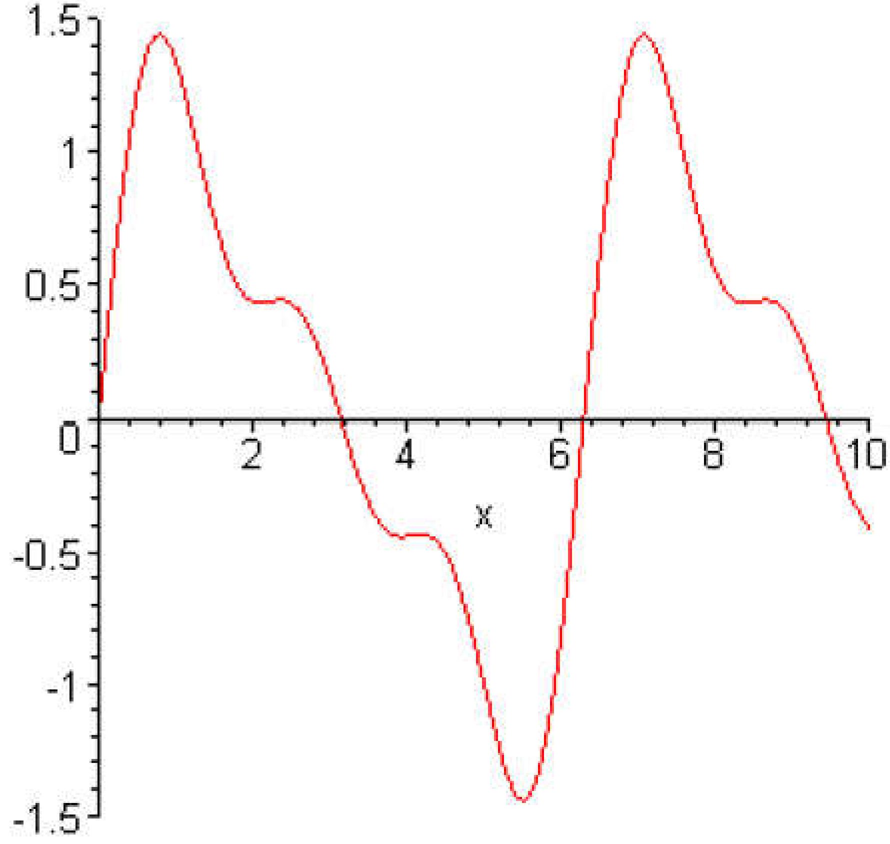
$$b_n = \frac{4}{6}\int_0^3 (-0.5t + 1.5)sen\frac{n\pi}{3}tdt \quad \Rightarrow b_n = \frac{3}{n\pi}$$



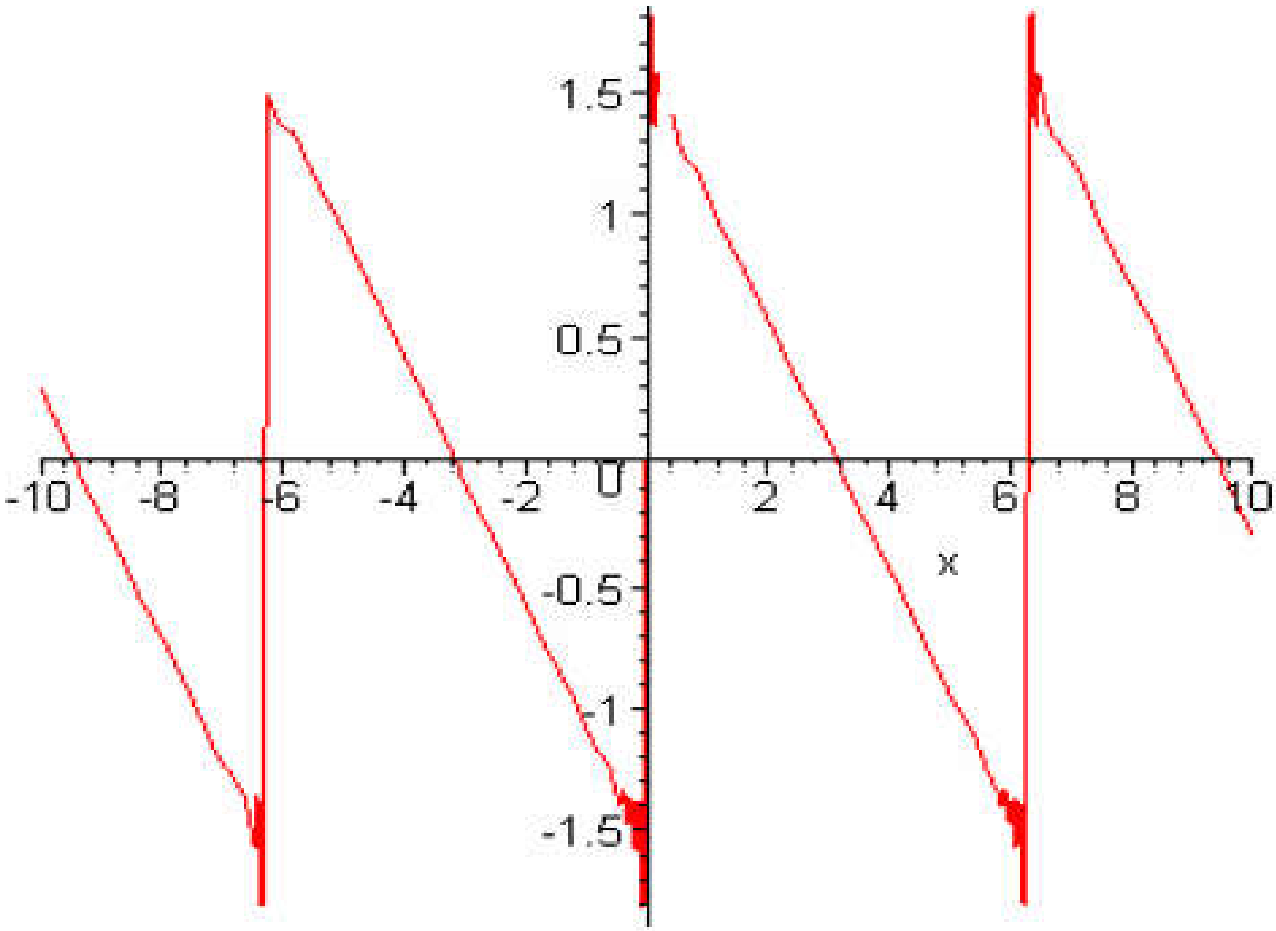
$$f(t)=\frac{3}{\pi}sen\frac{\pi}{3}t+\frac{3}{2\pi}sen\frac{2\pi}{3}t+\frac{1}{\pi}sen\frac{3\pi}{3}t+\frac{3}{4\pi}sen\frac{4\pi}{3}t+\ldots$$



Suma de los tres primeros armónicos



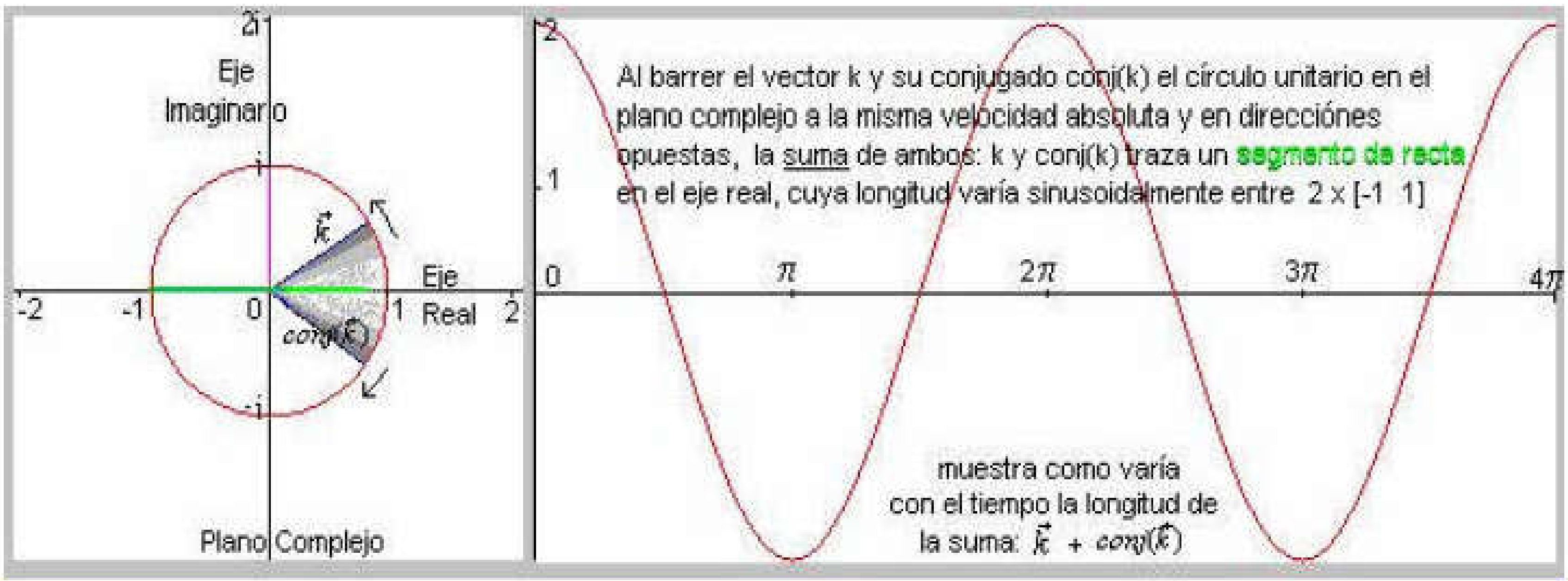
Fenómeno de Gibbs



Forma compleja

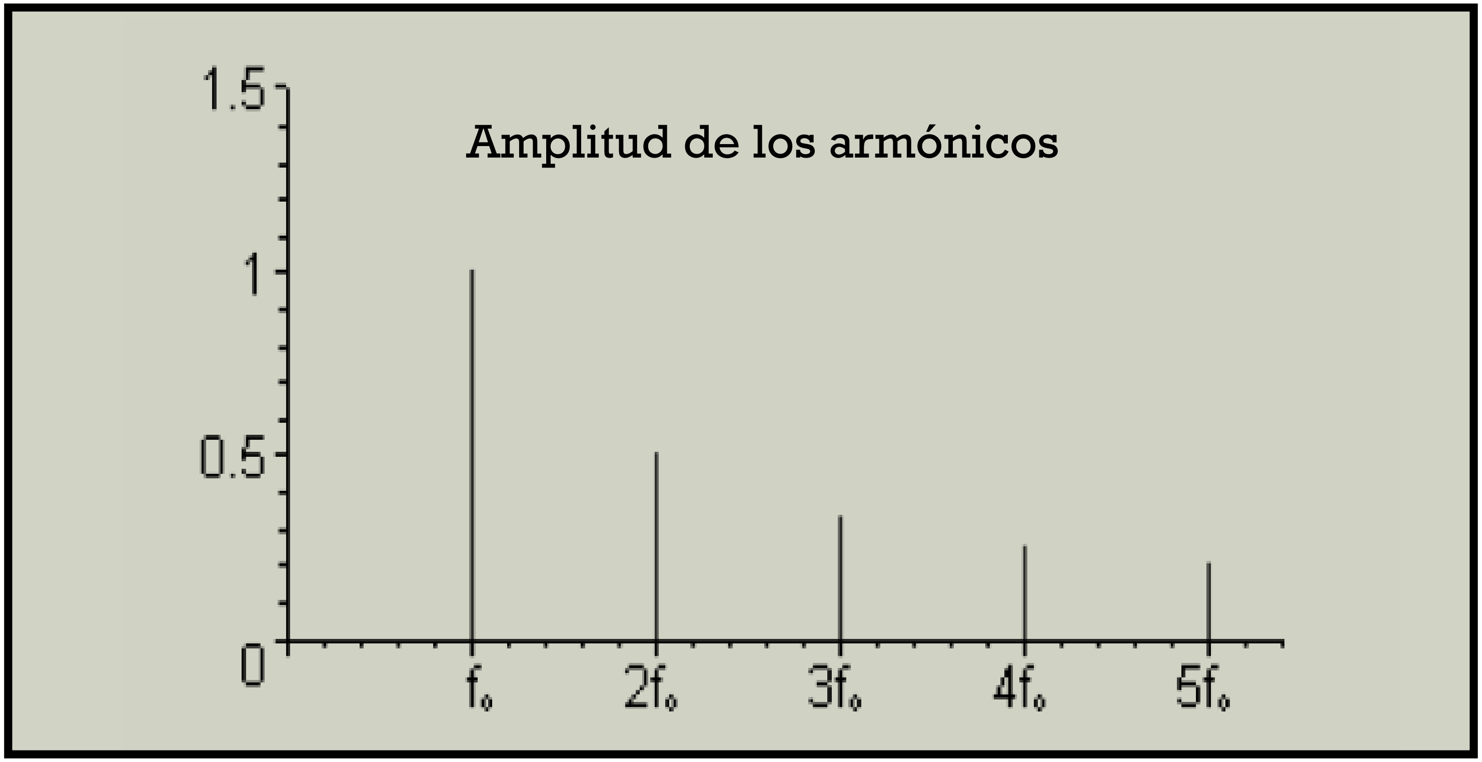
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega t}$$



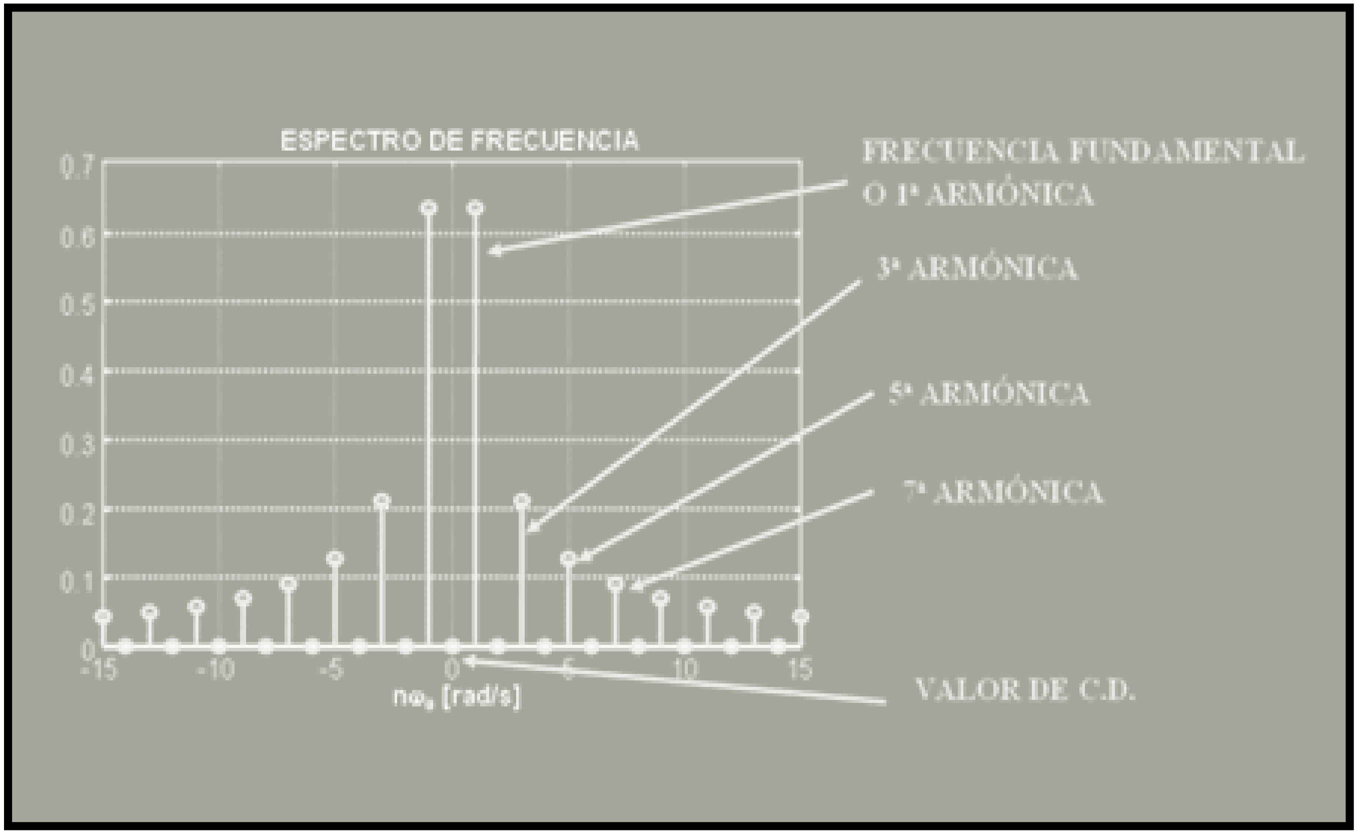
Forma Exponencial
Ejemplo espectro de frecuencia

$$f(t) = \sum_{n=0}^{\infty} A_n e^{jn\omega t} + B_n e^{-jn\omega t}$$



Forma Compleja
Ejemplo de espectro de frecuencia

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$



FORMA COMPLEJA:

Si f(t) es par entonces a_n = 2 real de C_n y b_n= -2 Img. C_n

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-jnx}dx \qquad C_n = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -e^{-jnx}dx + \int_{0}^{\pi} e^{-jnx}dx\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{-jnx}\right)_{-\pi}^{0} - \frac{1}{jn}\left(e^{-jnx}\right)_{0}^{\pi}\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{1}{jn}(1-e^{-jn\pi}) - \frac{1}{jn}(e^{-jn\pi} - 1\right] = \frac{1}{jn\pi}[1-\cos(n\pi)]$$

Comparamos: $C_n = \frac{1}{jn\pi}[1-\cos(n\pi)]$ $\quad b_n = \frac{2}{n\pi}[1-\cos(n\pi)]$

ESPECTRO DISCRETO O DE LINEA

$$f(t) = \ldots C_{-3}e^{-j3\omega t} + C_{-2}e^{-j2\omega t} + C_{-1}e^{-j\omega t} + C_0 + C_1 e^{j\omega t} + C_2 e^{j2\omega t} + C_3 e^{j3\omega t} + \ldots$$