Problema 1)

a)

$x := -3, -2.99 .. 3$

$f(x) := \sin(x)$

$F(x) = a + bx + cx^2 = af1(x) + bf2(x) + cf3(x)$

$f1(x) := 1$ B={f1,f2,f3}

$f2(x) := x$

$f3(x) := x^2$

$w(x) := 1$

$$A := \begin{pmatrix} \int_{-1}^{1} f1(x)\cdot f1(x)\cdot w(x)\,dx & \int_{-1}^{1} f1(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f1(x)\cdot f3(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f1(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f2(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f1(x)\cdot f3(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0.6666667 \\ 0 & 0.6666667 & 0 \\ 0.6666667 & 0 & 0.4 \end{pmatrix}$$

$$B := \begin{pmatrix} \int_{-1}^{1} f(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix} = \begin{pmatrix} 0 \\ 0.6023374 \\ 0 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} 0 \\ 0.903506 \\ 0 \end{pmatrix}$$

$$E := \left| \sqrt{\int_{-1}^{1} (f(x))^2\,dx - \sum_{i=0}^{2} \left(B_i\cdot X_i\right)} \right| = 0.0337023$$

$$Fa(x) := X_1\cdot f2(x) \rightarrow 0.90350603681927044\cdot x$$

b)

$F(x) = ax + bx^3 + cx^5 = af1(x) + bf2(x) + cf3(x)$

$f1(x) := x$ B={f1,f2,f3}

$f2(x) := x^3$

$f3(x) := x^5$

$w(x) := 1$

$$A := \begin{pmatrix} \int_{-1}^{1} f1(x)\cdot f1(x)\cdot w(x)\,dx & \int_{-1}^{1} f1(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f1(x)\cdot f3(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f1(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f2(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f1(x)\cdot f3(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f2(x)\cdot w(x)\,dx & \int_{-1}^{1} f3(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix} = \begin{pmatrix} 0.6666667 & 0.4 & 0.2857143 \\ 0.4 & 0.2857143 & 0.2222222 \\ 0.2857143 & 0.2222222 & 0.1818182 \end{pmatrix}$$

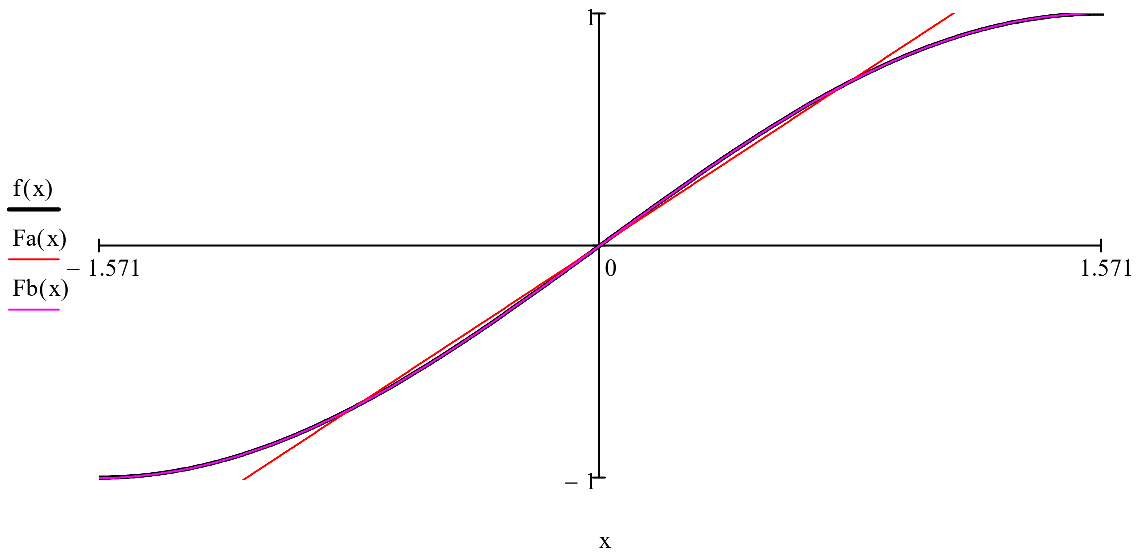
$$B := \begin{pmatrix} \int_{-1}^{1} f(x)\cdot f1(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f(x)\cdot f2(x)\cdot w(x)\,dx \\ \int_{-1}^{1} f(x)\cdot f3(x)\cdot w(x)\,dx \end{pmatrix} = \begin{pmatrix} 0.6023374 \\ 0.3541971 \\ 0.2501622 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} 0.9999842 \\ -0.1665242 \\ 8.0181104 \times 10^{-3} \end{pmatrix}$$

$$E := \left| \sqrt{\int_{-1}^{1} (f(x))^2\,dx - \sum_{i=0}^{2} \left(B_i \cdot X_i\right)} \right| = 2.620974 \times 10^{-6}$$

$$Fb(x) := f1(x)\cdot X_0 + f2(x)\cdot X_1 + f3(x)\cdot X_2 \rightarrow 0.0080181103647021246\cdot x^5 + 0.99998421244531954\cdot x + -0.16652418106578182\cdot x^3$$

d)



Problema 2)

$f(x) := x\cdot e^{-x}$

Intervalo = [0, 1]

$x := -3, -2.99 .. 3$

$F(x) = a + bx = af1(x) + bf2(x)$

$f1(x) := 1$ B={f1,f2}

$f2(x) := x$

$$A := \begin{bmatrix} \int_0^1 \left(f1(x)\cdot f1(x)\cdot e^x\right) dx & \int_0^1 \left(f1(x)\cdot f2(x)\cdot e^x\right) dx \\ \int_0^1 \left(f2(x)\cdot f1(x)\cdot e^x\right) dx & \int_0^1 \left(f2(x)\cdot f2(x)\, e^x\right) dx \end{bmatrix} = \begin{pmatrix} 1.718 & 1 \\ 1 & 0.718 \end{pmatrix}$$

$$B := \begin{bmatrix} \int_0^1 \left(f(x)\cdot f1(x)\, e^x\right) dx \\ \int_0^1 \left(f(x)\cdot f2(x)\, e^x\right) dx \end{bmatrix} = \begin{pmatrix} 0.5 \\ 0.333 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} 0.11 \\ 0.311 \end{pmatrix}$$

$$E := \left| \sqrt{\int_0^1 f(x)\cdot f(x)\cdot e^x\, dx - \sum_{i=0}^{1} \left(B_i\cdot X_i\right)} \right| = 0.0441995$$

$$Fa(x) := f1(x)\cdot X_0 + f2(x)\, X_1 \rightarrow 0.31066316074409439\cdot x + 0.11018963019919914$$

PARABOLA

$$F(x) = a + bx + cx^2 = af1(x) + bf2(x) + cf3(x)$$

$f1(x) := 1$ B={f1,f2,f3}

$f2(x) := x$

$f3(x) := x^2$

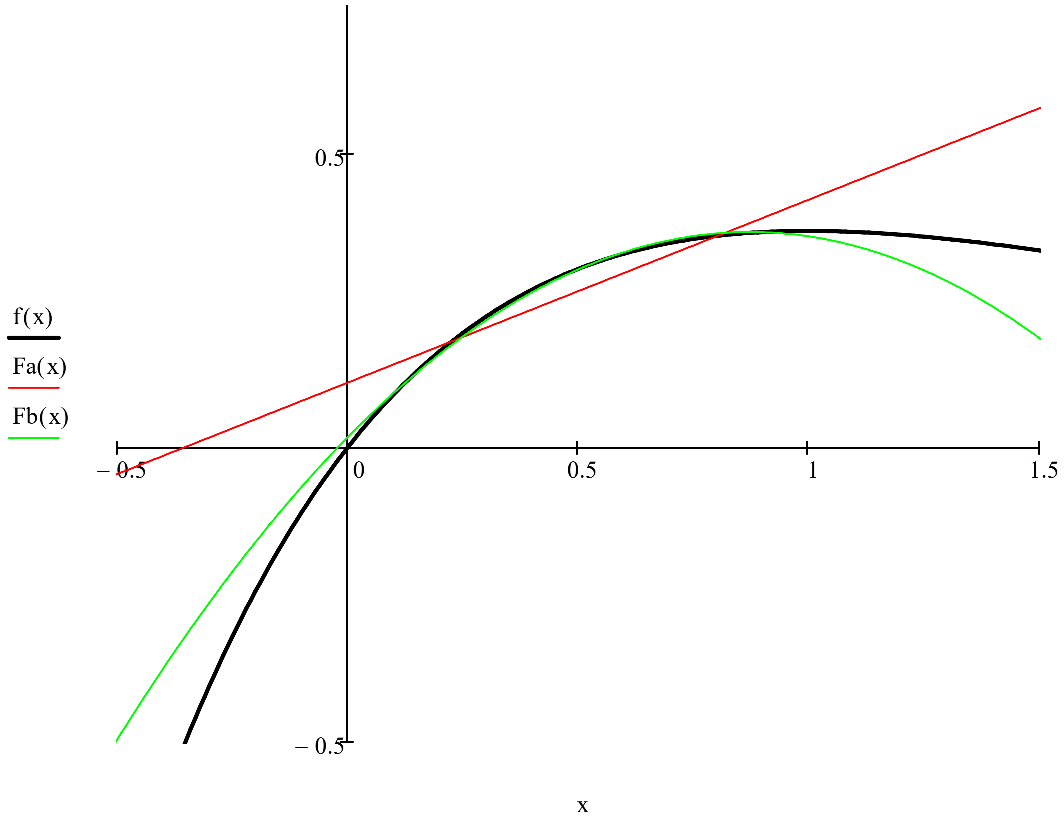
$$A := \begin{bmatrix} \int_0^1 \left(f1(x)\cdot f1(x)\, e^x\right) dx & \int_0^1 \left(f1(x)\cdot f2(x)\, e^x\right) dx & \int_0^1 \left(f1(x)\cdot f3(x)\, e^x\right) dx \\ \int_0^1 \left(f2(x)\cdot f1(x)\, e^x\right) dx & \int_0^1 \left(f2(x)\cdot f2(x)\, e^x\right) dx & \int_0^1 \left(f2(x)\cdot f3(x)\, e^x\right) dx \\ \int_0^1 \left(f3(x)\cdot f1(x)\, e^x\right) dx & \int_0^1 \left(f3(x)\cdot f2(x)\, e^x\right) dx & \int_0^1 \left(f3(x)\cdot f3(x)\, e^x\right) dx \end{bmatrix} = \begin{pmatrix} 1.718 & 1 & 0.718 \\ 1 & 0.718 & 0.563 \\ 0.718 & 0.563 & 0.465 \end{pmatrix}$$

$$B := \begin{bmatrix} \int_0^1 \left(f(x)\cdot f1(x)\, e^x\right) dx \\ \int_0^1 \left(f(x)\cdot f2(x)\, e^x\right) dx \\ \int_0^1 \left(f(x)\cdot f3(x)\, e^x\right) dx \end{bmatrix} = \begin{pmatrix} 0.5 \\ 0.333 \\ 0.25 \end{pmatrix}$$

$$X := A^{-1} \cdot B = \begin{pmatrix} 0.017 \\ 0.799 \\ -0.458 \end{pmatrix}$$

$$E := \left| \sqrt{\int_0^1 f(x) \cdot f(x) \cdot e^x \, dx - \sum_{i=0}^{2} \left(B_i \cdot X_i\right)} \right| = 6.1721191 \times 10^{-3}$$

$$Fb(x) := f1(x) \cdot X_0 + f2(x) X_1 + f3(x) X_2 \rightarrow 0.79882135604427518 \cdot x + -0.45758246723882223 \cdot x^2 + 0.017373061101153553$$



Problema 3)

a)

$$f(x) := e^{-x^2}$$

$$F(x) = a + bx + cx^2 = af1(x) + bf2(x) + cf3(x)$$

$f1(x) := 1$ B={f1,f2,f3}

$f2(x) := x$ $w(x) := 1$

$f3(x) := x^2$

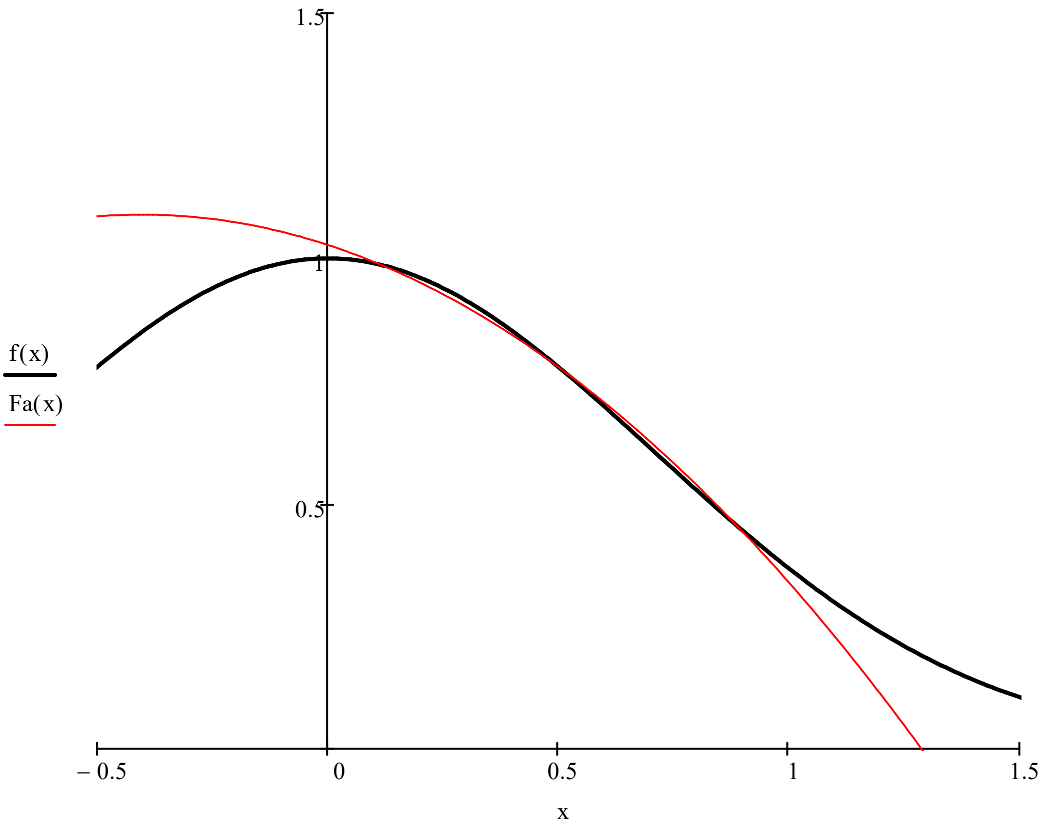
$$A := \begin{bmatrix} \int_0^1 (f1(x) \cdot f1(x)\, w(x))\, dx & \int_0^1 (f1(x) \cdot f2(x)\, w(x))\, dx & \int_0^1 (f1(x) \cdot f3(x)\, w(x))\, dx \\ \int_0^1 (f2(x) \cdot f1(x)\, w(x))\, dx & \int_0^1 (f2(x) \cdot f2(x)\, w(x))\, dx & \int_0^1 (f2(x) \cdot f3(x)\, w(x))\, dx \\ \int_0^1 (f3(x) \cdot f1(x)\, w(x))\, dx & \int_0^1 (f3(x) \cdot f2(x)\, w(x))\, dx & \int_0^1 (f3(x) \cdot f3(x)\, w(x))\, dx \end{bmatrix} = \begin{pmatrix} 1 & 0.5 & 0.333 \\ 0.5 & 0.333 & 0.25 \\ 0.333 & 0.25 & 0.2 \end{pmatrix}$$

$$B := \begin{bmatrix} \int_0^1 (f(x)\cdot f1(x)\, w(x))\, dx \\ \int_0^1 (f(x)\cdot f2(x)\, w(x))\, dx \\ \int_0^1 (f(x)\cdot f3(x)\, w(x))\, dx \end{bmatrix} = \begin{pmatrix} 0.747 \\ 0.316 \\ 0.189 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} 1.027 \\ -0.307 \\ -0.381 \end{pmatrix}$$

$$E := \left| \sqrt{\int_0^1 f(x)\cdot f(x)\cdot w(x)\, dx - \sum_{i=0}^{2} \left(B_i\cdot X_i\right)} \right| = 0.0109591$$

$$Fa(x) := f1(x)\cdot X_0 + f2(x)\, X_1 + f3(x)\, X_2 \rightarrow -0.30711738139444122\cdot x + -0.38110406250876849\cdot x^2 + 1.0274175110125716$$



b)

$$XV := (0 \quad 0.5 \quad 1)$$

$$YV := \left[f\left[\left(XV^T\right)_0\right] \quad f\left[\left(XV^T\right)_1\right] \quad f\left[\left(XV^T\right)_2\right] \right] = (1 \quad 0.779 \quad 0.368)$$

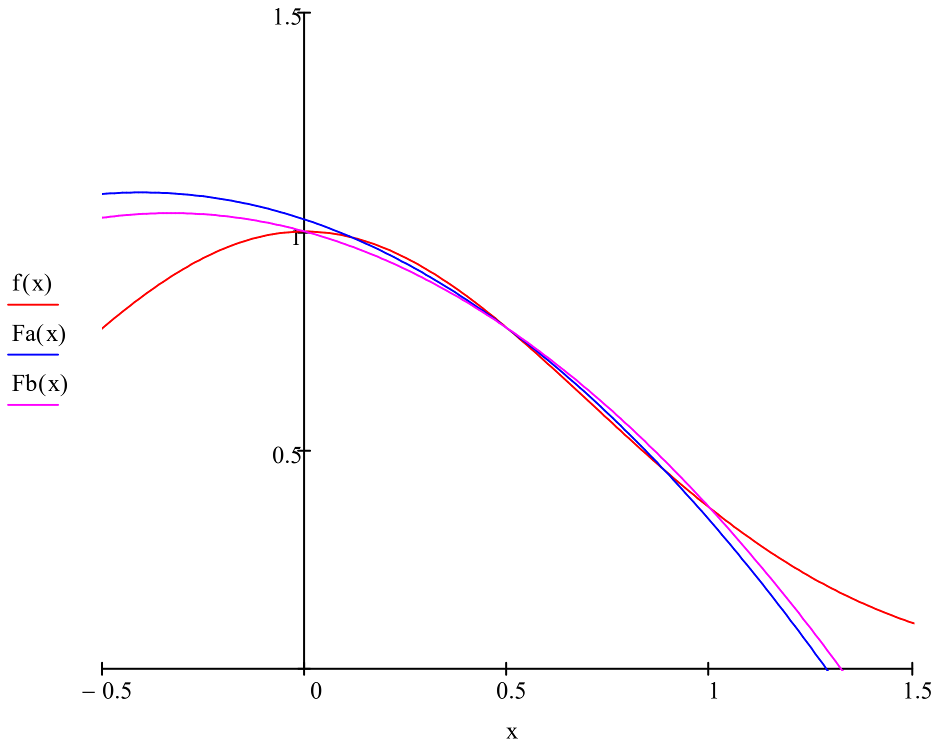
$$A := \begin{bmatrix} \sum_{i=0}^{2} \left[f1\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f2\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f3\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{2} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f2\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f3\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{2} \left[f1\left[\left(XV^T\right)_i\right]\cdot f3\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f2\left[\left(XV^T\right)_i\right]\cdot f3\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{2} \left[f3\left[\left(XV^T\right)_i\right]\cdot f3\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 3 & 1.5 & 1.25 \\ 1.5 & 1.25 & 1.125 \\ 1.25 & 1.125 & 1.063 \end{pmatrix}$$

$$B := \begin{bmatrix} \sum_{i=0}^{2} \left[\left(YV^T\right)_i \cdot f1\left[\left(XV^T\right)_i\right] \cdot w(x)\right] \\ \sum_{i=0}^{2} \left[\left(YV^T\right)_i \cdot f2\left[\left(XV^T\right)_i\right] \cdot w(x)\right] \\ \sum_{i=0}^{2} \left[\left(YV^T\right)_i \cdot f3\left[\left(XV^T\right)_i\right] \cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 2.1466802 \\ 0.7572798 \\ 0.5625796 \end{pmatrix}$$

$$X := A^{-1} \cdot B = \begin{pmatrix} 1 \\ -0.2526763 \\ -0.3794442 \end{pmatrix}$$

$$E := \left| \sqrt{\sum_{i=0}^{2} \left[\left(YV^T\right)_i \cdot \left(YV^T\right)_i \cdot w(x)\right] - \sum_{i=0}^{1} \left(B_i \cdot X_i\right)} \right| = 0.4620255$$

$$Fb(x) := f1(x) \cdot X_0 + f2(x) X_1 + f3(x) X_2 \rightarrow -0.25267630888582104 \cdot x + -0.37944424994273618 \cdot x^2 + 1.0000000000000004$$



d)

$$\int_0^1 f(x)\,dx = 0.7468241328124271$$ Funcion continua

$$\int_0^1 Fa(x)\,dx = 0.7468241328124282$$ Funcion continua aproximada

$$\int_0^1 Fb(x)\,dx = 0.7471804289$$ Funcion discreta aproximada

Problema 4)

$$y = a\ln(x) + b \cdot e^{-x} = a \cdot f1(x) + b \cdot f2(x)$$

B={f1, f2} $f1(x) := \ln(x)$ $f2(x) := e^{-x}$ $w(x) := 1$

$YV := (0.2420 \;\; 0.1942 \;\; 0.1497 \;\; 0.1109 \;\; 0.079)$

$XV := (1 \;\; 1.2 \;\; 1.4 \;\; 1.6 \;\; 1.8)$

$$A := \begin{bmatrix} \sum_{i=0}^{4} \left[f1\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{4} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{4} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{4} \left[f2\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 0.7128513 & 0.3299398 \\ 0.3299398 & 0.3549492 \end{pmatrix}$$

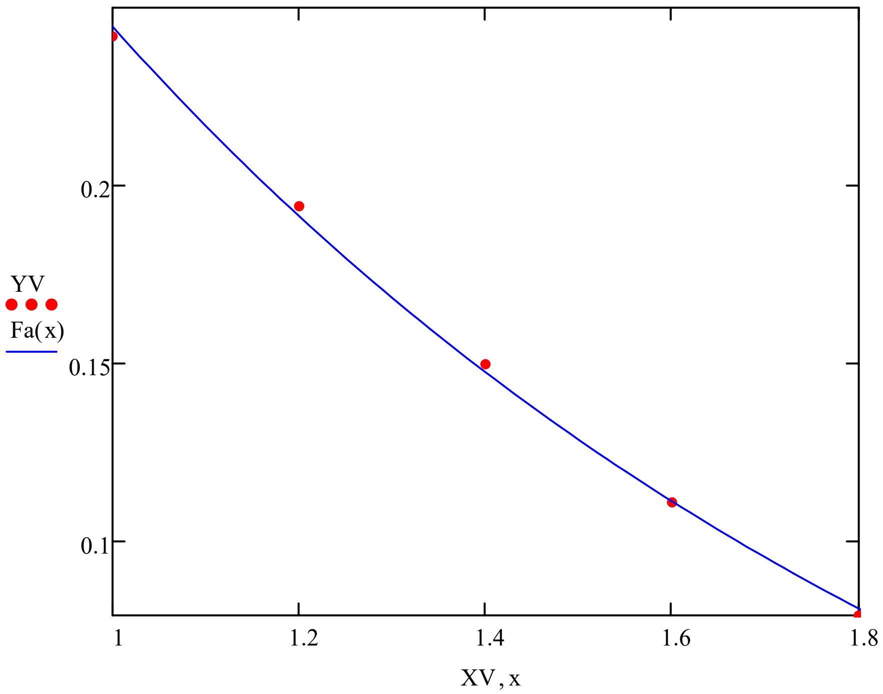
$$B := \begin{bmatrix} \sum_{i=0}^{4} \left[\left(YV^T\right)_i\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{4} \left[\left[\left(YV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 0.1843353 \\ 0.2198832 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} -0.0493777 \\ 0.6653766 \end{pmatrix}$$

$$Fa(x) := X_0\cdot f1(x) + X_1\cdot f2(x) \rightarrow -0.049377659037275368\cdot\ln(x) + 0.66537656813755142\cdot e^{-x}$$

$Fa(1.30) = 0.1683813$

$Fa(2) = 0.0558229$



$$E := \left|\sqrt{\sum_{i=0}^{4} \left[\left(YV^T\right)_i\cdot\left(YV^T\right)_i\cdot w(x)\right] - \sum_{i=0}^{1} \left(B_i\cdot X_i\right)}\right| = 4.9424638 \times 10^{-3}$$

Problema 5)

$p(x) = a\cdot e^{m\cdot x}$

$\ln(p(x)) = \ln\left(a\cdot e^{m\cdot x}\right)$

$(\ln(p(x)) = \ln(a)) + \ln\left(e^{m\cdot x}\right)$

$(\ln(p(x)) = \ln(a)) + m\cdot x$

$f(x) = h\cdot f1(x) + k\cdot f2(x)$

$f1(x) := 1 \quad f2(x) := x \quad w(x) := 1 \quad f(x) = \ln(p(x)) \quad h = \ln(a) \quad k = m$

$XV := (0 \;\; 0.4 \;\; 0.8 \;\; 1.2 \;\; 1.6 \;\; 2)$

$YV := (3.1437 \;\; 4.4169 \;\; 6.0203 \;\; 8.6512 \;\; 11.0078 \;\; 16.2161)$

$$A := \begin{bmatrix} \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{5} \left[f2\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 6 & 6 \\ 6 & 8.8 \end{pmatrix}$$

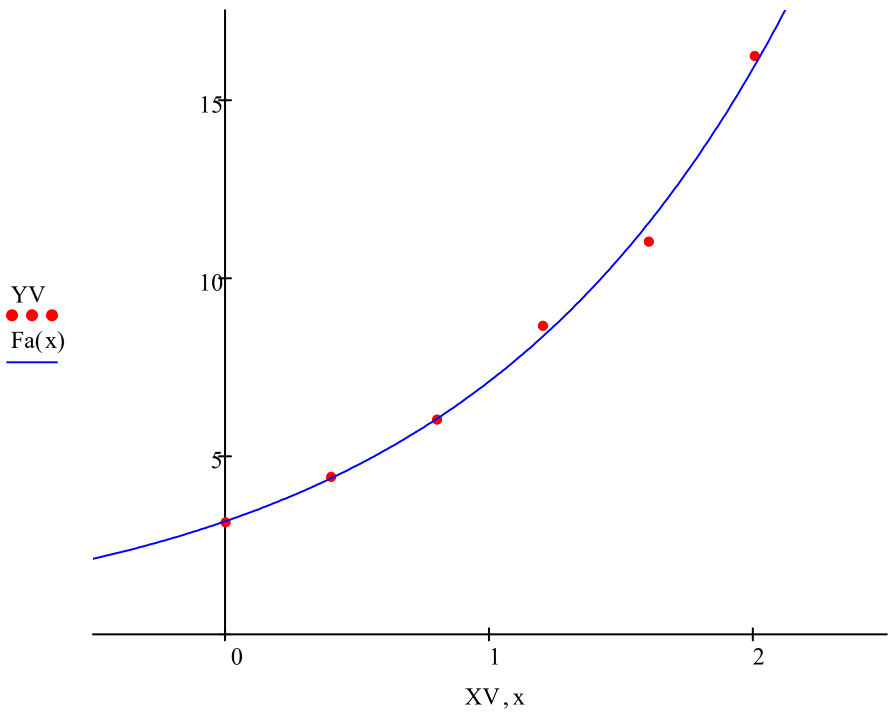
$$B := \begin{bmatrix} \sum_{i=0}^{5} \left[\ln\left(\left(YV^T\right)\right)_i\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{5} \left[\left[\ln\left(\left(YV^T\right)\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 11.7682824 \\ 14.0292983 \end{pmatrix}$$

$$X := A^{-1}\cdot B = \begin{pmatrix} 1.1538747 \\ 0.8075057 \end{pmatrix}$$

$a := e^{X_0} = 3.1704537$

$b := X_1 = 0.8075057$

$Fa(x) := a\cdot e^{b\cdot x} \rightarrow 3.1704537152892573\cdot e^{0.80750568827391955\cdot x}$



Problema 6)

$$y = \alpha\cdot\frac{x}{\beta + x} \quad \text{-->} \quad \frac{1}{y} = \frac{\beta}{\alpha}\cdot\frac{1}{x} + \frac{1}{\alpha}$$

$$x1 = \frac{\beta}{\alpha} \qquad x2 = \frac{1}{\alpha}$$

$$f1(x) := \frac{1}{x} \qquad f2(x) := 1 \qquad w(x) := 1$$

$$XV := (1 \;\; 3 \;\; 5 \;\; 10 \;\; 15 \;\; 21)$$

$$YV := (0.89 \;\; 1.32 \;\; 1.46 \;\; 1.59 \;\; 1.64 \;\; 1.66)$$

$$A := \begin{bmatrix} \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{5} \left[f1\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] & \sum_{i=0}^{5} \left[f2\left[\left(XV^T\right)_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 1.1678231 & 1.747619 \\ 1.747619 & 6 \end{pmatrix}$$

$$B := \begin{bmatrix} \sum_{i=0}^{5} \left[\left[\frac{1}{\left(\left(YV^T\right)\right)}\right]_i\cdot f1\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \\ \sum_{i=0}^{5} \left[\left[\left[\frac{1}{\left(\left(YV^T\right)\right)}\right]_i\right]\cdot f2\left[\left(XV^T\right)_i\right]\cdot w(x)\right] \end{bmatrix} = \begin{pmatrix} 1.6453367 \\ 4.4071993 \end{pmatrix}$$

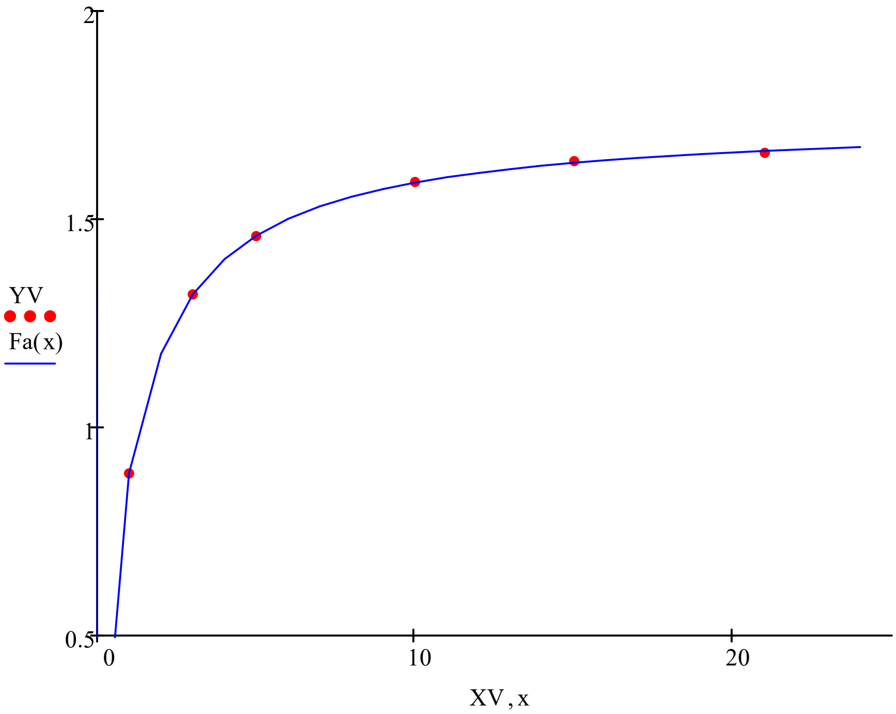
$$X := A^{-1}\cdot B = \begin{pmatrix} 0.5489611 \\ 0.5746374 \end{pmatrix}$$

$$\alpha := \frac{1}{X_1} = 1.7402278$$

$$\beta := X_0\cdot\alpha = 0.9553173$$

$$Fa(x) := \alpha\cdot\frac{x}{\beta + x} \rightarrow \frac{1.7402277745807557\cdot x}{x + 0.95531727787568843}$$

$$x := -3., -2.999..25$$



2
1.5
1
0.5
YV
Fa(x)
0
10
20
XV , x