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Chaos Control for Colpitts Oscillator with State Feedback

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Abstract:

In this paper, a state feedback controller is designed for a well known chaotic Colpitts oscillator. In particular, two kinds of chaos families are investigated under the same structure with pole-placement technique. Simulation results using the fourth-order Runge-Kutta method and SPICE are shown and analyzed.

Key word: chaos control, pole-placement, Colpitts oscillator.

1. Introduction

The well-known Colpitts oscillator has been widely used in electronic devices and communication systems for many years. Its frequencies of operation cover from a few hertz up to the microwave bands.
In this work we consider the Colpitts oscillator as a paradigm for sinusoidal oscillation, as soon as it exhibits rich dynamical behavior (shown in the literature [1-6] by theoretical predictions and experimental verifications) will be a disaster of these electronic devices and communication systems.
In general, there are two ways to eliminate chaos: feedback control [7-9] and non-feedback control [10, 11]. Since 1990, the pioneering work of Ott, Grebogi, and Yorke (OGY) presented a method to make small time-dependent perturbations in an accessible parameter and thereby to achieve a desired periodic output [12]. There are many methods of controlling chaos have been presented by extending and modifying the OGY method [8,9]. However, the pole placement method has been presented a more general form than the OGY method for controlling chaos [8, 13, 14].
In this paper, a state feedback controller with pole placement technique is designed to eliminate chaotic behavior of Colpitts oscillator. In particular, two kinds of chaos families, the Feigenbaum-like and Shil'nikov-like chaos, are controlled. Simulation results using the fourth-order Runge-Kutta method and SPICE are shown and described.

2. Dynamic equations of Colpitts oscillator

The schematic circuit of the chaotic Colpitts oscillator is shown in Figure 1 and its complicated behavior is presented in [4]. It contains voltage supply, resistor and the current source for bias, an inductor and two capacitors for resonance, and a transistor for providing enough gain to sustain the oscillation.

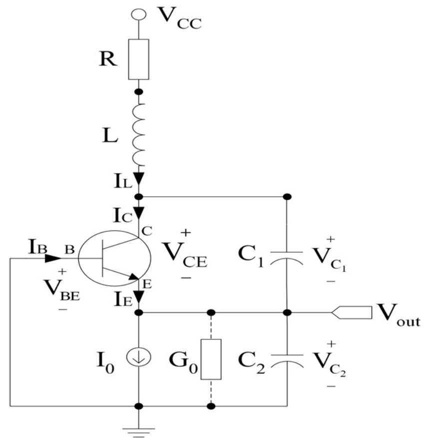

Figure 1. The schematic circuit of the chaotic Colpitts oscillator

In this paper, a practical exponential model is used, which is a common type of the mathematical model of the transistor:

$$I_E = f\left(V_{BE}\right) = \frac{I_S}{\alpha_F}\exp\left(-\frac{V_{C_2}}{V_T}\right) \tag{1}$$

where V_T is thermal voltage ($\approx 26mV$) at room temperature, I_S is the reverse saturation current of the B-E junction (typically $I_S \approx 10^{-15}A$), α_F is the common-base forward short-circuit current gain of the transistor.

The nonlinear Colpitts oscillator is referred to [4, 6] as shown in figure 1 and its state equations are described as:

$$\begin{aligned} C_1\frac{dV_{C_1}}{dt} &= -\alpha_F f\left(-V_{C_2}\right)+I_L \\ C_2\frac{dV_{C_2}}{dt} &= \left(1-\alpha_F\right) f\left(-V_{C_2}\right)-G_0V_{C_2}+I_L-I_0 \\ L\frac{dI_L}{dt} &= -V_{C_1}-V_{C_2}-RI_L+V_{CC} \end{aligned} \tag{2}$$

where the state variables V_{C_1} and V_{C_2} are the voltages across the capacitors C_1 and C_2, respectively, and the current I_L in the inductor L, $G_0(\approx 0)$ is characterized by a Norton-equivalent conductance in ideal model.

By means of normalizing the state and time variables, the new open-loop dimensionless variables, the state equations of (2) can be rewritten as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \frac{g^*}{Q(1-k)}\left[-\alpha_F n(x_2)+x_3\right] \\ \frac{g^*}{Qk}\left[(1-\alpha_F)n(x_2)+x_3\right] \\ -\frac{Qk(1-k)}{g^*}\left[x_1+x_2\right]-\frac{1}{Q}x_3 \end{pmatrix} \tag{3}$$

where the nonlinear function is given by $n(x_2)=e^{-x_2}-1$, while the dimensionless variables (x_1,x_2,x_3) are obtained by translating the equilibrium point of (2) to the new coordinate system and by normalizing voltages, currents, and time with respect to $V_{ref}=V_T$, $I_{ref}=I_0$, and $t_{ref}=1/\omega_0$, respectively, with $\omega_0=1/\sqrt{L\left(C_1C_2/C_1+C_2\right)}$. The parameters Q, k and g^* are defined by:

$Q=\dfrac{\omega_0 L}{R}$, $k=\dfrac{C_2}{C_1+C_2}$, and $g^*=\dfrac{I_0L}{V_TR\left(C_1+C_2\right)}$.

Where Q represents the quality factor of the resonant network, while g^* is the loop gain of the oscillator.

In addition, we may assume that the bias current I_0 can be treated as the input term in the state equations (3). As mentioned before, if the equilibrium point O and corresponding parameters I_0 are chosen. The small perturbations caused by the current bias I_0' from state feedback. Thus the linearized state equation is modeled by using the first terms of a Taylor expansion:

$$\begin{pmatrix} \dot{x}_1' \\ \dot{x}_2' \\ \dot{x}_3' \end{pmatrix} = \begin{bmatrix} 0 & \frac{g^*}{Q(1-k)} & \frac{g^*}{Q(1-k)} \\ 0 & 0 & \frac{g^*}{Qk} \\ -\frac{Qk(1-k)}{g^*} & -\frac{Qk(1-k)}{g^*} & -\frac{1}{Q} \end{bmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} + \begin{bmatrix} 0 \\ \frac{g^*}{Qk} \\ 0 \end{bmatrix} I_0' \tag{4}$$

The corresponding characteristic equation of (4), $d\left(\lambda I-A\right)=0$ is given by

$$\lambda^3+\frac{1}{Q}\lambda^2+\lambda+\frac{g^*}{Q}=0 \tag{5}$$

In general, the complicated function of the circuit parameters can be shown in [15]. It is interesting that the loop gain g^* increases far from unity with the quality factor Q fixed, in the meanwhile, the real part of complex conjugate pair eigenvalues of (5) will shift from zero to the right half s-plane simultaneously.

3. State feedback controller design

The chaotic dynamical system will be considered as a weakly nonlinear, and assume that dynamic behavior at the neighborhood nearby equilibrium points is linear.

As expressed above, the approach allows us to manipulate that a single-loop feedback schematic can model a chaotic dynamical system with negative state feedback, as shown in figure 2(a), where S is the third-order chaotic Colpitts oscillator and K is the state feedback control gain, which consists of one parallel and two series conductance which determining the feedback gain is treated as a constant value for chaos control, as shown in figure 2 (b).

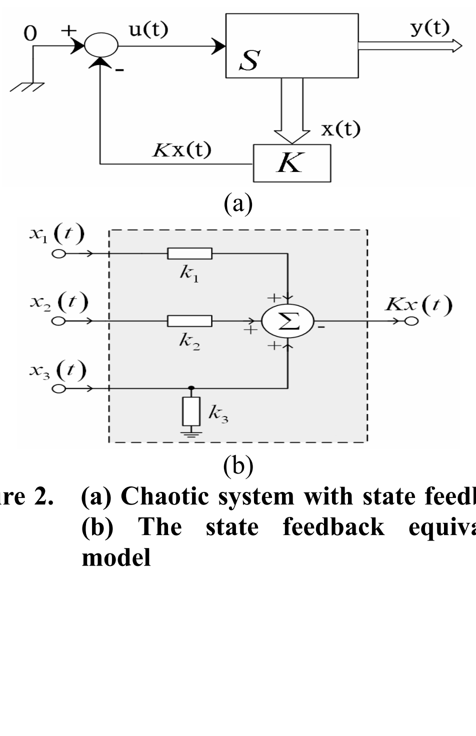

Figure 2. (a) Chaotic system with state feedback (b) The state feedback equivalent model

In order to be able to use state feedback control, the following controllability matrix of the linear system must be full-rank:

$$P=\begin{bmatrix} B & AB & A^2B \end{bmatrix} \neq 0 \tag{6}$$

Where A and B are the state and input matrices of (4), respectively. It can be easily verified by Gauss elimination method that the controllability of chaotic Colpitts oscillator can be manipulated into:

$$\tilde{P}=\begin{bmatrix} 1 & 0 & k-1 \\ 0 & 1 & \dfrac{Qk(k-1)}{g^*} \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

This has rank 3 and indicates that S is controllable. Thus the close-loop circuit $\dot{x}'=(A-BK)x'$ may set up by:

$$A-BK=\begin{bmatrix} 0 & \dfrac{g^*}{Q(1-k)} & \dfrac{g^*}{Q(1-k)} \\ -\dfrac{k_1 g^*}{Qk} & -\dfrac{k_2 g^*}{Qk} & \dfrac{g^*}{Qk}(1-k_3) \\ -\dfrac{Qk(1-k)}{g^*} & -\dfrac{Qk(1-k)}{g^*} & -\dfrac{1}{Q} \end{bmatrix} \tag{8}$$

Where $K=[k_1 \quad k_2 \quad k_3]$ is a row matrix, the equivalent circuit model is shown in figure 2(b). Furthermore, the natural modes of close-loop are characterized by the characteristic equations of $d(\lambda I-(A-BK))=0$ is given by

$$\lambda^3+\left(1+\frac{g^*}{k}k_2\right)\frac{1}{Q}\lambda^2+\left(1+\frac{g'^2}{Q^2k(1-k)}k_1+\frac{g^*}{Q^2k}k_2-(1-k)k_3\right)\lambda$$
$$+\left(1+\left(\frac{g^*}{Q^2k(1-k)}-1\right)k_1+k_2-k_3\right)\frac{g^*}{Q}=0 \tag{9}$$

According to Shilinikov's theorem [16], if the third-order dynamic system possesses a single equilibrium point, it can be shown a saddle focus with a stable pair of complex conjugate eigenvalues and an unstable real eigenvalue.

This theorem also applies when the characteristic equations of equilibrium point with a stable real eigenvalue and an unstable pair of complex conjugate eigenvalues, which trajectories will be shown to be a horseshoes and chaos [3].

In practice, K is chosen so that the largest unstable eigenvalue of (9) will move backward to imaginary axis of **s**-plane and admits a pair of pure imaginary eigenvalues. Therefore, one possible application of the pole position would be as follow

$$\left(j,-j,-\left(1+\frac{g^*Q^2(g^*-1)}{Q^2k^2-Q^2k-Q^2kg^*+Q^2g^*+g^*}\right)\frac{1}{Q}\right) \tag{10}$$

Then substitute the pole position into (9), consequently, the controller K is given by:

$$K=\begin{bmatrix} \rho * \dfrac{(k-1)}{g^*} & \rho & 0 \end{bmatrix} \tag{11}$$

Where $\rho=\dfrac{Q^2k(g^*-1)}{Q^2k^2-Q^2k-Q^2kg^*+Q^2g^*+g^*}$, thus the close-loop system will return to nearly-sinusoidal behavior.

4. Simulation results

In order to demonstrate quantitative analysis, some numerical simulations are presented in this section.

4.1 Quantitative analysis

In what follows, the fourth-order Runge-Kutta method and fast Fourier transform (FFT) algorithm are applied to solve the system of state equations, and represented by phase portrait, power spectrum, and time series in all numerical simulation results.

We first assume that the typical value $\alpha_F=0.996$ of CB short-circuit forward current gain in a 2N2222A transistor. The parameters of corresponding simulation results of model (3) with the normalized values: $L=10mH$, $C_1=1\mu F$, $C_2=1\mu F$, $V_{CC}=3V$.

In chaotic Colpitts oscillator, two kinds of chaotic behavior [6] are under investigated: One is the Feigenbaum-like chaos, as shown in figure 3 with $g^*=2.8009$, $Q=1.7678$ and K=0. The corresponding controlled results are reported in figure 4 with control gain K=[-0.0785 0.4399 0] which is calculated from (11).

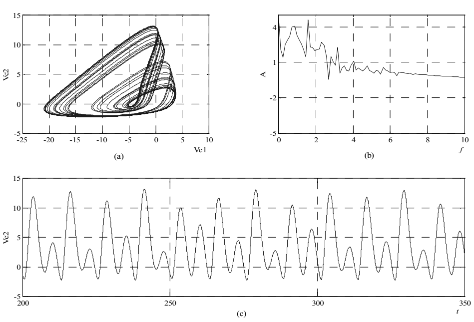

Figure 3. Feigenbaum-like chaos with $g^*=2.8009$ and $Q=1.7678$. (a) Phase portrait; (b) Power spectrum; (c) Time series

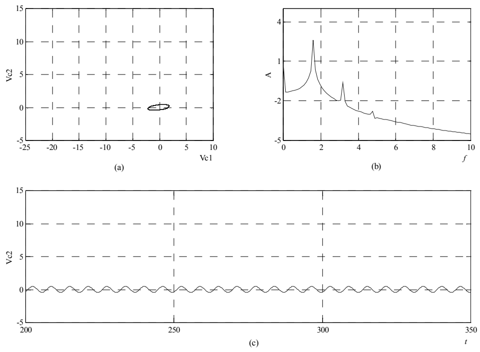

Figure 4. Periodic-1 response converted from Feigenbaum-like chaos with control gain *K*=[-0.0785 0.4399 0]. (a) Phase portrait; (b) Power spectrum; (c) Time series

The other is the Shil'nikov-like chaos, as shown in figure 5 with $g^* = 3.268$, $Q = 1.3860$ and *K*=0. The corresponding controlled results are reported in figure 6 with control gain *K*=[-0.0539, 0.3523, 0].

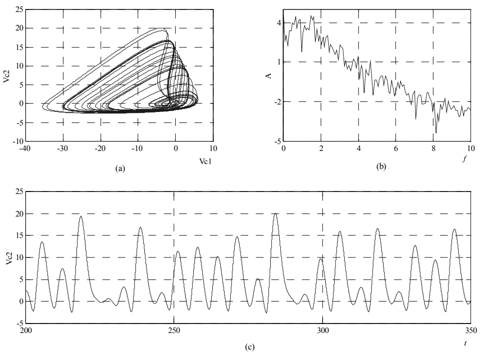

Figure 5. Shil'nikov-like chaos with $g^* = 3.268$ and $Q = 1.3860$. (a) Phase portrait; (b) Power spectrum; (c) Time series

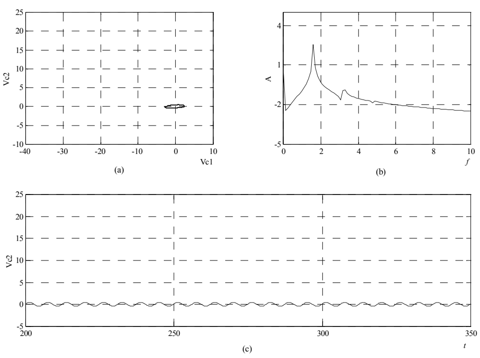

Figure 6. Periodic-1 response converted from Shil'nikov-like chaos with control gain *K*=[-0.0539 0.3523 0]. (a) Phase portrait; (b) Power spectrum; (c) Time series

We observed that the control gain is slightly smaller than ideal value of (11) in figure 6, that is a parameter mismatch with respect to the theoretical predictions is observed ($\alpha_F = 1$ in theoretical analysis and $\alpha_F = 0.996$ in numerical integration) [6].

4.2 SPICE simulation

The SPICE simulation setup in this section is shown in figure 7, with the same parameter values described above, while the BJT used a standard 2N2222A.
The ideal current generator which provides the reference bias current is an implementation of Howland current source [17]. Furthermore, the relationship between parameters and chaotic behaviors are shown in table 1.

Table 1. The relationship between chaos families and parameters

Chaos families	I_0	R	g^*	Q
Feigenbaum-like chaos	1.21mA	80Ω	2.8009	1.7678
Shil'nikov-like chaos	1.80mA	102Ω	3.2680	1.3860

In practical configuration, one is to avoid the operating point of the chaotic Colpitts oscillator changing by feedback controller. Two capacitors ($96\,\mu F$) used as AC couplers; the other, two high input impedance OP amplifiers are designed as voltage-to-current converter to avoid the loading effects [17].
When the state feedback controller *K* is connected as shown in figure 7, the chaos control analyses are processed.
Two kinds of chaotic behavior control are simulated in this section: One is the Feigenbaum-like chaos, as shown in figure 8 with parameters $g^* = 2.8009$, $Q = 1.7678$ and the control gain *K*=0. The corresponding results are reported in figure 9 with control gain *K*=[-0.078536 0.439946 0].

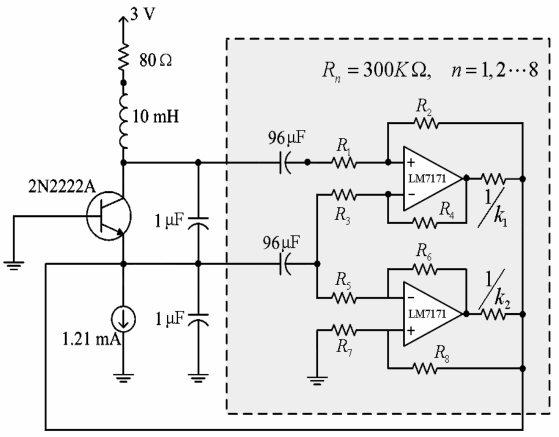

Figure 7. The close loop circuit with the component values

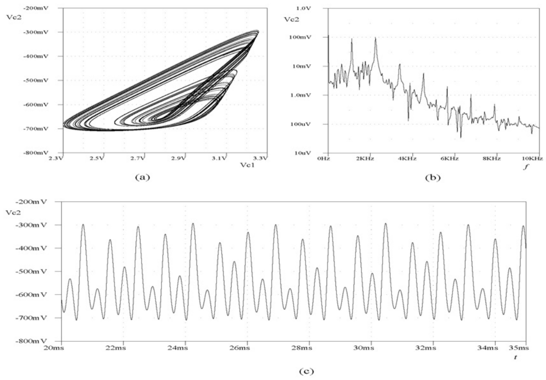

Figure 8. Feigenbaum-like chaos with $g^* = 2.8009$ and $Q = 1.7678$ simulated by SPICE. (a) Phase portrait; (b) Power spectrum; (c) Time series

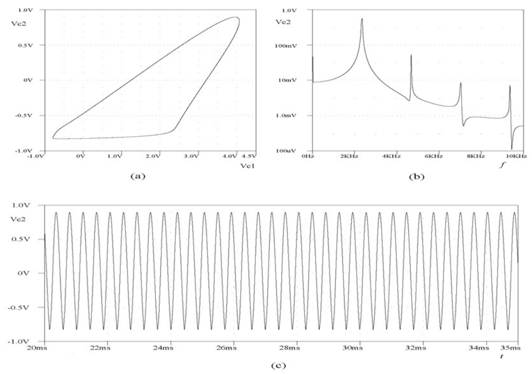

Figure 9. Periodic-1 response converted from Feigenbaum-like chaos with control gain *K*=[-0.078536 0.439946 0]. (a) Phase portrait; (b) Power spectrum; (c) Time series

The other is the Shil'nikov-like chaos, as shown in figure 10, with following parameters $g^* = 3.268$, $Q = 1.3860$ and *K*=0. The corresponding controlled results are reported in figure 11 with control gain *K*=[-0.056258 0.367702 0].

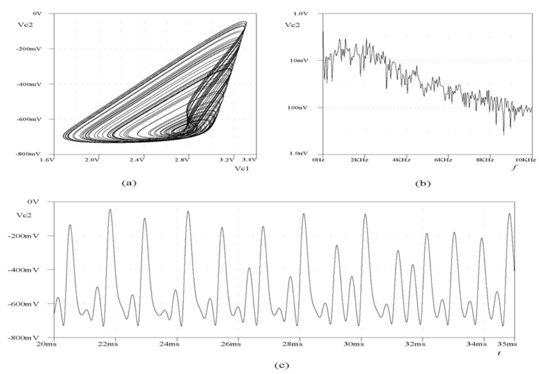

Figure 10. Shil'nikov-like chaos with $g^* = 3.268$ and $Q = 1.3860$ simulated by SPICE. (a) Phase portrait; (b) Power spectrum; (c) Time series

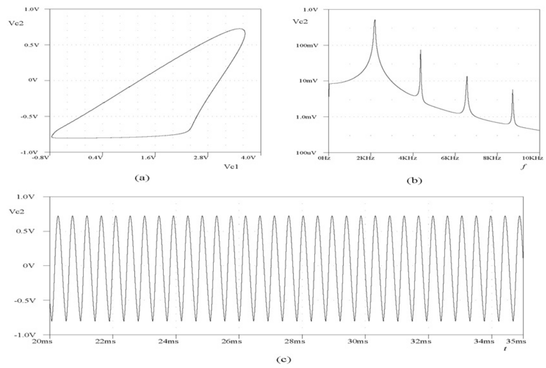

Figure 11. Periodic-1 response converted from Shil'nikov-like chaos with control gain *K*=[-0.056258 0.367702 0]. (a) Phase portrait; (b) Power spectrum; (c) Time series

The observed feedback controller is robust in the sense that it persists even if the chaotic Colpitts oscillator is replaced with one of a dual-voltage supply type which is describe in [2].

5. Conclusion

In this work a state feedback controller applied with the pole-placement method is designed for chaos control in the chaotic Colpitts oscillator.
The nearly-sinusoidal phenomenon is analyzed from the mathematical model. It has been also verified both by using the fourth-order Runge-Kutta algorithm and SPICE simulations in time series, power spectrum and phase portrait diagrams. The simulation results show a good quantitative agreement with pole-placement technique on the chaos control.

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