Package ‘Manifoldgstat’

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Type Package

Title Kriging prediction for manifold-valued data.

Version 1.0.0

Description Predictive analysis for manifold-valued data. This package provides a C++ implementation of functions to create a model for spatial dependent manifold-valued data, in order to perform kriging.
In each location, specified by a vector of coordinates ([lat,long], [x,y] or [x,y,z]), the datum is supposed to be a symmetric positive definite matrix.
The user is provided with three main functions: model_kriging, full_RDD, mixed_RDD, each designed to deliver kriging predictions following the corresponding algorithm (GlobalModel, FullRDD and MixedRDD), as presented in the reference dissertation. They exploit, to different extents, tangent space approximations, Random Domain Decomposition and advanced differential geometry concepts like parallel transport.

Reference Ilaria Sartori Luca Torriani (2019): Mixed Random Domain Decomposition: an innovative approach for kriging prediction of manifold valued data, Master Degree Thesis

Depends R (>= 3.2.0), Rcpp (>= 0.12.16), RcppEigen (>= 0.3.3.4.0), plyr(>= 1.8.4)

LinkingTo Rcpp, RcppEigen

NeedsCompilation yes

SystemRequirements C++11

License What license is it under?

Encoding UTF-8

LazyData true

RoxygenNote 6.1.1

R topics documented:

`assign2center` *Assign a point to the cell with the closest center*

Description

...

Usage

```
assign2center(distance.vector)
```

Arguments

`distance.vector`
vector of length K containing the distances between the point and the centers

Details

...

Value

it returns the index(es) of the cell(s) with the closest center(s). Returns 0 it the minimum does not exists `bootstrapVar` *Compute the bootstrap variance*

Description

Compute the bootstrap variance

Usage

```
bootstrapVar(res.boot, res.aggr, K, metric_manifold)
```

Arguments

`res.boot`	A list of length B. Each field contains a list with the M predictions generated by the corresponding iteration
`res.aggr`	A list of lenght M. Each field a single prediction, computed aggregating the corresponding data in res.boot
`K`	number of cells the domain is subdivided in
`metric_manifold`	metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot"

Value

It returns a vector of length M (i.e. the number of locations where we predict), containing the the prediction variance in the corresponding location

`create.rdd` *Divide the domain in K subregions*

Description

...

Usage

```
create.rdd(K, method.rdd = "Voronoi", data_coords, graph.distance,
  nk_min, grid, data.grid.distance, suppressMes = T)
```

Arguments

`K`	number of regions the domain is divided in
`method.rdd`	method used to define the subregions. So far only "Voronoi" has been implemented
`data_coords`	`N*2` or `N*3` matrix of [lat,long], [x,y] or [x,y,z] coordinates of the locations where data has been measured
`graph.distance`	`N*N` distance matrix (the [i,j] element is the length of the shortest path between points i and j)

`nk_min`	minimum number of observations within a cell
`grid`	prediction grid, i.e. `M*2` or `M*3` matrix of coordinates where to predict
`data.grid.distance`	`N*M` distance matrix between locations where the datum has been observed and locations where the datum has to be predicted
`suppressMes`	{`TRUE`, `FALSE`} controls the level of interaction and warnings given

Details

...

Value

it returns a list with the following fields

- `assign` vector of length N that indicates, for every known location, the cell it has been assigned to
- `centers` `K*3` matrix reporting the coordinates and the index of the locations drawn as centers of the K cells
- `assigng` vector of length M that indicates, for every new location, the cell it has been assigned to
- `gridk` list of K elements. The i-th element contains the coordinates of the grid points assigned to the i-th cell
- `graph.distance.grid.centers` `K*M` matrix containing the distances between each grid points and the K centers

---

`distance_manifold` *Distance on the manifold*

Description

Compute the manifold distance between symmetric positive definite matrices

Usage

```
distance_manifold(data1, data2, metric_manifold = "Frobenius")
```

Arguments

`data1`	Either a list/array `[p,p,B1]` of `B1` symmetric positive definite matrices of dimension `p*p`, or a single `p*p` matrix
`data2`	Either a list/array `[p,p,B2]` of `B2` symmetric positive definite matrices of dimension `p*p`, or a single `p*p` matrix.
`metric_manifold`	metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"

Details

If B2=B1 then the result is a vector of length B1=B2 containing in position i the manifold distance beetween data1[,,i] and data2[,,i]. Instead if B2=1 and B1!=1 the result is a vector of length B1 containing in position i the manifold distance between data1[,,i] and data2[,,1]

Value

A vector of distances, or a double if data1 and data2 are single matrices.

Examples

```
data_manifold_model <- Manifoldgstat::rCov
distances <-distance_manifold(data_manifold_model, diag(2), metric_manifold = "Frobenius")
print(distances)
```

Eucldist *Compute the Euclidean distance between two points*

Description

...

Usage

```
Eucldist(c1, c2)
```

Arguments

c1	coordinates of the first point.
c2	coordinates of the second point.

Details

...

Value

the Euclidean distance between c1 and c2 full_RDD *Perform full_RDD*

Description

Perform kriging prediction using FullRDD procedure

Usage

```
full_RDD(data_coords, data_val, K, grid, nk_min = 1, B = 100,
  suppressMes = F, tol = 1e-12, max_it = 100, n_h = 15,
  tolerance_intrinsic = 10^(-6), X = NULL, X_new = NULL,
  ker.width.intrinsic = 0, ker.width.vario = 1.5,
  graph.distance.complete, data.grid.distance, aggregation_mean,
  aggregation_kriging, method.analysis = "Local mean", metric_manifold,
  metric_ts, model_ts, vario_model, distance = NULL)
```

Arguments

data_coords	N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
data_val	array [p,p,N] of N symmetric positive definite matrices of dimension p*p
K	number of cells the domain is subdivided in
grid	prediction grid, i.e. M*2 or M*3 matrix of coordinates where to predict
nk_min	minimum number of observations within a cell
B	number of *divide* iterations to perform
suppressMes	{TRUE, FALSE} controls the level of interaction and warnings given
tol	tolerance for the main loop of model_kriging
max_it	maximum number of iterations for the main loop of model_kriging
n_h	number of bins in the empirical variogram
tolerance_intrinsic	tolerance for the computation of the intrinsic mean
X	matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL
X_new	matrix (with the same number of rows of new_coords) of additional covariates for the new locations, possibly NULL
ker.width.intrinsic	parameter controlling the width of the Gaussian kernel for the computation of the local mean (if 0, a "step kernel" is used, giving weight 1 to all the data within the cell and 0 to those outside of it)
ker.width.vario	parameter controlling the width of the Gaussian kernel for the computation of the empirical variogram (if 0, a "step kernel" is used, giving weight 1 to all the data within the cell and 0 to those outside of it)
graph.distance.complete	N*N distance matrix (the [i,j] element is the length of the shortest path between points i and j)

`data.grid.distance`
: `N*M` distance matrix between locations where the datum has been observed and locations where the datum has to be predicted

`aggregation_mean`
: "Weighted" to aggregate the mean predictions using kernel-based weights, "Equal" to use equal weights

`aggregation_kriging`
: "Weighted" to aggregate the Kriging predictions using kernel-based weights, "Equal" to use equal weights

`method.analysis`
: "Local mean" to predict just with the mean, "Kriging" to predict via Kriging procedure

`metric_manifold`
: metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot"

`metric_ts`
: metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"

`model_ts`
: type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"

`vario_model`
: type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"

`distance`
: type of distance between coordinates. It must be either "Eucldist" or "Geodist"

Details

It uses a repetition of local analyses, through a *divide* et *impera* strategy. In the *divide* step, the domain is randomly decomposed in subdomains where local tangent-space models are estimated in order to predict at new locations (in each subregion is performed exactly the analysis described in the `model_kriging` function). This is repeated `B` times with different partitions of the domain. Then, in the *impera* step, the results of these iterations are aggregated, by means of the intrinsic mean, to provide a final prediction.

Value

According to the analysis chosen:

- If `method.analysis` = "Local mean" it returns a list with the following fields
  - `resBootstrap` A list consisting of
    - `fmean` list of length `B`. Each field contains the prediction (at iteration `b`) for each new location, obtained as the intrinsic mean of the data within the tile it belongs to
    - `kervalues_mean` Weights used for aggregating `fmean`
  - `resAggregated` Predictions, for each new location, obtained aggregating `fmean` using `kervalues_mean` as weights
- If `method.analysis` = "Kriging" it returns a list with the following fields
  - `resBootstrap` A list consisting of
    - `fmean` list of length `B`. Each field contains the prediction (at iteration `b`) for each new location, obtained as the intrinsic mean of the data within the tile it belongs to
    - `fpred` list of length `B`. Each field contains the prediction (at iteration `b`) for each new location, obtained through kriging
    - `kervalues_mean` Weights used for aggregating `fmean`

    * `kervalues_krig` Weights used for aggregating `fpred`
    * `variofit` list of length `B`. Each field contains, for each datum, the parameters of the variogram fitted in the tile it belongs to
- `resAggregated` Predictions, for each new location, obtained aggregating `fpred` using `kervalues_krig` as weights
- `resLocalMean` Predictions, for each new location, obtained aggregating `fmean` using `kervalues_mean` as weights

---

`Geodist` *Compute the great-circle distance between two points*

Description

...

Usage

```
Geodist(c1, c2)
```

Arguments

c1	coordinates [lat, long] of the first point.
c2	coordinates [lat, long] of the second point.

Details

The distance is computed using the Haversine formula

Value

the great-circle distance between `c1` and `c2`

`intrinsic_mean` *Intrinsic mean*

Description

Evaluate the intrinsic mean of a given set of symmetric positive definite matrices

Usage

```
intrinsic_mean(data, metric_manifold = "Frobenius",
  metric_ts = "Frobenius", tolerance = 1e-06,
  weight_intrinsic = NULL, weight_extrinsic = weight_intrinsic,
  tolerance_map_cor = 1e-06)
```

Arguments

`data`	list or array [`p,p,B`] of `B` symmetric positive definite matrices of dimension `p*p`
`metric_manifold`	metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
`metric_ts`	metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
`tolerance`	tolerance for the computation of the intrinsic mean
`weight_intrinsic`	vector of length `B` to weight the matrices in the computation of the intrinsic mean. If NULL a vector of ones is used
`weight_extrinsic`	vector of length `B` to weight the matrices in the computation of the extrinsic mean. If NULL `weight_intrinsic` is used
`tolerance_map_cor`	tolerance to use in the maps. Required only if `metric_manifold== "Correlation"`

Value

A matrix representing the intrinsic mean of the `data`

References

X. Pennec, P. Fillard, and N. Ayache. A riemannian framework for tensor computing. International Journal of computer vision, 66(1):41-66, 2006.

Examples

```
data_manifold_tot <- Manifoldgstat::fieldCov
Sigma <-intrinsic_mean(data_manifold_tot, metric_manifold = "Frobenius",
            metric_ts = "Frobenius")
print(Sigma)
```

`kerfn` *Evaluate a gaussian kernel*

Description

...

Usage

```
kerfn(newdata, center, ker.type = "Gau", param)
```

Arguments

`newdata`	coordinates of the locations where we want to compute the kernel values
`center`	coordinates of the reference center for the kernel
`ker.type`	type of kernel. So far only "Gau" (i.e gaussian kernel) has been implemented
`param`	parameters that define the kernel. For the gaussian kernel it is the sigma parameter

Details

...

Value

the values of the kernel function corresponding to the vector of Euclidean distances between `newdata` and the `center`

`kriging` *Kriging prediction given the model*

Description

Given the GLS model, kriging prediction on new location(s) is performed.

Usage

```
kriging(GLS_model, coords, new_coords, model_ts = "Additive",
  vario_model = "Gaussian", metric_manifold = "Frobenius",
  X_new = NULL, distance = "Geodist", tolerance_map_cor = 1e-06)
```

Arguments

`GLS_model` the object returned by `model_GLS`, or a list containing the fields: `Sigma` (tangent point), `beta` (vector of the beta matrices of the fitted model), `gamma_matrix` (`N*N` covariogram matrix), `residuals` (vector of the `N` residual matrices), `fitted_par_vario` (estimates of *nugget*, *sill-nugget* and *practical range*)

`coords` `N*2` or `N*3` matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees

`new_coords` matrix of coordinates for the M new locations where to perform kriging

`model_ts` type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"

`vario_model` type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"

`metric_manifold` metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"

`X_new` matrix (with the same number of rows of `new_coords`) of additional covariates for the new locations, possibly NULL

`distance` type of distance between coordinates. It must be either "Eucldist" or "Geodist"

`tolerance_map_cor` tolerance to use in the maps.
Required only if `metric_manifold=="Correlation"`

`data_grid_dist_mat` Matrix of dimension `N*M` of distances between data points and grid points. If not provided it is computed using `distance`

Details

The model provided is used to perform simple kriging on the tangent space in correspondence of the new locations. The estimates are then mapped to the manifold to produce the actual prediction.

Value

A list with a single field:

`prediction` vector of matrices predicted at the new locations

References

D. Pigoli, A. Menafoglio & P. Secchi (2016): Kriging prediction for manifold-valued random fields. Journal of Multivariate Analysis, 145, 117-131.

Examples

```
data_manifold_tot <- Manifoldgstat::fieldCov
data_manifold_model <- Manifoldgstat::rCov
coords_model <- Manifoldgstat::rGrid
coords_tot <- Manifoldgstat::gridCov
Sigma <- matrix(c(2,1,1,1), 2,2)

model = model_GLS(data_manifold = data_manifold_model, coords = coords_model, Sigma = Sigma,
              metric_manifold = "Frobenius", metric_ts = "Frobenius", model_ts = "Coord1",
                vario_model = "Spherical", n_h = 15, distance = "Eucldist", max_it = 100,
                  tolerance = 1e-7, plot = TRUE)
result = kriging (GLS_model = model, coords = coords_model, new_coords = coords_model,
              model_ts="Coord1", vario_model= "Spherical", metric_manifold = "Frobenius",
                  distance="Eucldist")
result_tot = kriging (GLS_model = model, coords = coords_model, new_coords = coords_tot,
               model_ts="Coord1", vario_model= "Spherical", metric_manifold = "Frobenius",
                      distance="Eucldist")

x.min=min(coords_tot[,1])
x.max=max(coords_tot[,1])
y.min=min(coords_tot[,2])
y.max=max(coords_tot[,2])
dimgrid=dim(coords_tot)[1]
radius = 0.02

par(cex=1.25)
plot(0,0, asp=1, col=fields::tim.colors(100), ylim=c(y.min,y.max), xlim=c(x.min, x.max),
     pch='', xlab='', ylab='', main = "Real Values")
for(i in 1:dimgrid){
  if(i %% 3 == 0)
     car::ellipse(c(coords_tot[i,1],coords_tot[i,2]), data_manifold_tot[,,i],
                    radius=radius, center.cex=.5, col='navyblue')
}
rect(x.min, y.min, x.max, y.max)

for(i in 1:250)
{ car::ellipse(c(coords_model[i,1],coords_model[i,2]), data_manifold_model[,,i],
               radius=radius, center.cex=.5, col='green')}
rect(x.min, y.min, x.max, y.max)
```

```
par(cex=1.25)
plot(0,0, asp=1, col=fields::tim.colors(100), ylim=c(y.min,y.max),xlim=c(x.min, x.max),
     pch='', xlab='', ylab='',main = "Predicted values")
for(i in 1:dimgrid){
  if(i %% 3 == 0)
     car::ellipse(c(coords_tot[i,1],coords_tot[i,2]), result_tot$prediction[[i]],
                  radius=radius, center.cex=.5, col='navyblue' )
}
rect(x.min, y.min, x.max, y.max)

for(i in 1:250)
{ car::ellipse(c(coords_model[i,1],coords_model[i,2]), result$prediction[[i]],
                radius=radius, center.cex=.5, col='red')}
rect(x.min, y.min, x.max, y.max)
```

---

mixed_RDD *Perform mixed_RDD*

Description

Perform kriging prediction using MixedRDD procedure

Usage

```
mixed_RDD(data_coords, data_val, K, grid, nk_min = 1, B = 100,
  suppressMes = F, ker.width.intrinsic = 0, graph.distance.complete,
  data.grid.distance, N_samples, aggregation_mean, metric_ts,
  tol = 1e-12, max_it = 100, n_h = 15,
  tolerance_intrinsic = 10^(-6), max_sill = NULL, max_a = NULL,
  X = NULL, X_new = NULL, create_pdf_vario = FALSE,
  pdf_parameters = NULL, metric_manifold, model_ts, vario_model,
  distance)
```

Arguments

data_coords	N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
data_val	array [p,p,N] of N symmetric positive definite matrices of dimension p*p
K	number of cells the domain is subdivided in
grid	prediction grid, i.e. M*2 or M*3 matrix of coordinates where to predict
nk_min	minimum number of observations within a cell
B	number of *divide* iterations to perform
suppressMes	{TRUE, FALSE} controls the level of interaction and warnings given
ker.width.intrinsic	parameter controlling the width of the Gaussian kernel for the computation of the local mean (if 0, a "step kernel" is used, giving weight 1 to all the data within the cell and 0 to those outside of it)
graph.distance.complete	N*N distance matrix (the [i,j] element is the length of the shortest path between points i and j)

`data.grid.distance`
: `N*M` distance matrix between locations where the datum has been observed and locations where the datum has to be predicted

`N_samples`
: number of data N

`aggregation_mean`
: "Weighted" to aggregate the mean predictions using kernel-based weights, "Equal" to use equal weights

`metric_ts`
: metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"

`tol`
: tolerance for the main loop of `model_kriging`

`max_it`
: maximum number of iterations for the main loop of `model_kriging`

`n_h`
: number of bins in the empirical variogram

`tolerance_intrinsic`
: tolerance for the computation of the intrinsic mean

`max_sill`
: max value allowed for `sill` in the fitted variogram. If NULL it is defined as `1.15*max(emp_vario_values)`

`max_a`
: maximum value for `a` in the fitted variogram. If NULL it is defined as `1.15*h_max`

`X`
: matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL

`X_new`
: matrix (with the same number of rows of `new_coords`) of additional covariates for the new locations, possibly NULL

`create_pdf_vario`
: boolean. If `TRUE` the empirical and fitted variograms are plotted in a pdf file

`pdf_parameters`
: list with the fields `test_nr` and `sample_draw`. Additional parameters to name the pdf

`metric_manifold`
: metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot"

`model_ts`
: type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"

`vario_model`
: type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"

`distance`
: type of distance between coordinates. It must be either "Eucldist" or "Geodist"

Details

It employs a *divide* et *impera* strategy to provide an estimate of a "fictional" field of tangent points, used to encode the information regarding the drift of the field. To this end in the *divide* step, the domain is randomly decomposed and in each subdomain a tangent point (assigned to each location in that subregion) is estimated as the intrinsic mean of the data belonging to it. This is repeated `B` times with different partitions of the domain and the results are then aggregated in the *impera* stage by means of the intrinsic mean. Eventually, exploiting this "fictional" field of tangent points and the concept of parallel transport, a kriging analysis over the whole domain is performed to predict the field values at new locations.

Value

it returns a list with the following fields

- resBootstrap list of length B. Each field contains a tangent point estimate (at iteration b) for each new location, obtained as the intrinsic mean of the data within the tile it belongs to
- resAggregated field of tangent points computed, for each location (both those where data are measured and where they must be predicted), aggregating the corresponding resBootstrap
- model_pred list with the details of the global model fitted on the common Hibert space and the resulting kriging predictions. Namely it contains the following fields: beta vector of the beta matrices of the fitted model gamma_matrix N*N covariogram matrix residuals vector of the N residual matrices emp_vario_values vector of empircal variogram values in correspondence of h_vec h_vec vector of positions at which the empirical variogram is computed fitted_par_vario estimates of *nugget*, *sill-nugget* and *practical range* iterations number of iterations of the main loop prediction vector of matrices predicted at the new locations

---

model_GLS *Create a GLS model*

Description

Given the coordinates and corresponding manifold values, this function creates a GLS model on the tangent space.

Usage

```
model_GLS(data_manifold, coords, X = NULL, Sigma = NULL,
  metric_manifold = "Frobenius", metric_ts = "Frobenius",
  model_ts = "Additive", vario_model = "Gaussian", n_h = 15,
  distance = "Geodist", max_it = 100, tolerance = 1e-06,
  weight_intrinsic = NULL, tolerance_intrinsic = 1e-06,
  max_sill = NULL, max_a = NULL, param_weighted_vario = NULL,
  plot = FALSE, suppressMes = FALSE, weight_extrinsic = NULL,
  tolerance_map_cor = 1e-06)
```

Arguments

data_manifold	list or array [p,p,N] of N symmetric positive definite matrices of dimension p*p
coords	N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
X	matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL
Sigma	p*p matrix representing the tangent point. If NULL the tangent point is computed as the intrinsic mean of data_manifold
metric_manifold	metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
metric_ts	metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"

- `model_ts` type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
- `vario_model` type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
- `n_h` number of bins in the emprical variogram
- `distance` type of distance between coordinates. It must be either "Eucldist" or "Geodist"
- `max_it` max number of iterations for the main loop
- `tolerance` tolerance for the main loop
- `weight_intrinsic` vector of length `N` to weight the locations in the computation of the intrinsic mean. If NULL a vector of ones is used. Not needed if `Sigma` is provided
- `tolerance_intrinsic` tolerance for the computation of the intrinsic mean. Not needed if Sigma is provided
- `max_sill` max value allowed for *sill* in the fitted variogram. If NULL it is defined as `1.15*max(emp_vario_values)`
- `max_a` maximum value for *a* in the fitted variogram. If NULL it is defined as `1.15*h_max`
- `param_weighted_vario` List of 7 elements to be provided to consider Kernel weights for the variogram (significant only within an RDD procedure). Indeed in this case the N_tot data regarding the whole domain must be provided to the algorithm, not only the N in the cell under consideration. Therefore the list must contain the following fields: `weight_vario` (vector of length `N_tot` to weight the locations in the computation of the empirical variogram), `distance_matrix_tot` (`N_tot*N_tot` matrix of distances between the locations), `data_manifold_tot` (list or array `[p,p,N_tot]` of `N_tot` symmetric positive definite matrices of dimension `p*p`), `coords_tot` (`N_tot*2` or `N_tot*3` matrix of [lat,long], [x,y] or [x,y,z] coordinates), `X_tot` (matrix with N_tot rows and unrestricted number of columns of additional covariates for the tangent space model, possibly NULL), `h_max` (maximum value of distance for which the variogram is computed), `indexes_model` (indexes of the N_tot data corresponding to the N data in the cell).
- `plot` boolean. If `TRUE` the empirical and fitted variograms are plotted
- `suppressMes` boolean. If `TRUE` warning messagges are not printed
- `weight_extrinsic` vector of length `N` to weight the locations in the computation of the extrinsic mean. If NULL weight_intrinsic are used. Needed only if `Sigma` is not provided and `metric_manifold== "Correlation"`
- `tolerance_map_cor` tolerance to use in the maps.
  Required only if `metric_manifold== "Correlation"`
- `data_dist_mat` Matrix of dimension `N*N` of distances between data points. If not provided it is computed using `distance`

Details

The manifold values are mapped on the tangent space and then a GLS model is fitted to them. A first estimate of the `beta` coefficients is obtained assuming spatially uncorrelated errors. Then, in the main the loop, new estimates of the `beta` are obtained as a result of a weighted least square problem where the weight matrix is the inverse of `gamma_matrix`. The residuals

(residuals = data_ts - fitted) are updated accordingly. The parameters of the variogram fitted to the residuals (and used in the evaluation of the gamma_matrix) are computed using Gauss-Newton with backtrack method to solve the associated non-linear least square problem. The stopping criteria is based on the absolute value of the variogram residuals' norm if ker.width.vario=0, while it is based on its increment otherwise.

Value

A list with the following fields:

- `beta` vector of the beta matrices of the fitted model
- `gamma_matrix` N*N covariogram matrix
- `residuals` vector of the N residual matrices
- `emp_vario_values` vector of empircal variogram values in correspondence of h_vec
- `h_vec` vector of positions at which the empirical variogram is computed
- `fitted_par_vario` estimates of *nugget*, *sill-nugget* and *practical range*
- `iterations` number of iterations of the main loop
- `Sigma` tangent point

References

D. Pigoli, A. Menafoglio & P. Secchi (2016): Kriging prediction for manifold-valued random fields. Journal of Multivariate Analysis, 145, 117-131.

Examples

```
data_manifold_model <- Manifoldgstat::rCov
coords_model <- Manifoldgstat::rGrid
Sigma <- matrix(c(2,1,1,1), 2,2)
model = model_GLS(data_manifold = data_manifold_model, coords = coords_model, Sigma = Sigma,
            metric_manifold = "Frobenius", metric_ts = "Frobenius", model_ts = "Coord1",
              vario_model = "Spherical", n_h = 15, distance = "Eucldist", max_it = 100,
               tolerance = 1e-7, plot = TRUE)
```

---

model_kriging *Create a GLS model and directly perform kriging*

Description

Given the coordinates and corresponding manifold values, this function firstly creates a GLS model on the tangent space, and then performs kriging on the new locations.

Usage

```
model_kriging(data_manifold, coords, X = NULL, Sigma = NULL,
  metric_manifold = "Frobenius", metric_ts = "Frobenius",
  model_ts = "Additive", vario_model = "Gaussian", n_h = 15,
  distance = NULL, data_dist_mat = NULL, data_grid_dist_mat = NULL,
  max_it = 100, tolerance = 1e-06, weight_intrinsic = NULL,
  tolerance_intrinsic = 1e-06, max_sill = NULL, max_a = NULL,
  param_weighted_vario = NULL, new_coords, X_new = NULL,
  create_pdf_vario = TRUE, pdf_parameters = NULL,
  suppressMes = FALSE, weight_extrinsic = NULL,
  tolerance_map_cor = 1e-06)
```

Arguments

- `data_manifold` list or array [p,p,N] of N symmetric positive definite matrices of dimension p*p
- `coords` N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
- `X` matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL
- `Sigma` p*p matrix representing the tangent point. If NULL the tangent point is computed as the intrinsic mean of `data_manifold`
- `metric_manifold` metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
- `metric_ts` metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
- `model_ts` type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
- `vario_model` type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
- `n_h` number of bins in the emprical variogram
- `distance` type of distance between coordinates. It must be either "Eucldist" or "Geodist"
- `data_dist_mat` Matrix of dimension N*N of distances between data points. If not provided it is computed using `distance`
- `data_grid_dist_mat` Matrix of dimension N*M of distances between data points and grid points. If not provided it is computed using `distance`
- `max_it` max number of iterations for the main loop
- `tolerance` tolerance for the main loop
- `weight_intrinsic` vector of length N to weight the locations in the computation of the intrinsic mean. If NULL a vector of ones is used. Not needed if Sigma is provided
- `tolerance_intrinsic` tolerance for the computation of the intrinsic mean. Not needed if Sigma is provided
- `max_sill` max value allowed for `sill` in the fitted variogram. If NULL it is defined as `1.15*max(emp_vario_values)`

max_a maximum value for a in the fitted variogram. If NULL it is defined as 1.15*h_max

param_weighted_vario
List of 7 elements to be provided to consider Kernel weights for the variogram (significant only within an RDD procedure). Indeed in this case the N_tot data regarding the whole domain must be provided to the algorithm, not only the N in the cell under consideration. Therefore the list must contain the following fields: weight_vario (vector of length N_tot to weight the locations in the computation of the empirical variogram), distance_matrix_tot (N_tot*N_tot matrix of distances between the locations), data_manifold_tot (list or array [p,p,N_tot] of N_tot symmetric positive definite matrices of dimension p*p), coords_tot (N_tot*2 or N_tot*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates), X_tot (matrix with N_tot rows and unrestricted number of columns of additional covariates for the tangent space model, possibly NULL), h_max (maximum value of distance for which the variogram is computed), indexes_model (indexes of the N_tot data corresponding to the N data in the cell).

new_coords matrix of coordinates for the M new locations where to perform kriging

X_new matrix (with the same number of rows of new_coords) of additional covariates for the new locations, possibly NULL

create_pdf_vario
boolean. If TRUE the empirical and fitted variograms are plotted in a pdf file

pdf_parameters list with the fields test_nr and sample_draw. Additional parameters to name the pdf

suppressMes boolean. If TRUE warning messagges are not printed

weight_extrinsic
vector of length N to weight the locations in the computation of the extrinsic mean. If NULL weight_intrinsic are used. Needed only if Sigma is not provided and metric_manifold== "Correlation"

tolerance_map_cor
tolerance to use in the maps.
Required only if metric_manifold== "Correlation"

Details

The manifold values are mapped on the tangent space and then a GLS model is fitted to them. A first estimate of the beta coefficients is obtained assuming spatially uncorrelated errors. Then, in the main the loop, new estimates of the beta are obtained as a result of a weighted least square problem where the weight matrix is the inverse of gamma_matrix. The residuals (residuals = data_ts - fitted) are updated accordingly. The parameters of the variogram fitted to the residuals (and used in the evaluation of the gamma_matrix) are computed using Gauss-Newton with backtrack method to solve the associated non-linear least square problem. The stopping criteria is based on the absolute value of the variogram residuals' norm if ker.width.vario=0, while it is based on its increment otherwise. Once the model is computed, simple kriging on the tangent space is performed in correspondence of the new locations and eventually the estimates are mapped to the manifold.

Value

list with the following fields:

beta vector of the beta matrices of the fitted model

gamma_matrix N*N covariogram matrix

`residuals` vector of the `N` residual matrices

`emp_vario_values` vector of empircal variogram values in correspondence of `h_vec`

`h_vec` vector of positions at which the empirical variogram is computed

`fitted_par_vario` estimates of *nugget*, *sill-nugget* and *practical range*

`iterations` number of iterations of the main loop

`Sigma` tangent point

`prediction` vector of matrices predicted at the new locations

References

D. Pigoli, A. Menafoglio & P. Secchi (2016): Kriging prediction for manifold-valued random fields. Journal of Multivariate Analysis, 145, 117-131.

Examples

```
data_manifold_tot <- Manifoldgstat::fieldCov
data_manifold_model <- Manifoldgstat::rCov
coords_model <- Manifoldgstat::rGrid
coords_tot <- Manifoldgstat::gridCov
Sigma <- matrix(c(2,1,1,1), 2,2)

result = model_kriging (data_manifold = data_manifold_model, coords = coords_model,
                        Sigma = Sigma, metric_manifold = "Frobenius",
                        metric_ts = "Frobenius", model_ts = "Coord1",
                        vario_model = "Spherical", n_h = 15, distance = "Eucldist",
                        max_it = 100, tolerance = 10e-7, new_coords = coords_model)
result_tot = model_kriging (data_manifold = data_manifold_model, coords = coords_model,
                            metric_ts = "Frobenius", Sigma = Sigma,
                            metric_manifold = "Frobenius", model_ts = "Coord1",
                            vario_model = "Spherical", n_h = 15, distance = "Eucldist",
                            max_it = 100, tolerance = 10e-7, new_coords = coords_tot,
                            create_pdf_vario = FALSE)
x.min=min(coords_tot[,1])
x.max=max(coords_tot[,1])
y.min=min(coords_tot[,2])
y.max=max(coords_tot[,2])
dimgrid=dim(coords_tot)[1]
radius = 0.02

par(cex=1.25)
plot(0,0, asp=1, col=fields::tim.colors(100), ylim=c(y.min,y.max), xlim=c(x.min, x.max),
      pch='', xlab='', ylab='', main = "Real Values")
for(i in 1:dimgrid){
 if(i %% 3 == 0)
    car::ellipse(c(coords_tot[i,1],coords_tot[i,2]) , data_manifold_tot[,,i],
                                 radius=radius, center.cex=.5, col='navyblue')
}
rect(x.min, y.min, x.max, y.max)

for(i in 1:250)
{ car::ellipse(c(coords_model[i,1],coords_model[i,2]) , data_manifold_model[,,i],
               radius=radius, center.cex=.5, col='green')}
rect(x.min, y.min, x.max, y.max)
```

```
par(cex=1.25)
plot(0,0, asp=1, col=fields::tim.colors(100), ylim=c(y.min,y.max),xlim=c(x.min, x.max),
      pch='',  xlab='', ylab='',main = "Predicted values")

for(i in 1:dimgrid){
  if(i %% 3 == 0)
     car::ellipse(c(coords_tot[i,1],coords_tot[i,2]), result_tot$prediction[[i]],
                                  radius=radius, center.cex=.5, col='navyblue' )
}
rect(x.min, y.min, x.max, y.max)

for(i in 1:250)
{ car::ellipse(c(rGrid[i,1],rGrid[i,2]), result$prediction[[i]],radius=radius,
               center.cex=.5, col='red')
}
rect(x.min, y.min, x.max, y.max)
```

`model_kriging_mixed` *Perform main routine for mixed_RDD*

Description

...

Usage

```
model_kriging_mixed(data_manifold, coords, X = NULL, Sigma_data,
  metric_manifold = "Frobenius", model_ts = "Additive",
  vario_model = "Gaussian", n_h = 15, distance = NULL,
  data_dist_mat = NULL, data_grid_dist_mat = NULL, max_it = 100,
  tolerance = 1e-06, max_sill = NULL, max_a = NULL, new_coords,
  Sigma_new, X_new = NULL, create_pdf_vario = TRUE,
  pdf_parameters = NULL, suppressMes = FALSE)
```

Arguments

`data_manifold`	array [p,p,N] of N symmetric positive definite matrices of dimension p*p
`coords`	N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates.
`X`	matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL
`Sigma_data`	List of the N fictional tangent points in correspondence with the data used to build the model
`metric_manifold`	metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot"
`model_ts`	type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
`vario_model`	type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
`n_h`	number of bins in the empirical variogram

distance	type of distance between coordinates. It must be either "Eucldist" or "Geodist"
data_dist_mat	N*N distance matrix (the [i,j] element is the length of the shortest path between points i and j)
data_grid_dist_mat	N*M distance matrix between locations where the datum has been observed and locations where the datum has to be predicted
max_it	maximum number of iterations for the main loop of `model_kriging`
tolerance	tolerance for the main loop of `model_kriging`
max_sill	max value allowed for `sill` in the fitted variogram. If NULL it is defined as `1.15*max(emp_vario_values)`
max_a	maximum value for `a` in the fitted variogram. If NULL it is defined as `1.15*h_max`
new_coords	prediction grid, i.e. `M*2` or `M*3` matrix of coordinates where to predict
Sigma_new	List of the `M` fictional tangent points in correspondence with the locations where we want to predict
X_new	matrix (with the same number of rows of `new_coords`) of additional covariates for the new locations, possibly NULL
create_pdf_vario	boolean. If `TRUE` the empirical and fitted variograms are plotted in a pdf file
pdf_parameters	list with the fields `test_nr` and `sample_draw`. Additional parameters to name the pdf
suppressMes	{`TRUE`, `FALSE`} controls the level of interaction and warnings given

Details

...

Value

it returns a list with the following fields

beta	vector of the beta matrices of the fitted model
gamma_matrix	N*N covariogram matrix
residuals	vector of the N residual matrices
emp_vario_values	vector of empircal variogram values in correspondence of `h_vec`
h_vec	vector of positions at which the empirical variogram is computed
fitted_par_vario	estimates of *nugget*, *sill-nugget* and *practical range*
iterations	number of iterations of the main loop
prediction	vector of matrices predicted at the new locations

plot_ker_rect *Plot kernel*

Description

Plot kernel

Usage

```
plot_ker_rect(data_coords, id, xmax, ymax, m, n, ker.width)
```

Arguments

`data_coords`	coordinates of the data
`id`	the index of the row of `data_coords` that will be used as center
`xmax`	the maximum value for the x-coordinate (the minimum is 0)
`ymax`	the maximum value for the y-coordinate (the minimum is 0)
`m`	number of points on the grid in horizontal direction
`n`	number of points on the grid in vertical direction
`ker.width`	kernel width

plot_variogram *Plot empirical and fitted variogram*

Description

Plot empirical and fitted variogram

Usage

```
plot_variogram(empirical_variogram, fitted_variogram, model, distance)
```

Arguments

`empirical_variogram`	A list containing the two following fields: - h_vec: vector of positions at which the empirical variogram is computed - emp_vario_values: vector of empircal variogram values in correspondence of `h_vec`
`fitted_variogram`	A list containing the two following fields: - hh: dense vector of positions at which `fit_vario_values` is computed - fit_vario_values: Vector of fitted variogram values in correspondence of `hh`
`model`	Type of variogram used for fitting (it will be reported on the y-axis). It can be "Gaussian", "Spherical" or "Exponential"
`distance`	Type of distance used to compute `h_vec` (it will be reported on the x-axis). It must be either "Eucldist" or "Geodist"

RDD_OOK_aggr_man *Aggregate the results of the bootrap iterations*

Description

...

Usage

```
RDD_OOK_aggr_man(fOKBV, weights_intrinsic, ker.width.intrinsic)
```

Arguments

`fOKBV` list of length `B`, containing the results obtained, for each location, at the `B` bootstrap iterations (Usually it is the `fmean` or `fpred` returned by `RDD_OOK_boot_man` or `RDD_OOK_boot_man_mixed`)

`weights_intrinsic` weights to use to aggregate the results

`ker.width.intrinsic` width of the kernel used to compute `weights_intrinsic`. 0 if we use equal weights

Details

If `ker.width.intrinsic!=0` the data are aggregated using the normalized `weights_intrinsic`. Otherwise equal weights are used

Value

the aggregated result, obtained as the intrinsic_mean of `fOKBV`

RDD_OOK_boot_man *Main routine for full_RDD*

Description

...

Usage

```
RDD_OOK_boot_man(data_coords, data_val, K, grid, nk_min, B, suppressMes,
  tol, max_it, n_h, tolerance_intrinsic, X, X_new, ker.width.intrinsic,
  ker.width.vario, graph.distance.complete, data.grid.distance,
  method.analysis, metric_manifold, metric_ts, model_ts, vario_model,
  distance)
```

Arguments

- `data_coords` — N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates.
- `data_val` — list of N symmetric positive definite matrices of dimension p*p
- `K` — Number of neighborhood (i.e., centers) to sample at each iteration
- `grid` — M*2 or M*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates of the new locations where to predict
- `nk_min` — Minimum number of observations within a neighborhood
- `B` — Number of bootstap iterations
- `suppressMes` — boolean. If TRUE warning messagges are not printed
- `tol` — tolerance for each main loop
- `max_it` — max number of iterations for each main loop
- `n_h` — number of bins in the emprical variogram
- `tolerance_intrinsic` — tolerance for the computation of the intrinsic mean
- `X` — Additional covariates for the locations used to create the model
- `X_new` — Additional covariates for the M locations where to perform kriging
- `ker.width.intrinsic` — Parameter controlling the width of the Gaussian kernel for the computation of the local mean (if 0, no kernel is used)
- `ker.width.vario` — Parameter controlling the width of the Gaussian kernel for the computation of the empirical variogram (if 0, no kernel is used)
- `graph.distance.complete` — N*N distance matrix (the [i,j] element is the length of the shortest path between points i and j)
- `data.grid.distance` — N*M distance matrix between locations where the datum has been observed and locations where the datum has to be predicted
- `method.analysis` — "Local mean" to predict just with the mean, "Kriging" to predict via Kriging procedure
- `metric_manifold` — Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot" and "Correlation"
- `metric_ts` — Metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
- `model_ts` — Type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
- `vario_model` — Type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
- `distance` — Type of distance between coordinates. It must be either "Eucldist" or "Geodist"

Details

...

Value

According to the analysis chosen:

- If method.analysis = "Local mean" it returns a list with the following fields
  - fmean list of length B. Each field contains the prediction (at iteration b) for each new location, obtained as the intrinsic mean of the data within the tile it belongs to
  - kervalues_mean Weights used for aggregating fmean
- If method.analysis = "Kriging" it returns a list with the following fields
  - fmean list of length B. Each field contains the prediction (at iteration b) for each new location, obtained as the intrinsic mean of the data within the tile it belongs to
  - fpred list of length B. Each field contains the prediction (at iteration b) for each new location, obtained through kriging
  - kervalues_mean Weights used for aggregating fmean
  - kervalues_krig Weights used for aggregating fpred
  - variofit list of length B. Each field contains, for each datum, the parameters of the variogram fitted in the tile it belongs to

---

RDD_OOK_boot_man_mixed

Main routine for mixed_RDD

Description

...

Usage

```
RDD_OOK_boot_man_mixed(data_coords, data_val, K, grid, nk_min, B,
  suppressMes, ker.width.intrinsic, graph.distance.complete,
  data.grid.distance, metric_ts, vario_model, tol, max_it, n_h,
  tolerance_intrinsic, X, X_new, metric_manifold, model_ts, distance)
```

Arguments

data_coords	N*2 or N*3 matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
data_val	array [p,p,N] of N symmetric positive definite matrices of dimension p*p
K	number of cells the domain is subdivided in
grid	prediction grid, i.e. M*2 or M*3 matrix of coordinates where to predict
nk_min	minimum number of observations within a cell
B	number of *divide* iterations to perform
suppressMes	{TRUE, FALSE} controls the level of interaction and warnings given
ker.width.intrinsic	parameter controlling the width of the Gaussian kernel for the computation of the local mean (if 0, a "step kernel" is used, giving weight 1 to all the data within the cell and 0 to those outside of it)

graph.distance.complete
: N*N distance matrix (the [i,j] element is the length of the shortest path between points i and j)

data.grid.distance
: N*M distance matrix between locations where the datum has been observed and locations where the datum has to be predicted

metric_ts
: metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"

vario_model
: type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"

tol
: tolerance for the main loop of `model_kriging`

max_it
: maximum number of iterations for the main loop of `model_kriging`

n_h
: number of bins in the empirical variogram

tolerance_intrinsic
: tolerance for the computation of the intrinsic mean

X
: matrix (N rows and unrestricted number of columns) of additional covariates for the tangent space model, possibly NULL

X_new
: matrix (with the same number of rows of `new_coords`) of additional covariates for the new locations, possibly NULL

metric_manifold
: metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot"

model_ts
: type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"

distance
: type of distance between coordinates. It must be either "Eucldist" or "Geodist"

Details

...

Value

it returns a list with the following fields

- `fmean` list of length `B`. Each field contains the prediction (at iteration `b`) for each location, obtained as the intrinsic mean of the data within the tile it belongs to
- `kervalues_mean` Weights used for aggregating `fmean`

`return_ith_list_element` *Return a given element of a list*

Description

Return a given element of a list

Usage

```
return_ith_list_element(lista, i)
```

Arguments

`lista`	A list
`i`	The index of the element to extract

Value

It returns the i-th element of `lista`

`return_ith_row` *Return a given row of a matrix*

Description

Return a given row of a matrix

Usage

```
return_ith_row(mat, i)
```

Arguments

`mat`	A matrix
`i`	The index of the row to extract

Value

It returns the i-th row of `mat`

Index