Kriging prediction for manifold-valued data

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## **Chapter 1**

## What is Manifoldgstat Package

R Package to make inference and prediction for manifold-valued data analysis. This package provides a C++ implementation of the functions to create a model, for spatial dependent manifold valued data, in order to perform kriging. In each location, specified by a vector of coordinates ([lat,long], [x,y] or [x,y,z]), the datum is supposed to be a symmetric and positive definite matrix (possibly a correlation matrix). The algorithm exploits a projection of these data on a tangent space, where the tangent point is either provided by the user or computed as intrinsic mean of the data in input.

#### References

JD. Pigoli, A. Menafoglio & P. Secchi (2016). Kriging prediction for manifold-valued random fields. Journal of Multivariate Analysis, 145, 117-131.

# **Chapter 2**

## **Hierarchical Index**

## 2.1 Class Hierarchy

This inheritance list is sorted roughly, but not completely, alphabetically:

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## **Chapter 4**

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## **Chapter 5**

## **Class Documentation**

## 5.1 Coordinates Class Reference

Class to store the coordinates of the data points.

```
#include <Coordinates.hpp>
```

#### **Public Member Functions**

- Coordinates (const std::shared\_ptr< const MatrixXd > coords)
   Constructor.
- unsigned int get\_N\_station () const

Return the number of data points in the analysis.

- const std::shared\_ptr< const MatrixXd > get\_coords () const
  - Return the matrix of the coordinates.

unsigned int get\_n\_coords () const

Return the number of coordinates for each data point.

## **Private Attributes**

const std::shared\_ptr< const MatrixXd > \_coords

#### 5.1.1 Detailed Description

Class to store the coordinates of the data points.

#### 5.1.2 Constructor & Destructor Documentation

## 5.1.2.1 Coordinates()

```
Coordinates::Coordinates ( {\tt const \ std::shared\_ptr<\ const \ MatrixXd > coords}\ ) \quad [inline]
```

## Constructor.

#### **Parameters**

coords Matrix co	ntaing the coordinates of the data points
------------------	---

#### 5.1.3 Member Data Documentation

#### 5.1.3.1 \_coords

```
const std::shared_ptr<const MatrixXd> Coordinates::_coords [private]
```

Matrix of the coordinates

The documentation for this class was generated from the following files:

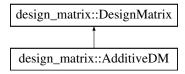
- · Coordinates.hpp
- · Coordinates.cpp

## 5.2 design\_matrix::AdditiveDM Class Reference

Class for the computation of the design\_matrix when model\_ts=="Additive"

```
#include <DesignMatrix.hpp>
```

Inheritance diagram for design\_matrix::AdditiveDM:



## **Public Member Functions**

- MatrixXd compute\_design\_matrix (const Coordinates &coords) const override
  - Compute the design matrix with no covariates besides the coordinates.
- MatrixXd compute\_design\_matrix (const Coordinates &coords, const MatrixXd &X) const override
   Compute the design matrix with extra covariates besides the coordinates.
- ∼AdditiveDM ()=default

Destructor.

## 5.2.1 Detailed Description

Class for the computation of the design\_matrix when model\_ts=="Additive"

#### 5.2.2 Member Function Documentation

#### 5.2.2.1 compute\_design\_matrix() [1/2]

Compute the design matrix with no covariates besides the coordinates.

#### **Parameters**

coords Matrix of the coordinates of the loca
--

#### **Returns**

Design matrix

Implements design\_matrix::DesignMatrix.

### 5.2.2.2 compute\_design\_matrix() [2/2]

Compute the design matrix with extra covariates besides the coordinates.

#### **Parameters**

coords	Matrix of the coordinates of the locations
X	Matrix of the additional covariates for the locations

#### Returns

Design matrix

 $Implements\ design\_matrix:: DesignMatrix.$ 

The documentation for this class was generated from the following files:

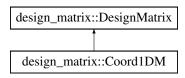
- DesignMatrix.hpp
- DesignMatrix.cpp

## 5.3 design\_matrix::Coord1DM Class Reference

Class for the computation of the design\_matrix when model\_ts=="Coord1"

```
#include <DesignMatrix.hpp>
```

Inheritance diagram for design\_matrix::Coord1DM:



#### **Public Member Functions**

- MatrixXd compute\_design\_matrix (const Coordinates &coords) const override

  Compute the design matrix with no covariates besides the coordinates.
- MatrixXd compute\_design\_matrix (const Coordinates &coords, const MatrixXd &X) const override
   Compute the design matrix with extra covariates besides the coordinates.
- ~Coord1DM ()=default
   Destructor.

## 5.3.1 Detailed Description

Class for the computation of the design\_matrix when model\_ts=="Coord1"

#### 5.3.2 Member Function Documentation

```
5.3.2.1 compute_design_matrix() [1/2]
```

Compute the design matrix with no covariates besides the coordinates.

#### **Parameters**

coords Matrix of the coordinates of the locations
---

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

#### 5.3.2.2 compute\_design\_matrix() [2/2]

Compute the design matrix with extra covariates besides the coordinates.

#### **Parameters**

coords	Matrix of the coordinates of the locations
Χ	Matrix of the additional covariates for the locations

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

The documentation for this class was generated from the following files:

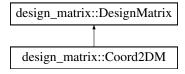
- DesignMatrix.hpp
- DesignMatrix.cpp

## 5.4 design\_matrix::Coord2DM Class Reference

Class for the computation of the design\_matrix when model\_ts=="Coord2"

```
#include <DesignMatrix.hpp>
```

Inheritance diagram for design\_matrix::Coord2DM:



## **Public Member Functions**

- MatrixXd compute\_design\_matrix (const Coordinates &coords) const override
   Compute the design matrix with no covariates besides the coordinates.
- MatrixXd compute\_design\_matrix (const Coordinates &coords, const MatrixXd &X) const override
   Compute the design matrix with extra covariates besides the coordinates.
- Coord2DM ()=default

Destructor.

## 5.4.1 Detailed Description

Class for the computation of the design\_matrix when model\_ts=="Coord2"

#### 5.4.2 Member Function Documentation

```
5.4.2.1 compute_design_matrix() [1/2]
```

Compute the design matrix with no covariates besides the coordinates.

#### **Parameters**

coords	Matrix of the coordinates of the locations
--------	--

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

#### 5.4.2.2 compute\_design\_matrix() [2/2]

Compute the design matrix with extra covariates besides the coordinates.

## **Parameters**

coords	Matrix of the coordinates of the locations
X	Matrix of the additional covariates for the locations

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

The documentation for this class was generated from the following files:

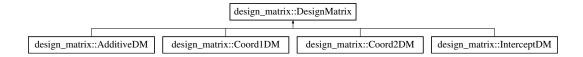
- DesignMatrix.hpp
- DesignMatrix.cpp

## 5.5 design\_matrix::DesignMatrix Class Reference

Abstract class for the computation of the design\_matrix.

```
#include <DesignMatrix.hpp>
```

Inheritance diagram for design\_matrix::DesignMatrix:



#### **Public Member Functions**

- virtual MatrixXd compute\_design\_matrix (const Coordinates &coords) const =0

  Compute the design matrix with no covariates besides the coordinates.
- virtual MatrixXd compute\_design\_matrix (const Coordinates &coords, const MatrixXd &X) const =0
   Compute the design matrix with extra covariates besides the coordinates.
- virtual ~DesignMatrix ()=default Destructor.

### 5.5.1 Detailed Description

Abstract class for the computation of the design matrix.

#### 5.5.2 Member Function Documentation

```
5.5.2.1 compute_design_matrix() [1/2]
```

Compute the design matrix with no covariates besides the coordinates.

## **Parameters**

```
coords Matrix of the coordinates of the locations
```

#### Returns

Design matrix

Implemented in design\_matrix::AdditiveDM, design\_matrix::Coord2DM, design\_matrix::Coord1DM, and design\_matrix::InterceptDM.

#### 5.5.2.2 compute\_design\_matrix() [2/2]

Compute the design matrix with extra covariates besides the coordinates.

#### **Parameters**

coords	Matrix of the coordinates of the locations
Χ	Matrix of the additional covariates for the locations

#### Returns

Design matrix

Implemented in design\_matrix::AdditiveDM, design\_matrix::Coord2DM, design\_matrix::Coord1DM, and design\_matrix::InterceptDM.

The documentation for this class was generated from the following file:

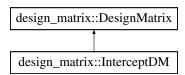
DesignMatrix.hpp

## 5.6 design\_matrix::InterceptDM Class Reference

Class for the computation of the design\_matrix when model\_ts=="Intercept"

```
#include <DesignMatrix.hpp>
```

Inheritance diagram for design\_matrix::InterceptDM:



## **Public Member Functions**

- MatrixXd compute\_design\_matrix (const Coordinates &coords) const override
   Compute the design matrix with no covariates besides the coordinates.
- MatrixXd compute\_design\_matrix (const Coordinates &coords, const MatrixXd &X) const override
   Compute the design matrix with extra covariates besides the coordinates.
- ∼InterceptDM ()=default

Destructor.

## 5.6.1 Detailed Description

Class for the computation of the design\_matrix when model\_ts=="Intercept"

#### 5.6.2 Member Function Documentation

#### 5.6.2.1 compute\_design\_matrix() [1/2]

Compute the design matrix with no covariates besides the coordinates.

#### **Parameters**

coords	Matrix of the coordinates of the locations
--------	--

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

#### 5.6.2.2 compute\_design\_matrix() [2/2]

Compute the design matrix with extra covariates besides the coordinates.

#### **Parameters**

С	oords	Matrix of the coordinates of the locations
λ	(	Matrix of the additional covariates for the locations

#### Returns

Design matrix

Implements design\_matrix::DesignMatrix.

The documentation for this class was generated from the following files:

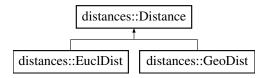
- · DesignMatrix.hpp
- DesignMatrix.cpp

### 5.7 distances::Distance Class Reference

Abstract class for the computation of the distance between data locations.

```
#include <Distance.hpp>
```

Inheritance diagram for distances::Distance:



#### **Public Member Functions**

- virtual double compute\_distance (const Vec &P1, const Vec &P2) const =0
   Compute the distance between two locations.
- std::shared\_ptr< const MatrixXd > create\_distance\_matrix (const Coordinates &coordinates, unsigned int N) const

Compute the distance matrix among a set of locations.

std::vector< double > create\_distance\_vector (const Coordinates &coordinates, const Vec &new\_coord)
 const

Compute the vector of distances between a point and a set of locations.

virtual ∼Distance ()=default

Destructor.

## 5.7.1 Detailed Description

Abstract class for the computation of the distance between data locations.

#### 5.7.2 Member Function Documentation

#### 5.7.2.1 compute\_distance()

Compute the distance between two locations.

#### **Parameters**

P1	Vector of coordinates for the first location
P2	Vector of coordinates for the second location

#### Returns

Points' distance

Implemented in distances::GeoDist, and distances::EuclDist.

#### 5.7.2.2 create\_distance\_matrix()

```
std::shared_ptr< const MatrixXd > Distance::create_distance_matrix ( const Coordinates & coordinates, unsigned int N ) const
```

Compute the distance matrix among a set of locations.

#### **Parameters**

coordinates	Matrix of the coordinates of the locations
N	Number of locations

#### Returns

Matrix of distances

## 5.7.2.3 create\_distance\_vector()

Compute the vector of distances between a point and a set of locations.

#### **Parameters**

coordinates	Matrix of the coordinates of the locations
new_coord	Vector of coordinates of the new location

#### Returns

Vector of distances

The documentation for this class was generated from the following files:

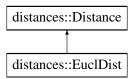
- · Distance.hpp
- Distance.cpp

## 5.8 distances::EuclDist Class Reference

Class for the computation of the distance between data locations when distance=="Eucldist"

```
#include <Distance.hpp>
```

Inheritance diagram for distances::EuclDist:



#### **Public Member Functions**

- double compute\_distance (const Vec &P1, const Vec &P2) const override
   Compute the distance between two locations.
- ~EuclDist ()=default
   Destructor.

## 5.8.1 Detailed Description

Class for the computation of the distance between data locations when distance=="Eucldist"

## 5.8.2 Member Function Documentation

#### 5.8.2.1 compute\_distance()

Compute the distance between two locations.

#### **Parameters**

P1	Vector of coordinates for the first location
P2	Vector of coordinates for the second location

#### Returns

Points' distance

Implements distances::Distance.

The documentation for this class was generated from the following files:

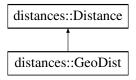
- Distance.hpp
- · Distance.cpp

## 5.9 distances::GeoDist Class Reference

Class for the computation of the distance between data locations when distance=="Geodist"

```
#include <Distance.hpp>
```

Inheritance diagram for distances::GeoDist:



## **Public Member Functions**

- double compute\_distance (const Vec &P1, const Vec &P2) const override
   Compute the distance between two locations.
- ~GeoDist ()=default
   Destructor.

#### **Static Private Attributes**

- static double constexpr Earth R = 6371.0
- static double constexpr eps\_dbl = std::numeric\_limits<double>::epsilon()

### 5.9.1 Detailed Description

Class for the computation of the distance between data locations when distance=="Geodist"

## 5.9.2 Member Function Documentation

## 5.9.2.1 compute\_distance()

Compute the distance between two locations.

#### **Parameters**

P1	Vector of coordinates for the first location
P2	Vector of coordinates for the second location

#### Returns

Points' distance

Implements distances::Distance.

#### 5.9.3 Member Data Documentation

#### 5.9.3.1 Earth\_R

double constexpr distances::GeoDist::Earth\_R = 6371.0 [static], [private]

#### Earth radius

## 5.9.3.2 eps\_dbl

double constexpr distances::GeoDist::eps\_dbl = std::numeric\_limits<double>::epsilon() [static],
[private]

## Machine epsilon

The documentation for this class was generated from the following files:

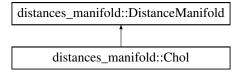
- · Distance.hpp
- · Distance.cpp

## 5.10 distances\_manifold::Chol Class Reference

Class for the computation of the distance on the manifold when manifold\_metric=="Chol"

#include <DistanceManifold.hpp>

Inheritance diagram for distances\_manifold::Chol:



#### **Public Member Functions**

double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const override

Compute the distance between two matrices on the manifold.

∼Chol ()=default

Destructor.

#### 5.10.1 Detailed Description

Class for the computation of the distance on the manifold when manifold\_metric=="Chol"

Note

The data on the manifold must be correlation matrices

#### 5.10.2 Member Function Documentation

### 5.10.2.1 compute\_distance()

Compute the distance between two matrices on the manifold.

#### **Parameters**

M1	First matrix
M2	Second matrix

#### Returns

Distance

Implements distances manifold::DistanceManifold.

The documentation for this class was generated from the following files:

- · DistanceManifold.hpp
- · DistanceManifold.cpp

## 5.11 distances\_manifold::DistanceManifold Class Reference

Abstract class for the computation of the distance on the manifold.

#include <DistanceManifold.hpp>

Inheritance diagram for distances\_manifold::DistanceManifold:



#### **Public Member Functions**

- virtual double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const =0
   Compute the distance between two matrices on the manifold.
- virtual ~DistanceManifold ()=default
   Destructor.

## 5.11.1 Detailed Description

Abstract class for the computation of the distance on the manifold.

#### 5.11.2 Member Function Documentation

#### 5.11.2.1 compute\_distance()

```
virtual double distances_manifold::DistanceManifold::compute_distance ( const MatrixXd & M1, const MatrixXd & M2) const [pure virtual]
```

Compute the distance between two matrices on the manifold.

#### **Parameters**

M1	First matrix
M2	Second matrix

#### Returns

Distance

Implemented in distances\_manifold::Chol, distances\_manifold::SqRoot, distances\_manifold::LogEuclidean, and distances\_manifold::Frobenius.

The documentation for this class was generated from the following file:

• DistanceManifold.hpp

# 5.12 distances\_manifold::Frobenius Class Reference

Class for the computation of the distance on the manifold when manifold\_metric=="Frobenius"

```
#include <DistanceManifold.hpp>
```

Inheritance diagram for distances\_manifold::Frobenius:

```
distances_manifold::DistanceManifold

distances_manifold::Frobenius
```

## **Public Member Functions**

- double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const override
   Compute the distance between two matrices on the manifold.
- ~Frobenius ()=default
   Destructor.

## 5.12.1 Detailed Description

Class for the computation of the distance on the manifold when manifold\_metric=="Frobenius"

## 5.12.2 Member Function Documentation

## 5.12.2.1 compute\_distance()

Compute the distance between two matrices on the manifold.

#### **Parameters**

M1	First matrix
M2	Second matrix

#### Returns

Distance

Implements distances\_manifold::DistanceManifold.

The documentation for this class was generated from the following files:

- DistanceManifold.hpp
- · DistanceManifold.cpp

# 5.13 distances\_manifold::LogEuclidean Class Reference

Class for the computation of the distance on the manifold when manifold\_metric=="LogEuclidean"

```
#include <DistanceManifold.hpp>
```

Inheritance diagram for distances\_manifold::LogEuclidean:

```
distances_manifold::DistanceManifold

distances_manifold::LogEuclidean
```

## **Public Member Functions**

- double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const override
   Compute the distance between two matrices on the manifold.
- ~LogEuclidean ()=default Destructor.

# 5.13.1 Detailed Description

Class for the computation of the distance on the manifold when manifold\_metric=="LogEuclidean"

## 5.13.2 Member Function Documentation

## 5.13.2.1 compute\_distance()

Compute the distance between two matrices on the manifold.

## **Parameters**

M1	First matrix
M2	Second matrix

Returns

Distance

Implements distances\_manifold::DistanceManifold.

The documentation for this class was generated from the following files:

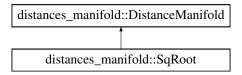
- · DistanceManifold.hpp
- · DistanceManifold.cpp

# 5.14 distances\_manifold::SqRoot Class Reference

Class for the computation of the distance on the manifold when manifold\_metric=="SqRoot"

```
#include <DistanceManifold.hpp>
```

Inheritance diagram for distances manifold::SqRoot:



## **Public Member Functions**

- double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const override
   Compute the distance between two matrices on the manifold.
- ~SqRoot ()=default
   Destructor.

## 5.14.1 Detailed Description

Class for the computation of the distance on the manifold when manifold\_metric=="SqRoot"

#### 5.14.2 Member Function Documentation

## 5.14.2.1 compute\_distance()

Compute the distance between two matrices on the manifold.

#### **Parameters**

M1	First matrix
M2	Second matrix

#### Returns

Distance

Implements distances manifold::DistanceManifold.

The documentation for this class was generated from the following files:

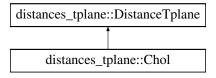
- DistanceManifold.hpp
- · DistanceManifold.cpp

# 5.15 distances\_tplane::Chol Class Reference

Class for the computation of the distance on the tangent space when ts\_metric=="Correlation"

```
#include <DistanceTplane.hpp>
```

Inheritance diagram for distances\_tplane::Chol:



## **Public Member Functions**

∼Chol ()=default

Destructor.

• double norm (const MatrixXd &M1) const override

Compute the norm of a matrix on the tangent space.

void set\_members (const MatrixXd &Sigma) override

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

## 5.15.1 Detailed Description

Class for the computation of the distance on the tangent space when ts\_metric=="Correlation"

## 5.15.2 Member Function Documentation

```
5.15.2.1 norm()
```

Compute the norm of a matrix on the tangent space.

M1	Matrix
	i i i i a ci i i i

## Returns

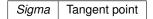
Norm

Implements distances\_tplane::DistanceTplane.

## 5.15.2.2 set\_members()

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

## **Parameters**



Implements distances\_tplane::DistanceTplane.

The documentation for this class was generated from the following files:

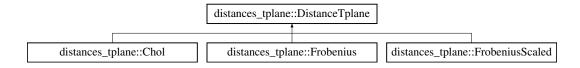
- · DistanceTplane.hpp
- · DistanceTplane.cpp

# 5.16 distances\_tplane::DistanceTplane Class Reference

Abstract class for the computation of the distance on the tangent space.

```
#include <DistanceTplane.hpp>
```

Inheritance diagram for distances\_tplane::DistanceTplane:



## **Public Member Functions**

double compute\_distance (const MatrixXd &M1, const MatrixXd &M2) const

Compute the distance between two matrices on the tangent space.

• virtual double norm (const MatrixXd &M1) const =0

Compute the norm of a matrix on the tangent space.

• virtual void set\_members (const MatrixXd &Sigma)=0

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

virtual ∼DistanceTplane ()=default

Destructor.

## 5.16.1 Detailed Description

Abstract class for the computation of the distance on the tangent space.

## 5.16.2 Member Function Documentation

## 5.16.2.1 compute\_distance()

Compute the distance between two matrices on the tangent space.

## **Parameters**

M1	First matrix
M2	Second matrix

#### Returns

Distance

## 5.16.2.2 norm()

Compute the norm of a matrix on the tangent space.

M1	Matrix
	i i i i a ci i i i

## Returns

Norm

 $Implemented \ in \ distances\_tplane::FrobeniusScaled, \ and \ an$ 

## 5.16.2.3 set\_members()

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

#### **Parameters**



Implemented in distances\_tplane::Chol, distances\_tplane::FrobeniusScaled, and distances\_tplane::Frobenius.

The documentation for this class was generated from the following files:

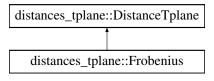
- DistanceTplane.hpp
- · DistanceTplane.cpp

# 5.17 distances\_tplane::Frobenius Class Reference

Class for the computation of the distance on the tangent space when  $ts\_metric=="Frobenius"$ 

```
#include <DistanceTplane.hpp>
```

Inheritance diagram for distances\_tplane::Frobenius:



## **Public Member Functions**

∼Frobenius ()=default

Destructor.

• double norm (const MatrixXd &M1) const override

Compute the norm of a matrix on the tangent space.

• void set\_members (const MatrixXd &Sigma) override

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

## 5.17.1 Detailed Description

Class for the computation of the distance on the tangent space when ts\_metric=="Frobenius"

## 5.17.2 Member Function Documentation

```
5.17.2.1 norm()
```

Compute the norm of a matrix on the tangent space.

## **Parameters**

```
M1 Matrix
```

## Returns

Norm

Implements distances tplane::DistanceTplane.

## 5.17.2.2 set\_members()

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

Sigma	Tangent point
-------	---------------

Implements distances\_tplane::DistanceTplane.

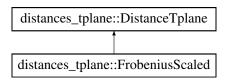
The documentation for this class was generated from the following files:

- DistanceTplane.hpp
- · DistanceTplane.cpp

# 5.18 distances\_tplane::FrobeniusScaled Class Reference

Class for the computation of the distance on the tangent space when ts\_metric=="FrobeniusScaled" #include <DistanceTplane.hpp>

Inheritance diagram for distances\_tplane::FrobeniusScaled:



## **Public Member Functions**

∼FrobeniusScaled ()=default

Destructor.

• double norm (const MatrixXd &M1) const override

Compute the norm of a matrix on the tangent space.

· void set members (const MatrixXd &Sigma) override

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

## **Private Attributes**

- MatrixXd SigmaInv
- unsigned int \_p

## 5.18.1 Detailed Description

Class for the computation of the distance on the tangent space when ts\_metric=="FrobeniusScaled"

## 5.18.2 Member Function Documentation

## 5.18.2.1 norm()

Compute the norm of a matrix on the tangent space.

## **Parameters**

M1 N	∕latrix
------	---------

## Returns

Norm

Implements distances\_tplane::DistanceTplane.

## 5.18.2.2 set\_members()

Set the members that will be used in the computation of the norms and distances, according to the metric on the tangent space.

## **Parameters**



Implements distances\_tplane::DistanceTplane.

## 5.18.3 Member Data Documentation

```
5.18.3.1 _SigmaInv
```

```
MatrixXd distances_tplane::FrobeniusScaled::_SigmaInv [private]
```

Inverse of the tangent point Sigma

```
5.18.3.2 _p
```

```
unsigned int distances_tplane::FrobeniusScaled::_p [private]
```

Dimension of the matrices on the tangent space

The documentation for this class was generated from the following files:

- DistanceTplane.hpp
- DistanceTplane.cpp

# 5.19 generic\_factory::Factory< AbstractProduct, Identifier, Builder > Class Template Reference

## A generic factory.

```
#include <Factory.hpp>
```

## **Public Types**

using AbstractProduct\_type = AbstractProduct

The container for the rules.

• using Identifier\_type = Identifier

The identifier.

• using Builder\_type = Builder

The builder type.

## **Public Member Functions**

• std::unique\_ptr< AbstractProduct > create (Identifier const &name) const

Get the rule with given name.

void add (Identifier const &, Builder\_type const &)

Register the given rule.

std::vector< Identifier > registered () const

Returns a list of registered rules.

void unregister (Identifier const &name)

Unregister a rule.

∼Factory ()=default

Destructor.

## **Static Public Member Functions**

• static Factory & Instance ()

Method to access the only instance of the factory. We use Meyer's trick to istantiate the factory.

## **Private Types**

typedef std::map< Identifier, Builder\_type > Container\_type

Type of the object used to store the object factory.

## **Private Member Functions**

• Factory ()=default

Constructor made private since it is a Singleton.

Factory (Factory const &)=delete

Copy constructor deleted since it is a Singleton.

• Factory & operator= (Factory const &)=delete

Assignment operator deleted since it is a Singleton.

## **Private Attributes**

· Container\_type \_storage

It contains the actual object factory.

## 5.19.1 Detailed Description

 $template < typename\ AbstractProduct,\ typename\ Identifier,\ typename\ Builder = std::function < std::unique\_ptr < AbstractProduct > () >>$ 

class generic\_factory::Factory < AbstractProduct, Identifier, Builder >

A generic factory.

It is implemented as a Singleton. The compulsory way to access a method is Factory::Instance().method(). Typycally to access the factory one does

```
auto& myFactory = Factory<A,I,B>::Instance();
myFactory.add(...)
```

#### 5.19.2 Member Function Documentation

## 5.19.2.1 create()

Get the rule with given name.

The pointer is null if no rule is present.

The documentation for this class was generated from the following file:

· Factory.hpp

# 5.20 generic\_factory::Proxy < Factory, ConcreteProduct > Class Template Reference

```
#include <Proxy.hpp>
```

## **Public Types**

typedef Factory::AbstractProduct\_type AbstractProduct\_type

The container for the rules.

typedef Factory::ldentifier\_type Identifier\_type

The identifier.

· typedef Factory::Builder\_type Builder\_type

The builder type.

typedef Factory Factory\_type

The factory type.

## **Public Member Functions**

• Proxy (Identifier\_type const &)

The constructor does the registration.

## **Static Public Member Functions**

static std::unique\_ptr< AbstractProduct\_type > Build ()
 The builder.

#### **Private Member Functions**

• Proxy (Proxy const &)=delete

Copy onstructor deleted since it is a Singleton.

Proxy & operator= (Proxy const &)=delete

Assignment operator deleted since it is a Singleton.

## 5.20.1 Detailed Description

```
{\it template}{<} {\it typename Factory, typename ConcreteProduct}{>} \\ {\it class generic\_factory::Proxy}{<} {\it Factory, ConcreteProduct}{>} \\ {\it typename ConcreteProduct}{>} \\ {\it
```

A simple proxy for registering into a factory.

It provides the builder as static method and the automatic registration mechanism.

## **Parameters**

Factory	The type of the factory.
ConcreteProduct	Is the derived (concrete) type to be registered in the factory

## Note

I have to use the default builder provided by the factory. No check is made to verify it

The documentation for this class was generated from the following file:

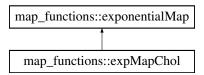
• Proxy.hpp

# 5.21 map\_functions::expMapChol Class Reference

Class for the computation of the exponential map when manifold\_metric=="Correlation"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::expMapChol:



#### **Public Member Functions**

∼expMapChol ()=default

Destructor.

• MatrixXd map2manifold (const MatrixXd &M) const override

Map a tangent space matrix to the manifold.

• void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

- double \_tolerance\_map\_cor
- MatrixXd \_Sigma

## 5.21.1 Detailed Description

Class for the computation of the exponential map when manifold\_metric=="Correlation"

#### 5.21.2 Member Function Documentation

## 5.21.2.1 map2manifold()

```
\label{eq:matrixXd} $$\operatorname{MatrixXd} \exp\operatorname{MapChol}:::map2manifold ($$\operatorname{const} \operatorname{MatrixXd} \& M$)$ const [override], [virtual]
```

Map a tangent space matrix to the manifold.

```
M Tangent space matrix to map
```

## Returns

Manifold matrix identifying the mapped data

Implements map\_functions::exponentialMap.

## 5.21.2.2 set\_members()

Set class members.

## **Parameters**

oigina   Tangoni ponii	Sigma	Tangent point
------------------------	-------	---------------

Implements map\_functions::exponentialMap.

# 5.21.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

 $Implements\ map\_functions:: exponential Map.$ 

# 5.21.3 Member Data Documentation

## 5.21.3.1 \_tolerance\_map\_cor

```
double map_functions::expMapChol::_tolerance_map_cor [private]
```

Tolerance on the norm of the columns to avoid Nan

## 5.21.3.2 \_Sigma

```
MatrixXd map_functions::expMapChol::_Sigma [private]
```

Tangent point Sigma

The documentation for this class was generated from the following files:

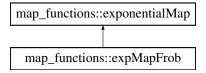
- MapFunctions.hpp
- · MapFunctions.cpp

# 5.22 map\_functions::expMapFrob Class Reference

Class for the computation of the exponential map when manifold\_metric=="Frobenius"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map functions::expMapFrob:



## **Public Member Functions**

∼expMapFrob ()=default

Destructor.

MatrixXd map2manifold (const MatrixXd &M) const override

Map a tangent space matrix to the manifold.

· void set\_members (const MatrixXd &Sigma) override

Set class members.

· void set tolerance (double tolerance map cor) override

Set tolerance.

## **Private Attributes**

- MatrixXd \_sqrtSigma
- MatrixXd \_sqrtSigmaInv

## 5.22.1 Detailed Description

Class for the computation of the exponential map when manifold\_metric=="Frobenius"

## 5.22.2 Member Function Documentation

## 5.22.2.1 map2manifold()

Map a tangent space matrix to the manifold.

M Tangent space matrix to map

## Returns

Manifold matrix identifying the mapped data

Implements map\_functions::exponentialMap.

## 5.22.2.2 set\_members()

Set class members.

## **Parameters**

Sigma   Tangent point	Sigma	Tangent point
-----------------------	-------	---------------

Implements map\_functions::exponentialMap.

## 5.22.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

 $Implements\ map\_functions:: exponential Map.$ 

## 5.22.3 Member Data Documentation

## 5.22.3.1 \_sqrtSigma

MatrixXd map\_functions::expMapFrob::\_sqrtSigma [private]

Square root of the tangent point Sigma

## 5.22.3.2 \_sqrtSigmaInv

```
MatrixXd map_functions::expMapFrob::_sqrtSigmaInv [private]
```

Inverse of the square root of the tangent point Sigma

The documentation for this class was generated from the following files:

- MapFunctions.hpp
- MapFunctions.cpp

# 5.23 map\_functions::expMapLogEucl Class Reference

Class for the computation of the exponential map when manifold\_metric=="LogEuclidean"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::expMapLogEucl:

```
map_functions::exponentialMap

map_functions::expMapLogEucl
```

## **Public Member Functions**

∼expMapLogEucl ()=default

Destructor.

MatrixXd map2manifold (const MatrixXd &M) const override

Map a tangent space matrix to the manifold.

void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

· MatrixXd Sigma

## 5.23.1 Detailed Description

Class for the computation of the exponential map when manifold\_metric=="LogEuclidean"

## 5.23.2 Member Function Documentation

## 5.23.2.1 map2manifold()

Map a tangent space matrix to the manifold.

```
M Tangent space matrix to map
```

## Returns

Manifold matrix identifying the mapped data

Implements map\_functions::exponentialMap.

## 5.23.2.2 set\_members()

Set class members.

## **Parameters**

Sigma   Tangent point
-----------------------

Implements map\_functions::exponentialMap.

## 5.23.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

Implements map\_functions::exponentialMap.

## 5.23.3 Member Data Documentation

## 5.23.3.1 \_Sigma

MatrixXd map\_functions::expMapLogEucl::\_Sigma [private]

Tangent point Sigma

The documentation for this class was generated from the following files:

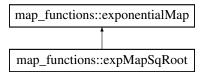
- MapFunctions.hpp
- · MapFunctions.cpp

# 5.24 map\_functions::expMapSqRoot Class Reference

Class for the computation of the exponential map when manifold\_metric=="SqRoot"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map functions::expMapSqRoot:



## **Public Member Functions**

∼expMapSqRoot ()=default

Destructor.

MatrixXd map2manifold (const MatrixXd &M) const override

Map a tangent space matrix to the manifold.

· void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

MatrixXd \_Sigma

# 5.24.1 Detailed Description

Class for the computation of the exponential map when manifold\_metric=="SqRoot"

## 5.24.2 Member Function Documentation

## 5.24.2.1 map2manifold()

Map a tangent space matrix to the manifold.

M Tangent space matrix to map

## Returns

Manifold matrix identifying the mapped data

Implements map\_functions::exponentialMap.

## 5.24.2.2 set\_members()

Set class members.

## **Parameters**

Sigma   Tangent point
-----------------------

Implements map\_functions::exponentialMap.

## 5.24.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

Implements map\_functions::exponentialMap.

## 5.24.3 Member Data Documentation

## 5.24.3.1 \_Sigma

MatrixXd map\_functions::expMapSqRoot::\_Sigma [private]

Tangent point Sigma

The documentation for this class was generated from the following files:

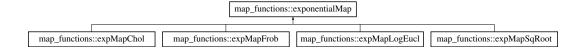
- MapFunctions.hpp
- · MapFunctions.cpp

# 5.25 map\_functions::exponentialMap Class Reference

Abstract class for the computation of the exponential map.

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::exponentialMap:



## **Public Member Functions**

virtual ∼exponentialMap ()=default

Destructor.

virtual MatrixXd map2manifold (const MatrixXd &M) const =0

Map a tangent space matrix to the manifold.

virtual void set\_members (const MatrixXd &Sigma)=0

Set class members.

virtual void set\_tolerance (double tolerance\_map\_cor)=0
 Set tolerance.

# 5.25.1 Detailed Description

Abstract class for the computation of the exponential map.

## 5.25.2 Member Function Documentation

#### 5.25.2.1 map2manifold()

Map a tangent space matrix to the manifold.

M Tangent space matrix to map

#### Returns

Manifold matrix identifying the mapped data

 $Implemented \ in \ map\_functions::expMapChol, \ map\_functions::expMapSqRoot, \ map\_functions::expMapLogEucl, \ and \ map\_functions::expMapFrob.$ 

## 5.25.2.2 set\_members()

Set class members.

#### **Parameters**

 $Implemented \ in \ map\_functions::expMapChol, \ map\_functions::expMapSqRoot, \ map\_functions::expMapLogEucl, \ and \ map\_functions::expMapFrob.$ 

#### 5.25.2.3 set tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor | Tolerance
```

 $Implemented \ in \ map\_functions::expMapChol, \ map\_functions::expMapSqRoot, \ map\_functions::expMapLogEucl, \ and \ map\_functions::expMapFrob.$ 

The documentation for this class was generated from the following file:

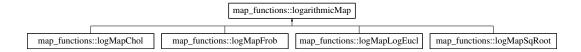
MapFunctions.hpp

# 5.26 map\_functions::logarithmicMap Class Reference

Abstract class for the computation of the logarithmic map.

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::logarithmicMap:



#### **Public Member Functions**

virtual ∼logarithmicMap ()=default

Destructor.

- virtual MatrixXd map2tplane (const MatrixXd &M) const =0
  - Map a manifold matrix to the tangent space.
- virtual void set\_members (const MatrixXd &Sigma)=0

Set class members.

virtual void set\_tolerance (double tolerance\_map\_cor)=0
 Set tolerance.

## 5.26.1 Detailed Description

Abstract class for the computation of the logarithmic map.

## 5.26.2 Member Function Documentation

## 5.26.2.1 map2tplane()

Map a manifold matrix to the tangent space.

## **Parameters**

M Manifold matrix to map

## Returns

Tangent space matrix identifying the mapped data

Implemented in map\_functions::logMapChol, map\_functions::logMapSqRoot, map\_functions::logMapLogEucl, and map\_functions::logMapFrob.

#### 5.26.2.2 set\_members()

Set class members.

#### **Parameters**

Sigma	Tangent point
-------	---------------

Implemented in map\_functions::logMapChol, map\_functions::logMapSqRoot, map\_functions::logMapLogEucl, and map\_functions::logMapFrob.

## 5.26.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

tolerance_map_cor	Tolerance

Implemented in map\_functions::logMapChol, map\_functions::logMapSqRoot, map\_functions::logMapLogEucl, and map\_functions::logMapFrob.

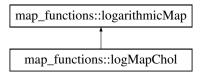
The documentation for this class was generated from the following file:

• MapFunctions.hpp

# 5.27 map\_functions::logMapChol Class Reference

Class for the computation of the logarithmic map when manifold\_metric=="Correlation" #include <MapFunctions.hpp>

Inheritance diagram for map\_functions::logMapChol:



## **Public Member Functions**

∼logMapChol ()=default

Destructor.

• MatrixXd map2tplane (const MatrixXd &M) const override

Map a manifold matrix to the tangent space.

• void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Member Functions**

• Vec proj2tspace (const Vec &, const Vec &) const Project a matrix in Chol(p) to the tangent space.

## **Private Attributes**

- MatrixXd \_Sigma
- double \_tolerance\_map\_cor

## 5.27.1 Detailed Description

Class for the computation of the logarithmic map when manifold\_metric=="Correlation"

## 5.27.2 Member Function Documentation

## 5.27.2.1 map2tplane()

Map a manifold matrix to the tangent space.

#### **Parameters**

M Manifold matrix to map

## Returns

Tangent space matrix identifying the mapped data

Implements map\_functions::logarithmicMap.

## 5.27.2.2 set\_members()

Set class members.

**Parameters** 

```
Sigma Tangent point
```

Implements map\_functions::logarithmicMap.

## 5.27.2.3 set\_tolerance()

Set tolerance.

**Parameters** 

```
tolerance_map_cor Tolerance
```

Implements map\_functions::logarithmicMap.

## 5.27.3 Member Data Documentation

## 5.27.3.1 \_Sigma

```
MatrixXd map_functions::logMapChol::_Sigma [private]
```

Tangent point Sigma

# 5.27.3.2 \_tolerance\_map\_cor

```
double map_functions::logMapChol::_tolerance_map_cor [private]
```

Tolerance on the norm of the columns to avoid Nan

The documentation for this class was generated from the following files:

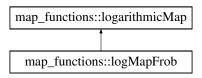
- MapFunctions.hpp
- MapFunctions.cpp

# 5.28 map\_functions::logMapFrob Class Reference

Class for the computation of the logarithmic map when manifold\_metric="Frobenius"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::logMapFrob:



## **Public Member Functions**

∼logMapFrob ()=default

Destructor.

MatrixXd map2tplane (const MatrixXd &M) const override

Map a manifold matrix to the tangent space.

void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

- MatrixXd \_sqrtSigma
- MatrixXd \_sqrtSigmaInv

# 5.28.1 Detailed Description

Class for the computation of the logarithmic map when manifold\_metric=="Frobenius"

## 5.28.2 Member Function Documentation

## 5.28.2.1 map2tplane()

Map a manifold matrix to the tangent space.

```
M Manifold matrix to map
```

## Returns

Tangent space matrix identifying the mapped data

Implements map\_functions::logarithmicMap.

## 5.28.2.2 set\_members()

Set class members.

## **Parameters**

Sigiria   Tangent point	Sigma	Tangent point
-------------------------	-------	---------------

Implements map\_functions::logarithmicMap.

# 5.28.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

 $Implements\ map\_functions::logarithmic Map.$ 

# 5.28.3 Member Data Documentation

## 5.28.3.1 \_sqrtSigma

MatrixXd map\_functions::logMapFrob::\_sqrtSigma [private]

Square root of the tangent point Sigma

## 5.28.3.2 \_sqrtSigmaInv

```
MatrixXd map_functions::logMapFrob::_sqrtSigmaInv [private]
```

Inverse of the square root of the tangent point Sigma

The documentation for this class was generated from the following files:

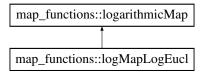
- MapFunctions.hpp
- MapFunctions.cpp

# 5.29 map\_functions::logMapLogEucl Class Reference

Class for the computation of the logarithmic map when  $manifold\_metric == "LogEuclidean"$ 

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::logMapLogEucl:



## **Public Member Functions**

• ~logMapLogEucl ()=default

Destructor.

MatrixXd map2tplane (const MatrixXd &M) const override

Map a manifold matrix to the tangent space.

void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

· MatrixXd Sigma

## 5.29.1 Detailed Description

Class for the computation of the logarithmic map when manifold\_metric=="LogEuclidean"

## 5.29.2 Member Function Documentation

## 5.29.2.1 map2tplane()

Map a manifold matrix to the tangent space.

M Manifold matrix to map

## Returns

Tangent space matrix identifying the mapped data

Implements map\_functions::logarithmicMap.

## 5.29.2.2 set\_members()

Set class members.

## **Parameters**

Sigma Tangent p	ooint
-----------------	-------

Implements map\_functions::logarithmicMap.

## 5.29.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

Implements map\_functions::logarithmicMap.

## 5.29.3 Member Data Documentation

## 5.29.3.1 \_Sigma

```
MatrixXd map_functions::logMapLogEucl::_Sigma [private]
```

Tangent point Sigma

The documentation for this class was generated from the following files:

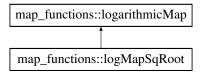
- MapFunctions.hpp
- · MapFunctions.cpp

# 5.30 map\_functions::logMapSqRoot Class Reference

Class for the computation of the logarithmic map when manifold\_metric=="SqRoot"

```
#include <MapFunctions.hpp>
```

Inheritance diagram for map\_functions::logMapSqRoot:



## **Public Member Functions**

∼logMapSqRoot ()=default

Destructor.

MatrixXd map2tplane (const MatrixXd &M) const override

Map a manifold matrix to the tangent space.

void set\_members (const MatrixXd &Sigma) override

Set class members.

 void set\_tolerance (double tolerance\_map\_cor) override Set tolerance.

## **Private Attributes**

· MatrixXd Sigma

## 5.30.1 Detailed Description

Class for the computation of the logarithmic map when manifold\_metric=="SqRoot"

## 5.30.2 Member Function Documentation

# 5.30.2.1 map2tplane()

```
\label{eq:matrixXd} $$\operatorname{MatrixXd logMapSqRoot::map2tplane (} $$\operatorname{const MatrixXd \& $M$ ) const [override], [virtual]$}
```

Map a manifold matrix to the tangent space.

M Manifold matrix to map

## Returns

Tangent space matrix identifying the mapped data

Implements map\_functions::logarithmicMap.

## 5.30.2.2 set\_members()

Set class members.

## **Parameters**

Sigma Tangent point
---------------------

 $Implements\ map\_functions::logarithmic Map.$ 

## 5.30.2.3 set\_tolerance()

Set tolerance.

## **Parameters**

```
tolerance_map_cor Tolerance
```

Implements map\_functions::logarithmicMap.

## 5.30.3 Member Data Documentation

## 5.30.3.1 \_Sigma

```
MatrixXd map_functions::logMapSqRoot::_Sigma [private]
```

Tangent point Sigma

The documentation for this class was generated from the following files:

- MapFunctions.hpp
- MapFunctions.cpp

# 5.31 model\_fit::Model Class Reference

Class to compute and store the linear model on the tangent space.

```
#include <Model.hpp>
```

## **Public Member Functions**

Model (const std::shared\_ptr< const MatrixXd > data\_tspace, const std::shared\_ptr< const MatrixXd > design\_matrix\_model, unsigned int p, const std::string &distance\_Manifold\_name)

Constructor.

Model (const std::shared\_ptr< const MatrixXd > data\_tspace, const std::shared\_ptr< const MatrixXd > design\_matrix\_model, const std::shared\_ptr< const MatrixXd > design\_matrix\_tot, unsigned int p, const std::string &distance\_Manifold\_name)

Constructor.

void update\_model (const MatrixXd &gamma\_matrix)

Update\_beta\_matrix,\_fitted\_values and\_residuals according to the new covariogram matrix.

· MatrixXd get beta () const

**Return**\_beta\_matrix

MatrixXd get\_residuals () const

Return\_residuals

MatrixXd get\_fitted\_values () const

```
Return_fitted_values
```

## **Private Attributes**

- const std::shared\_ptr< const MatrixXd > \_data\_tspace
- const std::shared\_ptr< const MatrixXd > \_design\_matrix\_model
- const std::shared\_ptr< const MatrixXd > \_design\_matrix\_tot
- const std::string \_distance\_Manifold\_name
- · const unsigned int \_N
- const unsigned int \_p
- const unsigned int \_num\_cov
- const unsigned int \_num\_coeff
- MatrixXd <u>beta\_matrix</u>
- · MatrixXd fitted values
- MatrixXd <u>residuals</u>

## 5.31.1 Detailed Description

Class to compute and store the linear model on the tangent space.

## 5.31.2 Member Function Documentation

## 5.31.2.1 update\_model()

Update \_beta\_matrix, \_fitted\_values and \_residuals according to the new covariogram matrix.

#### **Parameters**

gamma_matrix	Covariogram matrix	
--------------	--------------------	--

#### 5.31.3 Member Data Documentation

# 5.31.3.1 \_data\_tspace

```
const std::shared_ptr<const MatrixXd> model_fit::Model::_data_tspace [private]
```

BigMatrix of the data on the tangent space

## 5.31.3.2 \_design\_matrix\_model

```
const std::shared_ptr<const MatrixXd> model_fit::Model::_design_matrix_model [private]
```

Design matrix for the data in the cell (used to compute the beta)

## 5.31.3.3 \_design\_matrix\_tot

```
const std::shared_ptr<const MatrixXd> model_fit::Model::_design_matrix_tot [private]
```

Design matrix for all the data in the domain (used ti compute the residuals)

## 5.31.3.4 \_distance\_Manifold\_name

```
\verb|const| std::string| model_fit::Model::\_distance\_Manifold\_name | [private]| \\
```

Name of the metric on the manifold

```
5.31.3.5 N
const unsigned int model_fit::Model::_N [private]
Number of stations in the cell
5.31.3.6 _p
const unsigned int model_fit::Model::_p [private]
Dimension of the matrices on the manifold
5.31.3.7 _num_cov
const unsigned int model_fit::Model::_num_cov [private]
Number of covariates in the model
5.31.3.8 _num_coeff
const unsigned int model_fit::Model::_num_coeff [private]
Number of significant entries in a symmetric (p * p) matrix. _num_coeff = \frac{p*(p+1)}{2}
5.31.3.9 _beta_matrix
MatrixXd model_fit::Model::_beta_matrix [private]
( num cov * num coeff) matrix where the i^{th} row contains the upper triangular part of \beta_{..i}, the i^{th} coefficient of
the tangent space linear model
5.31.3.10 _fitted_values
MatrixXd model_fit::Model::_fitted_values [private]
(N tot * num coeff) matrix where the i^{th} row contains the upper triangular part of the matrix fitted in the i^{th}
location on the tangent space by the linear model. _fitted_values = (* (_design_matrix_tot)) * _beta_matrix
5.31.3.11 _residuals
MatrixXd model_fit::Model::_residuals [private]
```

 $(N_{tot} * _num_coeff)$  matrix where the  $i^{th}$  row contains the upper triangular part of the residual matrix in the  $i^{th}$ 

The documentation for this class was generated from the following files:

location.  $\_$ residuals  $= (*(\_$ data $\_$ tspace $)) - \_$ fitted $\_$ values

- Model.hpp
- Model.cpp

# 5.32 variogram\_evaluation::EmpiricalVariogram Class Reference

Class for computation and storage of the empirical variogram.

```
#include <EmpiricalVariogram.hpp>
```

#### **Public Member Functions**

• EmpiricalVariogram (const std::shared\_ptr< const MatrixXd >, unsigned int, const Coordinates &, const distances::Distance &)

Constructor.

EmpiricalVariogram (const std::shared\_ptr< const MatrixXd >, unsigned int, unsigned int, const Vec &, double)

Constructor.

 void update\_emp\_vario (const std::vector< MatrixXd > &res, const distances\_tplane::DistanceTplane &distanceTplane)

```
\textit{Update} \_\textit{emp\_vario\_values}, \_\textit{hvec} \textit{ and } \_\textit{N}\_\textit{hvec} \textit{ according to the new residuals } \Delta \left( \boldsymbol{s_i} \right) i = 1, \cdot, \_\textit{N}.
```

std::vector< double > get\_emp\_vario\_values () const

```
Return _emp_vario_values
```

std::vector< unsigned int > get\_N\_hvec () const

**Return**\_N\_hvec

• std::vector< double > get\_hvec () const

Return\_hvec

unsigned int get\_card\_h () const

Return\_card\_h

• unsigned int get\_N () const

 $Return _N$ 

double get\_hmax () const

 $Return\_hmax$ 

# **Private Member Functions**

• void compute\_hmax (const Coordinates &, const distances::Distance &)

Compute the maximum distance to be considered.

# **Private Attributes**

- const unsigned int \_n\_h
- const unsigned int \_N
- const std::shared ptr< const MatrixXd > distanceMatrix
- std::vector< double > \_emp\_vario\_values
- std::vector< double > \_hvec
- std::vector< unsigned int > \_N\_hvec
- · unsigned int card h
- Vec d
- double hmax
- · Vec \_weights

# 5.32.1 Detailed Description

Class for computation and storage of the empirical variogram.

### 5.32.2 Member Function Documentation

# 5.32.2.1 update\_emp\_vario()

Update \_emp\_vario\_values, \_hvec and \_N\_hvec according to the new residuals  $\Delta\left(s_{i}\right)i=1,\cdot,\_N$ .

\_emp\_vario\_values are estimated through the method of moments:

$$\widehat{\gamma}\left(h\right) = \frac{\sum\limits_{N(h)} w\left(s_{i}\right) w\left(s_{j}\right) \left\|\Delta\left(s_{i}\right) - \Delta\left(s_{j}\right)\right\|^{2}}{2\sum\limits_{N(h)} w\left(s_{i}\right) w\left(s_{j}\right)}$$

where 
$$N\left(h\right) = \left\{\left(\boldsymbol{s}_{i}, \boldsymbol{s}_{j} \in D\right) : h - \Delta h \leq \|\boldsymbol{s}_{i} - \boldsymbol{s}_{j}\| \leq h - \Delta h\right\}$$

### **Parameters**

res	Vector of the $N$ residual matrices
distanceTplane	Distance on the tangent space

# 5.32.3 Member Data Documentation

```
5.32.3.1 _n_h
```

const unsigned int variogram\_evaluation::EmpiricalVariogram::\_n\_h [private]

Number of bins

5.32.3.2 \_N

Number of data points used to compute the empirical variogram

```
5.32.3.3 _distanceMatrix
```

```
\verb|const| std::shared_ptr < const| MatrixXd > variogram_evaluation::EmpiricalVariogram::_distance \leftarrow Matrix| [private]
```

Matrix of distances among the N locations

```
5.32.3.4 _emp_vario_values
```

```
std::vector<double> variogram_evaluation::EmpiricalVariogram::_emp_vario_values [private]
```

Vector storing the variogram estimates.  $\_emp\_vario\_values[i] = \hat{\gamma} (\_hvec[i])$ 

```
5.32.3.5 _hvec
```

```
std::vector<double> variogram_evaluation::EmpiricalVariogram::_hvec [private]
```

Vector storing the distances at which the variogram is estimated

```
5.32.3.6 _N_hvec
```

```
\verb|std::vector<| unsigned int> variogram_evaluation:: \verb|EmpiricalVariogram::_N_hvec [private]| \\
```

Vector storing the number of data used in the estimation of the corresponding empirical variogram value

```
5.32.3.7 _card_h
```

```
unsigned int variogram_evaluation::EmpiricalVariogram::_card_h [private]
```

Number of distances for which the variogram has been evaluated.  $\_card_h = \_hvec.size()$ 

5.32.3.8 \_d

```
Vec variogram_evaluation::EmpiricalVariogram::_d [private]
```

Vector of equispaced distances whose midpoints are the candidates to enter in \_hvec

```
5.32.3.9 _hmax
```

```
double variogram_evaluation::EmpiricalVariogram::_hmax [private]
```

Maximum distance considered

5.32.3.10 \_weights

```
Vec variogram_evaluation::EmpiricalVariogram::_weights [private]
```

Vector of the weights for the N data

The documentation for this class was generated from the following files:

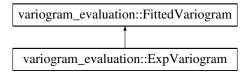
- · EmpiricalVariogram.hpp
- · EmpiricalVariogram.cpp

# 5.33 variogram\_evaluation::ExpVariogram Class Reference

Class for computation and storage of the fitted variogram when vario\_model == "Exponential"

```
#include <FittedVariogram.hpp>
```

Inheritance diagram for variogram\_evaluation::ExpVariogram:



# **Public Member Functions**

- double get\_vario\_univ (const double &h) const override
  - Compute the value of the model variogram at a given distance, according to the variogram type.
- $\sim$ ExpVariogram ()=default

Destructor.

# **Private Member Functions**

• void get\_init\_par (const EmpiricalVariogram &) override

*Initialize*\_parameters

• MatrixXd compute\_jacobian (const std::vector< double > &, unsigned int) const override

Compute the jacobian of the variogram residuals (which coincides with the one of the model variogram), according to the variogram type.

# **Additional Inherited Members**

# 5.33.1 Detailed Description

Class for computation and storage of the fitted variogram when vario\_model=="Exponential"

# 5.33.2 Member Function Documentation

### 5.33.2.1 get\_init\_par()

```
\label{local_parameters} $$\operatorname{Init\_par}(\ \operatorname{const}\ \operatorname{EmpiricalVariogram}\ \&\ \operatorname{emp\_vario})\ [\operatorname{override}],\ [\operatorname{private}],\ [\operatorname{virtual}]$$ Initialize \_parameters $$ \operatorname{Initialize}\ \operatorname{Initialize}\ \operatorname{are}\ \operatorname{initialize}\ \operatorname{as}\ \operatorname{follows}:\ \operatorname{parameters}(0) = \operatorname{weighted\_median}(\ \operatorname{emp\_vario\_values.head}(2),\ \operatorname{N\_hvec.head}(2))$$ _parameters $(1) = \operatorname{weighted\_median}(\ \operatorname{emp\_vario\_values.tail}(4),\ \operatorname{N\_hvec.tail}(4)) - \operatorname{parameters}(0)$$ _parameters $(2) = \ \operatorname{hvec}(i^*)$$ where $i^*$ is the first $i$ s.t. $|\ \operatorname{emp\_vario\_values}(i) - (\ \operatorname{parameters}(0) + \ \operatorname{parameters}(1))| < \operatorname{tol}(i^*)$$ and $i^* = i^* = i^*
```

#### **Parameters**

```
emp_vario | Empirical variogram
```

Implements variogram\_evaluation::FittedVariogram.

### 5.33.2.2 get\_vario\_univ()

Compute the value of the model variogram at a given distance, according to the variogram type.

#### **Parameters**

h The distance where to evaluate the variogram

### Returns

Variogram value at distance h:  $\gamma_m(h)$ 

Implements variogram\_evaluation::FittedVariogram.

The documentation for this class was generated from the following files:

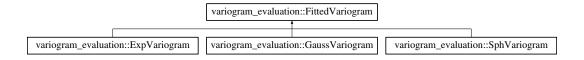
- · FittedVariogram.hpp
- · FittedVariogram.cpp

# 5.34 variogram\_evaluation::FittedVariogram Class Reference

Abstract class for computation and storage of the fitted variogram.

```
#include <FittedVariogram.hpp>
```

Inheritance diagram for variogram\_evaluation::FittedVariogram:



#### **Public Member Functions**

• double get\_tau2 () const

Return \_parameters (0), i.e. the nugget.

• double get\_sigma2 () const

Return \_parameters (1), i.e. the sill-nugget.

• double get\_a () const

Return\_parameters (2), i.e. the practical range.

void evaluate\_par\_fitted\_E (const EmpiricalVariogram &emp\_vario, double max\_sill, double max\_a)

Compute the parameters of the fitted variogram.

void evaluate\_par\_fitted\_W (const EmpiricalVariogram &emp\_vario, double max\_sill, double max\_a)

Compute the parameters of the fitted variogram.

virtual double get\_vario\_univ (const double &h) const =0

Compute the value of the model variogram at a given distance, according to the variogram type.

• double get\_covario\_univ (const double &h) const

Compute the value of the model covariogram at a given distance, according to the variogram type.

 $\bullet \ \ \text{Vec get\_vario\_vec} \ (\text{const std::vector} < \text{double} > \& \text{h\_vec}, \text{unsigned int card\_h}) \ \text{const} \\$ 

Compute the values of the model variogram at a given vector of distances, according to the variogram type.

Vec get\_vario\_vec (const Vec &h\_vec, unsigned int card\_h) const

Compute the values of the model variogram at a given vector of distances, according to the variogram type.

MatrixXd compute\_gamma\_matrix (const std::shared\_ptr< const MatrixXd > distanceMatrix, unsigned int N)
const

Compute the covariogram matrix, according to the variogram type.

• Vector3d get\_parameters () const

**Return**\_parameters

void set\_parameters (const Vector3d &parameters)

Set\_parameters.

Vec get\_covario\_vec (const std::vector< double > &h\_vec, unsigned int card\_h) const

Compute the values of the model covariogram at a given vector of distances, according to the variogram type.

• virtual  $\sim$ FittedVariogram ()=default

Destructor.

### **Protected Member Functions**

- double weighted\_median (const std::vector< double > &values, const std::vector< unsigned int > &card)

  Compute weighted median.
- virtual void get\_init\_par (const EmpiricalVariogram &emp\_vario)=0
   Initialize\_parameters
- void backtrack (const Vector3d &dir, Vector3d &gk, Vec &res, const std::vector< double > &h\_vec, unsigned int card\_h, double c, double s, const Vec &emp\_vario\_values, double max\_sill, double max\_a)

Update \_ parametersmoving along dir

• virtual MatrixXd compute\_jacobian (const std::vector< double > &h\_vec, unsigned int card\_h) const =0

Compute the jacobian of the variogram residuals (which coincides with the one of the model variogram), according to the variogram type.

# **Protected Attributes**

Vector3d parameters

# 5.34.1 Detailed Description

Abstract class for computation and storage of the fitted variogram.

### 5.34.2 Member Function Documentation

# 5.34.2.1 weighted\_median()

Compute weighted median.

### **Parameters**

values	Values whose median must be computed	
card	Weights	

### Returns

Median of values, weighted using card

### 5.34.2.2 get\_init\_par()

Initialize \_parameters

#### **Parameters**

```
emp_vario | Empirical variogram
```

Implemented in variogram\_evaluation::SphVariogram, variogram\_evaluation::ExpVariogram, and variogram\_evaluation::GaussVariogram\_e

### 5.34.2.3 evaluate\_par\_fitted\_E()

Compute the parameters of the fitted variogram.

Note

Like variogram\_evaluation::FittedVariogram::evaluate\_par\_fitted\_W, but different stopping criteria. This function is used when equal weights are considered

The parameters are computed using Gauss-Newton with backtrack method to solve the non-linear least square problem:

The starting values for the \_parameters are obtained through variogram\_evaluation::FittedVariogram::get\_init\_par.

The stopping criteria is based on the decrease of the error norm.

### **Parameters**

emp_vario	Empirical variogram
max_sill	Maximum value for the sill
max_a	Maximum value for a

### 5.34.2.4 evaluate\_par\_fitted\_W()

Compute the parameters of the fitted variogram.

### Note

Like variogram\_evaluation::FittedVariogram::evaluate\_par\_fitted\_E, but different stopping criteria. This function is used when kernel weights are considered

The parameters are computed using Gauss-Newton with backtrack method to solve the non-linear least square problem:

The starting values for the \_parameters are obtained through variogram evaluation::FittedVariogram::get init par.

The stopping criteria is based on the difference in the decrease of the error norm between two consecutive iterations.

### **Parameters**

emp_vario	Empirical variogram
max_sill	Maximum value for the sill
max_a	Maximum value for a

### 5.34.2.5 get\_vario\_univ()

Compute the value of the model variogram at a given distance, according to the variogram type.

# **Parameters**

```
h The distance where to evaluate the variogram
```

### Returns

Variogram value at distance h:  $\gamma_m(h)$ 

Implemented in variogram\_evaluation::SphVariogram, variogram\_evaluation::ExpVariogram, and variogram\_evaluation::GaussVariogram

### 5.34.2.6 get\_covario\_univ()

```
double FittedVariogram::get_covario_univ (  {\tt const\ double\ \&\ h\ )\ const}
```

Compute the value of the model covariogram at a given distance, according to the variogram type.

### **Parameters**

```
h The distance where to evaluate the covariogram
```

### Returns

```
Covariogram value at distance h:C_{m}\left(h\right)=\left(\text{\_parameters}\left(0\right)+\text{\_parameters}\left(1\right)\right)-\gamma_{m}\left(h\right)
```

```
5.34.2.7 get_vario_vec() [1/2]
```

Compute the values of the model variogram at a given vector of distances, according to the variogram type.

### **Parameters**

h_vec	The distances where to evaluate the variogram	
card↩	Number of distances where the variogram must be computed. card_h = h_vec.size()	
_h		

### Returns

Vector of variogram values at distances  $h \in \underline{h}_{vec}$ 

# **5.34.2.8** get\_vario\_vec() [2/2]

Compute the values of the model variogram at a given vector of distances, according to the variogram type.

### **Parameters**

h_vec	The distances where to evaluate the variogram	
card←	Number of distances where the variogram must be computed. card_h = h_vec.size()	
h		

#### Returns

Vector of variogram values at distances  $h \in h\_vec$ 

# 5.34.2.9 compute\_gamma\_matrix()

```
MatrixXd FittedVariogram::compute_gamma_matrix (  {\tt const~std::shared\_ptr} < {\tt const~MatrixXd} > {\tt distanceMatrix}, \\ {\tt unsigned~int} ~N~)~{\tt const}
```

Compute the covariogram matrix, according to the variogram type.

### **Parameters**

distanceMatrix	Matrix of distances among the $N$ locations
N	Number of data locations

# Returns

Covariogram matrix

# 5.34.2.10 set\_parameters()

Set \_parameters.

### **Parameters**

parameters	Vector of parameters' values
------------	------------------------------

# 5.34.2.11 get\_covario\_vec()

Compute the values of the model covariogram at a given vector of distances, according to the variogram type.

# **Parameters**

h_vec	The distances where to evaluate the covariogram	
_card↔	Number of distances where the covariogram must be computed. card_h = h_vec.size()	
Generated by Doxygen		

#### Returns

Vector of covariogram values at distances  $h \in h\_vec$ 

### 5.34.3 Member Data Documentation

#### 5.34.3.1 \_parameters

Vector3d variogram\_evaluation::FittedVariogram::\_parameters [protected]

Vector storing the three parameters of the fitted varriogram (nugget, sill-nugget, practical range)

The documentation for this class was generated from the following files:

- · FittedVariogram.hpp
- FittedVariogram.cpp

# 5.35 variogram\_evaluation::GaussVariogram Class Reference

Class for computation and storage of the fitted variogram when vario\_model == "Gaussian"

```
#include <FittedVariogram.hpp>
```

Inheritance diagram for variogram\_evaluation::GaussVariogram:



# **Public Member Functions**

- double get\_vario\_univ (const double &h) const override
  - Compute the value of the model variogram at a given distance, according to the variogram type.
- ∼GaussVariogram ()=default

Destructor.

# **Private Member Functions**

void get\_init\_par (const EmpiricalVariogram &) override

Initialize \_ parameters

• MatrixXd compute\_jacobian (const std::vector< double > &, unsigned int) const override

Compute the jacobian of the variogram residuals (which coincides with the one of the model variogram), according to the variogram type.

### **Additional Inherited Members**

# 5.35.1 Detailed Description

Class for computation and storage of the fitted variogram when vario\_model=="Gaussian"

### 5.35.2 Member Function Documentation

### 5.35.2.1 get\_init\_par()

The \_parameters are initialized as follows:

```
 \begin{tabular}{ll} $\_$ parameters (0) = weighted\_median (\_emp\_vario\_values.head (2) , \_N\_hvec.head (2)) \\ $\_$ parameters (1) = weighted\_median (\_emp\_vario\_values.tail (4) , \_N\_hvec.tail (4)) - \_parameters (0) \\ $\_$ parameters (2) = \_hvec (i^*) \\ $$ where $i^*$ is the first $i$ s.t. $|\_emp\_vario\_values (i) - (\_parameters (0) + \_parameters (1))| < tol. $|\_emp\_vario\_values (i) - (\_parameters (0) + \_parameters (1))| < tol. $|\_emp\_vario\_values (i) - (\_parameters (i) + \_parameters (i))| < tol. $|\_emp\_vario\_values (i) - (\_parameters (i) + \_parameters (i) + \_parameter
```

#### **Parameters**

emp_vario   Empirical variogram
---------------------------------

 $Implements\ variogram\_evaluation:: Fitted Variogram.$ 

### 5.35.2.2 get\_vario\_univ()

Compute the value of the model variogram at a given distance, according to the variogram type.

#### **Parameters**

h The distance where to evaluate the variogram

### Returns

Variogram value at distance h:  $\gamma_m(h)$ 

Implements variogram\_evaluation::FittedVariogram.

The documentation for this class was generated from the following files:

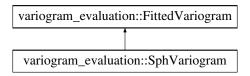
- · FittedVariogram.hpp
- · FittedVariogram.cpp

# 5.36 variogram\_evaluation::SphVariogram Class Reference

Class for computation and storage of the fitted variogram when  $vario\_model==$ "Spherical"

```
#include <FittedVariogram.hpp>
```

Inheritance diagram for variogram\_evaluation::SphVariogram:



### **Public Member Functions**

- double get\_vario\_univ (const double &h) const override
  - Compute the value of the model variogram at a given distance, according to the variogram type.
- ∼SphVariogram ()=default

Destructor.

#### **Private Member Functions**

- void get\_init\_par (const EmpiricalVariogram &) override
  - *Initialize*\_parameters
- MatrixXd compute\_jacobian (const std::vector< double > &, unsigned int) const override

Compute the jacobian of the variogram residuals (which coincides with the one of the model variogram), according to the variogram type.

### **Additional Inherited Members**

### 5.36.1 Detailed Description

Class for computation and storage of the fitted variogram when vario\_model == "Spherical"

# 5.36.2 Member Function Documentation

### **Parameters**

emp_vario	Empirical variogram
-----------	---------------------

Implements variogram\_evaluation::FittedVariogram.

# 5.36.2.2 get\_vario\_univ()

Compute the value of the model variogram at a given distance, according to the variogram type.

### **Parameters**

h The distance where to evaluate the variogram

### Returns

Variogram value at distance h:  $\gamma_m(h)$ 

Implements variogram\_evaluation::FittedVariogram.

The documentation for this class was generated from the following files:

- FittedVariogram.hpp
- · FittedVariogram.cpp

# **Chapter 6**

# **File Documentation**

# 6.1 Coordinates.hpp File Reference

# Coordinates class.

```
#include <iostream>
#include <string>
#include <memory>
#include "Helpers.hpp"
```

# Classes

• class Coordinates

Class to store the coordinates of the data points.

# 6.1.1 Detailed Description

Coordinates class.

# 6.2 DesignMatrix.hpp File Reference

Classes to create the design\_matrix according to the model on the tangent space.

```
#include <memory>
#include <iostream>
#include <utility>
#include <map>
#include <string>
#include <Eigen/Dense>
#include "Coordinates.hpp"
```

### Classes

```
    class design_matrix::DesignMatrix
        Abstract class for the computation of the design_matrix.
    class design_matrix::InterceptDM
        Class for the computation of the design_matrix when model_ts=="Intercept"
    class design_matrix::Coord1DM
        Class for the computation of the design_matrix when model_ts=="Coord1"
    class design_matrix::Coord2DM
        Class for the computation of the design_matrix when model_ts=="Coord2"
    class design_matrix::AdditiveDM
    Class for the computation of the design_matrix when model_ts=="Additive"
```

# 6.2.1 Detailed Description

Classes to create the design\_matrix according to the model on the tangent space.

# 6.3 Distance.hpp File Reference

Classes to create the compute the distances between data locations.

```
#include "Helpers.hpp"
#include "Coordinates.hpp"
#include <vector>
#include <utility>
#include <map>
#include <functional>
```

### **Classes**

· class distances::Distance

Abstract class for the computation of the distance between data locations.

· class distances::EuclDist

Class for the computation of the distance between data locations when distance=="Eucldist"

· class distances::GeoDist

Class for the computation of the distance between data locations when distance=="Geodist"

# 6.3.1 Detailed Description

Classes to create the compute the distances between data locations.

# 6.4 DistanceManifold.hpp File Reference

Classes to compute the distance on the manifold according to its metric.

```
#include "Helpers.hpp"
#include <vector>
#include <utility>
#include <map>
#include <functional>
#include <memory>
```

### Classes

· class distances manifold::DistanceManifold

Abstract class for the computation of the distance on the manifold.

· class distances\_manifold::Frobenius

Class for the computation of the distance on the manifold when manifold\_metric=="Frobenius"

· class distances manifold::LogEuclidean

Class for the computation of the distance on the manifold when manifold\_metric=="LogEuclidean"

class distances\_manifold::SqRoot

Class for the computation of the distance on the manifold when manifold\_metric=="SqRoot"

· class distances manifold::Chol

Class for the computation of the distance on the manifold when manifold\_metric="Chol"

# 6.4.1 Detailed Description

Classes to compute the distance on the manifold according to its metric.

# 6.5 DistanceTplane.hpp File Reference

Classes to compute the distance on the tangent space according to its metric.

```
#include "Helpers.hpp"
#include <vector>
#include <utility>
#include <map>
#include <functional>
#include <memory>
```

### **Classes**

class distances\_tplane::DistanceTplane

Abstract class for the computation of the distance on the tangent space.

class distances\_tplane::Frobenius

Class for the computation of the distance on the tangent space when ts\_metric=="Frobenius"

· class distances\_tplane::FrobeniusScaled

Class for the computation of the distance on the tangent space when ts\_metric=="FrobeniusScaled"

class distances\_tplane::Chol

Class for the computation of the distance on the tangent space when  $ts\_metric=="Correlation"$ 

# 6.5.1 Detailed Description

Classes to compute the distance on the tangent space according to its metric.

# 6.6 Empirical Variogram.hpp File Reference

Class to compute the empirical variogram.

```
#include "Helpers.hpp"
#include "DistanceTplane.hpp"
#include "Distance.hpp"
#include "Coordinates.hpp"
```

### Classes

• class variogram\_evaluation::EmpiricalVariogram

Class for computation and storage of the empirical variogram.

# 6.6.1 Detailed Description

Class to compute the empirical variogram.

# 6.7 Factory.hpp File Reference

# Factory class.

```
#include <map>
#include <vector>
#include <memory>
#include <functional>
#include <stdexcept>
#include <type_traits>
#include <Rcpp.h>
```

# Classes

 class generic\_factory::Factory < AbstractProduct, Identifier, Builder > A generic factory.

# 6.7.1 Detailed Description

Factory class.

# 6.8 FittedVariogram.hpp File Reference

Classes to fit a model variogram (Gaussian, Exponential or Spherical) to an empirical one.

```
#include "EmpiricalVariogram.hpp"
#include <iostream>
#include <Rcpp.h>
```

# Classes

• class variogram\_evaluation::FittedVariogram

Abstract class for computation and storage of the fitted variogram.

· class variogram\_evaluation::GaussVariogram

Class for computation and storage of the fitted variogram when vario\_model == "Gaussian"

• class variogram\_evaluation::ExpVariogram

Class for computation and storage of the fitted variogram when vario\_model == "Exponential"

• class variogram\_evaluation::SphVariogram

Class for computation and storage of the fitted variogram when vario\_model == "Spherical"

# 6.8.1 Detailed Description

Classes to fit a model variogram (Gaussian, Exponential or Spherical) to an empirical one.

# 6.9 Helpers.hpp File Reference

Functions to manipulate matrices.

```
#include <Eigen/Sparse>
#include <Eigen/Dense>
#include <Eigen/Eigenvalues>
#include <Eigen/Cholesky>
#include <Eigen/SparseCholesky>
#include <cmath>
#include <string>
```

# **Typedefs**

typedef Eigen::VectorXd Vec

Eigen dynamic vector of doubles.

### **Functions**

MatrixXd matrix\_manipulation::expMat (const MatrixXd &A)

Compute the exponential of a matrix.

MatrixXd matrix\_manipulation::logMat (const MatrixXd &A)

Compute the logarithm of a matrix.

MatrixXd matrix\_manipulation::sqrtMat (const MatrixXd &A)

Compute the square root of a matrix.

std::vector< MatrixXd > matrix\_manipulation::bigMatrix2VecMatrices (const MatrixXd &bigMatrix, unsigned int p, const std::string &distance\_Manifold\_name)

```
Transform a \left(\frac{p*(p+1)}{2},N\right) matrix in a vector of Np*p symmetric matrices.
```

• MatrixXd matrix\_manipulation::VecMatrices2bigMatrix (const std::vector< MatrixXd > &vecMatrices)

Transform a vector of Np \* p symmetric matrices in a  $\left(\frac{p*(p+1)}{2}, N\right)$  matrix.

MatrixXd matrix\_manipulation::Chol\_semidef (const MatrixXd &M1)

Compute the Cholesky decomposition of a matrix not positive definite.

MatrixXd matrix\_manipulation::Chol\_decomposition (const MatrixXd M1)

Compute the Cholesky decomposition of a matrix.

# 6.9.1 Detailed Description

Functions to manipulate matrices.

### 6.9.2 Function Documentation

```
6.9.2.1 expMat()
```

```
\label{lem:matrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatrixMatri
```

Compute the exponential of a matrix.

### **Parameters**

```
A Matrix whose exponential must be computed
```

# Returns

Expontial of A

### 6.9.2.2 logMat()

Compute the logarithm of a matrix.

### **Parameters**

A Matrix whose logarithm must be computed

# Returns

Logarithm of A

# 6.9.2.3 sqrtMat()

Compute the square root of a matrix.

# **Parameters**

A Matrix whose square root must be computed

### Returns

Square root of A

# 6.9.2.4 bigMatrix2VecMatrices()

Transform a  $\left(\frac{p*(p+1)}{2},N\right)$  matrix in a vector of Np\*p symmetric matrices.

# **Parameters**

bigMatrix	Matrix whose rows store the values of the upper triangular parts of $Np \ast p$ symmetric matrices	
ρ	Dimension of the matrices	
distance_Manifold_name	Name of the metric on the manifold to use. If	
	distance_Manifold_name=="Correlation" then only the upper	
	triangular parts of the matrices is filled	

#### Returns

Vector of Np \* p symmetric matrices corresponding to the transformation of bigMatrix

# 6.9.2.5 VecMatrices2bigMatrix()

Transform a vector of Np\*p symmetric matrices in a  $\left(\frac{p*(p+1)}{2},N\right)$  matrix.

#### **Parameters**

vecMatrices	Vector of symmetric matrices whose upper trinagular parts will be stored in the rows of the
	output matrix

### Returns

Matrix  $\left(\frac{p*(p+1)}{2},N\right)$  correponsing to the transformation of vecMatrices

### 6.9.2.6 Chol\_semidef()

Compute the Cholesky decomposition of a matrix not positive definite.

### **Parameters**

M1 Positive semidefinite matrix whose Cholesky decomposition must be computed

### Returns

Matrix in Chol(p), representing the upper trianglar matrix of A's Cholesky decomposition

# 6.9.2.7 Chol\_decomposition()

Compute the Cholesky decomposition of a matrix.

#### **Parameters**

M1 | Positive definite or semidefine matrix whose Cholesky decomposition must be computed

### Returns

Matrix in Chol(p), representing the upper trianglar matrix of A's Cholesky decomposition

# 6.10 HelpersFactory.hpp File Reference

### Typedefs for the factory.

```
#include "FittedVariogram.hpp"
#include "DesignMatrix.hpp"
#include "Distance.hpp"
#include "MapFunctions.hpp"
#include "DistanceManifold.hpp"
#include "DistanceTplane.hpp"
#include "Factory.hpp"
#include "Proxy.hpp"
```

### **Typedefs**

- typedef generic\_factory::Factory < variogram\_evaluation::FittedVariogram, std::string > vario\_factory::VariogramFactory Factory for the fitted variogram.
- template<typename ConcreteProduct >
   using vario\_factory::VariogramProxy = generic\_factory::Proxy
   VariogramFactory, ConcreteProduct >
   Proxy for the fitted variogram.
- typedef generic\_factory::Factory < design\_matrix::DesignMatrix, std::string > design\_factory::DesignFactory Factory for the design matrix.
- template<typename ConcreteProduct >
   using design\_factory::DesignProxy = generic\_factory::Proxy< DesignFactory, ConcreteProduct >
   Proxy for the design matrix.
- typedef generic\_factory::Factory< distances::Distance, std::string > distance\_factory::DistanceFactory Factory for the distance.
- template<typename ConcreteProduct >
   using distance\_factory::DistanceProxy = generic\_factory::Proxy< DistanceFactory, ConcreteProduct >
   Proxy for the distance.
- typedef generic\_factory::Factory < map\_functions::logarithmicMap, std::string > map\_factory::LogMapFactory Factory for the logarithmic map.
- typedef generic\_factory::Factory < map\_functions::exponentialMap, std::string > map\_factory::ExpMapFactory Factory for the exponential map.
- template<typename ConcreteProduct >
   using map\_factory::LogMapProxy = generic\_factory::Proxy< LogMapFactory, ConcreteProduct >
   Proxy for the logarithmic map.
- template<typename ConcreteProduct >
   using map\_factory::ExpMapProxy = generic\_factory::Proxy< ExpMapFactory, ConcreteProduct >
   Proxy for the exponential map.
- typedef generic factory::Factory < distances manifold::DistanceManifold, std::string > manifold factory::ManifoldFactory

Factory for the distance on the manifold.

- template<typename ConcreteProduct >
   using manifold\_factory::ManifoldProxy = generic\_factory::Proxy< ManifoldFactory, ConcreteProduct >
   Proxy for the distance on the manifold.
- typedef generic\_factory::Factory < distances\_tplane::DistanceTplane, std::string > tplane\_factory::TplaneFactory Factory for the distance on the tangent space.
- template<typename ConcreteProduct >
   using tplane\_factory::TplaneProxy = generic\_factory::Proxy< TplaneFactory, ConcreteProduct >
   Proxy for the distance on the tangent space.

# 6.10.1 Detailed Description

Typedefs for the factory.

# 6.11 interface\_function.cpp File Reference

Main functions to create the model and perform kriging, along with functions to compute the distance on the manifold and the intrinsic mean.

```
#include <iostream>
#include <vector>
#include <utility>
#include <memory>
#include <Rcpp.h>
#include <RcppEigen.h>
#include "Coordinates.hpp"
#include "DesignMatrix.hpp"
#include "DistanceManifold.hpp"
#include "DistanceTplane.hpp"
#include "Distance.hpp"
#include "EmpiricalVariogram.hpp"
#include "FittedVariogram.hpp"
#include "Helpers.hpp"
#include "HelpersFactory.hpp"
#include "MapFunctions.hpp"
#include "Model.hpp"
#include "IntrinsicMean.hpp"
```

# **Functions**

RcppExport SEXP get\_model (SEXP s\_data\_manifold, SEXP s\_coordinates, SEXP s\_X, SEXP s\_Sigma, SEXP s\_distance, SEXP s\_manifold\_metric, SEXP s\_ts\_metric, SEXP s\_ts\_model, SEXP s\_vario\_model, SEXP s\_n\_h, SEXP s\_max\_it, SEXP s\_tolerance, SEXP s\_max\_sill, SEXP s\_max\_a, SEXP s\_weight\_vario, SEXP s\_distance\_matrix\_tot, SEXP s\_data\_manifold\_tot, SEXP s\_coordinates\_tot, SEXP s\_X\_tot, SEXP s\_hmax, SEXP s\_indexes\_model, SEXP s\_weight\_intrinsic, SEXP s\_tolerance\_intrinsic, SEXP s\_weight\_\( \limits \) extrinsic, SEXP s\_suppressMes, SEXP s\_tolerance\_map\_cor)

Given the coordinates and corresponding manifold values, this function creates a GLS model on the tangent space.

RcppExport SEXP get\_kriging (SEXP s\_coordinates, SEXP s\_new\_coordinates, SEXP s\_Sigma, SEXP s
\_distance, SEXP s\_manifold\_metric, SEXP s\_ts\_model, SEXP s\_vario\_model, SEXP s\_beta, SEXP s\_
gamma\_matrix, SEXP s\_vario\_parameters, SEXP s\_residuals, SEXP s\_X\_new, SEXP s\_tolerance\_map\_
cor)

Given the GLS model kriging prediction on new location is performed.

RcppExport SEXP get\_model\_and\_kriging (SEXP s\_data\_manifold, SEXP s\_coordinates, SEXP s\_X, SEXP s\_Sigma, SEXP s\_distance, SEXP s\_manifold\_metric, SEXP s\_ts\_metric, SEXP s\_ts\_model, SEXP s\_wario\_model, SEXP s\_n\_h, SEXP s\_max\_it, SEXP s\_tolerance, SEXP s\_max\_sill, SEXP s\_max\_a, SEXP s\_weight\_vario, SEXP s\_distance\_matrix\_tot, SEXP s\_data\_manifold\_tot, SEXP s\_coordinates\_tot, SE XP s\_X\_tot, SEXP s\_hmax, SEXP s\_indexes\_model, SEXP s\_weight\_intrinsic, SEXP s\_tolerance\_intrinsic, SEXP s\_weight\_extrinsic, SEXP s\_new\_coordinates, SEXP s\_X\_new, SEXP s\_suppressMes, SEXP s\_weight\_extrinsic, SEXP s\_new\_coordinates

Given the coordinates and corresponding manifold values, this function firstly creates a GLS model on the tangent space, and then it performs kriging on the new locations.

 RcppExport SEXP intrinsic\_mean (SEXP s\_data, SEXP s\_N, SEXP s\_manifold\_metric, SEXP s\_ts\_metric, SEXP s\_tolerance, SEXP s\_weight\_intrinsic, SEXP s\_weight\_extrinsic, SEXP s\_tolerance\_map\_cor)

Evaluate the intrinsic mean of a given set of symmetric positive definite matrices.

RcppExport SEXP distance\_manifold (SEXP s\_data1, SEXP s\_data2, SEXP s\_N1, SEXP s\_N2, SEXP s\_
manifold\_metric)

Compute the manifold distance between symmetric positive definite matrices.

# 6.11.1 Detailed Description

Main functions to create the model and perform kriging, along with functions to compute the distance on the manifold and the intrinsic mean.

### 6.11.2 Function Documentation

# 6.11.2.1 get\_model()

```
RcppExport SEXP get_model (
             SEXP s_data_manifold,
             SEXP s_coordinates,
             SEXP s_X,
             SEXP s_Sigma,
             SEXP s_distance,
             SEXP s_manifold_metric,
             SEXP s_ts_metric,
             SEXP s_ts_model,
             SEXP s_vario_model,
             SEXP s_n_h,
             SEXP s_max_it,
             SEXP s_tolerance,
             SEXP s_max_sill,
             SEXP s_{max_a}
             SEXP s_weight_vario,
             SEXP s_distance_matrix_tot,
             SEXP s_data_manifold_tot,
             SEXP s\_coordinates\_tot,
             SEXP s_X_{tot}
             SEXP s_hmax,
             SEXP s_indexes_model,
             SEXP s_weight_intrinsic,
             SEXP s_tolerance_intrinsic,
```

```
SEXP s_weight_extrinsic,
SEXP s_suppressMes,
SEXP s_tolerance_map_cor )
```

Given the coordinates and corresponding manifold values, this function creates a GLS model on the tangent space.

The manifold values are mapped on the tangent space and then a GLS model is fitted to them. A first estimate of the beta coefficients is obtained assuming spatially uncorrelated errors. Then, in the main the loop, new estimates of the beta are obtained as a result of a weighted least square problem where the weight matrix is the inverse of  $gamma_matrix$ . The residuals (residuals =  $data_ts - fitted$ ) are updated accordingly. The parameters of the variogram fitted to the residuals (and used in the evaluation of the  $gamma_matrix$ ) are computed using Gauss-Newton with backtrack method to solve the associated non-linear least square problem. The stopping criteria is based on the absolute value of the variogram residuals' norm if ker.width.vario=0, while it is based on its increment otherwise.

#### Note

Reference: "Kriging prediction for manifold-valued random fields."

Authors: D. Pigoli, A. Menafoglio & P. Secchi (2016) Periodical: Journal of Multivariate Analysis, 145, 117-131.

#### **Parameters**

s_data_manifold	list of $N$ symmetric positive definite matrices of dimension $(p \ast p)$
s_coordinates	(N*2) or $(N*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
s_X	matrix Matrix with $N$ rows and unrestricted number of columns of additional covariates for the tangent space model, possibly $\mathtt{NULL}$
s_Sigma	Matrix $(p*p)$ representing the tangent point. If NULL the tangent point is computed as the intrinsic mean of s_data_manifold
s_distance	Type of distance between coordinates. It must be either "Eucldist" or "Geodist"
s_manifold_metric	Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
s_ts_metric	Metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
s_ts_model	Type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
s_vario_model	Type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
s_n_h	Number of bins in the emprical variogram
s_max_it	Max number of iterations for the main loop
s_tolerance	Tolerance for the main loop
s_max_sill	Maximum value allowed for $\emph{sill}$ in the fitted variogram. If <code>NULL</code> it is defined as $1.15*\max(\text{emp\_vario\_values})$
s_max_a	Maximum value for a in the fitted variogram. If $\mathtt{NULL}$ it is defined as $1.15*\mathtt{h\_max}$
s_weight_vario	Vector of length $N\_tot$ to weight the locations in the computation of the empirical variogram
s_distance_matrix_tot	$\label{eq:matrix} Matrix\;(N\_tot*N\_tot)\;of\;distances\;between\;the\;locations,$
s_data_manifold_tot	List of $N\_tot$ symmetric positive definite matrices of dimension $(p*p)$
s_coordinates_tot	$(N\_tot*2)$ or $(N\_tot*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees),
s_X_tot	Matrix with $N\_tot$ rows and unrestricted number of columns, of additional covariates for the tangent space model. Possibly $\mathtt{NULL}$

#### **Parameters**

s_hmax	Maximum value of distance for which the variogram is computed
s_indexes_model	<pre>Indexes corresponding to coords in coords_tot. Required only in the case metric_manifold=="Correlation"</pre>
s_weight_intrinsic	Vector of length $N$ to weight the locations in the computation of the intrinsic mean. If $\mathtt{NULL}$ a vector of ones is used. Not needed if $\mathtt{Sigma}$ is provided
s_tolerance_intrinsic	Tolerance for the computation of the intrinsic mean. Not needed if Sigma is provided
s_weight_extrinsic	Vector of length $N$ to weight the locations in the computation of the extrinsic mean. If NULL weight_intrinsic are used. Needed only if Sigma is not provided and metric_manifold=="Correlation"
s_suppressMes	Boolean. If TRUE warning messagges are not printed
s_tolerance_map_cor	Tolerance to use in the maps. Required only if metric_manifold=="Correlation"

### Returns

A list with the following fields:

- · beta Vector of the beta matrices of the fitted model
- fit\_vario\_values Vector of fitted variogram values in correspondence of hh
- hh Dense vector of positions at which fit\_vario\_values is computed
- gamma\_matrix Covariogram matrix (N \* N)
- ullet residuals Vector of the N residual matrices
- emp\_vario\_values Vector of empircal variogram values in correspondence of h\_vec
- h\_vec Vector of positions at which the empirical variogram is computed
- fitted\_par\_vario Estimates of nugget, sill-nugget and practical range
- iterations Number of iterations of the main loop
- Sigma Tangent point

### 6.11.2.2 get\_kriging()

```
RcppExport SEXP get_kriging (

SEXP s_coordinates,

SEXP s_new_coordinates,

SEXP s_Sigma,

SEXP s_distance,

SEXP s_manifold_metric,

SEXP s_ts_model,

SEXP s_vario_model,

SEXP s_beta,

SEXP s_gamma_matrix,

SEXP s_vario_parameters,

SEXP s_residuals,

SEXP s_X_new,

SEXP s_tolerance_map_cor)
```

Given the GLS model kriging prediction on new location is performed.

The model provided is used to perform simple kriging on the tangent space in correspondence of the new locations. The estimates are then mapped to the manifold to produce the actual prediction.

### Note

Reference: "Kriging prediction for manifold-valued random fields."

Authors: D. Pigoli, A. Menafoglio & P. Secchi (2016) Periodical: Journal of Multivariate Analysis, 145, 117-131.

### **Parameters**

s_coordinates	(N*2) or $(N*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
s_new_coordinates	(N*2) or $(N*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
s_Sigma	Matrix $(p*p)$ representing the tangent point. If NULL the tangent point is computed as the intrinsic mean of s_data_manifold
s_distance	Type of distance between coordinates. It must be either "Eucldist" or "Geodist"
s_manifold_metric	Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
s_ts_model	Type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
s_vario_model	Type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
s_beta	Vector of the beta matrices of the fitted model
s_gamma_matrix	Covariogram matrix $(N*N)$
s_vario_parameters	Estimates of nugget, sill-nugget and practical range
s_residuals	Vector of the $N$ residual matrices
s_X_new	Matrix (with the same number of rows of s_new_coordinates) of additional covariates for the new locations, possibly NULL
s_tolerance_map_cor	Tolerance to use in the maps. Required only if metric_manifold=="Correlation"

### Returns

# A list with the following field:

• prediction Vector of matrices predicted at the new locations

# 6.11.2.3 get\_model\_and\_kriging()

```
SEXP s_max_sill,
SEXP s_{max_a}
SEXP s_weight_vario,
SEXP s_distance_matrix_tot,
SEXP s_data_manifold_tot,
SEXP s_coordinates_tot,
SEXP s_X_{tot}
SEXP s_hmax,
SEXP s_indexes_model,
SEXP s_weight_intrinsic,
SEXP s_tolerance_intrinsic,
SEXP s_weight_extrinsic,
SEXP s_new_coordinates,
SEXP s_X_new,
SEXP s\_suppressMes,
SEXP s_tolerance_map_cor )
```

Given the coordinates and corresponding manifold values, this function firstly creates a GLS model on the tangent space, and then it performs kriging on the new locations.

The manifold values are mapped on the tangent space and then a GLS model is fitted to them. A first estimate of the beta coefficients is obtained assuming spatially uncorrelated errors. Then, in the main the loop, new estimates of the beta are obtained as a result of a weighted least square problem where the weight matrix is the inverse of gamma\_matrix. The residuals (residuals = data\_ts - fitted) are updated accordingly. The parameters of the variogram fitted to the residuals (and used in the evaluation of the gamma\_matrix) are computed using Gauss-Newton with backtrack method to solve the associated non-linear least square problem. The stopping criteria is based on the absolute value of the variogram residuals' norm if ker.width.vario=0, while it is based on its increment otherwise. Once the model is computed, simple kriging on the tangent space is performed in correspondence of the new locations and eventually the estimates are mapped to the manifold.

# Note

Reference: "Kriging prediction for manifold-valued random fields."

Authors: D. Pigoli, A. Menafoglio & P. Secchi (2016) Periodical: Journal of Multivariate Analysis, 145, 117-131.

### **Parameters**

s_data_manifold	list of $N$ symmetric positive definite matrices of dimension $(p \ast p)$
s_coordinates	(N*2) or $(N*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
s_X	matrix Matrix with $N$ rows and unrestricted number of columns of additional covariates for the tangent space model, possibly $\mathtt{NULL}$
s_Sigma	Matrix $(p*p)$ representing the tangent point. If NULL the tangent point is computed as the intrinsic mean of s_data_manifold
s_distance	Type of distance between coordinates. It must be either "Eucldist" or "Geodist"
s_manifold_metric	Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
s_ts_metric	Metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
s_ts_model	Type of model fitted on the tangent space. It must be chosen among "Intercept", "Coord1", "Coord2", "Additive"
s_vario_model	Type of variogram fitted. It must be chosen among "Gaussian", "Spherical", "Exponential"
s_n_h	Number of bins in the emprical variogram
s_max_it	Max number of iterations for the main loop

#### **Parameters**

s_tolerance	Tolerance for the main loop
s_max_sill	Maximum value allowed for $\emph{sill}$ in the fitted variogram. If <code>NULL</code> it is defined as $1.15*\max(\text{emp\_vario\_values})$
s_max_a	Maximum value for a in the fitted variogram. If $\mathtt{NULL}$ it is defined as $1.15*\mathtt{h\_max}$
s_weight_vario	Vector of length $N\_tot$ to weight the locations in the computation of the empirical variogram
s_distance_matrix_tot	$\label{eq:matrix} Matrix\; (N\_tot*N\_tot) \; of \; distances \; between \; the \; locations,$
s_data_manifold_tot	List of $N\_tot$ symmetric positive definite matrices of dimension $(p*p)$
s_coordinates_tot	$(N\_tot*2)$ or $(N\_tot*3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees),
s_X_tot	Matrix with $N\_tot$ rows and unrestricted number of columns, of additional covariates for the tangent space model. Possibly NULL
s_hmax	Maximum value of distance for which the variogram is computed
s_indexes_model	<pre>Indexes corresponding to coords in coords_tot. Required only in the case metric_manifold=="Correlation"</pre>
s_weight_intrinsic	Vector of length $N$ to weight the locations in the computation of the intrinsic mean. If NULL a vector of ones is used. Not needed if Sigma is provided
s_tolerance_intrinsic	Tolerance for the computation of the intrinsic mean. Not needed if Sigma is provided
s_weight_extrinsic	Vector of length $N$ to weight the locations in the computation of the extrinsic mean. If NULL weight_intrinsic are used. Needed only if Sigma is not provided and metric_manifold=="Correlation"
s_new_coordinates	$(N\ast2)$ or $(N\ast3)$ matrix of [lat,long], [x,y] or [x,y,z] coordinates. [lat,long] are supposed to be provided in signed decimal degrees
s_X_new	Matrix (with the same number of rows of s_new_coordinates) of additional covariates for the new locations, possibly NULL
s_suppressMes	Boolean. If TRUE warning messagges are not printed
s_tolerance_map_cor	Tolerance to use in the maps. Required only if metric_manifold=="Correlation"

### Returns

# A list with the following fields:

- beta Vector of the beta matrices of the fitted model
- fit\_vario\_values Vector of fitted variogram values in correspondence of hh
- hh Dense vector of positions at which  $fit\_vario\_values$  is computed
- gamma\_matrix Covariogram matrix (N\*N)
- $\bullet$  residuals Vector of the N residual matrices
- emp\_vario\_values Vector of empircal variogram values in correspondence of h\_vec
- h\_vec Vector of positions at which the empirical variogram is computed
- fitted\_par\_vario Estimates of nugget, sill-nugget and practical range
- $\bullet$  iterations Number of iterations of the main loop
- Sigma Tangent point
- prediction Vector of matrices predicted at the new locations

### 6.11.2.4 intrinsic\_mean()

Evaluate the intrinsic mean of a given set of symmetric positive definite matrices.

#### **Parameters**

s_data	list of $N$ symmetric positive definite matrices of dimension $(p \ast p)$
s_N	Number of data. N = s_data.size()
s_manifold_metric	Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean", "SquareRoot", "Correlation"
s_ts_metric	Metric used on the tangent space. It must be chosen among "Frobenius", "FrobeniusScaled", "Correlation"
s_tolerance	Tolerance for the computation of the intrinsic_mean
s_weight_intrinsic	Vector of length $N$ to weight the locations in the computation of the intrinsic mean. If $\mathtt{NULL}$ a vector of ones is used
s_weight_extrinsic	Vector of length $N$ to weight the locations in the computation of the extrinsic mean. If NULL weight_intrinsic are used
	NOLL Weight_Intlinste are used
s_tolerance_map_cor	Tolerance to use in the maps. Required only if
	metric_manifold=="Correlation"

### Returns

A matrix representing the intrinsic mean of the s\_data

# 6.11.2.5 distance\_manifold()

```
RcppExport SEXP distance_manifold (

SEXP s_data1,

SEXP s_data2,

SEXP s_N1,

SEXP s_N2,

SEXP s_manifold_metric )
```

Compute the manifold distance between symmetric positive definite matrices.

If N1 == N2 then the result is a vector of length N1 = N2 containing in position  $\mathtt{i}$  the manifold distance beetween datal <code>[[i]]</code> and datal <code>[[i]]</code>. Instead if N2 = 1 and N1! = 1 the result is a vector of length B1 containing in position  $\mathtt{i}$  the manifold distance between datal <code>[[i]]</code> and datal <code>[[i]]</code>

#### **Parameters**

s_data1	list of $N1$ symmetric positive definite matrices of dimension $(p \ast p)$	
s_data2	list of $N2$ symmetric positive definite matrices of dimension $(p \ast p)$	
s_N1	Number of data1. N1 = s_data1.size()	
s_N2	Number of data2. N2 = s_data2.size()	
s_manifold_metric	Metric used on the manifold. It must be chosen among "Frobenius", "LogEuclidean",	
	"SquareRoot", "Correlation"	

### Returns

A double or a vector of distances

# 6.12 IntrinsicMean.hpp File Reference

Functions to compute intrinsic and extrinsic mean for manifold data.

```
#include <Eigen/Dense>
#include <memory>
#include "DistanceTplane.hpp"
#include "Helpers.hpp"
#include "MapFunctions.hpp"
```

# **Functions**

MatrixXd intrinsic\_mean\_C (const std::vector< MatrixXd > &data\_manifold, std::string distance\_
 —
 Manifold\_name, map\_functions::logarithmicMap &logMap, map\_functions::exponentialMap &expMap,
 distances\_tplane::DistanceTplane &distanceTplane, double tolerance, const Vec &weight\_intrinsic, const
 Vec &weight\_extrinsic)

Compute the intrinsic mean of a set of manifold matrices.

MatrixXd extrinsic\_mean (const std::vector< MatrixXd > &data\_manifold, const Vec &weight\_extrinsic)
 Compute the extrinsic mean of a set of matrices in Chol (p).

# 6.12.1 Detailed Description

Functions to compute intrinsic and extrinsic mean for manifold data.

# 6.12.2 Function Documentation

# 6.12.2.1 intrinsic\_mean\_C()

Compute the intrinsic mean of a set of manifold matrices.

#### **Parameters**

data_manifold	Vector of manifold matrices
distance_Manifold_name	Name of the metric on the manifold
logMap	Logarithmic map
ехрМар	Exponential map
distanceTplane	Distance on the tangent space
tolerance	Tolerance for the algorithm's loop
weight_intrinsic	Weights
weight_extrinsic	Weights for the computation of the extrinsic mean (used if
	distance_Manifold_name=="Correlation")

### Returns

Matrix identifying the intrinsic mean of data\_manifold

### 6.12.2.2 extrinsic\_mean()

Compute the extrinsic mean of a set of matrices in Chol(p).

# Parameters

data_manifold	Vector of matrices in $Chol\left(p\right)$
weight_extrinsic	Weights

# Returns

Matrix identifying the extrinsic mean of data\_manifold

# 6.13 MapFunctions.hpp File Reference

Classes to compute the exponential and logarithmic map according to the manifold metric.

```
#include <memory>
#include "DistanceManifold.hpp"
```

# **Classes**

• class map\_functions::logarithmicMap

Abstract class for the computation of the logarithmic map.

class map\_functions::logMapFrob

Class for the computation of the logarithmic map when manifold\_metric=="Frobenius"

• class map\_functions::logMapLogEucl

Class for the computation of the logarithmic map when manifold\_metric=="LogEuclidean"

class map\_functions::logMapSqRoot

Class for the computation of the logarithmic map when manifold\_metric=="SqRoot"

class map\_functions::logMapChol

Class for the computation of the logarithmic map when manifold\_metric=="Correlation"

class map functions::exponentialMap

Abstract class for the computation of the exponential map.

class map\_functions::expMapFrob

Class for the computation of the exponential map when manifold\_metric="Frobenius"

· class map\_functions::expMapLogEucl

Class for the computation of the exponential map when manifold\_metric=="LogEuclidean"

class map\_functions::expMapSqRoot

Class for the computation of the exponential map when manifold\_metric=="SqRoot"

class map\_functions::expMapChol

Class for the computation of the exponential map when manifold\_metric=="Correlation"

# 6.13.1 Detailed Description

Classes to compute the exponential and logarithmic map according to the manifold metric.

# 6.14 Model.hpp File Reference

#### Model class.

```
#include <vector>
#include <iostream>
#include <string>
#include <utility>
#include <Eigen/IterativeLinearSolvers>
#include <memory>
#include "Helpers.hpp"
```

### Classes

· class model\_fit::Model

Class to compute and store the linear model on the tangent space.

# 6.14.1 Detailed Description

Model class.

# 6.15 Proxy.hpp File Reference

# Proxy class.

```
#include <string>
#include <memory>
#include <iostream>
#include <type_traits>
```

# Classes

class generic\_factory::Proxy< Factory, ConcreteProduct >

# 6.15.1 Detailed Description

Proxy class.