Introduction to Probability

- The study of randomness and the likelihood of various outcomes.

- **Lo studio della casualità e della probabilità di vari risultati.**

Random Variable
Variabile casuale

- A variable whose possible values are numerical outcomes of a random phenomenon. There are two types: discrete and continuous.
- **Una variabile i cui possibili valori sono risultati numerici di un fenomeno casuale. Ne esistono due tipi: discreti e continui.**

Sample Space
spazio campione

- The set of all possible outcomes of an experiment.
- **L’insieme di tutti i possibili risultati di un esperimento.**

Event
Evento

- A set of outcomes (a subset of the sample space) to which a probability is assigned.
- **Un insieme di risultati (un sottoinsieme dello spazio campionario) a cui è assegnata una probabilità.**

Probability
Probabilità

- A measure that quantifies the likelihood that an event will occur. It ranges from 0 (impossibility) to 1 (certainty).

- **Misura che quantifica la probabilità che un evento si verifichi. Si va da 0 (impossibilità) a 1 (certezza).**

Probability Distribution
Distribuzione di probabilità

- Describes how probabilities are distributed over the values of a random variable. For discrete variables, it's represented by a probability mass function (PMF), and for continuous variables, by a probability density function (PDF)
- **Descrive come vengono distribuite le probabilità sui valori di una variabile casuale. Per le variabili discrete, è rappresentato da una funzione di massa di probabilità (PMF) e per le variabili continue, da una funzione di densità di probabilità (PDF).**

Expected Value
Valore atteso

- The long-run average value of repetitions of the experiment it represents. For a discrete variable, it's the sum of the product of each outcome and its probability.
- **Il valore medio a lungo termine delle ripetizioni dell'esperimento che rappresenta. Per una variabile discreta, è la somma del prodotto di ciascun risultato e della sua probabilità.**

Variance and Standard Deviation
Varianza e deviazione standard

- Variance measures the dispersion of a set of data points around their mean value. Standard deviation is the square root of the variance.
- **La varianza misura la dispersione di un insieme di dati attorno al loro valore medio. La deviazione standard è la radice quadrata della varianza.**

Sample space and Events

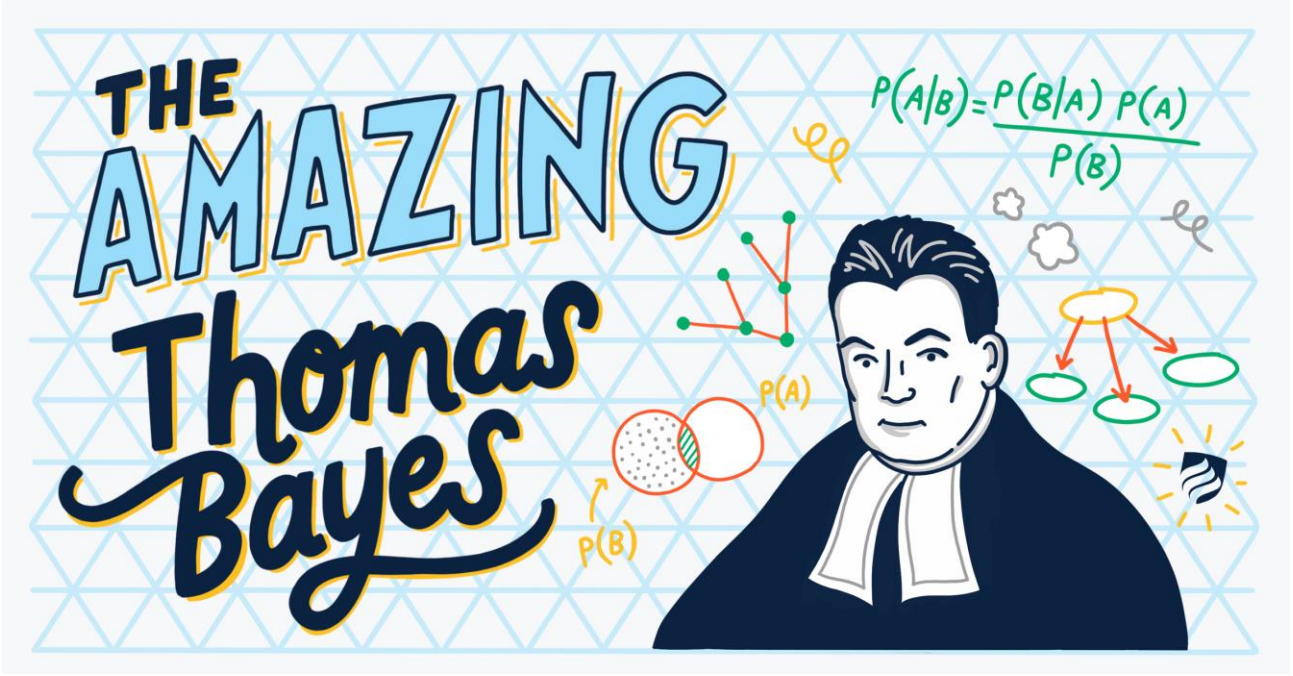
- Ω : Sample Space, result of an experiment
  - If you toss a coin twice $\Omega = \{\mathrm{HH},\mathrm{HT},\mathrm{TH},\mathrm{TT}\}$
- Event: a subset of Ω
  - First toss is head = {HH,HT}
- S: event space, a set of events:
  - Closed under finite union and complements
    - Entails other binary operation: union, diff, etc.
  - Contains the empty event and Ω

Probability Measure

- Defined over (Ω, S) s.t.
  - $P(\alpha) >= 0$ for all α in S
  - $P(\Omega) = 1$
  - If α, β are disjoint, then
    - $P(\alpha \cup \beta) = p(\alpha) + p(\beta)$
- We can deduce other axioms from the above ones
  - Ex: $P(\alpha \cup \beta)$ for non-disjoint event

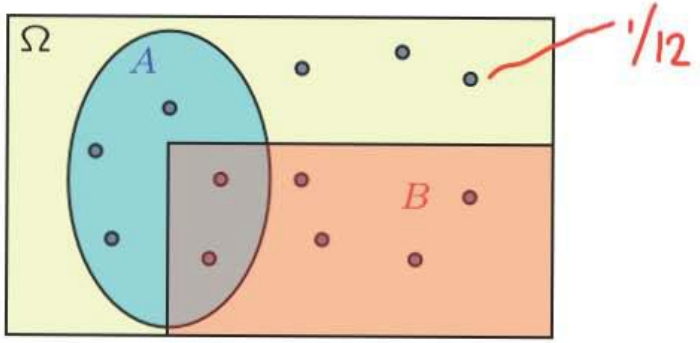
    $P(\alpha \cup \beta) = p(\alpha) + p(\beta) - p(\alpha \cap \beta)$

1701 – 7 April 1761

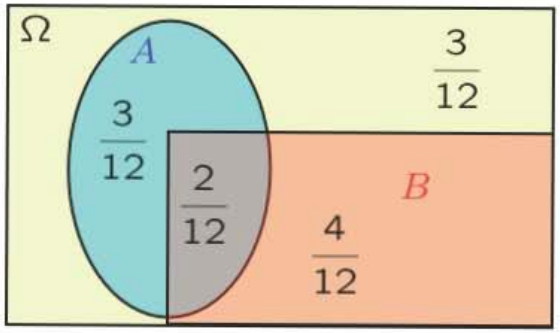


The idea of conditioning

Assume 12 equally likely outcomes

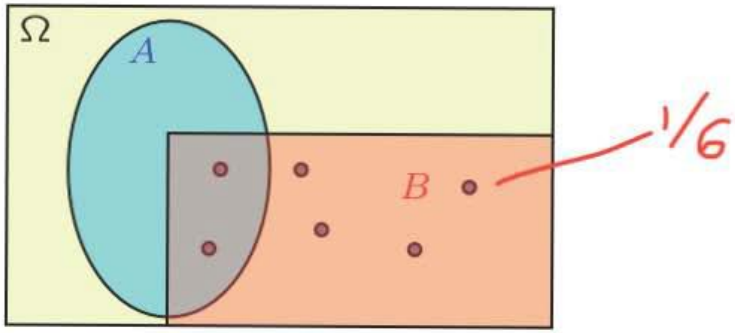


$$\mathbf{P}(A) = \frac{5}{12} \qquad \mathbf{P}(B) = \frac{6}{12}$$

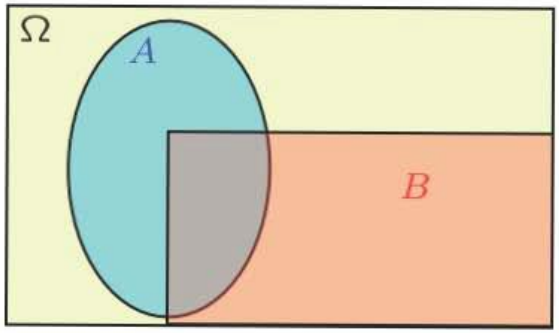


Use new information to revise a model

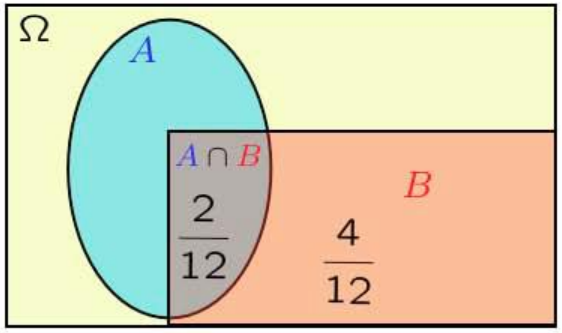
If told B occurred:



$$\mathbf{P}(A \mid B) = \frac{2}{6} = \frac{1}{3} \qquad \mathbf{P}(B \mid B) = 1$$



Definition of conditional probability



- $\mathbf{P}(A \mid B)$ = "probability of A, given that B occurred"

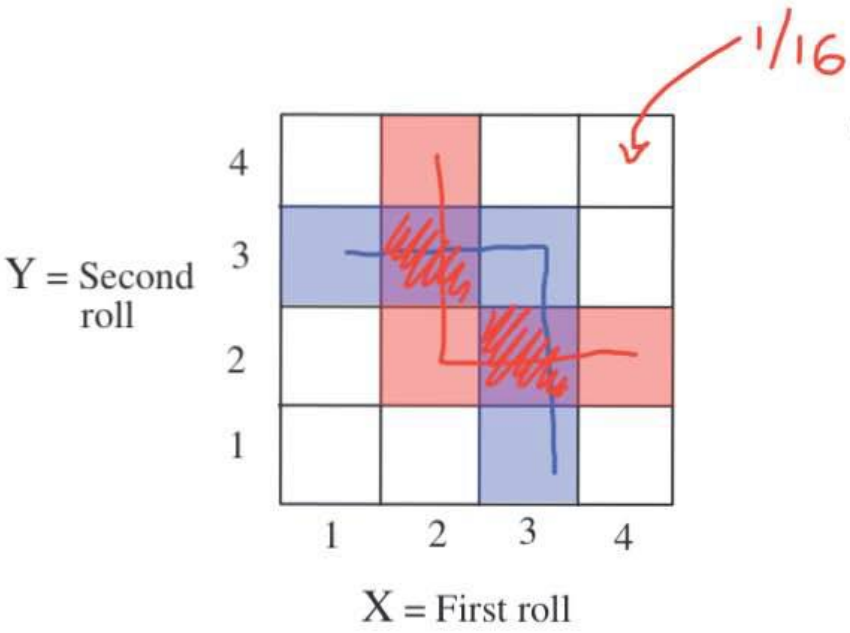
Def. →

$$\mathbf{P}(A \mid B) \triangleq \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

defined only when $\mathbf{P}(B) > 0$

$= \frac{2/12}{6/12} = \frac{1}{3}$

Example: two rolls of a 4-sided die



- Let B be the event: $\min(X, Y) = 2$

  Let $M = \max(X, Y)$

$\mathbf{P}(M = 1 \mid B) =$ 0

$$\mathbf{P}(M = 3 \mid B) = \frac{P(M=3 \text{ and } B)}{P(B)}$$

$$= \frac{2/16}{5/16} = \frac{2}{5}$$