Literature Review on Many-Body Ground States via Quantum Circuits

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1 Introduction

Quantum mechanics has not only revolutionized science and how we think of nature, but more recently ,in the second quantum revolution [1], it has opened the scene for quantum information and quantum computing. The NISQ era [2] presents us with many new technologies and practices like quantum cryptography [3] and quantum deep learning [4]. In order to run,test and develop new quantum technologies and algorithms one must first have a realisation of a quantum computer. Many proposals, using different physical systems, have been provided: qubits via superconducting materials [5, 6], quantum computation using Rydberg atoms [7, 8], photonic quantum computing [9, 10] and silicon based quantum computers [11, 12]. Quantum computation can also be used to simulate physical systems [13]. One area of physics that greatly benefits from this is the study of multi-body systems and as a result of this, the study of condensed matter physics. In our project we will tackle and study how quantum circuits can be used to find and approximate ground states of such systems.

The complexity of many-body physics lies in the fact that the dimension of the underlying Hilbert space grows exponentially as p^N, where here p is the dimension of each of the individual spaces and N is the number of particles. For most purposes we will consider two level systems $p = 2$, then we get a 2^N dimensional Hilbert space where a state $|\psi\rangle = \sum_{i_1,\cdots,i_N} a_{i_1\cdots i_N} |i_1 \cdots i_N\rangle$ where here $i_j \in \{0, 1\}$ for $j \in \{1, \cdots, N\}$, and we suppressed the tensor product symbol ($|00\rangle = |0\rangle \otimes |0\rangle$).

As mentioned above, optimization problems and multi-body quantum systems are a topic of great interest. The Variational quantum eigensolver (VQE) was used by IBM to find the ground state energy of molecules in quantum chemistry [14] and it was also implemented on a photonic quantum processing unit [15] to solve problems involving large systems of molecules. Other algorithms suited for different purposes are: the QAOA (quantum approximate optimization algorithm) which was used to produce approximate solutions to combinatorial optimization problems [16] and found to outperform other methods [17], the DMRG (density-matrix renormalization group) which is one of the most powerful algorithms for studying one dimensional quantum lattices [18, 19]. It turns out that DMRG can be linked to MPS (matrix product states) formalism [20], a powerful way of rewriting a quantum state in a multi multi body system, in fact DMRG can be formulated in the language of MPS as studied extensively in this

article [21]. In the recent years a lot of research was made concerning the above, and many other quantum algorithms. Most of these methods give performance speed-ups compared to competing classical methods solving the same problem; in the case of multi-body quantum systems studies this is due to the fact that quantum circuits that run on quantum computers, deal with a lot of entanglement fairly easily which is profitable for solving such systems numerically, whereas ,classically, a high amount of entanglement makes the classical methods struggle. On the contrary, not much is yet known on the limitations of the quantum algorithms via quantum circuits. Since the quantum algorithms solve the same problems as their classical counterparts and both these depend on the same number of parameters we have to assume that the quantum algorithms have certain limitations as well. In the case of multi-body systems, entanglement is the limitation for the classical methods, however entanglement is effectively unlimited in the quantum methods, thus it means some other property must carry the limitations.

We seek to analyse these systems by first solving the eigenvalue problem for certain Hamiltonians and looking at the exact ground states. To do this we will consider multiple models: ising, XXZ and ising with transverse and longitudinal field. These were used as benchmarks in the literature [22–24] thus form convenient examples. To solve these, we will adopt brute force and numerical methods. We then model certain quantum circuits using python and use these to solve an optimization problem, namely we want to optimize the parameters in the quantum circuits to maximize the overlap of the resulting states with the exact states (fidelity). Optimization of fidelity is carried out to find the most optimal quantum circuit described by unitaries; an example of this optimization procedure is described in appendix C of [23]. We will use the exact states found out using the methods above, however exact states can be found through other methods like MPS, and those can be used instead via certain algorithms [23]. We will then numerically calculate different correlation functions and other objective functions for the exact states and for the quantum circuit approximations, and study how the results differ from each other. For example we might look at the entanglement entropy [24]. We will plot the results to show the deviation of the ground states, generated by our quantum circuit ansatz, to the exact states; plots of this form for the ising model using a sequential ansatz were already provided in [23]. Moreover, We will look at different circuit architecture and do all the above for these different circuit structures and record the differences in the results.

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