Project Plan

27 October 2022 17:09

Introduction

The quantum many-body problem is a challenge in physics that has been the topic of a large amount of recent discussion due to the ability of a many-body problem to predict the behaviour of interacting particles. Using quantum circuits is one such approach which provides a more concise way to efficiently approximate the ground sates of a many-body system and this will be the topic of this project. We aim to compare numerical methods of calculating ground states to our approach, and deduce any issues of quantum circuits.

Objectives

One of the primary objectives would be to study the limitations of the quantum circuits. We will also seek to get a new understanding of correlations on quantum circuits, which might also help us with our first objective . Moreover, we want to propose new circuit structures for approximating ground states.

Task 1: Numerically exact ground states

Description:

In this task we consider different models of Hamiltonians, namely: ising, XXZ (integrable) and ising + transverse & longitudinal field (non integrable). We then solve for the exact eigen values and eigen vectors numerically using python NumPy. We can vary the parameters in the Hamiltonian and note how these change the resulting ground state solutions. We can do these calculations for multiple particle systems, however we will do it up to possibly $n = 16$ particles since the size of the matrices increases as 2^n and the numerical solver will start to struggle.

Deliverables:

D1 a) Code solving the various Hamiltonians for multiple particle systems
D1 b) Database storing the different ground states and corresponding ground state energies for the different Hamiltonians and different system size n

Task 2: Optimization of Quantum Circuits

Description:

Simulating and optimizing a quantum circuit using Python. This is a different way of solving a quantum many body system and will allow us to simulate many more qubits in a reasonable amount of time compared to exact diagonalization. Operators such as the Hadamard gate and the controlled not (CNOT) gate and general two-qubit gates will be used across qubits in this simulation.

Deliverables:

D2) Code allowing us to simulate and optimize a desired quantum circuit, adding any operators on a N dimensional qubit system

Task 3: Measurement of Correlation Functions and Entanglement Measures

Description:

Using the results from task 2, the aim is to measure certain quantities on the "brick wall" setup of gates, namely correlation functions and the amount of entanglement on a given number of qubits. This approach will be applied to both of the ways we have solved for ground states so far; exact diagonalization and quantum circuit approximations.

Deliverables:

D3) A working measurement of the entanglement and compile a list of correlation functions/

Task 4: Analysis and comparison between the methods

Description:

Once we have optimized the quantum circuit (QC), we are ready to compare the results of the QC with the exact results obtained numerically in Task 1. We can compare the time and space complexity of these two methods, also we can look at how these methods perform in different regimes, for example with varying n. We then ask ourselves what are the limitations for the exact numerical solution in task 1, and also how does the QC solve those limitations; more importanlty, we want to investigate what are the limitations for the QC method. Once we understand these limitations we can try to quantify them by deriving appropriate bounds.

Deliverables:

D4 a) Table showing the differences between the two methods
D4 b) Plotted diagrams of how different properties vary for the two methods e.g n with speed of algorithm.

Task 5: Test alternative objective functions

Description:

Instead of maximizing the fidelity, we could maximize other correlation functions or measure other physical properties of interest like entanglement entropy/energy. We will then carry out tasks 3 and 4 for these other correlation functions and compare to previous results.

Deliverables:

D5 a) Table comparing the different correlation functions, for example how the optimized parameters differ
D5 b) Python code for the optimization part.

Task 6: Propose and Test alternative circuit structures

Description:

We will investigate alternative circuit structures, for example using MERA. After implementing this methodology, we will evaluate how it changes tasks 3&4.

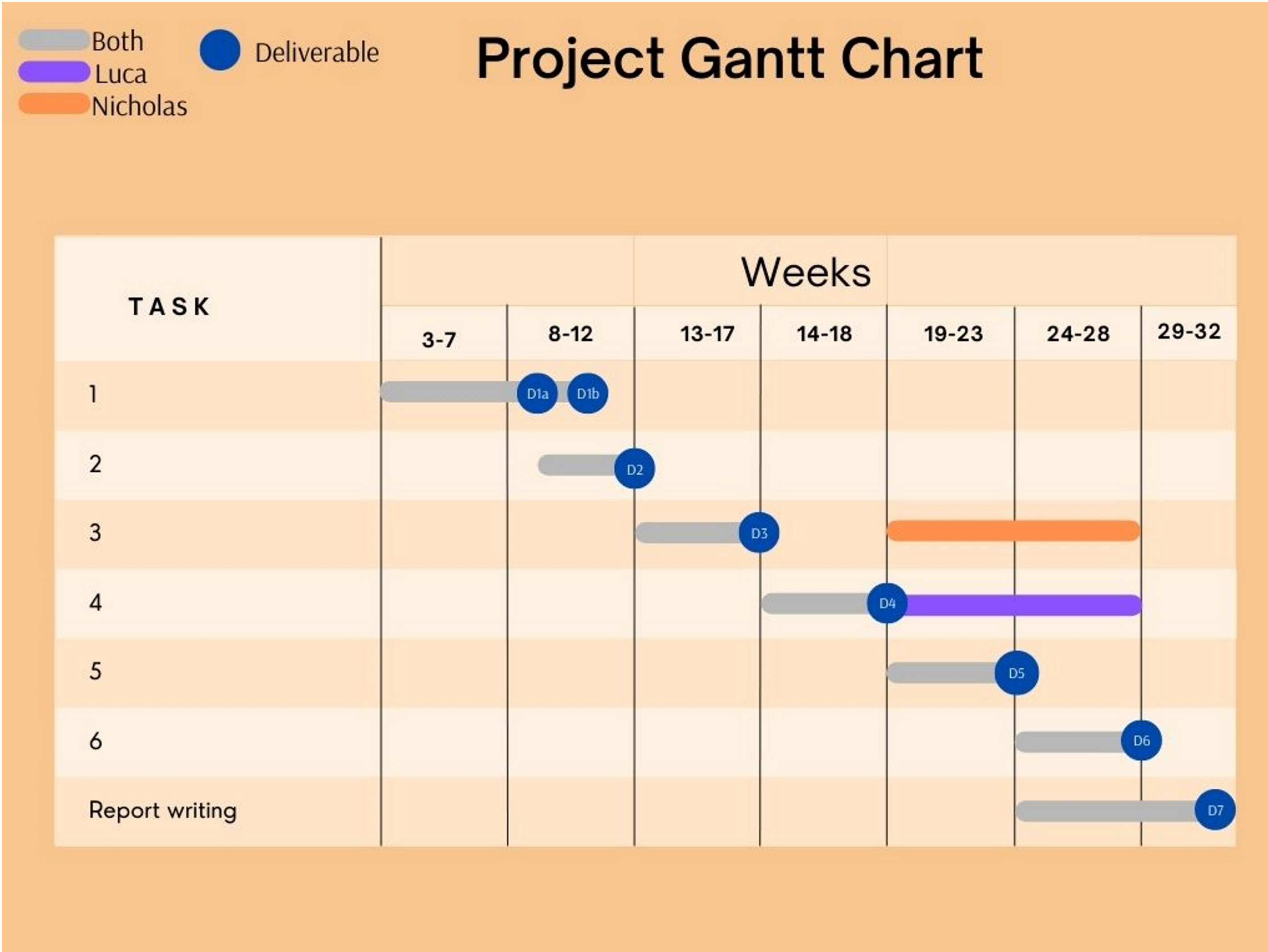
Deliverables:

D6) A comparison of how using a different setup for our quantum circuit impacts our previous work

Resources

Since the complexity of the physical systems in question increases rapidly with the number of particles, we will use a combination of personal and university computers, potentially we might also use Nottingham HPC. For the quantum circuits methods, we will use python libraries to simulate these as running on a quantum computer. Various other python libraries and other resources will be used, all summarised below:

- Personal computer
- Nottingham HPC
- Google Colaboratory
- Python
- Numpy,Scipy, Numba, Jax, Optax
- Database storage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Luca
Nicholas
Deliverable
Project Gantt Chart
TASK
Weeks
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8-12
13-17
14-18
19-23
24-28
29-32
1
2
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Report writing
D1a
D1b
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D4
D5
D6
D7