Exchange Rate Volatility and Monetary Policies

Fabio Comazzi, Stefano Meschi, Luca Russo, Luca Zerman

June 5, 2022

Contents

1 Introduction

Uncertainty plays a crucial role in the economic activities of a country. One of the most relevant is represented by the exchange rate volatility, given that nowadays economies are globally interlinked. Specifically, we focused on the EUR/USD exchange, with the aim of proposing different models for its volatility considering many financial indices, some of which represent monetary policy choices of ECB, some other quantities that can play a relevant role in the exchange market.

2 Literature Review

The **exchange rate** is the rate at which the currency of one country is converted to another currency of a different country. Its volatility is associated with unexpected exchange rate movements [1] and it has been a serious concern for those in the academia, policymakers, and participants in the financial market [2].

High fluctuation of the exchange rate in a short time horizon makes economic activity more risky as uncertainty rises. For example, Miyajima [3] observed that when the volatility in the rate of exchange spikes, it leads to an increase in core inflation. As it is not good for the economy, there should be a systematic and measured policy to mitigate the foreign exchange fluctuations and to minimize them, as well as to drive the rate to its fundamental value [4].

Monetary policy is how the monetary authority of each country (central bank) decides the amount of money supply for that country. It has been the "fundamental instrument over the years in attaining macroeconomic stability and a prerequisite to attaining sustainable output growth" [5] [6]. It represents, as well, the regulating amount of the supply of money in a country's economy in order to attain an optimal mix of output and inflation goal realization [7]; exchange rates can thus act as "automatic stabilisers" for the macro-economy of any nation [8].

Eze and Okotori [9] found that, in the long run, monetary policy instruments tend to have a significant nexus with the volatility in the exchange rate, meaning that some monetary policy variables have a significant long run correlation with the exchange rate. Furthermore, they found that while money supply and the exchange rate seem to have a significant short run impact on exchange rate volatility, other variables such as liquidity ratio or monetary policy rate did not show a significant short run relationship with it.

Adeoye and Saibu [10] analyzed this topic in the specific case of Nigeria, observing that there is a causal link between the past values of monetary policy variables (e.g. interest rates) and the exchange rate, such that a change in the level of previous values of monetary policy variables causes exchange rate volatility; they concluded that "inflation rate, reserves, interest rate and money supply depreciate and cause volatility in nominal exchange rate which further reinforce other findings that monetary policy is crucial to exchange rate management in Nigeria". They also found that "the short-run dynamics reveal that the changes in monetary policy instruments correlate to the variations in the rate of exchange via a process that is self-correcting, without the involvement of the CBN (the Central Bank of Nigeria)".

Focusing on Indonesia and more generally on all developing countries, it has been found [4] that "Indonesia as a small open economy tends to have a high and persistent exchange rate volatility, when this result holds in most open emerging countries", and that an increase in real interest rate parity causes appreciation of the domestic currency. Furthermore, Foreign-exchange-Sale Interventions (a monetary policy strategy implemented by the Central Bank of Indonesia) causes a slight decrease in the return of the exchange rate, meaning that the monetary policies actually have got an effect on the evolution of the exchange rate volatility.

Hassan [11] suggested that a possible way to reduce exchange rate volatility could be to increase the foreign exchange reserve, like several developing countries do: the findings of his research reveal that "there is still ample scope to accumulate reserves to absorb large inflows when the exchange rate is highly likely to be overvalued and to be contributing to a loss of competitiveness", since "there is a very strong relationship between a country's ratio of reserves to external financing requirement and the extent of the sell-off of its currency over this period".

Adusei and Gyapong [12], focusing on the Ghanaian situation, observed that inflation rate, monetary policy rate, current account balance, money and quasi money supply per GDP, annual GDP growth rate and the total external debt are significant predictors of the cedi-dollar exchange rate in Ghana, supporting the already present evidences that monetary policy tools can generally help driving the exchange rate volatility in a desired way.

An and Sun [13] investigated the linkage between monetary policy, the monetary authority's intrusion into the foreign exchange market, and the rate of exchange for Japan. Their results show that "the impact of intervention is not effective, possibly it has even a negative effect, and that normal monetary policy seems to exert a major impact on both the rate of exchange and interventions in foreign exchange". They also observed that in response to contractionary monetary policy shocks, the exchange rate appreciates for a short while with the maximum effect coming within several months, and then depreciates over time to the original level in Japan.

Cagliarini and Mckibbin [14] ascertained that in the US monetary policy tends to affect relative prices for up to four years, since "the effect of a temporary change in real interest rates varies across sectors. The effect depends on each sector's relative capital intensity as well as on the change in the demand for the output of each sector as consumption and investment adjust". Eventually, the effect of monetary policy on relative prices dissipates.

Regarding specifically the EU, it has been found [15] that ECB systematically responds to exchange rate movements but quantitative effects are small; this is consistent with the hypothesis that the central banks do not target the fluctuations in the exchange rate but consider them only to the impact they have on the expected inflation and output path. It has also been observed [16] that "the most defining element of the ECB's monetary policy framework, *i.e.* its characteristic definition of price stability with a hard 2% ceiling, worked as a key shock-absorber in the relatively high-inflation years prior to the crisis, but offered a softer defence in the face of the disinflationary forces that hit the euro area in its aftermath. The imperative to halt persistent disinflation in the post-crisis era therefore called for a radical, unprecedented policy response, comprising negative policy rates, enhanced forms of forward guidance, a large asset purchase program and targeted long-term loans to banks".

Finally, for what concerns methodological approaches to model time series volatility, William D. Lastrapes [17] have used the ARCH framework to model the conditional variance of five exchange rates. In particular, they used a dummy variable to account for monetary policy regime shifts and find a significant relationship in 4 out of 5 exchange rates [17].

3 Data and Methodology

3.1 Dataset

In order to create our dataset, we first downloaded, from the ECB statistical data warehouse and from the FRED platform, monthly observations from October 2004 up to March 2022 of the following quantities:

1. `EURUSD`: **EUR/USD exchange rate**;
2. `M3`: **Monetary aggregate**, namely the sum of Euro currency in circulation, overnight deposits, deposits with an agreed maturity of up to two years, deposits redeemable at notice of up to three months, repurchase agreements, money market fund shares/units and debt securities with a maturity of up to two years;
3. `HICP`: **Harmonized Index of Consumer Prices**, hence a measure of the inflation;
4. `MRO`: **interest rate on Main Refinancing Operations**, meaning the interest rate the banks pay when they borrow money from the ECB for one week;
5. `yieldEU_1y`: **Yield curve spot rate** of Euro Zone bonds (all issuers of all ratings included) with one year maturity;

6. `ExtRes`: **Foreign currency reserves**, so the quantity of reserves in dollars that ECB holds;
7. `yieldUSA_1y`: **Yield curve spot rate** of the government Treasury Bill with one year maturity;
8. `Oil_price`: **oil price per barrel**;
9. `inf_USA`: **inflation rate computed on the CPI index**.

After that, we created two other time series that we thought would be useful for our analysis:

1. `yield_diff`: the difference between yield curve spot rate of US bonds and yield curve spot rate of EU bonds with one year maturity. We created this time series because of its power in explaining when most probably investors switch from one country's bonds to the ones of the other country, as suggested by [19];
2. `EURUSD_vol`: **EUR/USD empirical volatility**. Since this is something not observable, we decided, between other methods, to perform a rough estimation of it, specifically taking daily data of the exchange rate and performing the empirical standard deviation on each month.

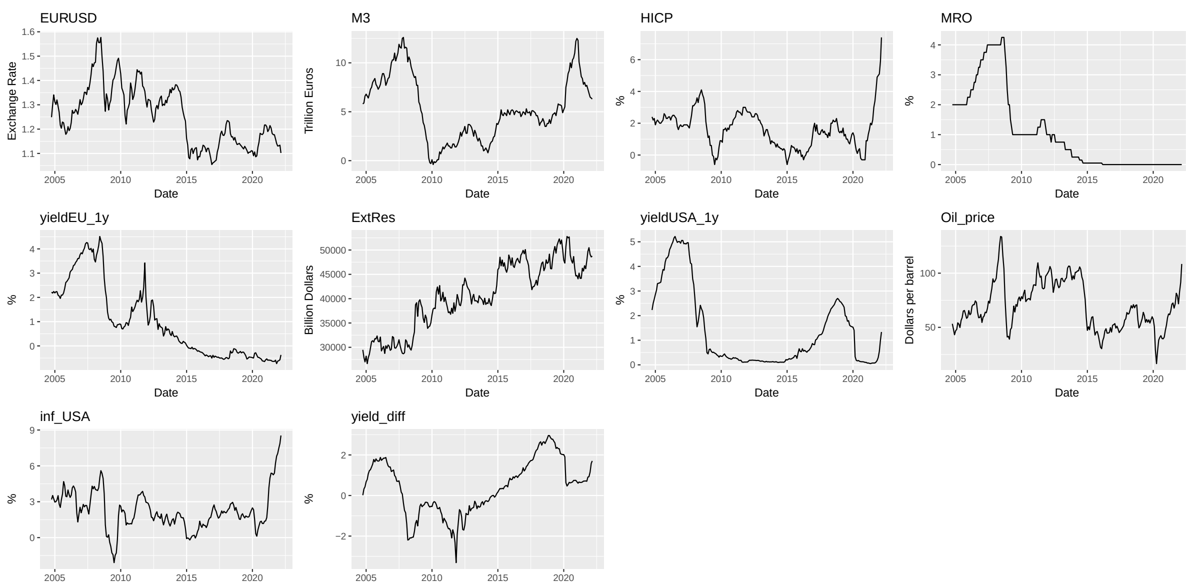

Figure 1: Plots of our ten time series.

3.2 Goals and Models

Regarding our proposals, first we focused on **modelling the EUR/USD exchange rate** through some economical variables we thought could be significantly linked to it. In particular, we started performing two regression models, one considering our time series and the other with them lagged of one month, in order to spot some correlations and short run relationships between our variables and the exchange rate, in addition to testing the forecasting ability of our model. Afterwards, we dedicated ourselves to finding a cointegration relationship, in order to account also for a long run relationship with the exchange rate, and, consequently, we built a Vector Error Correction Model (VECM) ([9]).

Subsequently, we addressed the issue of **estimating the EUR/USD exchange rate volatility** exploiting three different methods:

1. GARCH model for the residuals of the aforementioned VECM;
2. ARMA and GARCH model for the EUR/USD returns, as suggested by Abdullah et al. [18], together with a dummy variable accounting for decision in monetary policy of the ECB as in [17];
3. Empirical estimate of the volatility, obtained by the empirical standard deviation on daily EUR/USD exchange rate over each month.

4 Results

4.1 Models on EUR/USD exchange rate

4.1.1 Regression Models

We started implementing some regression models with the aim of explaining the EUR/USD exchange rate through other time series. Initially, in order to avoid some dangerous spurious regressions, we checked if our time series were stationary or not through the Augmented Dickey Fuller Test. Specifically, we considered this test using 4 lags, as written below:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + ... + \delta_3 \Delta y_{t-3} + \epsilon_t \,,$$

where $H0 : \gamma = 0$ vs $H1 : \gamma < 0$.

We performed all the tests considering the time trend coefficient β equal to 0, as we did not noticed relevant trends in our data, except for `ExtRes` which, instead, showed a time dependent pattern (as we can see in Figure 1). The results are shown in the table below:

Variable	p-value
EURUSD	0.211
M3	0.338
HICP	0.652
MRO	0.538
yieldEU_1y	0.641
ExtRes	0.054
yieldUSA_1y	0.426
Oil_price	0.066
inf_USA	0.336
yield_diff	0.524

Given that all the p-values are above 0.05, at 5% level we cannot reject the hypothesis of the presence of a unit root for any of these time series.

Since, as we said, we cannot use non-stationary time series for regression, we computed the differences of our time series and performed the same Augmented Dickey Fuller Test; now, as we can see from Figure 2, all our time series have no trend, thus we consider always $\beta = 0$.

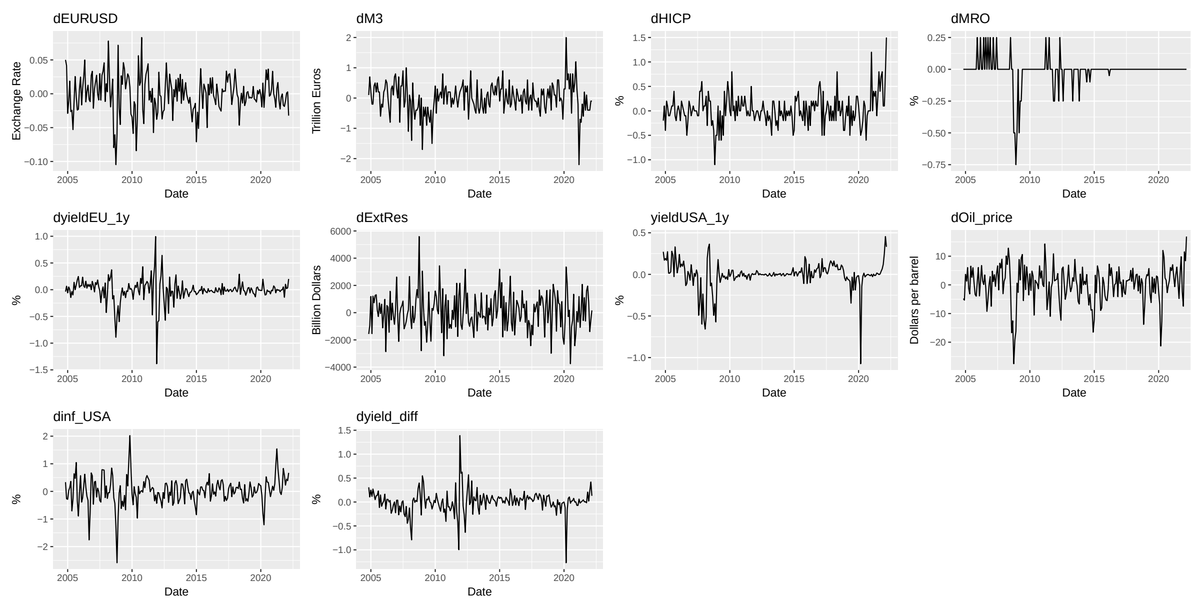

Figure 2: Plots of the differences of our ten time series.

The results of the tests are shown in the table below:

Variable	p-value
dEURUSD	0.01
dM3	0.01
dHICP	0.01
dMRO	0.01
dyieldEU_1y	0.01
dExtRes	0.01
dyieldUSA_1y	0.01
dOil_price	0.01
dinf_USA	0.01
dyield_diff	0.01

As we can see, all the p-values are below the threshold of 5%; thus, we accept the hypothesis of stationarity for all the time series and we can proceed with our analysis.

We first performed a regression model considering the differentiated variables at time t as regressors, in order to capture information about correlation between the differences of EUR/USD exchange rate and the differences of the other variables. After having excluded `dyieldEU_1y` and `dyieldUSA_1y` in place of `dyield_diff`, we implemented the following regression model:

$$\begin{aligned}\texttt{dEURUSD}_t = &\beta_0 + \beta_1 \texttt{dM3}_t + \beta_2 \texttt{dHICP}_t + \beta_3 \texttt{dMRO}_t + \beta_4 \texttt{dinf\_USA}_t + \\ &\beta_5 \texttt{dyield\_diff}_t + \beta_6 \texttt{dOil\_price}_t + \beta_7 \texttt{dExtRes}_t + \varepsilon_t \,,\end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma^2)$. From the t-tests of the single variables we do not have evidence to reject $\beta_1 = 0$ and $\beta_3 = 0$. In order to confirm so, we performed a Wald Test, considering jointly $H0 : \beta_1 = \beta_3 = 0$ vs $H1 : \beta_1 \neq 0$ or $\beta_3 \neq 0$; the p-value of the test is 0.1985, hence we cannot reject $H0$, removing the two related variables. The new regression model is:

$$\begin{aligned}\texttt{dEURUSD}_t = &\beta_0 + \beta_2 \texttt{dHICP}_t + \beta_4 \texttt{dinf\_USA}_t + \\ &\beta_5 \texttt{dyield\_diff}_t + \beta_6 \texttt{dOil\_price}_t + \beta_7 \texttt{dExtRes}_t + \varepsilon_t \,,\end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma^2)$. The results are shown in Figure 3:

```
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.186e-04  1.467e-03   0.149 0.881663    
dHICP       -2.772e-02  6.097e-03  -4.547 9.34e-06 ***
dinf_USA     1.562e-02  4.009e-03   3.895 0.000133 ***
dyield_diff -2.623e-02  6.151e-03  -4.265 3.06e-05 ***
dOil_price   1.180e-03  2.754e-04   4.285 2.83e-05 ***
dExtRes     -8.233e-06  1.113e-06  -7.398 3.57e-12 ***
```

Figure 3: Coefficients and p-values of regression with 0 lag.

As we can see, dHCIP, $\texttt{dinf\_USA}$, $\texttt{dyield\_diff}$, $\texttt{dOil\_price}$ and $\texttt{dExtRes}$ at time t show a significant correlation with $\texttt{dEURUSD}$ at time t. Furthermore, $R^2 = 0.4285$ and all the hypotheses of linear regression are satisfied.

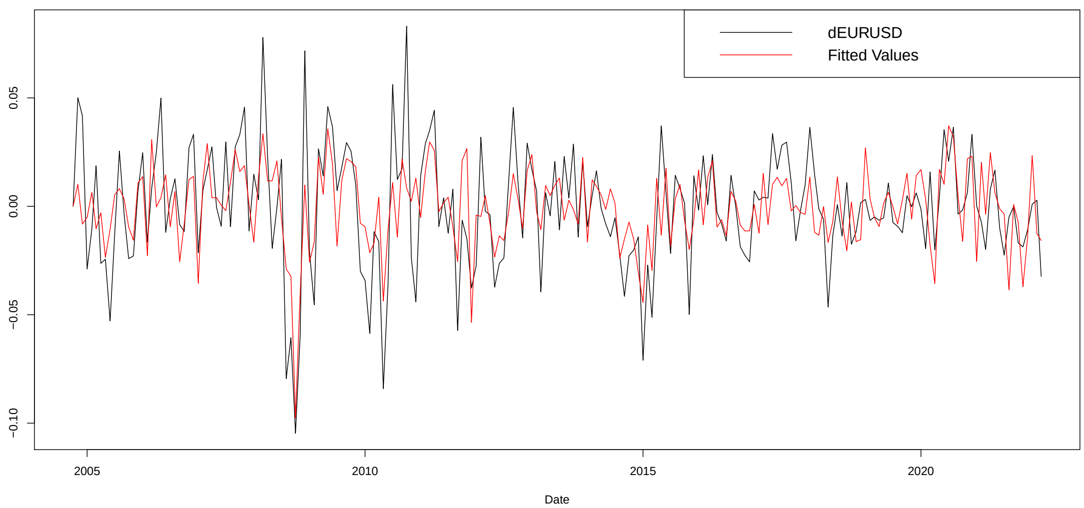

Figure 4: Difference of EUR/USD exchange rate against fitted values of the regression with 0 lag.

In addition to the significant result of R^2, Figure 4 confirms that the short-run relationship between these variables is appreciable.

Afterwards, we wanted to see if the differences of our time series have also prediction power on the differences of the exchange rate. To do so, we exploited the same regression framework of before but considering a one-month lag in the independent variables, in this way:

$$\begin{aligned}\texttt{dEURUSD}_t =& \beta_0 + \beta_1 \texttt{dM3}_{t-1} + \beta_2 \texttt{dHICP}_{t-1} + \beta_3 \texttt{dMRO}_{t-1} + \beta_4 \texttt{dinf\_USA}_{t-1} + \\ & \beta_5 \texttt{dyield\_diff}_{t-1} + \beta_6 \texttt{dOil\_price}_{t-1} + \beta_7 \texttt{dExtRes}_{t-1} + \varepsilon_t \,,\end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma^2)$. Through the same considerations and performing the same tests as before, the final model is:

$$\begin{aligned}\texttt{dEURUSD}_t =& \beta_0 + \beta_2 \texttt{dHICP}_{t-1} + \beta_4 \texttt{dinf\_USA}_{t-1} + \\ & \beta_5 \texttt{dyield\_diff}_{t-1} + \beta_6 \texttt{dOil\_price}_{t-1} + \beta_7 \texttt{dExtRes}_{t-1} + \varepsilon_t \,,\end{aligned}$$

where $\varepsilon_t \sim N(0, \sigma^2)$.

```
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)           -1.572e-04  1.720e-03  -0.091  0.92727
L(ts(dHICP), 1)       -2.318e-03  7.541e-03  -0.307  0.75882
L(ts(dinf_USA), 1)    -9.222e-03  4.762e-03  -1.936  0.05422 .
L(ts(dyield_diff), 1) -4.866e-03  7.219e-03  -0.674  0.50100
L(ts(dOil_price), 1)   9.516e-04  3.239e-04   2.938  0.00369 **
L(ts(dExtRes), 1)     -7.443e-06  1.304e-06  -5.708 4.03e-08 ***
```

Figure 5: Coefficients and p-values of regression with 1 lag.

The results are represented in Figure 5. In this case, the results are poorer than before: only `dOil_price` and `dExtRes` are significant at 5% and $R^2 = 0.207$; this, obviously, affects also the plot of the fitted values:

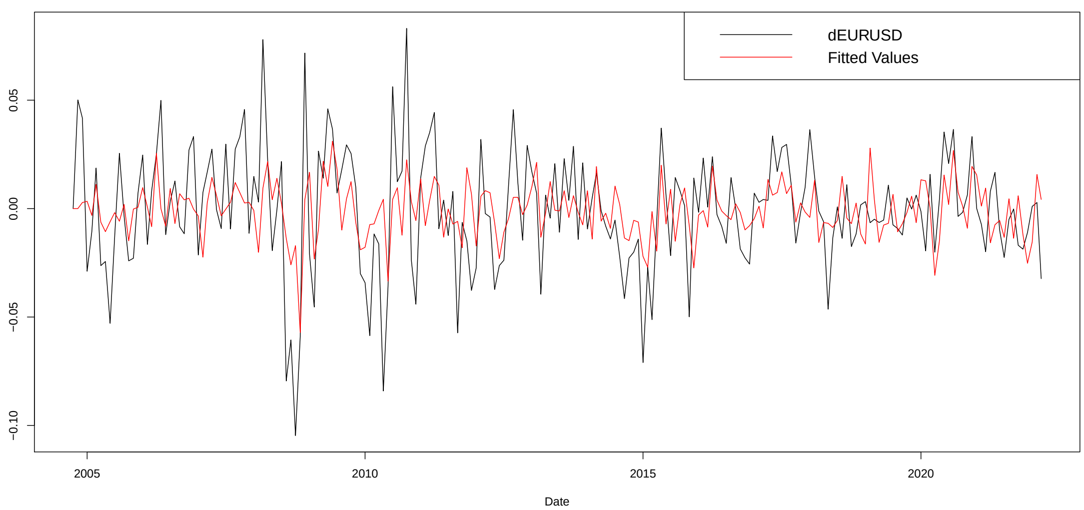

Figure 6: Difference of EUR/USD exchange rate against fitted values of the regression with one lag.

We could have expected these worse results, because now we are trying to predict the differences on the exchange rate, we are not only doing inference "on-time"; anyway, our findings are still noteworthy in explaining the short-run behaviours.

4.1.2 Cointegration and VECM

At this point, we wanted to create a model which takes into account not only short-run relationships but also long-run ones between the exchange rate and our time series; we did so through a cointegration model.

First, stated that all our time series are $I(1)$ thanks to the Augmented Dickey Fuller tests performed before, we decided to proceed also here with `yield_diff` instead of the two different rates and to exclude `MRO` from our analysis.
In order to find whether there is at least a cointegration relationship between EUR/USD exchange rate and the other time series, we exploited the Johansen method: we started selecting the optimal number of lags, 2, for the equation through some Information Criteria (AIC, HQ, FPE). Afterwards, we performed the Trace test which showed the presence, at level 5%, of one cointegration relationship (the value of our statistic, 136.89, was higher than the critical value at 5%, 131.70, thus we reject the hypothesis of rank = 0 but we cannot reject the hypothesis of rank = 1): this result is crucial, indeed we can state that there is a long run relationship between the EUR/USD exchange rate and the other time series.
In particular, we obtained the following cointegration equation:

$$\begin{aligned}\texttt{EURUSD} =& 1.302 + 7.497 \cdot 10^{-2}\texttt{M3} + 1.119 \cdot 10^{-1}\texttt{yield\_diff} + 4.189 \cdot 10^{-1}\texttt{HICP} + \\ & - 1.407 \cdot 10^{-5}\texttt{ExtRes} + 1.016 \cdot 10^{-2}\texttt{Oil\_price} - 6.047 \cdot 10^{-1}\texttt{inf\_USA} + u_t\,,\end{aligned}$$

where $u_t \sim I(0)\,.$
Afterwards, we can state the general Vector Error Correction Model as:

$$\Delta \mathbf{X}_t = \mathbf{\Gamma}_1 \Delta \mathbf{X}_{t-1} + \mathbf{\Pi} \mathbf{X}_{t-2} + \mu + \varepsilon_t\,, \tag{1}$$

where $\mathbf{X}_t = (\texttt{EURUSD}, \texttt{M3}, \texttt{yield\_diff}, \texttt{HICP}, \texttt{ExtRes}, \texttt{Oil\_price}, \texttt{inf\_USA}) \in \mathbb{R}^7\,.$
The numerical results are the followings:

```
Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
ect1           -1.501e-01  6.199e-02  -2.422 0.016328 *
EURUSD.dl1     -3.515e-02  1.372e+00  -0.026 0.979587
M3.dl1         -1.437e-01  5.630e-02  -2.553 0.011430 *
yield_diff.dl1  1.045e-01  1.220e-01   0.856 0.392820
HICP.dl1        1.541e-01  1.324e-01   1.164 0.245701
ExtRes.dl1     -3.438e-05  2.371e-05  -1.450 0.148625
Oil_price.dl1   1.879e-02  5.509e-03   3.411 0.000784 ***
inf_USA.dl1     2.555e-01  8.067e-02   3.167 0.001781 **
```

Figure 7: VECM coefficients and p-values.

As we can see, M3, oil price and inflation rate in USA are significant, as well as the error correction term which takes into account the divergence from the long term trend. Furthermore, $R^2 = 0.356$, thus this model clearly outperformed the regression on the lagged differences: the latter, indeed, considered only short-run dynamics instead of both short-run and long-run as in VECM.

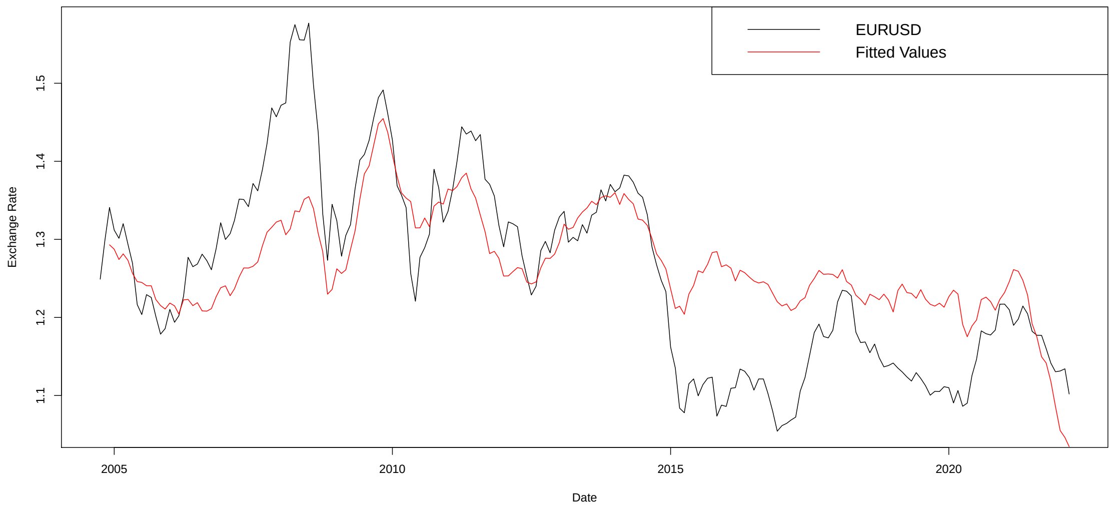

Figure 8: EUR/USD exchange rate against VECM fitted values.

We performed a Shapiro test on the residuals of VECM which led us to the rejection of the hypothesis of gaussianity (p-value = 0.0069). Furthermore, we tested the kurtosis and it turned out that the residuals are leptokurtic (estimated kurtosis = $4.27 > 3$), as we can deduce also from the histogram below; these outcomes directed us towards a GARCH-type model to describe the variance of the residuals. This intuition is supported by the plot of the squared residuals (Figure 9) and the Lagrange Multiplier test for autoregressive conditional heteroskedasticity; in particular, the latter rejects the null hypothesis of no ARCH effect with a p-value of $3.426 \cdot 10^{-6}$.

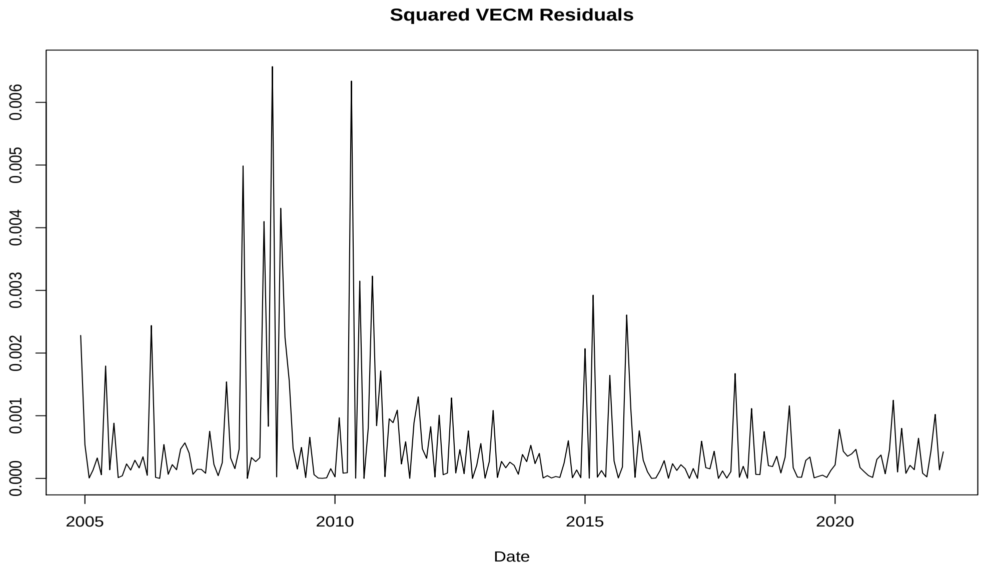

Figure 9: Squared residuals of the VECM model.

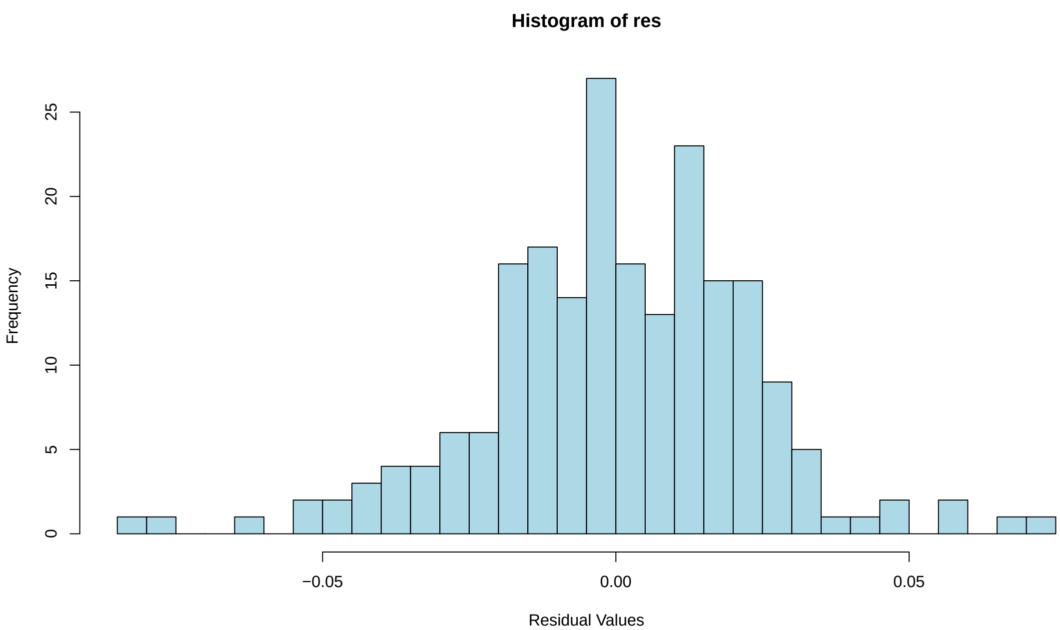

Figure 10: Histogram representing the residuals of VECM.

4.2 Estimate of EUR/USD exchange rate volatility

In this section we address the issue of modelling the exchange rate volatility in order to understand whether the central bank can influence its dynamics.
To do so we propose three different models: in the first one we model the residuals of the VECM with a GARCH model with external regressors: the aim of this analysis is to assess whether the central banks has influence on the variance of the VECM model implemented in the previous section.
In the second approach we repeat the analysis but considering an ARMA-GARCH model: in this case, the conditional mean of the EUR/USD exchange rate is modelled without any external regressor whereas, for the conditional variance, we consider regressors which are directly controlled by the central bank. The aim of this part is to assess the influence of the ECB when the model (for the conditional mean) of the EUR/USD exchange rate, unlike the VECM model, does not consider other correlated relevant variables.
To conclude, we propose a more empirical approach, which consists in modelling directly the monthly EUR/USD exchange rate volatility: in this part of the analysis we evaluate the correlation of US market volatility (through the VIX index) and ECB actions on the monthly volatility of the exchange rate.

4.2.1 GARCH through VECM

The starting point is to consider the residuals of the VECM of the previous section; as we said before, our aim is to model such residuals in GARCH fashion.
As we discussed at the end of the last section, there is evidence to take into account an ARCH behaviour in the residuals of the VECM. Therefore, we will model the variance of the residuals in the GARCH framework considering external regressors representing the ECB actions. In particular, we will consider changes in the MRO rate, external reserves and `M3`.
The complete model is the following:

$$\texttt{EURUSD}_t = \text{VECM}(\mathbf{X}_{t-1}, \mathbf{X}_{t-2}) + \varepsilon_t$$

$$\varepsilon_t | I_{t-1} \sim \mathcal{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma_1 \text{d}\texttt{ExtRes}_t + \gamma_2 \text{d}\texttt{MRO}_t + \gamma_3 \text{d}\texttt{M3}_t \,,$$

where VECM$(\cdot, \cdot)$ is the Vector Error Correction Model estimated in (1), $\mathbf{X}_t$ is the vector of variables we took into account and I_t is the information set at time t. The estimation is performed by maximum likelihood (here we are assuming that the residuals observed are the true error of the VECM model) and we end up with the following coefficients:

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
omega   0.000001    0.000007   0.088247  0.92968
alpha1  0.049037    0.002996  16.369066  0.00000
beta1   0.949054    0.005797 163.711647  0.00000
vxreg1  0.000000    0.000000   0.006192  0.99506
vxreg2  0.000000    0.000078   0.000128  0.99990
vxreg3  0.000000    0.000002   0.005514  0.99560
```

As we can see, all the external regressors are not significant apart from α and β. Overall we can conclude that, once considered all the regressors used in the previous section to model the conditional mean of the EUR/USD exchange rate, ECB actions do not have a significant influence on the variance of the model: this means that the ECB decisions, limited to MRO rate, M3 and external reserves, do not seem to increase the uncertainty in the prediction of the following month exchange rate. We will come back to this consideration in the conclusion of the report. Finally, dropping all the insignificant regressors, we produce the plot of the fitted variance (Figure 11).

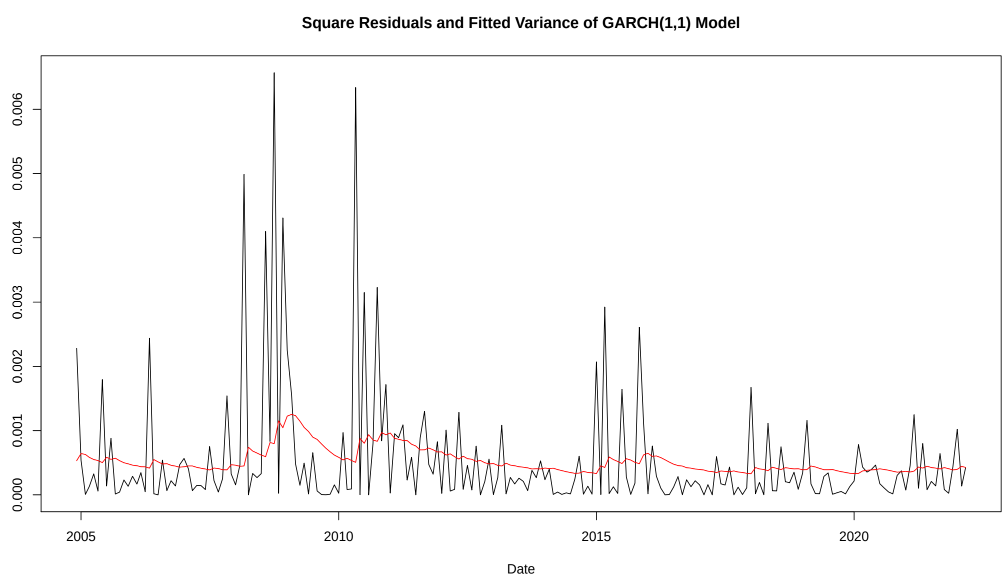

Figure 11: Fitted variance (red line) against squared residuals (black line).

The model can roughly capture the increase in the variance during the 2008 crisis and the following Euro Zone crisis but there are often huge spikes in the error which are not explainable under the normality assumption of the errors. As an example, one can see from the plot of the residuals standardized with the conditional standard deviation (Figure 12) that we can observe two event above three sigma. Such event occurs with probability of around 0.0027 and makes our normality assumption a bit critical (p-value of the Shapiro test is around 2%). Even thought most of this extreme errors were recorded around the 2008 crisis, in some future work we could address this issue adopting different heavy tailed distributions for the conditional distribution of the error ε_t such as a t-student.

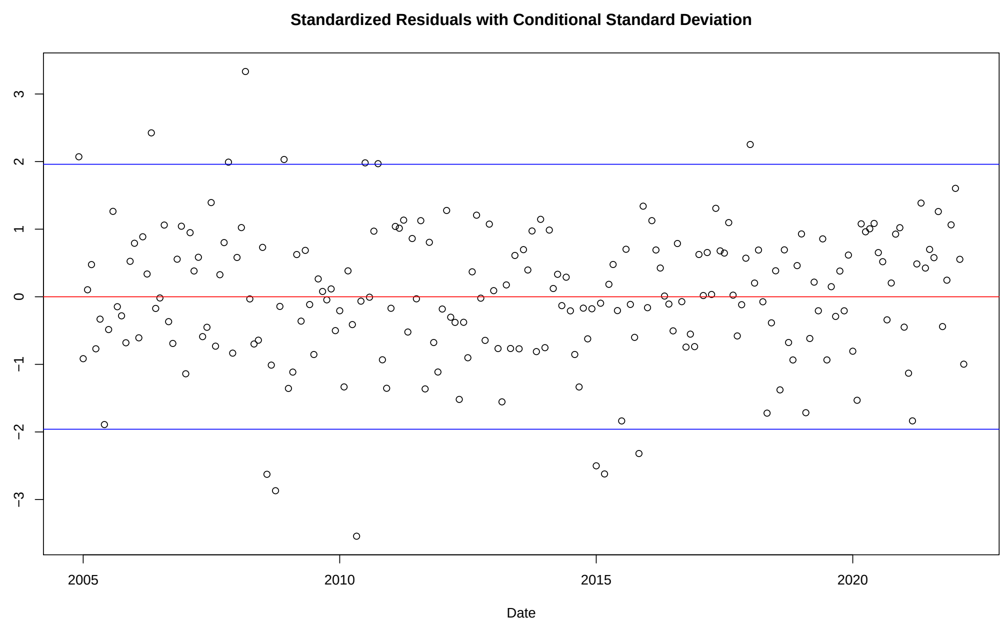

Figure 12: Residuals standardized with the conditional standard deviation. The inner part of the blue lines is the 95% confidence interval.

4.2.2 ARMA-GARCH

In this section we just modeled the exchange rate log returns via an ARMA-GARCH model [18] taking into account external variables such as difference in the `MRO`, external reserves (`ExtRes`) and money supply (`M3`), which represent monetary policy decisions of the ECB. The goal is to see if changes in these external variables influence the volatility of the exchange rate for the current month.
The starting point of this model is the log return of the `EURUSD` exchange rate. Namely, we consider

$$\Delta\tilde{y}_t = \log(y_t) - log(y_{t-1}) = log(\frac{y_t}{y_{t-1}}) \approx \frac{\Delta y_t}{y_{t-1}},$$

where y_t is the `EURUSD` exchange rate at time t. The advantage of working in this setting, among others, is that the obtained series is approximately normally distributed (the Shapiro test cannot reject the normality assumption at 5% level). Following standard practice, we study the autocorrelation function (Figure 13) and the partial autocorrelation function (Figure 14) to guess the order of the moving average and the autoregressive part of the ARMA model for the conditional mean of the series.

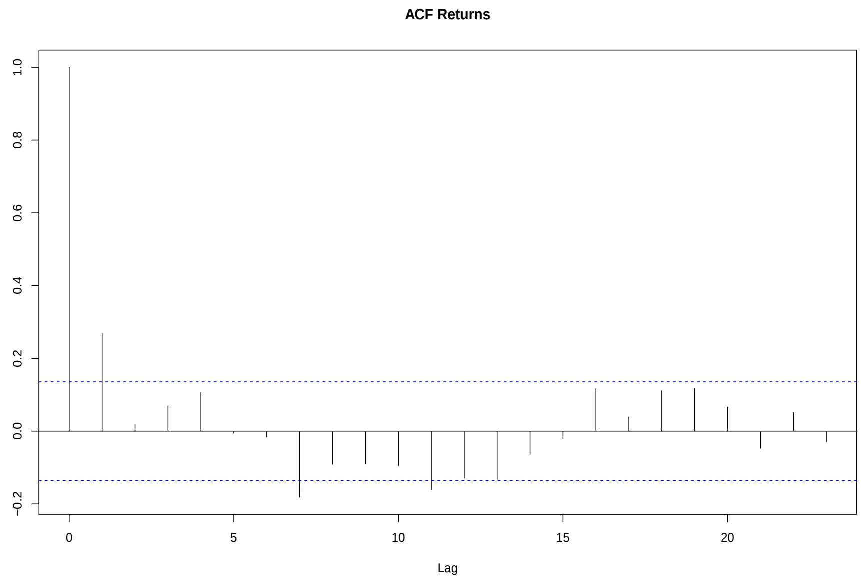

Figure 13: Autocorrelation function of the `EURUSD` log returns.

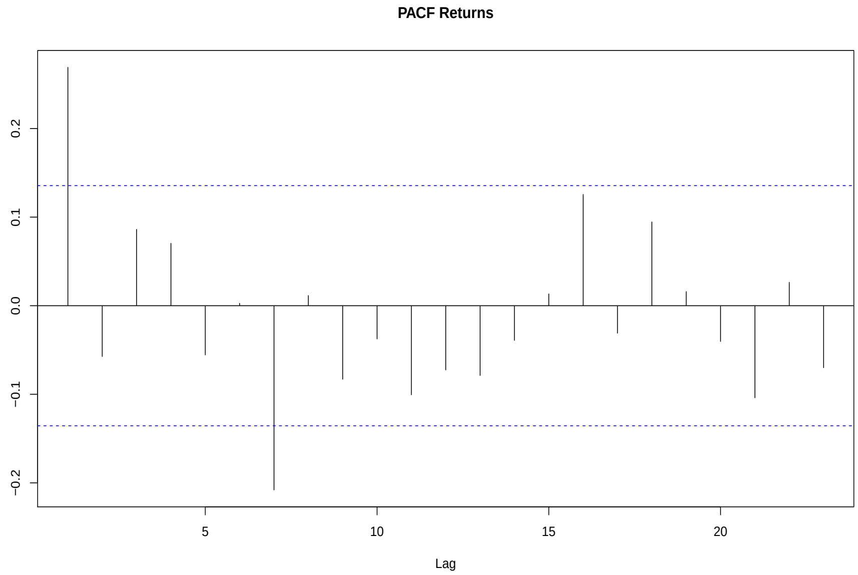

Figure 14: Partial autocorrelation function of the EURUSD log returns.

These plots suggest an ARMA(1,0) model and an extensive grid search aiming at minimizing the AIC Information Criterion over different moving average and auto regressive orders confirms this intuition.

Following this analysis, we proceeded considering an ARMA(1,0)-GARCH(1,1) model:

$$\Delta\tilde{y}_t = \alpha_1 \Delta\tilde{y}_{t-1} + \varepsilon_t$$

$$\varepsilon_t | I_{t-1} \sim \mathcal{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \tilde{\alpha}_1 \varepsilon_{t-1}^2 + \tilde{\beta}_1 \sigma_{t-1}^2 + \gamma x_{t-1} \text{ with } \tilde{\alpha}_1, \tilde{\beta}_1, \gamma > 0\,,$$

where I_t is the information set available at time t, x_t is an external regressor and $\Delta\tilde{y}_t$ are the log returns at time t.
The estimation of such parameters is done with ML estimators which are asymptotically Normal distributed.
We found out that the variance of the parameters is heavily affected by the number of external regressors; for this reason, we estimated four different models. In the first three we used as external regressors the difference in MRO, external reserves and money supply but we did not found any significant influence of this variable on the variance σ^2 of the model. In the fourth one, instead, we considered as external regressor a dummy variable which is equal to one whenever in the current month the ECB has changed the MRO rate and to zero otherwise. The estimate of the coefficients along with the p-values for this choice of external regressor is reported in Figure 15.

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.270955    0.067788   3.9971 0.000064
omega   0.000006    0.000001  11.3480 0.000000
alpha1  0.014654    0.007937   1.8463 0.064846
beta1   0.947338    0.015260  62.0779 0.000000
vxreg1  0.000061    0.000025   2.4815 0.013083
```

Figure 15: Estimate of the coefficients of the ARMA(1,0)-GARCH(1,1) model.

As we can see, all the coefficients are significant at 5% except for the coefficient $\tilde{\alpha}_1$. In particular, we can see that the change in the `MRO` has a significant influence on the conditional variance, even though the effect is quite contained. As depicted in the graph of the estimated conditional variance (Figure 16), every time we have a change in the `MRO` (red line) the model predicts a slightly higher volatility for the returns.

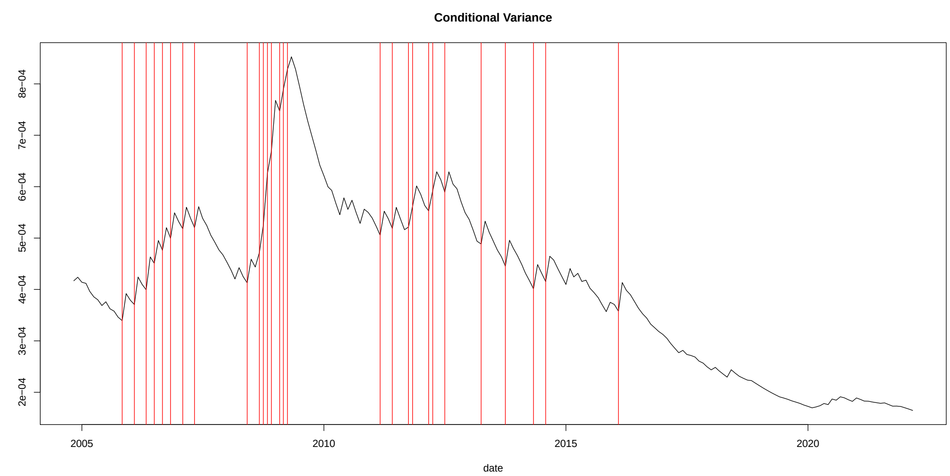

Figure 16: Fitted conditional variance for the ARMA(1,0)-GARCH(1,1) model, where red lines represent the time in which a change in the MRO rate occurs.

4.2.3 Empirical estimate models

In this section we try to model the monthly volatility directly through data. The goal is to assess whether there is a correlation between such volatility and some changes in the monetary policy of the ECB. Therefore, in this section we just consider M3 (proxy of the money supply), the MRO rate and the external reserves of the central bank as regressors (of course, for the same reasoning as in Section 4.1.1, we must consider their differences, in order to avoid spurious regressions).

A first regression of the volatility on the differences of such covariates yields the following results:

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)    1.222e-02  4.766e-04  25.634  < 2e-16 ***
diff(MRO)     -3.044e-02  4.196e-03  -7.254 8.15e-12 ***
diff(ExtRes)   3.692e-07  3.478e-07   1.061    0.290    
diff(M3)       1.043e-04  9.700e-04   0.108    0.914    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.006853 on 205 degrees of freedom
Multiple R-squared:  0.2196,	Adjusted R-squared:  0.2082 
F-statistic: 19.23 on 3 and 205 DF,  p-value: 4.995e-11
```

Figure 17: Regression coefficients and p-values.

The only significant variable is the difference in the MRO rate and this finding is consistent with what has been found in ARCH-GARCH analysis. In order to keep the analysis consistent with the previous one, we considered the dummy variable representing changes in the MRO rate and we choose an ARMA model to deal with autocorrelation in the error term. To choose the relevant ARMA orders we use again a grid search to find the one which minimizes AIC and we finally come up with an ARMAX(2,1) model with the difference of the MRO rate as the exogenous regressor. The estimated coefficients are the following:

```
            Estimate Std. Error z value  Pr(>|z|)    
ar1        0.5150280  0.1003297  5.1334 2.846e-07 ***
ar2        0.3697210  0.0752383  4.9140 8.924e-07 ***
ma1       -0.5274754  0.1001074 -5.2691 1.371e-07 ***
intercept  0.0120574  0.0016932  7.1210 1.072e-12 ***
dummy_MRO  0.0026333  0.0013153  2.0020   0.04528 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 18: ARMAX(2,1) coefficients and p-values.

Here, all the coefficients are significant and the R^2 is around 33.40%.
The fit of the model is quite good, but if we want to evaluate more in detail the influence of the ECB on the volatility we need to refine our argument.
In particular, one could think that the volatility in the exchange rates has a high correlation with the volatility in the capital markets. Therefore it is of particular interests to evaluate the influence of both market volatility and ECB intervention on the overall

exchange rate volatility.
In order to do so we include as external regressor the monthly average of the VIX index, which represents the implied volatility in the SP 500 index. The plot of `EURUSD` monthly volatility (standardized), MRO rate and VIX (standardized) is reported in this plot (19):

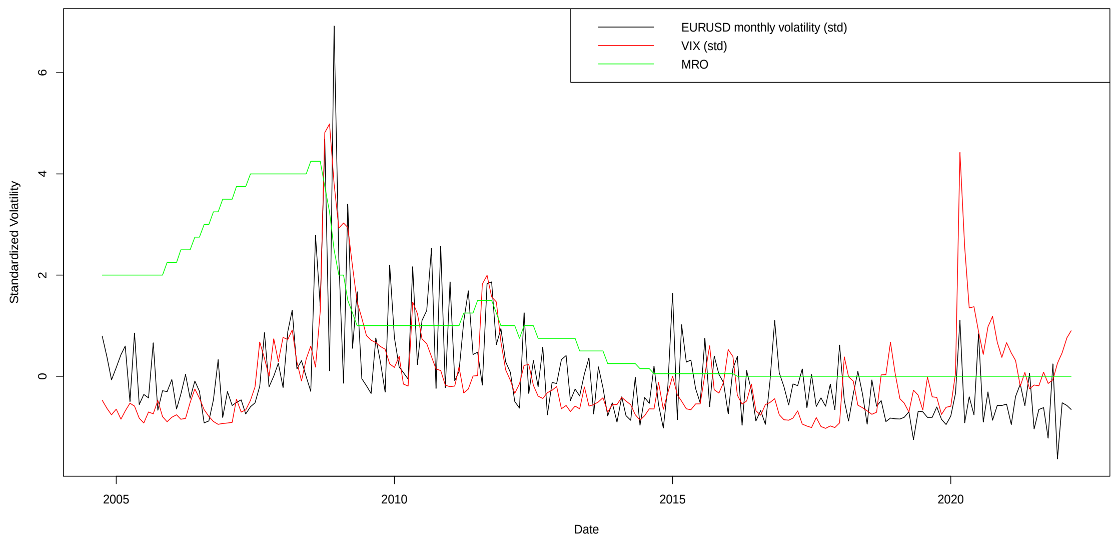

Figure 19: Monthly `EURUSD` volatility, VIX index and MRO rate.

As we can see, the volatility in the exchange rate closely mimics the one of the VIX. Given such considerations, we regress the monthly `EURUSD` volatility on both VIX and the dummy variable capturing MRO rate changes. The residuals of this model show high partial autocorrelation up to lag 5 (see Figure below) so we chose to build an ARMAX(5,0) model with `vix` and `dummy_MRO` as external regressors.

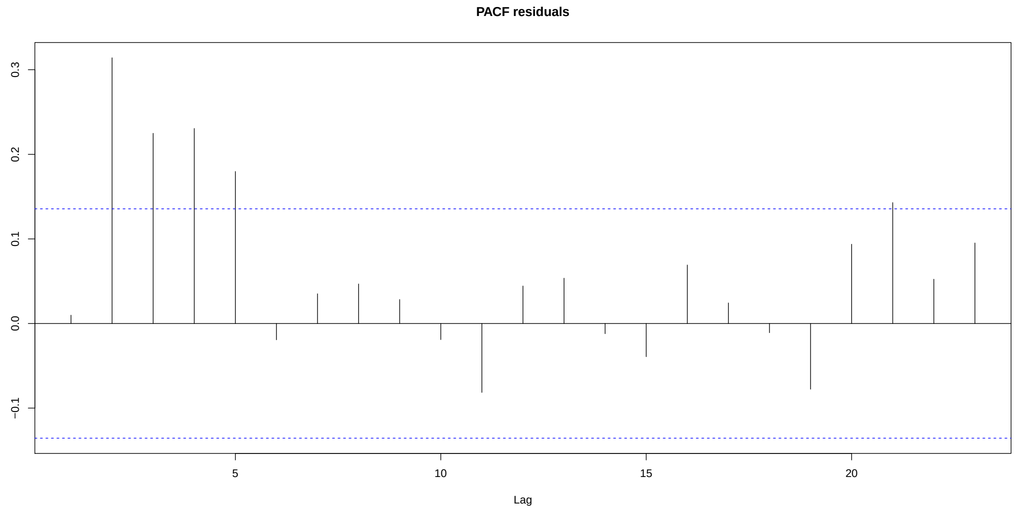

Figure 20: PACF of the residuals of EURUSD volatility regressed on VIX and dMRO.

The final estimated coefficients are:

```
z test of coefficients:

             Estimate  Std. Error z value  Pr(>|z|)
ar1       -1.6106e-01  6.8126e-02 -2.3642 0.0180699 *
ar2        2.1433e-01  6.7706e-02  3.1656 0.0015478 **
ar3        2.1382e-01  6.6439e-02  3.2183 0.0012894 **
ar4        2.5971e-01  6.7556e-02  3.8444 0.0001208 ***
ar5        1.9182e-01  6.8975e-02  2.7810 0.0054195 **
intercept  5.4155e-03  2.3962e-03  2.2600 0.0238193 *
vix        3.5145e-04  9.1698e-05  3.8327 0.0001268 ***
dummy_MRO  2.1031e-03  1.2487e-03  1.6843 0.0921245 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The R^2 is 42.49% (improved by ten percentage points from the model without VIX) and all the coefficients are significant. Therefore, the final model is the following:

$$\begin{aligned}
\texttt{EURUSD_vol} = &5.41 \cdot 10^{-3} - 0.161\texttt{EURUSD_vol}_{t-1} + 0.214\texttt{EURUSD_vol}_{t-2} \\
&+ 0.214\texttt{EURUSD_vol}_{t-3} + 0.260\texttt{EURUSD_vol}_{t-4} \\
&+ 0.192\texttt{EURUSD_vol}_{t-5} + 3.51 \cdot 10^{-4}\texttt{vix}_t + 2.10 \cdot 10^{-3}\texttt{dummy_MRO}_t + \varepsilon_t \\
&\text{where } \varepsilon_t \sim \mathcal{N}(0, 3.5 \cdot 10^{-5})\,.
\end{aligned}$$

It is of particular interest to notice that the coefficient corresponding to the difference in MRO rate goes down to $2.1 \cdot 10^{-3}$ (from the $2.6 \cdot 10^{-3}$ of the model without `vix`). Hence, we can say that when VIX is considered, the influence of changes in monetary policy rates on the exchange rates volatility decreases. Moreover, the magnitude of such coefficient, with respect to the intercept, is consistent with what we found in Section 4.2.2, namely the effect of `MRO` changes on exchange rate volatility is very limited.

5 Conclusion

The aim of this work was to assess the influence of the European Central Bank monetary policy on the exchange rate volatility. In order to do so, we followed three approaches: in the first one we built a cointegration model to predict the EUR/USD exchange rate and we evaluated whether actions of the ECB could explain the variance of such a model. In this first part of the analysis, we found out that ECB has no significant influence on the variance of that model, thus we can conclude that ECB actions have influence in predicting the mean exchange rate and not its variance.
In the second approach, we modeled directly the EUR/USD monthly returns with an ARMA-GARCH model without accounting for external regressors for the mean model of such process, but considering a dummy variable tracking whether a change in the MRO rate has occurred during a given month. We found that the decisions of ECB to increase or decrease the MRO rate have a statistically significant but somehow limited influence on the volatility of such model. Similarly, in the last approach we modeled directly the empirical monthly volatility of the exchange rate, finding that when overall market volatility is taken into account the influence of the ECB on the volatility of the exchange rate is again limited.
The results of the whole analysis can be summed up saying that monetary policy surely influences the EUR/USD exchange rate but, once this influence is considered, the actions of the ECB do not account for an extra uncertainty in such rate. Indeed, considering the monthly EUR/USD volatility, we found a positive correlation with ECB actions: this can be explained by the fact that the exchange rate is for sure influenced by such actions, so that in that month we find big movements around the mean. Once constructed a predictive model for the exchange rate which takes into account many variables related or not to the monetary policy, we were able to predict these movements and we found out that the variance of the error is not particularly correlated to actions of the ECB.
Finally, we point out that in this analysis we just considered measurable ECB actions, namely money supply, MRO rate and External Reserves, and we didn't found any particular relationship between a change in these variables and the uncertainty of the EUR/USD exchange rate; probably, other types of non measurable actions of the ECB (like interviews, reports etc.) can be more influent.

6 Bibliography

[1] Ozturk Ilhan, "Exchange rate volatility and trade: a literature survey", International Journal of Applied Econometrics and Quantitative Studies Vol.3-1 (2006).

[2] Rigobon Roberto, "Contagion, spillover and interdependence", ECB Working paper no. 1975 (2016).

[3] Miyajima K., "Exchange rate volatility and pass-through to inflation in South Africa", IMF Working Paper, WP/19/277. Washington DC: International Monetary Fund (2019)

[4] Ferry Syarifuddin, Noer Azam Achsani, Dedi Budiman Hakim, Toni Bakhtiar, "Monetary policy response to exchange rate volatility in Indonesia", Journal of Contemporary Economic and Business Issues (2014)

[5] Okotori TW, "Effect of monetary policy on inflation in Nigeria 2009-2014" (Master thesis), Niger Delta University (2017)

[6] Okotori TW, "The dynamics of monetary policy and inflation in Nigeria", IOSR, Journal of Economics and Finance (IOSR-JEF) (2019)

[7] Mathai K, "Monetary policy: stabilizing process and output", Washington DC: International Monetary Fund (2012)

[8] Gang Y, "Monetary policy retrospective and Outlook", Vol. 3, China Finance (2018)

[9] Gbalam Peter Eze and Tonprebofa Waikumo Okotori, "Exchange Rate Volatility and Monetary Policy Shocks", Macroeconomics (by Dr. Musa Jega Ibrahim) (2022)

[10] Babatunde Wasiu Adeoye and Olufemi Muibi Saibu, "Monetary policy shocks and exchange rate volatility in Nigeria", Asian Economic and Financial Review (2014)

[11] Shakill Hassan, "Speculative flows, exchange-rate volatility and monetary policy: the South African experience", prepared for and presented at the Conference on Fourteen Years of Inflation Targeting in South Africa and the Challenge of a Changing Mandate held at the South African Reserve Bank in Pretoria (2014)

[12] Michael Adusei and Eddie Yaw Gyapong," The impact of macroeconomic variables on exchange rate volatility in Ghana: The Partial Least Squares Structural Equation Modelling approach", Research in International Business and Finance (2017)

[13] An L, Sun W., "Monetary policy, foreign exchange intervention, and the exchange rate: The case of Japan", International Research Journal of Finance and Economics (2008)

[14] Cagliarini A, Mckibbin W., "Global relative price shocks: The role of macroeconomic policies", RBA Research (2009)

[15] İshak Demir, "Monetary policy responses to the exchange rate: Empirical evidence from the ECB", Economic Modelling (2014)

[16] Rostagno, Massimo et al. (2019) : A tale of two decades: The ECB's monetary policy at 20, ECB Working Paper, No. 2346, ISBN 978-92-899-3978-2, European Central Bank (ECB), Frankfurt a. M.

[17] Exchange Rate Volatility and U. S. Monetary Policy: An ARCH Application Author(s): William D. Lastrapes Source: Journal of Money, Credit and Banking , Feb., 1989, Vol. 21, No. 1 (Feb., 1989), pp. 66-77 Published by: Ohio State University Press Stable URL: https://www.jstor.org/stable/1992578

[18] Abdullah, S.M., Siddiqua, S., Siddiquee, M.S.H. et al. Modeling and forecasting exchange rate volatility in Bangladesh using GARCH models: a comparison based on normal and Student's t-error distribution. Financ Innov 3, 18 (2017). https://doi.org/10.1186/s40854-017-0071-z

[19] Journal Article, Engel, Charles, T Exchange Rates, Interest Rates, and the Risk Premium, American Economic Review, 106, 2, 2016, 436-74, 10.1257/aer.20121365 https://www.aeaweb.org/articles?id=10.1257/aer.20121365