

# HW1

## Question1

Assumption: matrix A is square and invertible.

```
A = [2 1 1;  
     4 -6 0;  
     -2 7 2];  
  
[P, L, D, U] = LDU_Decomposition(A);  
  
disp('P * A =');
```

P \* A =

```
disp(P*A);
```

```
  4   -6    0  
  2    1    1  
 -2    7    2
```

```
disp('L * D * U =');
```

L \* D \* U =

```
disp(L*D*U);
```

```
  4   -6    0  
  2    1    1  
 -2    7    2
```

```
err = norm(P*A - L*D*U);  
fprintf('Verification error = %.2e\n', err);
```

Verification error = 0.00e+00

## Test 2:

```
A2 = [5, -2, 3;  
      1,  0, 2;  
      4,  1, 1];  
  
[P, L, D, U] = LDU_Decomposition(A2);  
  
disp('P * A =');
```

P \* A =

```
disp(P*A2);
```

5	-2	3
4	1	1
1	0	2

```
disp('L * D * U =');
```

L \* D \* U =

```
disp(L*D*U);
```

5.0000	-2.0000	3.0000
4.0000	1.0000	1.0000
1.0000	0	2.0000

```
err = norm(P*A2 - L*D*U);
fprintf('Verification error = %.2e\n', err);
```

Verification error = 4.44e-16

## Question 2

```
A1 = [10, -10, 0;
      0, -4, 2;
      2, 0, -5];
[U, S, V] = SVD(A1);
```

```
disp('Matrix A1:');
```

Matrix A1:

```
disp(A1);
```

10	-10	0
0	-4	2
2	0	-5

```
disp('U matrix:');
```

U matrix:

```
disp(U);
```

-0.9750	-0.0193	0.2213
-0.2000	0.5095	-0.8369
-0.0966	-0.8603	-0.5006

```
disp('S matrix:');
```

S matrix:

```
disp(S);
```

14.4978	0	0
0	5.9473	0
0	0	1.8556

```
disp('V matrix:');
```

V matrix:

```
disp(V);
```

```
-0.6859   -0.3217    0.6528
 0.7277   -0.3102    0.6117
 0.0057    0.8946    0.4469
```

```
disp('Reconstructed A1 from U*S*V':');
```

Reconstructed A1 from U\*S\*V':

```
disp(U*S*V');
```

```
10.0000  -10.0000    0.0000
 0.0000   -4.0000    2.0000
 2.0000   -0.0000   -5.0000
```

```
disp('Difference between original and reconstructed A1:');
```

Difference between original and reconstructed A1:

```
disp(norm(A1 - U*S*V'));
```

```
4.3381e-15
```

A2

```
A2 = [5, -5, 0, 0;
      5, 5, 5, 0;
      0, -1, 4, 1;
      0, 4, -1, 2;
      0, 0, 2, 1];
```

```
[U, S, V] = SVD(A2);
```

```
% Display results
```

```
disp('Matrix A2:');
```

Matrix A2:

```
disp(A2);
```

```
5   -5    0    0
5    5    5    0
0   -1    4    1
0    4   -1    2
0    0    2    1
```

```
disp('U matrix:');
```

U matrix:

```
disp(U);
```

-0.1126	0.8671	0.3748	-0.3075	-0.0212
0.9322	0.1523	0.1626	0.2846	0.0212
0.2020	0.2258	-0.7497	-0.3169	-0.4957
0.2399	-0.4123	0.3833	-0.7709	-0.1770
0.1417	0.0637	-0.3522	-0.3603	0.8497

```
disp('S matrix:');
```

S matrix:

```
disp(S);
```

9.1449	0	0	0
0	7.7981	0	0
0	0	4.4207	0
0	0	0	2.2398
0	0	0	0

```
disp('V matrix:');
```

V matrix:

```
disp(V);
```

0.4481	0.6536	0.6078	-0.0511
0.6541	-0.6988	0.2765	0.0867
0.6028	0.2826	-0.7405	0.0919
0.0900	-0.0686	-0.0758	-0.9907

```
% Verify correctness
```

```
disp('Reconstructed A2 from U*S*V':');
```

Reconstructed A2 from U\*S\*V':

```
disp(U*S*V');
```

5.0000	-5.0000	-0.0000	0.0000
5.0000	5.0000	5.0000	-0.0000
-0.0000	-1.0000	4.0000	1.0000
0.0000	4.0000	-1.0000	2.0000
-0.0000	-0.0000	2.0000	1.0000

```
disp('Difference between original and reconstructed A2:');
```

Difference between original and reconstructed A2:

```
disp(norm(A2 - U*S*V'));
```

8.3107e-15

A3

```
A3 = [1, 1, 1, 1;  
      10, 2, 9, 0;  
      8, 0, 0, 7];
```

```
[U, S, V] = SVD(A3);
```

```
% Display results  
disp('Matrix A3:');
```

Matrix A3:

```
disp(A3);
```

1	1	1	1
10	2	9	0
8	0	0	7

```
disp('U matrix:');
```

U matrix:

```
disp(U);
```

0.1088	-0.0195	0.9939
0.8344	0.5453	-0.0806
0.5404	-0.8380	-0.0756

```
disp('S matrix:');
```

S matrix:

```
disp(S);
```

15.4773	0	0	0
0	7.8292	0	0
0	0	1.0748	0

```
disp('V matrix:');
```

V matrix:

```
disp(V);
```

0.8254	-0.1623	-0.3883	0.3762
0.1148	0.1368	0.7747	0.6066
0.4922	0.6243	0.2496	-0.5528
0.2514	-0.7518	0.4321	-0.4300

```
% Verify correctness  
disp('Reconstructed A from U*S*V':');
```

Reconstructed A from  $U*S*V'$ :

```
disp(U*S*V');
```

1.0000	1.0000	1.0000	1.0000
10.0000	2.0000	9.0000	0.0000
8.0000	0.0000	0.0000	7.0000

```
disp('Difference between original and reconstructed A:');
```

Difference between original and reconstructed A:

```
disp(norm(A3 - U*S*V'));
```

7.2975e-15

### Question 3

A1

```
A1 = [10 -10 0;  
      0 -4 2;  
      2 0 -5];  
b1 = [10; 2; 13];
```

```
det_A1 = det(A1)
```

```
det_A1 =  
160
```

```
[U1, S1, V1] = SVD(A1);
```

```
disp('U1 matrix:');
```

U1 matrix:

```
disp(U1);
```

-0.9750	-0.0193	0.2213
-0.2000	0.5095	-0.8369
-0.0966	-0.8603	-0.5006

```
disp('S1 matrix:');
```

S1 matrix:

```
disp(S1);
```

14.4978	0	0
0	5.9473	0
0	0	1.8556

```
disp('V1 matrix:');
```

V1 matrix:

```
disp(V1);
```

```
-0.6859   -0.3217    0.6528  
 0.7277   -0.3102    0.6117  
 0.0057    0.8946    0.4469
```

```
n1 = V1(:,rank(A1)+1:end)
```

n1 =

3×0 empty double matrix

```
X1 = V1 * (S1 \ ( U1' * b1))
```

```
X1 = 3×1  
-1.0000  
-2.0000  
-3.0000
```

A2

```
A2 = [1 1 1;  
      10 2 9;  
      8 0 7];  
b2 = [1; 3; 1];
```

```
det_A2 = det(A2)
```

```
det_A2 =  
1.7764e-15
```

```
[U2, S2, V2] = SVD(A2);
```

```
disp('U2 matrix:');
```

U2 matrix:

```
disp(U2);
```

```
 0.0869    0.5708    0.8165  
 0.7859    0.4644   -0.4082  
 0.6122   -0.6772    0.4082
```

```
disp('S2 matrix:');
```

S2 matrix:

```
disp(S2);
```

```
17.2832     0     0  
 0    1.5132     0  
 0     0     0
```

```
disp('V2 matrix:');
```

V2 matrix:

```
disp(V2);
```

```
    0.7431   -0.1339    0.6556  
    0.0960    0.9910    0.0937  
    0.6622    0.0067   -0.7493
```

```
rank2 = rank(A2)
```

```
rank2 =  
2
```

```
n2 = V2(:,rank2+1:end)
```

```
n2 = 3×1  
    0.6556  
    0.0937  
   -0.7493
```

```
s2 = diag(S2);  
tol = max(size(A)) * eps(max(s2));  
idx = (s2 > tol);  
  
x2 = V2(:, idx) * ((U2(:, idx)' * b2) ./ s2(idx))
```

```
x2 = 3×1  
    0.0175  
    0.8596  
    0.1228
```

A3

```
A3 = [1 1 1;  
      10 2 9;  
      8 0 7];  
b3 = [3; 2; 2];  
  
[U3, S3, V3] = SVD(A3);  
  
disp('U3 matrix:');
```

U3 matrix:

```
disp(U3);
```

```
    0.0869    0.5708    0.8165  
    0.7859    0.4644   -0.4082  
    0.6122   -0.6772    0.4082
```

```
disp('S3 matrix:');
```



S3 matrix:

```
disp(S3);
```

```
17.2832    0    0
    0    1.5132    0
    0    0    0
```

```
disp('V3 matrix:');
```

V3 matrix:

```
disp(V3);
```

```
0.7431 -0.1339 0.6556
0.0960 0.9910 0.0937
0.6622 0.0067 -0.7493
```

```
rank3 = rank(A3)
```

```
rank3 =
2
```

```
n3 = V3(:,rank3+1:end)
```

```
n3 = 3×1
    0.6556
    0.0937
   -0.7493
```

```
s3 = diag(S3);
tol = max(size(A)) * eps(max(s3));
idx = (s3 > tol);

x3 = V3(:, idx) * ((U3(:, idx)' * b3) ./ s3(idx))
```

```
x3 = 3×1
    0.0175
    0.8596
    0.1228
```

## Question 5

```
rng(0);
P = randn(3,3);

theta = deg2rad(30);
Rz = [cos(theta) -sin(theta) 0;
      sin(theta) cos(theta) 0;
      0 0 1];
t0 = [0.3; -0.4; 0.8];

Q = Rz*P + t0;
```

```
[A,t] = transform(P,Q);  
disp(A), disp(t), disp(det(A))
```

```
0.8660    -0.5000    0.0000  
0.5000    0.8660   -0.0000  
-0.0000   -0.0000    1.0000
```

```
0.3000  
-0.4000  
0.8000
```

```
1.0000
```

```
r_error = norm(A - Rz)
```

```
r_error =  
4.7421e-15
```

```
t_error = norm(t - t0)
```

```
t_error =  
2.7833e-15
```