HW1

Question1

Assumption: matrix A is square and invertible.

```
A = [2 1 1;
    4 -6 0;
    -2 7 2];

[P, L, D, U] = LDU_Decomposition(A);

disp('P * A =');
```

```
P * A =
```

```
disp(P*A);
```

```
     4    -6     0
     2     1     1
    -2     7     2
```

```
disp('L * D * U =');
```

```
L * D * U =
```

```
disp(L*D*U);
```

```
     4    -6     0
     2     1     1
    -2     7     2
```

```
err = norm(P*A - L*D*U);
fprintf('Verification error = %.2e\n', err);
```

```
Verification error = 0.00e+00
```

```

```

Test 2:

```
A2 = [5, -2, 3;
      1,  0, 2;
      4,  1, 1];

 [P, L, D, U] = LDU_Decomposition(A2);

disp('P * A =');
```

```
P * A =
```

```
disp(P*A2);
```

```
     5    -2     3
     4     1     1
     1     0     2
```

```
disp('L * D * U =');
```

```
L * D * U =
```

```
disp(L*D*U);
```

```
    5.0000   -2.0000    3.0000
    4.0000    1.0000    1.0000
    1.0000         0    2.0000
```

```
err = norm(P*A2 - L*D*U);
fprintf('Verification error = %.2e\n', err);
```

```
Verification error = 4.44e-16
```

Question 2

```
A1 = [10, -10,  0;
       0,  -4,  2;
       2,   0, -5];
[U, S, V] = SVD(A1);

disp('Matrix A1:');
```

```
Matrix A1:
```

```
disp(A1);
```

```
    10   -10     0
     0    -4     2
     2     0    -5
```

```
disp('U matrix:');
```

```
U matrix:
```

```
disp(U);
```

```
   -0.9750   -0.0193    0.2213
   -0.2000    0.5095   -0.8369
   -0.0966   -0.8603   -0.5006
```

```
disp('S matrix:');
```

```
S matrix:
```

```
disp(S);
```

```
   14.4978         0         0
         0    5.9473         0
         0         0    1.8556
```

```
disp('V matrix:');
```

```
V matrix:
```

```
disp(V);
```

```
   -0.6859   -0.3217    0.6528
    0.7277   -0.3102    0.6117
    0.0057    0.8946    0.4469
```

```
disp('Reconstructed A1 from U*S*V'':');
```

```
Reconstructed A1 from U*S*V':
```

```
disp(U*S*V');
```

```
   10.0000  -10.0000    0.0000
    0.0000   -4.0000    2.0000
    2.0000   -0.0000   -5.0000
```

```
disp('Difference between original and reconstructed A1:');
```

```
Difference between original and reconstructed A1:
```

```
disp(norm(A1 - U*S*V'));
```

```
   4.3381e-15
```

A2

```
A2 = [5, -5,  0, 0;
      5,  5,  5, 0;
      0, -1,  4, 1;
      0,  4, -1, 2;
      0,  0,  2, 1];

[U, S, V] = SVD(A2);

% Display results
disp('Matrix A2:');
```

```
Matrix A2:
```

```
disp(A2);
```

```
     5    -5     0     0
     5     5     5     0
     0    -1     4     1
```

```
     0     4    -1     2
     0     0     2     1
```

```
disp('U matrix:');
```

```
U matrix:
```

```
disp(U);
```

```
   -0.1126    0.8671    0.3748   -0.3075   -0.0212
    0.9322    0.1523    0.1626    0.2846    0.0212
    0.2020    0.2258   -0.7497   -0.3169   -0.4957
    0.2399   -0.4123    0.3833   -0.7709   -0.1770
    0.1417    0.0637   -0.3522   -0.3603    0.8497
```

```
disp('S matrix:');
```

```
S matrix:
```

```
disp(S);
```

```
    9.1449         0         0         0
         0    7.7981         0         0
         0         0    4.4207         0
         0         0         0    2.2398
         0         0         0         0
```

```
disp('V matrix:');
```

```
V matrix:
```

```
disp(V);
```

```
    0.4481    0.6536    0.6078   -0.0511
    0.6541   -0.6988    0.2765    0.0867
    0.6028    0.2826   -0.7405    0.0919
    0.0900   -0.0686   -0.0758   -0.9907
```

```
% Verify correctness
disp('Reconstructed A2 from U*S*V'':');
```

```
Reconstructed A2 from U*S*V':
```

```
disp(U*S*V');
```

```
    5.0000   -5.0000   -0.0000    0.0000
    5.0000    5.0000    5.0000   -0.0000
   -0.0000   -1.0000    4.0000    1.0000
    0.0000    4.0000   -1.0000    2.0000
   -0.0000   -0.0000    2.0000    1.0000
```

```
disp('Difference between original and reconstructed A2:');
```

```
Difference between original and reconstructed A2:
```

```
disp(norm(A2 - U*S*V'));
```

```
   8.3107e-15
```

A3

```
A3 = [1, 1, 1, 1;
     10, 2, 9, 0;
      8, 0, 0, 7];


[U, S, V] = SVD(A3);

% Display results
disp('Matrix A3:');
```

```
Matrix A3:
```

```
disp(A3);
```

```
     1     1     1     1
    10     2     9     0
     8     0     0     7
```

```

disp('U matrix:');
```

```
U matrix:
```

```
disp(U);
```

```
    0.1088   -0.0195    0.9939
    0.8344    0.5453   -0.0806
    0.5404   -0.8380   -0.0756
```

```

disp('S matrix:');
```

```
S matrix:
```

```
disp(S);
```

```
   15.4773         0         0         0
         0    7.8292         0         0
         0         0    1.0748         0
```

```

disp('V matrix:');
```

```
V matrix:
```

```
disp(V);
```

```
    0.8254   -0.1623   -0.3883    0.3762
    0.1148    0.1368    0.7747    0.6066
    0.4922    0.6243    0.2496   -0.5528
    0.2514   -0.7518    0.4321   -0.4300
```

```
% Verify correctness
disp('Reconstructed A from U*S*V'':');
```

```
Reconstructed A from U*S*V':
```

```
disp(U*S*V');
```

```
    1.0000    1.0000    1.0000    1.0000
   10.0000    2.0000    9.0000    0.0000
    8.0000    0.0000    0.0000    7.0000
```

```
disp('Difference between original and reconstructed A:');
```

```
Difference between original and reconstructed A:
```

```
disp(norm(A3 - U*S*V'));
```

```
   7.2975e-15
```

Question 3

A1

```
A1 = [10 -10 0;
      0  -4  2;
      2   0  -5];
b1 = [10; 2; 13];

det_A1 = det(A1)
```

```
det_A1 =
160
```

```
[U1, S1, V1] = SVD(A1);

disp('U1 matrix:');
```

```
U1 matrix:
```

```
disp(U1);
```

```
   -0.9750   -0.0193    0.2213
   -0.2000    0.5095   -0.8369
   -0.0966   -0.8603   -0.5006
```

```
disp('S1 matrix:');
```

```
S1 matrix:
```

```
disp(S1);
```

```
   14.4978         0         0
         0    5.9473         0
         0         0    1.8556
```

```
disp('V1 matrix:');
```

```
V1 matrix:
```

```
disp(V1);
```

```
   -0.6859   -0.3217    0.6528
    0.7277   -0.3102    0.6117
    0.0057    0.8946    0.4469
```

```
X1 = V1 * (S1 \ ( U1' * b1))
```

```
X1 = 3×1
   -1.0000
   -2.0000
   -3.0000
```

A2

```
A2 = [1 1 1;
      10 2 9;
      8 0 7];
b2 = [1; 3; 1];

det_A2 = det(A2)
```

```
det_A2 =
1.7764e-15
```

```
[U2, S2, V2] = SVD(A2);

disp('U2 matrix:');
```

```
U2 matrix:
```

```
disp(U2);
```

```
    0.0869    0.5708    0.8165
    0.7859    0.4644   -0.4082
    0.6122   -0.6772    0.4082
```

```
disp('S2 matrix:');
```

```
S2 matrix:
```

```
disp(S2);
```

```
   17.2832         0         0
         0    1.5132         0
         0         0         0
```

```
disp('V2 matrix:');
```

V2 matrix:

```
disp(V2);
```

```
    0.7431   -0.1339    0.6556
    0.0960    0.9910    0.0937
    0.6622    0.0067   -0.7493
```

```
rank2 = rank(A2)
```

```
rank2 =
2
```

```
s2   = diag(S2);
tol = max(size(A)) * eps(max(s2));
idx = (s2 > tol);

x2 = V2(:, idx) * ((U2(:, idx)' * b2) ./ s2(idx))
```

```
x2 = 3×1
    0.0175
    0.8596
    0.1228
```

A3

```
A3 = [1 1 1;
      10 2 9;
      8 0 7];
b3 = [3; 2; 2];

[U3, S3, V3] = SVD(A3);

disp('U3 matrix:');
```

U3 matrix:

```
disp(U3);
```

```
    0.0869    0.5708    0.8165
    0.7859    0.4644   -0.4082
    0.6122   -0.6772    0.4082
```

```
disp('S3 matrix:');
```

S3 matrix:

```
disp(S3);
```

```
   17.2832         0         0
         0    1.5132         0
         0         0         0
```

```
disp('V3 matrix:');
```

```
V3 matrix:
```

```
disp(V3);
```

```
    0.7431   -0.1339    0.6556
    0.0960    0.9910    0.0937
    0.6622    0.0067   -0.7493
```

```
rank3 = rank(A3)
```

```
rank3 =
2
```

```
s3   = diag(S3);
tol = max(size(A)) * eps(max(s3));
idx = (s3 > tol);

x3 = V3(:, idx) * ((U3(:, idx)' * b3) ./ s3(idx))
```

```
x3 = 3×1
    0.0175
    0.8596
    0.1228
```

Question 5

```
rng(0);
P = randn(3,3);

theta = deg2rad(30);
Rz = [cos(theta) -sin(theta) 0;
     sin(theta) cos(theta)   0;
     0               0       1];
t0 = [0.3; -0.4; 0.8];

Q = Rz*P + t0;

[A,t] = transform(P,Q);
disp(A), disp(t), disp(det(A))
```

```
    0.8660   -0.5000    0.0000
    0.5000    0.8660   -0.0000
   -0.0000    0.0000    1.0000

    0.3000
   -0.4000
    0.8000

     1
```

```
r_error = norm(A - Rz)
```

```
r_error =
9.9987e-16
```

```
t_error = norm(t - t0)
```

```
t_error =
6.8664e-16
```