24-773: Multivariable Linear Control
Copter Control Lab

Background: The goal of this lab is for you to test the various synthesis approaches in the class on a hardware plant. The DIDO Quanser Aero system will be used to demonstrate the controller's performance (Figure 1). You have been provided with a Simulink template for this laboratory. Before performing the tasks here, please read the user manual for the Aero system to understand the encoder / motor parameters and limitations.

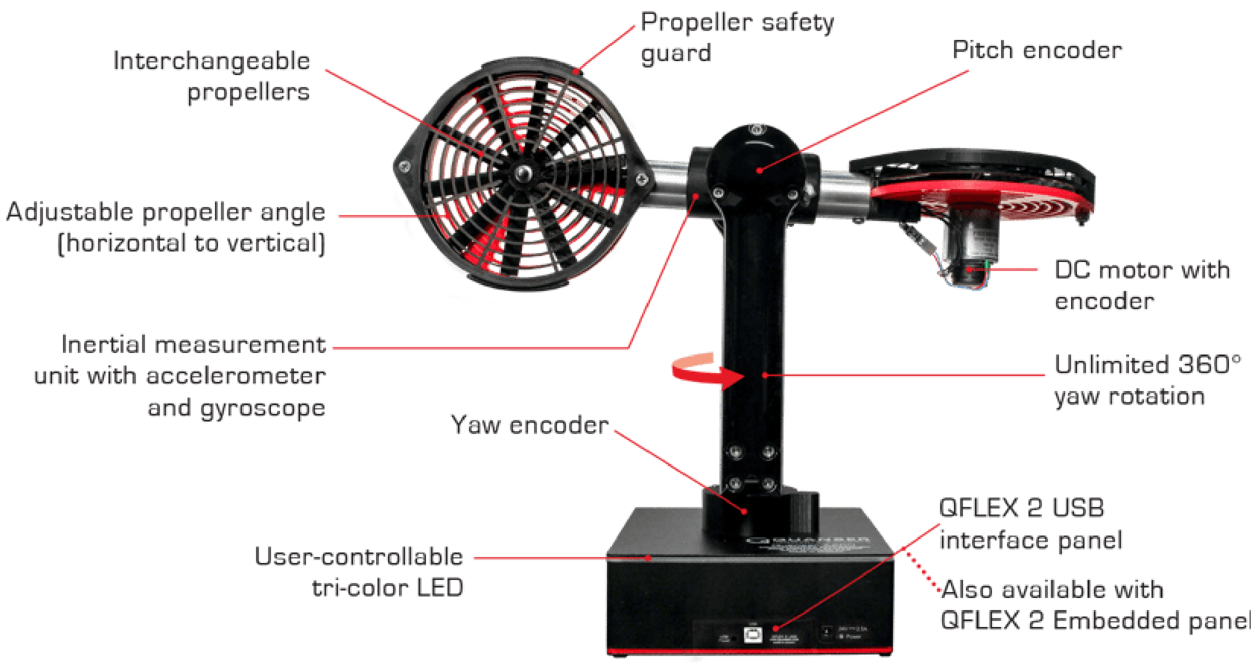

Figure 1: Quanser Aero System. The system has two degrees of freedom: pitch and yaw.

It is expected that you will work through this lab with some degree of parallelization, i.e., not everyone needs to be involved in every task. However, you must ensure that the work load is balanced, and that every team member understands the content that was generated by others. The report should also be written in parallel, but each team member is responsible for proof reading the final report. A suggested work breakdown is provided in Table 1 based on the task numbering provided in the Procedure.

Table 1: Suggested work breakdown. Note that Person 1's work on Part A needs to be completed before the others can start.

Person	Tasks
1	Parts A & B
2	Parts C & D
3	Part E + Lead Hardware

Please use the following to schedule your time in the lab in ANS 106: https://tinyurl.com/bdf5u64r. You have all been granted card access to the room – please let me know ASAP if you encounter any issues. To ensure fair access, each team will have a maximum of 4 hours / day in the lab; as a courtesy to others, please do not reserve times until you know you will use them. Much of the work can be done remotely (e.g. controller design). Your success will critically depend on starting early and putting in a consistent effort, rather than waiting until the last days before the due date.

The lab computer is a shared resource. **The password is acsi21**. To avoid cluttering the workspace for others, please store all data on a thumb drive or on the cloud and delete from the computer before leaving the lab.

Plant Model:
The files *quanser_aero_parameters.m* and *quanser_aero_state_space.m* describe the plant model for the system. Based on the Quanser documentation, the nominal plant model is given by

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1.7117 & 0 & -0.3249 & 0 \\ 0 & 0 & 0 & -1.0004 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0503 & 0.0959 \\ -0.1228 & 0.1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x$$

In this lab you will design and implement several controllers to test their performance on this system. Rather than dealing with the complexities of changing system parameters to test robustness, we will add uncertainty to the model directly in Simulink as shown in Figure 2. Here W_I is a weighting function that you will develop based on assumed parametric uncertainties and Δ_i is a sample from an uncertain system that is updated each time the simulation is run.

While you could certainly design an inverse-based controller for comparison, all of your designs in this lab will be generated using the synthesis techniques we have discussed in class. In this lab you will consider the simple block diagram shown in Figure 3 for synthesis.

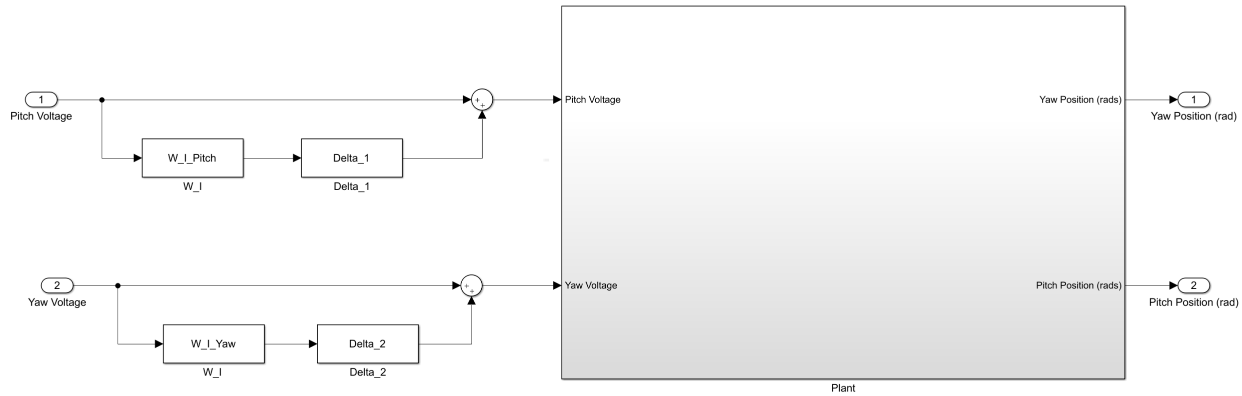

Figure 2: Input multiplicative uncertainty model. The weight is fixed, but the uncertainty element is changed each time the simulation is run.

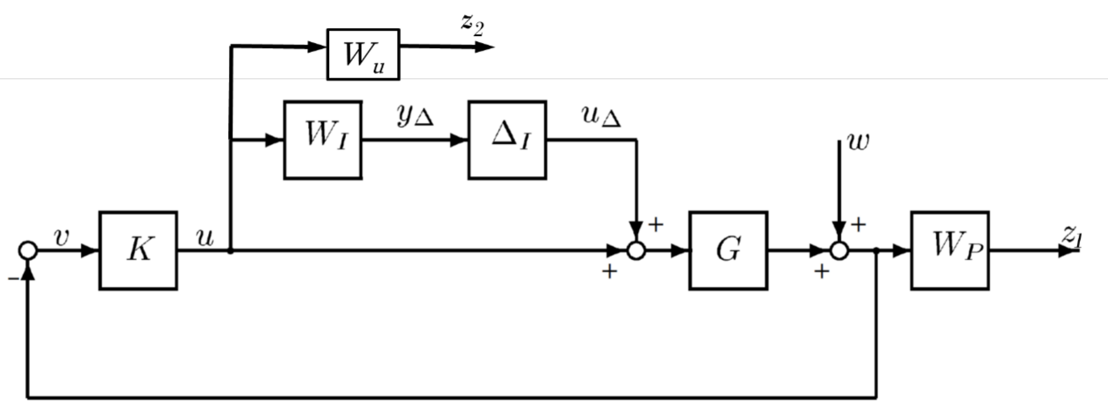

Figure 3: Control configuration for synthesis. Note that due to the MIMO nature of the plant, this DOES NOT conform to the form used in mixsyn.

The performance weight should be first order and diagonal, with each entry rejecting DC disturbances by a factor of 1000 while ensuring a sensitivity peak of no more than 3. The control weight should be a constant that is tuned to ensure that saturation does not break controller performance.

To test performance you will be following references and monitoring the response of the angles. For simplicity we will consider only low frequency square wave references (effectively step responses). The yaw reference has an amplitude of $\frac{\pi}{4}$ radians, whereas the pitch reference has an amplitude of $\frac{\pi}{6}$ radians. Both should have frequency 0.02 Hz.

Procedure

<u>A. Preliminaries</u>

1.) **(AT HOME)** Build a Simulink model based on the nominal plant and including a saturation nonlinearity that limits control voltages to ± 25 V. You will use this simulation to test your controllers' performance before you test them on the hardware.
2.) **(AT HOME)** Assuming 15% uncertainty in the inertia parameters J_p and J_y, build an uncertain model of the system. Fit multiplicative input uncertainty weights for the uncertain plant. Now augment your Simulink model from A.1 with an input multiplicative uncertainty model based on your weights (i.e., build a structure that looks like Figure 2).

B. H_∞ Loop Shaping

(AT HOME) Design an H_∞ loop shaping controller for the system that with desired diagonal loop shapes of $\frac{\omega_c}{s}$, where ω_c is a crossover frequency that you tune such that you see a good response in the face of saturation. Your design will certainly saturate when following step references, but that is not necessarily a deal breaker. A value of $\omega_c = 5\ rad/s$ is known to work well.

1.) **(AT HOME)** Plot the response from your simulation. Show the output of both angles, and the controller usage in a separate plot.
2.) **(AT HOME)** Check the robust stability and robust performance of your design. Run the simulation for 10 values of Δ taken from samples of an uncertain LTI system and plot the responses.
3.) **(IN LAB)** Implement your final controller on the hardware (using $\Delta = 0$). Plot the response from your hardware implementation (angles and voltages). If you are seeing instability here, perhaps decrease ω_c and retry.
4.) **(IN LAB)** Run the hardware simulation for 5 uncertain samples and plot the performance. Note that the system may hit the hard stops / become unstable. If this occurs, choose a new sample and rerun.

C. H_2 Optimal Control

(AT HOME) Solve the H_2 optimal control problem (assuming no uncertainty). Choose the sensitivity crossover to be ω_c rad/s and tune the control weights so that the simulated response behaves well. Note that you will need to add a high-frequency pole to the sensitivity weight to allow the H_2 problem to solve.

1.) **(AT HOME)** Plot the response from your simulation. Show the output of both angles, and the controller usage in a separate plot.
2.) **(AT HOME)** Check the robust stability and robust performance of your design.
3.) **(IN LAB)** Implement your final controller on the hardware (using $\Delta = 0$). Plot the response from your hardware implementation (angles and voltages). If you are running into stability issues, you again might want to drop ω_c.
4.) **(IN LAB)** Run the hardware simulation for 5 uncertain samples and plot the performance. Note that the system may hit the hard stops / become unstable. If this occurs, choose a new sample and rerun.

D. H_∞ Optimal Control

(AT HOME) Solve the H_∞ optimal control problem. Again use ω_c rad/s as the sensitivity crossover and tune controller weights to get good performance.

1.) **(AT HOME)** Plot the response from your simulation for Δ=0. Show the output of both angles, and the controller usage in a separate plot.
2.) **(AT HOME)** Check the robust stability and performance of your design. How do the results compare to the H_2 design?
3.) **(IN LAB)** Implement your final controller on the hardware, using $\Delta = 0$. Plot the response from your hardware implementation (angles and voltages). If you are running into stability issues, you again might want to drop ω_c.
4.) **(IN LAB)** Run the hardware simulation for 5 uncertain samples and plot the performance. Note that the system may hit the hard stops / become unstable. If this occurs, choose a new sample and rerun.

<u>E. μ-Synthesis</u>

(AT HOME) Solve the μ-synthesis problem. Use the same weights as in the H_∞ design, and reduce the controller order to match the order of the H_∞ controller.

1.) **(AT HOME)** Plot the response from your simulation. Show the output of both angles, and the controller usage in a separate plot.
2.) **(AT HOME)** Check the robust stability and robust performance of your design. Run the simulation for 10 values of Δ taken from samples of an uncertain LTI system and plot the responses.
3.) **(IN LAB)** Implement your final controller on the hardware (using $\Delta = 0$). Plot the response from your hardware implementation (angles and voltages).
4.) **(IN LAB)** Run the hardware simulation for 5 uncertain samples and plot the performance.

Tips and Tricks:

- The plant has a massive deadzone nonlinearity. DO NOT expect the hardware results to closely match your simulations. However, the trends between controllers may be comparable. This deadzone will also likely make your H_∞ loop shaping controller (which has a pure integrator) perform better than the synthesized controllers with finite DC gain.
- While the simulation may appear to be continuous-time, in fact the controller is discretized before implementation. The sampling rate of the model is 1 kHz, and hence the Nyquist frequency is 500 Hz (~3000 rad/s). As a rule of thumb, you should not have controller poles above ~300 rad/s to see good discrete-time reconstruction. This *shouldn't* be an issue for you, but might be if you really try to push performance.
- See the QuickStart instructions to see how to build and run Simulink models on the hardware. I would start by running the example controller given below.
    - You *can* build and run the model through a script using the following commands:
      ```
      qc_build_model('Quanser_Aero_Plant');
      qc_start_model('Quanser_Aero_Plant');
      ```
    - To record data using the "To Workspace" block, do the following to prevent data loss.
        - Code -> External Mode Control Panel -> Signal & Triggering -> Change Duration to 200000 or however many points you want to save.

    - Use "Array" as the save format.
- Be careful about signs. The nice thing is that the instability will not be catastrophic. If things aren't working, just flip the controller sign and try again.
- If you are having trouble getting anything to work on the hardware, the following controller should stabilize the system and have decent performance. Note: This contains black magic LQR tuning - yours probably won't be this good.

$$K = \begin{bmatrix} \frac{1711s + 4910}{s + 50} & \frac{-1557s - 5153}{s + 50} \\ \frac{2432s + 7817}{s + 50} & \frac{921.5s + 3308}{s + 50} \end{bmatrix}$$

Reporting

Compile a single PDF lab report for your team following the provided format. Place the PDF file and all associated files (including all Simulink models and Matlab scripts) in a .zip file and upload to Canvas.

MEMORANDUM

TO:	Dr. Bedillion
FROM:	24-773 Student Names
DATE:	March 26, 2025
RE:	MLC Laboratory

A paragraph or two here should transmit the laboratory report. You should think of this as the report abstract.

This report has been proofread by all members of the group:

Print Name	Signature and Date
Print Name	Signature and Date
Print Name	Signature and Date

Your "Informal Laboratory Report" should start here. It should be organized in terms of numbered items in the lab procedure. For each numbered item in the lab procedure you must address the following items at a minimum:

1.) A brief description of the goals of the lab exercise, and the equipment and procedure used to achieve those goals. The equipment can be specified once per subsection, i.e. describe the aero system only once, not *n* times.
2.) The details of all calculations involved in generating your results. Be sure to highlight the main results.
3.) Presentation of your results in the form of plots and tables. This should include all relevant plots and Simulink models. <u>Do not</u> present plots that use the black background that is the Simulink scope default. Place in-line links to your Youtube videos for video results.
4.) General discussion. What sense do you make of the results? What can you conclude?
5.) Answers to all of the discussion questions in the lab procedure.

After completing these tasks for all numbered items in the lab procedure, complete the following sections to finish your report:

- <u>Conclusions</u>: What were the main results? What did you learn (if anything) by completing the lab? What suggestions do you have to make the lab better or more interesting?
- <u>Work Distribution</u>: To what specific tasks did each team member contribute? Address this for both the laboratory exercises and the report writing.
- <u>References</u>: Compile all of your references into a single section at the end of the document. I highly recommend the use of a reference manager, e.g. Bibtex, EndNote, etc.
- <u>Appendix</u>: Attach scans of any hand calculations and copy and paste any Matlab / Arduino code. This is simply for ease of grading; you will also be posting the code files in your .zip file.

Lab Report Rubric

Front matter (5 points)	The front matter includes the transmission memorandum (signed by all students) along with the description of the experiments. • Report signed and dated by all students (1.5 pts) • Report abstract adequately describes the effort (1.5 pts) • Introduction and equipment specification well written (2 pts)
Preliminaries (5 points)	• Simulink model presented in a figure and described (2.5 pts) • Multiplicative uncertainty weight correctly calculated (2.5 pts)
H_∞ Loop Shaping (20 points)	• H_∞ Loop shaping controller designed correctly (5 pts) • Simulation performance (2.5 pts) • RS and RP calculation / simulation (2.5 pts) • Hardware results reasonable for nominal system (2.5 pts) • Hardware results reasonable for uncertain samples (2.5 pts) • All results supplemented with descriptive text so that reader understands importance of results. Figures are clear and readable. (5 pts)

H_2 Optimal Control (20 points)	• H_2 controller designed correctly (weights and system interconnection) (5 pts) • Simulation performance (2.5 pts) • RS and RP calculation / simulation (2.5 pts) • Hardware results reasonable for nominal system (2.5 pts) • Hardware results reasonable for uncertain samples (2.5 pts) • All results supplemented with descriptive text so that reader understands importance of results. Figures are clear and readable. (5 pts)
H_∞ Optimal Control (20 points)	• H_∞ controller designed correctly (weights and system interconnection) (5 pts) • Simulation performance (2.5 pts) • RS and RP calculation / simulation (2.5 pts) • Hardware results reasonable for nominal system (2.5 pts) • Hardware results reasonable for uncertain samples (2.5 pts) • All results supplemented with descriptive text so that reader understands importance of results. Figures are clear and readable. (5 pts)
μ Synthesis (20 points)	• μ controller designed correctly (weights and system interconnection, uses uncertain generalized plant) (5 pts) • Simulation performance (2.5 pts) • RS and RP calculation / simulation (2.5 pts) • Hardware results reasonable for nominal system (2.5 pts) • Hardware results reasonable for uncertain samples (2.5 pts) • All results supplemented with descriptive text so that reader understands importance of results. Figures are clear and readable. (5 pts)
Conclusions (2.5 points)	The main goals of the lab are summarized along with what the students took away.
Supplemental Material (2.5 points)	The group has provided a work distribution along with the requested Matlab scripts (you do NOT need to check the scripts – just whether they're present).
Grammar & Mechanics (5 points)	The paper is free of grammatical and spelling errors and has clearly been proofread.