Spatial Operations: Single Pixel Transforms

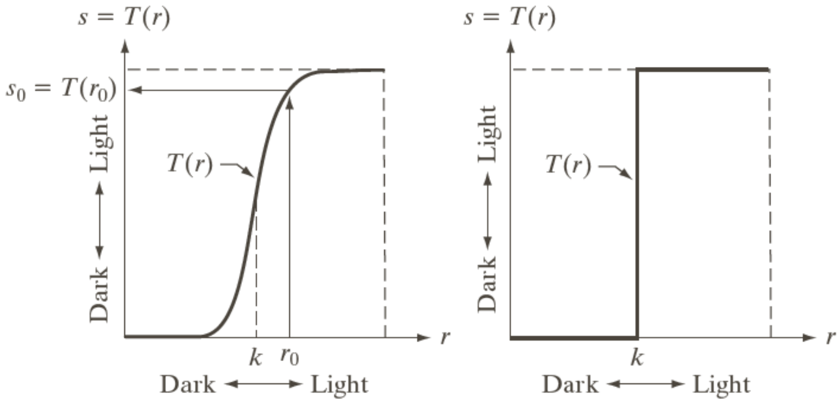
Computer Vision 2017

Figures from “Digital Image Processing” (Gonzalez-Woods)
Slides: P.Zanuttigh

Spatial Operations

- Geometric transforms
- Single-pixel operations
- Operations on the neighborhood of a point

Intensity Transforms



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

The intensity of the pixel in the output image is a function *s=T(r)* of the corresponding pixel in the input image

- Negative
- Logarithm
- Gamma
- Contrast stretching
- Intensity slicing
- Histogram equalization

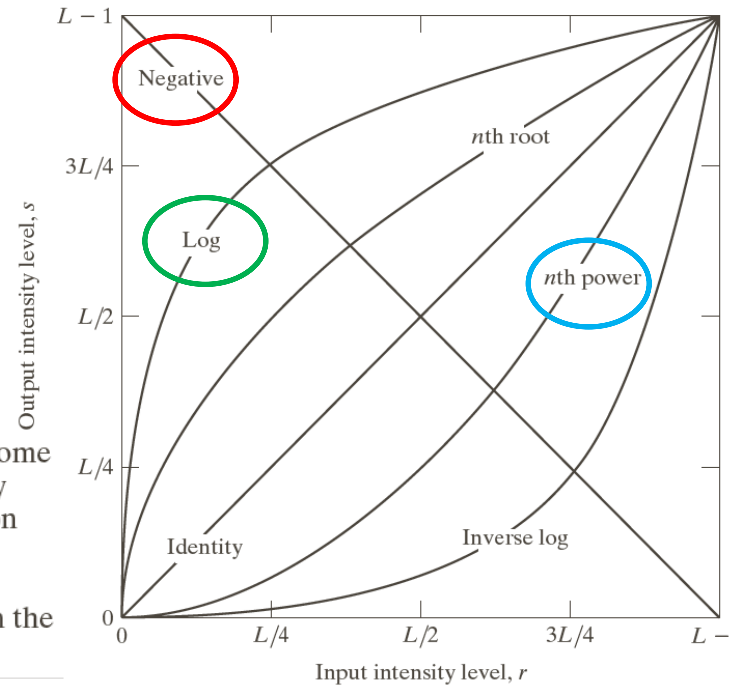
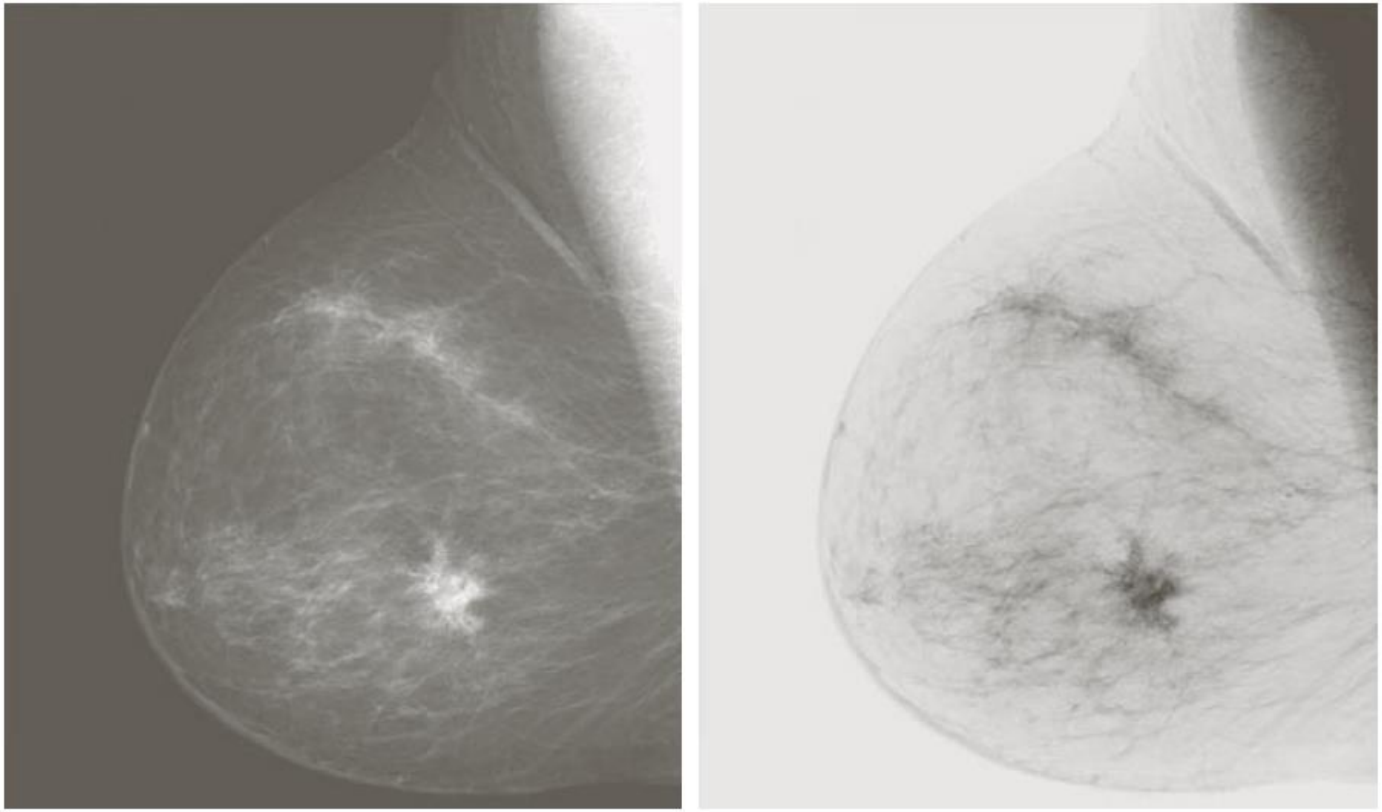


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Negative of an Image



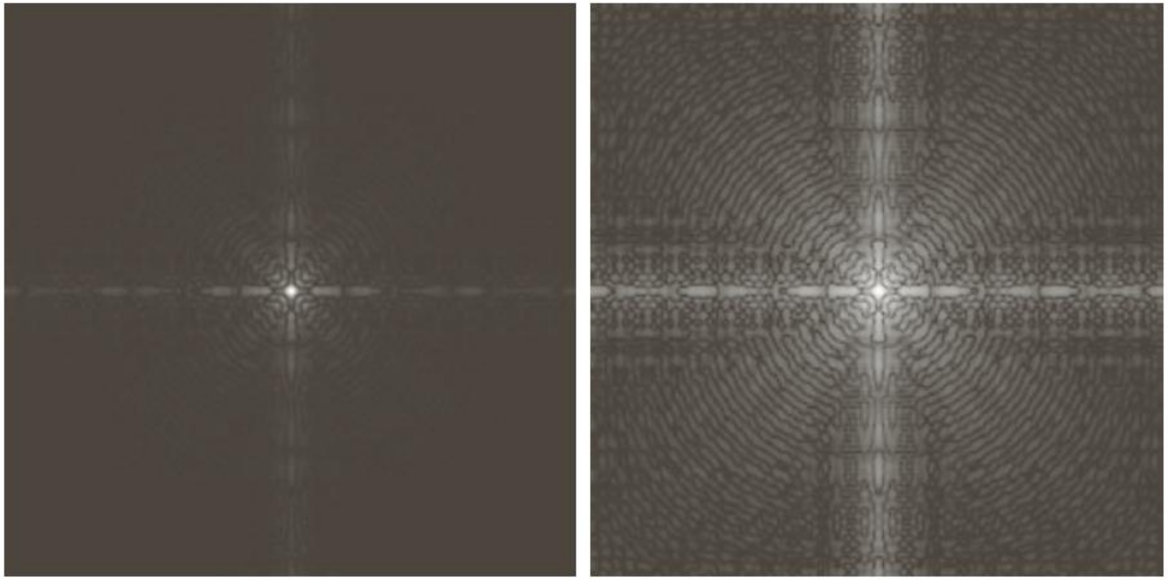
a b

FIGURE 3.4
(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

$$s = (L-1) - r$$

r : input value
s : output value
$r, s \in [0, \mathrm{L}-1]$

Log Transform

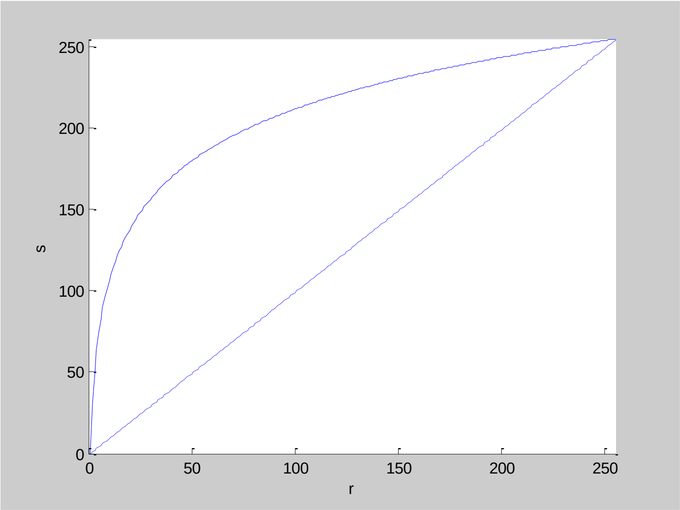


a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

$$s = c\log(1+r)$$

$$c = \frac{L-1}{\log(L)}$$



Gamma Transformation

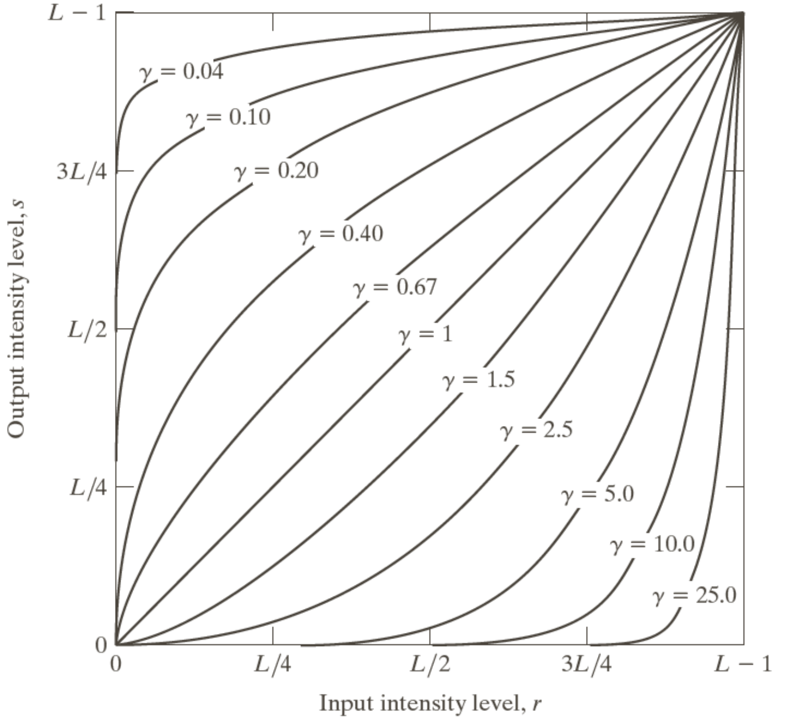


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ . All curves were scaled to fit in the range shown.

$$s = cr^{\gamma}$$

$$c = (L-1)^{1-\gamma}$$

Gamma Correction (Monitor)

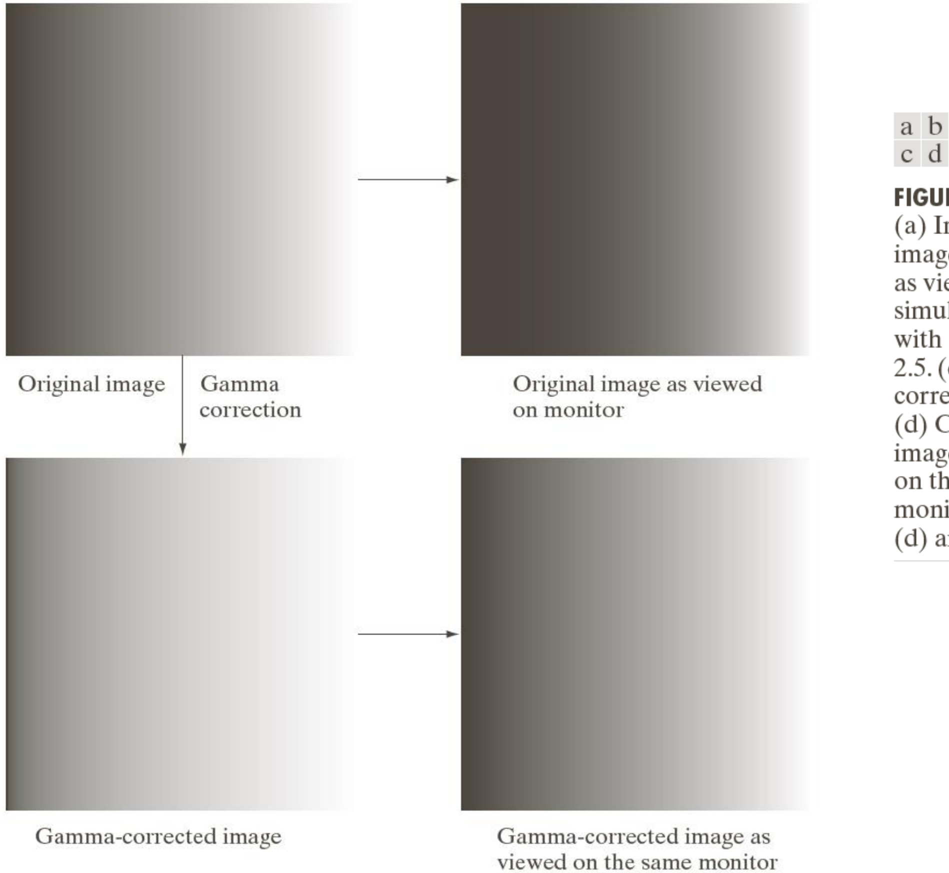


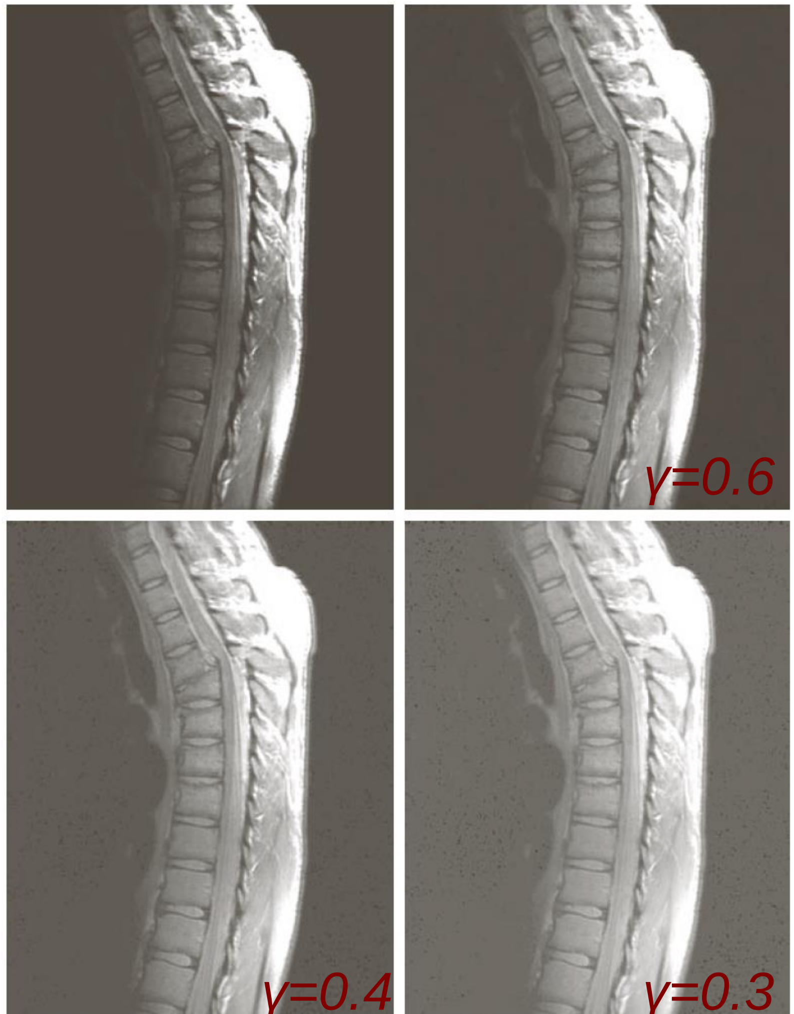
FIGURE 3.7 (a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

- Luminous intensity of monitor and cameras : $I = V^{\gamma}$
- Can be compensated with the gamma transform $s=cr^{1/\gamma}$
- This transform is used in the sRGB color space

Example of Gamma Correction (*1*)

$\boxed{\gamma<1}$

Image gets brighter



a b
c d

FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Example of Gamma Correction (2)

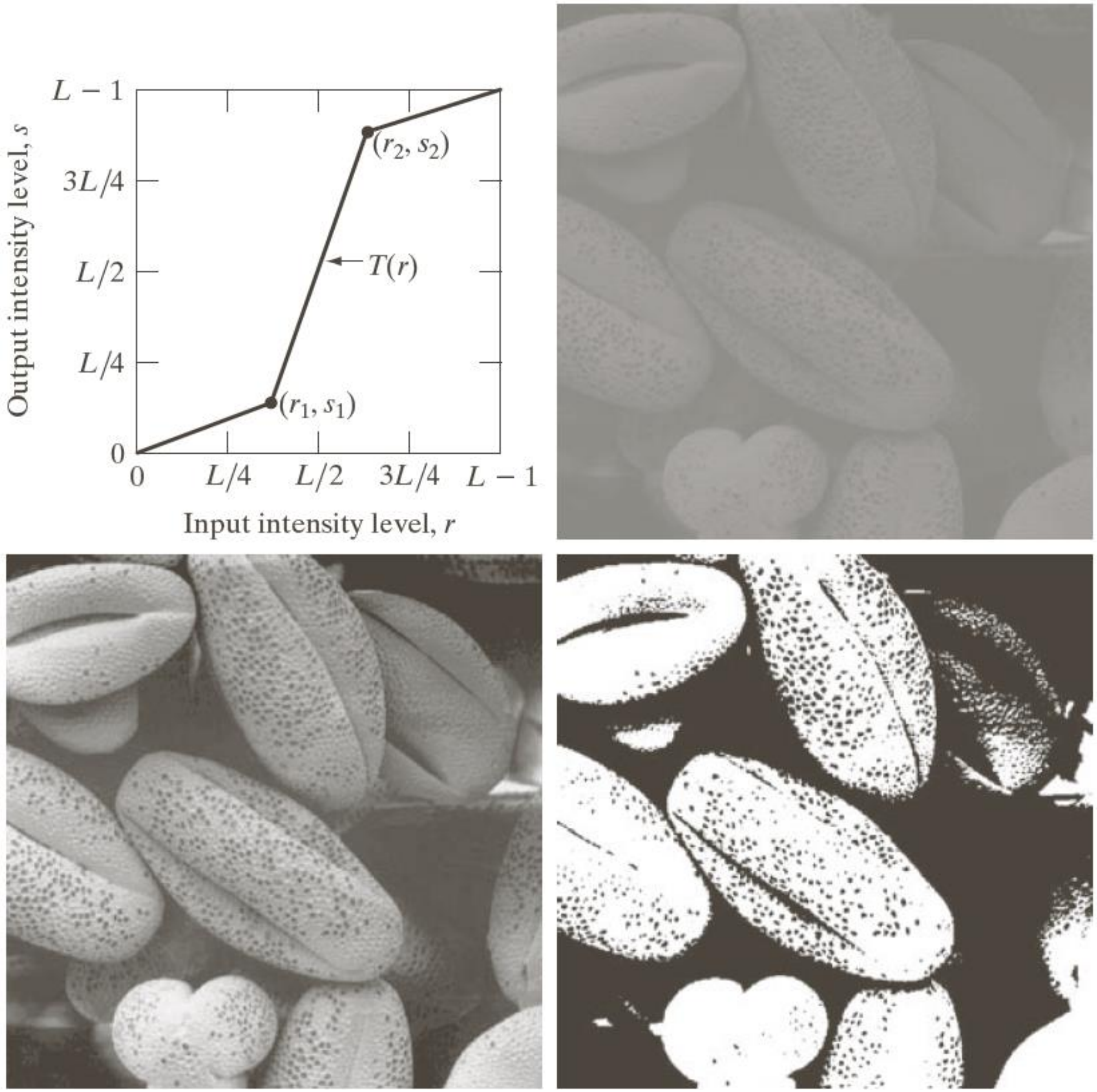
$\boxed{\gamma>1}$

Image gets darker

a	b
c	d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0$, 4.0, and 5.0, respectively. (Original image for this example courtesy of NASA.)

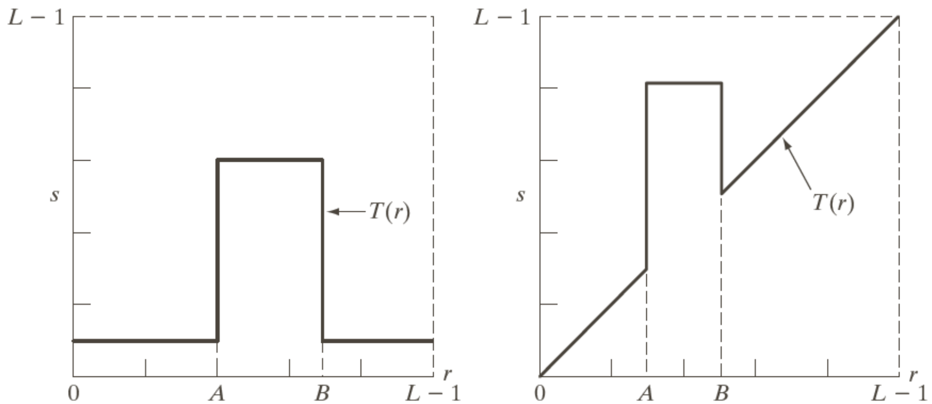
Contrast Stretching



a	b
c	d

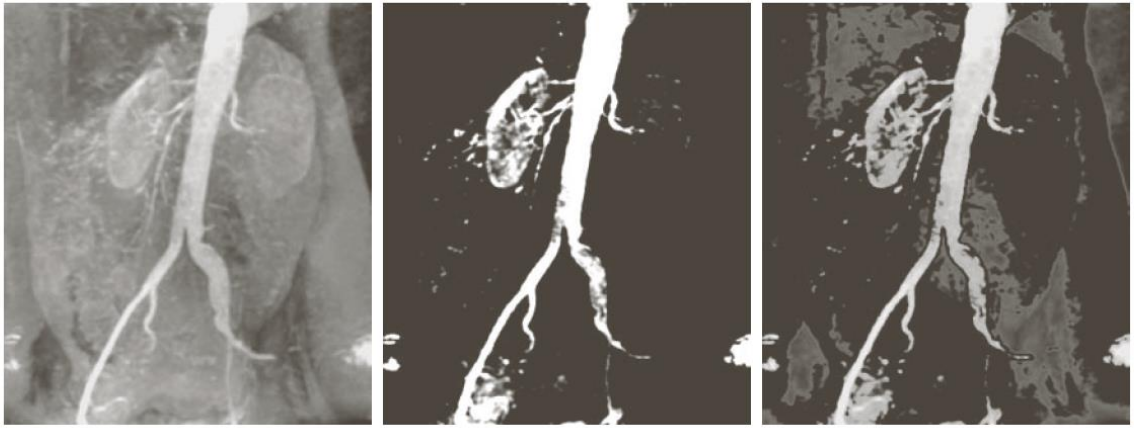
FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Intensity Slicing



a b

FIGURE 3.11 (a) This transformation highlights intensity range [*A*, *B*] and reduces all other intensities to a lower level. (b) This transformation highlights range [*A*, *B*] and preserves all other intensity levels.



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Bit Plane Slicing

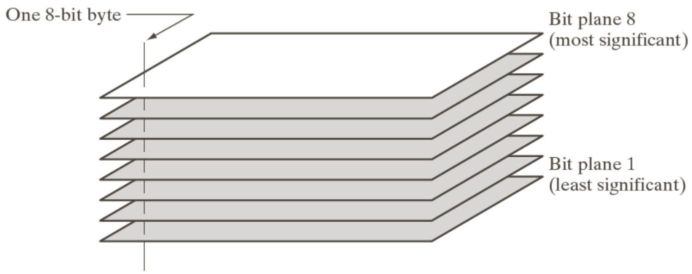


FIGURE 3.13 Bit-plane representation of an 8-bit image.

$$I = \sum_{k=0}^{\log L - 1} b_i 2^k$$

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Histogram of an Image

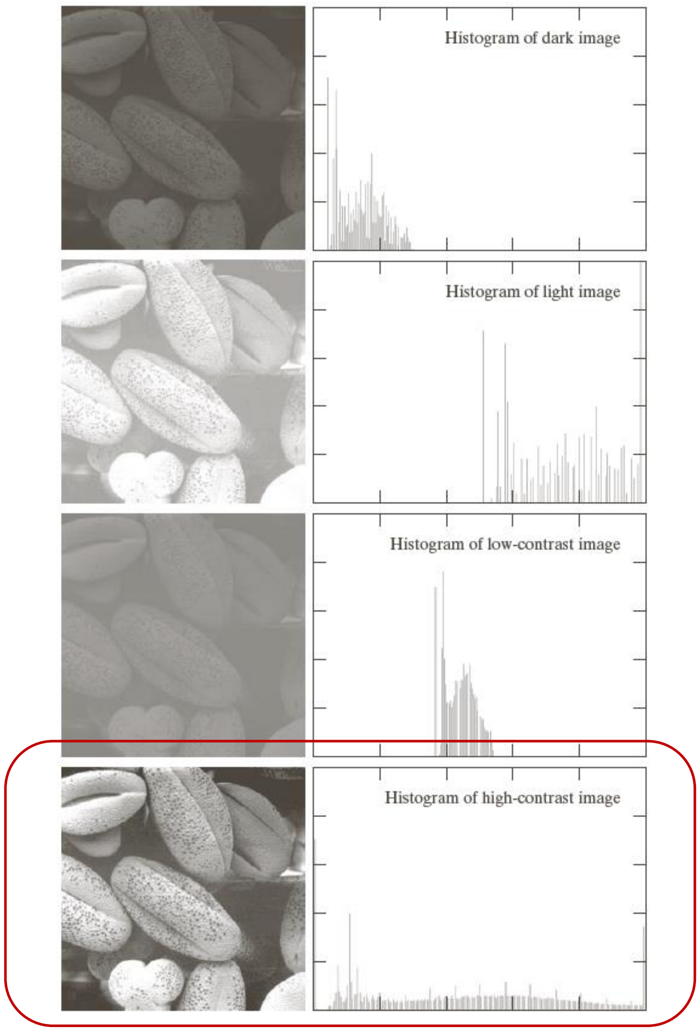


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Normalized Histogram

$$p(r_k) = \frac{h(r_k)}{MN} = \frac{n_k}{MN}$$

$h(r_k)$= n_k = number of pixels with intensity equal to r_k

Can be viewed as a *probability density*

Usage:

1. Image statistics
2. Compression
3. Segmentation
4. Image enhancement

Histogram Equalization Function

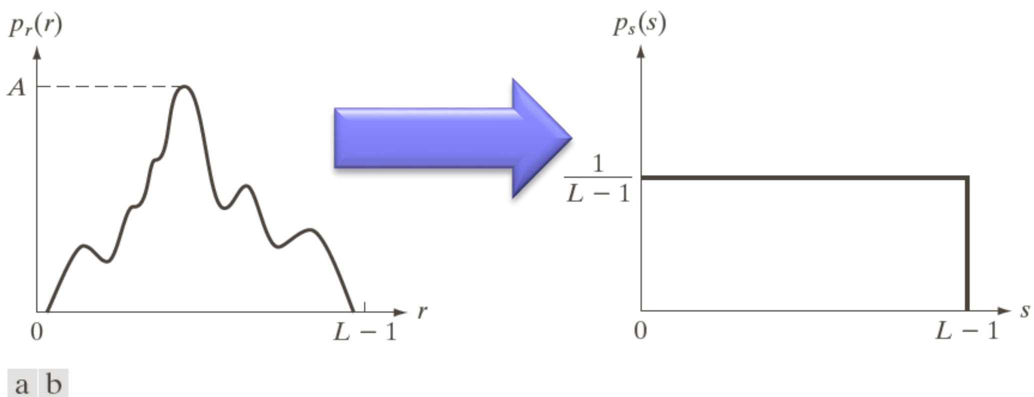
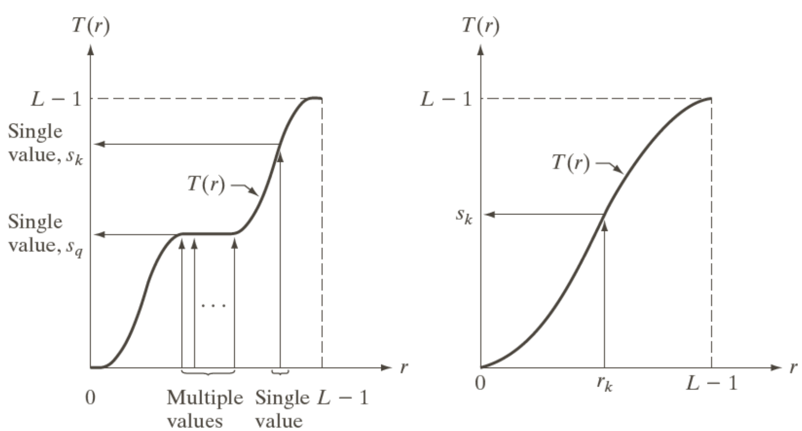


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Transform the intensity values in order to obtain an histogram as flat as possible

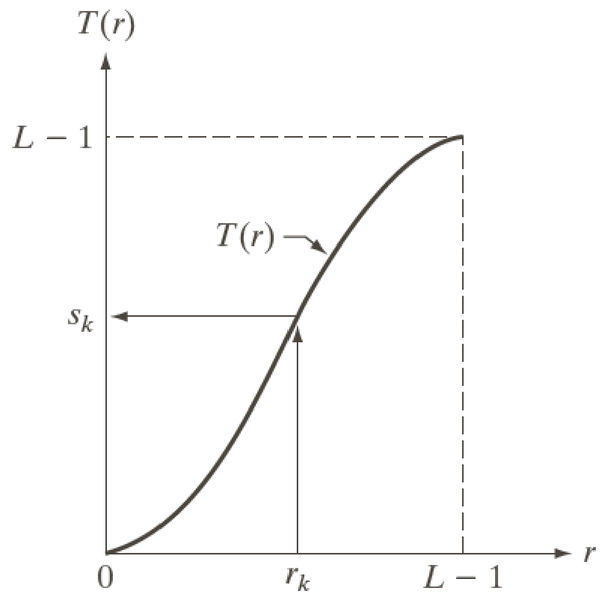


a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

The transform is invertible only if strictly monotonically increasing

Conditions for the Equalization Function



$s = T(r)$

1. *T(r)* monotonic not decreasing*
2. $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$
3. *T(r)* continuous and differentiable

*Monotonic increasing for invertibility in histogram specification

Histogram Equalization

1

2

3

4

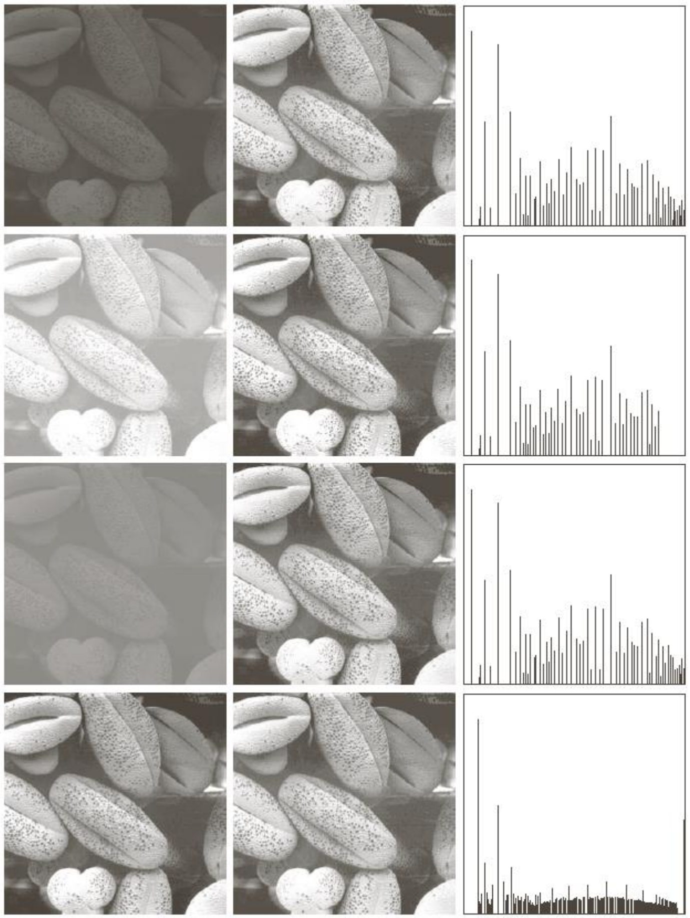


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

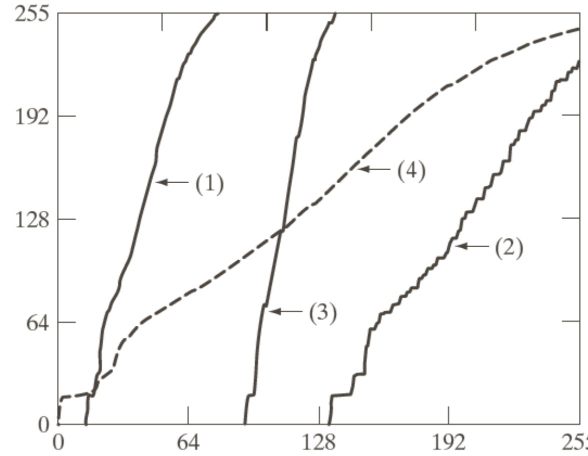


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

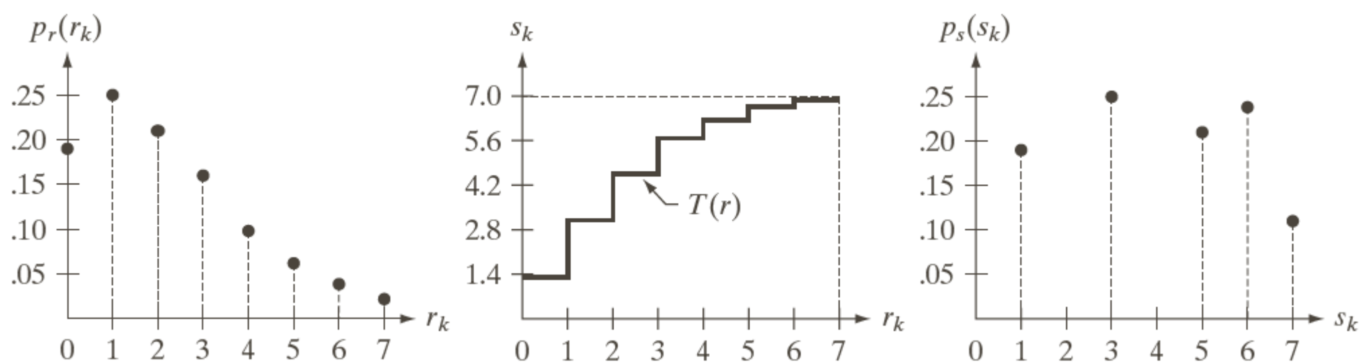
Histogram Equalization: Example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

$$s_i = 7\sum_{j=0}^{i} p_r(r_j)$$

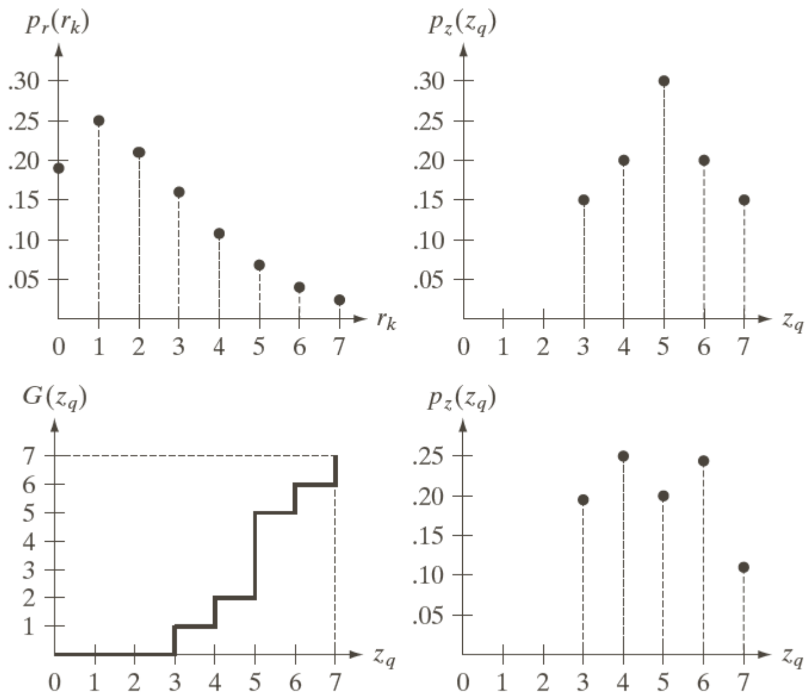
r		s_i	round
0	S_0	1.33	1
1	S_1	3.08	3
2	S_2	4.55	5
3	S_3	5.67	6
4	S_4	6.23	6
5	S_5	6.65	7
6	S_6	6.86	7
7	S_7	7.00	7



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Specification



a b
c d

FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Continuous

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

$$z = G^{-1}(s)$$

Discrete

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

$$z_q = G^{-1}(s_k)$$

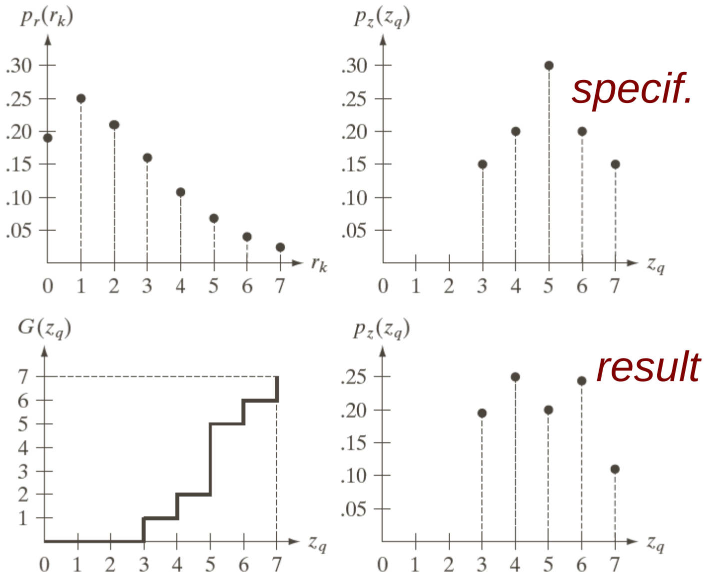
1: compute $p_r(r)$ and $T(r) = (L-1)\int_0^r p_r(w)dw$

2: Given $p_z(z)$, compute $G(z) = (L-1)\int_0^z p_z(t)dt$

3: Compute $z = G^{-1}(s) = G^{-1}(T(r))$

4: Apply G^{-1} to the equalized image

Histogram Specification (Example)



z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2 Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

1

S=T(r)	val	rounded
S_0	1.33	1
S_1	3.08	3
S_2	4.55	5
S_3	5.67	6
S_4	6.23	6
S_5	6.65	7
S_6	6.86	7
S_7	7.00	7

$G(z_q)$	val	rounded
$G(z_0)$	0	0
$G(z_1)$	0	0
$G(z_2)$	0	0
$G(z_3)$	1.05	1
$G(z_4)$	2.45	2
$G(z_5)$	4.55	5
$G(z_6)$	5.95	6
$G(z_7)$	7.00	7

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

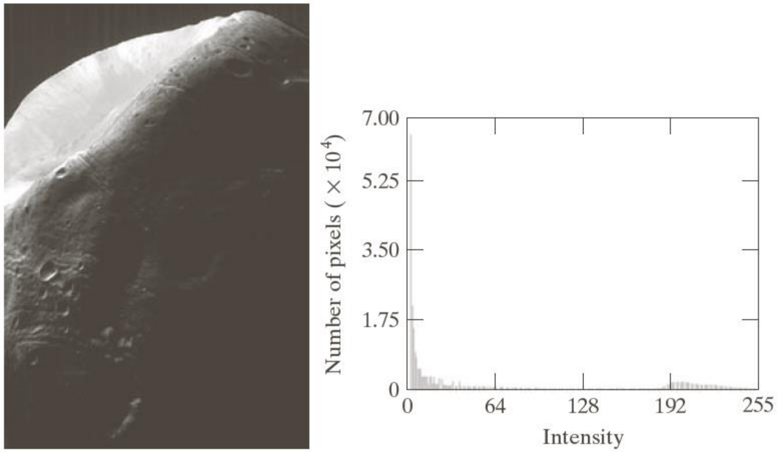
$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

$$z_q = G^{-1}(s_k)$$

Histogram Specification Procedure

1. Compute $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$
2. Compute $G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$ and round the values
3. Create table $z_q \leftrightarrow G(z_q)$
4. For each r_k get the corresponding s_k and search for the closest $G(z_q)$
5. Build the equalized image by mapping each r_k in the corresponding z_q

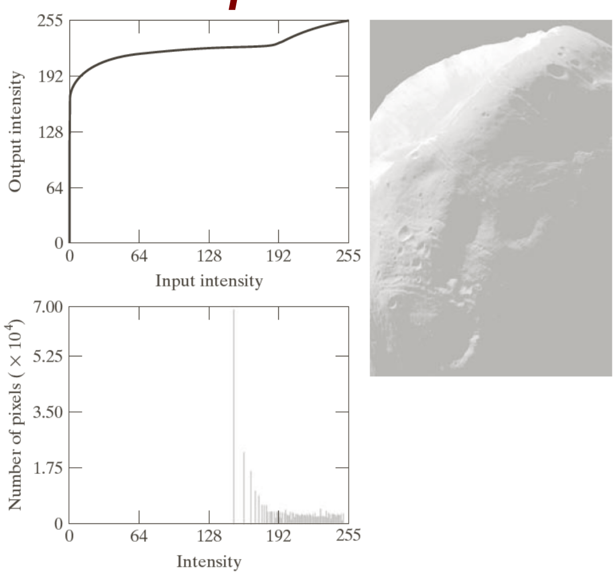
Histogram Equalization and Specification



a b

FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

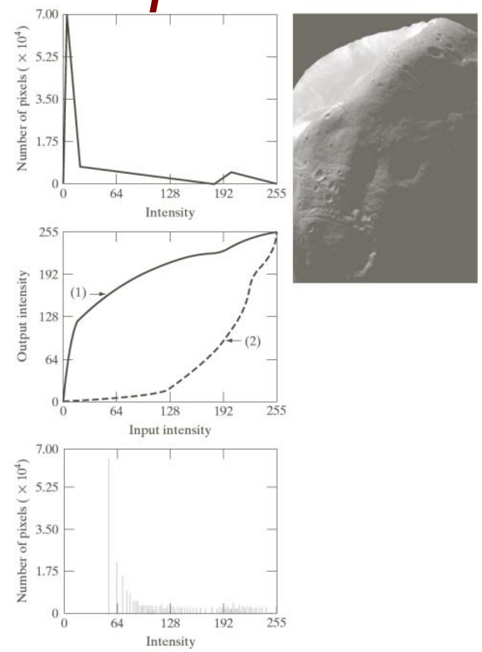
Equalization



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

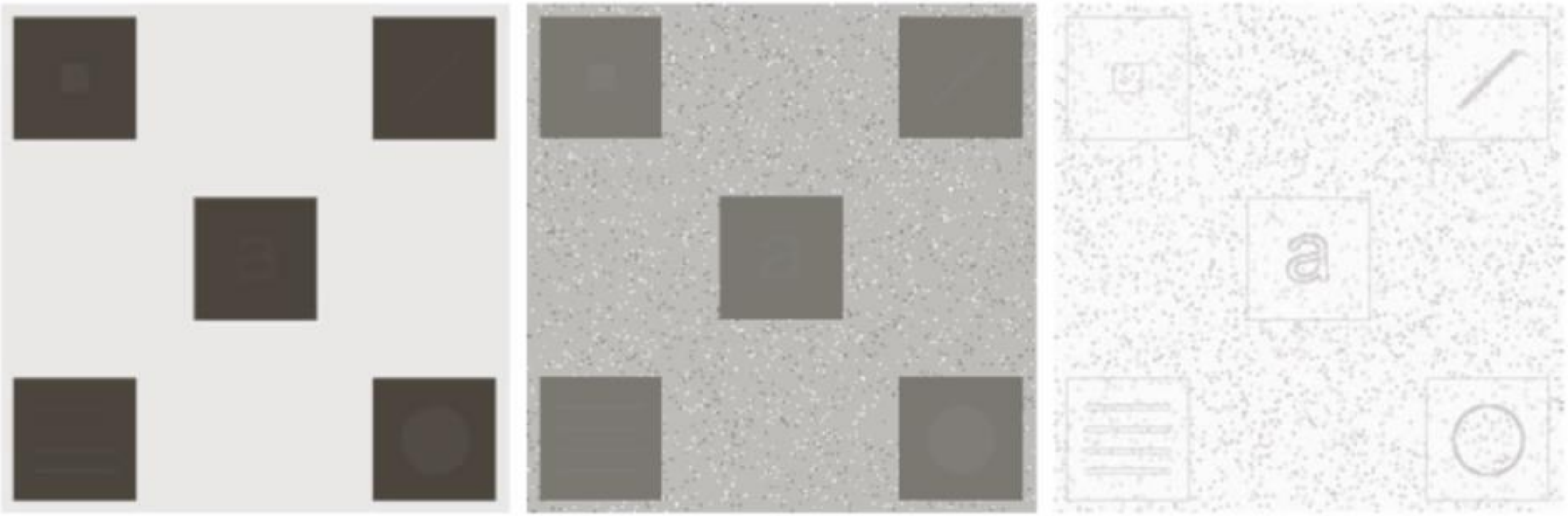
Specification



a c
b
d

FIGURE 3.25
(a) Specified histogram. (b) Transformations. (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).

Local Histogram Processing



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.