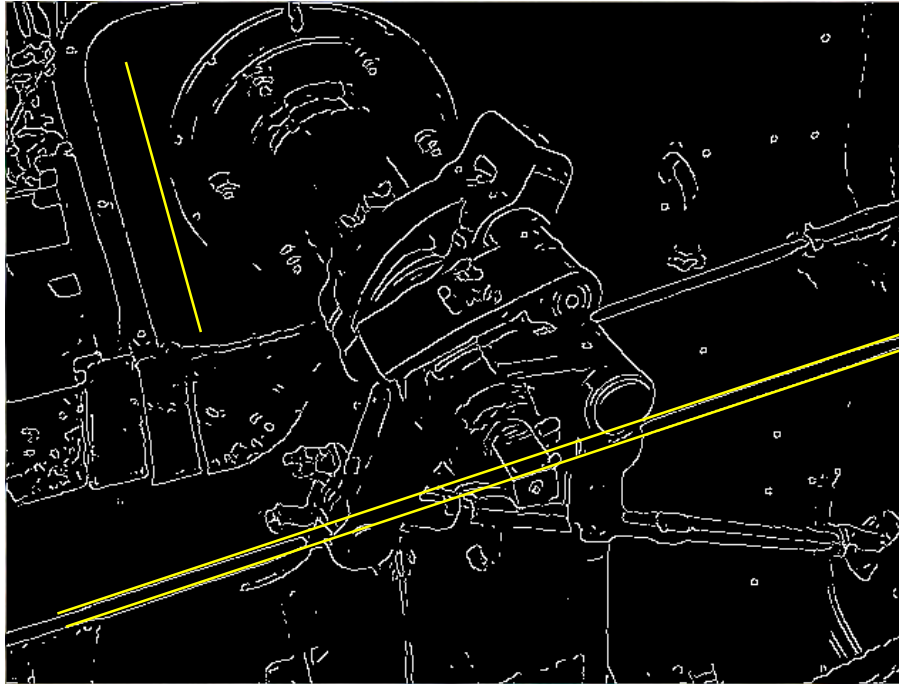




Hough Transform

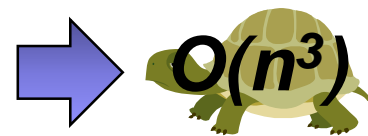
Computer Vision 2017

Find the Lines in an Image

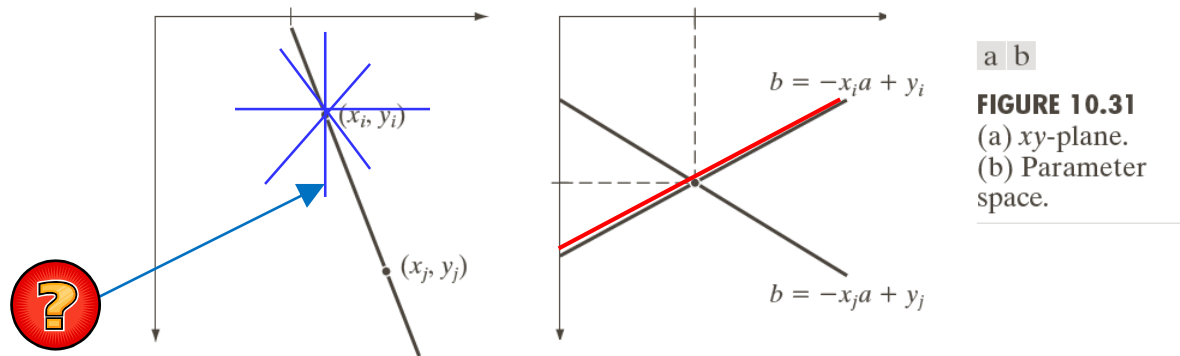


The “*brute-force*” approach:

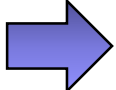
1. Find the edges (e.g., with Canny)
2. Find the line passing for each couple of edge points
3. Count the number of edge points compatible with each considered line



Hough Transform



Bundle of lines passing through a point  *Line in the parameter space*

Line through 2 points  *Intersection of the 2 corresponding lines in the parameter space*

Hough Transform: Work in the parameter space

$$y_i = ax_i + b$$



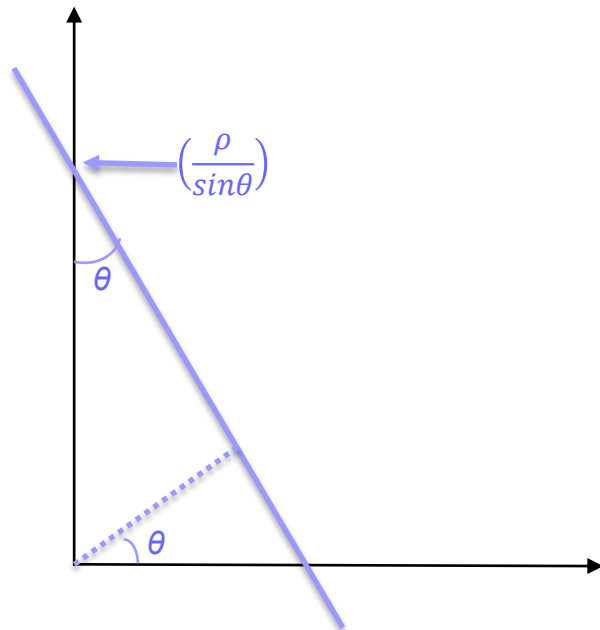
$$b = -x_i a + y_i$$

Normalized Representation

$$x \cos \theta + y \sin \theta = \rho$$

$$y = \left(-\frac{\cos \theta}{\sin \theta} \right) x + \left(\frac{\rho}{\sin \theta} \right)$$

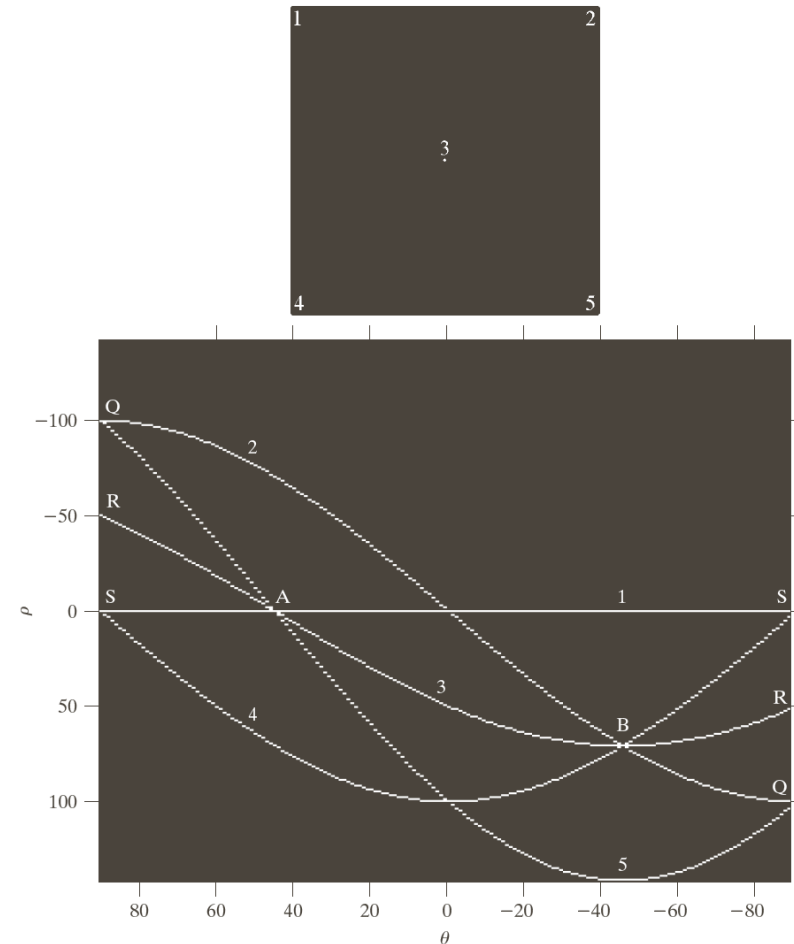
- *Works also with vertical lines*
- *Sinusoidal curves*
- *Intersections: line through both points*



a
b

FIGURE 10.33

(a) Image of size 101×101 pixels, containing five points.
(b) Corresponding parameter space. (The points in (a) were enlarged to make them easier to see.)

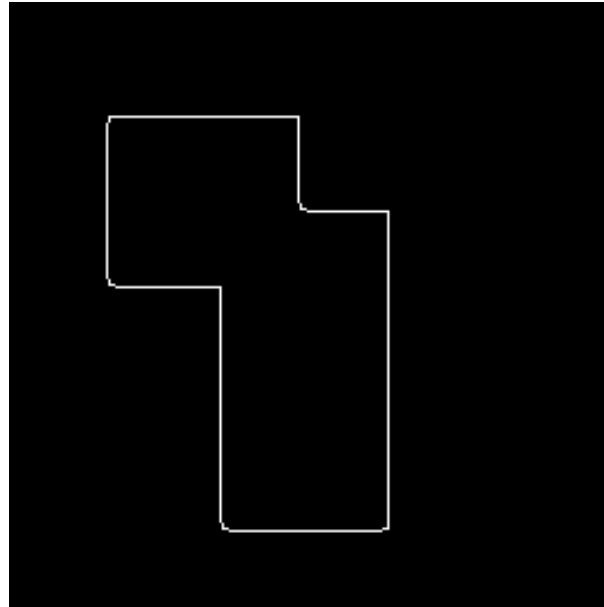


Interactive
Example

Hough Transform: Example



Image

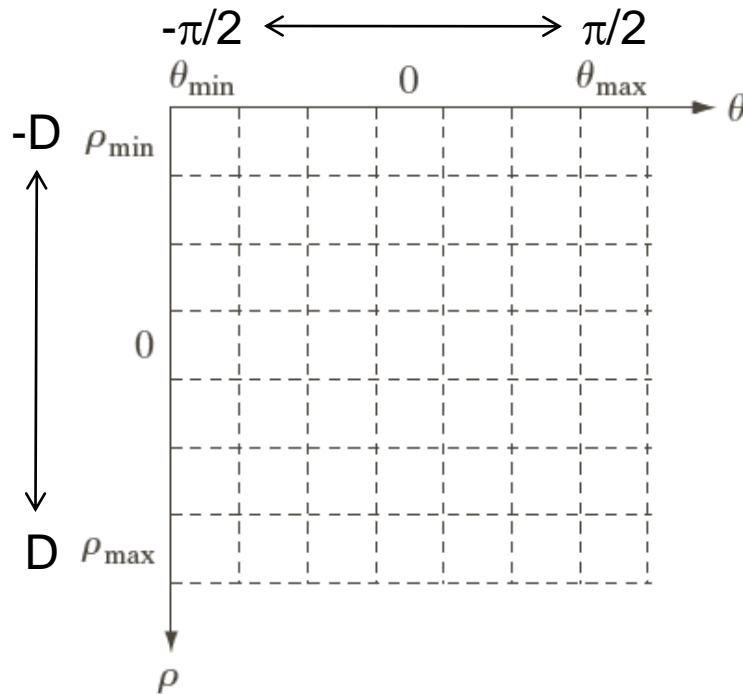


Edges



Hough Transform

Accumulation Cells



Few Cells	Many Cells
Handle pixels not perfectly aligned	Requires precise alignment
Poor lines localization	Accurate lines localization

- ❑ The parameter space is quantized along ρ and θ
- ❑ The cell subdivision allows to handle points not perfectly aligned

Algorithm

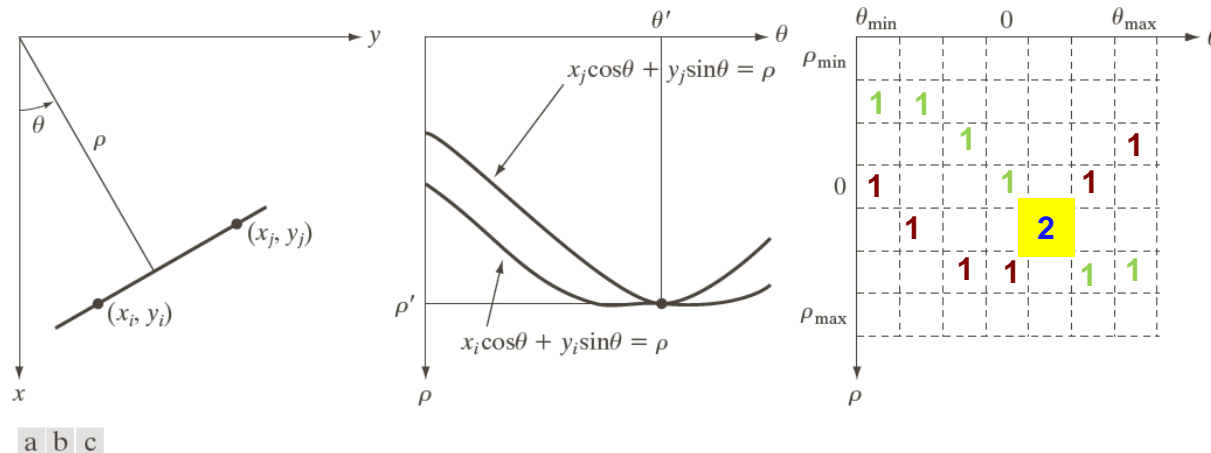
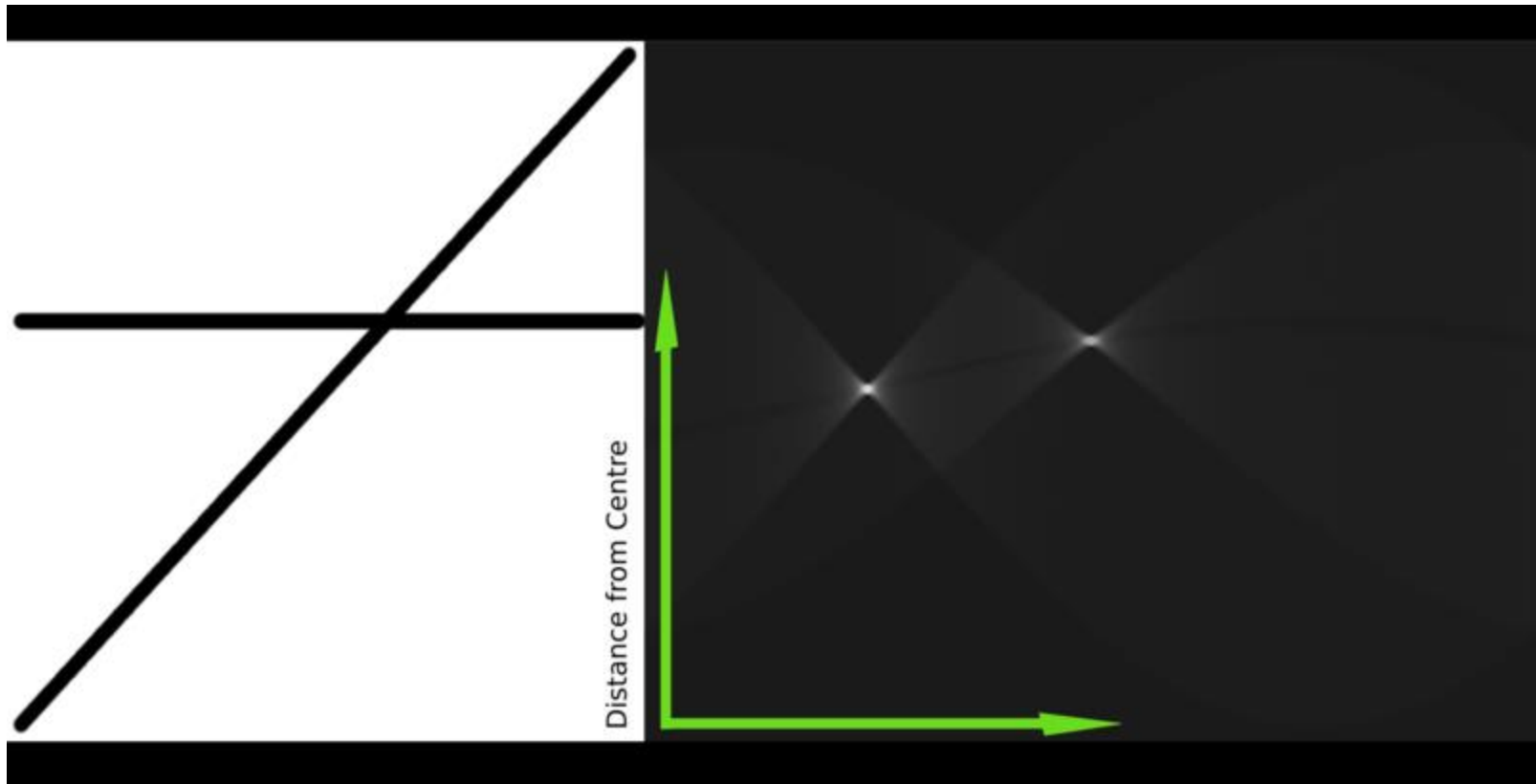


FIGURE 10.32 (a) (ρ, θ) parameterization of line in the xy -plane. (b) Sinusoidal curves in the $\rho\theta$ -plane; the point of intersection (ρ', θ') corresponds to the line passing through points (x_i, y_i) and (x_j, y_j) in the xy -plane. (c) Division of the $\rho\theta$ -plane into accumulator cells.

1. The parameter space is quantized in cells, there is a counter for each cell
2. For each edge pixel:
 - A. Let θ vary on the quantized interval and compute the corresponding ρ values
 - B. For each crossed cell increment by 1 the counter
3. The counter for each cell contains the number of pixels collinear on that line

Example (1)



Example (2)

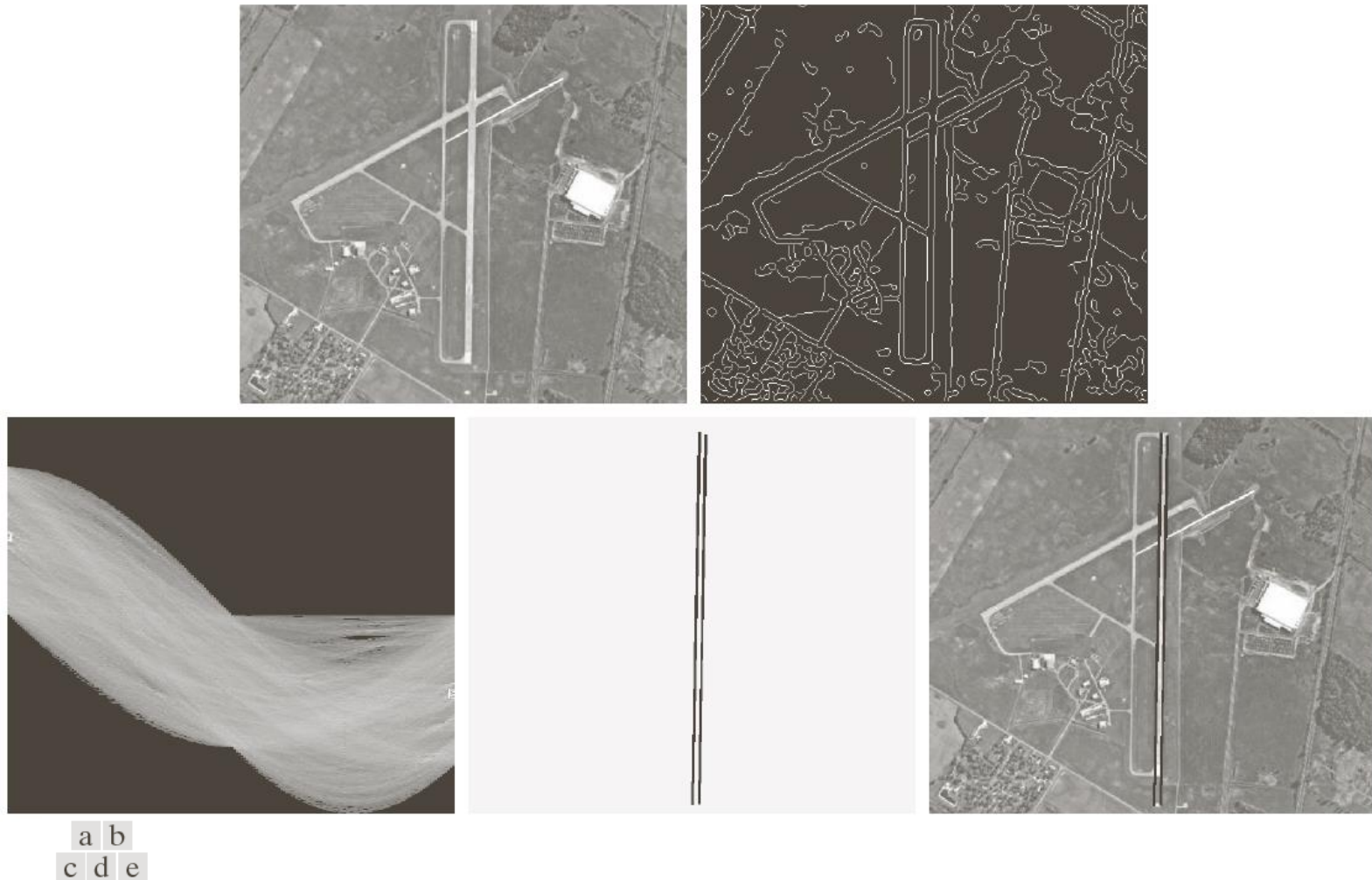
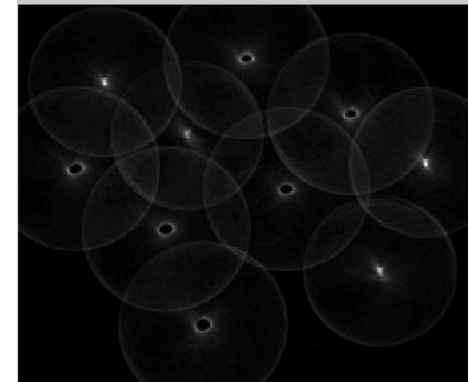
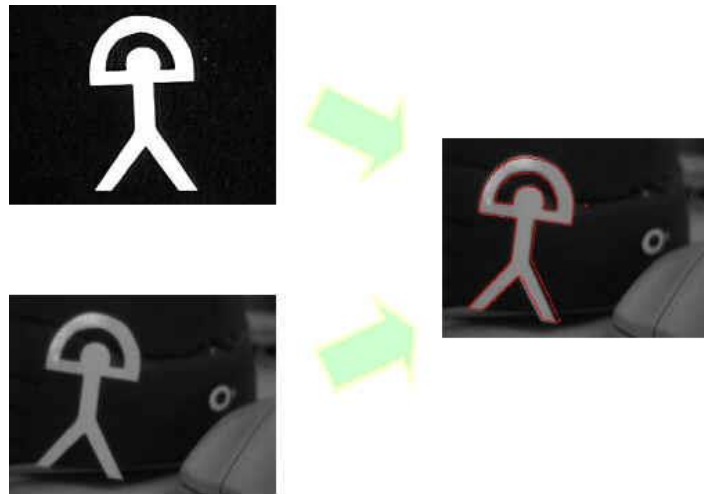


FIGURE 10.34 (a) A 502×564 aerial image of an airport. (b) Edge image obtained using Canny's algorithm. (c) Hough parameter space (the boxes highlight the points associated with long vertical lines). (d) Lines in the image plane corresponding to the points highlighted by the boxes. (e) Lines superimposed on the original image.



Generalised Hough Transform



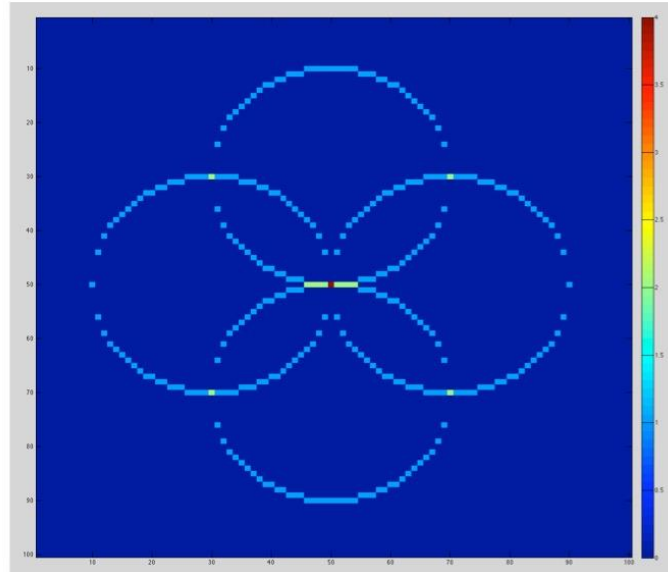
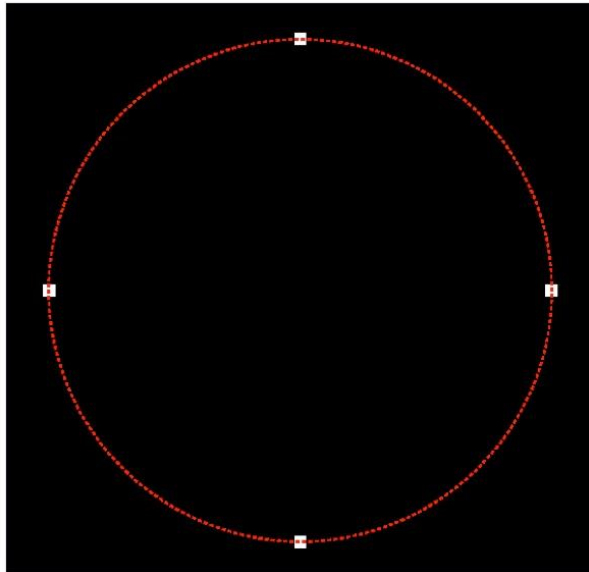
$$g(\mathbf{v}, \mathbf{c}) = 0 \rightarrow (x - c_1)^2 + (y - c_2)^2 = c_3^2$$

Generic form:
table with the function values

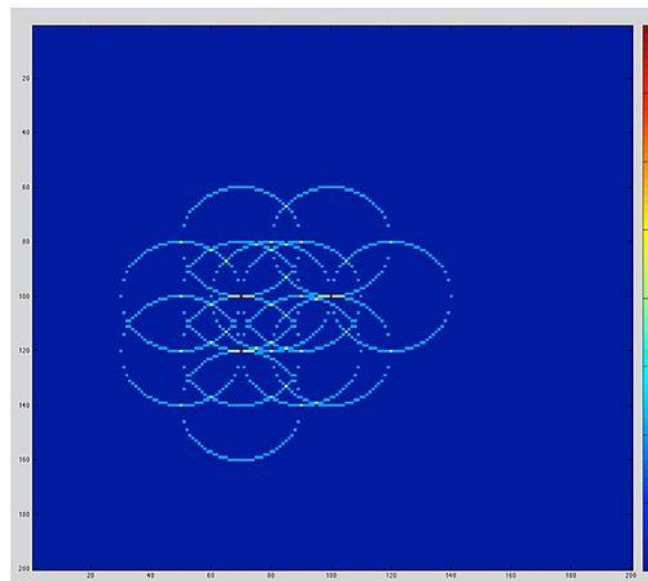
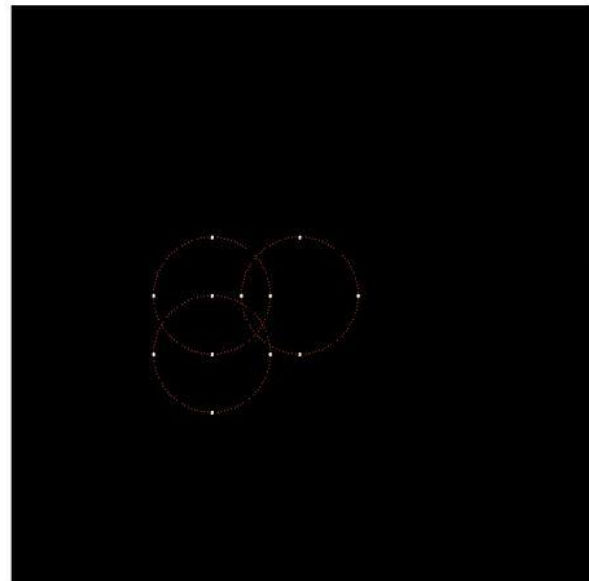


Dimensionality of vector \mathbf{c} (parameter space)
→ if large the approach is of limited practical application

Example: Find the Circles



$$(x - c_1)^2 + (y - c_2)^2 = r_{fixed}^2$$



*Simple example
with fixed radius*