

Laboratory 2: Difference Equations. Equilibrium Points. Periodic Points. Stability

1. Build a numerical solution for the following initial value problems. Plot your data to observe patterns in the solutions. Is there an equilibrium solution? Is it stable or unstable?

- (a) $a_{n+1} = -1.2a_n + 50, a_0 = 1000;$
- (b) $a_{n+1} = 0.8a_n - 100, a_0 = 500;$
- (c) $a_{n+1} = 0.8a_n - 100, a_0 = -500;$
- (d) $a_{n+1} = a_n - 100, a_0 = 1000;$

2. For the following problems find the solution to the difference equation and the equilibrium value if one exists. Discuss the long-term behaviour of the solutions for various initial data. Classify the equilibrium values as stable or unstable. Draw the Cobweb diagram for each equation with different initial starting points.

- (a) $a_{n+1} = -a_n + 2;$
- (b) $a_{n+1} = a_n + 2;$
- (c) $a_{n+1} = a_n + 3.2;$
- (d) $a_{n+1} = -3a_n + 4;$
- (e) $a_{n+1} = a_n^2 + 3a_n;$

3. (Newton's Method of Computing the Square Root of a Positive Number)

The equation $x^2 = a$ can be written in the form $x = \frac{1}{2}(x + \frac{a}{x})$. This form leads to Newton's method

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

- (a) Show that this difference equation has two equilibrium points \sqrt{a} and $-\sqrt{a}$;
 - (b) Sketch a cobweb diagram for $a = 3, x_0 = 1$ and $x_0 = -1$.
4. Let $f(x) = -\frac{1}{2}x^2 - x + \frac{1}{2}$ and the difference equation

$$x_{n+1} = f(x_n).$$

Show that 1 is an asymptotically stable 2-periodic point of f .

5. Find the solution for the following difference equations:

- (a) $x_{n+2} - 5x_{n+1} + 6x_n = 0, x_0 = 1, x_1 = 1;$
- (b) $y_{n+2} - 3y_{n+1} + 2y_n = 2n^2 + 6n, y_0 = 1, y_1 = 2;$
- (c) $y_{n+2} + 3y_n + 2y_n = 3^n(2n^2 + 4n), y_0 = 2, y_1 = 1;$

6. Consider the simple interest formula $S_n = (1 + np)S_0$ and the compound interest formula $S_n = (1 + p/r)^n S_0$. There are three options to earn interest. Company A offers simple interest at a rate of 6%. Company B offers compound interest at a 4% rate with a conversion period of one month. Company C offers compound interest at a 4% rate with a conversion period of three months.
- (a) Calculate for the three cases the amount on deposit after 5, 10, 15, and 20 years for any principal S_0 .
 - (b) Which interest offer maximizes the amount on deposit after 5, 10, 15, and 20 years?
7. The loan on a house is \$200,000.
- (a) Calculate the monthly repayment needed to have the loan repaid after 30 years. The interest rate is 5%.
 - (b) Calculate the total amount paid back on the loan.