Oceland: A conceptual model for ocean-land-atmosphere interactions based on water balance equations

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ABSTRACT: Enter the text of your abstract here.

1. Introduction

2. Model description

a. Design goals

Conceptual models do not try to explain natural processes in an exact, quantitative manner. Rather, they aim at helping us understand the dominant physical relationships that give rise to a certain natural phenomenon. These dominant factors often get modulated and thereby obscured by a plethora of other processes acting simultaneously in the real world and are therefore difficult to disentangle in observations or output of sophisticated climate models. Conceptual models can provide clarity at the expense of realism and with the danger of missing out on relevant physical processes. The successful development of a conceptual model is therefore an iterative process that begins with the most basic assumptions and ends when "the model is only as elaborate as it needs to be to capture the essence of a particular source of complexity, but is no more elaborate than this", as Held (2005) puts it. It is our hope that the proposed model of this study meets this balance and that the assumptions and choices we made in the model development process become clear.

- simplicity over realism (–> for versatile application and easier understanding)
- BUT: based on fundamental conservation laws
- where more complexity is needed: parametrizations based on established empirical relationships

b. Closed model setup

We propose a 2D box model as sketched in Figure [*****], which consists of an ocean domain, denoted by subscript ‘o’, and a land domain, denoted by subscript ‘l’. Each of the two domains contains a ground box at the bottom and an atmospheric box aloft. However, this spatial arrangement is only relevant in so far as we are interested in the water fluxes across the boundaries connecting any two boxes. We assume that all fluxes can be expressed as functions of the mean states of the model boxes so that an explicit dependence on the spatial variables (x,z) is obsolete. This choice trades some realism for the ease of working with ordinary differential equations (ODEs) instead of partial differential equations (PDEs).

The mean states of the model boxes represent their water content. For atmospheric boxes, we use the mean integrated water vapour pass w, and for the land box the mean relative soil moisture saturation s to describe the state. As the ocean is considered fully saturated at all times, the influence of the ocean state on fluxes is constant in time and can be prescribed. This means that the full information on the moisture state of the model at any given moment in time t is given by the set of state variables $\{w_o(t), w_l(t), s(t)\}$.

We limit the modelled water exchange between boxes to the following four flux types, denoted by arrows in Fig. [*****]: Evapo(transpi)ration E from ground to atmosphere, precipitation P from atmosphere to ground, advection A between the atmospheric boxes and runoff R from land to ocean. Expressions for these fluxes as functions of the state variables are provided in Section [*****].

It is important to note that we assume the model to have closed boundaries at the top of the atmosphere and the bottom of the ground boxes, while periodic boundary conditions are used in horizontal direction. This is, the model topologically resembles the walls of a cylinder and the right boundary of the land domain connects to the left boundary of the ocean domain. Motivated by a net easterly wind in the Tropics, ...? A constant mean background wind is introduced to facilitate advection and gives the atmospheric moisture transport a fixed directionality.

Lastly, the relative size of the ocean and land domain is set by the land fraction parameter α. The spatial extent of the land in x-direction is given by αL, where L denotes the full model length. Conversely, the ocean has a horizontal extent of $(1-\alpha)L$.

- box model, no spatial dimension resolved (?)
- type of fluxes, land fraction, domain size, pseudo-wind

c. Water balance equations

To a good approximation, the total amount of water is conserved within the tropical band. If we further assume that the mean water holding capacity of the atmosphere does not vary significantly over time, we can apply these properties of the tropics to our model and formulate a set of coupled differential equations that describe the rate of change of the water content in each of our model boxes. As we assume the moisture state of the ocean to be constant in time, the number of equations reduces by one and we are left with the following expressions for the changes in soil moisture saturation and land and ocean mean water vapour passes:

$$\frac{ds}{dt} = \frac{1}{nz_{\mathrm{r}}}\left[P(w_{\mathrm{l}}) - R(s, w_{\mathrm{l}}) - E(s)\right] \tag{1}$$

$$\frac{dw_{\mathrm{l}}}{dt} = E(s) - P(w_{\mathrm{l}}) + A_{\mathrm{l}}(w_{\mathrm{l}}, w_{\mathrm{o}}) \tag{2}$$

$$\frac{dw_{\mathrm{o}}}{dt} = e_{\mathrm{o}} - P(w_{\mathrm{o}}) + A_{\mathrm{o}}(w_{\mathrm{l}}, w_{\mathrm{o}}). \tag{3}$$

The water transfer terms P, R, E, e_{o}, A_{l} and A_{o} are expressed as mean flux rates in mm/day. Ocean evaporation rate e_{o}, dimensionless soil porosity n and hydrologically active soil depth z_{r} [mm] are constant parameters of the system.

d. Parametrizations

While the conservation of water is a rather fundamental condition, there are no simple, fundamental laws governing the water fluxes between the model boxes. Instead, we need to draw inspiration from existing literature that provides empirical relationships between the flux quantities and our model state variables.

Bretherton et al. (2004) provide such an empirical parametrisation for oceanic, tropical precipitation in mm/day as a function of the mean water vapor pass,

$$P(w) = \exp\left[a\left(\frac{w}{w_{\mathrm{sat}}} - b\right)\right]. \tag{4}$$

The parametrization introduces three parameters, two empirical dimensionless parameters $a \approx 15.6$ and $b \approx 0.6$ and the saturated water vapor pass w_{sat} [mm]. Lacking a corresponding expression for tropical land regions, we will make the explicit assumption that the oceanic precipitation formulation can also be applied to land atmospheres. This assumption has major implications for the results presented in Section 4 as will be discussed in greater detail later.

Runoff gets parametrized as the fraction R_f of precipitation that does not infiltrate the soil. This approach was, for instance, used in Rodriguez-Iturbe et al. (1991). The runoff fraction,

$$R_\mathrm{f}(s) = \epsilon s^r, \tag{5}$$

contains two empirical dimensionless parameters $\epsilon \approx 1$ and $r \approx 2$. It tells us that runoff intensifies as the soil moistens. The complete expression for the runoff rate reads

$$R(s, w_\mathrm{l}) = R_\mathrm{f} P(w_\mathrm{l}), \tag{6}$$

but it proves to be convenient to combine precipitation and runoff in Eqn. (1) to $P(w_\mathrm{l}) - R(s, w_\mathrm{l}) = P(w_\mathrm{l})\Phi(s)$, where we introduced the infiltration function $\Phi(s) = 1 - R_\mathrm{f} = 1 - \epsilon s^r$.

The qualitative dependence of evapotranspiration (ET) on soil moisture saturation is long-known and was first introduced by Budyko (1956). ET is close to zero for soil moisture saturation values below the permanent wilting point, $s < s_\mathrm{pwp}$, increases approximately linearly in a transition range between the permanent wilting point and a critical $s-$value close to the field capacity value, $s_\mathrm{pwp} < s < s_\mathrm{fc}$ and reaches a plateau for higher s-values, $s > s_\mathrm{fc}$ where evaporation is nearly constant. For computational convenience, we parametrize this sometimes piece-wise defined behaviour by the following smooth equivalent:

$$E(s) = \frac{E_\mathrm{p}}{2}\left[\tanh\left(10\left(s - \frac{s_\mathrm{pwp} + s_\mathrm{fc}}{2}\right)\right) + 1\right]. \tag{7}$$

The potential evapotranspiration E_p signifies the value of the plateau beyond s_fc and s_pwp and s_fc are the soil moisture saturation values for the permanent wilting point and field capacity, respectively.

It remains to find expressions for the advection rates into the land and ocean atmospheres. The total advection rate into a given box is the difference between the moisture entering and leaving the box. It can be written as the windward boundary water vapour pass times wind speed minus the analogous term at the leeward boundary of the box,

$$A_\mathrm{tot} = (w_\mathrm{in} - w_\mathrm{out})u. \tag{8}$$

TABLE 1. Parameter ranges for closed model Monte Carlo simulations with uniform sampling.

Parameter	Minimum	Maximum	Range choice motivated by
s_{pwp}	0.2	0.54	Hagemann and Stacke (2015)
s_{fc}	0.3	0.84	Hagemann and Stacke (2015)
e_{p} [mm/day]	4.1	4.5	Rodriguez-Iturbe et al. (1991)
nZr [mm]	90.0	110.0	Rodriguez-Iturbe et al. (1991)
e_{o} [mm/day]	2.8	3.2	C. Hohenegger, private communications
ϵ	0.9	1.1	Rodriguez-Iturbe et al. (1991)
r	2.0	2.0	fixed due to computational method, Rodriguez-Iturbe et al. (1991)
a	11.4	15.6	Bretherton et al. (2004)
b	0.522	0.603	Bretherton et al. (2004)
w_{sat} [mm]	65.0	80.0	Bretherton et al. (2004)
α	0.0	1.0	full possible range
u [m/s]	1.0	10.0	needs more research/thoughts
L [km]	1000.0	40000.0	needs more research/thoughts
$\tau = u/L$ [s^{-1}]	0.00216	0.864	computed from extreme u and L

Figure [****] illustrates the assumed water vapour pass distribution across the full model domain. Since we only have two boxes and periodic boundary conditions, the total advection rate A_{tot} into the land and ocean atmospheres are identical in magnitude but with opposite signs. If the ocean has a net advection outflux, then the land atmosphere gains this moisture as advection influx. What is left to do is to apply Eqn. (8) to the w-distribution in Figure [***] for the land and ocean atmosphere box, respectively, and to translate the obtained total advection fluxes into *mean* advection rates per land/ocean length unit. We obtain

$$A_{\mathrm{l}} = \frac{(w_{\mathrm{o}} - w_{\mathrm{l}})u}{\alpha L} \tag{9}$$

$$A_{\mathrm{o}} = -\frac{(w_{\mathrm{o}} - w_{\mathrm{l}})u}{(1-\alpha)L}, \tag{10}$$

where α and L are the land fraction and full model length, respectively, as introduced in Section b.

With these parametrizations, the model has a total of 14 free parameters which we can reduce to 12 if we treat nz_{r} as one combined parameter and $\tau = u/L$ [day−1] as a characteristic timescale parameter. Table 1 provides sensible ranges for the 12 parameters. These ranges are used to constrain the precipitation ratio and test the sensitivity of the model results to a variation of the different parameters.

- mathematical expressions
- table with values that define the parameter space

e. Further implicit assumptions

- same function for precipitation over land and ocean
- fluxes between boxes are characterized by mean moisture contents
- uniform (mean) soil type
- uniform drainage of runoff across land box
- uniform pseudo-wind

3. Evaluation methods

In this study, our primary interest is directed at equilibrium states of the model and the magnitude and ratio of different flux quantities that result from the equilibrium values of $\{s, w_\mathrm{l}, w_\mathrm{o}\}$. Further, we want to know how sensitive the ratio between land and ocean precipitation rates is to a variation in the model parameters. In the following, we outline the analysis methods to address these questions.

a. Equilibrium states

Mathematically, these states are fixed points of the system of coupled ODEs presented in Eqns. (1) to (3), i.e. the set for state variables $\{s, w_\mathrm{l}, w_\mathrm{o}\}$ for which all three time-derivatives are zero. Due to the nonlinear complexity of the equations, no analytical solution exists and we need to resort to numerical methods. A number of numerical algorithms exists to find fixed points of systems of ODEs. Our code is written in `Julia` (Bezanson et al. (2012)) and we use the `JuliaDynamics.jl` library (Datseris (2018)) to find all roots of the model equations along with the information whether each root represents a stable or unstable fixed point of the system. The advantage of this approach over other solution strategies such as, for example, sufficiently long time evolution, is the independence of the result on initial conditions and that we are guaranteed to find all equilibrium states within a defined range of possible state variable values. We define this range as [*****] Ask Adam how to do that in mathematical language. It turns out that our model has only one equilibrium state for any given set of parameters. With the equilibrium values for s, w_l and w_o and corresponding parameter choices at hand, we can compute all fluxes and flux ratios of interest using the parametrisations introduced in Section 2.d.

b. Scanning the parameter space

Adopting an agnostic view on the plausibility of each combination of parameter values from the ranges given in Tab. 1, we are confronted with a uniformly sampled, 12-dimensional parameter space. To answer the question of how sensitive the equilibrium state is to a variation of the model parameters, we need to scan the full parameter space. In order to minimize computational costs and systematic biases when solving for "all" possible combinations of parameter values, we run n Monte Carlo simulations which are each composed of three steps:

1. Compute a random set of parameter values, assuming the parameter ranges from Tab. 1 with uniform distribution for each range.

2. Find the fixed point for the system using the random parameter set from step 1.

3. Compute flux quantities from the obtained equilibrium state and store them together with the corresponding parameter values in a dataset.

c. Scatter plot analysis

Having obtained a sufficiently large dataset from scanning the parameter space, the sensitivity of a computed quantity Q such as the precipitation ratio to a given parameter p can be visually evaluated with scatter plots. With p on one axis and Q on the other, each of the n simulations can be represented by one data point in the scatter plot. A random distribution of data points across the entire p-range indicates insensitivity of Q to a variation in p. In this case, the choice of a certain p-value has no predictive power for the value of Q. In contrast, a scatter plot distribution where data points cluster in a non-uniform way points to a stronger sensitivity. Various degrees of sensitivity exist, ranging from a slight trend of Q across the range of p-values with considerable scatter for weak sensitivity to a clear, nearly functional dependency $Q(p)$ with narrow scatter range in the case of strong sensitivity. In either case, the influence of all other parameter variations combined determines the spread of the data points around some mean value of Q at any given p-value.

- equilibrium solution
- Monte Carlo sampling
- scatter plot analysis

4. Analysis of the precipitation ratio

- general features of the model behaviour (introduce the reader to how the model behaves, e.g. with state plot or fluxes as function of certain parameters)

a. PR < 1

- runoff as a characteristic property of the land
- compensation of runoff through advection (together with our way to parametrise precipitation) demands a moister atmosphere over ocean than over land –> PR <1

b. Parameter sensitivity

- land fraction - the smile
- spatial scaling u/L - how advection makes the equilibrium states scale-dependent
- permanent wilting point - the soil type matters

5. Open model formulation

a. Open model equations

b. Open model results

- How the open model relaxes the condition that PR<1 (PR>1 only under certain conditions)
- The role of synoptic moisture conditions in the atmosphere
- Transforming the open model into the closed model

6. Discussion and summary

- Which conditions need to be met to end up with a precipitation ratio larger one?
- What are possible use cases for the models?
- What can the model(s) tell us and what not and why? (e.g. land distribution not representative for the Tropics)

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Data availability statement.

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