Oceland: A conceptual model for ocean-land-atmosphere interactions based on water balance equations

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ABSTRACT: Which parameters control the partitioning of precipitation between land and ocean? Do large-scale constraints exist? We address these questions using a conceptual box model based on water balance equations. With a small number of empirical but physically motivated parametrizations of the water balance components, we construct a set of coupled ordinary differential equations which describe the dynamical behavior of the water vapor content of the land and ocean atmospheres as well as the land's soil moisture content. For a model configuration with one ocean and one land box and periodic boundary conditions, we compute equilibrium solutions across the parameter space and analyze the sensitivity of the equilibrium state to parameter choices. The precipitation partitioning is quantified by the precipitation ratio χ, defined as the ratio between the spatially averaged land and ocean precipitation rates in equilibrium. We find that χ is confined to a range between zero and one and that it is primarily controlled by an atmospheric transport parameter, followed by two soil parameters and the land fraction. Since precipitation ratios larger than one have been observed for local systems such as tropical islands, we modify the model such that it allows for net advection into/out of the domain, and change the box configuration to a land box between two ocean boxes. This open model version is able to produce precipitation ratios larger than one but such solutions arise from only a small subset of model parameter choices characterised by rather small land fractions and a sufficiently large external moisture influx through the windward boundary. All in all, given our model assumptions, we conclude that small-scale systems cannot be adequately described without a diurnal cycle and that the land is likely to affect the relationship between water vapor path and precipitation.

1. Introduction

All water that evaporates from Earth's land and ocean surfaces must eventually return to the surface as precipitation. But which physical processes or parameters of the system determine how precipitation gets partitioned between land and ocean? Ought land or ocean receive more rain on average? And do large-scale constraints on the partitioning exist based on fundamental water balance equations? Modern observations provide comprehensive descriptions of real-world precipitation partitioning but typically do not reveal the underlying reasons. Similarly, the complexity of sophisticated climate models limits clear process understanding and, apart from that, these models frequently fail to reproduce observed precipitation patterns (Fiedler et al. (2020)). It is therefore the aim of this study to explain precipitation partitioning with a simple box model describing a coupled land-ocean-atmosphere system.

In a recent study, Hohenegger and Stevens (2022) computed the ratio of spatially averaged precipitation rates over tropical land and ocean, χ, from different observational datasets and found values between $\chi = 0.9$ and 1.04. These values, reflecting nearly equal mean rain rates over land and ocean, are somewhat surprising given that the two surface types have very different characteristics and interact with the atmosphere in different ways. Most notably, the land can dry out and thereby significantly reduce the moisture input into the atmosphere through evapotranspiration while ocean evaporation is never moisture-limited.

The interaction between soil moisture and precipitation has been widely studied with the help of conceptual models. In a seminal paper, Budyko and Drozdov (1953) introduced a one-dimensional land-atmosphere model in which a horizontal airstream traverses a continental region. Starting from a given moisture content at the windward boundary the airstream moistens or dries along the passage depending on the relative magnitude of the region's mean precipitation and surface evapotranspiration. Since this framework distinguishes between precipitation from advected moisture from outside and locally evaporated moisture from within the region, it formed the basis for several moisture recycling studies which assessed the importance of locally evaporated moisture for precipitation in the same region (see Burde and Zangvil (2001) for a review of adaptations of the Budyko model and its general limitations).

Brubaker et al. (1993) and Eltahir and Bras (1994) are two examples which extended the Budyko model to two dimensions and used observations to quantify the water balance components. They

found varying but moderate contributions from local moisture to precipitation for a number of innercontinental regions. Using a global accounting technique, Ent et al. (2010) demonstrated that reevaporated moisture from continental surfaces is indeed important for land precipitation in many parts of the world and sometimes even the predominant contribution. For the question of precipitation partitioning, however, the origin of precipitated moisture is secondary. What matters is the amount of moisture that the land receives from the ocean in the first place and how much of this moisture is turned into precipitation in subsequent recycling loops before the moist air mass leaves the land again. This is best assessed through statistical analyses.

Bretherton et al. (2004) were the first to establish a statistical relationship between spatio-temporal mean precipitation rate P and water vapor pass w over tropical oceans. Based on observations, they found an exponential relationship between the atmospheric moisture content and precipitation, $P(w)$, in which precipitation increases strongly beyond some pick-up value of w. Similar results were obtained by Schiro et al. (2016) and Schiro et al. (2020) for the Amazon, suggesting that the processes which turn moisture into precipitation are not significantly affected by the underlying surface type. However, Ahmed and Schumacher compared the $P(w)$-relationship over land and ocean regions across the tropics and for different times of the day and found lower pick-up values for land regions. They also showed that pick-up values over land get shifted during the course of a day, presumably due to stronger surface heating compared to the ocean and orographic effects. Bergemann and Jakob (2016) added that the relationship between tropical *coastal* precipitation and water vapor pass differs from both the open ocean and inland areas but they also found an exponential shape in all cases.

The fact that moisture fluxes are found to be directly related to available moisture suggests the usage of box models which treat the land and atmosphere as well-mixed moisture reservoirs. Moisture balance equations which describe the change of soil moisture and atmospheric water content as the sum of moisture sinks and sources were formulated, for instance, by Horton (1943) and Peixóto and Oort (1983). Rodriguez-Iturbe et al. (1991) and Entekhabi et al. (1992), studying the stochastical-dynamical behaviour of continental hydrology, utilized the soil moisture balance equation in equilibrium to express the moisture fluxes in the Budyko framework as functions of relative soil moisture saturation. Brubaker and Entekhabi (1995) developed a box model of continental land-atmosphere interactions consisting of coupled water and energy balance equations

for soil and atmosphere reservoirs of both moisture and heat. They used this conceptual model to study the system's equilibrium behavior and in a follow-up publication the role of land-atmosphere feedbacks in the response to stochastic perturbations of horizontal wind speed and solar irradiation (Entekhabi and Brubaker (1995)).

All mentioned studies looked at land regions in isolation and prescribed advection from outside the domain. An exception to this is the simplified climate model of Entekhabi (1994) which was developed as a testbed for new parametrizations for General Circulation Models. This model consists of a land and ocean domain and regulates advection depending on the moisture difference between the two atmospheric columns. While their model is much more complex in its representation of the vertical structure, we will adopt here the idea of a coupled land-ocean-atmosphere system but restrict the scope to basic water balance equations with suitable empirical but physically motivated parametrizations for the moisture fluxes between the boxes. This approach allows us to analyze equilibrium solutions across the sensible parameter space, to put constraints on the ratio between mean land and ocean precipitation and to test the sensitivity of this ratio to the choice of model parameter values.

2. Model description

To understand the controlling factors and constraints for precipitation partitioning between land and ocean, we propose a box model as sketched in Figure 1. The model consists of an ocean domain denoted by subscript 'o', and a land domain denoted by subscript 'ℓ' whose sizes are determined by the length of the full domain L and the land fraction α. Each of the two domains contains a ground box at the bottom (ocean or land) and an atmospheric box aloft. The vertical extent of the ground and atmospheric boxes is taken to be infinite in downward and upward direction, respectively. The model has periodic boundary conditions in the horizontal direction, i.e. topologically, it resembles the wall of a cylinder with the right boundary of the land domain connecting to the left boundary of the ocean domain. This turns the model into a closed system within which water is conserved and which does not interact with any external environment. Such a *closed* model (CM) is suitable to describe, for example, the entire Earth or - if net moisture exchange with the extratropics is negligible - the tropics. Later, in section 6, we discuss a possible formulation for an *open* model (OM) which allows for non-zero net advection into or out of the modelled domain.

FIG. 1. Model sketch of the land-ocean-atmosphere system with periodic boundary conditions. Black arrows represent the box-averaged water fluxes between the four model boxes. With the exception of ocean evaporation E_o, which is treated as a constant model parameter, all fluxes are functions of the relative soil moisture saturation, s, or water vapor passes of land and ocean atmospheres, w_ℓ and w_o, respectively. The atmospheric moisture transport is driven by a constant horizontal background wind speed, u.

a. Water balance equations

To formulate the underlying water balance equations of the system, we express all water fluxes between the boxes as functions of the system's moisture state. For atmospheric boxes, we describe the moisture state by the mean water vapour pass w in mm, and for the land box by the unitless mean relative soil moisture saturation s. As the ocean is considered fully saturated at all times, we don't need to assign a moisture variable to it. Hence, the full information on the moisture state of the land-ocean-atmosphere system at any moment in time t is given by the set of state variables $\{w_o(t), w_\ell(t), s(t)\}$.

The time-evolution of the state variables can be expressed by coupled water balance equations in which the water fluxes between the boxes represent moisture sinks and sources (see e.g. Brubaker et al. (1991)):

$$\frac{ds}{dt} = \frac{1}{nz_\mathrm{r}}\left[P(w_\ell) - R(s, w_\ell) - E_\ell(s)\right] \tag{1}$$

$$\frac{dw_\ell}{dt} = E_\ell(s) - P(w_\ell) + A_\ell(w_\ell, w_\mathrm{o}) \tag{2}$$

$$\frac{dw_\mathrm{o}}{dt} = E_\mathrm{o} - P(w_\mathrm{o}) + A_\mathrm{o}(w_\ell, w_\mathrm{o}). \tag{3}$$

Note that the time-dependence of s, w_ℓ and w_o is implicit in Equations (1) to (3). The relevant fluxes, which are indicated by black arrows in Figure 1, are precipitation P from atmosphere to ground boxes, evapotranspiration E_ℓ from soil to land atmosphere, ocean evaporation E_o to the ocean atmosphere, runoff R from soil to ocean and moisture advection A between the atmospheric boxes. The advection terms A_ℓ and A_o in Eqs. (2) and (3) refer to the *net* advection rate into the land and ocean atmosphere, respectively, and are positive for a net moisture import and negative for net moisture export. In a closed system, the domain-mean advection vanishes, i.e. $\alpha A_\ell + (1-\alpha)A_\mathrm{o} = 0$. All fluxes are given as spatial mean flux rates in mm/day ('mean' being frequently omitted in the remainder of this text). Dimensionless soil porosity n and hydrologically active soil depth z_r in mm are combined to one model parameter.

b. Empirical relationships for water fluxes

To solve the water balance equations (Eqs. (1) - (3)), we need expressions for the water fluxes. While the conservation of water is a fundamental condition, there are no simple fundamental laws governing the moisture exchange between the model boxes. Instead, we turn to empirical relationships between the flux quantities and moisture state variables, as has been done in previous studies, e.g. by Rodriguez-Iturbe et al. (1991). As in their study, we express runoff through the fraction R_f of precipitation that does not infiltrate into the soil,

$$R(s, w_\ell) = R_\mathrm{f}(s)P(w_\ell), \tag{4}$$

with

$$R_\mathrm{f}(s) = \epsilon s^r, \tag{5}$$

and the two dimensionless parameters ϵ and r. Equation (5) tells us that runoff intensifies as the soil moistens and allows us to combine precipitation and runoff in Eqn. (1) as $P(w_\ell) - R(s, w_\ell) = P(w_\ell)\Phi(s)$, where we introduced the infiltration function $\Phi(s) = 1 - R_\mathrm{f} = 1 - \epsilon s^r$.

For precipitation, Rodriguez-Iturbe et al. (1991) followed the approach of Budyko and Drozdov (1953) which assumes that the advected part of land precipitation is known. This is not a desirable assumption in our case where the focus is on the factors controlling the land-to-ocean precipitation ratio which requires a free interaction between the two subdomains. Instead, we parametrize precipitation as a function of mean water vapor pass, w, as first established by Bretherton et al. (2004),

$$P(w) = \exp\left[a\left(\frac{w}{w_\mathrm{sat}} - b\right)\right]. \tag{6}$$

Equation (6) introduces two dimensionless parameters a and b and the saturated water vapor pass w_sat in mm.

The qualitative dependence of evapotranspiration E_ℓ on soil moisture saturation is long-known, see e.g. Budyko (1956) or – more recently and slightly modified – Seneviratne et al. (2010). E_ℓ is close to zero for soil moisture saturation values below the permanent wilting point, $s < s_\mathrm{pwp}$, increases approximately linearly in a transition range between the permanent wilting point and a critical value close to the field capacity, $s_\mathrm{pwp} < s < s_\mathrm{fc}$, and reaches a plateau for higher s-values, $s > s_\mathrm{fc}$, where evapotranspiration is nearly constant at its potential rate e_p. For computational convenience, we parametrize evapotranspiration by the following smooth function which has the qualitative properties described above,

$$E_\ell(s) = \frac{e_\mathrm{p}}{2}\left[\tanh\left(10\left(s - \frac{s_\mathrm{pwp} + s_\mathrm{fc}}{2}\right)\right) + 1\right]. \tag{7}$$

Unlike the land, the ocean is always fully saturated and we assume the resulting ocean evaporation rate to be a constant model parameter $E_\mathrm{o} = e_\mathrm{o}$.

It remains to find expressions for the *mean net* advection rates into the land and ocean atmospheres, hereafter just land/ocean advection. The total net advection A_tot into a given box is the difference between the moisture entering and leaving the box per unit time which is computed from the windward and leeward boundary water vapor pass, w_in and w_out, respectively, multiplied by

the mean horizontal background wind speed u in m/s (and converted mm/day when used in computations),

$$A_{\mathrm{tot}} = (w_{\mathrm{in}} - w_{\mathrm{out}})u. \tag{8}$$

The assumed water vapour pass distribution is characterized by one value w_{o} across the ocean atmosphere and another value w_ℓ across the land atmosphere. Hence, wind transports the moisture amount $w_{\mathrm{o}}u$ from the ocean to the land domain and $w_\ell u$ from land to ocean. Translating A_{tot} into mean advection rates *per unit length* in mm/day gives

$$A_\ell = \frac{(w_{\mathrm{o}} - w_\ell)u}{\alpha L} = \frac{(w_{\mathrm{o}} - w_\ell)}{\alpha}\tau \tag{9}$$

for land advection and

$$A_{\mathrm{o}} = -\frac{(w_{\mathrm{o}} - w_\ell)u}{(1-\alpha)L} = -\frac{(w_{\mathrm{o}} - w_\ell)}{(1-\alpha)}\tau, \tag{10}$$

for ocean advection. On the right hand sides of Eqs. (9) and (10), we introduced the atmospheric transport parameter $\tau = u/L$ in day^{-1}. Its inverse value τ^{-1} represents a characteristic timescale for atmospheric transport. Similar concepts were used by Brubaker et al. (1991), Sobel and Bellon (2009) and Lintner et al. (2013) to model advection.

c. Parameter ranges

The chosen ranges for all free model parameters are provided in Table 1. These ranges are used to construct the parameter space throughout which we constrain the partitioning of precipitation and test the sensitivity of the model results to parameter choices.

The range for s_{pwp} is taken from data for different soil types presented in Tab. 1 in Hagemann and Stacke (2015), where we discard the extreme cases of pure sand and peat. After converting the volumetric data of permanent wilting points and field capacities to relative soil moisture saturation values, s_{pwp} ranges between 0.15 and 0.55 and we notice the fairly consistent relationship, $s_{\mathrm{fc}} = s_{\mathrm{pwp}} + 0.3$, which we use to reduce the number of free model parameters by one. Entekhabi et al. (1992) provide values for e_{p}, nz_{r}, r and ϵ for both a semihumid and a semiarid climate. We take the values from these two cases as the limits for the respective parameter ranges and vary e_{p} between 4 and 6 mm/day and nz_{r} between 50 and 120 mm. Entekhabi et al. (1992) set $r = 6$ for both climates but we let the runoff exponent range between 2 and 6, motivated by Rodriguez-Iturbe

et al. (1991) who used $r = 2$ in an illustrative example. Since both studies agree on $\epsilon = 1$, we only vary this parameter slightly between 0.9 and 1.1. The precipitation parameters are taken from Bretherton et al. (2004) (where our b is called r). We use their fitting parameter values for monthly data as the lower bounds for a and b, and the values for daily data as the upper bounds. Bretherton et al. (2004) also give a typical value for the saturation water vapor pass in regions of tropical convection of $w_{\mathrm{sat}} = 72\,\mathrm{mm}$ and we deem it appropriate to vary w_{sat} between 65 and 80 mm. Ocean evaporation can be constrained from observed latent heat fluxes. For instance, Kumar et al. (2017) find the mean flux of latent heat from tropical ocean surfaces to lie around 75 W/m^2 while an earlier study by Zhang and Mcphaden (1995) indicates somewhat higher values of about 100 W/m^2. Based on these findings, we let e_{o} range between 2.5 and 3.5 mm/day. Lastly, τ is constrained by computing the smallest and largest value of u/L, respectively, by assuming plausible wind speeds in the lower troposphere between 1 and 10 m/s, and letting the domain length vary between 1000 and 40000 km, where the upper limit corresponds to Earth's circumference at the equator. In this way, we obtain a range for τ between 0.00216 and 0.864 day^{-1}. Brubaker et al. (1991), whose transport parameter is equivalent to our $\alpha\tau^{-1}$, looked at two regions centered on the coast shared by the Amazon basin and the Atlantic ocean with a land fraction of about 0.5. From observations, they estimated monthly values for their transport parameter which translate to τ values between 0.01 and 10 – a range slightly higher then ours but nevertheless qualitatively similar.

d. Model assumptions

The simplicity of the proposed model owes to a number of assumptions, some of which are important to be made explicit. Foremost, we assume that each model box has well-mixed physical properties so that all interactions are adequately described in terms of spatial mean quantities. Second, we assume that the atmospheric and soil water storage does not change with time on large enough temporal and spatial scales which implies that the moisture state of the land-ocean-atmosphere system is steady and that it is sufficient to analyse the equilibrium solutions of the model equations (1) to (3). Third, we determine the system's moisture state by water balance equations only, ignoring the potential effects of energy balance considerations such as the influence of a diurnal cycle. Energetic conditions are kept constant and only enter indirectly through values of energy-

TABLE 1. Parameter ranges for closed model simulations with uniform random sampling of parameter values.

Parameter	Minimum	Maximum	Range choice motivated by
s_{pwp}	0.15	0.55	Hagemann and Stacke (2015)
e_{p} [mm/day]	4.0	6.0	Entekhabi et al. (1992)
$nZ\mathrm{r}$ [mm]	50.0	120.0	Entekhabi et al. (1992)
e_{o} [mm/day]	2.5	3.5	Kumar et al. (2017), Zhang and Mcphaden (1995)
ϵ	0.9	1.1	Rodriguez-Iturbe et al. (1991), Entekhabi et al. (1992)
r	2	6	Rodriguez-Iturbe et al. (1991), Entekhabi et al. (1992)
a	11.4	15.6	Bretherton et al. (2004)
b	0.5	0.6	Bretherton et al. (2004)
w_{sat} [mm]	65.0	80.0	Bretherton et al. (2004)
α	0.0	1.0	full possible range
$\tau = u/L$ [day^{-1}]	0.00216	0.864	plausible ranges for L and u

dependent parameters such as the ocean evaporation rate. Last, we assume that runoff water does not participate in any secondary process that could moisten the soil, that the background wind has only a constant, horizontal component and that the functional relationship between precipitation and water vapor pass from Eqn. (6) holds over both land and ocean with the same choice of parameter values.

3. Evaluation methods

In this section, we present the different analysis methods that are employed to evaluate the model behavior and to assess the sensitivity of the precipitation partitioning to variations of the model parameter values. Precipitation partitioning is quantified by the precipitation ratio,

$$\chi = \frac{P_\ell}{P_{\mathrm{o}}} = \frac{P(w_\ell)}{P(w_{\mathrm{o}})}. \tag{11}$$

The equilibrium solution to the model equations (1) to (3) has to be found numerically. We use the DynamicalSystems.jl library from Datseris (2018) to find all roots of the model equations which represent stable fixed points of the system. Adopting an agnostic view on the plausibility of different combinations of parameter values from the ranges given in Table 1, we perform 50000 model simulations for randomly chosen points in the 11-dimensional parameter space, each yielding a corresponding equilibrium state. Based on the data for these equilibrium states, we use scatter plots to analyse the sensitivity of χ to a variation of the different model parameters.

A suitable measure for the correlation between a parameter p_i and some quantity Q which accounts for the possibility of a non-linear and non-monotonic relationship between p_i and Q is their mutual information $MI(p_i, Q)$. Mutual information quantifies how much the knowledge of p_i reduces the uncertainty about Q. The mutual information of p_i and Q is computed as

$$MI(p_i, Q) = H(p_i) + H(Q) - H(p_i, Q), \tag{12}$$

where $H(p_i)$, $H(Q)$ and $H(p_i, Q)$ are the Shannon entropies of p_i and Q values, and of their joint distribution, respectively (Shannon (1948)). To enable a direct comparison of different parameter sensitivities, we rescale p_i and Q so that their values range between 0 and 1 and call the rescaled versions $\hat{p}_i$ and $\hat{Q}$. We use amplitude binning to ascribe probability distributions and follow the approach from Datseris and Parlitz (2022), p.106, to assess whether the obtained mutual information values are significantly different from the null-hypothesis of uncorrelated $\hat{p}_i$ and $\hat{Q}$. From the mutual information values of 10000 shuffled surrogates of $\hat{p}_i$ and $\hat{Q}$, we compute a probability distribution of MI values for the uncorrelated case and define $MI_{\text{uncorr},3\sigma}(\hat{p}_i, \hat{Q})$ as the value that deviates by three standard deviations σ from the mean of this distribution. The mutual information index,

$$I_{MI}(p_i, Q) = \frac{MI(\hat{p}_i, \hat{Q})}{MI_{\text{uncorr},3\sigma}(\hat{p}_i, \hat{Q})}, \tag{13}$$

can then be used to determine whether the actual mutual information $MI(\hat{p}_i, \hat{Q})$ is high enough to pass the significance threshold, $I_{MI} = 1$. The higher $I_{MI}(p_i, Q)$, the more sensitive Q is to a variation of parameter p_i.

4. Basic model behavior and implications for χ

In this section, we analyze the output of the 50000 closed model simulations with different parameter choices, henceforth referred to as "CM data", with the aim of determining the range of possible equilibrium values for the precipitation ratio. We start by characterizing the obtained equilibrium states and associated moisture fluxes.

Figure 2 shows the probability density functions (PDF) of equilibrium soil moisture in panel a) and b), and water vapor pass values for land and ocean atmospheres in panel c) to illustrate

FIG. 2. Smoothed probability density functions of equilibrium soil moisture values in panel a), rescaled soil moisture values with vertical lines denoting the beginning and end of the transition regime of E_ℓ in panel b), water vapor pass of the land and ocean atmospheres in panel c) and values of the precipitation ratio in panel d).

the characteristics of possible equilibria. In panel b), s is rescaled to $\tilde{s} = (s - s_{\mathrm{pwp}})/(s_{\mathrm{fc}} - s_{\mathrm{pwp}})$ in order to demonstrate the location of the equilibrium values relative to the different regimes of evapotranspiration, E_ℓ, which are separated by dashed vertical lines. The bulk of all simulations equilibrates to intermediate soil moisture values between $s = 0.25$ and 0.75 with a sharp peak in the center part of the E_ℓ-transition regime around $\tilde{s} = 0.5$. Similarly, the atmospheres equilibrate to intermediate w_{o} and w_ℓ values between 40 and 50 mm, well below the saturation water vapor pass values given in Tab. 1. As a matter of comparison, in the tropics, a w-value of 48 mm is often employed to distinguished the deep moist tropics with deep convection from the dry subtropics (Masunaga and Mapes (2020)). Only few simulations result in very dry or moist soil moisture and atmospheric states. We will come back to these cases when discussing parameter sensitivities. Figure 2 also shows that land and ocean exhibit similar atmospheric moisture states but that the land is slightly drier than the ocean.

Figure 3 shows rolling averages of the equilibrium fluxes from all simulations as functions of the equilibrium soil moisture saturation values s. Note that the ocean advection rate A_{o} has negative values for all equilibrium solutions and is therefore multiplied by -1 to simplify the comparison of its magnitude with other fluxes. Figure 3 contains the entire CM data, i.e. solutions for all different combinations of parameter values. Therefore, one should not confuse the plotted curves with the

FIG. 3. Symmetric rolling average of equilibrium fluxes versus soil moisture saturation computed from CM data. For evapotranspiration (green line), the individual data points are shown as scatter plots and the range of $E_\ell(s)$ parametrizations for different parameter combinations is shaded in light green.

well-defined parametrizations of the water fluxes as functions of s for a fixed set of parameter values.

Figure 3 allows us to draw first conclusions about the magnitude and relative importance of the fluxes in different regimes of equilibrium states as well as about the partitioning of precipitation in the modelled land-ocean-atmosphere system:

1. Land advection, A_ℓ (purple line) is strictly positive, meaning that moisture is always supplied by the ocean atmosphere to the land atmosphere. Net advection to the land compensates for the soil's constant loss of water through runoff, i.e. $A_\ell = R$. This compensation mechanism in equilibrated hydrological cycles has been described in earlier studies, e.g. by Horton (1943) or Peixóto and Oort (1983).

2. According to the advection equations (9) and (10), net water transport from ocean to land requires the ocean atmosphere to be moister than the land atmosphere, $w_o > w_\ell$. Since the same, monotonically increasing function $P(w)$ from Eqn. (6) is used over land and ocean, $P_o = P(w_o)$ is necessarily larger than $P_\ell = P(w_\ell)$. In other words, mean precipitation is always stronger over ocean than over land. This is confirmed by Fig. 3 where the dark blue curve for

P_o always lies above the bright blue curve for P_ℓ. Hence, our water balance model reveals that an upper bound on the precipitation ratio exists, this upper bound being $\chi = 1$. That χ is bounded by one is also visible in Fig. 2d) that shows the PDF of χ values. The likelihood for values of χ increases towards its upper limit and 82.9 % of the parameter combinations yield values larger than 0.9, thereby falling into the range obtained by Hohenegger and Stevens (2022) from observations.

3. The ocean evaporation rate E_o (orange line) is always the largest flux. This can be explained as follows: E_o is partitioned into ocean precipitation P_o and ocean advection $|A_o|$ so that each of these two components needs to be smaller than E_o. Since we know from the previous point that the land atmosphere is drier than the ocean atmosphere, it follows that $P_\ell < P_o < E_o$. Land precipitation can be written as the sum of evapotranspiration and runoff or evapotranspiration and advection so that again, each of these components needs to be smaller than E_o.

4. The shapes of the lines in Fig. 3 indicate that three soil moisture-precipitation regimes can be distinguished: For low soil moisture values up to $s \approx 0.35$, runoff and land advection are negligible and precipitation nearly equals evapotranspiration which rises sharply with s. In an intermediate soil moisture regime, $0.35 \lesssim s \lesssim 0.65$, precipitation decreases due to a strong negative trend of evapotranspiration. The precipitation decrease is slightly dampened by a steadily growing contribution from land advection (purple line). Lastly, above $s \approx 0.65$, precipitation increases again as the evapotranspiration trend reverses and advection keeps intensifying. Apparent contradictions such as the overall decline of advection out of the ocean atmosphere (red line) while advection into the land atmosphere is monotonically increasing or the decrease of evapotranspiration with increasing soil moisture in the intermediate regime can be resolved by understanding the parameter sensitivities and how different parameters interact.

5. Parameter sensitivity of χ

Our water balance model revealed that the precipitation ratio is bounded by one. Figure 3 is nevertheless difficult to interpret as different combinations of parameter values can lead to the same equilibrium soil saturation, but the sensitivity of the χ to a change of each parameter value can be different. To understand the controls on the precipitation ratio more clearly, we quantify

FIG. 4. Relative sensitivity of the precipitation ratio to the different model parameters p_i measured by the mutual information index I_{MI}. Values larger than $I_{MI} = 1$ (dashed line) lie above the significance threshold.

the sensitivity of χ to each individual model parameter by the mutual information index $I_{MI}(p_i)$ defined in Eqn. (13). A comparison of the results for all model parameters p_i is provided in Figure 4. The atmospheric transport parameter τ is by far the most sensitive parameter, followed by the soil parameters permanent wilting point s_{pwp} and runoff exponent r, and the land fraction α. Some of the remaining parameters, most notably potential evapotranspiration E_{p}, also have I_{MI}-values above or close to the significance threshold of one but we do not discuss them in detail due to their rather small contributions to the overall sensitivity.

a. Atmospheric transport parameter τ

Figure 5 shows scatter plots of the precipitation ratio versus the four most sensitive model parameters. The relationship between χ and τ in Fig. 5a is strongly nonlinear and leads to variations of the χ mean value between 0.46 and 0.97, confirming the high sensitivity determined in the mutual information analysis. The precipitation ratio tend to be low for low values of τ and converges towards one as τ increases. The spread of χ values around the mean (dark grey line) decreases along increasing τ, making τ a better predictor for the precipitation partitioning as its value increases.

FIG. 5. Precipitation ratio χ versus atmospheric transport parameter τ, land fraction α, permanent wilting point s_{pwp} and runoff exponent r. The dark grey lines show the rolling average of χ values.

Physically, τ corresponds to the fraction of the total domain length that moisture can travel horizontally in the atmosphere in one day. As such, it can be interpreted as an efficiency of atmospheric moisture transport. High values of τ imply a fast exchange of moisture between land and ocean atmospheres with the result of relatively low moisture differences, $\Delta w = w_{\mathrm{o}} - w_{\ell}$, between the two. Since we assume the same precipitation parametrization $P(w)$ for land and ocean, small Δw means that the precipitation rates over land and ocean are similar and that χ is close to one. In contrast, small values of τ can sustain larger moisture differences between land and ocean and lead to smaller values of χ.

In fact, the distribution of data points in Fig. 5a indicates that τ mostly controls the lower limit of χ which increases with τ while the upper limit, $\chi = 1$, seems independent of τ and must therefore be controlled by a different parameter. We can understand how τ determines the lower bound of the precipitation ratio by returning to the advection equations (9) and (10), each of which can be rephrased as the product of Δw and an advection efficiency, namely τ/α for land advection

and $\tau/(1-\alpha)$ for ocean advection. Only if both advection efficiencies are low, a large moisture difference and low χ value can be sustained. The fact that α ranges between zero and one and that it affects ocean and land advection efficiencies in opposite ways suggests a "sweet spot" of overall low advection efficiency at $\alpha = 0.5$, leaving it to τ to set the final value of this lowest advection efficiency and thereby the lower bound of χ. In the next section, we will see that the complexity of land-atmosphere interactions leads to an asymmetry in the relationship between χ and α such that the postulated "sweet spot" of lowest χ is found at a larger land fraction than $\alpha = 0.5$.

b. Land fraction α

Fig. 5b illustrates how α impacts the partitioning of precipitation. The χ mean varies between 0.89 and 1.0. These variations form a u-shaped line between the extreme cases of an ocean-only, $\alpha = 0$, and land-only, $\alpha = 1$, scenario where in both cases atmospheric conditions over land and ocean are approximately the same and χ is close to one. As for τ, the impact of land fraction changes on the precipitation ratio is rooted in their control on the advection efficiencies, τ/α and $\tau/(1-\alpha)$. However, the role of α in the advection efficiencies is to differentiate between the efficiency of moisture export out of the ocean domain and the efficiency of moisture import into the land domain. If α is small, ocean advection is inefficient because the average time for moisture from the large ocean surface to reach the land is long. In contrast, it takes only little time to distribute the imported moisture over the small land size so that land advection is efficient. The reverse applies for a large land fraction.

While the model cannot handle the exact endpoints of the α-range, we can examine them in a thought experiment: Imagine a domain fully covered by ocean. Water balance would require the ocean precipitation to exactly balance ocean evaporation, $P_o = E_o$. If we now introduced an infinitesimal patch of land, some of the evaporated moisture would be advected into the tiny land atmosphere. Since τ/α is high, the atmospheric conditions would assimilate rapidly without significantly altering the moisture conditions over the vast ocean. Hence, for such small α, the system is expected to behave as if the land didn't exist with moist atmospheric conditions and similarly high precipitation rates close to E_o. At the other extreme end, imagine a pure land domain but with the assumption that runoff remains possible. The runoff would continuously reduce the soil moisture saturation and with it evapotranspiration and precipitation until the trivial

equilibrium solution $\{s = 0, w_\ell = 0\}$ was reached. Allowing for advection from an infinitesimal ocean would not change the picture much. With very high efficiency $\tau/(1-\alpha)$, almost the entire but nevertheless small amount of evaporated oceanic moisture would be exported to the land, leaving behind a fairly dry ocean atmosphere and hardly affecting the dry moisture state of the large land atmosphere and soil. Hence, we expect a situation in which the system behaves as if the ocean didn't exist with similarly low precipitation rates close to zero in both domains.

The transition between the ocean-only and land-only scenario is best understood by examining the behavior of the individual rain rates, P_o and P_ℓ, over the range of α which is shown in Figure 6 and confirms the developed arguments. The two extreme moisture conditions are connected by a regime of monotonic drying as the ocean surface - the only true source of moisture for the system's hydrological cycle - shrinks. Although not shown, the overall reduction of available moisture as the land fraction increases also manifests itself in a decrease of soil moisture. We can investigate the shapes of P_ℓ and P_o theoretically, assuming all parameters except α to be fixed. Expressions for the rain rates are found by imposing equilibrium conditions on Eqns. (2) and (3), yielding

$$P_\ell = E_\ell(s(\alpha)) + \Delta w(\alpha)\frac{\tau}{\alpha} \tag{14}$$

$$P_\mathrm{o} = e_\mathrm{o} - \Delta w(\alpha)\frac{\tau}{1-\alpha}. \tag{15}$$

Note that $s(\alpha)$ and $\Delta w(\alpha)$ are implicit functions of the land fraction but that we lack analytical expressions for them. As α increases in Fig. 6, P_ℓ decreases more strongly than P_o because land precipitation is not only affected by (initially sharply) decreasing $1/\alpha$ but in addition by a reduction in evapotranspiration as s declines. In contrast, ocean precipitation is only reduced by (initially weakly) increasing advection. The additional influence of α on P_ℓ through E_ℓ causes the asymmetry of the $\chi(\alpha)$-mean in Figure 5b with a minimum around $\alpha = 0.75$ instead of 0.5. Although $\chi(\alpha)$ is u-shaped for every combination of the other parameter values, the exact location of the minimum is different for each combination. Generally, the width of the spread of χ values indicates that the land fraction is the dominant predictor for χ near its extreme values but that it loses much of its predictive power for intermediate values.

FIG. 6. Ocean precipitation rate P_o and land precipitation rate P_ℓ versus land fraction α. The transition from an ocean-only to a land-only model scenario is marked by a decrease of both precipitation fluxes, indicating overall moist conditions for small land fractions and dry conditions for large land fractions.

c. Permanent wilting point s_{pwp} and field capacity s_{fc}

Figure 5c and 5d show that variations in the soil parameters lead to mean variations of χ on the order of the effects from land fraction changes, with mean values varying between $\chi = 0.87$ and 0.96. In Fig. 5c, the precipitation ratio shows an almost linear, negative trend as the permanent wilting point increases. To understand this trend, it is helpful to consider a system in equilibrium for some value of the permanent wilting point, e.g. $s_{\mathrm{pwp}} = 0.3$, and examine how the system reacts when this value is suddenly changed to $s_{\mathrm{pwp}} = 0.4$. The presented arguments assume that the other parameter values are kept fixed when varying s_{pwp} but we can see from the right panel of Figure 7 that the influence of the permanent wilting point also leaves its imprint on the mean soil moisture state (dark grey line) in the form of a clear positive trend. We therefore make us of the mean value for illustration purposes. The left panel of Figure 7 depicts the evapotranspiration curves for $s_{\mathrm{pwp}} = 0.3$ and 0.4, respectively, and a fixed potential evapotranspiration value $e_{\mathrm{p}} = 4.0$ mm/day. We see that a change of s_{pwp} merely shifts the evapotranspiration curve along s.

The mean equilibrium soil moisture value in the CM data for $s_{\mathrm{pwp}} = 0.3$ is $s = 0.49$. This initial state and its corresponding evapotranspiration value are displayed as blue dots in Figure 7. An abrupt increase of the permanent wilting point to $s_{\mathrm{pwp}} = 0.4$ leads to an instantaneous drop ΔE_{inst} in the evapotranspiration as illustrated by the first red arrow connecting the blue and green

dot in the left panel of Fig. 7. The green dot represents a temporary state where the model is not in equilibrium because the state variables have not yet been adjusted to the new situation. At this point, the soil receives the same amount of precipitation but loses less water through evapotranspiration. As a result, the soil moistens. As time progresses, the system attains a new equilibrium state at a higher s value. This moister state after the transition is marked by orange dots in both panels. However, as the soil moistens, not only evapotranspiration but also runoff and, hence, land advection increase. Assuming that τ and α are kept fixed, Δw has to increase to allow for the increase of advection. The water that is advected to the land atmosphere is taken from the ocean atmosphere, where w_{o} decreases as a consequence. Therefore, an increase in advection from increased Δw is only possible, if w_ℓ decreases more strongly than w_{o}. As a decrease in w_ℓ reduces P_ℓ, the new equilibrium is characterised by weaker precipitation than the initial equilibrium state. At the same time, the runoff is larger, thus leaving less water to evapotranspiration. This response to different values of the permanent wilting point is responsible for the shape of the fluxes in the regime of intermediate s values in Figure 3 where the precipitation and evapotranspiration rates decline with increasing soil moisture while the mean value of s_{pwp} increases linearly (not shown). All in all, the attained equilibrium state for $s_{\mathrm{pwp}} = 0.4$ has a moister soil but an evapotranspiration rate which is by ΔE_{eq} lower compared to the initial equilibrium state for $s_{\mathrm{pwp}} = 0.3$. The fact that w_ℓ must decrease more strongly than w_{o} in the adaptation process is the reason why χ declines with increasing s_{pwp} and s_{fc}.

d. Runoff exponent r

The relationship between χ and runoff exponent r in Figure 5d resembles the relationship of $\chi(s_{\mathrm{pwp}})$ but with opposite trend: χ increases with r while it decreases with s_{pwp}. Indeed, the similarity originates from a physical mechanism similar to the one discussed for the permanent wilting point.

In the formulation of the runoff fraction, $R_{\mathrm{f}} = \epsilon s^r$, r enters as the exponent. As in the discussion of the s_{pwp}-dependence, we can think of an equilibrium state but this time suddenly increase the value of r. As a consequence, runoff drops to a lower value and soil moisture begins to increase. As a response, also both atmospheres start to moisten: First, w_ℓ increases through increased evapotranspiration from the wetter soil, reducing the moisture difference between land and ocean.

FIG. 7. Influence of an increase in s_{pwp} on the equilibrium state. Left: Higher values of s_{pwp} shift the graph of the E_ℓ parametrization towards larger s. Right: Equilibrium values of the soil moisture saturation from CM data plotted over s_{pwp} values. The blue dots mark the initial equilibrium state, the green dot represents a temporary state of soil moisture imbalance due to a sudden increase of s_{pwp} and the orange dot marks the the eventually attained equilibrium for the new s_{pwp} value.

Second, the reduction of Δw implies reduced advection, allowing the ocean atmosphere to keep more of its moisture to itself, and w_o increases. At some point, the decreasing advection matches the runoff which started to intensify again after the initial drop as the soil moistened. A new equilibrium is attained in which runoff is reduced compared to the initial state that had a lower r-value. The equally reduced land advection with lower Δw reflect assimilated atmospheric moisture conditions and, hence, increased χ.

Equilibrium states with higher r value also have stronger precipitation rates due to the higher w_ℓ and w_o. We see the influence of r in both the low and the high soil moisture regime in Figure 3, where it works in tandem with the land fraction to shape the moisture fluxes. Both the initial and final precipitation increase for low and high soil moisture values, respectively, (blue curves) are caused by the combination of increasing r and decreasing α values.

6. Under which circumstances can χ become larger than 1?

So far, we concluded that the precipitation ratio is constrained to an upper limit of one due to the necessity of net moisture transport from ocean to land and the assumption that the efficiency with which atmospheric moisture is turned into precipitation is the same over land and ocean.

However, we know of local systems in the real world for which higher rain rates are observed over land compared to the adjacent ocean. For instance, Qian (2008), Sobel et al. (2011), Cronin et al. (2015), Wang and Sobel (2017) and Ulrich and Bellon (2019) found precipitation enhancement over tropical islands and attributed this observation mainly to the development of sea breezes triggering precipitating convection over the islands. Even for the full tropics, some observational datasets suggest a precipitation ratio slightly larger than one (Hohenegger and Stevens (2022)). For a land fraction of $\alpha \approx 0.35$, corresponding to the tropical land fraction, our model gives a mean ratio of 0.92 which matches the lower end of observed values but ratios of individual simulations reach as low as 0.17, mostly depending on the value for τ. Hence, the model tends to underestimate the actual precipitation ratio and it certainly does not produce values larger than one. In the following, we explore different ways in which our model framework could me modified to obtain precipitation enhancement over land, i.e. precipitation ratios larger than one.

a. Relaxing assumptions of the closed model

The first direct limitation for the precipitation ratio in our conceptual model is the assumption that the surface differences between land and ocean do not affect the way it rains. For a given water vapor path, it rains the same amount over land and over ocean. While the qualitative statement that a moister atmosphere yields more rainfall seems robust, the exact shape of this relationship might differ over land and ocean. While Ahmed and Schumacher found different shapes for land and ocean precipitation, Schiro et al. (2020) did not find a distinct difference, leaving the picture inconclusive. If different parametrizations $P_\ell(w)$ and $P_\mathrm{o}(w)$ were used for land and ocean precipitation, respectively, and $P_\ell(w)$ was larger than $P_\mathrm{o}(w)$ in the relevant range of w-values, it is conceivable that an equilibrium state could exhibit a moister ocean atmosphere supporting net advection to the land and at the same time stronger land precipitation, so that $\chi > 1$.

Second, we treat the four model boxes as being homogeneous and well-mixed which allows us to work with mean fluxes rather than resolving the horizontal direction $\vec{x}$ explicitly. It is well-known that an airstream traversing an oceanic region will moisten along its trajectory since mean ocean evaporation typically exceeds mean ocean precipitation and the reverse is true for land regions. Ogino et al. (2016) and Ogino et al. (2017) further found that the conventional view in which Earth's surface gets divided into ocean and land misses out on particular interactions driven by

the land-ocean contrast which are confined to a coastal region reaching a few hundred kilometers seaward and landward from the coast. These coastal regions receive higher amounts of precipitation than both the open ocean and inland continental regions. Also Bergemann and Jakob (2016) found that tropical rainfall over land associated with coastal effects such as sea breezes can occur under drier atmospheric conditions than rainfall over the open ocean. Hence, adding coastal zones with a dedicated coastal precipitation parametrization to the model is another flavor of the argument that - for precipitation enhancement over land - it has to rain differently over certain areas of the model. Coastal zones might also capture the fact that precipitation enhancement is particularly strong over relatively small land masses, where coastal effects are expected to be more influential.

Third, following up on the coastal argument, sea breezes as the primary coastal process enhancing precipitation over islands in reality are not captured by the presented model. Since sea breezes are diurnally forced, in order to account for them, the model would need to be extended by a diurnal cycle – either fundamentally by coupling the water balance equations to energy balance equations, or indirectly by expressing the state variables and fluxes as time-dependent quantities that vary over the course of a day with a distinct diurnal cycle over land and over ocean. Even with the same $P(w)$ for land and ocean, a diurnal cycle may make the mean precipitation ratio larger than one if the increased moisture transport from ocean to land during the day leads to higher temporary land precipitation rates than over the ocean, and if the reverse transport during the night does not fully compensate for the daytime precipitation signal. In other words, the diurnal cycle may explain why precipitation ratios are close to one and can be larger than one in reality.

Last, our model configuration with one land and one ocean box makes the comparison with observed values difficult. It is likely that a different land distribution with more boxes would lead to higher precipitation ratios because of increased advection efficiencies for shorter box lengths. If, in addition, the boxes were differently sized, it might even be possible to create instances where $\chi > 1$. To attain equilibrium, the leeward ocean atmospheres would still need to be moister than the neighbouring land atmospheres but it cannot be precluded that the weighting by the different box sizes leads to higher averaged land than ocean rain rates.

b. Opening the closed model

The previous arguments still treat the land-ocean-atmosphere system as a closed model. This assumption may be valid over the full tropics, assuming a negligible moisture transport to the extratropics, but it is definitely not valid over islands, where land precipitation enhancement is typically observed. Hence, allowing for atmospheric inflow and outflow out of the considered model domain might permit land-to-ocean precipitation ratios larger than one. We test this hypothesis in two ways.

First, moisture import or export from an external environment outside the model boundaries can be incorporated by an additional advection term, A_{ext} in mm/day in Equation (3) for the ocean atmospheric water balance. A positive A_{ext} denotes inflow of external moisture, while a negative A_{ext} means that the ocean atmosphere loses moisture through the model boundaries. This construction mimics the case of an island surrounded by an ocean under the influence of lange-scale convergence or divergence. But is this change in the model framework sufficient to create scenarios for which $\chi > 1$?

We argue that the answer is no: Regardless of whether the system is gaining or losing moisture through its boundaries, an equilibrium state still requires a net transport of moisture from ocean to land, and therefore $w_{\mathrm{o}} > w_{\ell}$ with $\chi < 1$. The term A_{ext} acts in a similar fashion as ocean evaporation. Under the influence of moisture convergence, the positive A_{ext} is equivalent to an increase in E_{o} and merely increases the overall relative moisture saturation in all boxes. In the case of moisture divergence, negative A_{ext} acts as a reduction of E_{o} and an overall drier equilibrium state is attained as long as $|A_{\mathrm{ext}}| < E_{\mathrm{o}}$. If the loss through the model boundary is stronger than the moisture input from the ocean surface, the system will undergo drying until the trivial solution $\{w_{\mathrm{o}} = 0, w_{\ell} = 0, s = 0\}$ is reached.

Second, instead of having an island surrounded by an ocean with convergence/divergence of moisture from the ocean into the land atmosphere, we open the closed model by allowing moisture to enter the domain from one side and leave it on the other side. As an illustrative example for this *open model*, we consider the simplest configuration with a land box of length L_{ℓ}, placed between two equally big ocean boxes of lengths $L_{\mathrm{o1}} = L_{\mathrm{o2}}$. All lengths are given in km. The subscript 'o1' refers to the ocean in front of the island as seen by an airflow horizontally traversing the model domain at constant wind speed, while 'o2' refers to the ocean behind the island. The model

equations governing the evolution of the three mean atmospheric water vapor passes w_{o1}, w_ℓ and w_{o2} and soil saturation s are formulated in analogy to the closed model equations presented in Section 2 and can be found in Appendix A along with a model sketch. Note that this open model requires an additional parameter w_0 which represents the windward boundary water vapor pass and enters the advection term in the atmospheric equation for the first ocean. As for the closed model, 50000 simulations with randomly chosen combinations of parameter values are performed, yielding a dataset of open model equilibrium solutions that we refer to as "OM data".

The precipitation ratio is now defined in terms of mean precipitation rates $\bar{P}_\ell$ and $\bar{P}_o$ as

$$\chi_{om} = \frac{\bar{P}_\ell}{\bar{P}_o} = \frac{2P_\ell}{P_{o1} + P_{o2}}. \tag{16}$$

As in the case of the closed model, equilibrium can only be attained if the land atmosphere receives advected moisture from the ocean. In the open model, this means that the first ocean atmosphere has to be moister than the land atmosphere which – sticking to the assumption of the same parametrization for precipitation across the model domain – implies $P_{o1} > P_\ell$. It follows from Equation (16) that a precipitation ratio larger than one is possible if the second ocean atmosphere is dry enough to compensate for the relatively moist first ocean atmosphere and thereby reduces the mean ocean precipitation to a value smaller than P_ℓ. Mathematically, a precipitation ratio larger than one requires the inequality, $P_{o2} < 2P_\ell - P_{o1}$, to be fulfilled. From the OM data, we find that only about 6.8 % of all simulations meet this condition and yield $\chi_{om} > 1$. As expected, these simulations have in common that atmospheric moisture reduces along the wind trajectory, i.e. $w_0 > w_{o1} > w_\ell > w_{o2}$.

We perform the same sensitivity analysis as for the closed model to understand which parameter combinations lead to ratios larger than one and to which parameters χ is most sensitive. Opening the model does not fundamentally change the principal sensitivities but modifies their order of importance: The most sensitive parameter is now w_0 with $I_{MI}(w_0, \chi) = 130$, followed by α with $I_{MI}(\alpha, \chi) = 104$ and τ with $I_{MI}(\tau, \chi) = 73$. All other parameters, including the formerly relevant soil parameters, have I_{MI} values lower than 4 and can be neglected as predictors for χ. To understand which parameter combinations lead to $\chi > 1$, Figure 8 shows PDFs of the possible values of w_0, α and τ for simulations with precipitation ratios smaller and larger than one in blue and orange, respectively. Even though the distributions have a significant overlap, $\chi > 1$ requires

FIG. 8. Probability density functions of parameter values for w_0, α and τ. Blue and orange graphs contain data from simulations with precipitation ratios smaller and larger than one, respectively. Ratios larger than one are only found for 6.8 % of all simulations.

a large enough boundary water vapor pass $w_0 \gtrsim 40$ mm and becomes more likely with smaller land fraction. States with $\chi > 1$ do not exist for $\alpha > 0.7$. In contrast, a value of $\chi > 1$ seems to be possible with any value for τ. Further inspection of the state variable values shows that equilibrium states with $\chi > 1$ are states with moist conditions. The soil moisture saturation values cluster around values that correspond to the upper end of the transitional E_ℓ-regime and beyond the field capacity where E_ℓ reaches the plateau. Water vapor pass values peak around 50 mm and get no lower than 38 mm. Simulations with $\chi < 1$ in contrast exhibit equilibrium states across the entire moisture spectrum, including very dry states with soil moisture values below the permanent wilting point.

As in the case of the closed model, the open model results are subject to the choice of the land distribution and may change for different numbers of land boxes. In contrast to the closed model, the open model is also sensitive to the location of the land box which affects the relative size of the two ocean boxes. However, a test of different asymmetric configurations indicates that the presented statements are only affected quantitatively, not qualitatively.

7. Conclusions

This study was motivated by our lack of theoretical understanding of how Earth's total precipitation gets partitioned between land and ocean. Estimates of the ratio between spatially averaged

land and ocean precipitation rates, $\chi = P_\ell / P_\mathrm{o}$, from different observational datasets range between 0.9 and 1.04 Hohenegger and Stevens (2022). We wanted to know which atmospheric and land parameters determine the partitioning and what constraints can be set on χ based on water balance equations.

We introduced a conceptual box model that describes the rate of change of soil moisture and atmospheric moisture over ocean and land, respectively. The water balance components are expressed as known empirical parametric functions of the mean water content of the land and atmospheric boxes. In particular, as a null hypothesis, we assumed that precipitation increases exponentially with water vapor path and that the presence of land does not affect this relationship. In order to investigate the bounds of χ and its parameter sensitivity, we analyzed a large number of equilibrium solutions for different combinations of model parameter values. The obtained results for the case of a closed model with one land and one ocean box can be summarized as follows:

- As long as the land does not affect the relationship between precipitation and water vapor path, the precipitation ratio is bound by an upper limit of one. This is a direct consequence of the equilibrium condition that the land's loss of water through runoff needs to be compensated for by an equally large net atmospheric moisture transport from a moister ocean atmosphere to a drier land atmosphere.

- The lower limit of χ is zero, in cases where the land precipitation is zero. Although χ can theoretically vary between zero and one, values between 0.7 and 1.0 appear most likely, with 95 % of the simulations falling into this range.

- The free model parameters are listed in Table 1. A sensitivity analysis shows that χ is most sensitive to a variation of the atmospheric transport parameter τ, followed by the two soil parameters permanent wilting point s_pwp and runoff exponent r and land fraction α. Efficient atmospheric transport, represented by high τ, helps to assimilate the land and ocean atmospheres and yields high values of χ. Land fraction is most influential near its extreme values, $\alpha \to 0$ and $\alpha \to 1$, where in both cases χ is close to one. Near these extremes, the highly efficient advection rate into or out of the respective tiny land or ocean atmosphere causes the atmospheric conditions in both boxes to assimilate. Finally, χ decreases with increasing permanent wilting point and increases with decreasing r. This can be understood

from the way in which these parameters control the amount of runoff and thereby the demand for advective moisture inflow into the land domain. With τ and α held constant, the advective inflow is regulated by the moisture difference Δw whose change is directly related to a change of χ in the same direction.

- The closed conceptual water balance model tends to underestimate the precipitation ratio. For a land fraction similar to the tropics and a domain length of 40000 km, an expected ratio of 0.59 is obtained which is lower than values obtained by Hohenegger and Stevens (2022). It also cannot explain observed island precipitation enhancement as reported by other studies because χ is bounded by one. Given the assumptions of our model, our interpretation of this finding is that precipitation enhancement over land requires the land to affect the relationship between precipitation and water vapor pass, that precipitation enhancement is linked to the presence of a diurnal cycle, that a different land distribution is required, or that it is only possible in an open model allowing for net advection into or out of the domain. We tested this last option with a model configuration in which moisture can enter the model through the windward boundary and leave on the other side. For this open model setup, we find enhanced land precipitation compared to the mean precipitation of the surrounding ocean for a small subset (6.8 %) of the performed simulations. These cases show that precipitation enhancement requires a large moisture inflow with a boundary water vapor pass of at least 40 mm/day, and small land sizes of at most $\alpha = 0.7$.

Even though the simple conceptual model does not capture the full range of physical processes that influence the precipitation partitioning between land and ocean, it is able to constrain the precipitation ratio and identifies the efficiency of atmospheric transport as the dominant factor governing the partitioning, while other parameters such as the soil type are shown to be less influential. Its limitations show that the presence of a diurnal cycle is indispensable for capturing the interaction between precipitation and rather small land masses and that land is likely to change the precipitation efficiency compared to over the ocean.

FIG. A1. Sketch of an land-ocean-atmosphere system consisting of an island between two ocean boxes with respective atmospheric boxes aloft. The lateral model boundaries are open, allowing moisture to enter and leave the modelled domain. The water vapor pass value at the windward model boundary is given by parameter w_0 and may reflect synoptic scale atmospheric conditions. Moisture fluxes between boxes are analogous to the closed model described in Section 2.

APPENDIX A

Open model formulation

The equations governing an open model configuration with an island (subscript 'ℓ') between two ocean boxes (subscripts 'o1' and 'o2' in the order of their appearance along the wind trajectory) are similar to the ones for the closed model introduced in Section 2. In contrast to the closed model, four instead of three equations are needed as the system contains an additional ocean subdomain. The equations read

$$\frac{ds}{dt} = \frac{1}{nz_{\mathrm{r}}}\left[P(w_\ell) - R(s, w_\ell) - E(s)\right] \tag{A1}$$

$$\frac{dw_{\mathrm{o}1}}{dt} = E_{\mathrm{o}} - P(w_{\mathrm{o}1}) + \frac{(w_0 - w_{\mathrm{o}1})u}{L_{\mathrm{o}1}}. \tag{A2}$$

$$\frac{dw_\ell}{dt} = E(s) - P(w_\ell) + \frac{(w_{\mathrm{o}1} - w_\ell)u}{L_\ell} \tag{A3}$$

$$\frac{dw_{\mathrm{o}2}}{dt} = E_{\mathrm{o}} - P(w_{\mathrm{o}2}) + \frac{(w_\ell - w_{\mathrm{o}2})u}{L_{\mathrm{o}2}},. \tag{A4}$$

The lengths of the model subdomains can be written as $L_{\mathrm{o}1} = L_{\mathrm{o}2} = (1-\alpha)L/2$ and $L_\ell = \alpha L$, where L denotes the full model domain length.

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Data availability statement.

References

Ahmed, F., and C. Schumacher, ????: Geographical differences in the tropical precipitation-moisture relationship and rain intensity onset. *Geophysical Research Letters*, **44 (2)**, 1114–1122, https://doi.org/10.1002/2016GL071980, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/2016GL071980.

Bergemann, M., and C. Jakob, 2016: How important is tropospheric humidity for coastal rainfall in the tropics? *Geophysical Research Letters*, **43 (11)**, 5860–5868, https://doi.org/10.1002/2016GL069255, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/2016GL069255.

Bretherton, C. S., M. E. Peters, and L. E. Back, 2004: Relationships between water vapor path and precipitation over the tropical oceans. *J. Climate*, **17**, 1517–1528, https://doi.org/10.1175/1520-0442(2004)017<1517:RBWVPA>2.0.CO;2.

Brubaker, K., D. Entekhabi, and P. Eagleson, 1991: Atmospheric water vapor transport: Estimation of continental precipitation recycling and parameterization of a simple climate model. URL https://ntrs.nasa.gov/citations/19910018381.

Brubaker, K. L., and D. Entekhabi, 1995: An analytic approach to modeling land-atmosphere interaction: 1. construct and equilibrium behavior. *Water Resources Research*, **31 (3)**, 619–632, https://doi.org/10.1029/94WR01772, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/94WR01772.

Brubaker, K. L., D. Entekhabi, and P. S. Eagleson, 1993: Estimation of continental precipitation recycling. *Journal of Climate*, **6**, 1077–1089, https://doi.org/10.1175/1520-0442(1993)006<1077:EOCPR>2.0.CO;2, URL https://journals.ametsoc.org/jcli/article/6/6/1077/39303/Estimation-of-Continental-Precipitation-Recycling.

Budyko, M. I., 1956: *Heat balance of the Earth's surface*. U.S. Dept. of Commerce, Weather Bureau.

Budyko, M. I., and O. A. Drozdov, 1953: Characteristics of the moisture circulation in the atmosphere. **4**, 5–14.

Burde, G. I., and A. Zangvil, 2001: The estimation of regional precipitation recycling. part i: Review of recycling models. *Journal of Climate*, **14 (12)**, 2497–2508, https://doi.org/10.1175/1520-0442(2001)014<2497:TEORPR>2.0.CO;2, URL https://journals.ametsoc.org/jcli/article/14/12/2497/29526/The-Estimation-of-Regional-Precipitation-Recycling.

Cronin, T. W., K. A. Emanuel, and P. Molnar, 2015: Island precipitation enhancement and the diurnal cycle in radiative-convective equilibrium. **141 (689)**, 1017–1034, https://doi.org/10.1002/qj.2443, URL https://rmets.onlinelibrary.wiley.com/doi/abs/10.1002/qj.2443.

Datseris, G., 2018: Dynamicalsystems.jl: A julia software library for chaos and nonlinear dynamics. *Journal of Open Source Software*, **3**, 598, https://doi.org/10.21105/joss.00598.

Datseris, G., and U. Parlitz, 2022: *Nonlinear Dynamics*. 2192-4791, Springer International Publishing, URL https://link.springer.com/book/9783030910334.

Eltahir, E. a. B., and R. L. Bras, 1994: Precipitation recycling in the amazon basin. *Quarterly Journal of the Royal Meteorological Society*, **120**, 861–880, https://doi.org/10.1002/qj.49712051806, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/qj.49712051806.

Ent, R. J. v. d., H. H. G. Savenije, B. Schaefli, and S. C. Steele-Dunne, 2010: Origin and fate of atmospheric moisture over continents. *Water Resources Research*, **46 (9)**, https://doi.org/10.1029/2010WR009127, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2010WR009127.

Entekhabi, D., 1994: A simple model of the hydrologic cycle and climate: 1. model construct and sensitivity to the land surface boundary. *Advances in Water Resources*, **17 (1)**, 79–91, https://doi.org/10.1016/0309-1708(94)90025-6, URL https://www.sciencedirect.com/science/article/pii/0309170894900256.

Entekhabi, D., and K. L. Brubaker, 1995: An analytic approach to modeling land-atmosphere interaction: 2. stochastic formulation. *Water Resources Research*, **31 (3)**, 633–643, https://doi.org/10.1029/94WR01773, URL https://onlinelibrary.wiley.com/doi/abs/10.1029/94WR01773.

Entekhabi, D., I. Rodriguez-Iturbe, and R. L. Bras, 1992: Variability in large-scale water balance with land surface-atmosphere interaction. *Journal of Climate*, **5**, 798–813, https://doi.org/10.1175/1520-0442(1992)005<0798:VILSWB>2.0.CO;2, URL https://journals.ametsoc.org/jcli/article/5/8/798/35919/Variability-in-Large-Scale-Water-Balance-with-Land.

Fiedler, S., and Coauthors, 2020: Simulated tropical precipitation assessed across three major phases of the coupled model intercomparison project (CMIP). *Monthly Weather Review*, **148 (9)**, 3653–3680, https://doi.org/10.1175/MWR-D-19-0404.1.

Hagemann, S., and T. Stacke, 2015: Impact of the soil hydrology scheme on simulated soil moisture memory. *Climate Dyn.*, **44**, 1731–1750, https://doi.org/10.1007/s00382-014-2221-6.

Hohenegger, C., and B. Stevens, 2022: Tropical continents rainier than expected from geometrical constraints.

Horton, R. e., 1943: Hydrologic interrelations between lands and oceans. *Eos, Transactions American Geophysical Union*, **24 (2)**, 753–764, https://doi.org/10.1029/TR024i002p00753, URL https://onlinelibrary.wiley.com/doi/abs/10.1029/TR024i002p00753.

Kumar, B. P., M. F. Cronin, S. Joseph, M. Ravichandran, and N. Sureshkumar, 2017: Latent heat flux sensitivity to sea surface temperature: Regional perspectives. *Journal of Climate*, **30 (1)**, 129–143, https://doi.org/10.1175/JCLI-D-16-0285.1, URL https://journals.ametsoc.org/view/journals/clim/30/1/jcli-d-16-0285.1.xml.

Lintner, B. R., P. Gentine, K. L. Findell, F. D'Andrea, A. H. Sobel, and G. D. Salvucci, 2013: An idealized prototype for large-scale land–atmosphere coupling. *Journal of Climate*, **26 (7)**, 2379–2389, https://doi.org/10.1175/JCLI-D-11-00561.1, URL https://journals.ametsoc.org/view/journals/clim/26/7/jcli-d-11-00561.1.xml.

Masunaga, H., and B. E. Mapes, 2020: A mechanism for the maintenance of sharp tropical margins. *Journal of the Atmospheric Sciences*, **77 (4)**, 1181–1197, https://doi.org/10.1175/JAS-D-19-0154.1, URL https://journals.ametsoc.org/view/journals/atsc/77/4/jas-d-19-0154.1.xml.

Ogino, S.-Y., M. D. Yamanaka, S. Mori, and J. Matsumoto, 2016: How much is the precipitation amount over the tropical coastal region? *Journal of Climate*, **29 (3)**, 1231–1236,

https://doi.org/10.1175/JCLI-D-15-0484.1, URL https://journals.ametsoc.org/view/journals/clim/29/3/jcli-d-15-0484.1.xml.

Ogino, S.-Y., M. D. Yamanaka, S. Mori, and J. Matsumoto, 2017: Tropical coastal dehydrator in global atmospheric water circulation. *Geophysical Research Letters*, **44 (22)**, 11,636–11,643, https://doi.org/https://doi.org/10.1002/2017GL075760, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2017GL075760%4010.1002/%28ISSN%291944-8007.GRLHIGHLIGHTS2017.

Peixóto, J. P., and A. H. Oort, 1983: The atmospheric branch of the hydrological cycle and climate. *Variations in the Global Water Budget*, Springer Netherlands, 5–65, https://doi.org/10.1007/978-94-009-6954-4_2, URL https://doi.org/10.1007/978-94-009-6954-4_2.

Qian, J.-H., 2008: Why precipitation is mostly concentrated over islands in the maritime continent. *Journal of the Atmospheric Sciences*, **65 (4)**, 1428–1441, https://doi.org/10.1175/2007JAS2422.1, URL https://journals.ametsoc.org/jas/article/65/4/1428/26793/Why-Precipitation-Is-Mostly-Concentrated-over.

Rodriguez-Iturbe, I., D. Entekhabi, and R. L. Bras, 1991: Nonlinear dynamics of soil moisture at climate scales: 1. stochastic analysis. *Water Resources Research*, **27**, 1899–1906, https://doi.org/10.1029/91WR01035.

Schiro, K. A., J. D. Neelin, D. K. Adams, and B. R. Lintner, 2016: Deep convection and column water vapor over tropical land versus tropical ocean: A comparison between the amazon and the tropical western pacific. *Journal of the Atmospheric Sciences*, **73 (10)**, 4043–4063, https://doi.org/10.1175/JAS-D-16-0119.1, URL https://journals.ametsoc.org/view/journals/atsc/73/10/jas-d-16-0119.1.xml.

Schiro, K. A., S. C. Sullivan, Y.-H. Kuo, H. Su, P. Gentine, G. S. Elsaesser, J. H. Jiang, and J. D. Neelin, 2020: Environmental controls on tropical mesoscale convective system precipitation intensity. *Journal of the Atmospheric Sciences*, **77 (12)**, 4233–4249, https://doi.org/10.1175/JAS-D-20-0111.1, URL https://journals.ametsoc.org/view/journals/atsc/77/12/jas-d-20-0111.1.xml.

Seneviratne, S. I., T. Corti, E. L. Davin, M. Hirschi, E. B. Jaeger, I. Lehner, B. Orlowsky, and A. J. Teuling, 2010: Investigating soil moisture–climate interactions in a changing climate: A review. **99 (3)**, 125–161, https://doi.org/10.1016/j.earscirev.2010.02.004, URL https://www.sciencedirect.com/science/article/pii/S0012825210000139.

Shannon, C. E., 1948: A mathematical theory of communication. *The Bell System Technical Journal*, **27 (3)**, 379–423, https://doi.org/10.1002/j.1538-7305.1948.tb01338.x.

Sobel, A. H., and G. Bellon, 2009: The effect of imposed drying on parameterized deep convection. *Journal of the Atmospheric Sciences*, **66 (7)**, 2085–2096, https://doi.org/10.1175/2008JAS2926.1, URL https://journals.ametsoc.org/view/journals/atsc/66/7/2008jas2926.1.xml.

Sobel, A. H., C. D. Burleyson, and S. E. Yuter, 2011: Rain on small tropical islands. *Journal of Geophysical Research: Atmospheres*, **116**, https://doi.org/10.1029/2010JD014695, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2010JD014695.

Ulrich, M., and G. Bellon, 2019: Superenhancement of precipitation at the center of tropical islands. *Geophysical Research Letters*, **46 (24)**, 14 872–14 880, https://doi.org/10.1029/2019GL084947, URL https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019GL084947.

Wang, S., and A. H. Sobel, 2017: Factors controlling rain on small tropical islands: Diurnal cycle, large-scale wind speed, and topography. *Journal of the Atmospheric Sciences*, **74 (11)**, 3515–3532, https://doi.org/10.1175/JAS-D-16-0344.1, URL https://journals.ametsoc.org/jas/article/74/11/3515/42168/Factors-Controlling-Rain-on-Small-Tropical-Islands.

Zhang, G. J., and M. J. Mcphaden, 1995: The relationship between sea surface temperature and latent heat flux in the equatorial pacific. *Journal of Climate*, **8 (3)**, 589–605, https://doi.org/10.1175/1520-0442(1995)008<0589:TRBSST>2.0.CO;2, URL https://journals.ametsoc.org/view/journals/clim/8/3/1520-0442_1995_008_0589_trbsst_2_0_co_2.xml.