# Running Putnam Notes

Jack Ceroni

October 2020

## Contents



## 1 Introduction

*These notes are my attempt at keeping a log of any useful Putnam-related notes and solutions during undergrad.*

## 2 Solutions

**Proposition 1.** *Every non-zero coefficient of the Maclauren series of:*

$$f(x) = (1 - x + x^2)e^x$$

*is rational, with the numerator being either* 1 *or a prime number (when fully reduced).*

*Proof.* Clearly:

$$\begin{aligned} f(x) = (1 - x + x^2)e^x = (1 - x + x^2) \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n - x^{n+1} + x^{n+2}}{n!} \\ = \sum_{n=0}^{\infty} \frac{x^n}{n!} - \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!} + \sum_{n=2}^{\infty} \frac{x^n}{(n-2)!} \end{aligned} \tag{1}$$

In the case that $n \geq 2$, the $n$-th coefficient of the series(which we denote by $c_n$) is thus given by:

$$c_n = \frac{1}{n!} - \frac{1}{(n+1)!} + \frac{1}{(n+2)!} = \frac{1 - n + n(n+1)}{n!} = \frac{n^2 + 1}{n!} \tag{2}$$

□