

# Artin Algebra: Chapter 1 Solutions

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## Contents

<a href="#">1 Notes</a>	1
<a href="#">2 Solutions</a>	1

## 1 Notes

**Proposition 1.** *If matrices  $A$  and  $B$  are invertible, then so is the matrix  $AB$ , and  $(AB)^{-1} = B^{-1}A^{-1}$*

*Proof.*

$$\mathbb{I} = AA^{-1} = AIA^{-1} = A(BB^{-1})A^{-1} = (AB)(B^{-1}A^{-1}) \quad (1)$$

Therefore, by definition of the inverse,  $(AB)^{-1}$  exists and is equal to  $B^{-1}A^{-1}$ .  $\square$

**Proposition 2.** *The equation  $AB - BA = \mathbb{I}$  has no solutions, for  $n \times n$  matrices with real entries.*

*Proof.*

$$\text{Tr}(AB - BA) = \text{Tr}(AB) - \text{Tr}(BA) = 0 \quad (2)$$

But we also know that  $\text{Tr}(\mathbb{I}) \neq 0$ , so it follows that  $AB - BA \neq \mathbb{I}$ .  $\square$

## 2 Solutions