

Qubit Complexity Notes

(Dated: January 19, 2022)

I. AUTOMATIC COMPUTATION OF THE METRIC TENSOR AND CHRISTOFFEL SYMBOLS

In order to determine the complexity of some unitary U , denoted $C[U]$, we must determine the length of the geodesic extending from I to U with respect to some metric on the space of unitaries. In general, a metric on the space of unitaries is given by:

$$ds^2 = \text{Tr}(idUU^\dagger T_I) \mathcal{I}_{IJ} \text{Tr}(idUU^\dagger T_J) \quad (1)$$

where the T_I are the generators of $SU(2^N)$, for some N -qubit system. Given some parametrized unitary $U(\boldsymbol{\theta})$, we can write this metric explicitly in terms of derivatives with respect to the parameters, as $dU = \partial_{\boldsymbol{\theta}} U \, d\boldsymbol{\theta}$.

The case considered in the paper [CITE] considers a two-qubit system, with a parametrization as follows:

$$U(\boldsymbol{\theta}) = \exp(-iH(\boldsymbol{\theta})) = \exp(-i[\theta_x X + \theta_y Y + \theta_z Z]) \quad (2)$$