# ***Computer Vision***

Course 2

## Fundamental Steps in Image Processing

- methods whose input and output are images
- methods whose inputs are images but whose outputs are attributes extracted from those images

# Course 2

## Outputs are images

- image acquisition
- image filtering and enhancement
- image restoration
- color image processing
- wavelets and multiresolution processing
- compression
- morphological processing

**Outputs are attributes**

- morphological processing
- segmentation
- representation and description
- object recognition

**Image acquisition -** may involve preprocessing such as scaling

**Image enhancement**

- manipulating an image so that the result is more suitable than the original for a specific operation
- enhancement is problem oriented
- there is no general ‚theory' of image enhancement
- enhancement use subjective methods for image improvement
- enhancement is based on human subjective preferences regarding what is a „good" enhancement result

**Image restoration**

- improving the appearance of an image
- restoration is objective - the techniques for restoration are based on mathematical or probabilistic models of image degradation

**Color image processing**

- fundamental concept in color models
- basic color processing in a digital domain

**Wavelets and multiresolution processing**

- representing images in various degree of resolution

**Compression**

- reducing the storage required to save an image or the bandwidth required to transmit it

**Morphological processing**

- tools for extracting image components that are useful in the representation and description of shape
- a transition from processes that output images to processes that output image attributes

**Segmentation**

- partitioning an image into its constituents parts or objects
- autonomous segmentation is one of the most difficult tasks of DIP
- the more accurate the segmentation, the more likely recognition is to succeed

**Representation and description** (almost always follows segmentation)

- segmentation produces either the boundary of a region or all the points in the region itself
- converting the data produced by segmentation to a form suitable for computer processing
- boundary representation: the focus is on external shape characteristics such as corners or inflections

- complete region: the focus is on internal properties such as texture or skeletal shape

- description is also called feature extraction – extracting attributes that result in some quantitative information of interest or are basic for differentiating one class of objects from another

**Object recognition** (Machine Learning techniques)

- the process of assigning a label (e.g. „vehicle”) to an object based on its descriptors

**Knowledge database**

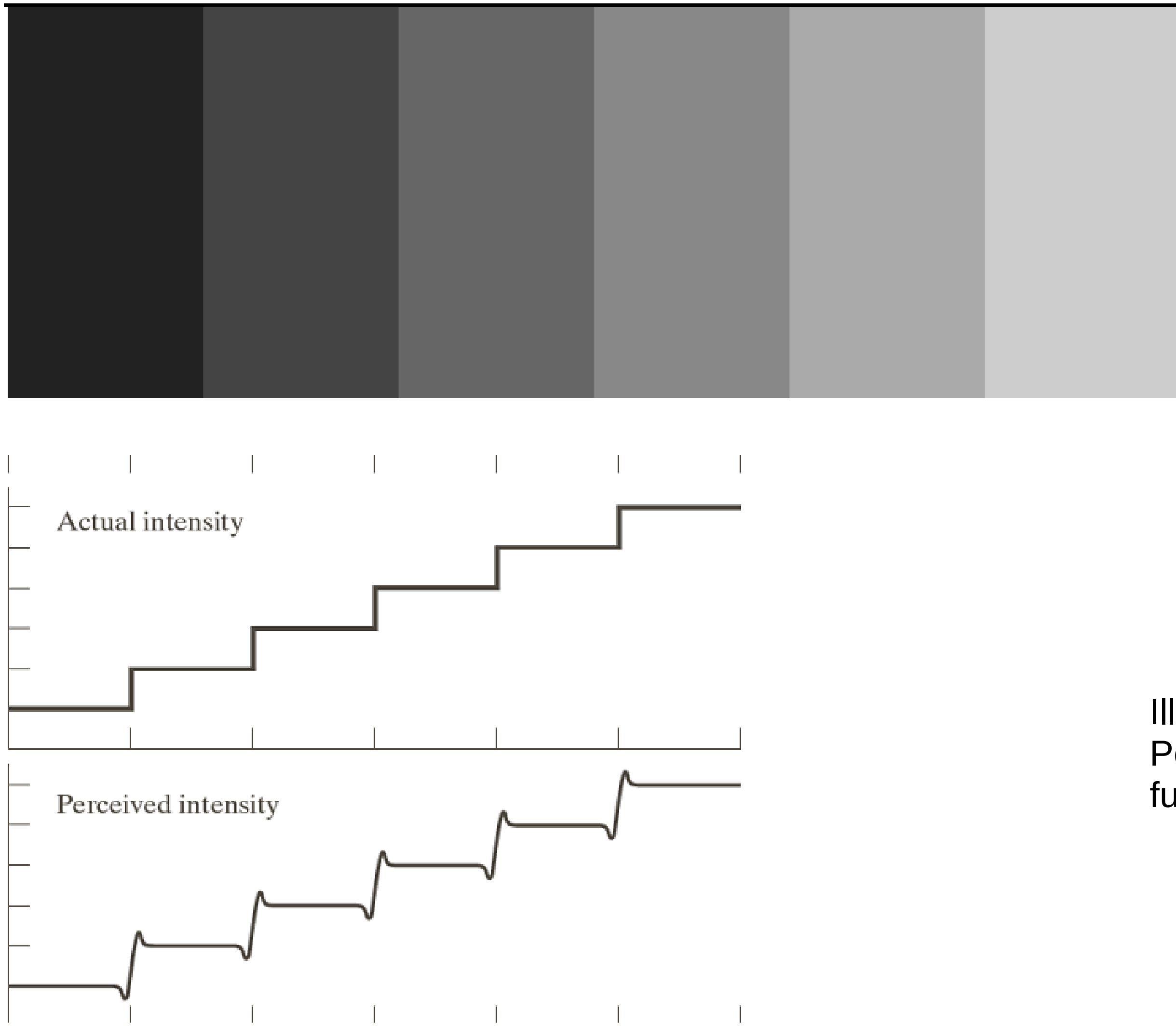
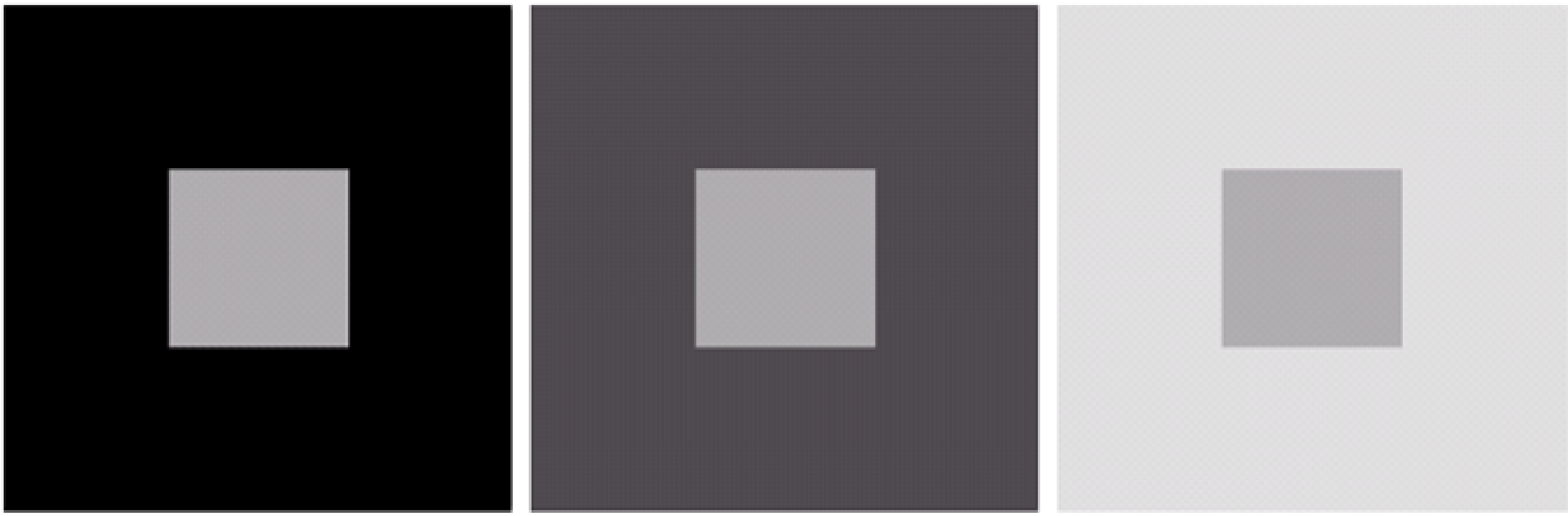


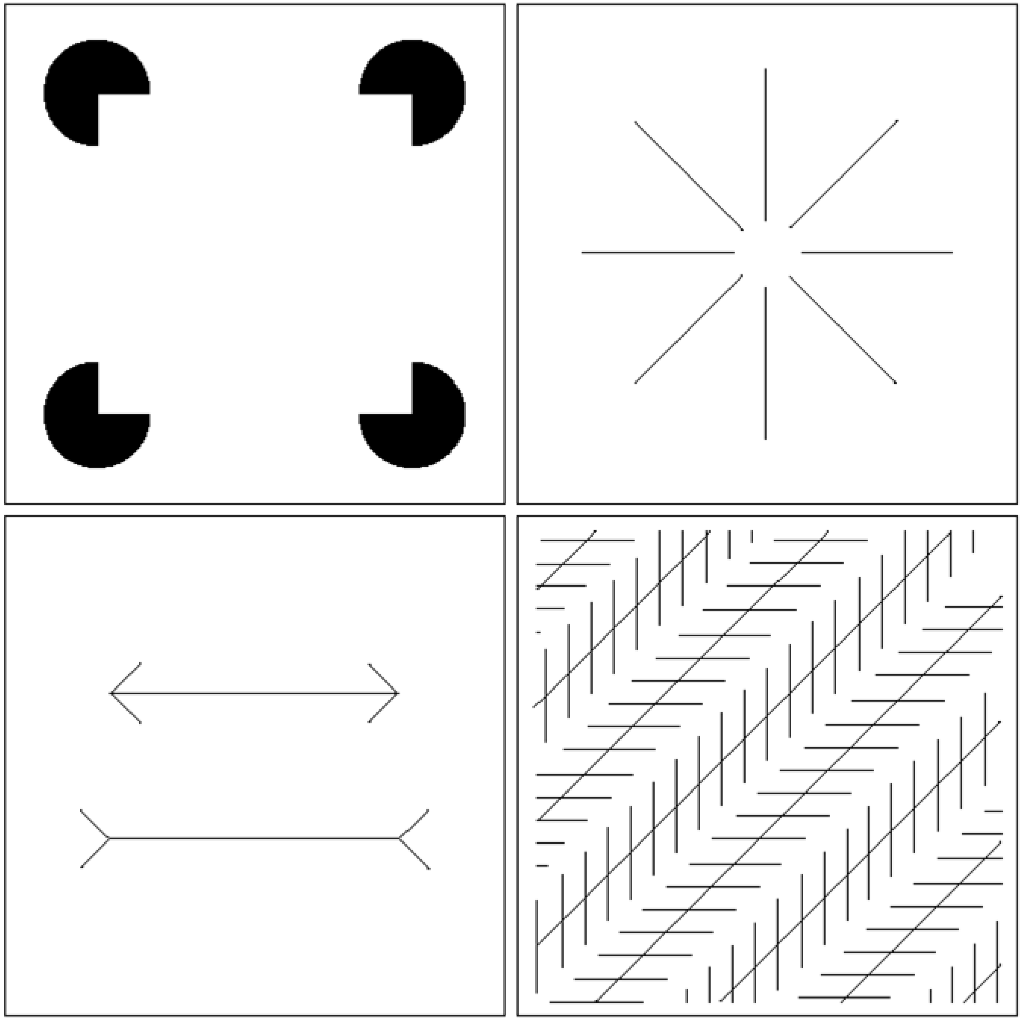

Illustration of Mach band effect
Perceived intensity is not a simple function of the actual intensity

**Course 2**



All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter

**Course 2**



Optical illusions

| | |
|---|---|
| ALBASTRU | BLUE |
| VERDE | GREEN |
| GALBEN | YELLOW |
| ROSU | RED |
| PORTOCALIU | ORANGE |
| GALBEN | YELLOW |
| VISINIU | BURGUNDY |
| ALB | WHITE |

**Course 2**

# Light

- *achromatic* or *monochromatic* light - light that is void of color the attribute of such light is its *intensity*, or amount. *Gray level* is used to describe monochromatic intensity because it ranges from black, to grays, and to white.

- *chromatic light* spans the electromagnetic energy spectrum from approximately 0.43 to 0.79 μm quantities that describe the quality of a chromatic light source:

  - *radiance* - the total amount of energy that flows from the light source, and it is usually measured in watts (W)

  - *Luminance* - measured in lumens (lm), gives a measure of the amount of energy an observer *perceives* from a light source.

For example, light emitted from a source operating in the far infrared region of the spectrum could have significant energy (radiance), but an observer would hardly perceive it; its luminance would be almost zero.

❖ *brightness* - a subjective descriptor of light perception that is practically impossible to measure. It embodies the achromatic notion of intensity and is one of the key factors in describing color sensation.

**Course 2**

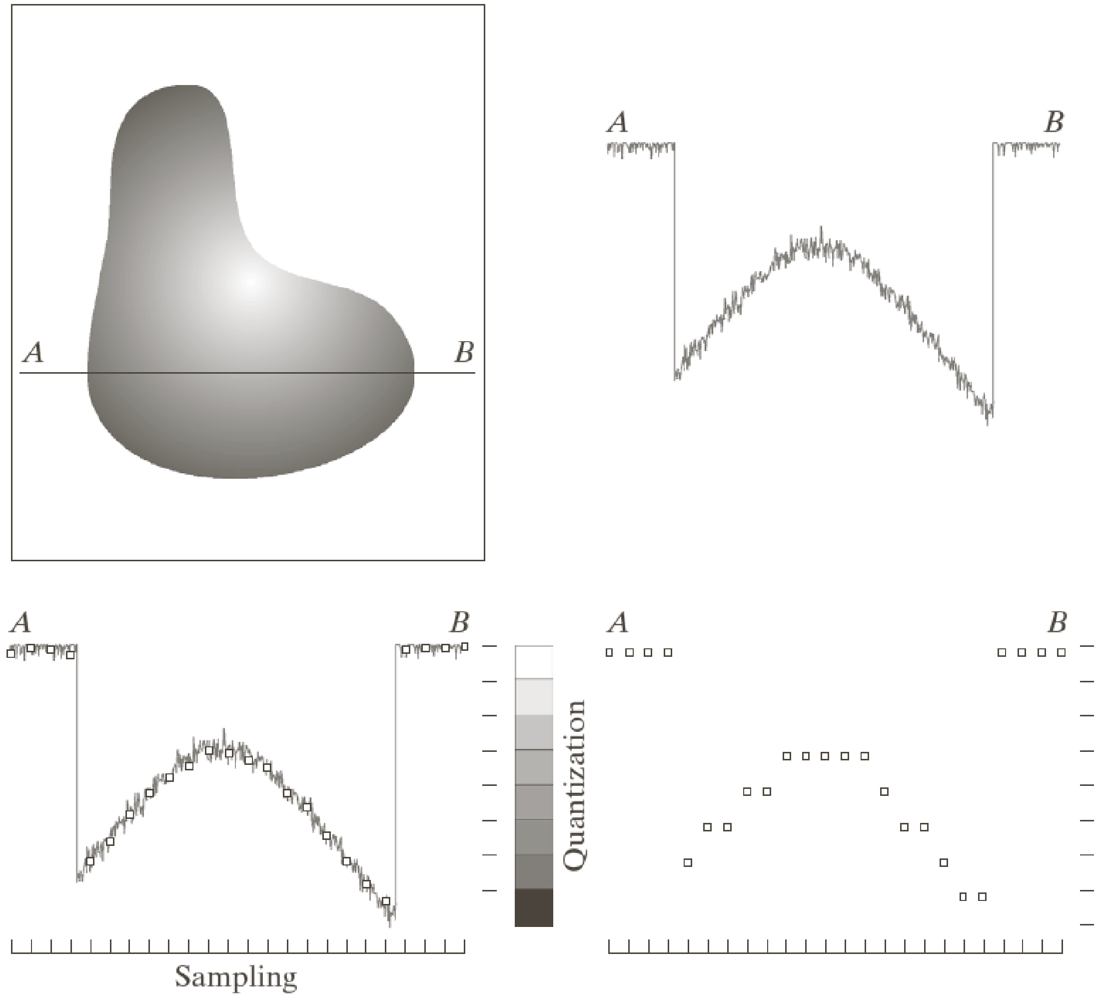
## Image Sampling and Quantization

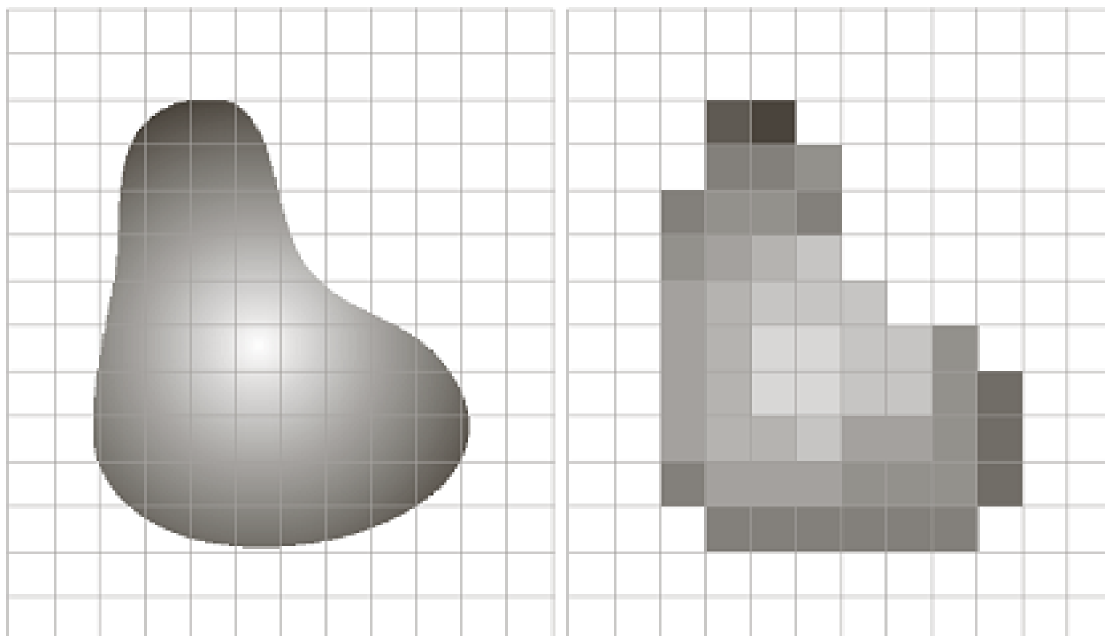
- the output of the sensors is a continuous voltage waveform related to the sensed scene

→ converting a continuous image ***f*** to digital form

- digitizing ***(x, y)*** is called ***sampling***
- digitizing ***f(x, y)*** is called ***quantization***

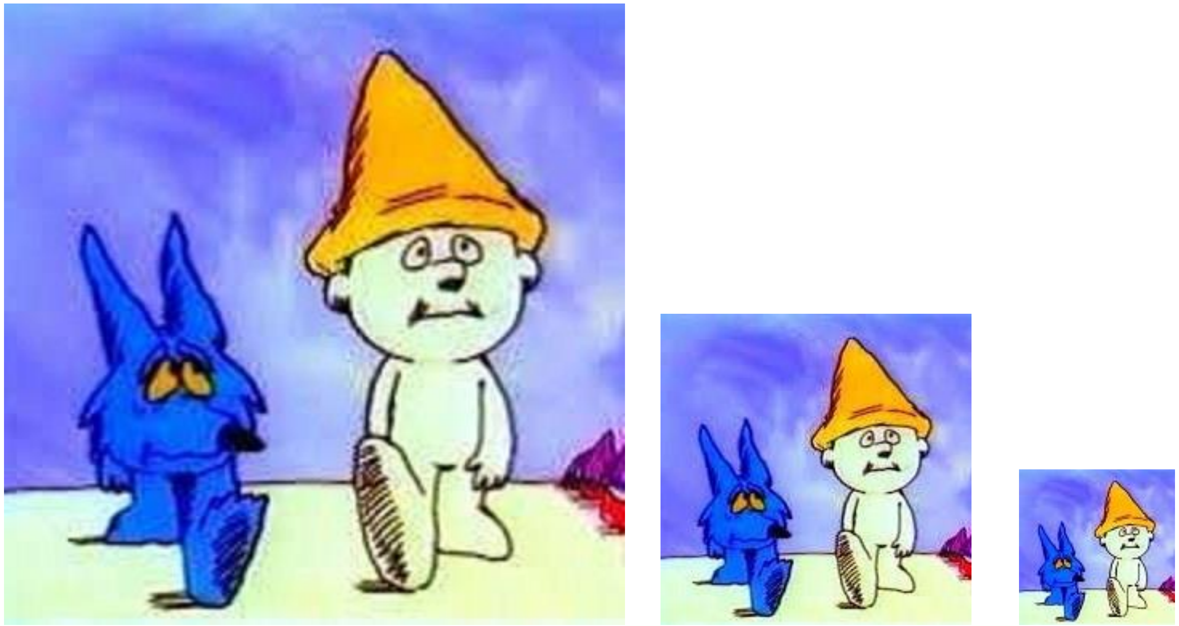
# Course 2





Continuous image projected onto a sensor array   Result of image sampling and quantization

# Course 2



## Representing Digital Images

***(x,y)*** → ***x = 0,1,…,M-1 ,  y = 0,1,…,N-1*** – *spatial variables* or *spatial coordinates*

$$f(x,y) \rightarrow \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix} \in \mathbb{R}^{M\times N} , \quad a_{i,j} = f(x=i, y=j) = f(i,j)$$

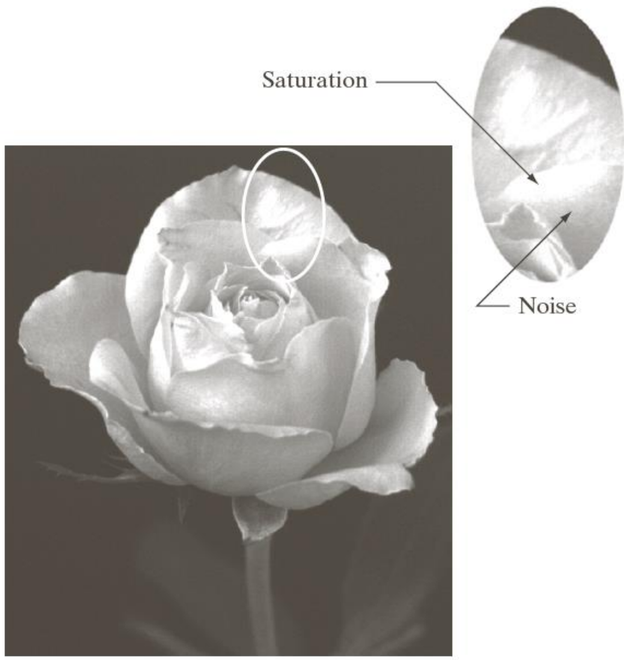
$a_{i,j}$ – image element, pixel

***f(0,0)*** – the upper left corner of the image

$$M, N \geq 0, \quad L = 2^k$$

$$a_{i,j} \in \mathbb{N} \ , \quad a_{i,j} \in [0, L-1]$$

***Dynamic range*** of an image = the ratio of the maximum measurable intensity to the minimum detectable intensity level in the system
Upper limit – determined by *saturation,* lower limit - *noise*



**Course 2**

http://xahlee.info/img/image_editing.html

## Course 2

# Course 2

# Course 2

## Course 2

# Course 2

# Course 2

Number of bits required to store a digitized image:

$$b = M \times N \times k \ , \quad \text{for } M = N \ , \ b = N^2 k$$

When an image can have $2^k$ intensity levels, the image is referred as a ***k-bit image***
256 discrete intensity values – 8-bit image

**TABLE 2.1**
Number of storage bits for various values of $N$ and $k$.

| $N/k$ | 1 ($L$ = 2) | 2 ($L$ = 4) | 3 ($L$ = 8) | 4 ($L$ = 16) | 5 ($L$ = 32) | 6 ($L$ = 64) | 7 ($L$ = 128) | 8 ($L$ = 256) |
|---|---|---|---|---|---|---|---|---|
| 32 | 1,024 | 2,048 | 3,072 | 4,096 | 5,120 | 6,144 | 7,168 | 8,192 |
| 64 | 4,096 | 8,192 | 12,288 | 16,384 | 20,480 | 24,576 | 28,672 | 32,768 |
| 128 | 16,384 | 32,768 | 49,152 | 65,536 | 81,920 | 98,304 | 114,688 | 131,072 |
| 256 | 65,536 | 131,072 | 196,608 | 262,144 | 327,680 | 393,216 | 458,752 | 524,288 |
| 512 | 262,144 | 524,288 | 786,432 | 1,048,576 | 1,310,720 | 1,572,864 | 1,835,008 | 2,097,152 |
| 1024 | 1,048,576 | 2,097,152 | 3,145,728 | 4,194,304 | 5,242,880 | 6,291,456 | 7,340,032 | 8,388,608 |
| 2048 | 4,194,304 | 8,388,608 | 12,582,912 | 16,777,216 | 20,971,520 | 25,165,824 | 29,369,128 | 33,554,432 |
| 4096 | 16,777,216 | 33,554,432 | 50,331,648 | 67,108,864 | 83,886,080 | 100,663,296 | 117,440,512 | 134,217,728 |
| 8192 | 67,108,864 | 134,217,728 | 201,326,592 | 268,435,456 | 335,544,320 | 402,653,184 | 469,762,048 | 536,870,912 |

## Course 2

### Spatial and Intensity Resolution

*Spatial resolution* – the smallest discernible detail in an image

Measures: *line pairs per unit distance*, *dots (pixels) per unit distance*

Image resolution = the largest number of discernible line pairs per unit distance

(e.g. 100 line pairs per mm)

Dots per unit distance are commonly used in printing and publishing

In U.S. the measure is expressed in *dots per inch* (***dpi***)

(newspapers are printed with 75 dpi, glossy brochures at 175 dpi)

*Intensity resolution* – the smallest discernible change in intensity level

The number of intensity levels ($L$) is determined by hardware considerations

$L=2^k$ – most common $k = 8$

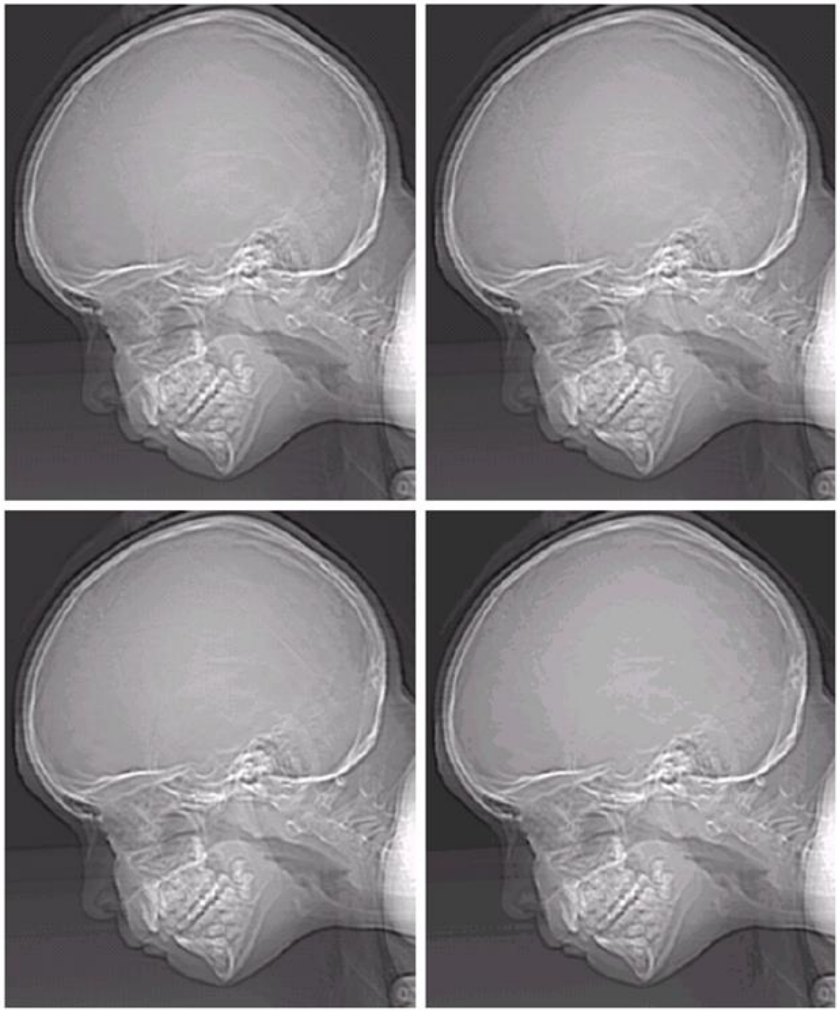
Intensity resolution, in practice, is given by $k$ (number of bits used to quantize intensity)

# Course 2

Fig.1 Reducing spatial resolution: 1250 dpi(upper left), 300 dpi (upper right)
150 dpi (lower left), 72 dpi (lower right)

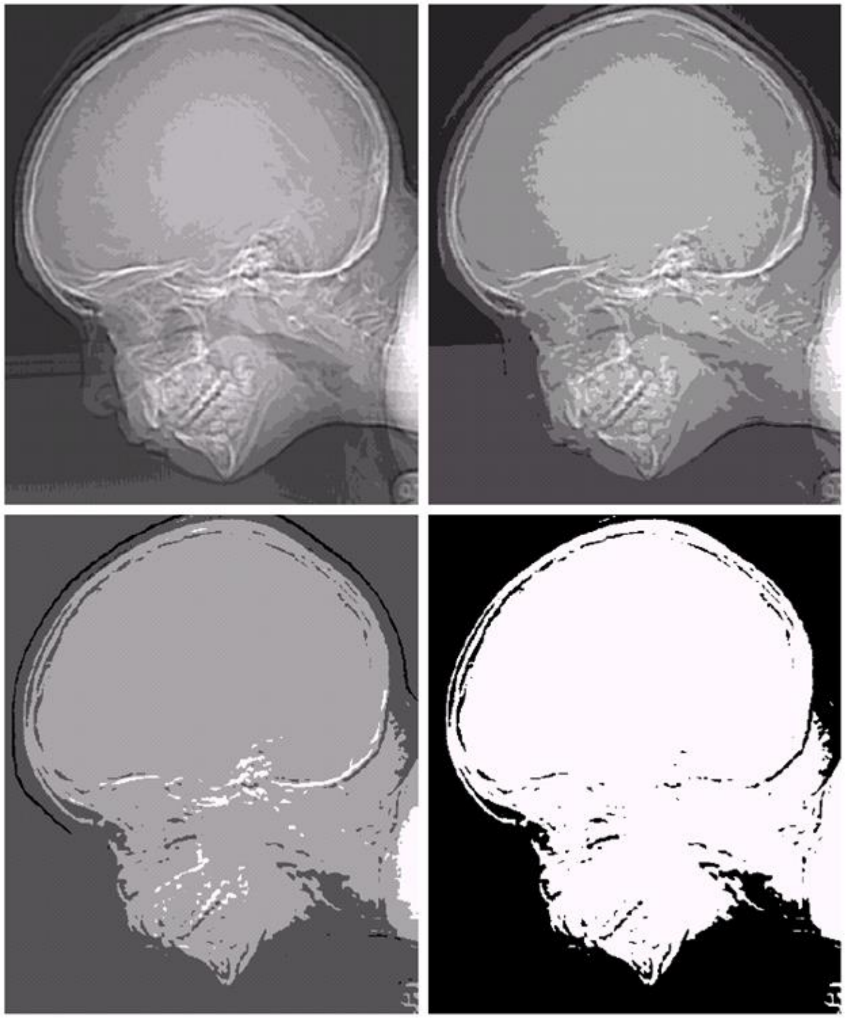
# Course 2

Reducing the number of gray levels: 256, 128, 64, 32



# Course 2

Reducing the number of gray levels: 16, 8, 4, 2



## Course 2

# Image Interpolation

**-** used in zooming, shrinking, rotating, and geometric corrections

Shrinking, zooming – image resizing – image resampling methods

***Interpolation*** is the process of using known data to estimate values at unknown locations

Suppose we have an image of size ***500* × *500*** pixels that has to be enlarged 1.5 times to ***750×750*** pixels. One way to do this is to create an imaginary ***750* × *750*** grid with the same spacing as the original, and then shrink it so that it fits exactly over the original image. The pixel spacing in the ***750* × *750*** grid will be less than in the original image.

*Problem*: assignment of intensity-level in the new ***750* × *750*** grid

**Course 2**

$$
\begin{matrix}
255 & 250 & 200 & 220 \\
245 & 200 & 180 & 200 \\
138 & 145 & 150 & 160 \\
124 & 130 & 125 & 129
\end{matrix}
\rightarrow
\begin{matrix}
255 & ? & 250 & ? & 200 & ? & 220 & ? \\
? & ? & ? & ? & ? & ? & ? & ? \\
245 & ? & 200 & ? & 180 & ? & 200 & ? \\
? & ? & ? & ? & ? & ? & ? & ? \\
138 & ? & 145 & ? & 150 & ? & 160 & ? \\
? & ? & ? & ? & ? & ? & ? & ? \\
124 & ? & 130 & ? & 125 & ? & 129 & ? \\
? & ? & ? & ? & ? & ? & ? & ?
\end{matrix}
$$

**Course 2**

***Nearest neighbor interpolation:*** assign for every point in the new grid (***750 × 750***) the intensity of the closest pixel (nearest neighbor) from the old/original grid (***500 × 500***). This technique has the tendency to produce undesirable effects, like severe distortion of straight edges.

***Bilinear interpolation*** – assign for the new ***(x, y)*** location the following intensity:

$$v(x,y) = a\,x + b\,y + c\,x\,y + d$$

where the four coefficients are determined from the 4 equations in 4 unknowns that can be written using the 4 nearest neighbors of point ***(x, y).***

Bilinear interpolation gives much better results than nearest neighbor interpolation, with a modest increase in computational effort.

## Course 2

***Bicubic interpolation*** – assign for the new ***(x, y)*** location an intensity that involves the 16 nearest neighbors of the point:

$$v(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}c_{i,j}x^i y^j$$

The coefficients $c_{i,j}$ are obtained solving a ***16x16*** linear system:

$$\sum_{i=0}^{3}\sum_{j=0}^{3}c_{i,j}x^i y^j = \text{intensity levels of the 16 nearest neighbors of } (x,y)$$

Generally, bicubic interpolation does a better job of preserving fine detail than the bilinear technique. Bicubic interpolation is the standard used in commercial image editing programs, such as Adobe Photoshop and Corel Photopaint.

Figure 2 (a) is the same as Fig. 1 (d), which was obtained by reducing the resolution of the 1250 dpi in Fig. 1(a) to 72 dpi (the size shrank from ***3692 × 2812*** to ***213 × 162***) and then zooming the reduced image back to its original size. To generate Fig. 1(d) nearest neighbor interpolation was used (both for shrinking and zooming).

Figures 2(b) and (c) were generated using the same steps but using bilinear and bicubic interpolation, respectively. Figures 2(d)+(e)+(f) were obtained by reducing the resolution from 1250 dpi to 150 dpi (instead of 72 dpi)

# Course 2

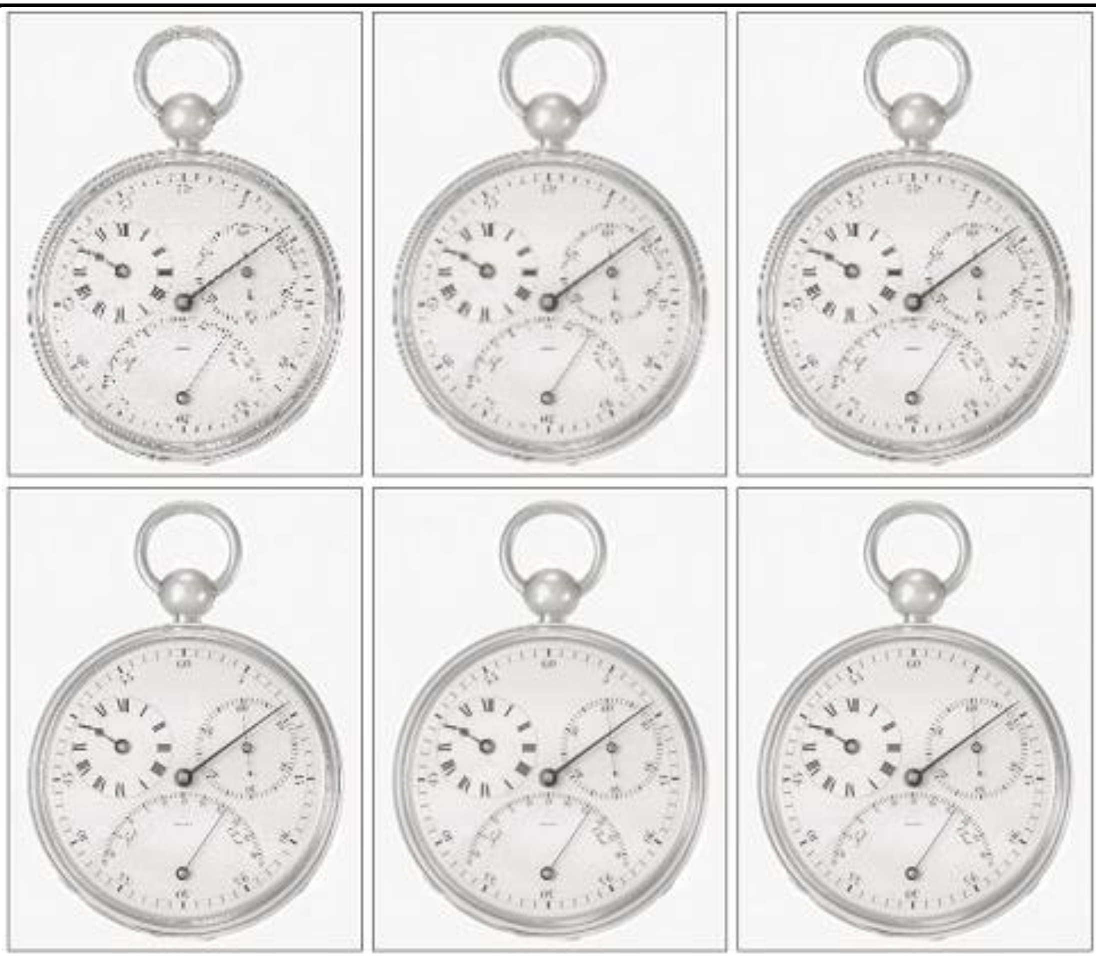
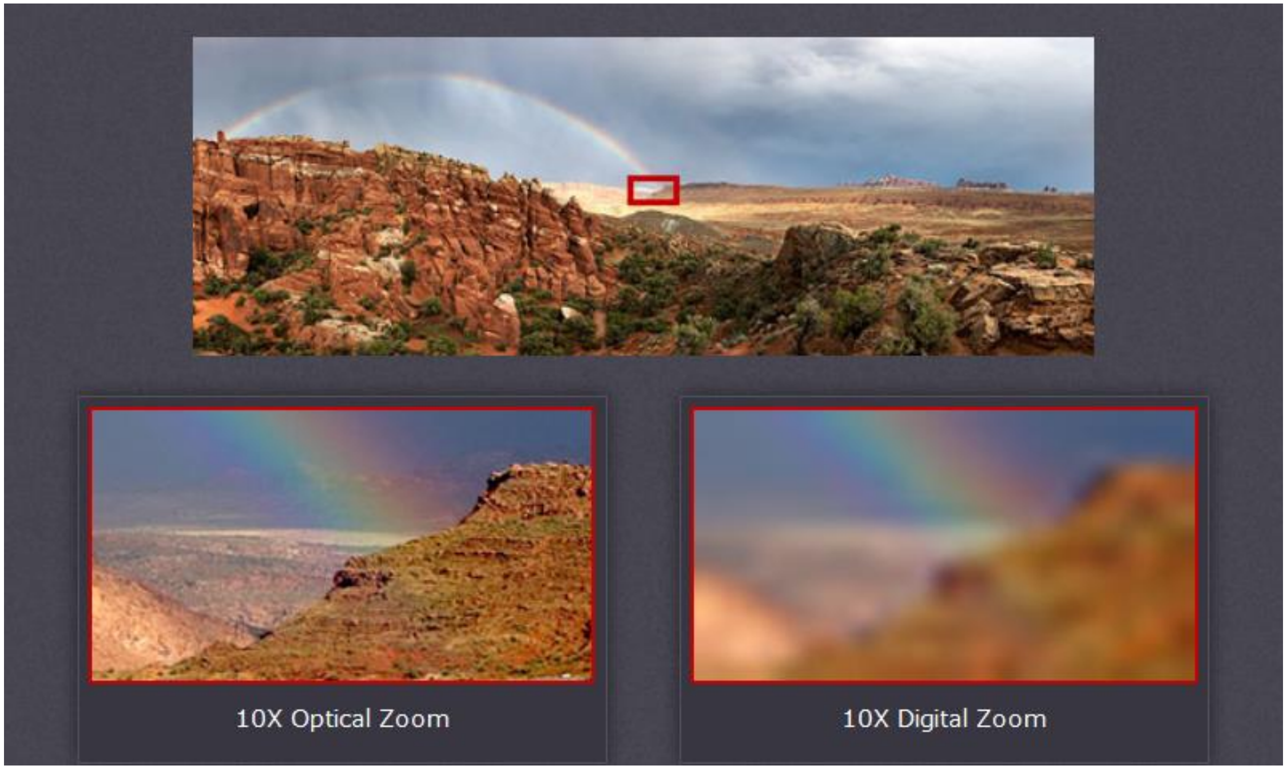


Fig. 2 – Interpolation examples for zooming and shrinking (nearest neighbor, linear, bicubic)

# Course 2




https://www.cambridgeincolour.com/tutorials/image-interpolation.htm

## Course 2

### Neighbors of a Pixel

A pixel ***p*** at coordinates ***(x, y)*** has 4 *horizontal* and *vertical* neighbors:

$$\text{horizontal: } (x+1, y)\ ,\ (x-1, y)\quad ;\quad \text{vertical: } (x, y+1)\ ,\ (x, y-1)$$

This set of pixels, called the *4-neighbors* of ***p***, denoted by $N_4(p)$.

The 4 *diagonal* neighbors of ***p*** have coordinates:

$$(x+1, y+1)\ ,\ (x+1, y-1)\ ,\ (x-1, y+1)\ ,\ (x-1, y-1)$$

are denoted $N_D(p)$.

The horizontal, vertical and diagonal neighbors are called the ***8-neighbors*** of ***p***, denoted $N_8(p)$.

If ***(x,y)*** is on the border of the image some of the neighbor locations in $N_D(p)$ and $N_8(p)$ fall outside the image.

## Course 2

### Adjacency, Connectivity, Regions, Boundaries

Denote by ***V*** the set of intensity levels used to define adjacency.

- in a binary image $V \subseteq \{0,1\}$ $(V=\{0\}, V=\{1\})$
- in a gray-scale image with 256 possible gray-levels, ***V*** can be any subset of $\{0,255\}$

We consider 3 types of adjacency:

(a) *4-adjacency* : two pixels ***p*** and ***q*** with values from ***V*** are *4-adjacent* if $q \in N_4(p)$

(b) *8-adjacency* : two pixels ***p*** and ***q*** with values from ***V*** are *8-adjacent* if $q \in N_8(p)$

(c) *m-adjacency* (mixed adjacency) : two pixels ***p*** and ***q*** with values from ***V*** are *m-adjacent* if :

- $q \in N_4(p)$ or
- $q \in N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from ***V***.

Mixed adjacency is a modification of *8*-adjacency. It is introduced to eliminate the ambiguities that often arise when *8*-adjacency is used. Consider the example:

**Course 2**

$$\boldsymbol{V}=\{\mathbf{1}\}\ -\ \text{binary image}\quad \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}, \begin{pmatrix} \mathbf{0} & \mathbf{1} & \cdots & \mathbf{1} \\ & \vdots & \iddots & \\ \mathbf{0} & \mathbf{1} & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \mathbf{0} & & \mathbf{1} \end{pmatrix}, \begin{pmatrix} \mathbf{0} & \mathbf{1} & \cdots & \mathbf{1} \\ & \vdots & & \\ \mathbf{0} & \mathbf{1} & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \mathbf{0} & & \mathbf{1} \end{pmatrix}$$

The three pixels at the top (first line) in the above example show multiple (ambiguous) *8*-adjacency, as indicated by the dashed lines. This ambiguity is removed by using *m*-adjacency.

## Course 2

A *(digital) path* (or *curve*) from pixel ***p*** with coordinates ***(x,y)*** to ***q*** with coordinates ***(s,t)*** is a sequence of distinct pixels with coordinates:

$$(x_0, y_0) = (x, y),\ (x_1, y_1), \ldots, (x_n, y_n) = (s, t)$$

$$(x_{i-1}, y_{i-1}) \text{ and } (x_i, y_i) \text{ are adjacent, } i = 1, 2, \ldots, n$$

The *length* of the path is ***n***. If $(x_0, y_0) = (x_n, y_n)$ the path is *closed*.

Depending on the type of adjacency considered the paths are: *4*-, *8*-, or *m*-paths.

Let ***S*** denote a subset of pixels in an image. Two pixels ***p*** and ***q*** are said to be *connected* in ***S*** if there exists a path between them consisting only of pixels from ***S***.

***S*** is a *connected set* if there is a path in ***S*** between any 2 pixels in ***S.***

Let ***R*** be a subset of pixels in an image. ***R*** is a *region* of the image if ***R*** is a connected set.

Two regions $R_1$ and $R_2$ are said to be *adjacent* if $R_1 \cup R_2$ form a connected set. Regions that are not adjacent are said to be *disjoint*. When referring to regions only *4*- and *8*-adjacency are considered.

## Course 2

Suppose that an image contains $K$ disjoint regions, $R_k$, $k = 1, ..., K$, none of which touches the image border.

$$R_u = \bigcup_{k=1}^{K} R_k \quad , \quad (R_u)^c - \text{the complement of } R_u$$

We call all the points in $R_u$ the *foreground* of the image and the points in $(R_u)^c$ the *background* of the image.

The *boundary (border* or *contour*) of a region $R$ is the set of points that are adjacent to points in the complement of $R$, $(R)^c$. The border of an image is the set of pixels in the region that have at least one background neighbor. This definition is referred to as the *inner border* to distinguish it from the notion of *outer border* which is the corresponding border in the background.

## Distance measures

For pixels ***p, q***, and ***z***, with coordinates ***(x,y), (s,t)*** and ***(v,w)*** respectively, ***D*** is a *distance function* or *metric* if:

(a) $D(p, q) \geq 0$ , $D(p, q) = 0$ iff $p=q$

(b) $D(p, q) = D(q, p)$

(c) $D(p, z) \leq D(p, q) + D(q, z)$

The *Euclidean distance* between ***p*** and ***q*** is defined as:

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{\frac{1}{2}} = \sqrt{(x-s)^2 + (y-t)^2}$$

The pixels ***q*** for which $D_e(p,q) \leq r$ are the points contained in a disk of radius ***r*** centered at ***(x, y)***.

The $\boldsymbol{D_4}$ *distance (*also called *city-block distance)* between $\boldsymbol{p}$ and $\boldsymbol{q}$ is defined as:

$$D_4(p,q) = |x-s| + |y-t|$$

The pixels $\boldsymbol{q}$ for which $D_4(p,q) \le r$ form a diamond centered at $\boldsymbol{(x,y)}$.

$$D_4 \le 2 \rightarrow \quad \begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

The pixels with $\boldsymbol{D_4 = 1}$ are the *4*-neighbors of $\boldsymbol{(x, y)}$.

## Course 2

The $D_8$ *distance (*called the *chessboard distance)* between $p$ and $q$ is defined as:

$$D_8(p,q) = \max\{|x-s|, |y-t|\}$$

The pixels $q$ for which $D_8(p,q) \leq r$ form a square centered at $(x, y)$.

$$D_8 \leq 2 \rightarrow \quad \begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

The pixels with $D_8 = 1$ are the *8*-neighbors of $(x, y)$.

$D_4$ and $D_8$ distances are independent of any paths that might exist between $p$ and $q$ because these distances involve only the coordinates of the point.

## Array versus Matrix Operations

An array operation involving one or more images is carried out on a *pixel-by-pixel* basis.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{21} \end{bmatrix}$$

Matrix product:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{21} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

***We assume array operations unless stated otherwise!***

## Course 2

### Linear versus Nonlinear Operations

One of the most important classifications of image-processing methods is whether it is linear or nonlinear.

$$\boldsymbol{H}\left[ f(x,y)\right] = g(x,y)$$

***H*** is said to be a *linear operator* if:

$$\boldsymbol{H}\left[a\, f_1(x,y) + b\, f_2(x,y)\right] = a\, \boldsymbol{H}\left[ f_1(x,y)\right] + b\, \boldsymbol{H}\left[ f_2(x,y)\right]$$

$$\forall a,b \in \mathbb{R},\ \forall f_1, f_2 - \text{ images}$$

Example of nonlinear operator:

$$\boldsymbol{H}\left[ f\right] = \max\{ f(x,y)\} = \text{ the maximum value of the pixels of image } f$$

$$f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix},\ f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix},\ a = 1,\ b = -1$$

## Course 2

$$\max\{a\, f_1 + b\, f_2\} = \max\left\{1\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix} + (-1)\begin{bmatrix}6 & 5\\ 4 & 7\end{bmatrix}\right\} = \max\left\{\begin{bmatrix}-6 & -3\\ -2 & -4\end{bmatrix}\right\} = -2$$

$$1\cdot\max\left\{\begin{bmatrix}0 & 2\\ 2 & 3\end{bmatrix}\right\} + (-1)\cdot\max\left\{\begin{bmatrix}6 & 5\\ 4 & 7\end{bmatrix}\right\} = 3 + (-1)7 = -4$$

## Course 2

## Arithmetic Operations in Image Processing

Let $g(x,y)$ denote a corrupted image formed by the *addition* of noise:

$$g(x,y) = f(x,y) + \eta(x,y)$$

$f(x,y)$ – the noiseless image ; $\eta(x,y)$ the noise, uncorrelated and has 0 average value.

For a random variable $z$ with mean $m$, $E[(z-m)^2]$ is the variance ($E(\cdot)$ is the expected value). The covariance of two random variables $z_1$ and $z_2$ is defined as $E[(z_1-m_1)(z_2-m_2)]$. The two random variables are *uncorrelated* when their covariance is 0.

Objective: reduce noise by adding a set of noisy images $\{g_i(x,y)\}$ (technique frequently used in image enhancement)

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

If the noise satisfies the properties stated above, we have:

## Course 2

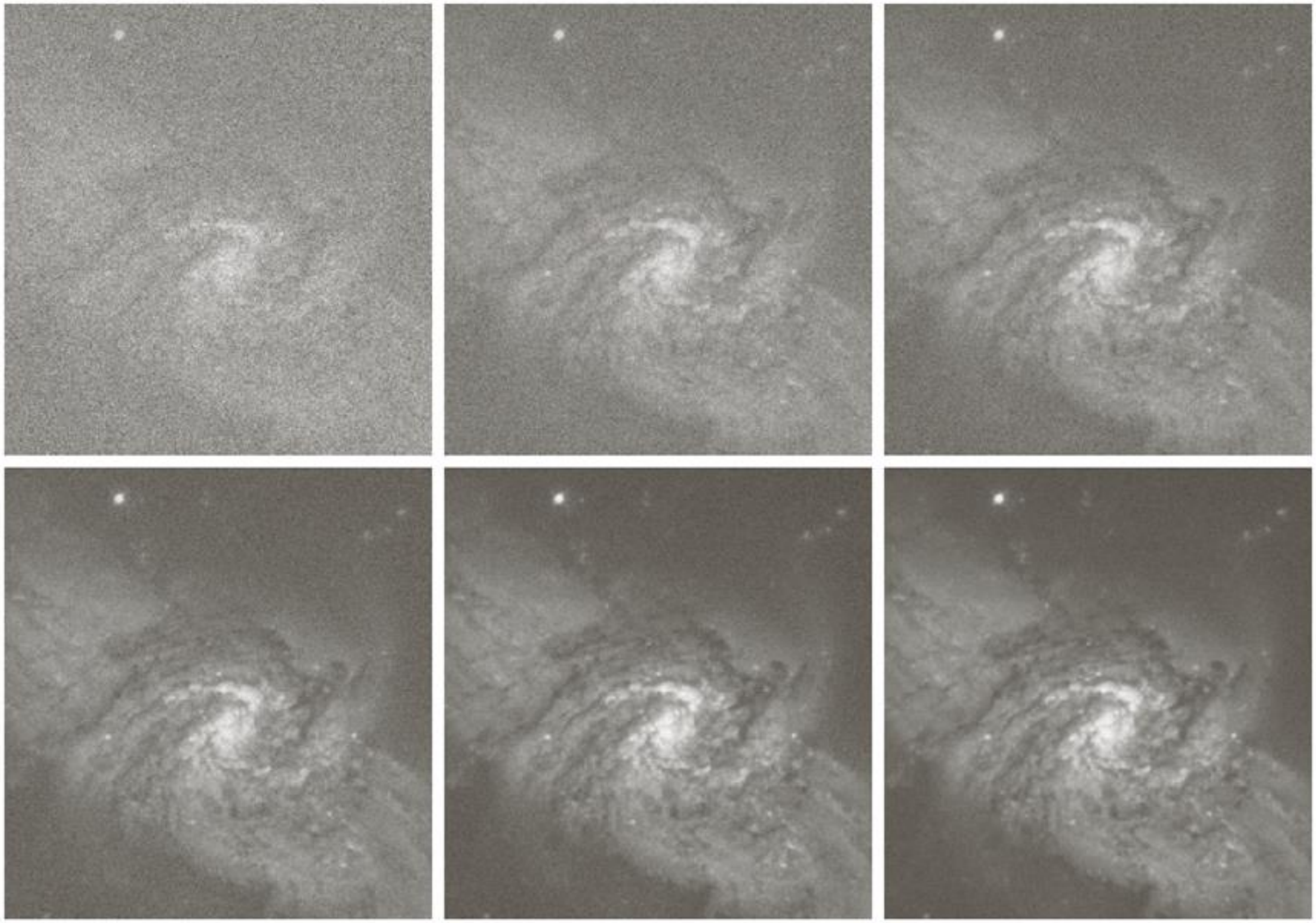
$$E\left(\bar{g}(x,y)\right) = f(x,y) \qquad , \qquad \sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

$E(\bar{g}(x,y))$ is the expected value of $\bar{g}$, and $\sigma^2_{\bar{g}(x,y)}$ and $\sigma^2_{\eta(x,y)}$ are the variances of $\bar{g}$ and $\eta$, respectively. The standard deviation (square root of the variance) at any point in the average image is:

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

As $K$ increases, the variability (as measured by the variance or the standard deviation) of the pixel values at each location $(x, y)$ decreases. Because $E\left(\bar{g}(x,y)\right) = f(x,y)$, this means that $\bar{g}(x,y)$ approaches $f(x, y)$ as the number of noisy images used in the averaging process increases.

An important application of image averaging is in the field of astronomy, where imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis. Figure 3(a) shows an 8-bit image in which corruption was simulated by adding to it Gaussian noise with zero mean and a standard deviation of 64 intensity levels. Figures 3(b)-(f) show the result of averaging 5, 10, 20, 50 and 100 images, respectively.

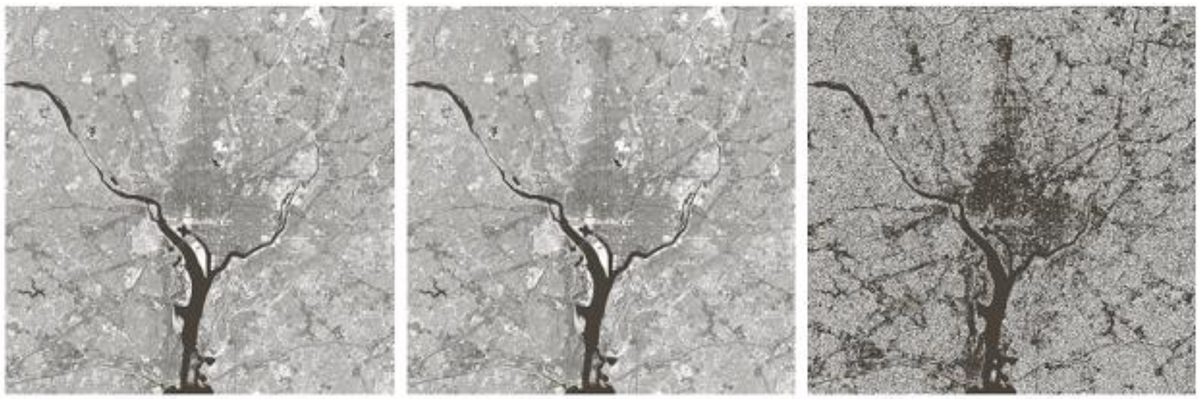


a b c
d e f

Fig. 3 Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise (left corner); Results of averaging 5, 10, 20, 50, 100 noisy images

## Course 2

A frequent application of image *subtraction* is in the enhancement of *differences* between images.



(a) (b) (c)

Fig. 4 (a) Infrared image of Washington DC area; (b) Image obtained from (a) by setting to zero the least significant bit of each pixel; (c) the difference between the two images

Figure 4(b) was obtained by setting to zero the least-significant bit of every pixel in Figure 4(a). The two images seem almost the same. Figure 4(c) is the difference between images (a) and (b). Black (0) values in Figure (c) indicate locations where there is no difference between images (a) and (b).

## Course 2

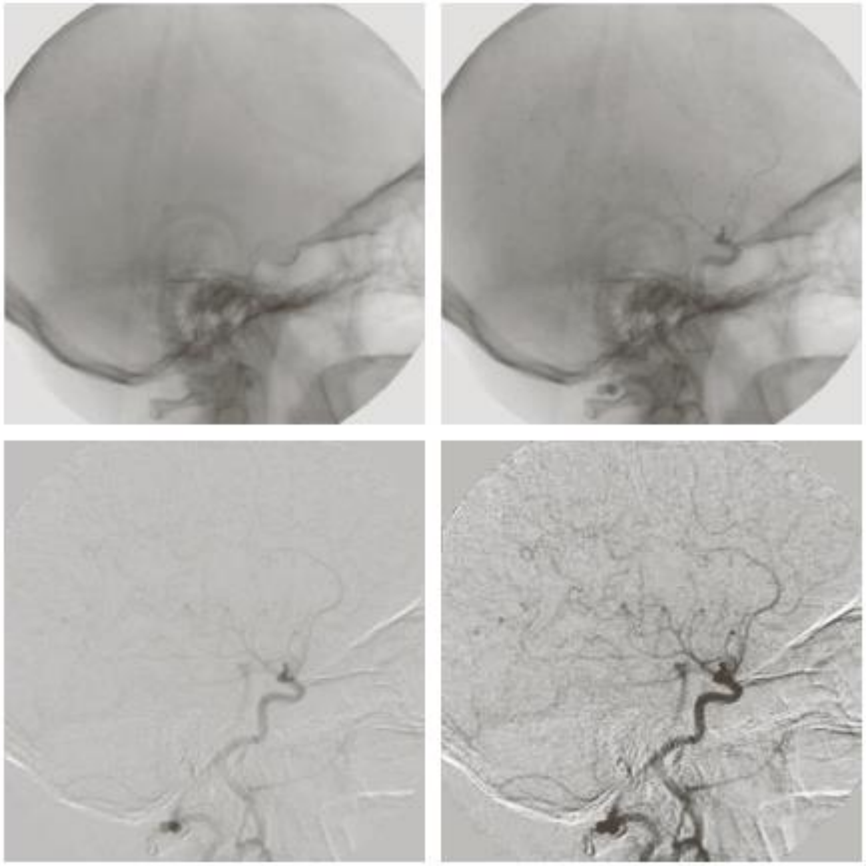
*Mask mode radiography*

$$g(x, y) = f(x, y) - h(x, y)$$

***h(x, y)***, the *mask*, is an *X*-ray image of a region of a patient's body, captured by an intensified TV camera (instead of traditional *X*-ray film) located opposite an *X*-ray source. The procedure consists of injecting an *X*-ray contrast medium into the patient's bloodstream, taking a series of images called *live images* (denoted ***f(x, y)***) of the same anatomical region as ***h(x, y)***, and subtracting the mask from the series of incoming live images after injection of the contrast medium.

In ***g(x, y)*** we can find the differences between ***h*** and ***f***, as enhanced detail.

Images being captured at TV rates, we obtain a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a b
c d
Fig. 5 – Angiography – subtraction example
(a) – mask image; (b) – live image ;
(c) – difference between (a) and (b);
(d) - image (c) enhanced

## Course 2

An important application of image *multiplication* (and *division*) is *shading correction*.

Suppose that an imaging sensor produces images in the form:

$$g(x,y) = f(x,y) \cdot h(x,y)$$

***f(x, y)*** – the "perfect image" , ***h(x, y)*** – the shading function

When the shading function is known:

$$f(x,y) = \frac{g(x,y)}{h(x,y)}$$

***h(x, y)*** is unknown but if we have access to the imaging system, we can obtain an approximation to the shading function by imaging a target of constant intensity. When the sensor is not available, often the shading pattern can be estimated from the image.

(a) (b) (c)

Fig. 6 Shading correction (a) – Shaded image of a tungsten filament, magnified 130 ×; (b) - shading pattern ; (c) corrected image

Another use of image *multiplication* is in *masking*, also called *region of interest (ROI*), operations. The process consists of multiplying a given image by a mask image that has 1's (white) in the ROI and 0's elsewhere. There can be more than one ROI in the mask image and the shape of the ROI can be arbitrary, but usually is a rectangular shape.

(a) (b) (c)

Fig. 7 (a) – digital dental X-ray image; (b) - ROI mask for teeth with fillings; (c) product of (a) and (b)

## Course 2

In practice, most images are displayed using *8* bits → the image values are in the range ***[0,255]***.

TIFF, JPEG images – conversion to this range is automatic. The conversion depends on the system used.

Difference of two images can produce image with values in the range ***[-255,255]***

Addition of two images – range ***[0,510]***

Many software packages simply set the negative values to 0 and set to 255 all values greater than 255.

A more appropriate procedure: compute

$$f_m = f - \min(f)$$

$$f_s = K \frac{f_m}{\max(f_m)} \in [0\ ,\ K] \quad (K = 255)$$