λ- calcul

$x \qquad y \qquad z$

$\lambda x.x \qquad \lambda x.y$

$(\lambda x.x) \sqcup (\lambda x.x)$

β-reducere

$$(\lambda x.t) \sqcup t_1 \longrightarrow_\beta t[\![x/t_1]\!]$$

subst. care evită captura

I

$$(\lambda x.x) \sqcup (\lambda y.z) \longrightarrow_\beta x[x/\lambda y.z]$$

$$\| $$

II

$$(\lambda y.z)$$

Codări Church

Bool

Nat

Recursive

cum reprezint s.d. „interesante" ca λ-termeni.

Bool

$\mathrm{TRUE} = \lambda x.\lambda y.x \rightarrow \lambda x.\lambda y.((\lambda z.z)\ x)$ — ac. comp.

$\mathrm{FALSE} = \lambda x.\lambda y.y$

→ orice functie cu doua arg. care intoarce primul dintre ele este o repr. a lui TRUE

$\mathrm{AND}\ \mathrm{TRUE}\ \mathrm{FALSE} \rightarrow_\beta - - \mathrm{FALSE}$

$\mathrm{AND} = \lambda b_1.\lambda b_2.\ b_1\ b_2\ b_1$

$A\ T\ F \rightarrow_\beta (\lambda b_2.\ T\ b_2\ T)\ F \rightarrow_\beta$

$\rightarrow_\beta T\ F\ T = (\lambda x.\lambda y.x)\ F\ T \rightarrow_\beta (\lambda y.F)\ T \rightarrow_\beta F.$

$$OR = \lambda b_1.\ \lambda b_2.\ b_1\ b_1\ b_2$$

$$ITE = \lambda b.\ \lambda u.\ \lambda v.\ b\ u\ v$$

$$PAIR = \lambda u.\ \lambda v.\ \lambda b.\ b\ u\ v$$

↳ repr. a unei perechi

$$FST = \lambda p.\ p\ TRUE$$

$$SND = \lambda p.\ p\ FALSE$$

$$\forall t_1 t_2 \qquad FST\ (PAIR\ t_1\ t_2) \equiv_\beta t_1$$

Nr. nat.

$0 = \lambda s.\lambda z.\, z$

$1 = \lambda s.\lambda z.\, s\ z$

$2 = \lambda s.\lambda z.\, s(s\ z)$

$3 = \lambda s.\lambda z.\, s(s\ (s\ z))$

$\vdots$

$SUCC = \lambda n.\lambda s.\lambda z.\, s\ (n\ s\ z)$

$\rightarrow_\beta s(s(s \ldots (s\ z)))$

V_1

V_2

$= \lambda n.\lambda s.\lambda z.\, n\ s\ (s\ z)$

$\underbrace{s(s \ldots s}_{n \text{ ori}}(s\ z))$

$PLUS = \lambda n_1.\lambda n_2.\lambda s.\lambda z.\, n_1\ s\ (n_2\ s\ z)$

aplic n_1+n_2 ori s asupra lui z

$TIMES = \lambda n_1.\lambda n_2.\lambda s.\lambda z.\, n_1\ (\lambda m.\, PLUS\ n_2\ m)\ z$

Alt 1

aplic de $n_1 \times n_2$ ori s asupra lui z

Alt 2

$= \lambda n_1.\lambda n_2.\lambda s.\lambda z.\, n_1\ (n_2\ s)\ z$

aplică de n_2 ori s asupra arg.

EXP = exercițiu

$ISZERO = \lambda n.\, n\ (\lambda a.\, FALSE)\ TRUE$

$ISZERO\ 0 \rightarrow_\beta^* TRUE$

$ISZERO\ 3 \rightarrow_\beta^* FALSE$

$$\text{PRED} = \lambda n.\,\text{FST}\,(n\ \text{AUX}\ (\text{PAIR}\ 0\ 0))$$

$$\text{AUX} = \lambda p.\,\text{PAIR}\ (\text{SND}\ p)\ (\text{SUCC}\ (\text{SND}\ p))$$

$(0, 0)$

$\downarrow \; = \text{AUX}$

$(0, 1)$

$\downarrow \; \text{AUX}$

$(1, 2)$

$\downarrow \; \text{AUX}$

$(2, 3)$

3ork

OBS. 1

$$0 = \lambda s.\lambda z.z$$

$$FALSE = \lambda u.\lambda v.v$$

! 0 și $FALSE$ au aceeași codare Church

OBS. 2

$$AND\ 2\ 5 = \_\_\_\_$$

OBS. 3: codările se referă la comportamentul termenului în [illegible]

Y-combinator

Combinator = λ-termen fără var. libere

$$(\lambda x.\, x\ x)\ (\lambda x.\, x\ x)$$

Combinator de punct fix

$$\mathrm{Fix}\ t \equiv_\beta t\ (\mathrm{Fix}\ t)$$

adică comp

$$t\ (t\ (t\ \dots))$$

$$Y = \lambda f.(\lambda x. f(\lambda y. x\,x\,y))\ (\lambda x. f(\lambda y. x\,x\,y))$$

$$Y\ t \xrightarrow{\beta} (\lambda x. t(\lambda y. x\,x\,y))\ (\lambda x. t(\lambda y. x\,x\,y))$$

$$\xrightarrow{\beta} t\ (\lambda y. (\lambda x. t(\lambda y. x\,x\,y))\ (\lambda x. t(\lambda y. x\,x\,y))\ y)$$

$$\equiv_\beta t\ (\lambda y. (Y\ t)\ y)$$

$$\equiv_\beta t\ (Y\ t).$$

$$FACT = Y \sqcup FACT'$$

$$\overset{\text{conception}}{\equiv} (FACT' \, (FACT' \, (FACT' \, \_\_\_ \,)))$$

$$FACT\ 3 \overset{\text{Conception}}{\equiv} (FACT' \sqcup (FACT' \, (FACT' \, \_\_\_ \,))) \sqcup 3$$

#1

#2

"THEN"

$$FACT' = \lambda f.\ \lambda n.\ ITE \sqcup (ISZERO\ n) \sqcup 1 \sqcup$$
$$(TIMES\ n\ (f\ (PRED\ n)))$$

$$FACT = Y \sqcup FACT'$$

$$\mathrm{ISEVEN} = Y\ \mathrm{ISEVEN}'$$

$$\mathrm{ISEVEN}' = \lambda f\, \lambda n.\ \mathrm{ITE}\ (\mathrm{ISZERO}\ n)\ \mathrm{TRUE}$$

$$\Big(\mathrm{ITE}\ (\mathrm{ISZERO}\ (\mathrm{PRED}\ n))\ \mathrm{FALSE}$$

$$f\ (\mathrm{PRED}\ (\mathrm{PRED}\ n)))\Big)$$

LISTE

$$\{1,2,5,7\} = (1, \{2,5,7\})$$
$$= (1, (2, \{5,7\}))$$

$$\{7\} = (7, \text{FALSE})$$

$$\{\} = \text{FALSE}$$

$$\text{ISEMPTY} = \lambda e.\ \text{ISZERO}\ e$$

$$0 \rightarrow \text{ISZERO}\ (\text{PAIR}\ \_\ \_) \neq \text{TRUE}$$

$$\text{ISZERO}\ (\lambda u.\lambda v.\lambda b.\ b\ u\ v)$$

$$(\lambda n.n\ (\lambda a.\text{FALSE})\ \text{TRUE})\ (\lambda u.\lambda v.\lambda b.\ b\ u\ v)$$

$$\downarrow_\beta$$

$$(\lambda u.\lambda v.\lambda b.\ b\ u\ v)\ \underbrace{(\lambda a.\text{FALSE})}_{\#1}\ \underbrace{\text{TRUE}}_{\#2}$$

$$\downarrow_\beta^\alpha$$

$$(\lambda b.b\ (\lambda a.\text{FALSE})\ \text{TRUE})$$

$$\text{TRUE} = \lambda u.\lambda v.\ u$$

$$\Theta_u\, t \xrightarrow[\beta]{\qquad} t\,(\Theta t)$$