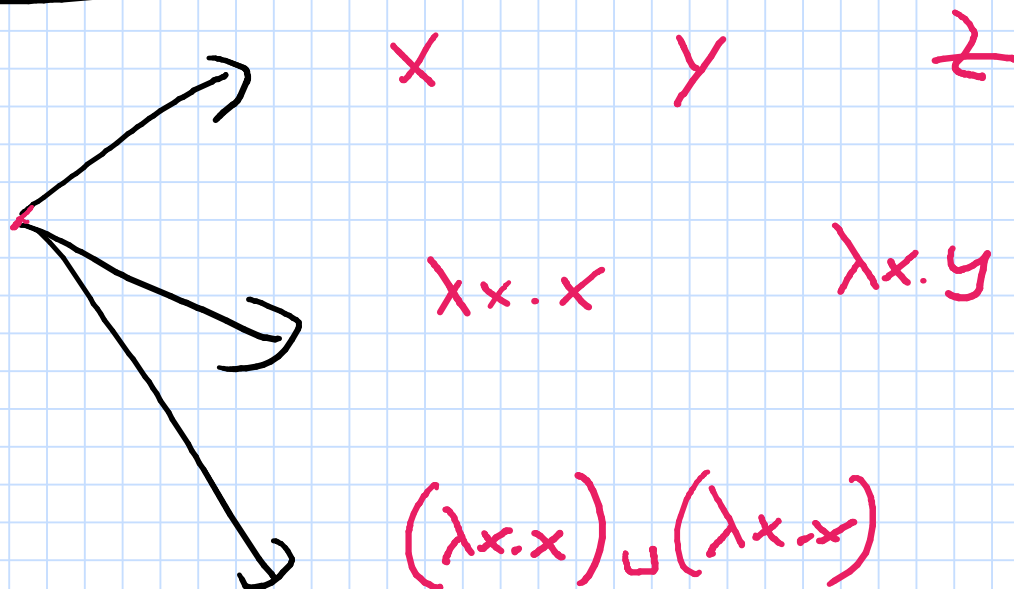


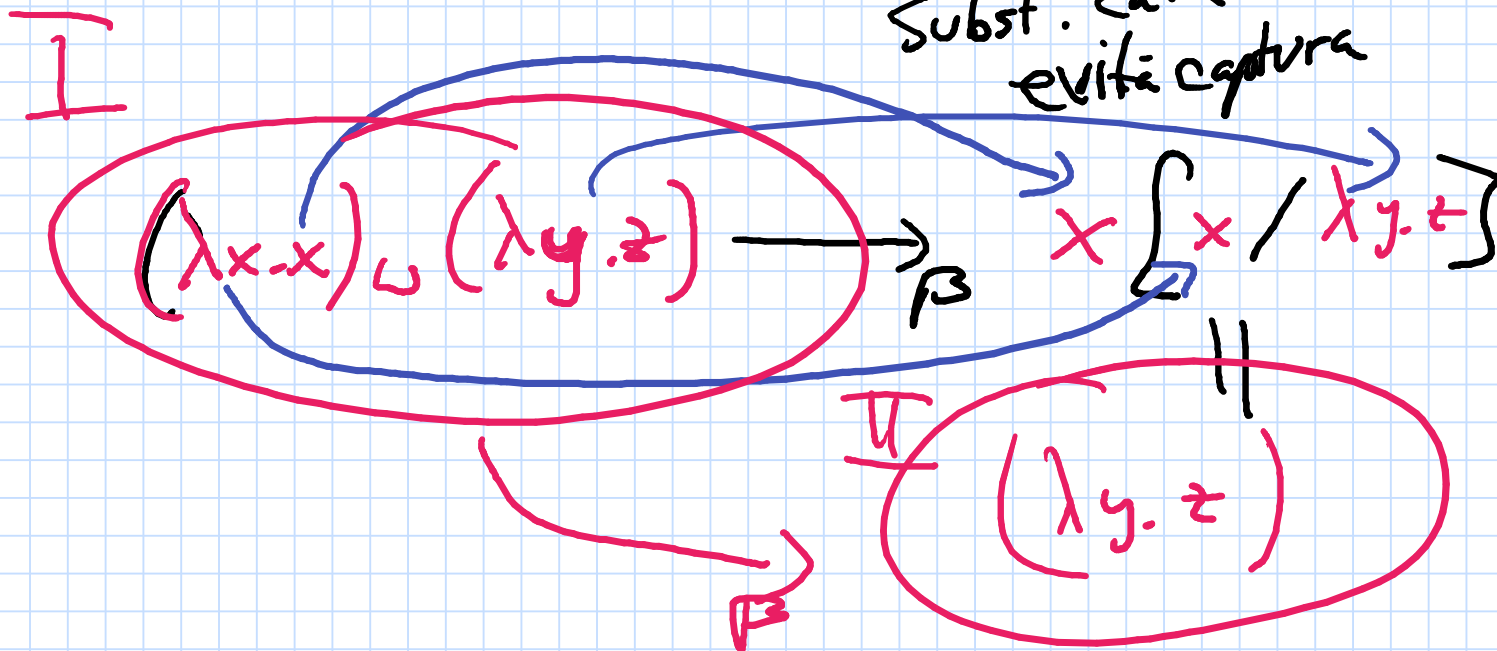
λ -calcul



β -reducere

$$(\lambda x.t) \circ t_1 \xrightarrow{\beta} t \llbracket x / t_1 \rrbracket$$

Subst. can
evita captura



Codāri Church

Bool

Nat

Recursive

→ cum reprezent
s.d. „intersecante”
Ca λ -termen.

Bool

TRUE =
FALSE =

$\lambda x. \lambda y. x$
 $\lambda x. \lambda y. y$

$\lambda x. \lambda y. ((\lambda z. z) x)$

acc.
comp.

orice functie
cu doua arg.
care returneaza
prima dintre
ele este o repr
a lui true

AND \sqsubseteq TRUE \sqsubseteq FALSE \rightarrow β
----- FALSE

AND = $\lambda b_1. \lambda b_2. b_1 \sqcup b_2 \sqcup b_1$

A \sqcup (T \sqsubseteq F) \rightarrow_{β} ($\lambda b_2. T \sqcup b_2 \sqcup T$) \sqsubseteq F \rightarrow_{β}

\rightarrow_{β} T F T =
($\lambda x. \lambda y. x$) \sqsubseteq (F) \sqsubseteq (T) \rightarrow_{β}
($\lambda y. F$) \sqsubseteq T \rightarrow_{β}
(F)

$$\text{OR} = \lambda b_1. \lambda b_2. b_1 \cup b_2$$

$$\text{ITE} = \lambda b. \lambda v. \lambda u. b \cup v \cup u$$

$$\text{PAIR} = \lambda v. \lambda u. \underbrace{\lambda b. b \cup v}_{\text{repr. a une 'peredw'}}$$

$$\text{FST} = \lambda p. p \cup \text{TRUE}$$

$$\text{SND} = \lambda p. p \cup \text{FALSE}$$

$$\forall t_1 t_2 \quad \text{FST} (\text{PAIR } t_1 t_2) \equiv_{\beta} t_1$$

Nr. nat.

$$0 = \lambda s. \lambda z. z$$

$$1 = \lambda s. \lambda z. s z$$

$$2 = \lambda s. \lambda z. s(s z)$$

$$3 = \lambda s. \lambda z. s(s(s z))$$

...

$$\text{SUCC} = \lambda n. \lambda s. \lambda z. s(\underbrace{n s z}_{\rightarrow s(s(s \dots (s z)))})$$

$$\text{PRED} = \lambda n. \lambda s. \lambda z. \underbrace{n s}_{\text{ori}}(s z)$$

$$\text{PLUS} = \lambda n_1. \lambda n_2. \lambda s. \lambda z. \underbrace{n_1 s (n_2 s z)}_{\substack{\text{aplic } n_1 + n_2 \text{ ori} \\ s \text{ asupra } n_2 z}}$$

$$\text{TIMES} = \lambda n_1. \lambda n_2. \lambda s. \lambda z. \underbrace{n_1 (\lambda m. \text{PLUS } n_2 m) z}_{\substack{\text{aplic de } n_1 \times n_2 \\ \text{ori } s \text{ asupra } z}}$$

\nearrow $\text{AR} + 1$
 \searrow $\text{AR} + 2$

$$= \lambda n_1. \lambda n_2. \lambda s. \lambda z. n_1 (\underline{n_2 s}) z$$

\hookrightarrow aplica de n_2 ori s asupra arg.

EXP = exerciții

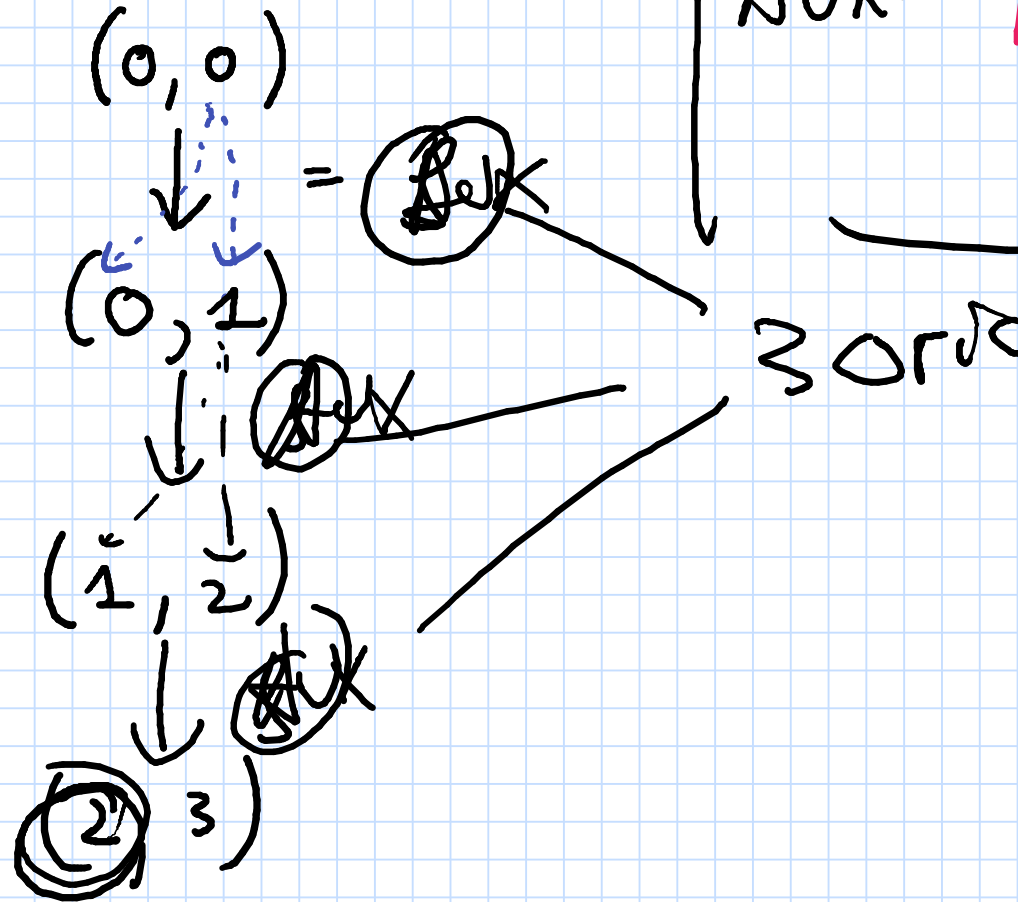
$$\text{ISZERO} = \lambda n. n _ (\lambda a. \text{FALSE}) _ \underline{\text{TRUE}}$$

$$\text{ISZERO } 0 \longrightarrow \text{TRUE}$$

$$\text{ISZERO } 3 \longrightarrow \text{FALSE}$$

$PRED = \lambda n. \text{fst}(\text{PAIR } 0\ 0)$

$AUX = \lambda p. \text{PAIR}_L$
 $(\text{SND } p)_L$
 $(\text{succ } (\text{SND } p))$



OBS. 1

$$0 = \lambda s. \lambda z. z$$

$$\text{FALSE} = \lambda v. \lambda u. v$$

!

$$0 \neq \text{FALSE}$$

are a ceeaşi codare
Church

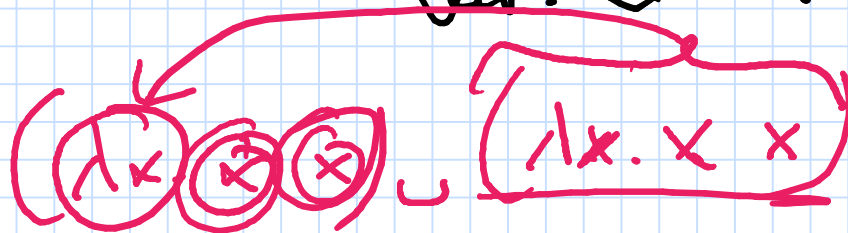
OBS. 2

$$\text{AND } 2 \ 5 = \underline{\quad}$$

OBS. 3: codările se referă la
comportamentul
termenului
în calculare

Y-combinator

Combinator = λ -termen for
var. Gebere



Combinator de punct-fix

$$\text{Fix}_\omega t \equiv t_\omega (\text{Fix } t)$$

$\uparrow \beta$
adap
comp

$$t_\omega (t_\omega (t_\omega \uparrow \dots))$$

$$Y \equiv \lambda f. (\lambda x. f(\lambda y. x \times y)) \cup (\lambda x. f(\lambda y. x \times y))$$

$$Y \cup t \xrightarrow{\beta} (\lambda x. t(\lambda y. x \times y)) \cup (\lambda x. t(\lambda y. x \times y))$$

$$\xrightarrow{\beta} t(\lambda y. (\lambda x. t(\lambda y. x \times y)) (\lambda x. t(\lambda y. x \times y)) y)$$

$$\downarrow$$

$$Y \cup t \xrightarrow{\beta}$$

$$\equiv t(\lambda y. (Y \cup t) \cup y)$$

$$\equiv t(Y \cup t).$$

$$FACT = Y \cup FACT'$$

$$\stackrel{\text{conceptual}}{=} (FACT' (FACT' (FACT' _ _)))$$

$$FACT \supset \stackrel{\text{conceptual}}{=} (FACT' (FACT' (FACT' _ _))) \cup _ _ _$$

#1

#2

"T+REN"

$$FACT' = \lambda f. \lambda n. ITE_{\perp}(isZERO \textcolor{red}{n}) \cup \textcircled{1}$$

$$(TIMES \textcolor{red}{n} (f (PRED \textcolor{red}{n})))$$

$$FACT = Y \cup FACT'$$

ISEVEN = λ ISEVEN'

ISEVEN' = λn . ITE (ISZERO n) TRUE
 (~~ITE (ISZERO (PREDE n)) FALSE~~
 $\&$ (PREP (PREP n)))

LISTE

$$\{1, 2, 5, 7\} = (1, \{2, 5, 7\}) \\ = (1, (2, \{5, 7\}))$$

$$\{7\} = (7, \text{FALSE})$$

$$\{\} = \text{FALSE} \quad \boxed{\text{isEmpty} = \lambda e. \text{isZERO } e}$$

$$0 \rightarrow \text{isZERO} (\text{PAIR } _ _) \neq \text{TRUE}$$

$$\text{isZERO} (\lambda v. \lambda u. \lambda b. b \vee u)$$

$$(\lambda n. n) (\lambda a. \text{FALSE}) \text{ TRUE} (\lambda v. \lambda u. \lambda b. b \vee u)$$

$$(\lambda v. \lambda u. \lambda b. b \vee u)$$

$$(\lambda a. \text{FALSE}) \text{ TRUE} \neq 1 \quad \neq 2$$

$$(\lambda b. b (\lambda a. \text{FALSE}) \text{ TRUE})$$

$$\text{TRUE} = \lambda v. \lambda u. u$$

$$\theta_t \xrightarrow{\mathbb{R}} t(\theta_t)$$