3. TRANSVERSE OSCILLATIONS OF CORONAL LOOPS

– Part 3: Damping by resonant absorption –

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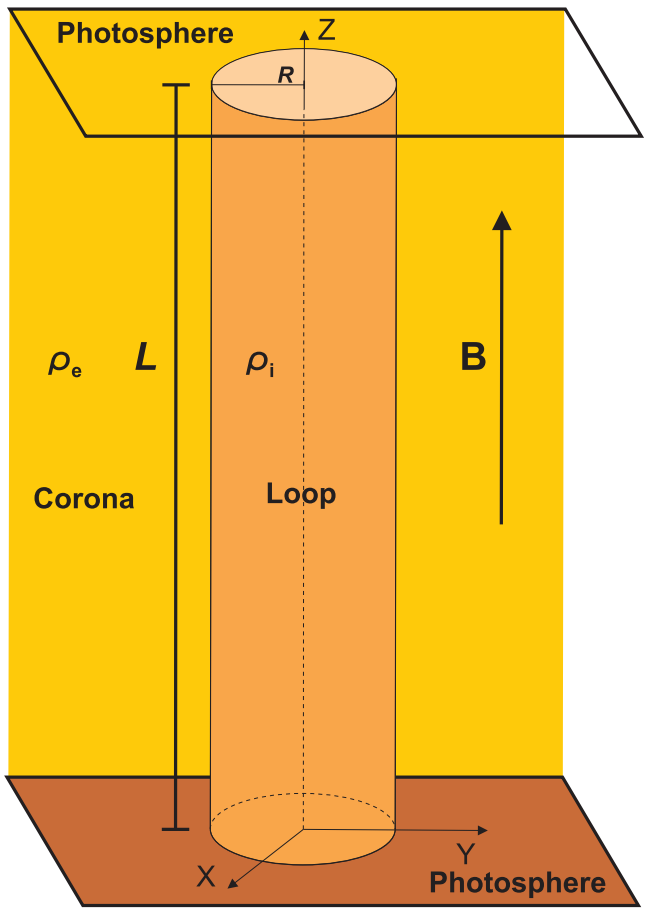
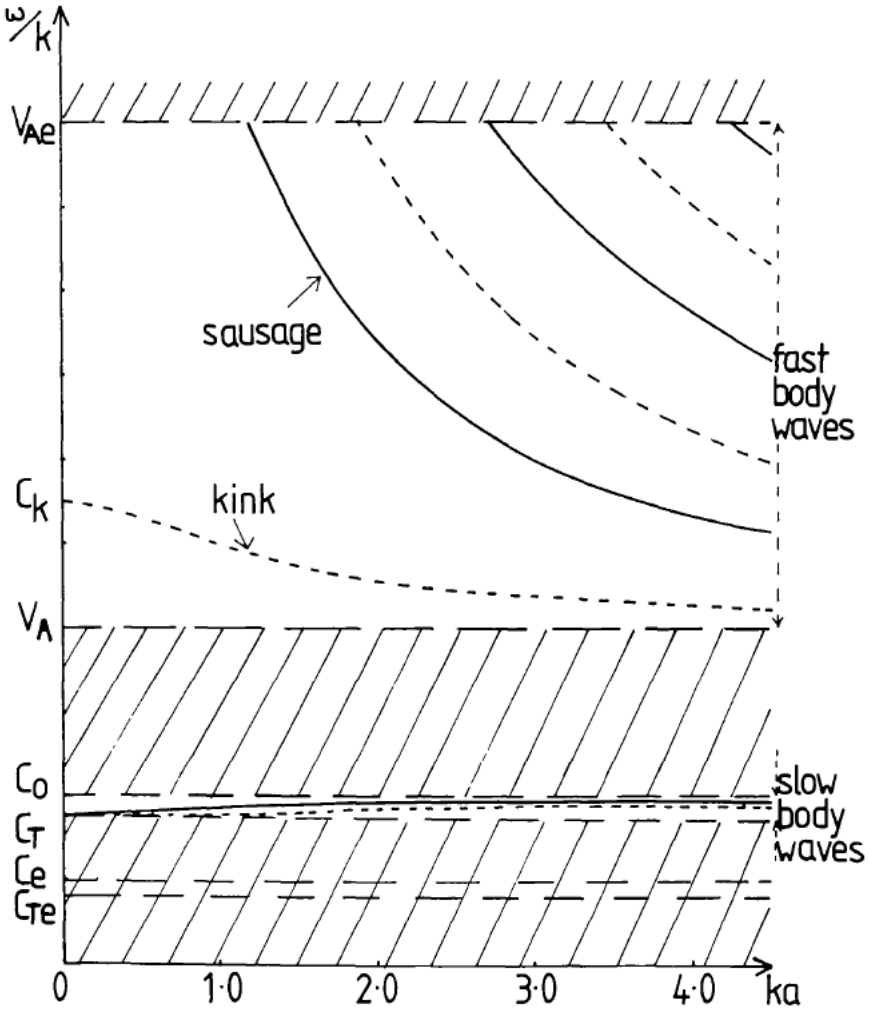
Solar Magnetohydrodynamics: Applications
2020–2021

Homogeneous magnetic cylinder

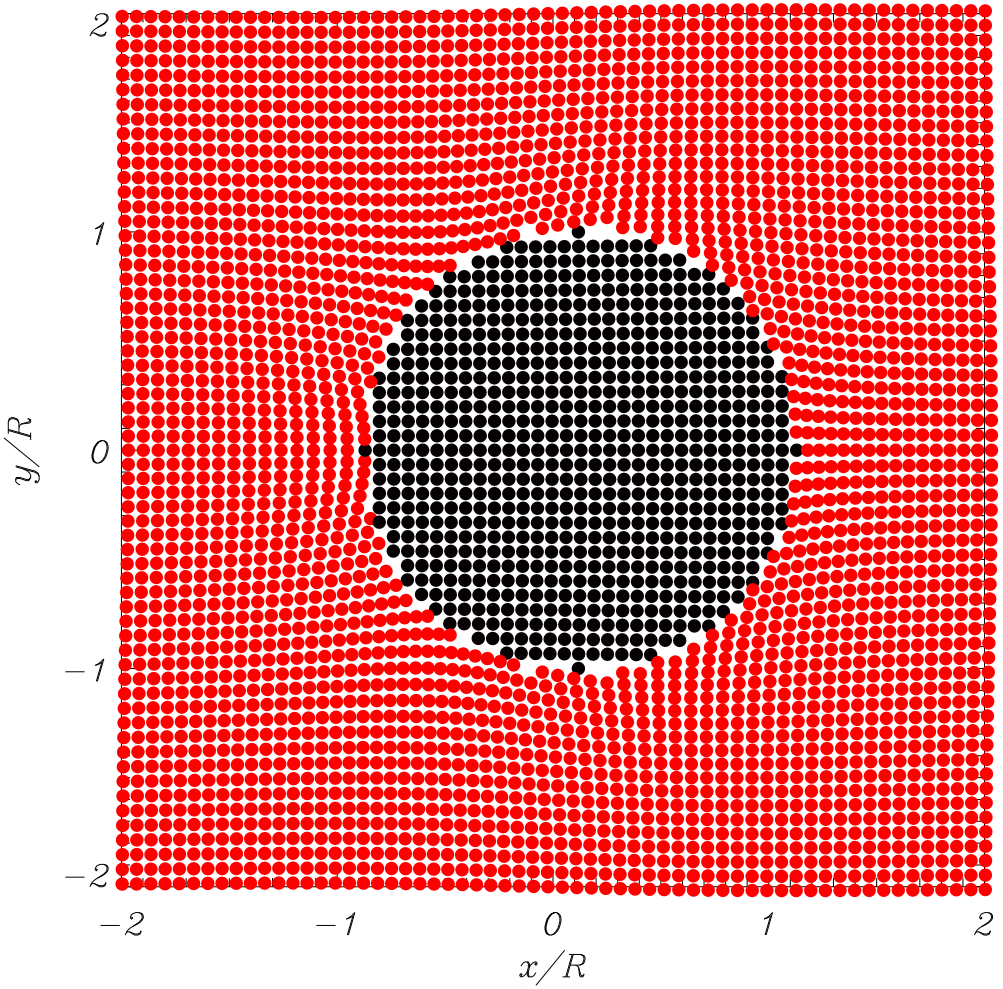




Edwin & Roberts (1983)

The kink mode of a homogeneous tube

- Global transverse motion of the flux tube
- No damping. . . but observations of loop oscillations show damping!

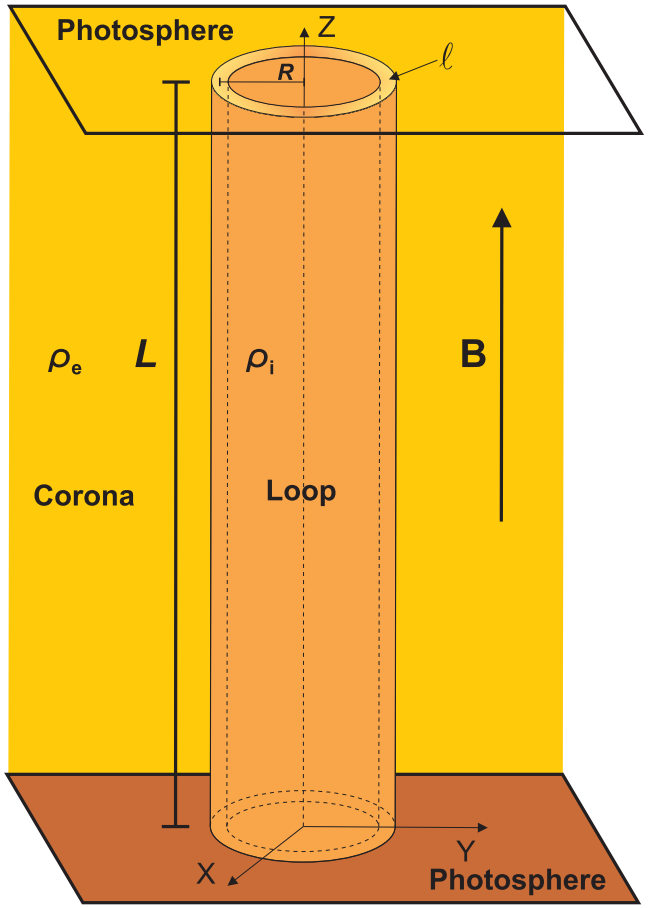


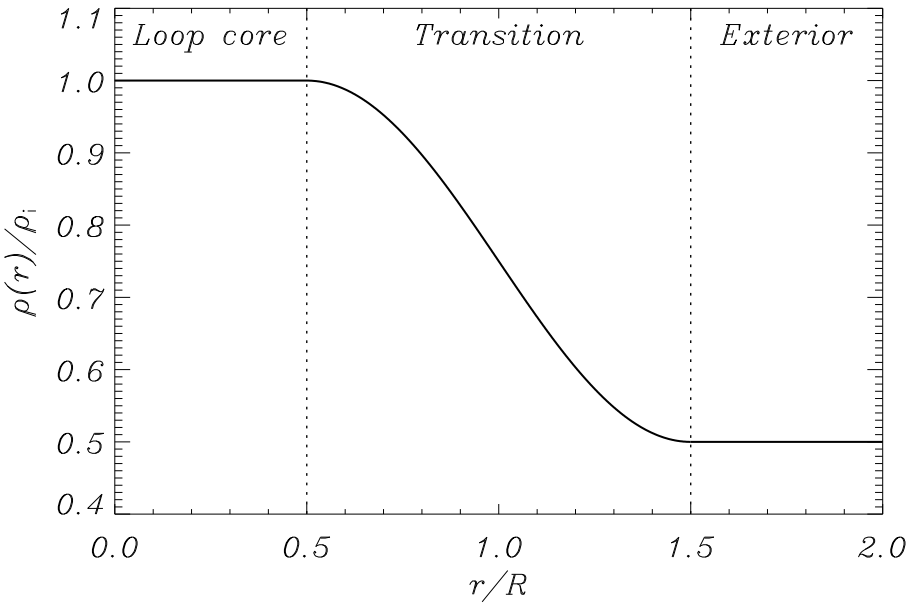
- Thin tube ($L/R \gg 1$) approximation:

$$P = \frac{2L}{v_{\mathrm{A,i}}}\sqrt{\frac{(\rho_{\mathrm{i}} + \rho_{\mathrm{e}})}{2\rho_{\mathrm{i}}}}$$

Transversely non-uniform cylinder

- We consider an improved model: a nonuniform layer is added in the transverse direction

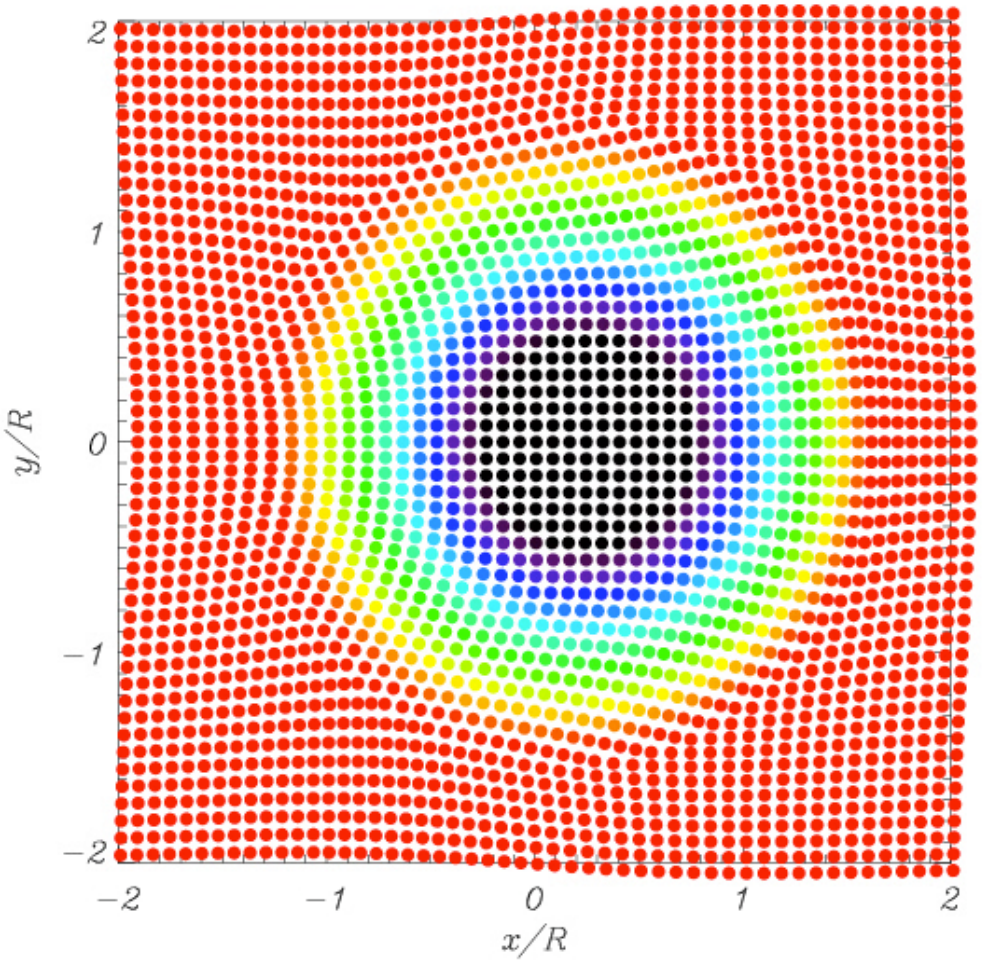




- $l = 0 \rightarrow$ Abrupt density jump
- $l = 2R \rightarrow$ Fully nonuniform tube

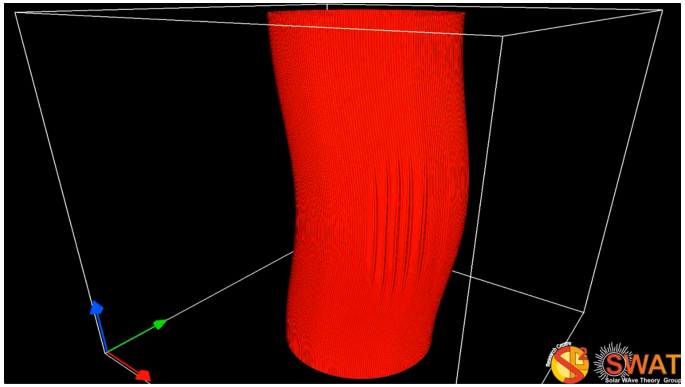
The kink mode of a non-uniform tube: new physics

- Phase 1: Damping of the global transverse motion
- Phase 2: Motions become rotational in the nonuniform layer
- Phase 3: Generation of small scales, dissipation, and plasma heating

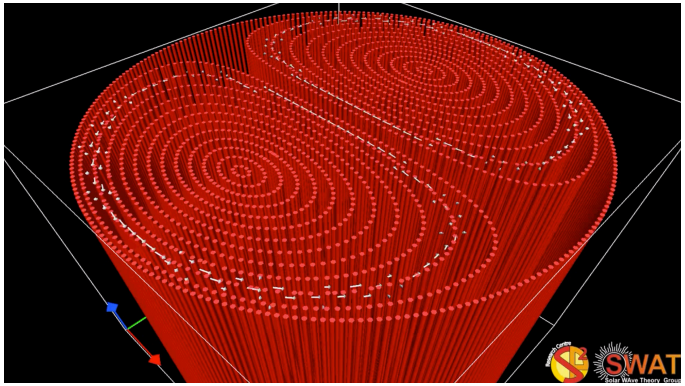


Resonant coupling: from Phase 1 to Phase 2

The transverse kink mode



couples with
the rotational Alfvén mode (with $m = 1$)

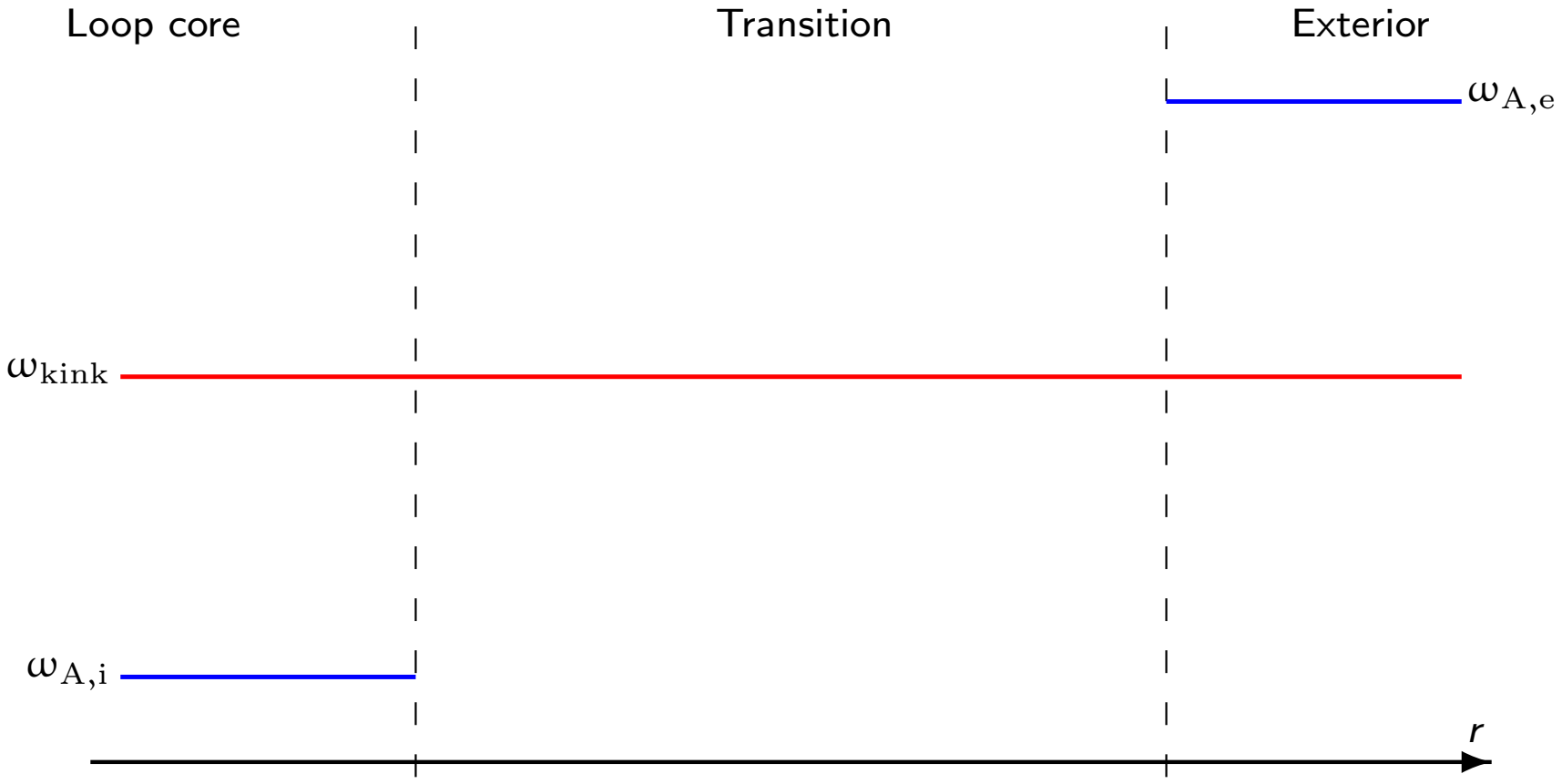


- Because of this coupling, there is a transfer of energy from the kink mode to the Alfvén modes
- The net result is the damping the the kink mode

The concept of the Alfvén resonance

Assumptions

1. The kink mode still exists as a global mode (quasi-mode) with a frequency in between in the internal and external Alfvén frequencies
2. In the nonuniform layer there is a continuum of Alfvén frequencies and the kink mode is resonant.



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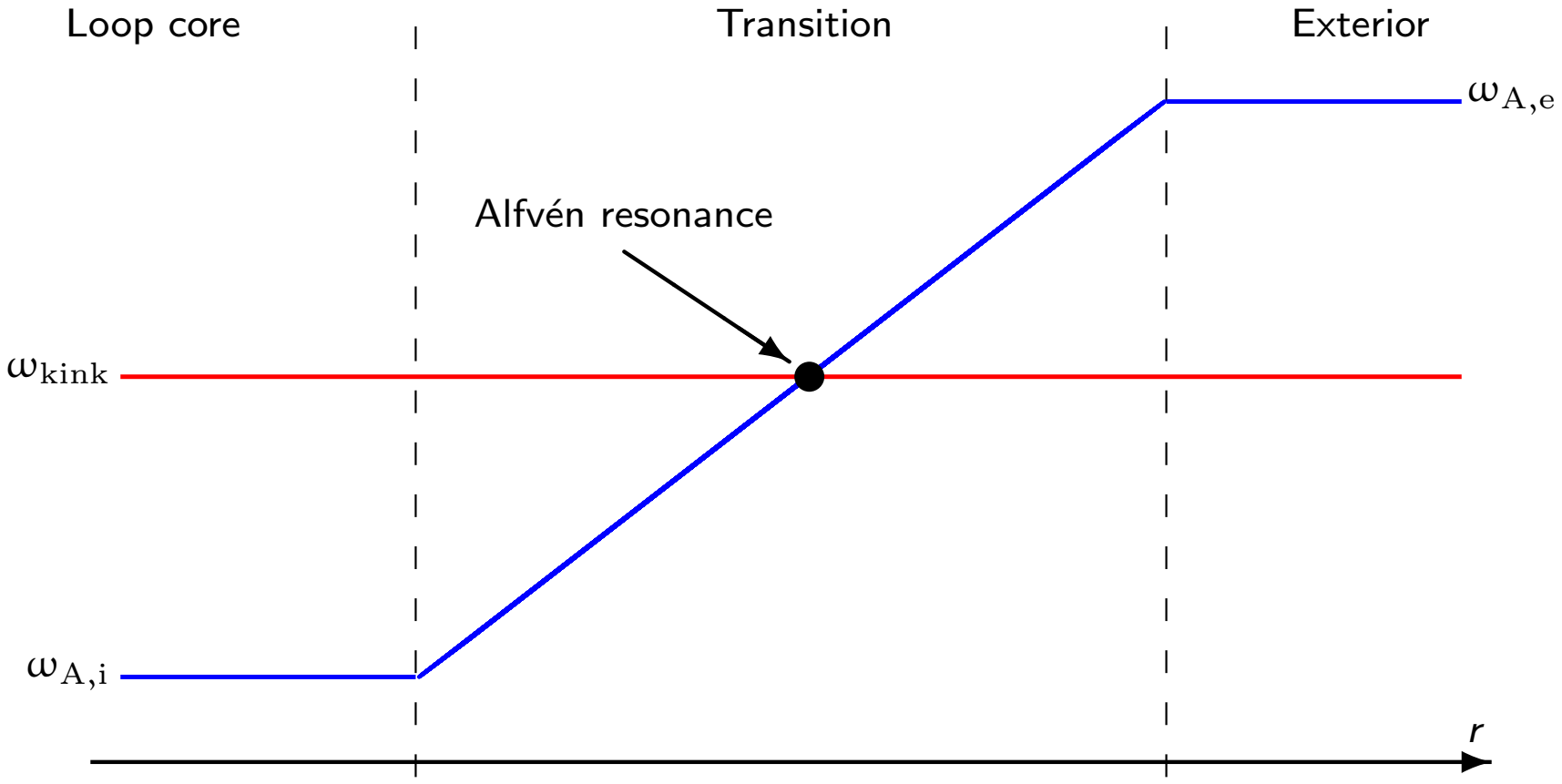


Mathematical description of resonant absorption

- Governing equation for P_1 when $\rho = \rho(r)$

$$\frac{\partial^2 P_1}{\partial r^2} + \left[\frac{1}{r} - \frac{\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho(r)\left(\omega^2 - \omega_{\mathrm{A}}^2(r)\right)\right)}{\rho(r)\left(\omega^2 - \omega_{\mathrm{A}}^2(r)\right)}\right]\frac{\partial P_1}{\partial r} + \left(\frac{\rho(r)\left(\omega^2 - \omega_{\mathrm{A}}^2(r)\right)}{B^2/\mu} - \frac{m^2}{r^2}\right)P_1 = 0$$

- If density is uniform ($\rho =$ constant) we recover the Bessel Equation

$$\frac{\partial^2 P_1}{\partial r^2} + \frac{1}{r}\frac{\partial P_1}{\partial r} + \left(\frac{\rho\left(\omega^2 - \omega_{\mathrm{A}}^2\right)}{B^2/\mu} - \frac{m^2}{r^2}\right)P_1 = 0, \qquad k_\perp^2 = \frac{\rho\left(\omega^2 - \omega_{\mathrm{A}}^2\right)}{B^2/\mu}$$

- Solution in the internal plasma ($r \leq R - \frac{l}{2}$), uniform density ρ_{i}

$$P_1 = A_{\mathrm{i}} J_m\left(k_{\perp,\mathrm{i}} r\right)$$

- Solution in the external plasma ($r \geq R + \frac{l}{2}$), uniform density ρ_{e}

$$P_1 = A_{\mathrm{e}} K_m\left(\tilde{k}_{\perp,\mathrm{e}} r\right)$$

Solution in the non-uniform transitional layer

- The full equation for non-uniform density has to be solved:

$$\frac{\partial^2 P'}{\partial r^2}+\left[\frac{1}{r}-\frac{\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)\right)}{\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)}\right]\frac{\partial P'}{\partial r}+\left(\frac{\rho(r)\left(\omega^2-\omega_{\mathrm{A}}^2(r)\right)}{B^2/\mu}-\frac{m^2}{r^2}\right)P'=0$$

- The solution method has to take into account the presence of a resonance at $r = r_{\mathrm{A}}$ where $\omega^2 = \omega_{\mathrm{A}}^2(r_{\mathrm{A}})$
- The solution can be computed with the **Frobenius Method**: series expansion around the resonance position, $\omega = \omega_{\mathrm{A}}(r_{\mathrm{A}})$

$$P_1(r) = (r - r_{\mathrm{A}})^2 \sum_{k=0}^{\infty} a_k \left(r - r_{\mathrm{A}}\right)^k$$

$$P_2(r) = \sum_{k=0}^{\infty} s_k \left(r - r_{\mathrm{A}}\right)^k + \frac{m^2}{2r_{\mathrm{A}}^2} P_1'(r) \ln\left(r - r_{\mathrm{A}}\right)$$

- General solution: $P_1(r) = A_0 P_1(r) + S_0 P_2(r)$

Solution in the non-uniform transitional layer

$$
\begin{aligned}
a_0 &= 1,\\
a_1 &= -\frac{2\rho_1 - 2\rho_2 r_{\mathrm{A}}}{3\rho_1 r_{\mathrm{A}}} a_0,\\
a_2 &= -\frac{9\rho_1 r_{\mathrm{A}} a_1 + \left(2\rho_1 - 2\rho_2 r_{\mathrm{A}} - 4\rho_3 r_{\mathrm{A}}^2 - m^2\rho_1\right) a_0}{8\rho_1 r_{\mathrm{A}}^2},\\
a_3 &= -\frac{1}{15\rho_1 r_{\mathrm{A}}^2}\left[\left(4\rho_2 r_{\mathrm{A}}^2 + 20\rho_1 r_{\mathrm{A}}\right) a_2 + \left(-3\rho_3 r_{\mathrm{A}}^2 + 3\rho_2 r_{\mathrm{A}} + 6\rho_1 - m^2\rho_1\right) a_1\right.\\
&\quad \left. + \left(-6\rho_4 r_{\mathrm{A}}^2 - 6\rho_3 r_{\mathrm{A}} + \frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_1^2 - m^2\rho_2\right) a_0\right],\\
a_4 &= -\frac{1}{24\rho_1 r_{\mathrm{A}}^2}\left[\sum_{j=0}^{3}(j+2)(2j-4) r_{\mathrm{A}}^2 \rho_{5-j} a_j + \sum_{j=0}^{3}(j+2)(4j-5) r_{\mathrm{A}} \rho_{4-j} a_j\right.\\
&\quad \left. + \sum_{j=0}^{2}\left((j+2)(2j-1) - m^2\right)\rho_{3-j} a_j + \sum_{j=0}^{1}\sum_{l=0}^{1-j}\frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_{l+1}\rho_{2-j-l} a_j + 2\frac{\omega^2 r_{\mathrm{A}}}{B^2/\mu}\rho_1^2 a_0\right],\\
a_k &= -\frac{1}{k(k+2)\rho_1 r_{\mathrm{A}}^2}\left[\sum_{j=0}^{k-1}(j+2)(2j-k) r_{\mathrm{A}}^2 \rho_{k-j+1} a_j + \sum_{j=0}^{k-1}(j+2)(4j-2k+3) r_{\mathrm{A}} \rho_{k-j} a_j\right.\\
&\quad + \sum_{j=0}^{k-2}\left((j+2)(2j-k+3) - m^2\right)\rho_{k-j-1} a_j + \sum_{j=0}^{k-3}\sum_{l=0}^{k-j-3}\frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-2} a_j\\
&\quad \left. + \sum_{j=0}^{k-4}\sum_{l=0}^{k-j-4} 2\frac{\omega^2 r_{\mathrm{A}}}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-3} a_j + \sum_{j=0}^{k-5}\sum_{l=0}^{k-j-5}\frac{\omega^2}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-4} a_j\right], \quad \text{for} \quad k \geq 5,
\end{aligned}
$$

$$
\begin{aligned}
s_0 &= 1,\\
s_1 &= 0,\\
s_2 &= 0,\\
s_3 &= \frac{1}{3\rho_1 r_{\mathrm{A}}^2}\left[\left(m^2\rho_2 - \frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_1^2\right) s_0 - \mathcal{C}\left(4 r_{\mathrm{A}}^2 \rho_1 a_1 + (r_{\mathrm{A}}^2\rho_2 + 5 r_{\mathrm{A}}\rho_1) a_0\right)\right],\\
s_4 &= -\frac{1}{8\rho_1 r_{\mathrm{A}}^2}\left[9\rho_1 r_{\mathrm{A}} s_3 + \left(2\frac{\omega^2 r_{\mathrm{A}}}{B^2/\mu}\rho_1^2 + 2\frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_1\rho_2 - m^2\rho_3\right) s_0\right.\\
&\quad \left. + \mathcal{C}\left(6 r_{\mathrm{A}}^2 \rho_1 a_2 + 3 r_{\mathrm{A}}^2 \rho_2 a_1 + 9 r_{\mathrm{A}}\rho_1 a_1 + 3(r_{\mathrm{A}}\rho_2 + \rho_1) a_0\right)\right],\\
s_k &= -\frac{1}{k(k-2)\rho_1 r_{\mathrm{A}}^2}\left\{\sum_{j=0}^{k-1} j(2j-k-2) r_{\mathrm{A}}^2 \rho_{k-j+1} s_j + \sum_{j=0}^{k-1} j(4j-2k-1) r_{\mathrm{A}} \rho_{k-j} s_j\right.\\
&\quad + \sum_{j=0}^{k-2}\left[\left(j(2j-k+1) - m^2\right)\rho_{k-j-1} s_j + \mathcal{C}(3j-k+4) r_{\mathrm{A}}^2 \rho_{k-j-1} a_j\right]\\
&\quad + \sum_{j=0}^{k-3}\left[\sum_{l=0}^{k-j-3}\frac{\omega^2 r_{\mathrm{A}}^2}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-2} s_j + \mathcal{C}(6j-2k+11) r_{\mathrm{A}} \rho_{k-j-2} a_j\right]\\
&\quad + \sum_{j=0}^{k-4}\left[\sum_{l=0}^{k-j-4} 2\frac{\omega^2 r_{\mathrm{A}}}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-3} s_j + \mathcal{C}(3j-k+7)\rho_{k-j-3} a_j\right]\\
&\quad \left. + \sum_{j=0}^{k-5}\sum_{l=0}^{k-j-5}\frac{\omega^2}{B^2/\mu}\rho_{l+1}\rho_{k-j-l-4} s_j\right\}, \quad \text{for} \quad k \geq 5.
\end{aligned}
$$

TTTB approximation

- Thin boundary ($l/R \ll 1$) approximation: we only keep the first term in the Forbenius series and derive jump contions for P_1 and ξ_r across the boundary

$$[P_1] \approx 0, \qquad [\xi_r] \approx -i\pi \frac{m^2/R^2}{\omega^2 \left|\mathrm{d}\rho/\mathrm{d}r\right|_R} P_1$$

- TB dispersion relation:

$$\frac{\tilde{k}_{\perp,\mathrm{e}}}{\rho_\mathrm{e}\left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right)} \frac{K_m'\left(\tilde{k}_{\perp,\mathrm{e}} R\right)}{K_m\left(\tilde{k}_{\perp,\mathrm{e}} R\right)} - \frac{k_{\perp,\mathrm{i}}}{\rho_\mathrm{i}\left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right)} \frac{J_m'\left(k_{\perp,\mathrm{i}} R\right)}{J_m\left(k_{\perp,\mathrm{i}} R\right)} = -i\pi \frac{m^2/R^2}{\omega^2 \left|\mathrm{d}\rho/\mathrm{d}r\right|_R}$$

- Thin tube ($L/R \gg 1$) approximation:

$$\rho_\mathrm{i}\left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_\mathrm{e}\left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right) - i\pi \frac{m}{R} \frac{\rho_\mathrm{i}\rho_\mathrm{e}}{\left|\mathrm{d}\rho/\mathrm{d}r\right|_R} \frac{\left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right)\left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right)}{\omega^2} = 0$$

$$\left|\frac{\mathrm{d}\rho}{\mathrm{d}r}\right|_R = F \frac{\pi^2}{4} \frac{\rho_\mathrm{i} - \rho_\mathrm{e}}{l} \qquad m = 1$$

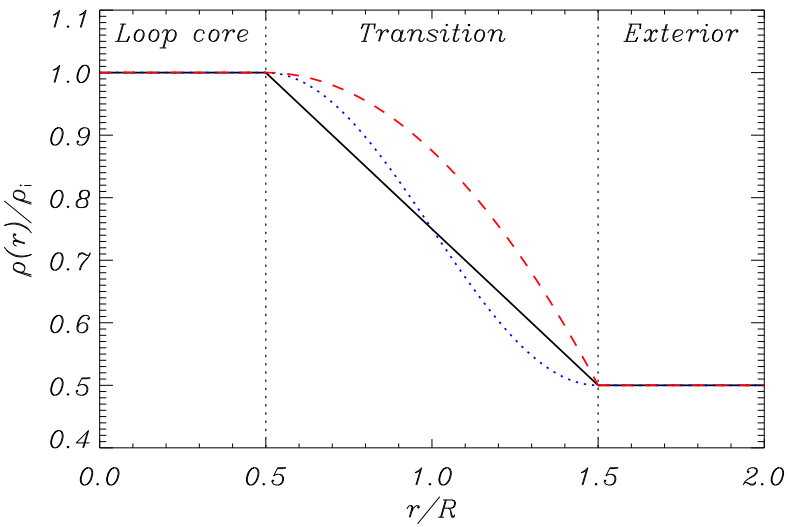
Analytic formulas for period and damping time

- We take $\omega = \omega_{\mathrm{R}} + i\omega_{\mathrm{I}}$ and assume $|\omega_{\mathrm{I}}| \ll |\omega_{\mathrm{R}}|$

$$\omega_{\mathrm{R}} \approx \omega_k, \qquad \omega_{\mathrm{I}} \approx -\frac{1}{2\pi F}\frac{l}{R}\frac{\rho_{\mathrm{i}} - \rho_{\mathrm{e}}}{\rho_{\mathrm{i}} + \rho_{\mathrm{e}}}\omega_{\mathrm{R}}$$

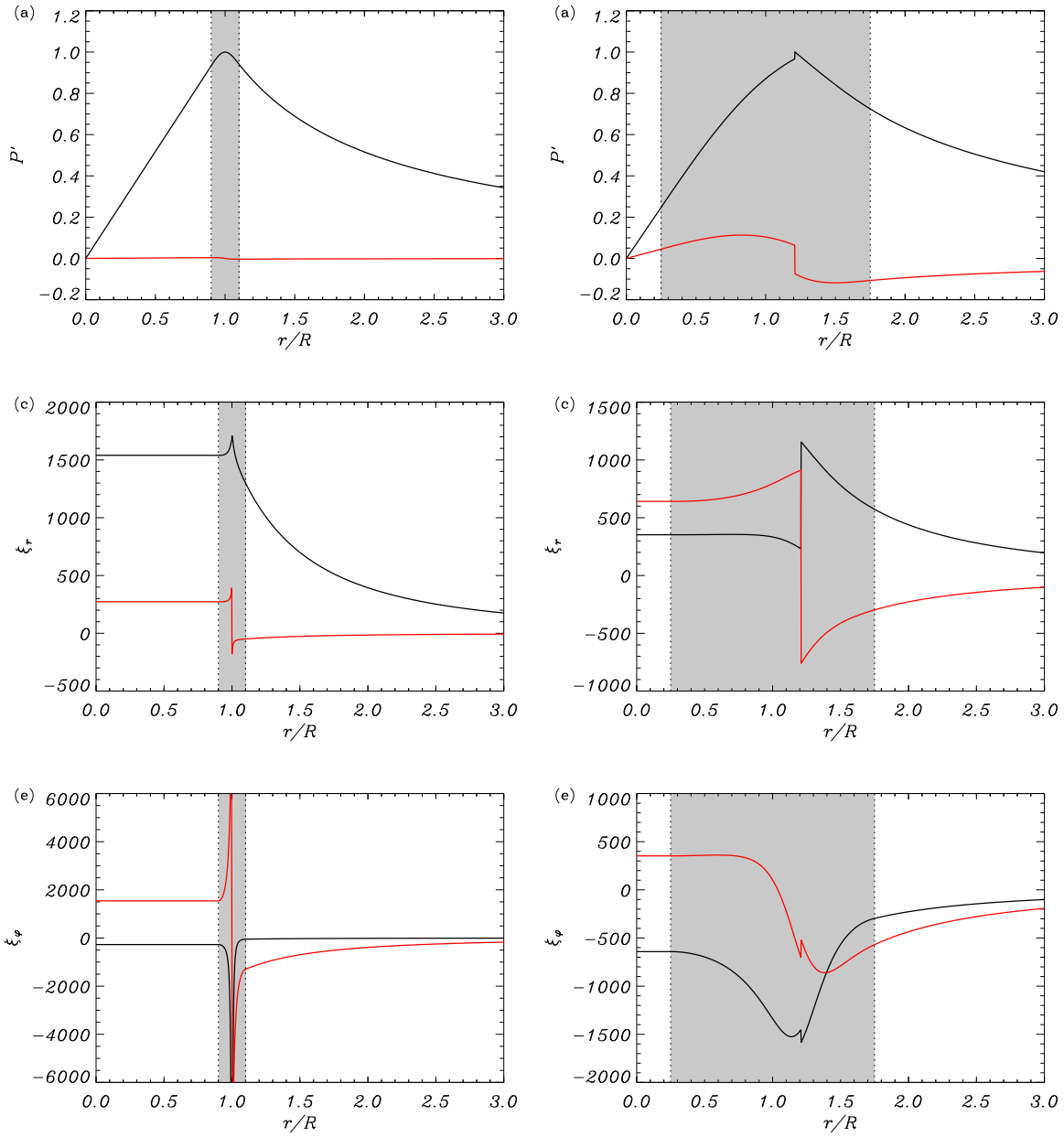
$$P = \frac{2\pi}{\omega_{\mathrm{R}}} = \frac{2L}{v_{\mathrm{A,i}}}\sqrt{\frac{(\rho_{\mathrm{i}} + \rho_{\mathrm{e}})}{2\rho_{\mathrm{i}}}}$$

$$\frac{\tau_{\mathrm{D}}}{P} = \frac{\omega_{\mathrm{R}}}{2\pi|\omega_{\mathrm{I}}|} = F\frac{R}{l}\frac{\rho_{\mathrm{i}} + \rho_{\mathrm{e}}}{\rho_{\mathrm{i}} - \rho_{\mathrm{e}}}$$



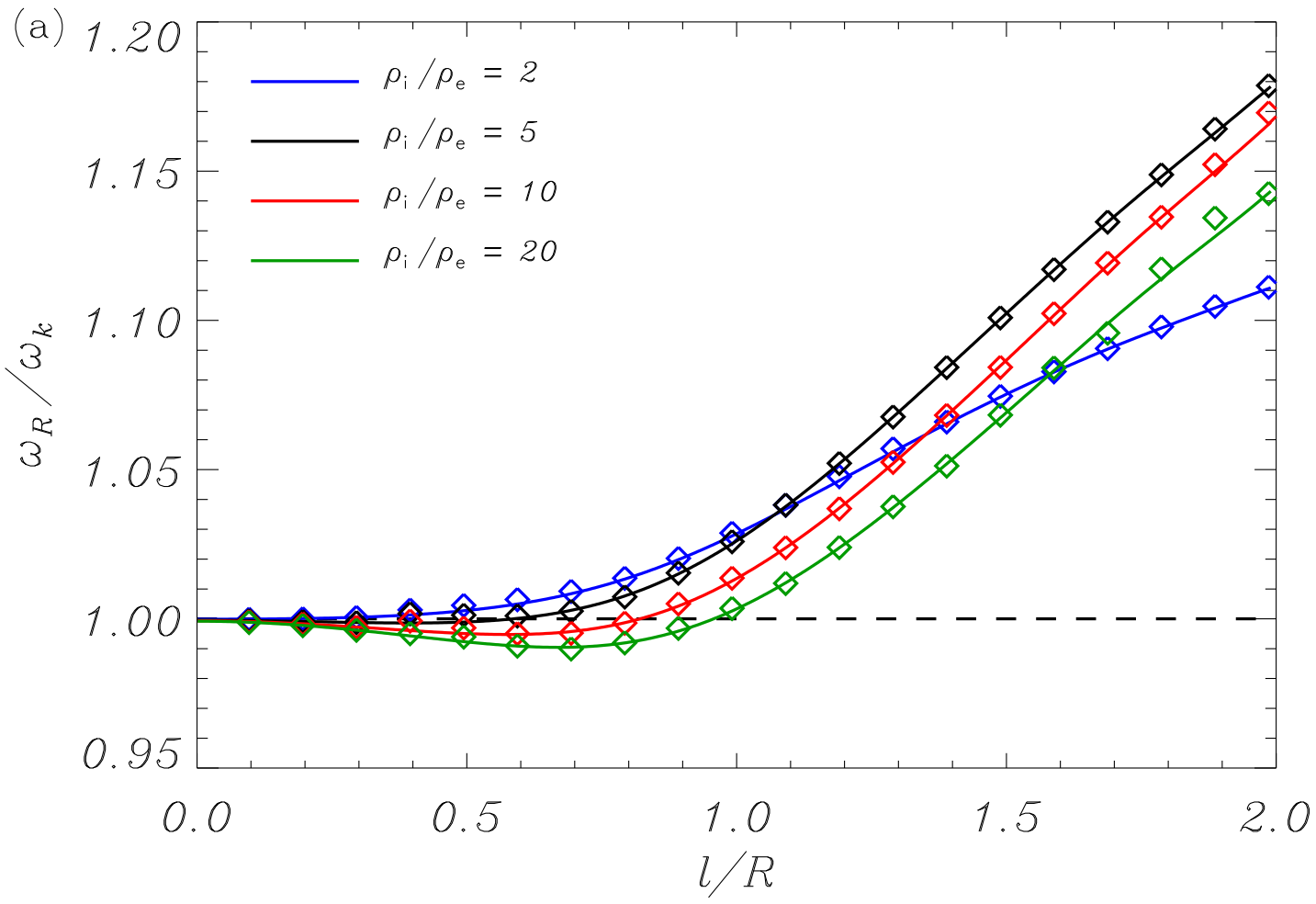
- Linear: $F = 4/\pi^2 \approx 0.405$
- Sinusoidal: $F = 2/\pi \approx 0.637$
- Parabolic: $F = 4\sqrt{2}/\pi^2 \approx 0.573$

Beyond TTTB: perturbations



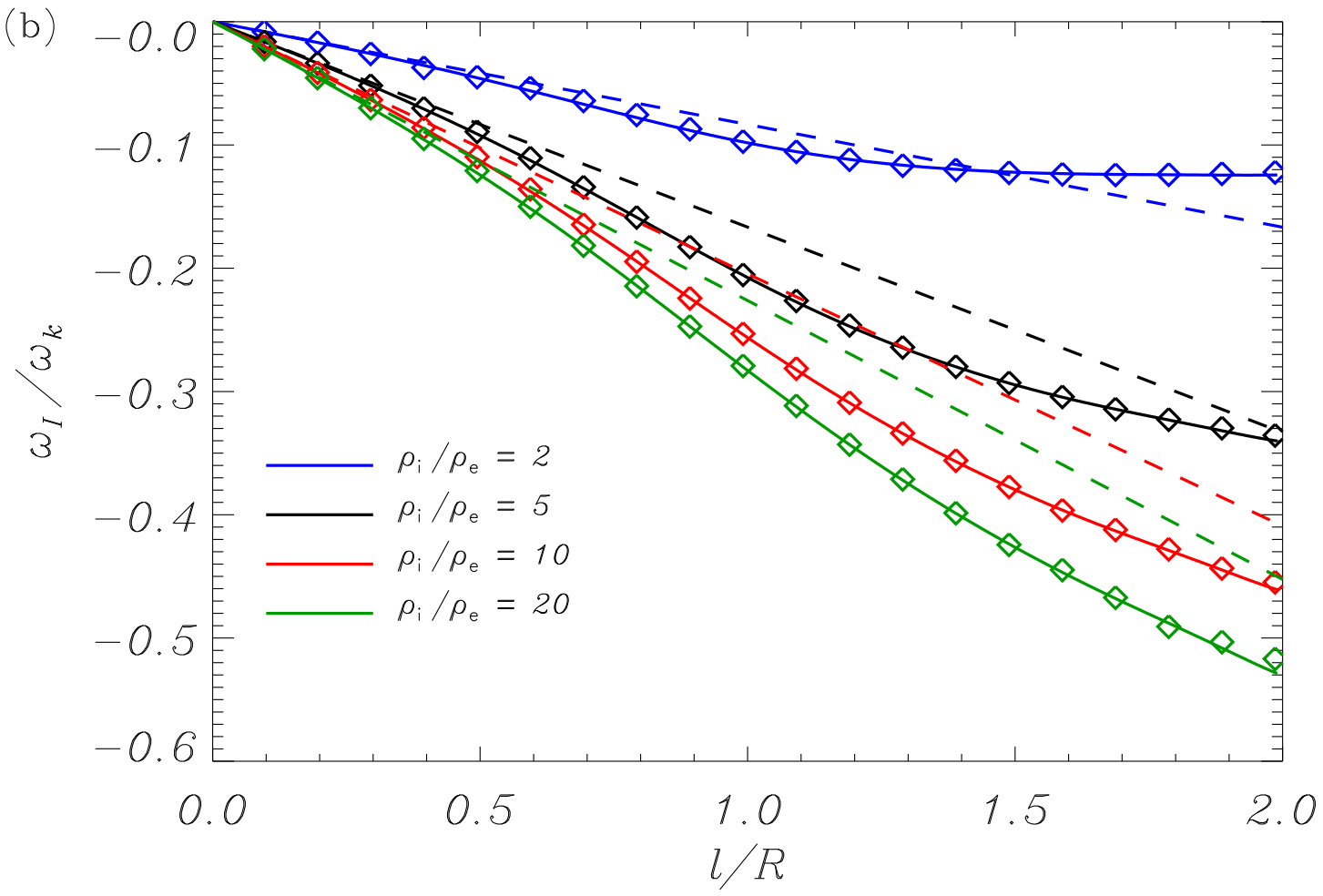
Beyond TTTB: frequency

- Sinusoidal transition

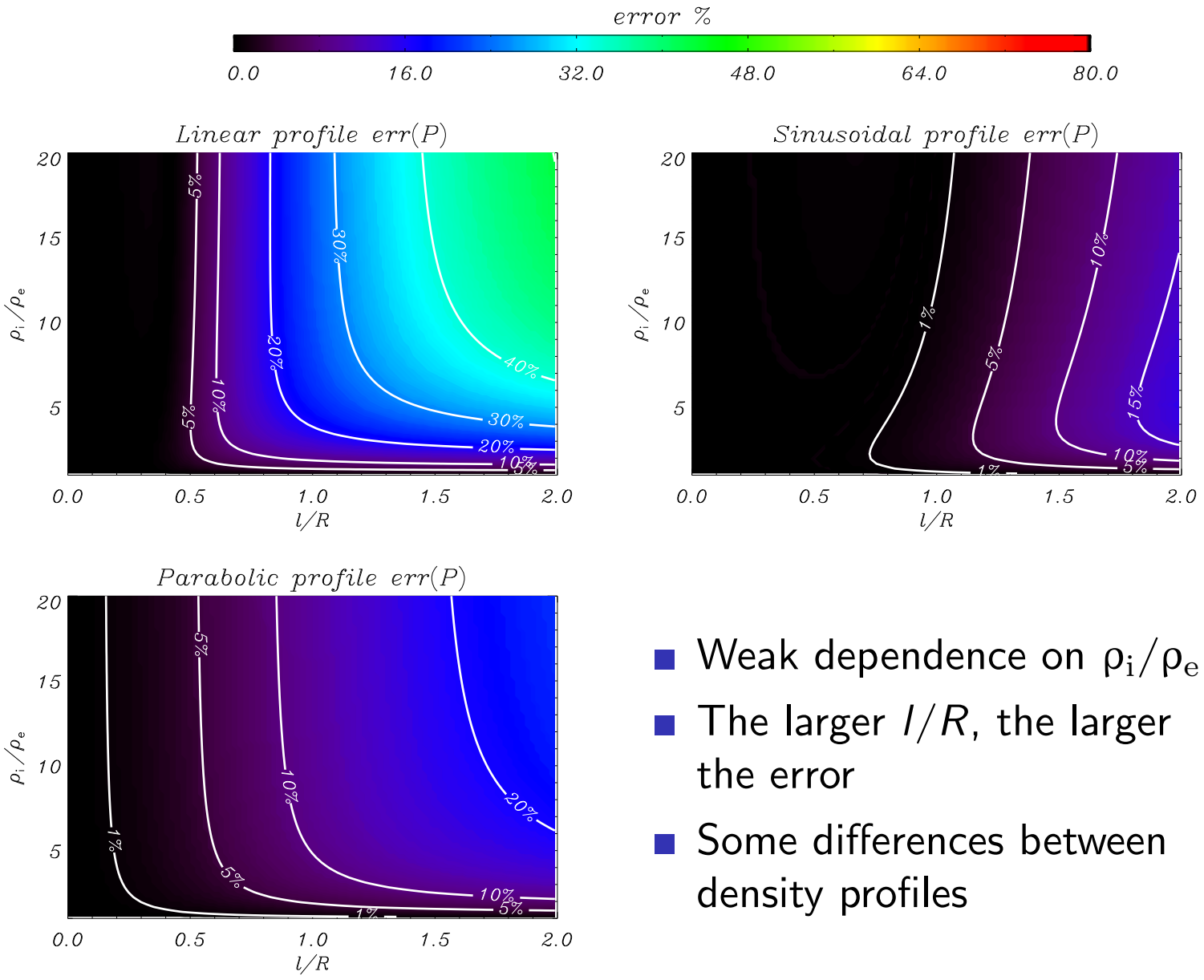


Beyond TTTB: damping rate

- Sinusoidal transition

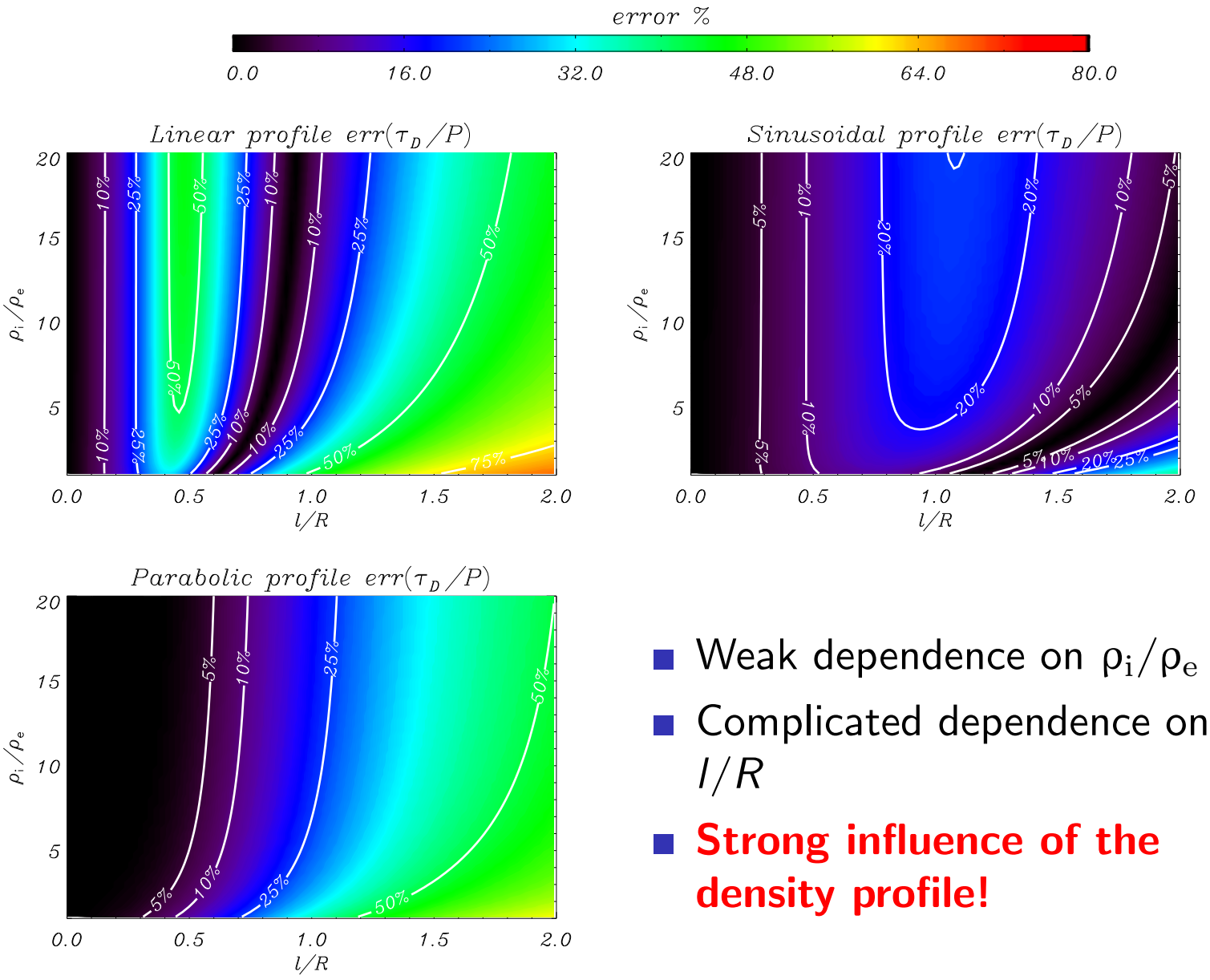


Error due to the TB approximation: P



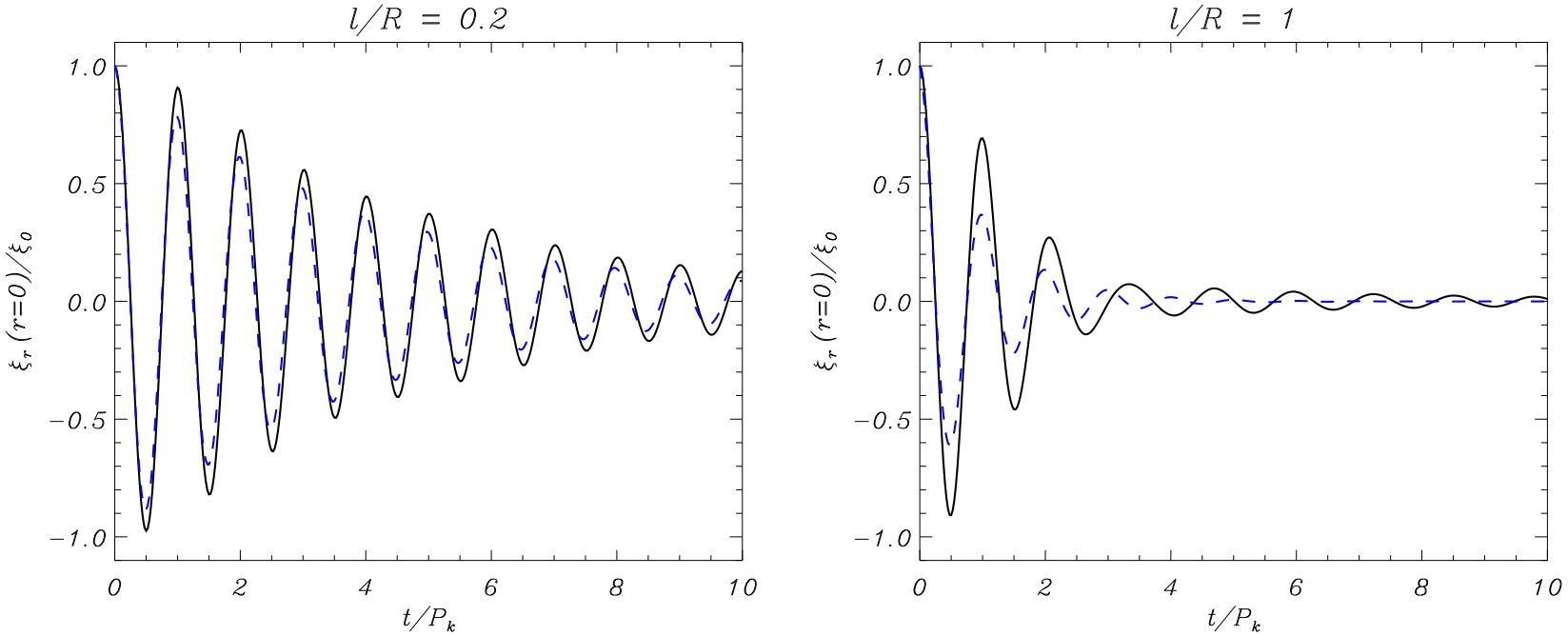
- Weak dependence on ρ_i/ρ_e
- The larger l/R, the larger the error
- Some differences between density profiles

Error due to the TB approximation: τ_D/P



- Weak dependence on ρ_i/ρ_e
- Complicated dependence on l/R
- **Strong influence of the density profile!**

Comparison with full temporal evolution

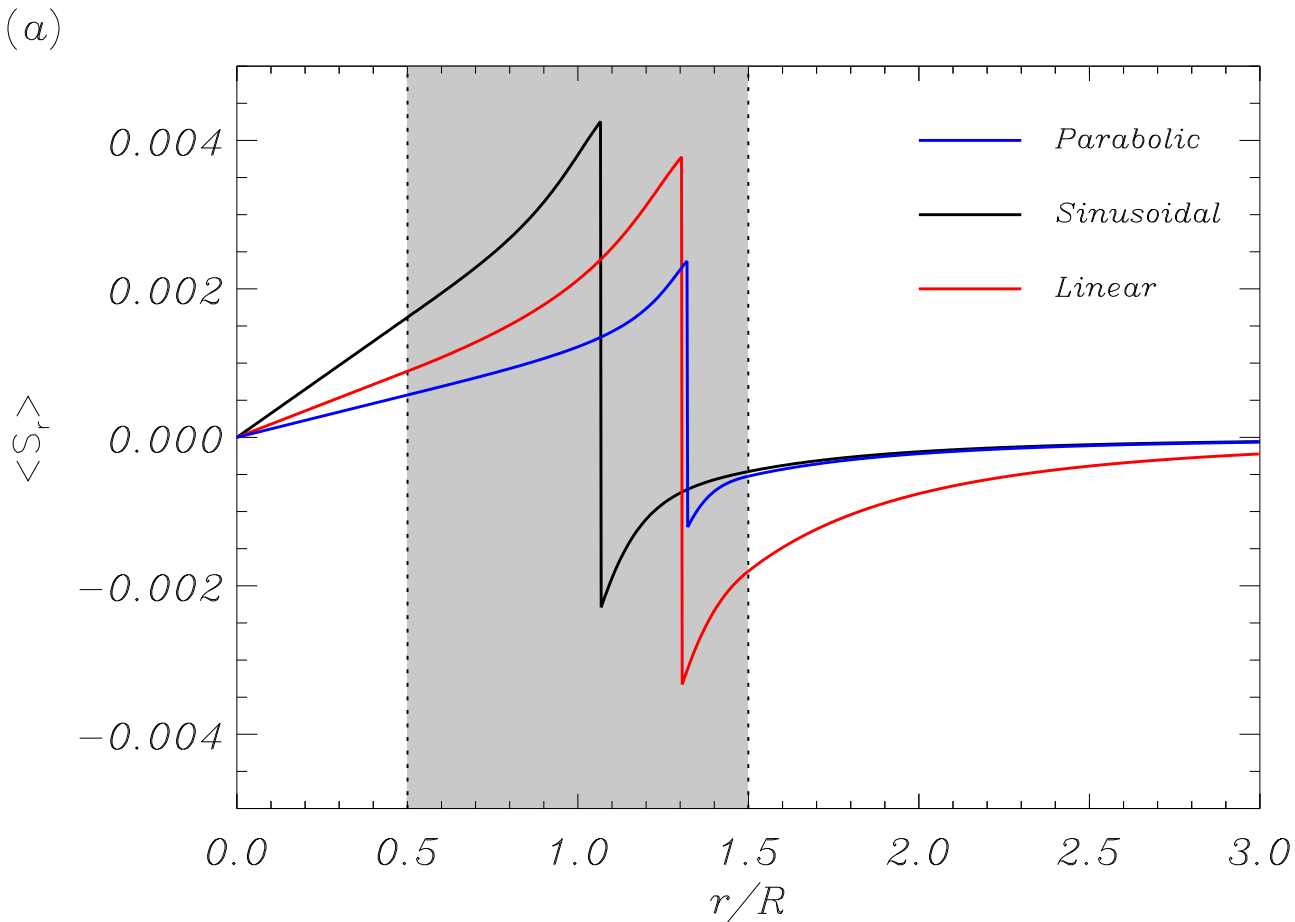


- Solid line: time-dependent simulation; Dashed line: Normal mode
- Excellent agreement, specially for thin nonuniform layers

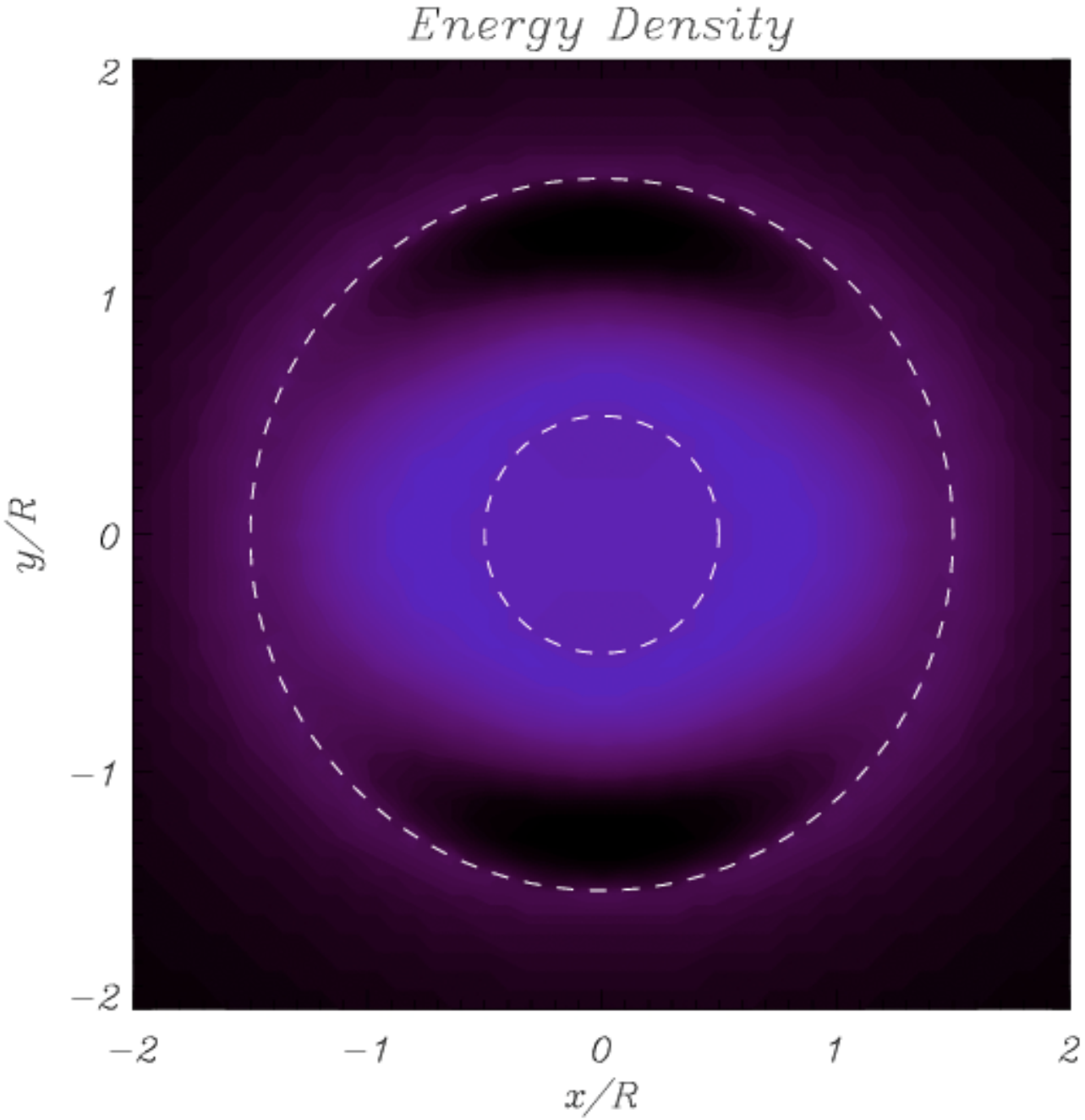
Flux of energy

- Resonant absorption is an ideal process $\rightarrow$ Wave energy is conserved
- Damping $\neq$ Dissipation
- Radial flux of energy that causes energy convergence to the nonuniform layer

$$\langle \Pi_r \rangle = -\frac{1}{2}\mathrm{Re}\left(i\omega\xi_r P'^*\right)$$

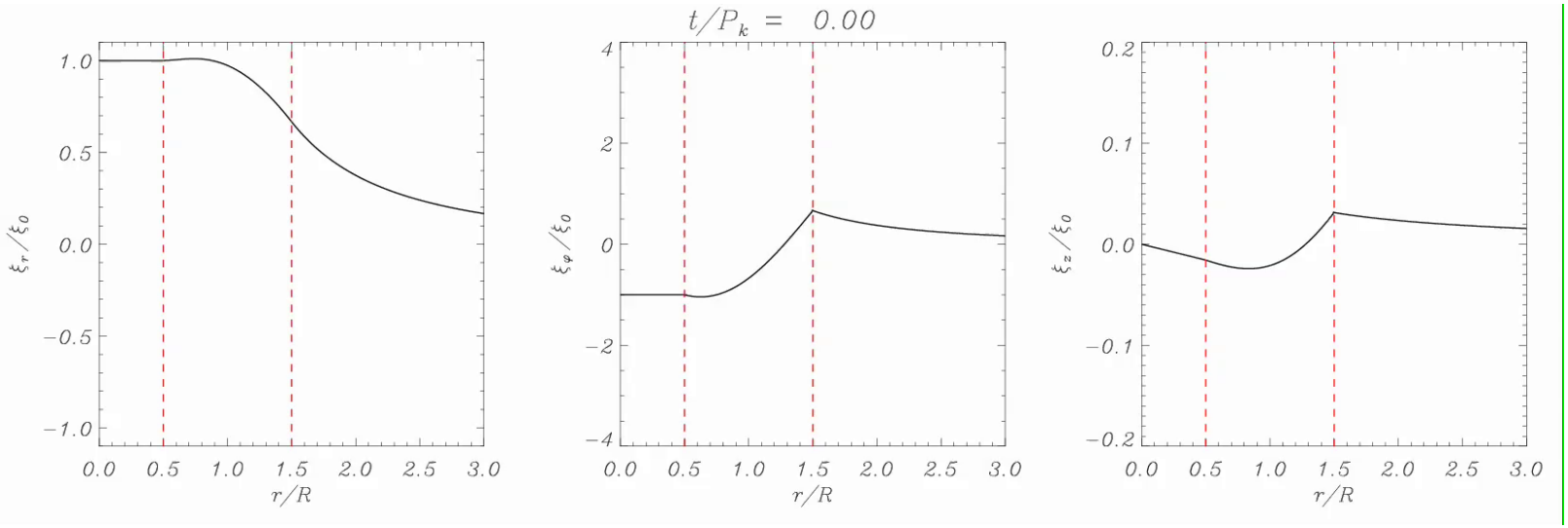


Flux of wave energy to the nonuniform boundary



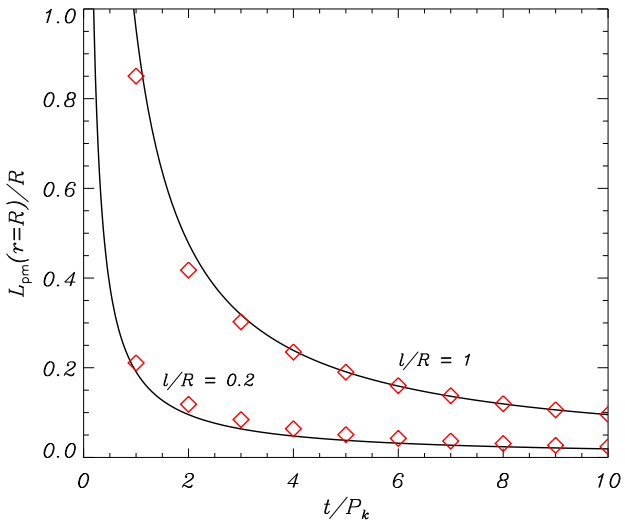
Phase mixing in the non-uniform layer

- Smaller and smaller spatial scales are generated as time increases



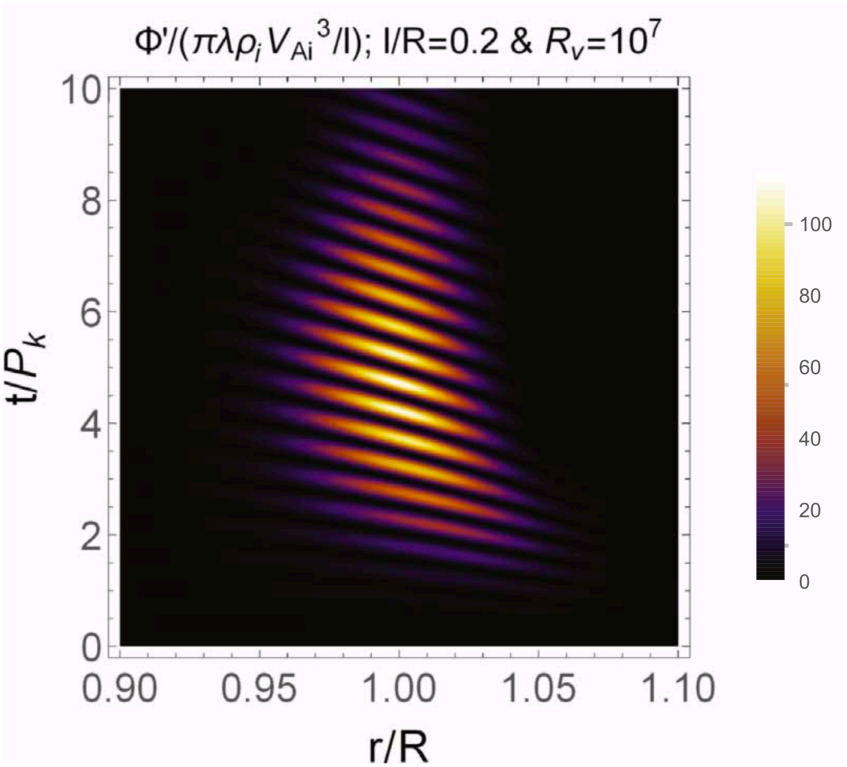
- Phase mixing length scale (Mann et al. 1995):

$$L_{\mathrm{ph}} = \frac{2\pi}{|\partial\omega_{\mathrm{A}}/\partial r|\, t}$$



Phase 3: Dissipation (finally!)

- The process of phase mixing will continuously work until the dissipative scales are reached.
- Time needed to reach the dissipative scale assuming Ohm's diffusion as a dissipation mechanism: $t_{\rm dis} \approx \eta^{-1/3} \left|\partial \omega_{\rm A}/\partial r\right|^{-2/3} \sim R_m^{1/3}$ (Heyvaerts & Priest 1983; Terradas & Arregui 2018)



- Magnetic Reynolds number in the corona: $R_m \approx 10^{12}$
- $t_{\rm dis} \gg P$ y $t_{\rm dis} \gg \tau_{\rm D}$ **Phase mixing is a slow process!**

The global picture

1. A disturbance excites the kink normal mode of the loop
2. Resonant absorption damps the global kink motion
3. Flux of energy towards the nonuniform boundary
4. Generation of rotational motions (Alfvén modes)
5. Phase mixing takes energy from large scales to small scales
6. The dissipatives scales are eventually reached (but a lot of time is needed!)
7. Wave energy is dissipated and the plasma is heated

Warning!

- All these calculations are based on LINEAR THEORY
- NONLINEARITY can dramatically change this picture
- Excitation of Kelvin-Helmholtz instabilities
- Transition to the turbulent regime
- Faster dissipation and heating!
- SEE PART 5!

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