3. TRANSVERSE OSCILLATIONS OF CORONAL LOOPS

– Part 4: Seismology –

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Solar Magnetohydrodynamics: Applications
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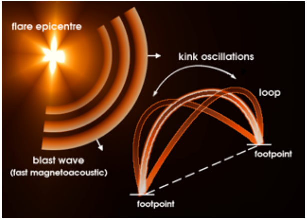
Transverse oscillations of coronal loops

- First observed with *TRACE* in 1999
  Nakariakov et al. (1999); Aschwanden et al. (1999)
- After an energetic disturbance (flare), the whole loop displays a damped transverse oscillation $\sim \cos(2\pi t/P + \phi)\exp(-t/\tau)$



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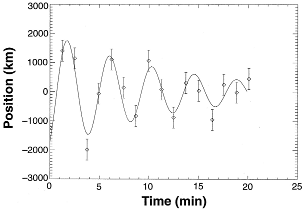

Nakariakov et al. (1999)

- Physical interpretation: Global kink MHD mode
  see, e.g., Edwin & Roberts (1983)
- Rapid attenuation consistent with damping by resonant absorption
  see, e.g., Ruderman & Roberts (2002); Goossens et al. (2002)

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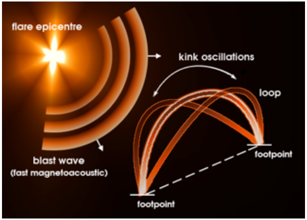

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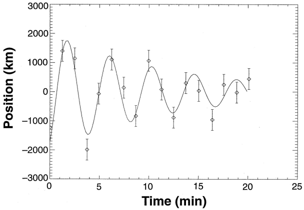

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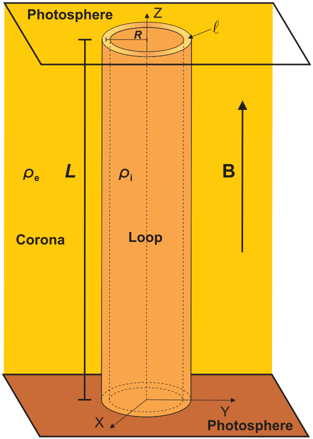
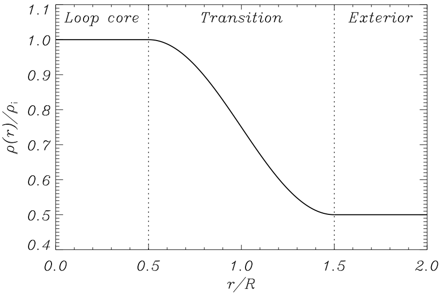
Theoretical model





- $l = 0 \longrightarrow$ Abrupt density jump
- $l = 2R \longrightarrow$ Fully nonuniform tube

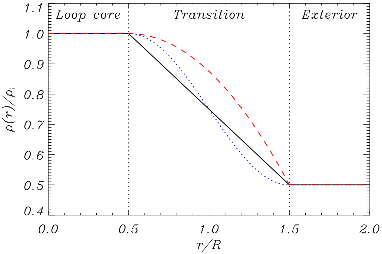
Kink mode: TTTB approximation

- We assume $L/R \gg 1$ and $l/R \ll 1$

$$P = \frac{2L}{v_{\mathrm{A,i}}}\sqrt{\frac{\zeta+1}{2\zeta}}$$

$$\frac{\tau_{\mathrm{D}}}{P} = F\,\frac{R}{l}\,\frac{\zeta+1}{\zeta-1}$$

$$\zeta = \frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{e}}}$$



- Linear: $F = 4/\pi^2 \approx 0.405$
- Sinusoidal: $F = 2/\pi \approx 0.637$
- Parabolic: $F = 4\sqrt{2}/\pi^2 \approx 0.573$

Observables and unknowns

Observables

- P
- τ_{D}
- L (and sometimes R)

Unknowns: seismic variables

- $v_{\mathrm{A,i}}$
- l/R (or l if R is known)
- ζ

Seismic problem

- 3 observables and 3 unknowns, but only 2 equations
- The seismological inversion has infinite possible solutions!
- 1D inversion curve in the 3D space formed by $v_{\mathrm{A,i}}$–l/R–ζ
- Any point on the inversion curve is equally compatible with the observations

Valid interval of $v_{\mathrm{A,i}}$

$$v_{\mathrm{A,i}} = \frac{L}{P}\sqrt{\frac{2\,(\zeta+1)}{\zeta}}$$

- We use the density contrast, ζ, as a parameter: $1 < \zeta < \infty$

$$\begin{aligned} \lim_{\zeta=1} v_{\mathrm{A},i} &= \lim_{\zeta=1} \frac{L}{P}\sqrt{\frac{2\,(1+\zeta)}{\zeta}} = \frac{2L}{P}, \\ \lim_{\zeta\to\infty} v_{\mathrm{A},i} &= \lim_{\zeta\to\infty} \frac{L}{P}\sqrt{\frac{2\,(1+\zeta)}{\zeta}} = \sqrt{2}\frac{L}{P}. \end{aligned}$$

- $v_{\mathrm{A},i}$ is constrained in a relatively narrow range:

$$v_{\mathrm{A},i} \in \left]\sqrt{2}\frac{L}{P}, \frac{2L}{P}\right]$$

Valid interval of l/R

$$\frac{l}{R} = F\frac{P}{\tau_{\mathrm{D}}}\frac{\zeta+1}{\zeta-1}$$

$$\begin{aligned}
\lim_{\zeta=1}\frac{l}{R} &= \lim_{\zeta=1}F\frac{P}{\tau_{\mathrm{D}}}\frac{\zeta+1}{\zeta-1} = \infty \qquad \text{(not useful)} \\
\lim_{\zeta\to\infty}\frac{l}{R} &= \lim_{\zeta\to\infty}F\frac{P}{\tau_{\mathrm{D}}}\frac{\zeta+1}{\zeta-1} = F\frac{P}{\tau_{\mathrm{D}}}
\end{aligned}$$

- Due to the geometry of the tube, l/R has a maximum value:

$$\frac{l}{R} \leq 2$$

$$\frac{l}{R} \in \left]F\frac{P}{\tau_{\mathrm{D}}}, 2\right]$$

Valid interval of ζ

- We rewrite the expression of I/R as an equation for ζ

$$\zeta = \frac{(I/R)\,C + 1}{(I/R)\,C - 1}, \qquad C = \frac{1}{F}\frac{\tau_{\mathrm{D}}}{P}$$

- ζ attains its absolute minimum for $I/R = 2$:

$$\zeta \geq \frac{2C + 1}{2C - 1} \neq 1$$

- ζ attains its maximal value of infinity in the limit $I/R \to 1/C$.
- With this information, we can see that ζ is constrained in the interval

$$\zeta \in \left[\frac{2C + 1}{2C - 1}, \infty\right[$$

or using the definition of C

$$\zeta \in \left[\frac{2\,(\tau_{\mathrm{D}}/P)\,/F + 1}{2\,(\tau_{\mathrm{D}}/P)\,/F - 1}, \infty\right[$$

Valid interval of $v_{\mathrm{A,i}}$ (again)

- We can use the information about the minimum value of ζ to obtain a more accurate interval for $v_{\mathrm{A},i}$

$$\lim_{\zeta \to \frac{2C+1}{2C-1}} v_{\mathrm{A},i} = \lim_{\zeta \to \frac{2C+1}{2C-1}} \frac{L}{P}\sqrt{\frac{2(1+\zeta)}{\zeta}} = \frac{2L}{P}\sqrt{\frac{2C}{2C+1}}.$$

- We find that $v_{\mathrm{A},i}$ is more accurately constrained in the interval

$$v_{\mathrm{A},i} \in \left]\sqrt{2}\frac{L}{P}, \frac{2L}{P}\sqrt{\frac{2C}{2C+1}}\right]$$

or using the definition of C

$$v_{\mathrm{A},i} \in \left]\sqrt{2}\frac{L}{P}, \frac{2L}{P}\sqrt{\frac{2(\tau_{\mathrm{D}}/P)/F}{2(\tau_{\mathrm{D}}/P)/F+1}}\right]$$

Summary of valid intervals

$$v_{\mathrm{A},i} \in \left] \sqrt{2}\frac{L}{P}, \frac{2L}{P}\sqrt{\frac{2\left(\tau_{\mathrm{D}}/P\right)/F}{2\left(\tau_{\mathrm{D}}/P/F\right)+1}} \right]$$

$$\frac{I}{R} \in \left] \frac{2}{\pi}\frac{P}{\tau_{\mathrm{D}}}, 2 \right]$$

$$\zeta \in \left[\frac{2\left(\tau_{\mathrm{D}}/P\right)/F+1}{2\left(\tau_{\mathrm{D}}/P\right)/F-1}, \infty \right[$$

Example

- Ofman & Aschwanden (2002), *The Astrophysical Journal* 576, L153

THE PARAMETERS OF THE OSCILLATING CORONAL LOOPS OBSERVED BY *TRACE*[a]

Loop	Date	Start Time (UT)	L (m)	w (m)	P (s)	V_A (m s^{-1})	t_{decay} (s)
1	1998 Jul 14	12:59:57	1.68E8	7.2E6	261	9.10E5	870
2	1998 Jul 14	12:57:38	7.20E7	6.7E6	265	3.84E5	300
3	1998 Jul 14	12:57:38	1.74E8	8.3E6	316	7.79E5	500
4	1998 Jul 14	12:56:32	2.04E8	7.9E6	277	1.04E6	400
5	1998 Jul 14	13:02:26	1.62E8	7.3E6	272	8.42E5	849
6	1998 Nov 23	06:35:57	3.90E8	16.8E6	522	1.06E6	1200
7	1999 Jul 04	08:33:17	2.58E8	7.0E6	435	8.39E5	600
8	1999 Oct 25	06:28:56	1.66E8	6.3E6	143	1.64E6	200
9	2001 Mar 21	02:32:44	4.06E8	9.2E6	423	1.36E6	800
10	2001 May 15	02:57:00	1.92E8	6.9E6	185	1.47E6	200
11	2001 Jun 15	06:32:29	1.46E8	15.8E6	396	5.21E5	400

[a] Values were obtained by Aschwanden et al. 2002 and Nakariakov et al. 1999.

Example

Observables

- Event #5
  - $P = 272$ s
  - $\tau_{\mathrm{D}} = 849$ s
  - $L = 162 \times 10^3$ km
- We assume a sinusoidal density profile: $F = 2/\pi$

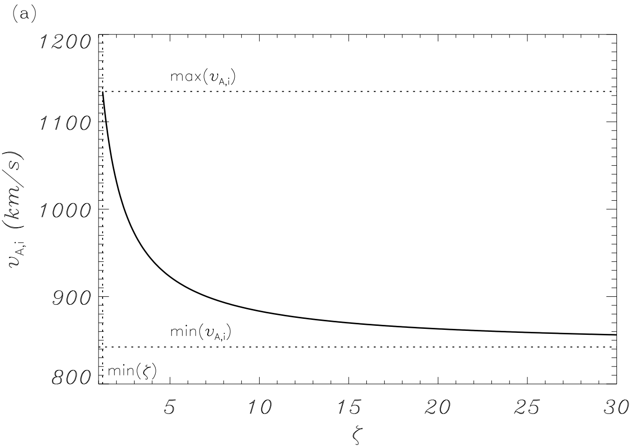
Seismic Intervals

- $842.29 \text{ km/s} < v_{\mathrm{A},i} \leq 1134.72 \text{ km/s}$
- $0.2 < l/R \leq 2$
- $1.23 \leq \zeta < \infty$

Example: inversion curve for $v_{\mathrm{A},i}$

$$v_{\mathrm{A,i}} = \frac{L}{P}\sqrt{\frac{2\,(\zeta+1)}{\zeta}}$$

- 842.29 km/s $< v_{\mathrm{A},i} \leq$ 1134.72 km/s

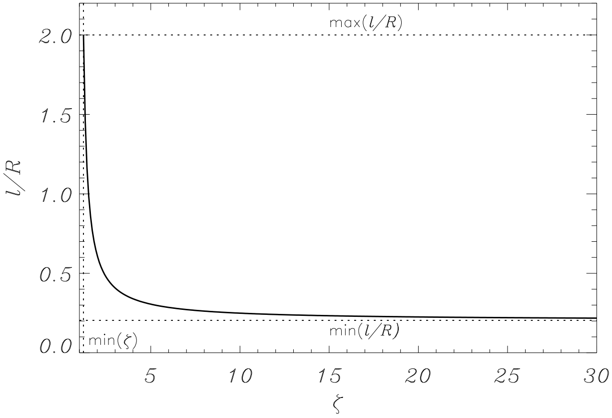


Example: inversion curve for l/R

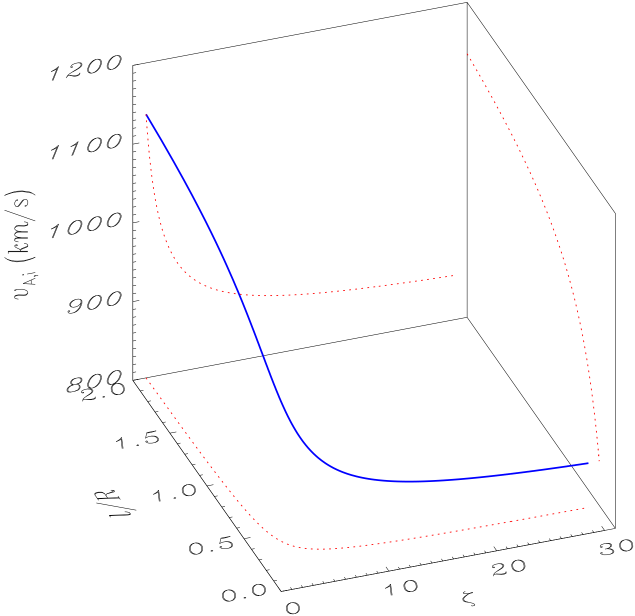
$$\frac{l}{R} = \frac{2}{\pi}\frac{P}{\tau_{\mathrm{D}}}\frac{\zeta+1}{\zeta-1}$$

- $0.2 < l/R \leq 2$

(b)



Example: full inversion curve



Seismic inversion with numerical results

- The error associated with the TTTB approximation affects the inversion of physical parameters in coronal loops
- Alternatively, we can also perform the numerical inversion using the full results of the Frobenius series
- The radius of the loop, R, is also needed in the numerical inversion

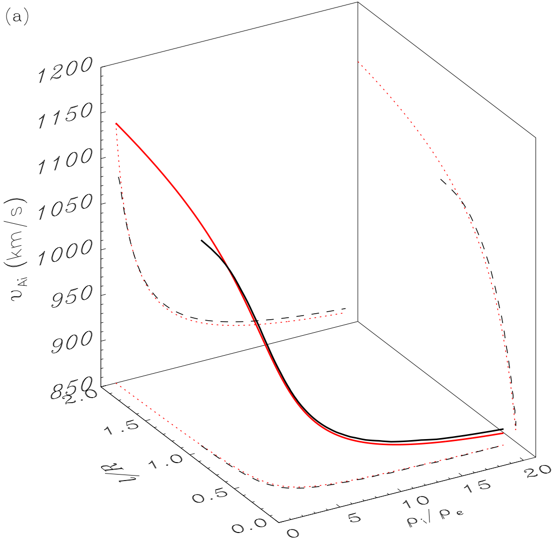
Table 2. Intervals of the seismic variables $v_{\mathrm{A,i}}$, l/R, and $\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$ for events #5 and #10 of Ofman & Aschwanden (2002) obtained from the analytic inversion (TTTB) and the numerical inversion.

Event	Inversion	$v_{\mathrm{A,i}}$	l/R	$\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$
#5	TTTB	842–1191	0.20–2	1.23–∞
#5	Numerical	868–1075	0.22–1.07	1.43–20
#10	TTTB	1468–2076	0.59–2	1.84–∞
#10	Numerical	1520–1646	0.58–1.49	2.41–20

Note. — $v_{\mathrm{A,i}}$ is given in km s^{-1}. The upper bound of the numerical $\rho_{\mathrm{i}}/\rho_{\mathrm{e}}$ is taken to be the maximum value used in the numerical inversions.

Seismic inversion with numerical results

- Event #5: weakly damped oscillation **Good agreement!**



Seismic inversion with numerical results

- Event #10: strongly damped oscillation **Not so good agreement!**