2. FUNDAMENTALS OF MHD WAVES

– Part 2: Structured medium –

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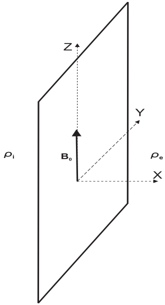
Effects of plasma structuring: interface

- The solar corona is highly structured due to the influence of the magnetic field.
- Coronal loops are an excellent example of coronal plasma structured because of the magnetic field.
- Let us see how plasma structuring adds new complexity to the MHD waves even in a very simple situation:
  - **interface between two homogeneous plasmas**
  - **constant magnetic field parallel to the interface**



$$\rho_0 = \begin{cases} \rho_i, & \text{if}, \quad x \leq 0 \\ \rho_e, & \text{if}, \quad x > 0 \end{cases}$$

$$\vec{B}_0 = B_0 \hat{e}_z$$

$$\frac{\partial}{\partial x}\left(p_0 + \frac{B_0^2}{2\mu}\right) = \frac{\partial P_0}{\partial x} = 0 \quad \text{at} \quad x = 0$$

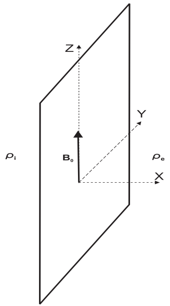
Local Alfvén waves

- Governing equation:

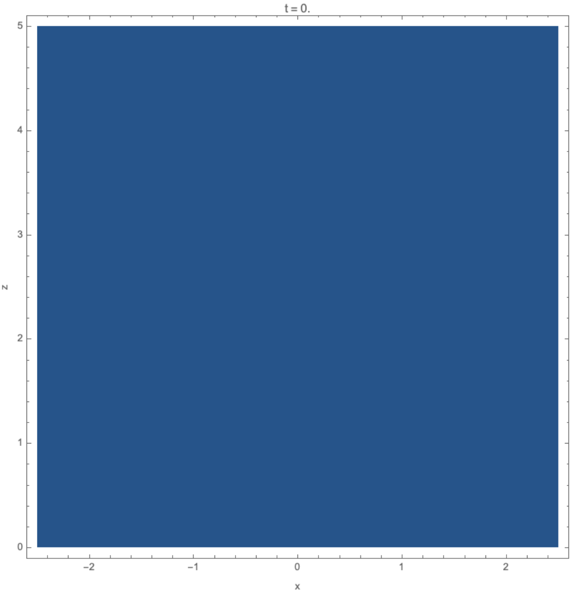
$$\frac{\partial^2 \Gamma}{\partial t^2} - v_{\mathrm{A}}^2 \frac{\partial^2 \Gamma}{\partial z^2} = 0$$

- **No derivatives across the interface**
- Alfvén waves on both sides of the interface are <u>uncoupled</u>
- Separate dispersion relations:
- Left plasma: $\omega^2 = k_z^2 v_{\mathrm{A,i}}^2$
- Right plasma: $\omega^2 = k_z^2 v_{\mathrm{A,e}}^2$

- **<u>Local Alfvén waves</u>**



Local Alfvén waves



Surface MHD waves

- In addition to local waves, a new type of MHD wave appears in a structured medium.

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HYDROMAGNETIC SURFACE WAVES*

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ABSTRACT

Plane and filamentary structures aligned with a magnetic field abound on the Sun and in both interplanetary and interstellar space. When the Alfvén speed changes across such boundaries, hydromagnetic surface waves can travel along them, carry energy, and provide heating. This paper surveys the nature of such surface waves, with emphasis on the dispersion relations, the spatial extent of the waves, the degree of gas compression, and the possibility of coupling to ordinary hydromagnetic waves. Explicit results are provided for the cases when the gas pressure is either much smaller or much larger than the magnetic pressure on either side of the surface. The waves are shear waves wherever $p \ll B^2/8\pi$. All surface waves involve finite gas compression, but this compression is negligible when $\boldsymbol{k}\cdot\boldsymbol{B} \ll kB$.

Subject headings: hydromagnetics — interstellar: magnetic fields — Sun: atmospheric motions

Wentzel (1979, ApJ)

WAVE PROPAGATION IN A MAGNETICALLY STRUCTURED ATMOSPHERE

I: *Surface Waves at a Magnetic Interface*

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Abstract. The solar atmosphere, from the photosphere to the corona, is structured by the presence of magnetic fields. We consider the nature of such inhomogeneity and emphasis that the usual picture of hydromagnetic wave propagation in a uniform medium may be misleading if applied to a structured field. We investigate the occurrence of magnetoacoustic surface waves at a single magnetic interface and consider in detail the case where one side of the interface is field-free. For such an interface, a *slow surface wave* can always propagate. In addition, a *fast surface wave* may propagate if the field-free medium is warmer than the magnetic atmosphere.

Roberts (1981, Sol. Phys.)

Incompressible surface waves

- The presence of the sharp discontinuity can give rise to the existence of **surface waves**.
- Unlike local waves, surface waves perturb and couple the plasmas at both sides of the interface.
- If the surface waves are incompressible, they are usually called **surface Alfvén waves**.

Linearized incompressible MHD equations

$$\frac{\partial^2 \mathbf{v}_1}{\partial t^2} - v_{\mathrm{A}}^2 \frac{\partial^2 \mathbf{v}_1}{\partial z^2} = -\frac{1}{\rho_0} \nabla \frac{\partial P_1}{\partial t}$$

$$\nabla \cdot \mathbf{v}_1 = 0$$

- The perturbation of the total pressure, P_1, is defined as the sum of the gas pressure perturbation, p_1, and the magnetic pressure perturbation, $\vec{B}_0 \cdot \vec{B}_1 / \mu$.
- Incompressibility DOES NOT MEAN $P_1 = 0$, it only means $\nabla \cdot \mathbf{v}_1 = 0$ so that $\rho_1 = 0$.

Incompressible surface waves

$$\rho_0 \left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right) \frac{\partial}{\partial x} \left(\frac{1}{\rho_0 \left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right)} \frac{\partial P_1}{\partial x} \right) - \left(k_y^2 + k_z^2\right) P_1 = 0$$

- For a sharp interface, $\rho_0 \left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right)$ is a constant quantity (but different) at both sides:

$$\frac{\partial^2 P_1}{\partial x^2} - \left(k_y^2 + k_z^2\right) P_1 = 0$$

- Solution:

$$P_1(x) = \begin{cases} A_{\mathrm{i}} \exp\left[\left(k_y^2 + k_z^2\right)^{1/2} x\right], & \text{if} \quad x \leq 0, \\ A_{\mathrm{e}} \exp\left[-\left(k_y^2 + k_z^2\right)^{1/2} x\right], & \text{if} \quad x > 0. \end{cases}$$

- The solution decays away from the interface $\rightarrow$ **surface wave**
- Interaction between the two plasmas separated by the interface

Incompressible surface waves

- The solutions at both sides of the discontinuity need to be matched together by ensuring:
  - **Mechanical equilibrium** $\rightarrow$ continuity of the total pressure perturbation, P_1
  - **Fluid continuity** $\rightarrow$ continuity of the normal component of the plasma Lagrangian displacement, ξ_x

$$\xi_x = \frac{i}{\omega} v_x = \frac{1}{\rho_0 \left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right)} \frac{\partial P_1}{\partial x}$$

- Suface wave dispersion relation:

$$\rho_i \left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right) = 0$$

- The analytic solution is

$$\omega^2 = \frac{\rho_i v_{\mathrm{A,i}}^2 + \rho_e v_{\mathrm{A,e}}^2}{\rho_i + \rho_e} k_z^2 \equiv \omega_k^2, \qquad v_k^2 = \frac{\rho_i v_{\mathrm{A,i}}^2 + \rho_e v_{\mathrm{A,e}}^2}{\rho_i + \rho_e}$$

- ω_k is called the **kink frequency** and is independent of k_y

The frequency of the incompressible surface wave is found to be the average of the frequency of the Alfvén waves at both sides of the interface: **surface Alfvén wave**.

Compressible surface waves

- Now $\Delta = \nabla \cdot \mathbf{v} \neq 0$, so that:

$$\frac{\partial^2}{\partial t^2}\left[\frac{\partial^2}{\partial t^2} - \left(v_\mathrm{s}^2 + v_\mathrm{A}^2\right)\nabla^2\right]\Delta + \frac{v_\mathrm{s}^2 v_\mathrm{A}^2}{|\vec{B}_0|^2}\left(\vec{B}_0 \cdot \nabla\right)^2 \nabla^2 \Delta = 0$$

- The functional dependence of Δ on x must remain unspecified

$$\Delta = X(x)\exp\left(ik_y y + ik_z z - i\omega t\right)$$

- We substitute this expression for Δ into the governing equation. We obtain

$$\frac{d^2 X(x)}{dx^2} + K^2 X(x) = 0$$

$$K^2 = \frac{\omega^4 - \left(v_\mathrm{s}^2 + v_\mathrm{A}^2\right)\left(\omega^2 - k_z^2 v_\mathrm{c}^2\right)\left(k_y^2 + k_z^2\right)}{\left(v_\mathrm{A}^2 + v_\mathrm{s}^2\right)\left(\omega^2 - k_z^2 v_\mathrm{c}^2\right)}, \qquad v_\mathrm{c} = \sqrt{\frac{v_\mathrm{A}^2 v_\mathrm{s}^2}{v_\mathrm{A}^2 + v_\mathrm{s}^2}}$$

- **The sign of K^2 determines the type of solution**

Compressible surface waves

$K^2 > 0$: Local slow and fast waves

$$X(x) = \begin{cases} A_{\rm i} \exp\left(iK_{\rm i}x\right), & \text{if}, \quad x \leq 0, \\ A_{\rm e} \exp\left(iK_{\rm e}x\right), & \text{if}, \quad x > 0, \end{cases}$$

- This solution has the form of plane waves and corresponds to the local slow and fast waves in a homogeneous plasma!
- K must be identified with the component of the wavenumber along the x-direction $= k_x$.

$K^2 < 0$: Surface waves

$$X(x) = \begin{cases} A_{\rm i} \exp\left(|K_{\rm i}|\, x\right), & \text{if}, \quad x \leq 0, \\ A_{\rm e} \exp\left(-\,|K_{\rm e}|\, x\right), & \text{if}, \quad x > 0. \end{cases}$$

- The solution decays away from the interface.
- **Surface MHD waves:** they are not present when the plasma is homogeneous and crucially involve the interaction between the two plasmas separated by the interface.

Compressible surface waves

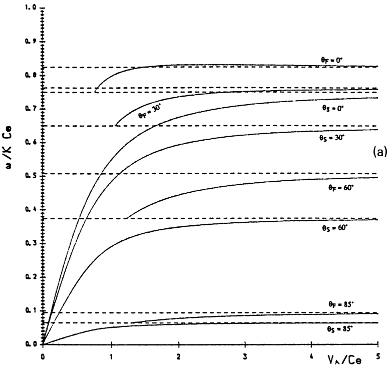
- The solutions at both sides of the discontinuity need to be matched together by ensuring:
  - **Mechanical equilibrium** $\rightarrow$ continuity of the total pressure perturbation, P_1
  - **Fluid continuity** $\rightarrow$ continuity of the normal component of the plasma Lagrangian displacement, ξ_x

$$\begin{aligned} P_1 &= -i\rho_0 \left(v_{\mathrm{s}}^2 + v_{\mathrm{A}}^2\right) \frac{\omega^2 - k_z^2 v_{\mathrm{c}}^2}{\omega^3} \Delta \\ \xi_x &= -i \frac{\left(v_{\mathrm{s}}^2 + v_{\mathrm{A}}^2\right)\left(\omega^2 - k_z^2 v_{\mathrm{c}}^2\right)}{\omega^3 \left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right)} \frac{d\Delta}{dx} \end{aligned}$$

Compressible surface waves

$$\rho_i \left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right) \frac{|K_{\mathrm{i}}|}{|K_{\mathrm{e}}|} = 0$$

$$K^2 = \frac{\omega^4 - \left(v_{\mathrm{s}}^2 + v_{\mathrm{A}}^2\right)\left(\omega^2 - k_z^2 v_{\mathrm{c}}^2\right)\left(k_y^2 + k_z^2\right)}{\left(v_{\mathrm{A}}^2 + v_{\mathrm{s}}^2\right)\left(\omega^2 - k_z^2 v_{\mathrm{c}}^2\right)}$$



Jain & Roberts (1991)

Compressible surface waves

- We take the limit or nearly perpendicular propagation, i.e., $k_y \gg k_z$.
- Then, $K^2 \to -k_y^2$, so that $|K_\mathrm{i}| / |K_\mathrm{e}| \to 1$
- The surface wave dispersion relation simplifies to the dispersion relation of the incompressible surface waces:

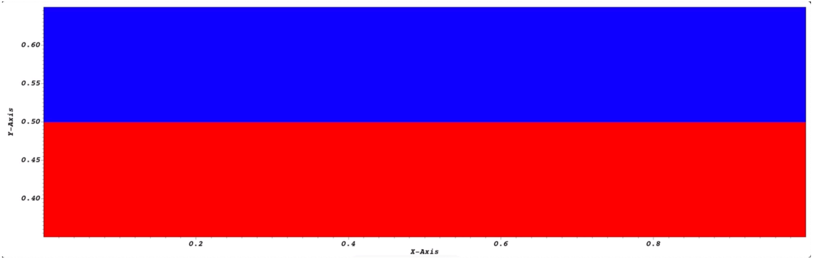
$$\rho_i \left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right) = 0$$

- The analytic solution is the **kink frequency**:

$$\omega^2 = \frac{\rho_i v_{\mathrm{A,i}}^2 + \rho_e v_{\mathrm{A,e}}^2}{\rho_i + \rho_e} k_z^2 \equiv \omega_k^2$$

The frequency of the compressible surface wave when $k_y \gg k_z$ is also the kink frequency. This frequency is independent of the sound velocity. The compressible surface wave behaves as the incompressible **surface Alfvén wave** for nearly perpendicular propagation.

Visualizing a surface wave



- Periodic disturbances of the interface.

Additional effect: mass flow

- Equilibrium velocity along the magnetic field direction, $\vec{v}_0 = v_0 \hat{e}_z$

$$v_0 = \begin{cases} U, & \text{if} \quad x \leq 0, \\ 0, & \text{if} \quad x > 0, \end{cases}$$

- The linearized version of the material derivative is now

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$$

- **Advection due to the flow**
- Simple recipe: replace $\frac{\partial}{\partial t}$ by $\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}$ in all the formulas for the static case derived before!
- The frequecy, ω, needs to be replaced by the **Doppler-shifted frequency:** $\Omega = \omega - k_z v_0$

Incompressible surface wave with flows

- Dispersion relation

$$\rho_i \left(\Omega_{\mathrm{i}}^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\Omega_{\mathrm{e}}^2 - k_z^2 v_{\mathrm{A,e}}^2\right) = 0$$

- Analytic solution when $\Omega_{\mathrm{i}} = \omega - k_z U$ and $\Omega_{\mathrm{e}} = \omega$

$$\omega = \frac{\rho_i}{\rho_i + \rho_e} k_z U \pm k_z \left(v_k^2 - \frac{\rho_i \rho_e}{\left(\rho_i + \rho_e\right)^2} U^2 \right)^{1/2}$$

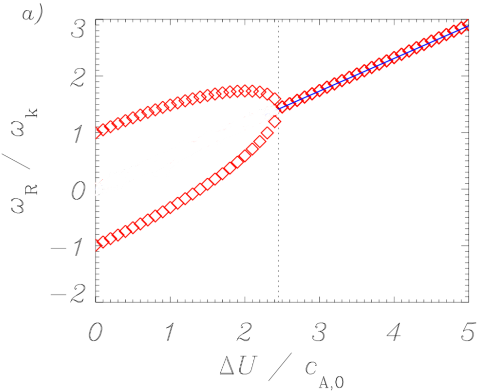
- Wave advection for slow flows: $U \ll v_k$

$$\omega \approx \frac{\rho_i}{\rho_i + \rho_e} k_z U \pm \omega_k, \qquad v_{\mathrm{ph}} \approx \frac{\rho_i}{\rho_i + \rho_e} U \pm v_k$$

- **Kelvin-Helmholtz instability** for fast (super-Alfvénic) flows

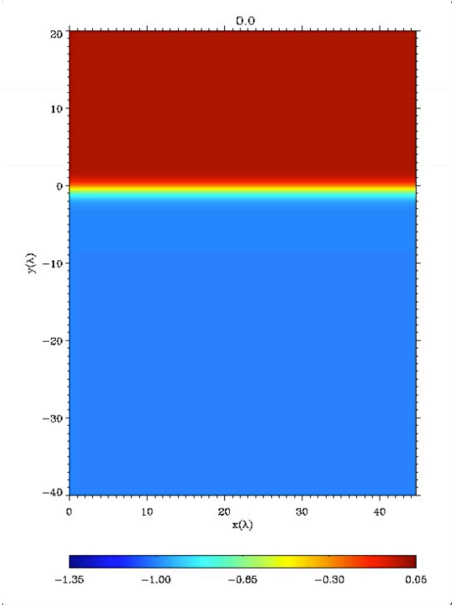
$$U > \frac{\rho_i + \rho_e}{\sqrt{\rho_i \rho_e}} v_k = v_{\mathrm{A,i}} \sqrt{2 \left(\frac{\rho_i}{\rho_e} + 1 \right)}$$

Kelvin-Helmholtz instability

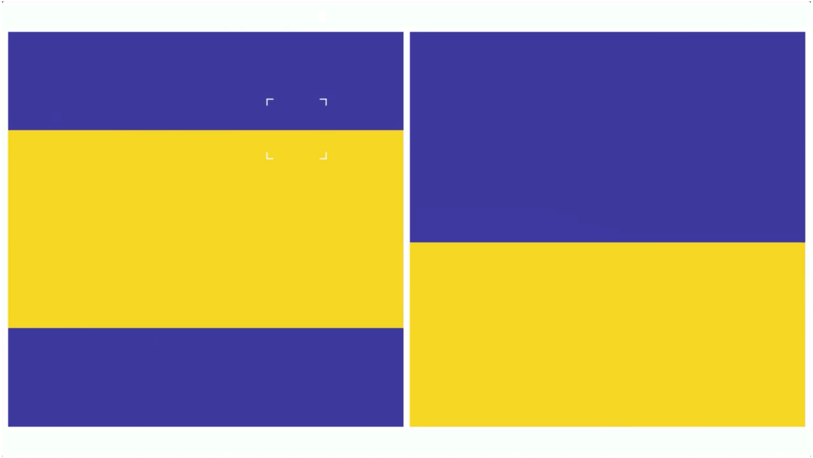


- Forward and backward surface waves coalescence at the velocity threshold.
- Unstable KHi mode: $\mathrm{Im}(\omega) > 0$
- Before that, backward wave reverses direction of propagation.

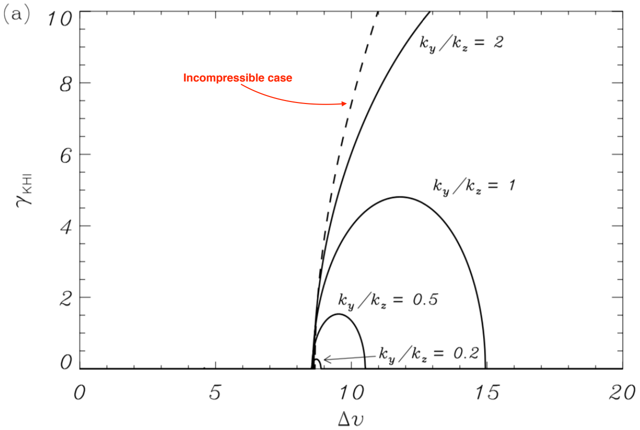
Visualizing the Kelvin-Helmholtz instability



Visualizing the Kelvin-Helmholtz instability



Effect of compressibility



- Velocity threshold is not modified $\longrightarrow$ we still need super-Alfvénic velocities.
- Growth rate is reduced and depends on k_y/k_z.
- Stabilization for fast enough flows and small k_y/k_z.

The case of a perpendicular flow

- Equilibrium velocity perpendicular to the magnetic field direction, $\vec{v}_0 = v_0 \hat{e}_y$

$$v_0 = \begin{cases} U, & \text{if} \quad x \leq 0, \\ 0, & \text{if} \quad x > 0, \end{cases}$$

- The linearized version of the material derivative is now

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \vec{v}_0 \cdot \nabla = \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial y}$$

- Now the simple recipe is: replace $\frac{\partial}{\partial t}$ by $\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial y}$ in all the formulas for the static case derived before!
- The frequecy, ω, needs to be replaced by the **Doppler-shifted frequency:** $\Omega = \omega - k_y v_0$

- **Note that, because of the flux freezing condition, this type of flow drags the magnetic field lines!**

Incompressible surface wave with perpendicular flow

- Dispersion relation (formally, same as before)

$$\rho_i \left(\Omega_{\mathrm{i}}^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\Omega_{\mathrm{e}}^2 - k_z^2 v_{\mathrm{A,e}}^2\right) = 0$$

- Analytic solution when $\Omega_{\mathrm{i}} = \omega - k_y U$ and $\Omega_{\mathrm{e}} = \omega$

$$\omega = \frac{\rho_i}{\rho_i + \rho_e} k_y U \pm \left(v_k^2 k_z^2 - \frac{\rho_i \rho_e}{\left(\rho_i + \rho_e\right)^2} U^2 k_y^2 \right)^{1/2}$$

- **Kelvin-Helmholtz instability**

$$U > \frac{\rho_i + \rho_e}{\sqrt{\rho_i \rho_e}} v_k \frac{k_z}{k_y} = v_{\mathrm{A,i}} \frac{k_z}{k_y} \sqrt{2 \left(\frac{\rho_i}{\rho_e} + 1 \right)}$$

Incompressible surface wave with perpendicular flow

- Critical (threshold) velocity:

$$U_{\mathrm{crit.}} = v_{\mathrm{A,i}} \frac{k_z}{k_y} \sqrt{2 \left(\frac{\rho_i}{\rho_e} + 1 \right)}$$

- In a coronal loop: $k_z \sim 1/L$ and $k_y \sim 1/R$, with $L/R \gg 1$ and $\sqrt{2 \left(\rho_i/\rho_e + 1 \right)} \sim 1$

$$U_{\mathrm{crit.}} \sim v_{\mathrm{A,i}} \frac{R}{L} \sqrt{2 \left(\frac{\rho_i}{\rho_e} + 1 \right)} \ll v_{\mathrm{A,i}}$$

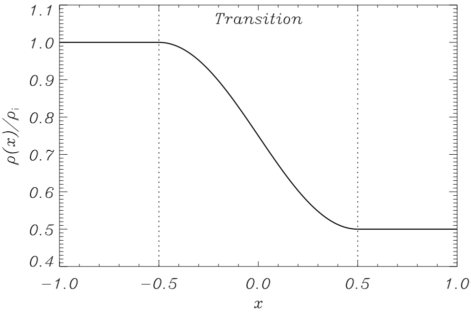
- **Slow perpendicular flows can trigger the Kelvin-Helmholtz instability in coronal loops** $\rightarrow$ SEE CHAPTER 3

Additional effect: nonuniformity

- Let us replace the sharp interface by a smooth transition of width l

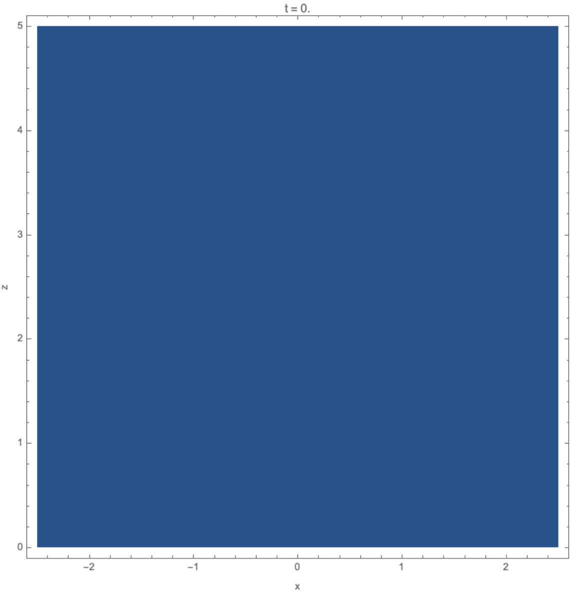
$$\rho_0 = \begin{cases} \rho_i, & \text{if,} \quad x < -l/2 \\ \rho_{tr}(x), & \text{if,} \quad -l/2 \leq x \leq l/2 \\ \rho_e, & \text{if,} \quad x > l/2 \end{cases}$$

- But still uniform magnetic field, $\vec{B}_0 = B_0 \hat{e}_z$



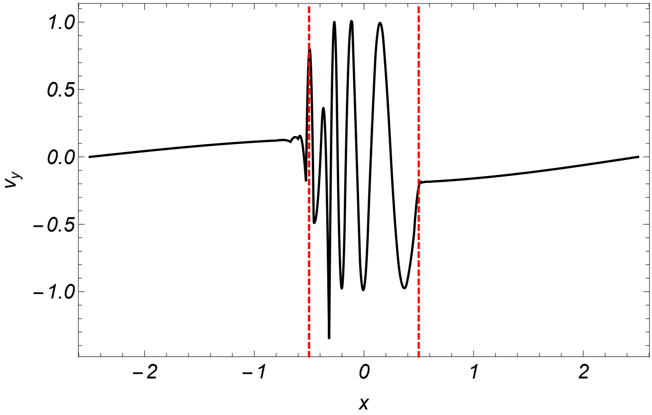
- If ρ_0 depends on position, $v_A = B_0/\sqrt{\mu\rho_0}$ depends on position

Effect of nonuniformity on local Alfvén waves



The phenomenon of phase mixing

- In the nonuniform transition, Alfvén waves in adjacent positions propagate at a slightly different velocity.
- As time increases, adjacent waves get out of phase.
- The result is the generation of small scales across the magnetic field.



The phenomenon of phase mixing

- Let us determine the effective length scale across the magnetic field.
- For a plane wave:

$$v_y \sim \exp\left(ik_z z - i\omega t\right)$$

- If the wave is a standing wave: $\omega = k_z v_{\rm A}(x)$

$$\frac{\partial v_y}{\partial x} \sim -ik_z \frac{\partial v_{\rm A}(x)}{\partial x} t\, v_y \sim -ik_x v_y$$

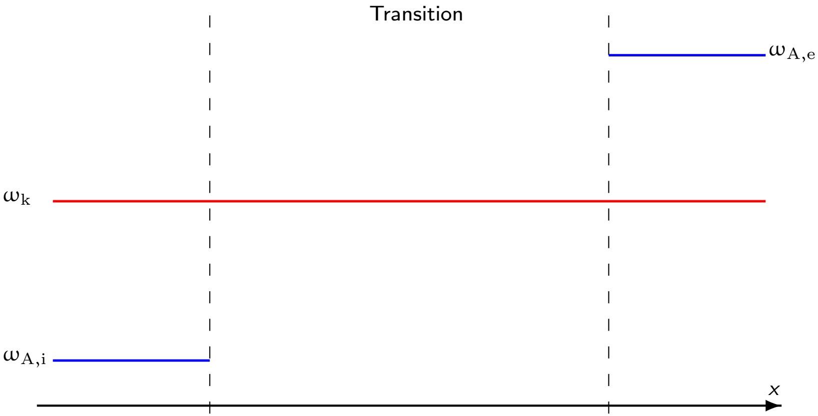
$$k_x = k_z \frac{\partial v_{\rm A}(x)}{\partial x} t = \frac{\partial \omega_{\rm A}(x)}{\partial x} t \qquad \rightarrow \qquad L_{\rm ph} = \frac{2\pi}{|k_x|} = \frac{2\pi}{|\partial\omega_{\rm A}/\partial x|\, t}$$

- If the wave is a propagating wave: $k_z = \omega / v_{\rm A}(x)$

$$\frac{\partial v_y}{\partial x} \sim -i \frac{\omega}{v_{\rm A}^2(x)} \frac{\partial v_{\rm A}(x)}{\partial x} z\, v_y \sim -ik_x v_y$$

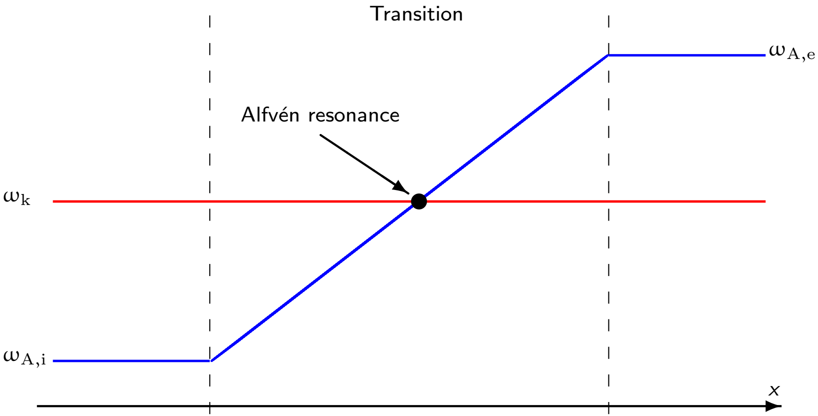
$$k_x = \frac{\omega}{v_{\rm A}^2(x)} \frac{\partial v_{\rm A}(x)}{\partial x} z = \frac{\partial \omega_{\rm A}(x)}{\partial x} \frac{z}{v_{\rm A}(x)} \qquad \rightarrow \qquad L_{\rm ph} = \frac{2\pi}{|k_x|} = \frac{2\pi v_{\rm A}(x)}{|\partial\omega_{\rm A}/\partial x|\, z}$$

Effect of nonuniformity on surface waves



$$\omega_k^2 = \frac{\rho_i \omega_{\mathrm{A,i}}^2 + \rho_e \omega_{\mathrm{A,e}}^2}{\rho_i + \rho_e}$$

Effect of nonuniformity on surface waves



$$\omega_k^2 = \frac{\rho_i \omega_{\mathrm{A,i}}^2 + \rho_e \omega_{\mathrm{A,e}}^2}{\rho_i + \rho_e}$$

The phenomenon of resonant absorption

Incompressible surface waves

$$\rho_0 \left(\omega^2 - k_z^2 v_\mathrm{A}^2\right) \frac{\partial}{\partial x} \left(\frac{1}{\rho_0 \left(\omega^2 - k_z^2 v_\mathrm{A}^2\right)} \frac{\partial P_1}{\partial x} \right) - \left(k_y^2 + k_z^2\right) P_1 = 0$$

- Now, $\rho_0 \left(\omega^2 - k_z^2 v_\mathrm{A}^2\right)$ is not a constant in the nonuniform transition.
- Resonance (singularity) at the specific location $x = x_\mathrm{A}$ where $\omega = \omega_\mathrm{A}(x_\mathrm{A})$
- Let us assume that $\rho_0 \left(\omega^2 - k_z^2 v_\mathrm{A}^2\right) = x - x_0$, so that

$$\frac{\partial^2 P_1}{\partial x^2} - \frac{1}{x - x_0} \frac{\partial P_1}{\partial x} - \left(k_y^2 + k_z^2\right) P_1 = 0$$

- Solution:

$$P_1 = (x - x_0) \left(A_1 I_1 \left[k(x - x_0)\right] + A_2 K_1 \left[k(x - x_0)\right]\right)$$

$$k = \sqrt{k_y^2 + k_z^2}$$

The phenomenon of resonant absorption

- In a thin nonuniform transition, $k(x - x_0) \ll 1$, so that we can perform a series expansion of I_1 and K_1 for small arguments:

$$P_1 = constant + \mathcal{O}(x - x_0)^2$$

- The jump of P_1 across the nonuniform transition up to $\mathcal{O}(x - x_0)^2$ is

$$[[P_1]]_{\mathrm{transition}} \approx 0$$

- Lagrangian displacement:

$$\xi_x = \frac{1}{\rho_0\left(\omega^2 - k_z^2 v_{\mathrm{A}}^2\right)} \frac{\partial P_1}{\partial x} = \frac{1}{x - x_0} \frac{\partial P_1}{\partial x}$$

- Using the expression for P_1 and performing again a series expansion:

$$\xi_x = constant + \frac{k^2 P_1}{\omega^2(\rho_i - \rho_e)/l} \ln x + \mathcal{O}(x - x_0)^2$$

- The jump of ξ_x across the nonuniform transition up to $\mathcal{O}(x - x_0)^2$ is

$$[[\xi_x]]_{\mathrm{transition}} \approx \frac{k^2 P_1}{\omega^2(\rho_i - \rho_e)/l} \ln(-1) = -i\pi \frac{k^2 l}{\omega^2(\rho_i - \rho_e)} P_1$$

The phenomenon of resonant absorption

- We use the jumps of P_1 and ξ_x to find a dispersion relation for the surface wave without specifying the wave perturbations in the thin nonuniform later

$$P_1(x) = \begin{cases} A_\mathrm{i} \exp\left[\left(k_y^2 + k_z^2\right) x\right], & \text{if} \quad x < -l/2, \\ A_\mathrm{e} \exp\left[-\left(k_y^2 + k_z^2\right) x\right], & \text{if} \quad x > l/2. \end{cases}$$

$$\xi_x = \frac{1}{\rho_0 \left(\omega^2 - k_z^2 v_\mathrm{A}^2\right)} \frac{\partial P_1}{\partial x}$$

$$[[P_1]]_\mathrm{transition} \approx 0, \qquad [[\xi_x]]_\mathrm{transition} \approx -i\pi \frac{k^2 l}{\omega^2(\rho_i - \rho_e)} P_1$$

- Dispersion relation:

$$\rho_i \left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right) + \rho_e \left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right)$$
$$-i\pi k l \frac{\rho_i \rho_e}{\rho_i - \rho_e} \frac{\left(\omega^2 - k_z^2 v_{\mathrm{A,i}}^2\right)\left(\omega^2 - k_z^2 v_{\mathrm{A,e}}^2\right)}{\omega^2} = 0$$

The phenomenon of resonant absorption

- The surface wave dispersion relation is complex, so that the frequency will be complex: $\omega = \omega_{\mathrm{R}} + i\omega_{\mathrm{I}}$
- The imaginary part of ω is a **damping rate** of the surface wave
- We seek an analytic solution when $\omega_{\mathrm{I}} \ll \omega_{\mathrm{R}}$ valid for thin nonuniform transitions

$$\omega_{\mathrm{R}} \approx k_z \sqrt{\frac{\rho_i v_{\mathrm{A,i}}^2 + \rho_e v_{\mathrm{A,e}}^2}{\rho_i + \rho_e}} = \omega_k$$

$$\omega_{\mathrm{I}} \approx -\frac{\pi}{8} kl \frac{\rho_i - \rho_e}{\rho_i + \rho_e} \omega_k$$

- Damping rate is proportional to the width of the nonuniform transition, l

- Resonant damping is an ideal process: NO DISSIPATION
- It physically represents that the motions on the interface loose coherence $\rightarrow$ Another manifestation of phase mixing!

Summary

- Plasma structuring affects MHD waves.
- New class of wave: surface wave.
- Surface waves are guided by the structure (interface).
- In the presence of flow, surface waves can be Kelvin-Helmholtz unstable.
- The Kelvin-Helmholtz instability drives turbulence $\rightarrow$ energy cascades to small scales $\rightarrow$ efficient dissipation.
- In the presence of nonuniformity, local Alfvén waves undergo phase mixing and surface waves undergo resonant absorption$\rightarrow$ energy cascades to small scales $\rightarrow$ efficient dissipation.