Tracking problems

A. Statement of the Problem

Given the plant as

$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$,

the output relation as

$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$,

the performance index as

$$J(k_0) = \frac{1}{2}\left[\mathbf{C}\mathbf{x}(k_f) - \mathbf{z}(k_f)\right]' \mathbf{F} \left[\mathbf{C}\mathbf{x}(k_f) - \mathbf{z}(k_f)\right] + \frac{1}{2}\sum_{k=k_0}^{k_f-1}\left\{\left[\mathbf{C}\mathbf{x}(k) - \mathbf{z}(k)\right]' \mathbf{Q}\left[\mathbf{C}\mathbf{x}(k) - \mathbf{z}(k)\right] + \mathbf{u}'(k)\mathbf{R}\mathbf{u}(k)\right\}$$

and the boundary conditions as

$\mathbf{x}(k_0) = \mathbf{x}_0$, $\mathbf{x}(k_f)$ is free, and k is fixed,

find the optimal control and state.

B. Solution of the Problem

Step 1	Solve the matrix difference Riccati equation $\mathbf{P}(k) = \mathbf{A}'\mathbf{P}(k+1)\left[\mathbf{I} + \mathbf{E}\mathbf{P}(k+1)\right]^{-1}\mathbf{A} + \mathbf{V}$ with $\mathbf{P}(k_f) = \mathbf{C}'\mathbf{F}\mathbf{C}$, where $\mathbf{V} = \mathbf{C}'\mathbf{Q}\mathbf{C}$ and $\mathbf{E} = \mathbf{B}\mathbf{R}^{-1}\mathbf{B}'$.
Step 2	Solve the vector difference equation $\mathbf{g}(k) = \mathbf{A}'\left\{\mathbf{I} - \left[\mathbf{P}^{-1}(k+1) + \mathbf{E}\right]^{-1}\mathbf{E}\right\}\mathbf{g}(k+1) + \mathbf{W}\mathbf{z}(k)$ with $\mathbf{g}(k_f) = \mathbf{C}'\mathbf{F}\mathbf{z}(k_f)$, where, $\mathbf{W} = \mathbf{C}'\mathbf{Q}$.
Step 3	Solve for the optimal state $\mathbf{x}^*(k)$ as $\mathbf{x}^*(k+1) = \left[\mathbf{A} - \mathbf{B}\mathbf{L}(k)\right]\mathbf{x}^*(k) + \mathbf{B}\mathbf{L}_g(k)\mathbf{g}(k+1)$ where, $\mathbf{L}(k) = \left[\mathbf{R} + \mathbf{B}'\mathbf{P}(k+1)\mathbf{B}\right]^{-1}\mathbf{B}'\mathbf{P}(k+1)\mathbf{A}$, $\mathbf{L}_g(k) = \left[\mathbf{R} + \mathbf{B}'\mathbf{P}(k+1)\mathbf{B}\right]^{-1}\mathbf{B}'$.
Step 4	Obtain the optimal control as $\mathbf{u}^*(k) = -\mathbf{L}(k)\mathbf{x}^*(k) + \mathbf{L}_g(k)\mathbf{g}(k+1)$.