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Define in Matlab/Simulink the control specified below for the following system.

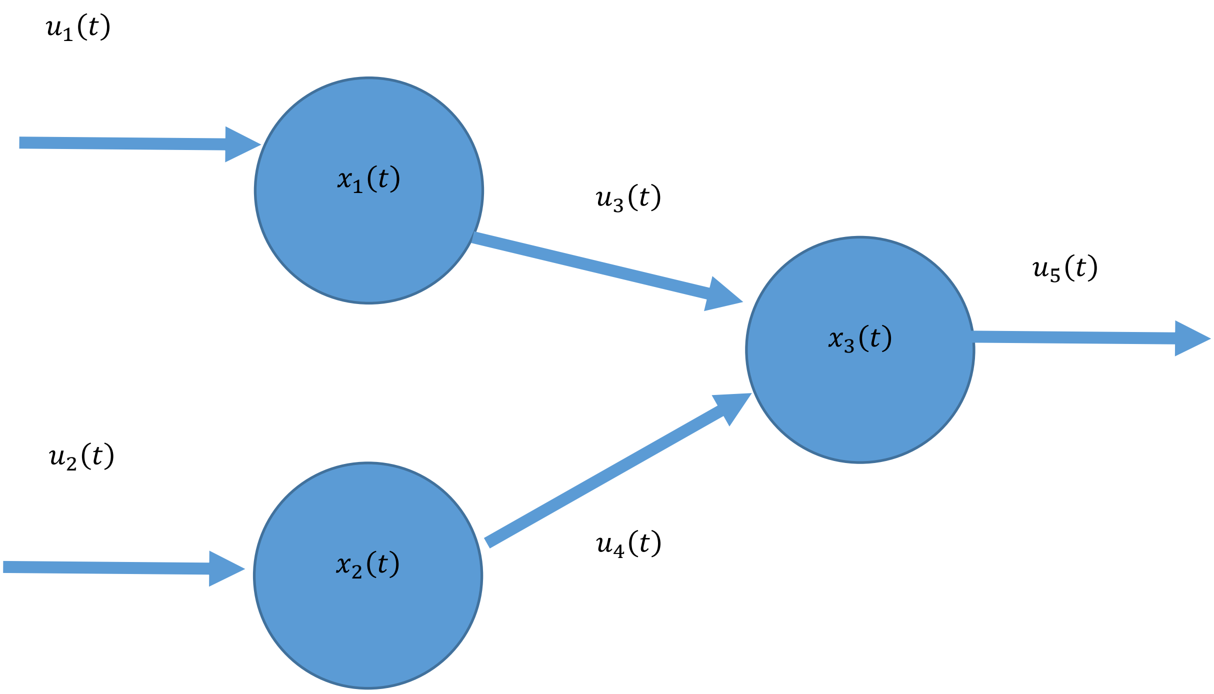
A network of inventories is defined by the following state equation.

$$\underline{x}(t+1) = \underline{x}(t) + \alpha\underline{u}(t)$$

where:

- *x(t)* is the vector of the quantities of each output product (inventories) in each production center on the *t*-th day. It is defined as the deviation from a given steady state condition. Each production center is supposed to produce just one product
- *u(t)* is the vector of product flows in input (raw material) and in output (final product for that production center). Some products are imported/exported in/from the network. Other products or part of them are used within the same network.
- α represents a matrix of the rate of materials for each production center.

In your example, please take into account the production network drawn below, consisting of three production centers. For example, production center 1 has an inventory described by $x_1(t)$, i.e. the first component of the vector $\underline{x}(t)$. The production center 1 requires the raw material $u_1(t)$ which is transformed in the same day in the final product and stored in the warehouse (so increasing the inventory). Similarly $u_4(t)$ is the rate of the final product in output from production center 2 and entering production 3 as raw material for a new production process.



For the production network, the following equations hold:

$$x_1(t+1) = x_1(t) + 2 * u_1(t) - u_3(t)$$
$$x_2(t+1) = x_2(t) + u_2(t) - 2u_4(t)$$
$$x_3(t+1) = x_3(t) + 3u_4(t) + u_3(t) - u_5(t)$$

In addition, it is required that:
$u_3(t) \sim 4 * u_1(t)$
$u_4(t) \sim 2 * u_5(t)$
$x_2(t) \sim x_1(t)$

1) Under the hypothesis of perfect observability of the state (i.e. the system output is the state), define the optimal control, in order to minimize the quadratic error of the state and of the control with respect to zero, on a time horizon of 7 days. Verify the solution with different parameters in the cost functions.
2) Under the hypothesis that it is not possible to measure $x_2(t)$ (see y(t) definition below), that the measure of the state is affected by a zero mean error with normal distribution (with known variance), and that the model itself is affected by a zero mean error with normal distribution (with known variance)
define the optimal control.
Suggestion. Use LQG. The system is so given by:

$x_1(t+1) = x_1(t) + 2 * u_1(t) - u_3(t) + w_1(t)$
$x_2(t+1) = x_2(t) + u_2(t) - 2u_4(t) + w_2(t)$
$x_3(t+1) = x_3(t) + 3u_4(t) + u_3(t) - u_5(t) + w_3(t)$

$u_3(t) \sim 4 * u_1(t)$
$u_4(t) \sim 2 * u_5(t)$
$x_2(t) \sim x_1(t)$

Output
$y_1(t) = x_1(t) - x_3(t) + v_1(t)$
$y_2(t) = x_3(t) - x_2(t) + v_2(t)$

3) Under the hypothesis of point 1, identify an optimal control which minimize the quadratic value of the state and of the control to zero, but for the following value to be tracked so that $x_1(t) \sim 10 \quad t = 3\,, t = 4\; e\; x_2(t) \sim -10\; \forall t\; e\; x3(5) \sim 0$