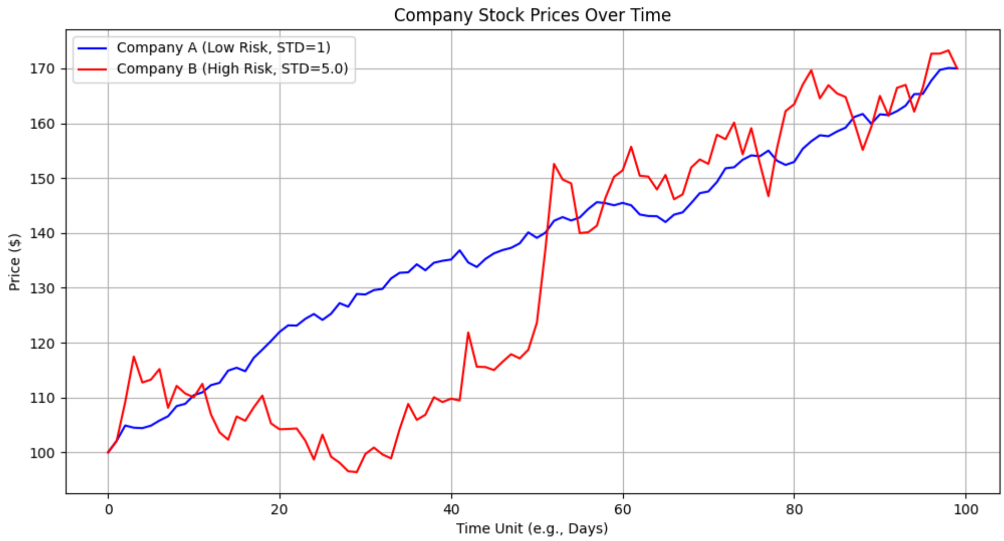
Risk Management and Portfolio Diversification: An Introductory Analysis

[Lucas-Bussinger/Portfolio_Risk_Management: This is a finance project on risk/earning optimization techniques]

1 - Introduction - Risk management

When making an investment, everyone seeks the highest possible return. However, returns always come with **risk** — the uncertainty in how prices change over time.

Let's look at two companies, A and B (see Figures below). Both start and end with the same stock price after 100 days. At first glance, they seem equally good. But if an investor needed to withdraw before day 100, the outcome would be very different. For Company A, the price moved smoothly upwards. For Company B, the price fluctuated a lot — meaning that if the investor sold on day 24, they would actually lose money.



This shows that **Company B is riskier than Company A**, even though both had the same final return.

In finance, we measure this risk as the **variation of returns** over time. Mathematically, it can be written as:

$$Var(Ri) = \beta^2 Var(Rm) + Var(\varepsilon_i)$$

- $\beta^2 Var(Rm)$: risk that comes from the market (systematic risk)
- $Var(\varepsilon_i)$: risk specific to the company (idiosyncratic risk)

- β : angular coefficient of the linear regression between a company and a market (e.g how the company responds to a market change)

The goal of this project is to explore how investors can **reduce risk without giving up returns**. To do this, we build portfolios with different combinations of companies. By combining them, we aim to create a risk management process in which we try to cancel out company-specific risks (idiosyncratic risks) and bring the total portfolio risk as close as possible to the **market risk**, which cannot be eliminated.

2 - Theoretical Background - Metrics

Evaluation Metrics

This project uses three main metrics to evaluate portfolio outcomes:

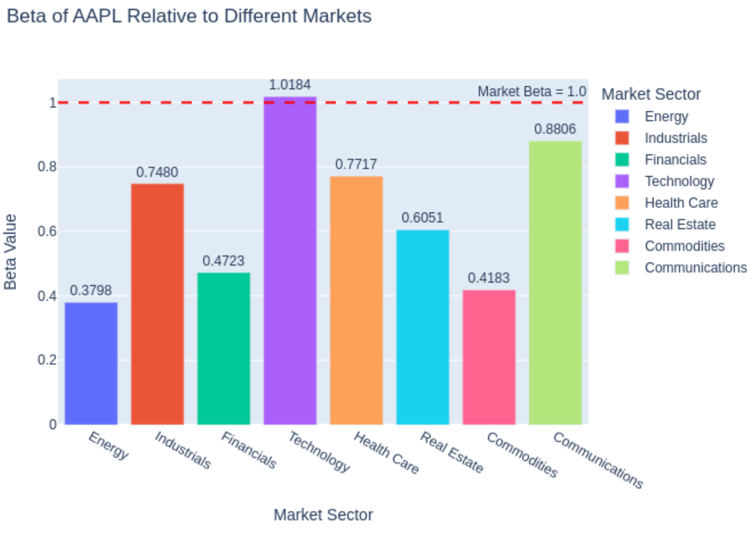
1. **Beta** – measures how sensitive a company (or portfolio) is to market movements.
2. **Sharpe Ratio** – compares return to risk, showing how efficiently a portfolio generates returns.
3. **Risk/Return Plots** – visualize the relationship between portfolio risk and expected earnings, helping to identify efficient portfolios.

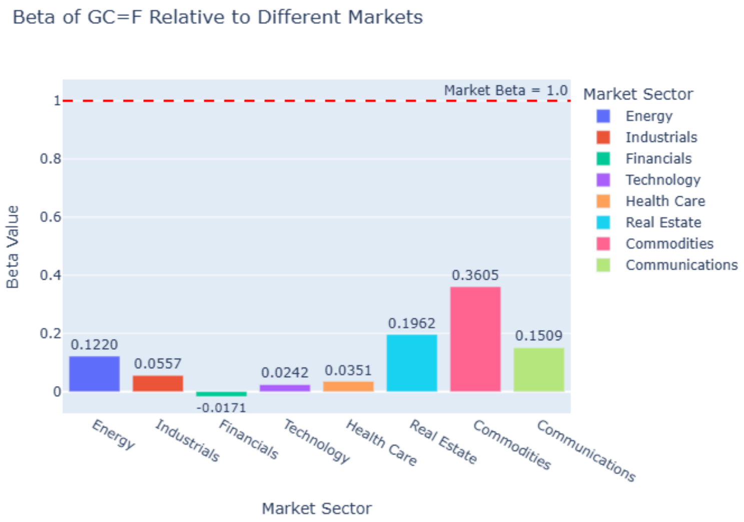
2.1) Beta - β

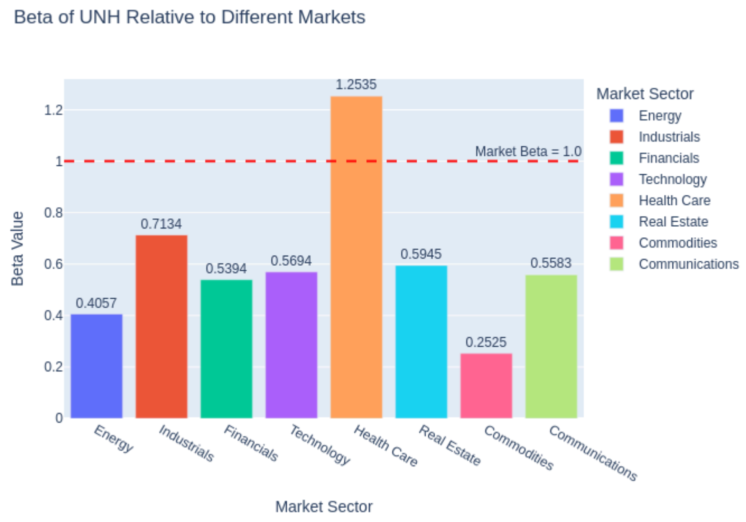
As explained before, **beta** is the slope (angular coefficient) of a linear regression between a company's/asset's price changes and the overall market's price changes. In practice, it shows **how much the company is influenced by market movements**:

- $\beta = 1$: The asset follows exactly the market change
- $\beta > 1$: The asset overreacts to the market change
- $0 < \beta < 1$: The asset underreacts to the market change.
- $\beta < 0$: The asset goes contrary to the market change

Examples:







AAPL = Apple - Follows the “Technology” market
CL=F = Oil - Overreacts to the “Commodities” market
GC=F = Gold - Underreacts to every market, Negative β in the “Financials” sector
UNH = United Heath Group - Follows (and overreacts) the “Health Care” market.

It’s interesting to note that some assets, like gold, often move differently from the market. Gold is considered a safe asset, meaning that when markets are falling, investors tend to buy gold (directly or indirectly) as protection. Because of this, gold may underreact to market movements, or even move in the opposite direction, showing a low or negative beta.

2.2) Sharpe Ratio - S

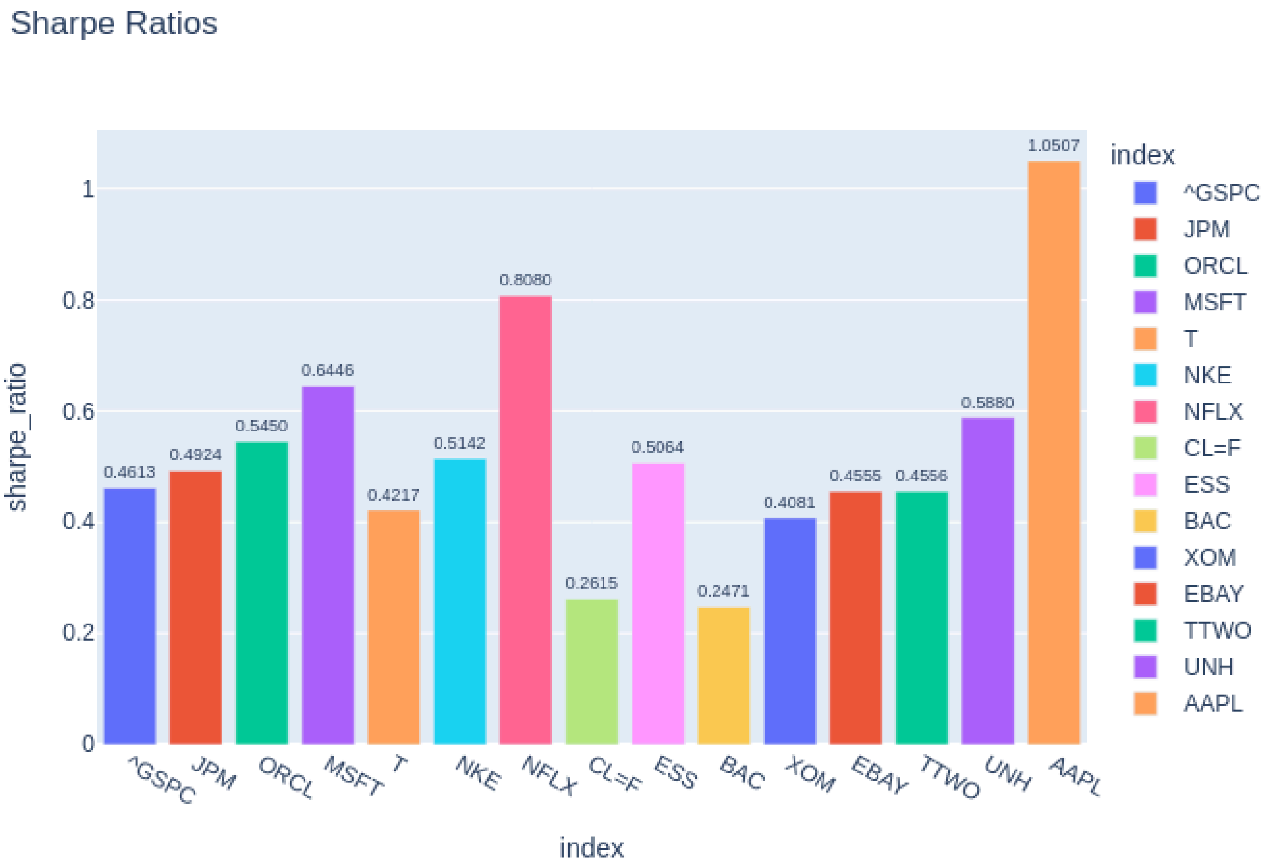
$$S = \frac{R_p - R_f}{\sigma_p}$$

- R_p = average asset return (annually, monthly, weekly…)
- R_f = risk free rate (annually, monthly, weekly…) .
- σ_p = standard deviation of the portfolio’s returns ($\sqrt{Var(R_i)}$)

The Sharpe Ratio shows **how much extra return an investment generates per unit of risk**, compared to a risk-free asset. In other words, it answers: *“For each unit of risk I take, how much am I earning above the risk-free rate?”* - The higher the better.

For Comparison: Warren Buffett's Berkshire Hathaway has a Sharpe Ratio of 0.79

Example:



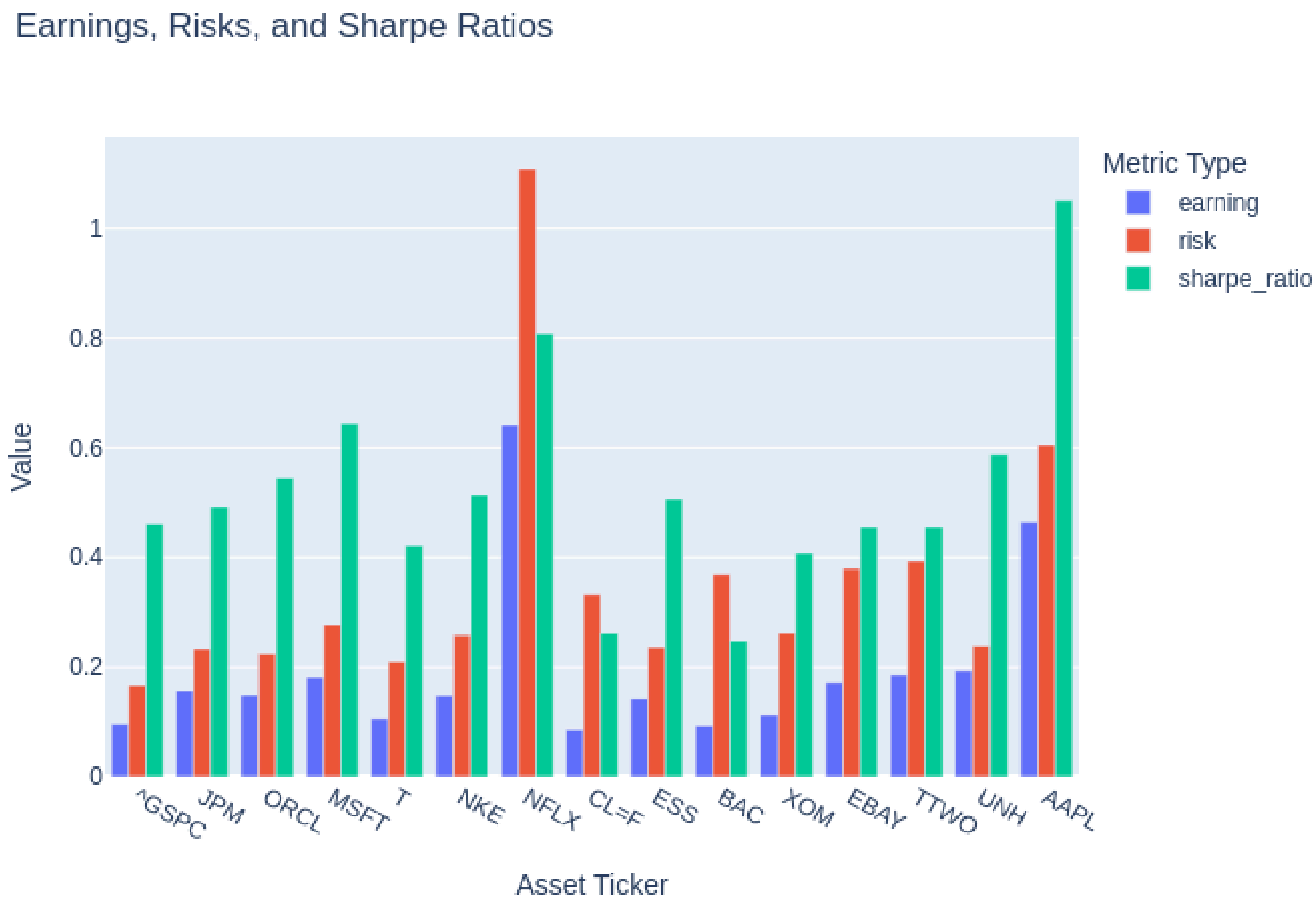
Note: those are the companies that will be used for **posterior** testing and **analysis** in the project

The Sharpe Ratios were calculated using weekly data (with annualized returns) over the period from 1995 to 2025. Each bar in the plot represents a company, illustrating its risk-adjusted performance.

Notably, some companies stand out as outliers, such as Apple and Netflix, due to their exceptional long-term success. Netflix, in particular, represents a special case that will be revisited in the **Efficient Frontier** chapter, given its unique impact on portfolio construction.

As a reference, the S&P 500 over this same period achieved a Sharpe Ratio of approximately 0.46, which serves as a reasonable estimate of the overall market's risk-adjusted return.

But the Sharpe Ratio is only meaningful when compared with other metrics, such as risk and return. For example, a company may have higher returns than the market but also much higher risk, which could result in a lower Sharpe Ratio, hiding the tremendous return aspect of the investment:



When looking only at the Sharpe Ratios (first chart), Apple appears to be the clear best investment choice, significantly outperforming the others. However, when we break the Sharpe Ratio down into its underlying components—mean annual returns and risk (second chart)—a more nuanced picture emerges.

For instance, Netflix exhibits considerably higher average annual returns than Apple, but these come at the cost of substantially greater risk. In this case, while Apple offers a more efficient return per unit of risk, a risk-tolerant investor might still prefer Netflix due to its higher absolute return potential.

This comparison highlights an important point: the Sharpe Ratio is not an absolute measure of attractiveness. It is only meaningful when interpreted alongside its base metrics—risk and return. Without considering these components, conclusions drawn from the Sharpe Ratio alone can be misleading.

2.3) **Risk/Return Plots**

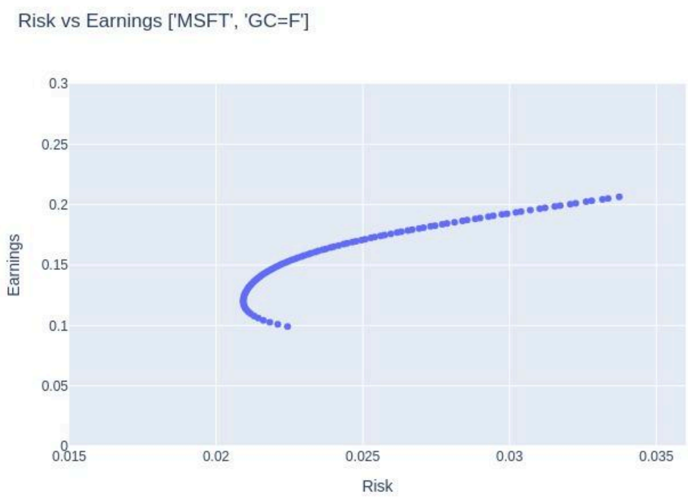
This metric is straightforward: it consists of plotting portfolio returns against their corresponding risks. Such a visualization allows investors to quickly identify the trade-off between risk and reward for different assets or portfolios. By observing the relative positioning of points on the plot, one can distinguish which assets provide higher returns for a given level of risk, and conversely, which assets involve greater risk without a proportional increase in return.

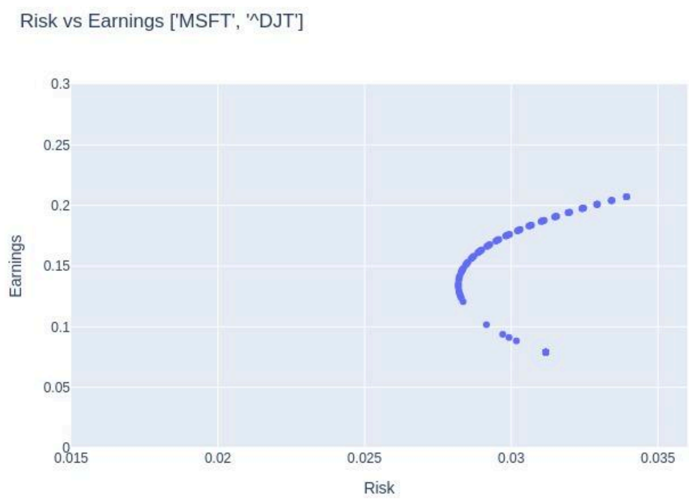
3 - Optimization - Feasible areas & Efficient Frontiers

In this section, our objective is to optimize the balance between risk and return when investing in a specific set of companies. To achieve this, we employ **portfolio simulations**, where the allocation of capital among the selected companies is systematically varied. For each allocation (e.g., 25% in Netflix, 25% in Apple, and 50% in the S&P500), we obtain a unique point on the **risk–return plot**, representing the portfolio's expected return and its corresponding risk. Collectively, these points allow us to visualize the trade-offs and identify efficient portfolios.

3.1 - Sets with 2 companies/assets

When dealing with sets of two companies we have a plots look like the following:

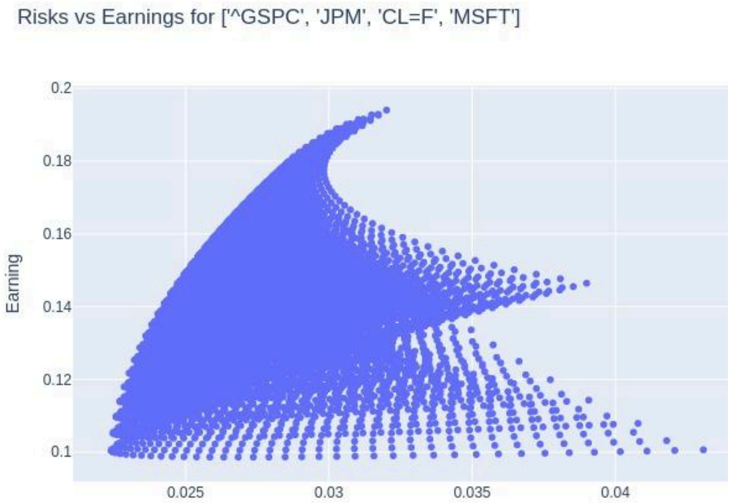


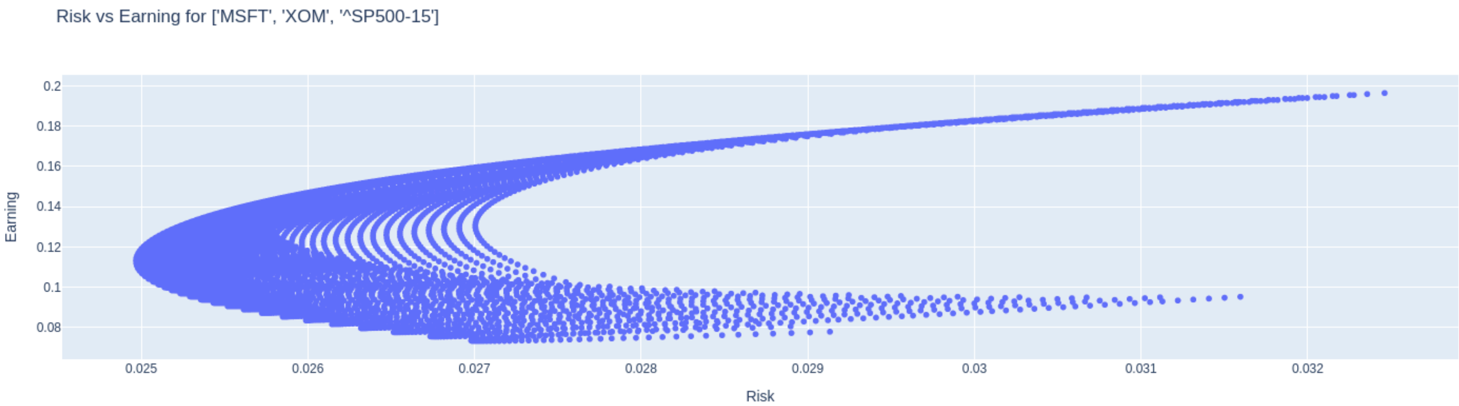


It is important to note that, for portfolios composed of two assets, certain regions represent inefficient choices regardless of the investor's risk profile—whether risk-averse or risk-seeking. In the lower-left areas of these plots, every portfolio point has an alternative allocation that achieves a strictly higher return for the same level of risk. Consequently, portfolios in these regions are considered dominated and should not be part of an optimal investment strategy. The upper boundary of the curve, on the other hand, defines the ***Efficient Frontier***—a set of portfolios that offers the maximum possible return for each given level of risk (to be further discussed in the next section).

3.2 - Sets with 3 or more companies/assets

When extending the analysis to portfolios composed of more than two assets, the resulting plots take on more complex shapes, as illustrated below.



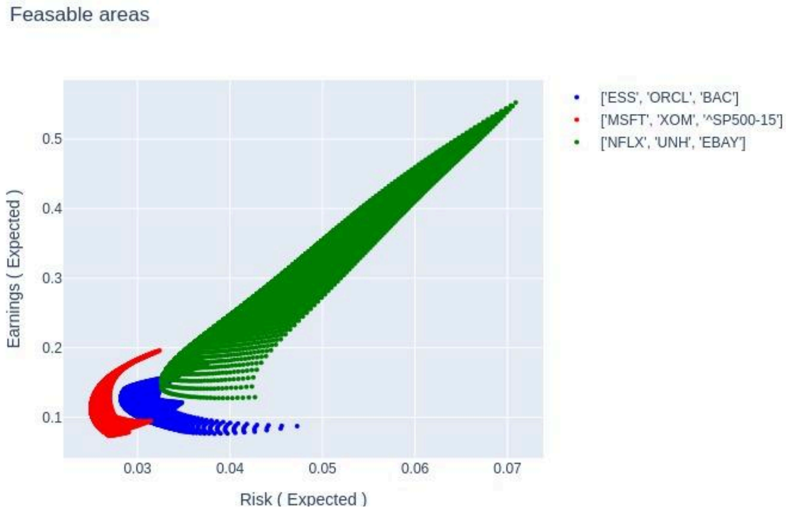


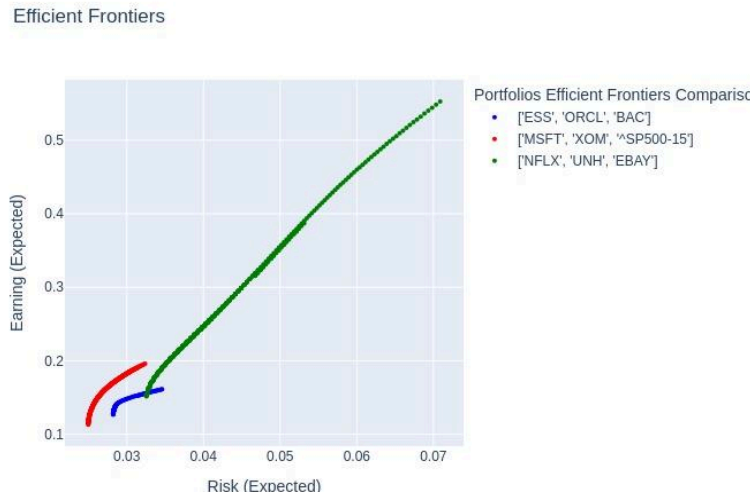
With more than two assets, the number of possible allocation combinations increases significantly. These allocations fall within what is known as the **Feasible Region**—the set of all achievable risk–return pairs given the chosen assets and their correlations. Within this region, only the upper boundary corresponds to the Efficient Frontier, representing the optimal trade-off between risk and return. Consequently, when investing in a given set of companies, the likelihood of making a suboptimal allocation—one that lies inside the feasible region but below the **Efficient Frontier**—is quite high without the use of optimization techniques.

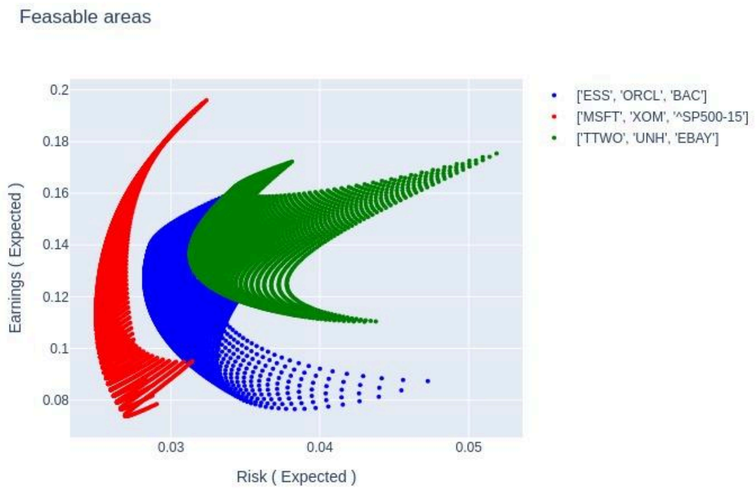
It is important to note that the plot including Netflix (NFLX) appears highly stretched. This distortion arises from the company's outlier behavior, characterized by exceptionally high returns accompanied by equally high levels of risk, as discussed before.

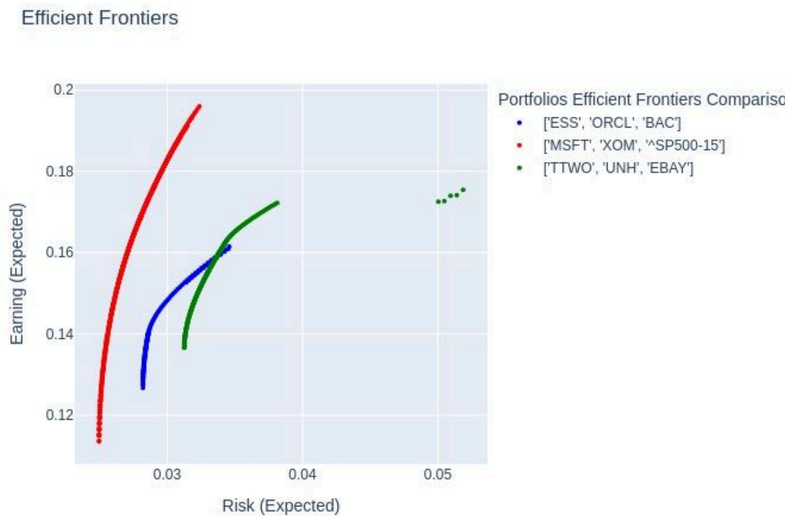
3.3 - Multiple Sets

Different sets of companies correspond to different feasible areas and efficient frontiers. In other words, the specific companies you choose to invest in directly determine the shape and position of the resulting efficient frontier:



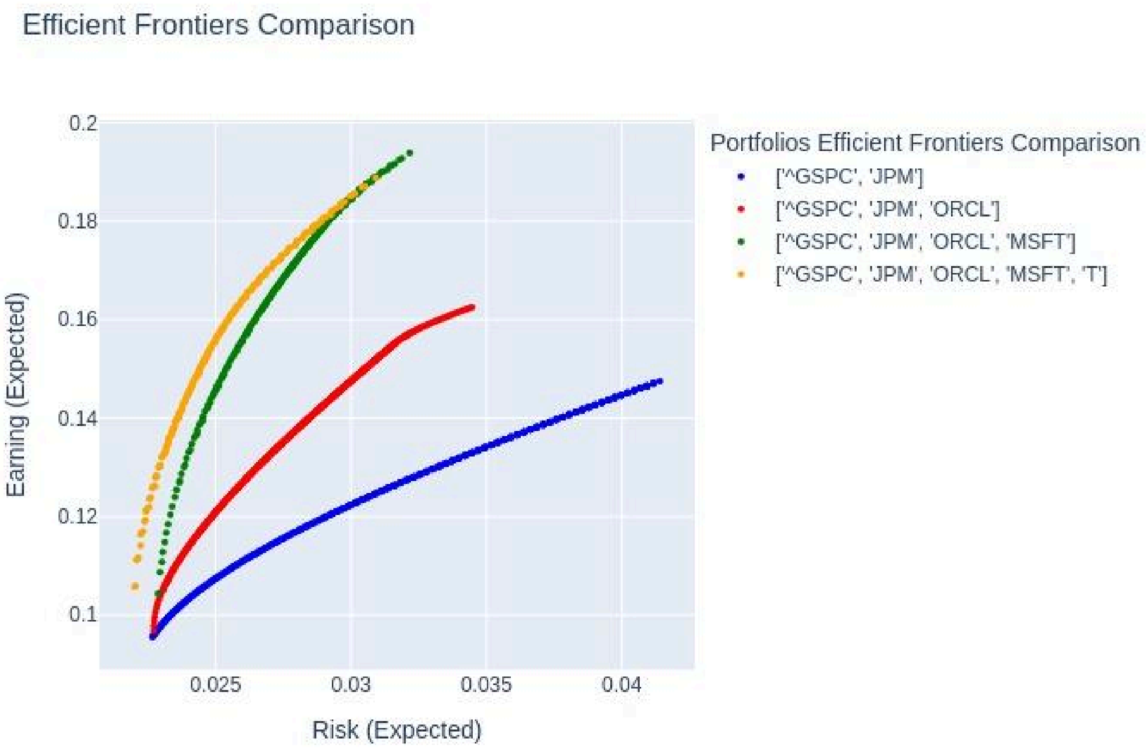




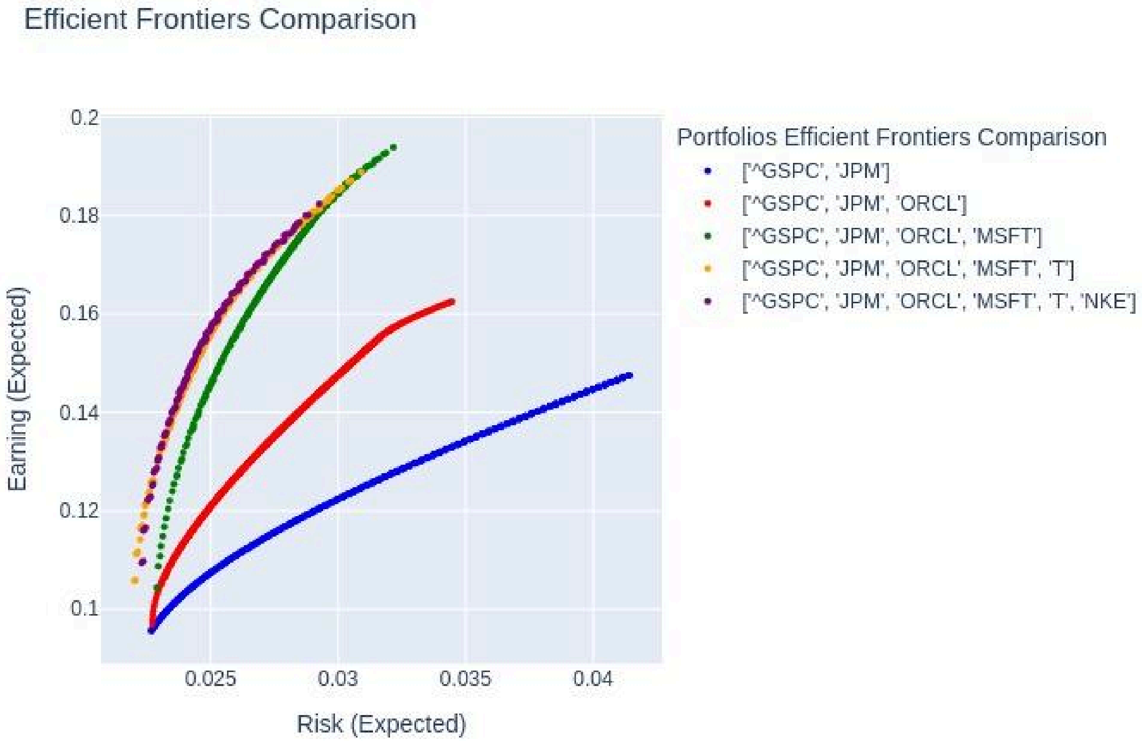


4 - Portfolio Diversification - mitigating idiosyncratic risks

To illustrate the impact of diversification on a portfolio's expected risk and return, we begin with a specific set of companies and plot its efficient frontier. Then, we add one additional company to the set, re-plot the efficient frontier, and repeat this process.



As shown in the figure, adding more companies to the portfolio generally shifts the efficient frontier upward, meaning that for a given level of risk, the expected return increases. However, the incremental gains diminish as more companies are added—the 'gap' between frontiers becomes smaller each time. This raises an important question: is there a point at which further diversification no longer provides a meaningful reduction in risk for a given level of return? See what happens when we add one more company to the set:



The efficient frontier is no longer shifting upward. To understand why, we must examine the risk equation.
For a portfolio of two assets, the variance of returns is given by:

$$Var\ (Rp)\ = X1^2\ *\ Var(r_1)\ +\ X2^2 * Var(r_2)\ +\ 2 * X1 * X2 * Cov(r_1, r_2)$$

Where X1 and X2 are the portfolio weights of each asset.
Generalizing to a portfolio with n assets:

$$Var\ (R_p)\ =\ \sum_{i=1}^{n}\sum_{j=1}^{n} X_i\ X_j Cov(r_i, r_j)$$

The idiosyncratic risk of an asset appears only when $i = j$: $X_i^2 * Var(r_i)$,
Thus, the total contribution of idiosyncratic risk is:

$$\sum_{k=1}^{n} X_k^2 * Var(\epsilon_k)$$ (see the equation in section 1),

Since each $X_k < 1$, adding more assets distributes portfolio weights across many firms. As a result, the aggregate impact of idiosyncratic risk diminishes rapidly, because it enters the formula in squared form. Moreover, firm-specific risks are typically uncorrelated across firms, meaning they tend to offset each other when the portfolio is sufficiently diversified.

To formalize this, consider the Capital Asset Pricing Model (CAPM):

$r_i = \beta_i * r_m + \epsilon_i$

where:

- r_i is the return from the asset
- r_m is the return from the associated market
- β_i is the asset sensitivity to the associated market (beta)
- ϵ_i is the idiosyncratic risk of the asset, with $E[\epsilon_i] = 0$ and $Var(\epsilon_i) = \sigma_{\epsilon_i}^2$

The variance of the portfolio can then be expressed as:

$$Var(R_p) = \beta_p^2 Var(r_m) + \sum_{g=1}^{n} X_i^2 \sigma_{\epsilon_i}^2, \text{ with } \beta_p = \sum_{i=1}^{n} X_i \beta_i$$

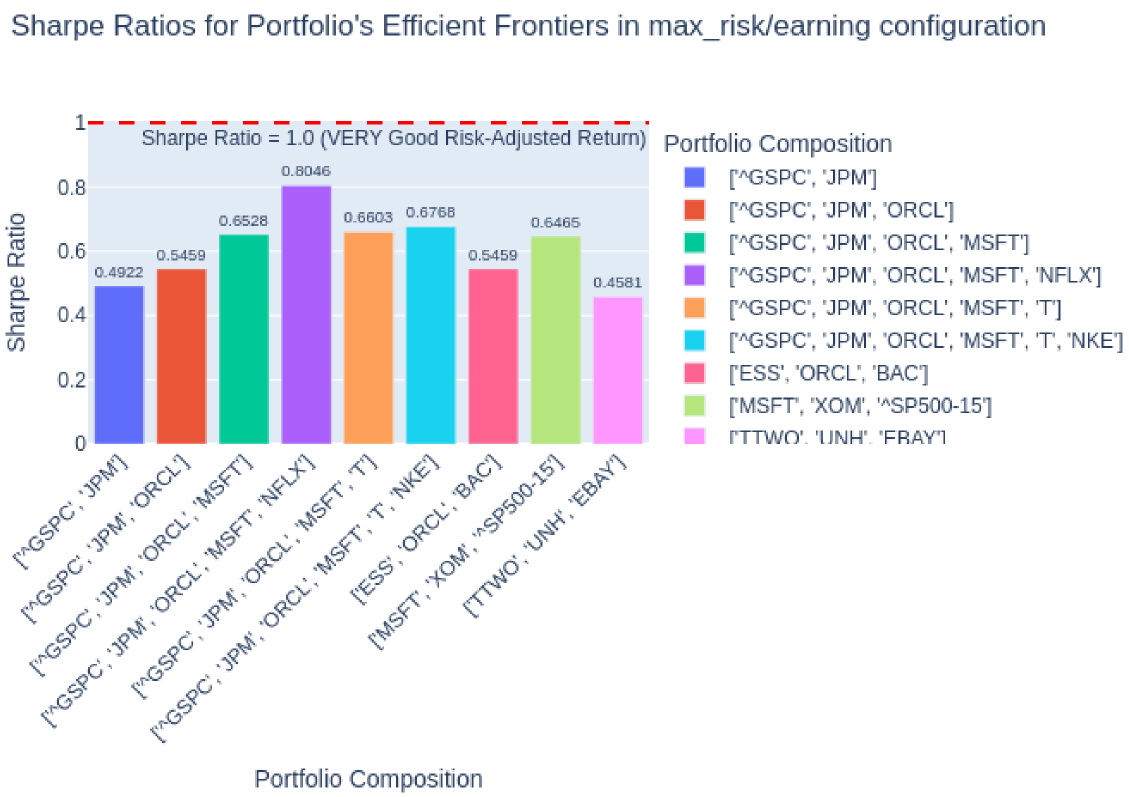
Where:

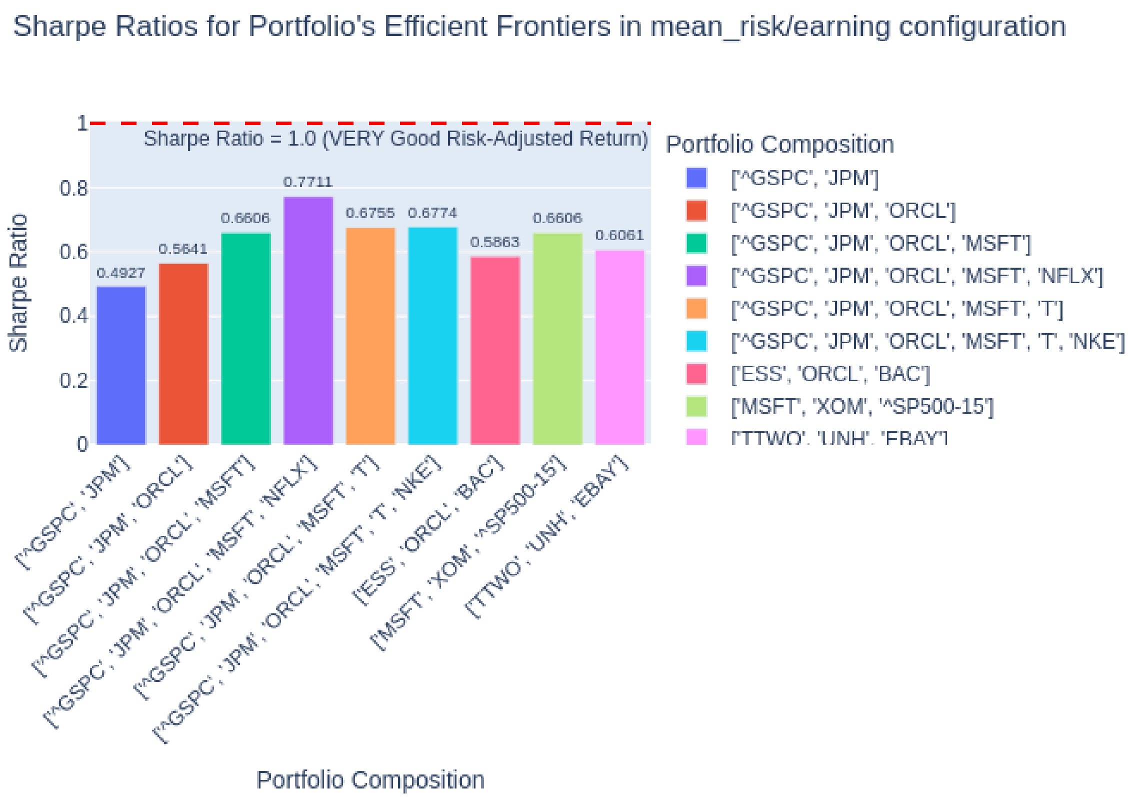
- $\beta_p^2 Var(r_m)$ - Market Risk
- $\sum_{g=1}^{n} X_i^2 \sigma_{\epsilon_i}^2$ - Idiosyncratic Risk

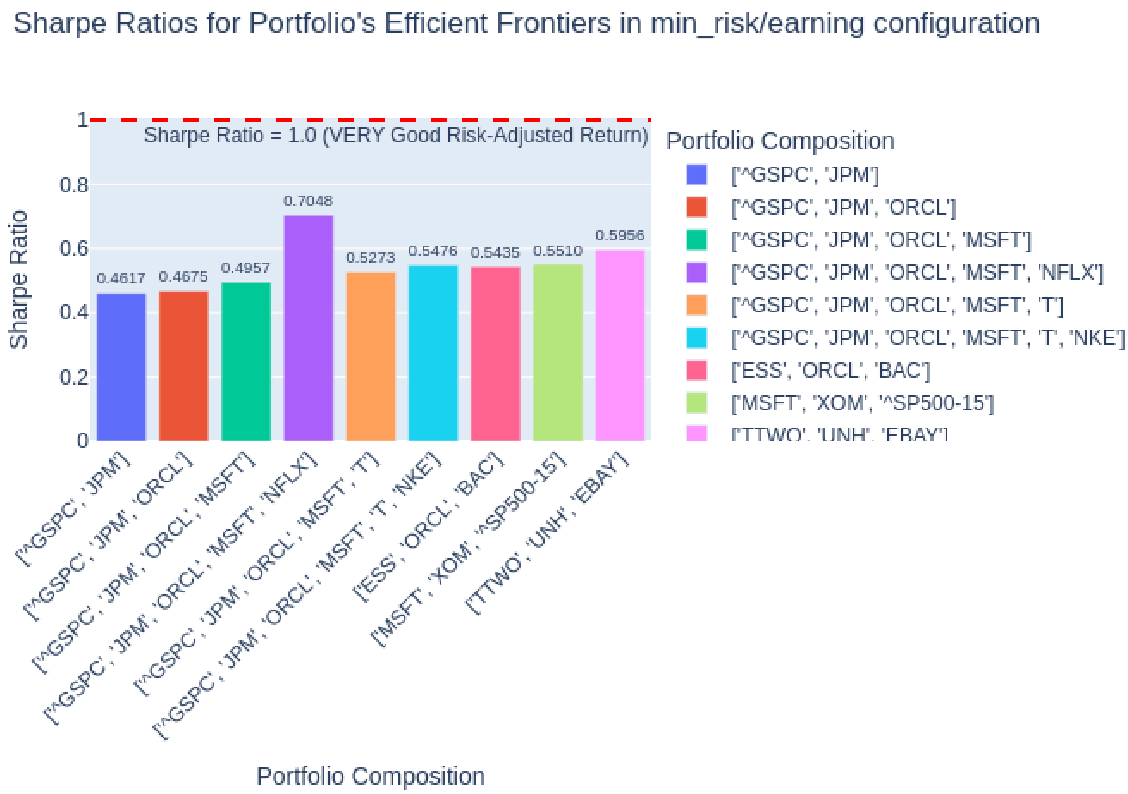
The second term is a weighted sum of idiosyncratic risks. Because it is scaled by X_i^2, this component shrinks as the portfolio becomes more diversified, tending toward zero. Consequently, once firm-specific risk has been diversified away, the total portfolio variance converges to market risk, which cannot be eliminated. This explains why, in our experiment, the addition of the final asset NKE (Nike) produced almost no improvement in the efficient frontier: the portfolio risk was already dominated by systematic market risk.

5 - Diversified portfolios - Sharpe Ratios of efficient frontiers

To conduct this analysis, we select three points from the distribution of portfolio weight combinations that lie on the efficient frontier: the minimum-risk point, the mean-risk point, and the maximum-risk point. We then calculate and compare the Sharpe ratios at these points.







From the plots, an interesting pattern emerges: the Sharpe ratios associated with the mean risk/return configuration are typically the highest among all points on the efficient frontier, suggesting that this configuration provides a strong risk-adjusted performance.

Another noteworthy observation is that the Sharpe ratios of diversified portfolios on the efficient frontier are, in most cases, higher than the Sharpe ratios of the individual assets that compose them (see Section 2.2). This highlights the benefits of diversification, particularly around the mean risk/return configuration.

The table below compares the Sharpe ratios of portfolio combinations with those of the individual assets (evaluated at the mean risk/return configuration):

Portfolio Combination	**Portfolio Sharpe Ratio**	**Individual Assets Sharpe Ratios**
[^GSPC, JPM]	**0.4927**	{ ^GSPC: 0.4613, JPM: **0.4924** }
[^GSPC, JPM, ORCL]	**0.5641**	{ ^GSPC: 0.4613, JPM: 0.4924, ORCL: **0.5450** }
[^GSPC, JPM, ORCL, MSFT, T]	**0.6755**	{ ^GSPC: 0.4613, JPM: 0.4924, ORCL: 0.5450, MSFT: **0.6446**, T: 0.4217 }