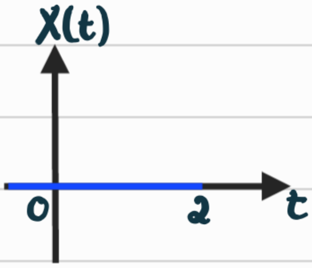
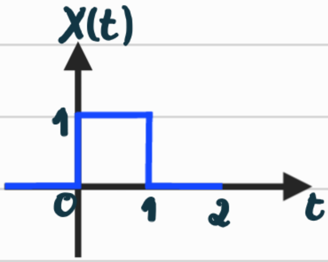
Lucas Coelho Raupp

8) a) $X(t) = A\,\mathrm{rect}(t-1/2) + B\,\mathrm{rect}(t-3/2), \quad A, B \overset{iid}{\sim} \mathrm{Unif}([0,4])$

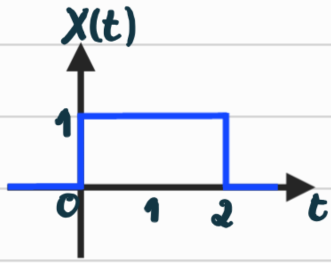
$A=0 \quad B=0$



$A=1 \quad B=0$



$A=1 \quad B=1$



b) $\mu_X(t) = E[X(t)]$

$\mu_X(t) = E[A\,\mathrm{rect}(t-1/2) + B\,\mathrm{rect}(t-3/2)]$

$\mu_X(t) = E[A\,\mathrm{rect}(t-1/2)] + E[B\,\mathrm{rect}(t-3/2)]$

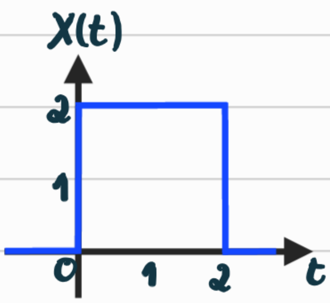
$\mu_X(t) = \mathrm{rect}(t-1/2)\cdot E[A] + \mathrm{rect}(t-3/2)\cdot E[B]$

$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

$E[A] = \int_0^4 a \cdot \frac{1}{4-0}\,da = \frac{1}{4}\cdot\int_0^4 a\cdot da = \frac{1}{4}\cdot\frac{a^2}{2}\Big|_0^4 = \frac{1}{8}(4^2-0^2) = \frac{1}{8}\cdot 16$

$E[A] = 2 \qquad E[B] = E[A]$

$\mu_X(t) = 2\,[0 \leq t \leq 2]$ //



c) $C_X(t_1,t_2) = \operatorname{cov}[X(t_1), X(t_2)]$

$C_X(t_1,t_2) = E[X(t_1)X(t_2)] - E[X(t_1)]\cdot E[X(t_2)]$

$E[X(t_1)X(t_2)] = E[(A\,\mathrm{rect}(t_1-1/2)+B\,\mathrm{rect}(t_1-3/2))\cdot(A\,\mathrm{rect}(t_2-1/2)+B\,\mathrm{rect}(t_2-3/2))] =$

$E[A^2\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-1/2)+AB\,\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-3/2)+AB\,\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-1/2)+$

$B^2\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-3/2)] = E[A^2]\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-1/2)+$

$E[AB](\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-3/2)+\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-1/2))+E[B^2]\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-3/2)$

$E[X^2] = \int_{-\infty}^{\infty} x^2\cdot f_X(x)\,dx$

$E[A^2] = \int_0^4 a^2\cdot\frac{1}{4-0}\,da = \frac{1}{4}\int_0^4 a^2\,da = \frac{1}{4}\cdot\frac{a^3}{3}\Big|_0^4 = \frac{1}{12}\cdot(4^3-0) = \frac{64}{13}$ $\quad E[A^2] = \frac{16}{3}$

$E[B^2] = \int_0^4 b^2\cdot\frac{1}{4-0}\,db = \frac{1}{4}\int_0^4 b^2\,db = \frac{1}{4}\cdot\frac{b^3}{3}\Big|_0^4 = \frac{1}{12}(4^3-0^3) = \frac{64}{13}$ $\quad E[B^2] = \frac{16}{3}$

$E[AB] = \int_0^4\int_0^4 a\cdot b\cdot\left(\frac{1}{4-0}\right)^2 da\,db = \frac{1}{16}\int_0^4 b\cdot\frac{a^2}{2}\Big|_0^4 db = \frac{1}{2}\cdot\frac{b^2}{2}\Big|_0^4 = \frac{4^2}{4}$ $\quad E[AB] = 4$

$E[X(t_1)X(t_2)] = \frac{16}{3}[\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-1/2)+\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-3/2)]+$

$4[\mathrm{rect}(t_1-1/2)\mathrm{rect}(t_2-3/2)+\mathrm{rect}(t_1-3/2)\mathrm{rect}(t_2-1/2)]$

$$E[X(t_1)] = 2\,\mathrm{rect}\left((t_1-1)/2\right)$$

$$E[X(t_2)] = 2\,\mathrm{rect}\left((t_2-1)/2\right)$$

$$C_X(t_1,t_2) = \frac{16}{3}\left[\mathrm{rect}(t_1-1/2)\,\mathrm{rect}(t_2-1/2) + \mathrm{rect}(t_1-3/2)\,\mathrm{rect}(t_2-3/2)\right] +$$

$$4\left[\mathrm{rect}(t_1-1/2)\,\mathrm{rect}(t_2-3/2) + \mathrm{rect}(t_1-3/2)\,\mathrm{rect}(t_2-1/2)\right] - 4\left[\mathrm{rect}\left((t_1-1)/2\right)\mathrm{rect}\left((t_2-1)/2\right)\right]_{//}$$