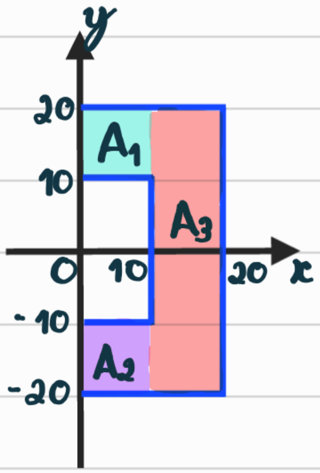
Lucas Coelho Raupp

10) a) $f_{x,y}(x,y) = k\left[(-20 \leq y \leq 20 \geq x \geq 0) \neq (-10 \leq y \leq 10 \geq x \geq 0)\right]$



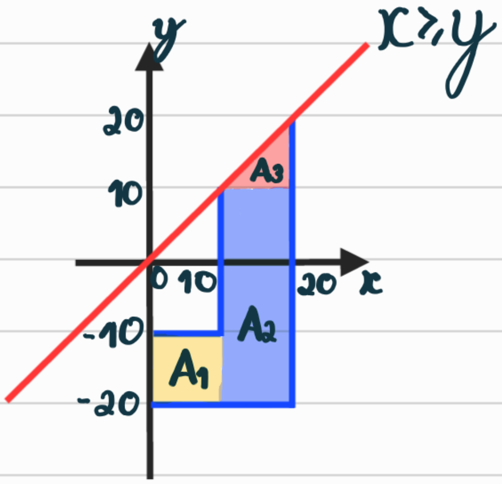
$A_1 = 10 \cdot (20-10) = 100\,u.A$

$A_2 = 10 \cdot (-10-(-20)) = 100\,u.A$

$A_3 = (20-10)(20-(-20)) = 400\,u.A$

$A_b = A_1 + A_2 + A_3 \qquad A_b = 100+100+400 \qquad A_b = 600\,u.A \qquad A_b \cdot k = 1 \qquad k = \dfrac{1}{600}$ //

b)



$A_1 = 10 \cdot (-10-(-20)) = 100$ u.A

$A_2 = (20-10)(10-(-20)) = 300$ u.A

$A_3 = (20-10)\cdot(20-10)\cdot \frac{1}{2} = 50$ u.A

$A_p = A_1 + A_2 + A_3 = 450$ u.A

600 u.A — 1
450 u.A — $Pr[X \geq Y]$

$600 \cdot Pr[X \geq Y] = 450 \qquad Pr[X \geq Y] = \frac{450}{600} \qquad Pr[X \geq Y] = 0{,}75$

c) $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

Caso $-\infty \leq y \leq -20$:

$$f_Y(y) = \int_{x=-\infty}^{x=\infty} 0\,dx = 0$$

Caso $-20 \leq y \leq -10$

$$f_Y(y) = \int_{x=0}^{x=20} \frac{1}{600}\,dx = \left[\frac{x}{600}\right]_0^{20} = \frac{20}{600} - \frac{0}{600} = \frac{2}{60} \qquad f_Y(y) = \frac{1}{30} = \frac{2}{60}$$

Caso $-10 \leq y \leq 10$

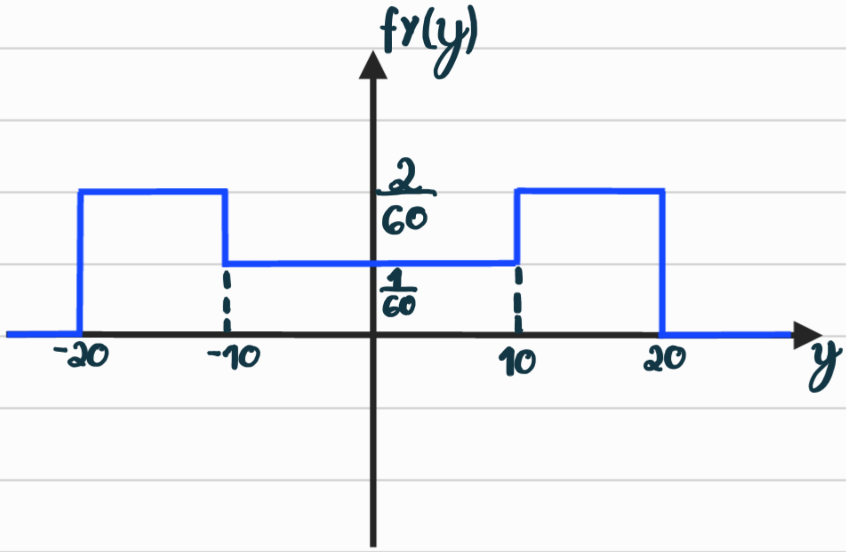
$$f_Y(y) = \int_{x=10}^{x=20} \frac{1}{600}\,dx = \left[\frac{x}{600}\right]_{10}^{20} = \frac{20}{600} - \frac{10}{600} = \frac{10}{600} \qquad f_Y(y) = \frac{1}{60}$$

Caso $10 \leq y \leq 20$

$$f_Y(y) = \int_{x=0}^{x=20} \frac{1}{600}\,dx = \left[\frac{x}{600}\right]_0^{20} = \frac{20}{600} - \frac{0}{600} = \frac{20}{600} \qquad f_Y(y) = \frac{1}{30} = \frac{2}{60}$$

Caso $20 \leq y \leq \infty$

$$f_Y(y) = \int_{x=-\infty}^{x=\infty} 0\,dx = 0$$



fY(y)
2/60
1/60
-20
-10
10
20
y

d) $F_Y(y)=\int_{-\infty}^{y^+} f_Y(u)du$

Caso $y<-20$

$$F_Y(y)=\int_{-\infty}^{y} 0\,du=0$$

Caso $-20\leq y<-10$

$$F_Y(y)=\int_{-20}^{y} \frac{2}{60}du=\frac{2}{60}[u]_{-20}^{y} \qquad F_Y(y)=\frac{2y}{60}+\frac{40}{60}$$

Caso $-10\leq y<10$

$$F_Y(y)=\frac{-20}{60}+\frac{40}{60}+\int_{-10}^{y}\frac{1}{60}du=\frac{20}{60}+\left[\frac{u}{60}\right]_{10}^{y} \qquad F_Y(y)=\frac{y}{60}+\frac{30}{60}$$

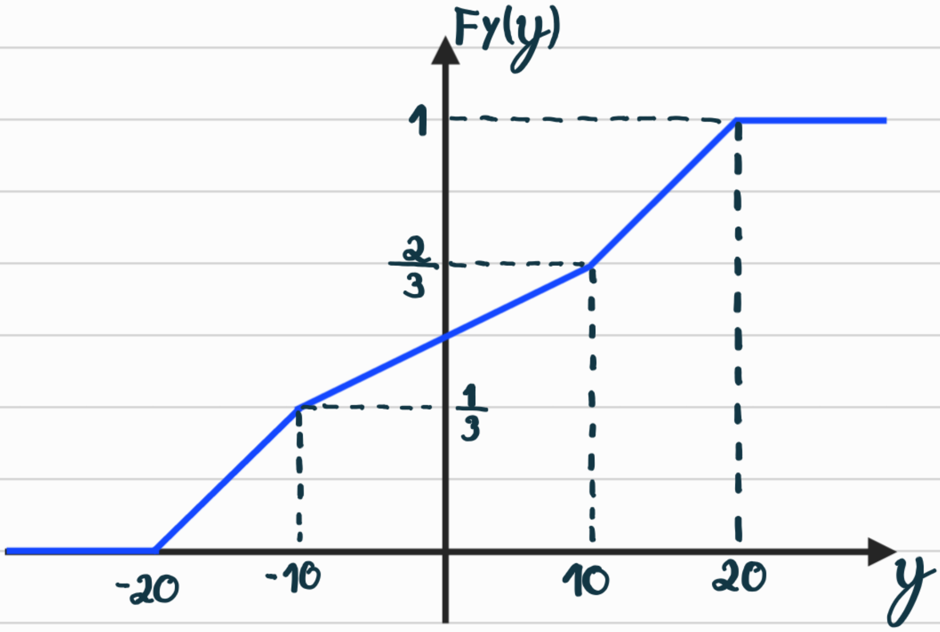
Caso $10\leq y<20$

$$F_Y(y)=\frac{10}{60}+\frac{30}{60}+\int_{10}^{y}\frac{2}{60}du=\frac{40}{60}+\left[\frac{2u}{60}\right]_{10}^{y} \qquad F_Y(y)=\frac{2y}{60}+\frac{20}{60}$$

Caso $20\leq y$

$$F_Y(y)=\frac{2\cdot 20}{60}+\frac{20}{60}+\int_{20}^{\infty}0\,du \qquad F_Y(y)=1$$

$$F_Y(y) = \begin{cases} 0 & , \; y < -20 \\ (2y+40)\cdot 1/60 & , \; -20 \leq y < -10 \\ (y+30)\cdot 1/60 & , \; -10 \leq y < 10 \\ (2y+20)\cdot 1/60 & , \; 10 \leq y < 20 \\ 1 & , \; y \geq 20 \end{cases}$$



e) $f_Y(y/X=x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$ $f_Y(y/X=5) = \dfrac{f_{X,Y}(5,y)}{f_X(5)}$

Caso $0 \leq x \leq 10$

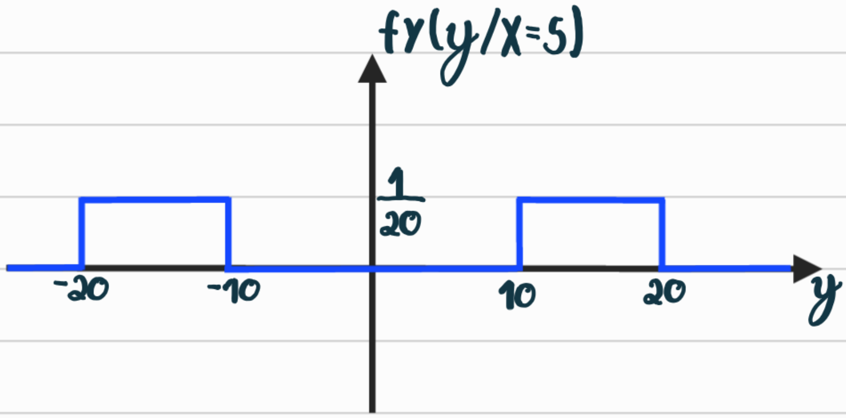
$$f_X(x) = \int_{y=-20}^{y=-10} \frac{1}{600}\,dy + \int_{y=10}^{y=20} \frac{1}{600}\,dy = \left[\frac{y}{600}\right]_{-20}^{-10} + \left[\frac{y}{600}\right]_{10}^{20}$$

$$f_X(x) = \left(\frac{-10}{600} + \frac{20}{600}\right) + \left(\frac{20}{600} - \frac{10}{600}\right) = \frac{10}{600} + \frac{10}{600} \qquad f_X(x) = \frac{1}{30}$$

$$f_{X,Y}(5,y) = \frac{1}{600}\left[(-20 \leq y \leq -10) \vee (10 \leq y \leq 20)\right] \qquad f_X(5) = \frac{1}{30}$$

$$f_Y(y/X=5) = \frac{1}{600}\left[(-20 \leq y \leq -10) \vee (10 \leq y \leq 20)\right] \cdot \frac{30}{1}$$

$$f_Y(y/X=5) = \begin{cases} 1/20 \,, & (-20 \leq y \leq -10) \vee (10 \leq y \leq 20) \\ 0 \,, & \text{C.C} \end{cases}$$



f) $\mathrm{cov}[X,Y] = E[XY] - E[X]\cdot E[Y]$

$$E[X] = \int_{x=-\infty}^{x=\infty} \int_{y=-\infty}^{y=\infty} x\, f_{X,Y}(x,y)\,dy\,dx$$

$$E[X] = \int_{x=0}^{x=10} \left(\int_{y=-20}^{y=-10} \frac{x}{600}\,dy + \int_{y=10}^{y=20} \frac{x}{600}\,dy \right) dx + \int_{x=10}^{x=20} \int_{y=-20}^{y=20} \frac{x}{600}\,dy\,dx$$

$$E[X] = \int_{x=0}^{x=10} \frac{x}{600}[y]_{-20}^{-10} + \frac{x}{600}[y]_{10}^{20}\,dx + \int_{x=10}^{x=20} \frac{x}{600}[y]_{-20}^{20}\,dx$$

$$E[X] = \int_{x=0}^{x=10} \frac{x}{600}(-10+20) + \frac{x}{600}(20-10)\,dx + \int_{x=10}^{x=20} \frac{x}{600}(20+20)\,dx$$

$$E[X] = \int_{x=0}^{x=10} \frac{x}{30}\,dx + \int_{x=10}^{x=20} \frac{x}{15}\,dx = \left[\frac{x^2}{60}\right]_0^{10} + \left[\frac{x^2}{30}\right]_{10}^{20} = \frac{1}{60}(10^2-0) + \frac{1}{30}(20^2-10^2)$$

$$E[X] = \frac{5}{3} + 10 \qquad E[X] = \frac{35}{3}$$

$$E[Y] = \int_{x=-\infty}^{x=\infty} \int_{y=-\infty}^{y=\infty} y \, f_{X,Y}(x,y) \, dy \, dx$$

$$E[Y] = \int_{x=0}^{x=10} \left(\int_{y=-20}^{y=-10} \frac{y}{600} \, dy + \int_{y=10}^{y=20} \frac{y}{600} \, dy \right) dx + \int_{x=10}^{x=20} \int_{y=-20}^{y=20} \frac{y}{600} \, dy \, dx$$

$$E[Y] = \int_{x=0}^{x=10} \frac{1}{1200} [y^2]_{-20}^{-10} + \frac{1}{1200} [y^2]_{10}^{20} \, dx + \int_{x=10}^{x=20} \frac{1}{1200} [y^2]_{-20}^{20} \, dx$$

$$E[Y] = \int_{x=0}^{x=10} \frac{1}{1200} \left[(-10)^2 - (-20)^2 \right] + \frac{1}{1200} \left[20^2 - 10^2 \right] dx + \int_{x=10}^{x=20} \frac{1}{1200} \left[20^2 - (-20)^2 \right] dx$$

$$E[Y] = \int_{x=0}^{x=10} -\frac{1}{4} + \frac{1}{4} \, dx + \int_{x=10}^{x=20} 0 \, dx \qquad E[Y] = \int_{x=0}^{x=10} 0 \, dx$$

$$E[Y] = 0$$

$$E[XY]=\int_{x=-\infty}^{x=\infty}\int_{y=-\infty}^{y=\infty} xy\cdot f_{X,Y}(x,y)\,dy\,dx$$

$$E[XY]=\int_{x=0}^{x=10}\left(\int_{y=-20}^{y=-10}\frac{xy}{600}\,dy+\int_{y=10}^{y=20}\frac{xy}{600}\,dy\right)dx+\int_{x=10}^{x=20}\int_{y=-20}^{y=20}\frac{xy}{600}\,dy\,dx$$

$$E[XY]=\int_{x=0}^{x=10}\frac{x}{1200}[y^2]_{-20}^{-10}+\frac{x}{1200}[y^2]_{10}^{20}dx+\int_{x=10}^{x=20}\frac{x}{1200}[y^2]_{-20}^{20}dx$$

$$E[XY]=\int_{x=0}^{x=10}\frac{x}{1200}[(-10)^2-(-20)^2]+\frac{x}{1200}(20^2-10^2)dx+\int_{x=10}^{x=20}\frac{x}{1200}[20^2-(-20)^2]dx$$

$$E[XY]=\int_{x=0}^{x=10}-\frac{x}{4}+\frac{x}{4}\,dx+\int_{x=10}^{x=20}0\,dx \qquad E[XY]=\int_{x=0}^{x=10}0\,dx \qquad E[XY]=0$$

$$\operatorname{cov}[XY]=E[XY]-E[X]\cdot E[Y]$$

$$\operatorname{cov}[XY]=0-\frac{35}{3}\cdot 0 \qquad \operatorname{cov}[XY]=0_{//}$$