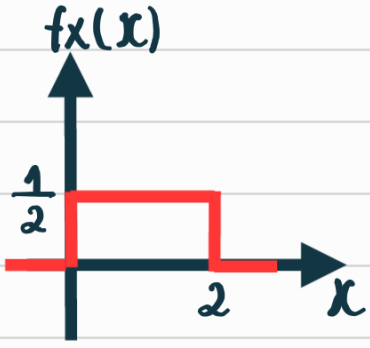
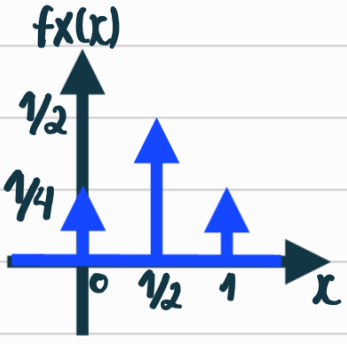
Lucas Coelho Raupp

3)a) $f_X(x/U=5 \text{ ou } U=6)$ $f_X(x/U\neq 5 \text{ e } U\neq 6)$

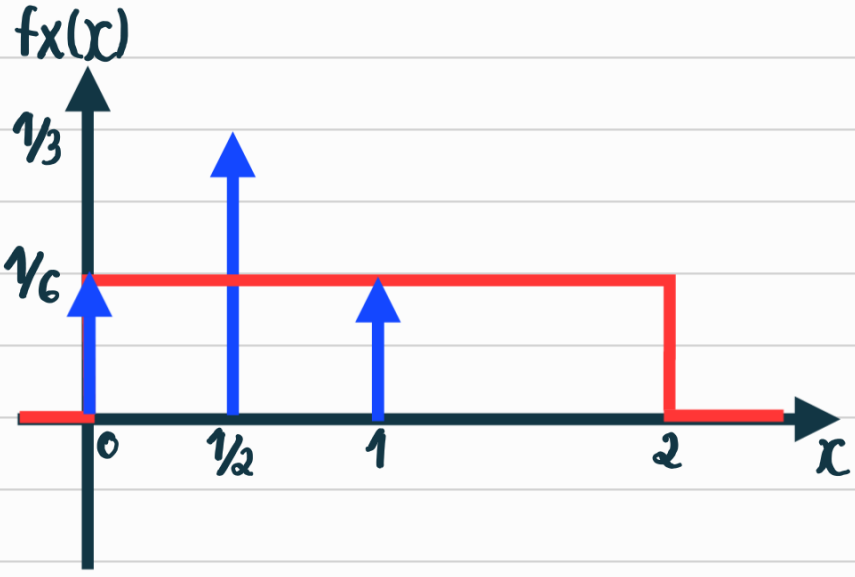




$$f_X(x)=f_X(x/U=5 \text{ ou } U=6)Pr[U=5 \text{ ou } U=6]+f_X(x/U\neq 5 \text{ e } U\neq 6)Pr[U\neq 5 \text{ e } U\neq 6]$$

$$f_X(x)=\frac{1}{2}[0\leq x\leq 2]\cdot\frac{2}{6}+\left[\frac{1}{4}\delta(x)+\frac{1}{2}\delta(x-1/2)+\frac{1}{4}\delta(x-1)\right]\cdot\frac{4}{6}$$

$$f_X(x)=\frac{1}{6}[0\leq x\leq 2]+\frac{1}{6}\delta(x)+\frac{1}{3}\delta(x-1/2)+\frac{1}{6}\delta(x-1)$$



b) $F_X(x) = \int_{-\infty}^{x^+} f_X(u)\,du$

$$F_X(x) = \int_{-\infty}^{x} 0\,du + \int_{0^-}^{0^+} \frac{1}{6}\cdot\delta(u)\,du + \int_{0^+}^{x} \frac{1}{6}\,du + \int_{1/2^-}^{1/2^+} \frac{1}{3}\,\delta(u-1/2)\,du + \int_{1/2^+}^{x} \frac{1}{6}\,du +$$

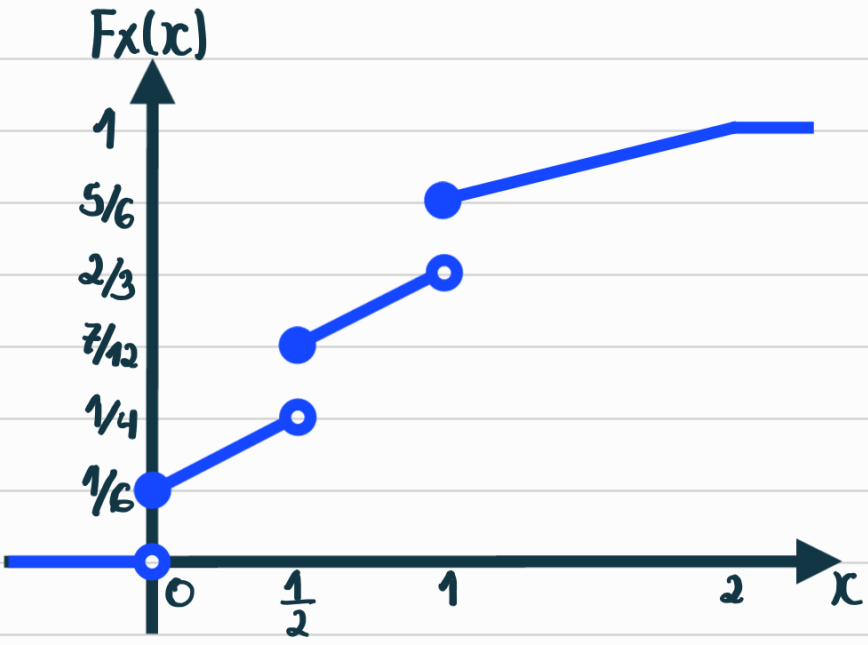
$$\int_{1^-}^{1^+} \frac{1}{6}\,\delta(u-1)\,du + \int_{1^+}^{x} \frac{1}{6}\,du + \int_{2}^{x} 0\,du$$

$$F_X(x) = [0]_{-\infty}^{0^-} + \left[\frac{1}{6}\right]_{0^-}^{0^+} + \left[\frac{u}{6}\right]_{0^+}^{1/2^-} + \left[\frac{1}{3}\right]_{1/2^-}^{1/2^+} + \left[\frac{u}{6}\right]_{1/2^+}^{1^-} + \left[\frac{1}{6}\right]_{1^-}^{1^+} + \left[\frac{u}{6}\right]_{1^+}^{2} + [0]_{2}^{\infty}$$

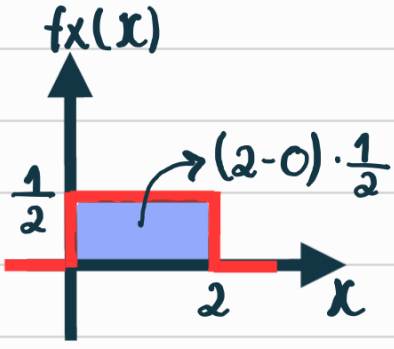
$$F_X(x) = 0 + \frac{1}{6} + \frac{1}{6}\left(\frac{1}{2} - 0\right) + \frac{1}{3} + \frac{1}{6}\left(1 - \frac{1}{2}\right) + \frac{1}{6} + \frac{1}{6}(2-1) + 0$$

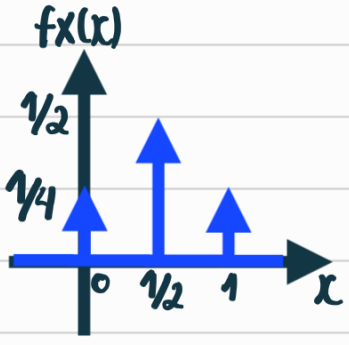
$$F_X(x) = \frac{1}{6} + \frac{1}{12} + \frac{1}{3} + \frac{1}{12} + \frac{1}{6} + \frac{1}{6} \qquad F_X(x) = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$$

$$F_X(x) = \begin{cases} 0 & , x<0 \\ 1/6 & , x=0 \\ 1/6 + x/6 & , 0<x<1/2 \\ 7/12 & , x=1/2 \\ 1/2 + x/6 & , 1/2<x<1 \\ 5/6 & , x=1 \\ 2/3 + x/6 & , 1<x<2 \\ 1 & , x\geq 2 \end{cases} \qquad F_X(x) = \begin{cases} 0 & , x<0 \\ 1/6 + x/6 & , 0\leq x<1/2 \\ 1/2 + x/6 & , 1/2\leq x<1 \\ 2/3 + x/6 & , 1\leq x<2 \\ 1 & , x\geq 2 \end{cases}$$


Fx(x)
1
5/6
2/3
7/12
1/4
1/6
0
1/2
1
2
x

c) $E[X] = E[X/U=5 \text{ ou } U=6] \cdot Pr[U=5 \text{ ou } U=6] + E[X/U\neq 5 \text{ e } U\neq 6] \cdot Pr[U\neq 5 \text{ e } U\neq 6]$



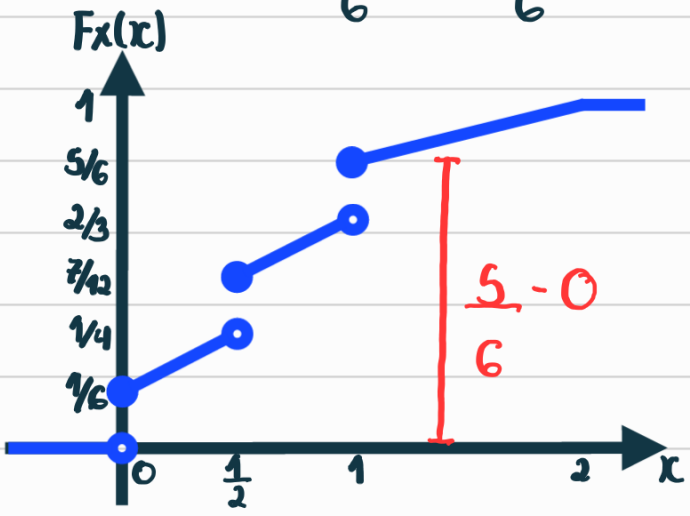


$$E[X] = \frac{2-0}{2} \cdot \frac{2}{6} + \left(0 \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} \right) \cdot \frac{4}{6}$$

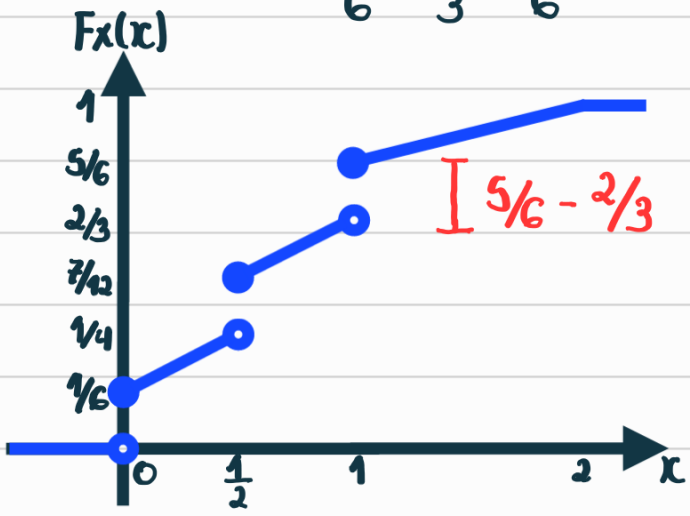
$$E[X] = \frac{1}{3} + \left(\frac{1}{4} + \frac{1}{4} \right) \cdot \frac{2}{3} = \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} + \frac{1}{3} \qquad E[X] = \frac{2}{3} \text{ //}$$

d) $Pr[X<1] = Pr[0 \le X \le 1] - Pr[X=1]$

$Pr[0 \le X \le 1] = \frac{5}{6} - 0 = \frac{5}{6}$



$Pr[X=1] = \frac{5}{6} - \frac{2}{3} = \frac{1}{6}$



$Pr[X<1] = \frac{5}{6} - \frac{1}{6} = \frac{4}{6}$

$Pr[X<1] = \frac{2}{3}$ //