Lucas Coelho Raupp

4)a)

X_1	X_2	$p_{X_1,X_2}(x)$	$Y_1 = X_1$	$Y_2 = X_2$	$Y_3 = X_1 \cdot X_2$
0	0	1/9	0	0	0
0	1	1/9	0	1	0
0	2	1/9	0	2	0
1	0	1/9	1	0	0
1	1	1/9	1	1	1
1	2	1/9	1	2	2
2	0	1/9	2	0	0
2	1	1/9	2	1	2
2	2	1/9	2	2	4

$$E[Y_1] = 3 \cdot \left(0 \cdot \frac{1}{9}\right) + 3 \cdot \left(1 \cdot \frac{1}{9}\right) + 3 \cdot \left(2 \cdot \frac{1}{9}\right) \qquad E[Y_1] = 1$$

$$E[Y_2] = 3 \cdot \left(0 \cdot \frac{1}{9}\right) + 3 \cdot \left(1 \cdot \frac{1}{9}\right) + 3 \cdot \left(2 \cdot \frac{1}{9}\right) \qquad E[Y_2] = 1$$

$$E[Y_3] = 5 \cdot \left(0 \cdot \frac{1}{9}\right) + 1 \cdot \frac{1}{9} + 2 \cdot \left(2 \cdot \frac{1}{9}\right) + 4 \cdot \frac{1}{9} \qquad E[Y_3] = 1$$

$$\vec{\mu}_{\vec{Y}} = \left[E[Y_1] \quad E[Y_2] \quad E[Y_3]\right]^T \qquad \vec{\mu}_{\vec{Y}} = [1 \quad 1 \quad 1]^T$$

$$E[Y_1^2] = 3\cdot\left(0^2\cdot\frac{1}{9}\right) + 3\cdot\left(1^2\cdot\frac{1}{9}\right) + 3\cdot\left(2^2\cdot\frac{1}{9}\right) \qquad E[Y_1^2] = \frac{5}{3}$$

$$E[Y_2^2] = 3\cdot\left(0^2\cdot\frac{1}{9}\right) + 3\cdot\left(1^2\cdot\frac{1}{9}\right) + 3\cdot\left(2^2\cdot\frac{1}{9}\right) \qquad E[Y_2^2] = \frac{5}{3}$$

$$E[Y_3^2] = 5\cdot\left(0^2\cdot\frac{1}{9}\right) + 1^2\cdot\frac{1}{9} + 2\cdot\left(2^2\cdot\frac{1}{9}\right) + 4^2\cdot\frac{1}{9} \qquad E[Y_3^2] = \frac{25}{9}$$

$$\mathrm{var}[Y_1] = E[Y_1^2] - E[Y_1]^2 = \frac{5}{3} - 1^2 \qquad \mathrm{var}[Y_1] = \frac{2}{3}$$

$$\mathrm{var}[Y_2] = E[Y_2^2] - E[Y_2]^2 = \frac{5}{3} - 1^2 \qquad \mathrm{var}[Y_2] = \frac{2}{3}$$

$$\mathrm{var}[Y_3] = E[Y_3^2] - E[Y_3]^2 = \frac{25}{9} - 1^2 \qquad \mathrm{var}[Y_3] = \frac{16}{9}$$

$$E[Y_1Y_2] = 1\cdot 1\cdot\frac{1}{9} + 1\cdot 2\cdot\frac{1}{9} + 2\cdot 1\cdot\frac{1}{9} + 2\cdot 2\cdot\frac{1}{9} \qquad E[Y_1Y_2] = 1$$

$$E[Y_1Y_3] = 1\cdot 1\cdot\frac{1}{9} + 1\cdot 2\cdot\frac{1}{9} + 2\cdot 2\cdot\frac{1}{9} + 2\cdot 4\cdot\frac{1}{9} \qquad E[Y_1Y_3] = \frac{5}{3}$$

$$E[Y_2Y_3] = 1\cdot 1\cdot\frac{1}{9} + 2\cdot 2\cdot\frac{1}{9} + 1\cdot 2\cdot\frac{1}{9} + 2\cdot 4\cdot\frac{1}{9} \qquad E[Y_2Y_3] = \frac{5}{3}$$

$$\mathrm{cov}[Y_n, Y_m] = E[Y_nY_m] - E[Y_n]\cdot E[Y_m]$$

$\operatorname{cov}[Y_1, Y_2] = 1 - 1 \cdot 1 \quad \operatorname{cov}[Y_1, Y_2] = 0$

$\operatorname{cov}[Y_1, Y_3] = \frac{5}{3} - 1 \cdot 1 \quad \operatorname{cov}[Y_1, Y_3] = \frac{2}{3}$

$\operatorname{cov}[Y_2, Y_3] = \frac{5}{3} - 1 \cdot 1 \quad \operatorname{cov}[Y_2, Y_3] = \frac{2}{3}$

$$C_{\vec{Y}} = \begin{bmatrix} 2/3 & 0 & 2/3 \\ 0 & 2/3 & 2/3 \\ 2/3 & 2/3 & 16/9 \end{bmatrix} //$$

b) $\begin{cases} Z_1 = Y_1 + Y_2 \\ Z_2 = Y_2 + Y_3 \\ Z_3 = Y_3 + Y_1 \end{cases}$ $\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\vec{\mu}_{\vec{z}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1+0 \\ 0+1+1 \\ 1+0+1 \end{bmatrix} \qquad \vec{\mu}_{\vec{z}} = \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T //$$

$$C_{\vec{z}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2/3 & 0 & 2/3 \\ 0 & 2/3 & 2/3 \\ 2/3 & 2/3 & 16/9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C_{\vec{z}} = \begin{bmatrix} 2/3 & 2/3 & 4/3 \\ 2/3 & 4/3 & 22/9 \\ 4/3 & 2/3 & 22/9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C_{\vec{z}} = \begin{bmatrix} 4/3 & 2 & 2 \\ 2 & 34/9 & 28/9 \\ 2 & 28/9 & 34/9 \end{bmatrix} //$$