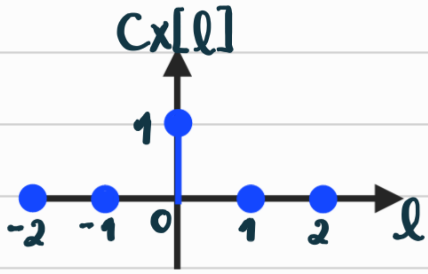
Lucas Coelho Raupp

10) a) $X[n] \overset{iid}{\sim} N(0,1)$

$$C_x[n_1,n_2] = cov[X_{n_1}, X_{n_2}] = \begin{cases} 1, & n_1 = n_2 \\ 0, & n_1 \neq n_2 \end{cases} //$$



b) $C_y[n_1,n_2] = cov[Y_{n_1}, Y_{n_2}] = E[Y_{n_1} \cdot Y_{n_2}] - E[Y_{n_1}] \cdot E[Y_{n_2}]$

$$E[Y_n] = E[X_n + X_{n-1} + X_{n-2}] = E[X_n] + E[X_{n-1}] + E[X_{n-2}]$$

$E[Y_n] = 0 + 0 + 0 \qquad E[Y_n] = 0$

$$C_y[n_1,n_2] = E[Y_{n_1} \cdot Y_{n_2}]$$

$$E[Y_{n_1} \cdot Y_{n_2}] = E[(X_{n_1} + X_{n_1-1} + X_{n_1-2})(X_{n_2} + X_{n_2-1} + X_{n_2-2})] =$$

$$E[X_{n_1} \cdot X_{n_2}] + E[X_{n_1} \cdot X_{n_2-1}] + E[X_{n_1} \cdot X_{n_2-2}] + E[X_{n_1-1} \cdot X_{n_2}] + E[X_{n_1-1} \cdot X_{n_2-1}] +$$

$$E[X_{n_1-1} \cdot X_{n_2-2}] + E[X_{n_1-2} \cdot X_{n_2}] + E[X_{n_1-2} \cdot X_{n_2-1}] + E[X_{n_1-2} \cdot X_{n_2-2}]$$

$$C_y[n_1,n_2] = C_x[n_1,n_2] + C_x[n_1,n_2-1] + C_x[n_1,n_2-2] + C_x[n_1-1,n_2] +$$

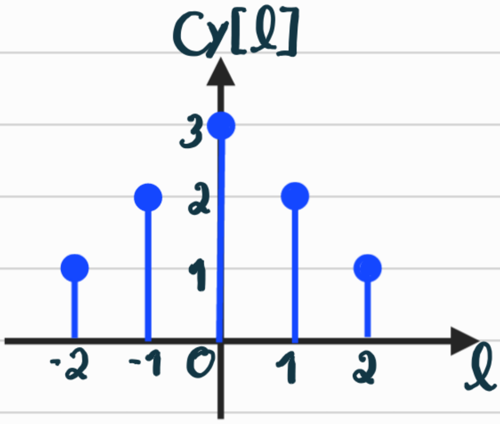
$$C_x[n_1-1,n_2-1] + C_x[n_1-1,n_2-2] + C_x[n_1-2,n_2] + C_x[n_1-2,n_2-1] +$$

$$C_x[n_1-2,n_2-2]$$

$$C_Y[n_1,n_2] = C_X[\ell]+C_X[\ell-1]+C_X[\ell-2]+C_X[\ell+1]+C_X[\ell]+C_X[\ell-1]+$$

$$C_X[\ell+2]+C_X[\ell+1]+C_X[\ell]$$

$$C_Y[n_1,n_2] = 3\delta[\ell]+2\delta[\ell-1]+\delta[\ell-2]+2\delta[\ell+1]+\delta[\ell+2]_{//}$$



c) $Y[3] = X[3]+X[2]+X[1]$

$$f_X(x) = \frac{1}{\sqrt{2\pi\cdot\sigma^2}}\cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$E[Y_3] = E[X_3]+E[X_2]+E[X_1]$ $\quad E[Y_3] = 0+0+0$ $\quad E[Y_3] = 0$

$\mathrm{var}[Y_3] = \mathrm{var}[X_3]+\mathrm{var}[X_2]+\mathrm{var}[X_1]$ $\quad \mathrm{var}[Y_3] = 1+1+1$ $\quad \mathrm{var}[Y_3] = 3$

$$Y[3] \overset{iid}{\sim} N(0,3)$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\cdot 3}}\cdot \exp\left(-\frac{(y-0)^2}{2\cdot 3}\right) \qquad f_Y(y) = \frac{1}{\sqrt{6\pi}}\cdot e^{-y^2/6}{}_{//}$$

d) $cov[Y_3, Y_4] = E[Y_3 \cdot Y_4] - E[Y_3] \cdot E[Y_4] \qquad E[Y_3] = E[Y_4] = 0$

$$E[Y_3 \cdot Y_4] = E[(X_3 + X_2 + X_1)(X_4 + X_3 + X_2)] = E[\cancelto{0}{X_3} X_4] + E[X_3^2] + E[\cancelto{0}{X_3} X_2] + E[\cancelto{0}{X_2} X_4] +$$

$$E[\cancelto{0}{X_2} X_3] + E[X_2^2] + E[\cancelto{0}{X_1} X_4] + E[\cancelto{0}{X_1} X_3] + E[\cancelto{0}{X_1} X_2]$$

$$cov[Y_3, Y_4] = E[X_3^2] + E[X_2^2]$$

$$E[X_3^2] = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \, dx = 1 \qquad E[X_2^2] = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \, dx = 1$$

$$cov[Y_3, Y_4] = 1 + 1 \qquad cov[Y_3, Y_4] = 2_{//}$$

e) $Pr[Y_3 > 0 / Y_0 = 1] = Pr[Y_3 > 0]$

$$Pr[Y_3 > 0] = \Phi\left(\frac{\infty - 0}{\sqrt{6}}\right) - \Phi\left(\frac{0 - 0}{\sqrt{6}}\right) \qquad Pr[Y_3 > 0] = 1 - 0{,}5$$

$$Pr[Y_3 > 0 / Y_0 = 1] = 0{,}5_{//}$$