Advanced Quantum Mechanics
Exercise Set 01

Due date: April 10^{th}, 2023

Exercise 1: Do the following problems from M. Le Bellac's book: 4.4.1, 4.4.2, 4.4.3, 4.4.4, 4.4.5, 4.4.6, 5.5.1, 5.5.2, 5.5.4, 5.5.5, 5.5.6.

Exercise 2: Avoided crossing, or level repulsion, or anticrossing.

Consider a two-level system, the energy levels being E_1 and E_2 and the corresponding eigenvectors being $|1\rangle$ and $|2\rangle$. In the basis $\{|1\rangle, |2\rangle\}$, the matrix representing the Hamiltonian, which we denote by H^0, is given by:

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

Suppose now that the system is perturbed by some external field $\hat{V}$, so that the Hamiltonian of the system becomes $\hat{H} = \hat{H}_0 + \hat{V}$, with $\hat{V}^\dagger = \hat{V}$. In the basis $\{|1\rangle, |2\rangle\}$, $\hat{H}$ has the matrix representation

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

where $V_{ij} = \langle i| \hat{V} |j\rangle$, $i, j = 1, 2$.

2.1 Show that one can always choose $V_{21} = V_{12} = v$, where v us a real number (see chapter 2 of Le Bellac). Make this choice for answering the following two questions.

2.2 Let $E_\pm$ denote the eigenvalues of H. Make a qualitative plot of $E_\pm$ as a function of v, by fixing the values of E_1, E_2, V_{11} and V_{22}. You should convince yourself that the curves $E_\pm(v)$ never cross for generic values of E_1, E_2, V_{11}, V_{22}.

2.3 Show that to have the levels crossing, one must have

$$v = 0 \quad \text{and} \quad E_1 - E_2 + V_{11} - V_{22} = 0.$$

2.4 Prove the following assertion: the eigenvalues of a two-level system depending on n continuous real parameters do not cross in general, except on a manifold (essentially a space) of $n-2$ dimensions.