Green's Function Solution for Many Functions

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1 Caso 1

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \tag{1.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= 0 \\ g &= 1 \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int_0^\infty \int_{-\infty}^\infty \frac{\mu_0 e^{-\frac{g\zeta}{RT}} \log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \, d\xi \, d\zeta \tag{1.2}$$

Isso resulta em

$$\psi = -\infty \operatorname{sign}(\mu_0) \operatorname{sign}\left(\frac{1}{\pi}\right) \int_0^\infty \operatorname{sign}\left(e^{-\frac{g\zeta}{RT}}\right) d\zeta \tag{1.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x \psi &= 0 \\ \partial_z \psi &= 0 \end{aligned}$$

Por fim, a corrente a pressão são:

$$
\begin{aligned}
I &= 0 \\
p &= 0
\end{aligned}
$$

2 Caso 2

Nosso problema tem a função de Green

$$
G = \frac{\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \tag{2.1}
$$

Consideraremos as funções:

$$
\begin{aligned}
f &= 1 \\
g &= 0
\end{aligned}
$$

Então, precisamos integrar

$$
\psi = -\int\limits_0^\infty \int\limits_{-\infty}^\infty \frac{\mu_0^2 \log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \, d\xi \, d\zeta \tag{2.2}
$$

Isso resulta em

$$
\psi = -\infty \operatorname{sign}\left(\mu_0^2\right) \operatorname{sign}\left(\frac{1}{\pi}\right) \int\limits_0^\infty 1 \, d\zeta \tag{2.3}
$$

com derivadas parciais:

$$
\begin{aligned}
\partial_x \psi &= 0 \\
\partial_z \psi &= 0
\end{aligned}
$$

Por fim, a corrente a pressão são:

$$
\begin{aligned}
I &= 0 \\
p &= 0
\end{aligned}
$$

3 Caso 3

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \tag{3.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= 1 \\ g &= 1 \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int_0^\infty \int_{-\infty}^\infty \frac{\left(\mu_0^2 + \mu_0 e^{-\frac{g\zeta}{RT}}\right) \log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \, d\xi \, d\zeta \tag{3.2}$$

Isso resulta em

$$\psi = -\infty \operatorname{sign}(\mu_0) \operatorname{sign}\left(\frac{1}{\pi}\right) \int_0^\infty \operatorname{sign}\left(\mu_0 e^{\frac{g\zeta}{RT}} + 1\right) \operatorname{sign}\left(e^{-\frac{g\zeta}{RT}}\right) d\zeta \tag{3.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x \psi &= 0 \\ \partial_z \psi &= 0 \end{aligned}$$

Por fim, a corrente a pressão são:

$$\begin{aligned} I &= 0 \\ p &= 0 \end{aligned}$$

4 Caso 4

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \tag{4.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= \xi \\ g &= \zeta \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int_{0}^{\infty}\int_{-\infty}^{\infty} \frac{\left(\mu_0^2\xi + \mu_0\zeta e^{-\frac{g\zeta}{RT}}\right)\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi}\, d\xi\, d\zeta \tag{4.2}$$

Isso resulta em

$$\psi = \mathrm{NaN} \tag{4.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x\psi &= 0 \\ \partial_z\psi &= 0 \end{aligned}$$

Por fim, a corrente a pressão são:

$$\begin{aligned} I &= 0 \\ p &= 0 \end{aligned}$$

5 Caso 5

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi} \tag{5.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= \xi^4 \\ g &= 1 \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int_{0}^{\infty}\int_{-\infty}^{\infty} \frac{\left(\mu_0^2\xi^4 + \mu_0 e^{-\frac{g\zeta}{RT}}\right)\log\left(\sqrt{(x-\xi)^2 + (z-\zeta)^2}\right)}{2\pi}\, d\xi\, d\zeta \tag{5.2}$$

Isso resulta em

$$\psi = -\infty \operatorname{sign}\left(\mu_0^2\right) \operatorname{sign}\left(\frac{1}{\pi}\right) \int_0^{\infty} 1\, d\zeta \tag{5.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x \psi &= 0 \\ \partial_z \psi &= 0 \end{aligned}$$

Por fim, a corrente a pressão são:

$$\begin{aligned} I &= 0 \\ p &= 0 \end{aligned}$$

6 Caso 6

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{\left(x-\xi\right)^2 + \left(z-\zeta\right)^2}\right)}{2\pi} \tag{6.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= \xi^3 + \xi^2 + \xi + 1 \\ g &= 0 \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int_0^{\infty} \int_{-\infty}^{\infty} \frac{\mu_0^2 \left(\xi^3 + \xi^2 + \xi + 1\right) \log\left(\sqrt{\left(x-\xi\right)^2 + \left(z-\zeta\right)^2}\right)}{2\pi}\, d\xi\, d\zeta \tag{6.2}$$

Isso resulta em

$$\psi = \mathrm{NaN} \tag{6.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x \psi &= 0 \\ \partial_z \psi &= 0 \end{aligned}$$

Por fim, a corrente a pressão são:

$$\begin{aligned} I &= 0 \\ p &= 0 \end{aligned}$$

7 Caso 7

Nosso problema tem a função de Green

$$G = \frac{\log\left(\sqrt{\left(x-\xi\right)^2+\left(z-\zeta\right)^2}\right)}{2\pi} \tag{7.1}$$

Consideraremos as funções:

$$\begin{aligned} f &= \sin\left(\xi\right) \\ g &= \cos\left(\zeta\right) \end{aligned}$$

Então, precisamos integrar

$$\psi = -\int\limits_{0}^{\infty}\int\limits_{-\infty}^{\infty} \frac{\left(\mu_0^2\sin\left(\xi\right)+\mu_0 e^{-\frac{g\zeta}{RT}}\cos\left(\zeta\right)\right)\log\left(\sqrt{\left(x-\xi\right)^2+\left(z-\zeta\right)^2}\right)}{2\pi}\,d\xi\,d\zeta \tag{7.2}$$

Isso resulta em

$$\psi = -\frac{\mu_0\left(\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\log\left(x^2-2x\xi+\xi^2+z^2-2z\zeta+\zeta^2\right)\cos\left(\zeta\right)d\xi\,d\zeta+\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\mu_0 e^{\frac{g\zeta}{RT}}\log\left(x\right.\right.}{4\pi} \tag{7.3}$$

com derivadas parciais:

$$\begin{aligned} \partial_x\psi &= -\frac{\mu_0\left(\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\frac{(2x-2\xi)\cos\left(\zeta\right)}{x^2-2x\xi+\xi^2+z^2-2z\zeta+\zeta^2}\,d\xi\,d\zeta+\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\frac{\mu_0\cdot(2x-2\xi)e^{\frac{g\zeta}{RT}}\sin\left(\xi\right)}{x^2-2x\xi+\xi^2+z^2-2z\zeta+\zeta^2}\,d\xi\,d\zeta\right)}{4\pi} \\ \partial_z\psi &= -\frac{\mu_0\left(\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\frac{(2z-2\zeta)\cos\left(\zeta\right)}{x^2-2x\xi+\xi^2+z^2-2z\zeta+\zeta^2}\,d\xi\,d\zeta+\int\limits_{0}^{\infty} e^{-\frac{g\zeta}{RT}}\int\limits_{-\infty}^{\infty}\frac{\mu_0\cdot(2z-2\zeta)e^{\frac{g\zeta}{RT}}\sin\left(\xi\right)}{x^2-2x\xi+\xi^2+z^2-2z\zeta+\zeta^2}\,d\xi\,d\zeta\right)}{4\pi} \end{aligned}$$

Por fim, a corrente a pressão são:

$$
\begin{aligned}
I &= \mathrm{NaN} \\
p &= \mathrm{NaN}
\end{aligned}
$$