Green's Function Solution for Many Functions

Lucas Froguel
Deparment of Physics, UFPR

September 4, 2022

1 Caso 1

Esse caso considerará as funções

$$\begin{aligned} f &= 0 \\ g &= 0 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \frac{d^2}{dx^2}X(x) = 0 \tag{1.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2}Z(z) = 0 \tag{1.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}}\right) \tag{1.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(-C_1\sqrt{-C}e^{-x\sqrt{-C}} + C_2\sqrt{-C}e^{x\sqrt{-C}}\right) \\ B_x &= -\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}}\right)\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z}\right) \end{aligned}$$

2 Caso 2

Esse caso considerará as funções

$$\begin{aligned} f &= 1 \\ g &= 1 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 + \frac{d^2}{dx^2}X(x) = 0 \tag{2.1}$$

E

$$-CZ(z) + \mu_0 e^{-\frac{gz}{RT}} + \frac{d^2}{dz^2}Z(z) = 0 \tag{2.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2}{C} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} - \frac{R^2T^2\mu_0 e^{-\frac{gz}{RT}}}{-CR^2T^2 + g^2} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} - \frac{R^2T^2\mu_0 e^{-\frac{gz}{RT}}}{-CR^2T^2 + g^2}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2}{C}\right) \tag{2.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(-C_1\sqrt{-C}e^{-x\sqrt{-C}} + C_2\sqrt{-C}e^{x\sqrt{-C}}\right)\left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} - \frac{R^2T^2\mu_0 e^{-\frac{gz}{RT}}}{-CR^2T^2 + g^2}\right) \\ B_x &= -\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2}{C}\right)\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z} + \frac{RT\mu_0 g e^{-\frac{gz}{RT}}}{-CR^2T^2 + g^2}\right) \end{aligned}$$

3 Caso 3

Esse caso considerará as funções

$$\begin{aligned} f &= x \\ g &= 0 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 x + \frac{d^2}{dx^2}X(x) = 0 \tag{3.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2}Z(z) = 0 \tag{3.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x}{C} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x}{C}\right) \tag{3.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(-C_1\sqrt{-C}e^{-x\sqrt{-C}} + C_2\sqrt{-C}e^{x\sqrt{-C}} - \frac{\mu_0^2}{C}\right) \\ B_x &= -\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x}{C}\right) \end{aligned}$$

4 Caso 4

Esse caso considerará as funções

$$\begin{aligned} f &= X(x) \\ g &= 0 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 X(x) + \frac{d^2}{dx^2}X(x) = 0 \tag{4.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2}Z(z) = 0 \tag{4.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}}\right) \tag{4.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(-C_1\sqrt{-C-\mu_0^2}e^{-x\sqrt{-C-\mu_0^2}} + C_2\sqrt{-C-\mu_0^2}e^{x\sqrt{-C-\mu_0^2}}\right) \\ B_x &= -\left(C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}}\right)\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z}\right) \end{aligned}$$

5 Caso 5

Esse caso considerará as funções

$$\begin{aligned} f &= x^3 + x^2 + x + 1 \\ g &= 0 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2\left(x^3 + x^2 + x + 1\right) + \frac{d^2}{dx^2}X(x) = 0 \tag{5.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2}Z(z) = 0 \tag{5.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^3}{C} - \frac{\mu_0^2 x^2}{C} - \frac{\mu_0^2 x}{C} - \frac{\mu_0^2}{C} + \frac{6\mu_0^2 x}{C^2} + \frac{2\mu_0^2}{C^2} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^3}{C} - \frac{\mu_0^2 x^2}{C} - \frac{\mu_0^2 x}{C} - \frac{\mu_0^2}{C} + \frac{6\mu_0^2 x}{C^2} + \frac{2\mu_0^2}{C^2}\right) \tag{5.3}$$

Isso leva aos campos magnéticos:

$$
\begin{aligned}
B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(-C_1\sqrt{-C}e^{-x\sqrt{-C}} + C_2\sqrt{-C}e^{x\sqrt{-C}} - \frac{3\mu_0^2 x^2}{C} - \frac{2\mu_0^2 x}{C} - \frac{\mu_0^2}{C} + \frac{6\mu_0^2}{C^2} \right.\\
B_x &= -\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^3}{C} - \frac{\mu_0^2 x^2}{C} - \frac{\mu_0^2 x}{C} - \frac{\mu_0^2}{C} + \right.
\end{aligned}
$$

6 Caso 6

Esse caso considerará as funções

$$
\begin{aligned}
f &= x^4\\
g &= 0
\end{aligned}
$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 x^4 + \frac{d^2}{dx^2}X(x) = 0 \tag{6.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2}Z(z) = 0 \tag{6.2}$$

Isso resulta na solução parcial:

$$
\begin{aligned}
X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^4}{C} + \frac{12\mu_0^2 x^2}{C^2} - \frac{24\mu_0^2}{C^3}\\
Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}
\end{aligned}
$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^4}{C} + \frac{12\mu_0^2 x^2}{C^2} - \frac{24\mu_0^2}{C^3}\right) \tag{6.3}$$

Isso leva aos campos magnéticos:

$$
\begin{aligned}
B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right)\left(-C_1\sqrt{-C}e^{-x\sqrt{-C}} + C_2\sqrt{-C}e^{x\sqrt{-C}} - \frac{4\mu_0^2 x^3}{C} + \frac{24\mu_0^2 x}{C^2}\right)\\
B_x &= -\left(-\sqrt{C}C_1 e^{-\sqrt{C}z} + \sqrt{C}C_2 e^{\sqrt{C}z}\right)\left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 x^4}{C} + \frac{12\mu_0^2 x^2}{C^2} - \frac{24\mu_0^2}{C^3}\right)
\end{aligned}
$$

7 Caso 7

Esse caso considerará as funções

$$\begin{aligned} f &= X(x) \\ g &= Z(z) \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 X(x) + \frac{d^2}{dx^2} X(x) = 0 \tag{7.1}$$

E

$$-CZ(z) + \mu_0 Z(z) e^{-\frac{gz}{RT}} + \frac{d^2}{dz^2} Z(z) = 0 \tag{7.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}} \\ Z &= C_2 \left(\frac{Cz^2}{2} - \frac{\mu_0 z^2 e^{-\frac{gz}{RT}}}{2} + \frac{z^4 \left(Ce^{\frac{gz}{RT}} - \mu_0\right)^2 e^{-\frac{2gz}{RT}}}{24} + 1 \right) + C_1 z \left(\frac{Cz^2}{6} - \frac{\mu_0 z^2 e^{-\frac{gz}{RT}}}{6} + 1 \right) + \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}} \right) \left(C_2 \left(\frac{Cz^2}{2} - \frac{\mu_0 z^2 e^{-\frac{gz}{RT}}}{2} + \frac{z^4 \left(Ce^{\frac{gz}{RT}} - \mu_0\right)^2 e^{-\frac{2gz}{RT}}}{24} + 1 \right) + C \right. \tag{7.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(-C_1 \sqrt{-C - \mu_0^2} e^{-x\sqrt{-C-\mu_0^2}} + C_2 \sqrt{-C - \mu_0^2} e^{x\sqrt{-C-\mu_0^2}} \right) \left(C_2 \left(\frac{Cz^2}{2} - \frac{\mu_0 z^2 e^{-\frac{gz}{RT}}}{2} + \frac{z^4}{} \right.\right. \\ B_x &= -\left(C_1 e^{-x\sqrt{-C-\mu_0^2}} + C_2 e^{x\sqrt{-C-\mu_0^2}} \right) \left(C_2 \left(Cz + \frac{Cgz^4 \left(Ce^{\frac{gz}{RT}} - \mu_0\right) e^{-\frac{gz}{RT}}}{12RT} - \mu_0 z e^{-\frac{gz}{RT}} + \frac{z}{} \right.\right. \end{aligned}$$

8 Caso 8

Esse caso considerará as funções

$$\begin{aligned} f &= \sin(x) + \cos(x) \\ g &= 0 \end{aligned}$$

Isso significa que precisamos resolver as EDOs:

$$CX(x) + \mu_0^2 (\sin(x) + \cos(x)) + \frac{d^2}{dx^2} X(x) = 0 \tag{8.1}$$

E

$$-CZ(z) + \frac{d^2}{dz^2} Z(z) = 0 \tag{8.2}$$

Isso resulta na solução parcial:

$$\begin{aligned} X &= C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 \sin(x)}{C-1} - \frac{\mu_0^2 \cos(x)}{C-1} \\ Z &= C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z} \end{aligned}$$

que implica no perfil:

$$\Psi = \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right) \left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 \sin(x)}{C-1} - \frac{\mu_0^2 \cos(x)}{C-1}\right) \tag{8.3}$$

Isso leva aos campos magnéticos:

$$\begin{aligned} B_z &= \left(C_1 e^{-\sqrt{C}z} + C_2 e^{\sqrt{C}z}\right) \left(-C_1\sqrt{-C} e^{-x\sqrt{-C}} + C_2\sqrt{-C} e^{x\sqrt{-C}} + \frac{\mu_0^2 \sin(x)}{C-1} - \frac{\mu_0^2 \cos(x)}{C-1}\right) \\ B_x &= -\left(-\sqrt{C} C_1 e^{-\sqrt{C}z} + \sqrt{C} C_2 e^{\sqrt{C}z}\right) \left(C_1 e^{-x\sqrt{-C}} + C_2 e^{x\sqrt{-C}} - \frac{\mu_0^2 \sin(x)}{C-1} - \frac{\mu_0^2 \cos(x)}{C-1}\right) \end{aligned}$$