Statistics 4.5 Exercises 1 and 2

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Problems

TABLE 4.5.2 Proportions of Unmarried Male-Female Couples Sharing a Household in the United States, 1991

	Female	
Male	**Never married**	**Divorced**
Never married	.401	.111
Divorced	.117	.195
Widowed	.006	.008
Married to other	.021	.022
Total	.545	.336

Explanation

This table shows the proportions of unmarried couples living together in the United States in 1991, categorized by their marital status. The table is organized as a cross-tabulation with: - Rows representing the male partner's marital status: never married, divorced, widowed, or married to someone else - Columns representing the female partner's marital status using the same categories - Each cell containing the proportion of all unmarried cohabiting couples that fall into that specific combination For example: - The largest proportion (.401 or 40.1%) represents couples where both partners have never been married - 19.5% of couples consist of both partners being divorced - Only 1.6% of couples have both partners being widowed - 1.6% of couples have both partners being married to other people The row and column totals show the overall distributions by gender. For instance, 55.4% of males in these arrangements have never been married, while 54.5% of females have never been married. This data provides insight into the relationship history and legal marital status of unmarried couples who were sharing households in the early 1990s.

Problem 1

A house needs to be reroofed in the spring. To do this a dry, windless day is needed. The probability of getting a dry day is 0.7, a windy day is 0.4, and a wet and windy day is 0.2. What is the probability of getting: (a) a wet day?

$$P(W) = 1 - P(D) = .3$$

(b) a day that is either wet, windy, or both?

$$P(wet \cup windy) = P(wet) + P(windy) - P(wet \cap windy)$$

$$.3 + .4 - .2 = \mathbf{.5}$$

(c) a day when the house can be reroofed?

$$= P(dry \cap windless)$$

$$= P(dry) * P(windless)$$

assuming they are independent

$$= .7 * .6 = \mathbf{.42}$$

Problem 2

Using the data in Table 4.5.2, what is the probability that for a randomly chosen unmarried couple: (a) the male is divorced or married to someone else?

$$P(M_D \cup M_{MO}) = P(M_D) + P(M_{MO})$$

$$= .353 + .062 = \mathbf{.415}$$

(b) both the male and the female are either divorced or married to someone else?

$$P((M_D \cup M_{MO}) \cap (F_D \cup F_{MO}))$$

$$= P((M_D \cup M_{MO}) \text{ and } (F_D \cup F_{MO}))$$

$$= .195 + .017 + .024 + .017 + .008 + .001 + .022 + .003 + .016 = \mathbf{.303}$$

(c) neither is married to anyone else? Such a collection of events is technically known as a partition of S.

$$P(\text{neither married to others}) = P(\text{not } M_{MO} \text{ and not } F_{MO})$$

$$= 1 - P(M_{MO} \cup F_{MO})$$

$$= 1 - (.062 + .059 - .016)$$

$$= 1 - .105 = \mathbf{.895}$$

(d) at least one is married to someone else?

$$P(M_{MO} \cup F_{MO}) = P(M_{MO}) + P(F_{MO}) - P(M_{MO} \cap F_{MO})$$

$$= .062 + .059 - .016 = \mathbf{.105}$$

(e) the male is married to someone else or the female is divorced or both?

$$P(M_{MO} \cup F_D) = P(M_{MO}) + P(F_D) - P(M_{MO} \cap F_D)$$

$$= .062 + .336 - .022 = \mathbf{.376}$$

(f) the female is divorced and the male is not divorced?

$$P(F_D \cap \text{not } M_D \cap \text{not } M_{MO}) = P(F_D \cap M_{NM} \cap M_W)$$

$$= .111 + .017 = \mathbf{.128}$$

Quiz on Section 4.5

Question 1

If A and B are mutually exclusive, what is the probability that both occur? What is the probability that at least one occurs?

For mutually exclusive events A and B:

Probability that both occur: $P(A \cap B) = 0$

This is because mutually exclusive events cannot occur simultaneously by definition.

Probability that at least one occurs: $P(A \cup B) = P(A) + P(B)$

Since the events are mutually exclusive, there is no overlap to subtract.

Question 2

If we have two or more mutually exclusive events, how do we find the probability that at least one of them occurs?

For mutually exclusive events $A_1, A_2, ..., A_n$:

$P(A_1 \cup A_2 \cup ... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$

We simply add the individual probabilities since there is no overlap between mutually exclusive events.

Question 3

Why is it sometimes easier to compute $P(A)$ from $1 - P(A^c)$?

Computing $P(A)$ from $1 - P(A^c)$ (where A^c is the complement of A) is often easier when:

1. The event A is complex with many possible outcomes
2. The complement A^c has fewer outcomes or a simpler structure
3. A can be defined as "at least one" of several events (making A^c "none of them")

This approach leverages the complement rule: $P(A) + P(A^c) = 1$, which can simplify calculations significantly when A^c is more straightforward to analyze than A directly.