Statistics 7.2.1 Exercises 1 and 2

Standard Error and Normal Distributions

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Exercise 1: Sample Size and Standard Error

We know that $\mathrm{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}}$, where σ is the population standard deviation and n is the sample size. Also, $\mathrm{sd}(\bar{X})$ decreases proportionally to $\frac{1}{\sqrt{n}}$.

Given information: - Initial sample size is 10 observations - At this sample size, $\mathrm{sd}(\bar{X}) = 9$

Let's solve for how many additional observations are needed in each scenario.

Part (a): Reducing sd($\bar{X}$) to 4.5

We need to find the new sample size n_2 such that $\mathrm{sd}_2(\bar{X}) = 4.5$.

Using the relationship between standard errors and sample sizes: $\frac{\mathrm{sd}_2(\bar{X})}{\mathrm{sd}_1(\bar{X})} = \sqrt{\frac{n_1}{n_2}}$

$\frac{4.5}{9} = \sqrt{\frac{10}{n_2}}$

$\left(\frac{4.5}{9}\right)^2 = \frac{10}{n_2}$

$\frac{20.25}{81} = \frac{10}{n_2}$

$n_2 = \frac{10 \times 81}{20.25} = \frac{810}{20.25} = 40$

Since we already have 10 observations, we need an additional 30 observations.

```
# Verify calculation
n1 <- 10
sd1 <- 9
sd2 <- 4.5

n2 <- n1 * (sd1/sd2)^2
additional_obs <- n2 - n1

cat("New sample size needed:", n2, "\n")
```

```
New sample size needed: 40
```

```
cat("Additional observations needed:", additional_obs, "\n")
```

```
Additional observations needed: 30
```

Part (b): Reducing sd($\bar{X}$) to 3

Using the same approach: $\frac{3}{9} = \sqrt{\frac{10}{n_2}}$

$\left(\frac{3}{9}\right)^2 = \frac{10}{n_2}$

$\frac{9}{81} = \frac{10}{n_2}$

$n_2 = \frac{10 \times 81}{9} = \frac{810}{9} = 90$

Since we already have 10 observations, we need an additional 80 observations.

```
sd2 <- 3
n2 <- n1 * (sd1/sd2)^2
additional_obs <- n2 - n1

cat("New sample size needed:", n2, "\n")
```

```
New sample size needed: 90
```

```r
cat("Additional observations needed:", additional_obs, "\n")
```

```
Additional observations needed: 80
```

Part (c): Reducing sd($\bar{X}$) to 1

Using the same approach: $\frac{1}{9} = \sqrt{\frac{10}{n_2}}$

$\left(\frac{1}{9}\right)^2 = \frac{10}{n_2}$

$\frac{1}{81} = \frac{10}{n_2}$

$n_2 = \frac{10 \times 81}{1} = 810$

Since we already have 10 observations, we need an additional 800 observations.

```r
sd2 <- 1
n2 <- n1 * (sd1/sd2)^2
additional_obs <- n2 - n1

cat("New sample size needed:", n2, "\n")
```

```
New sample size needed: 810
```

```r
cat("Additional observations needed:", additional_obs, "\n")
```

```
Additional observations needed: 800
```

Exercise 2: Normal Distribution and Probability Calculation

Problem Setup

Given information: - Monthly profits (X) follow Normal distribution: $X \sim \text{Normal}(\mu = 10, \sigma = 3.5)$ million dollars - Average is calculated over a six-month period: $\bar{X}$ - Company pays dividend if $\bar{X} > 8.5$ million - Need to find probability that company will pay a dividend

Finding the Distribution of $\bar{X}$

When individual observations follow a Normal distribution, the sampling distribution of the mean also follows a Normal distribution:

$\bar{X} \sim \text{Normal}(\mu_{\bar{X}}, \sigma_{\bar{X}})$

Where: - $\mu_{\bar{X}} = \mu = 10$ million (the mean of $\bar{X}$ equals the population mean) - $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{6}} = \frac{3.5}{2.45} \approx 1.429$ million

```
mu <- 10        # Population mean
sigma <- 3.5    # Population standard deviation
n <- 6          # Sample size (6 months)

mu_x_bar <- mu
sigma_x_bar <- sigma / sqrt(n)

cat("Mean of sampling distribution:", mu_x_bar, "million\n")
```

```
Mean of sampling distribution: 10 million
```

```
cat("Standard deviation of sampling distribution:", sigma_x_bar, "million\n")
```

```
Standard deviation of sampling distribution: 1.428869 million
```

Calculating the Probability of Paying Dividend

We need to find $P(\bar{X} > 8.5)$

First, we standardize to find the z-score: $z = \frac{8.5-10}{1.429} \approx -1.05$

Then, we find the probability: $P(\bar{X} > 8.5) = P(Z > -1.05) = 1 - P(Z \leq -1.05)$

```
threshold <- 8.5
z_score <- (threshold - mu_x_bar) / sigma_x_bar
probability <- 1 - pnorm(z_score)

cat("Z-score:", z_score, "\n")
```

```
Z-score: -1.049781
```

```r
cat("Probability of paying dividend:", probability, "\n")
```

```
Probability of paying dividend: 0.8530907
```

```r
cat("Probability as percentage:", probability * 100, "%\n")
```

```
Probability as percentage: 85.30907 %
```

The probability that the company will pay an annual dividend is approximately 0.853 or 85.3%.