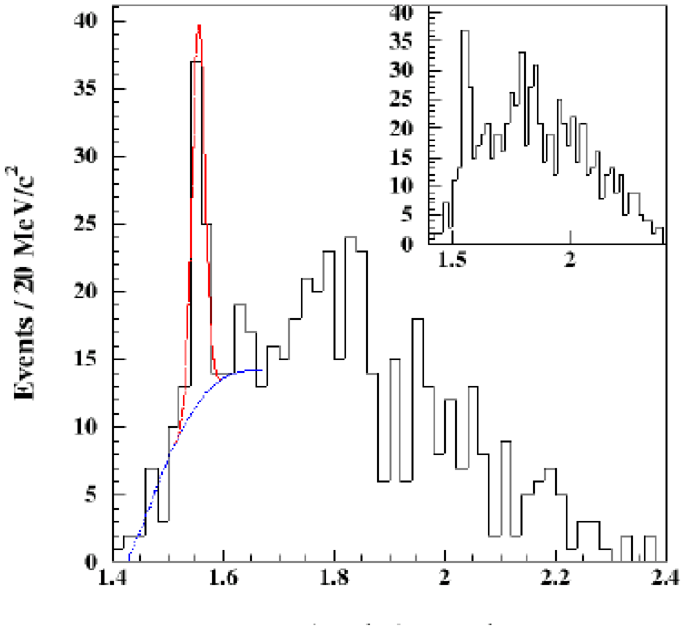
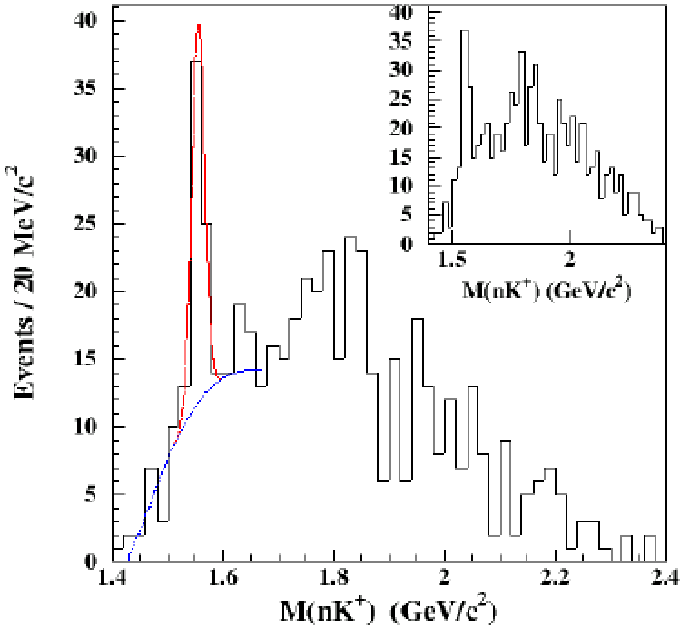
Is there evidence for a peak in this data?



Is there evidence for a peak in this data?



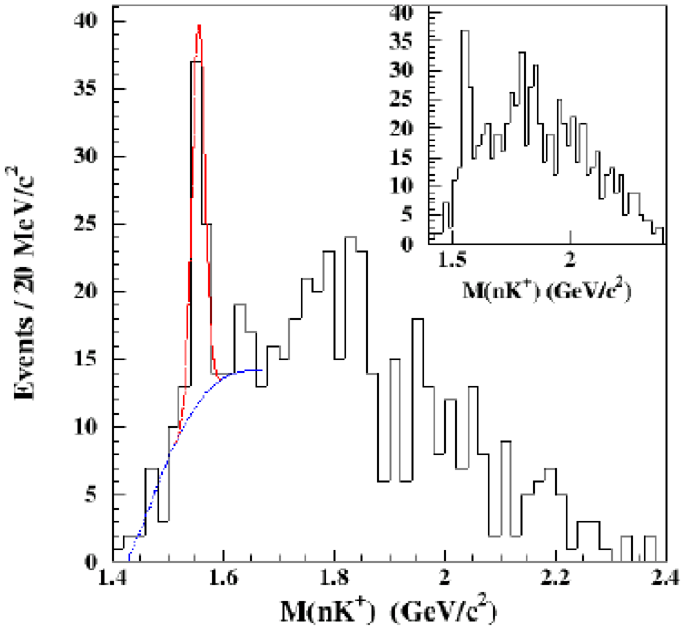
“Observation of an Exotic S=+1

Baryon in Exclusive Photoproduction from the Deuteron”

S. Stepanyan et al, CLAS Collab, Phys.Rev.Lett. 91 (2003) 252001

“The statistical significance of the peak is $5.2 \pm 0.6\ \sigma$”

Is there evidence for a peak in this data?



"Observation of an Exotic S=+1 Baryon in Exclusive Photoproduction from the Deuteron"

S. Stepanyan et al, CLAS Collab, Phys.Rev.Lett. 91 (2003) 252001

"The statistical significance of the peak is 5.2 ± 0.6 σ"

"A Bayesian analysis of pentaquark signals from CLAS data"

D. G. Ireland et al, CLAS Collab, Phys. Rev. Lett. 100, 052001 (2008)

"The ln(RE) value for g2a (-0.408) indicates weak evidence in favour of the data model without a peak in the spectrum."

Comment on "Bayesian Analysis of Pentaquark Signals from CLAS Data" Bob Cousins, http://arxiv.org/abs/0807.1330

Statistical issues in searches for New Phenomena:

p-values, Upper Limits and Discovery

Louis Lyons

IC and Oxford

l.lyons@physics.ox.ac.uk

CERN Summer Students,

July 2015

PHYSTAT 2011 Workshop at CERN, Geneva

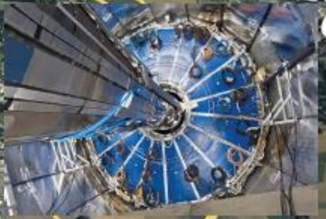
CERN

17-20 January 2011

http://indico.cern.ch/event/phystat2011

Contacts:
Albert de Roeck <albert.de.roeck@cern.ch>
Louis Lyons <l.lyons1@physics.ox.ac.uk>

LPCC
LHC Physics Centre at CERN



SPS 7 km

LHC 27 km

Jan 17-19: Statistical Issues for Search Experiments

Statistical issues related to discovery claims in search experiments, with emphasis on those at the LHC.

Jan 20: Unfolding

Unfolding of detector effects from experimental distributions

Programme committee
J. Berger (Duke)
V. Blobel (Hamburg)
R. Cousins (UCLA)
D. Cox (Oxford)
G. Cowan (Royal Holloway)
K. Cranmer (NYU)
L. Demortier (Rockefeller)
A. de Roeck (CERN)
B. Efron (Stanford)
G. Flucke (DESY)
E. Gross (Weizmann)
D. Hand (Imperial College)
J. Linnemann (MSU)
R. Lockhart (Simon Fraser)
L. Lyons (Imperial College)
M.L. Mangano (CERN)
S. Schmitt (DESY)
M. Williams (Imperial College)

TOPICS

Discoveries

H0 or H0 v H1

p-values: For Gaussian, Poisson and multi-variate data

What is p good for?

Errors of 1st and 2nd kind

What a p-value is not

Combining p-values

Significance

Look Elsewhere Effect

Blind Analysis

Why 5σ?

Setting Limits

Case study: Search for Higgs boson

DISCOVERIES

"Recent" history:

Charm	SLAC, BNL	1974
Tau lepton	SLAC	1977
Bottom	FNAL	1977
W, Z	CERN	1983
Top	FNAL	1995
{Pentaquarks	~Everywhere	2002 }
Higgs	CERN	2012
?	CERN	2015?

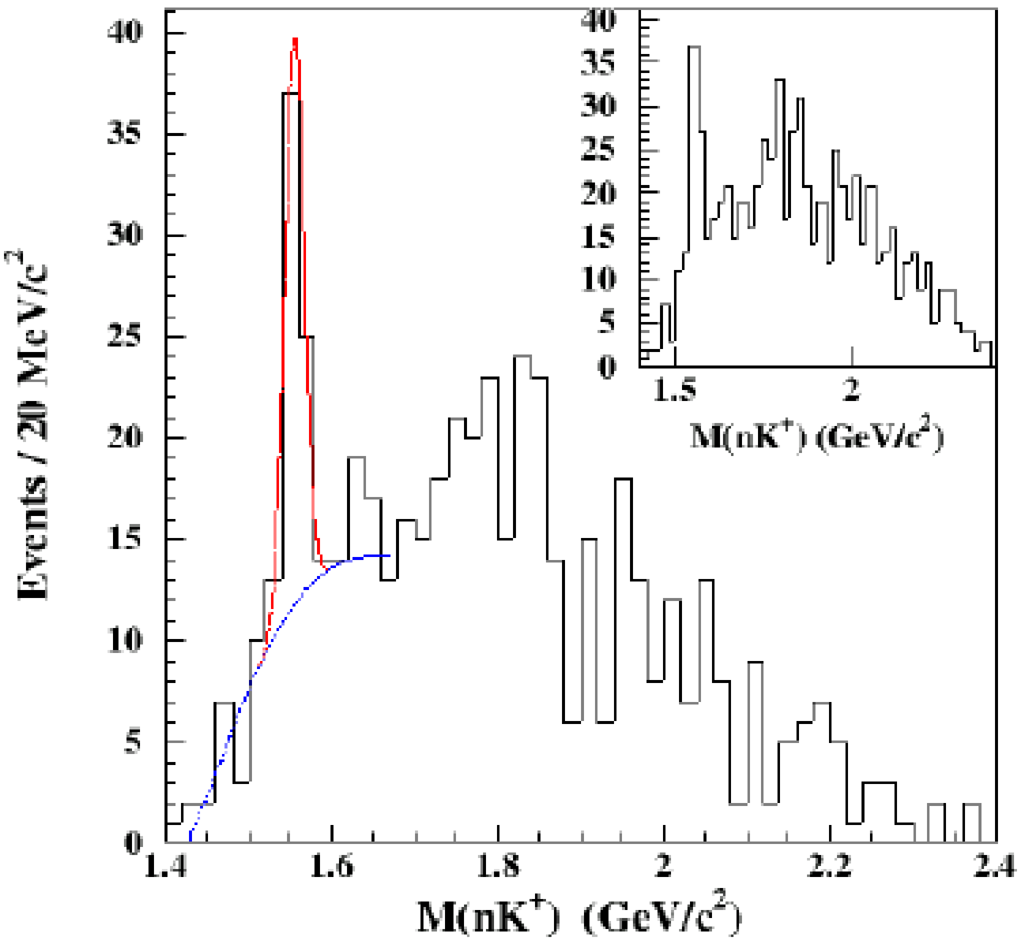
? = SUSY, q and l substructure, extra dimensions,

free q/monopoles, technicolour, 4^{th} generation, black holes,…..

QUESTION: How to distinguish discoveries from fluctuations?

Penta-quarks?

Hypothesis testing: New particle or statistical fluctuation?



H0 or H0 versus H1 ?

H0 = null hypothesis

e.g. Standard Model, with nothing new

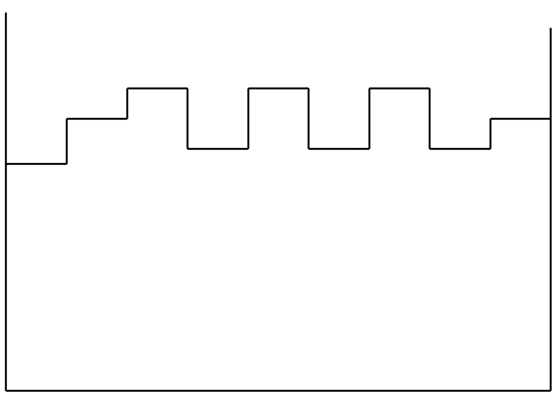
H1 = specific New Physics   e.g. Higgs with M_H = 125 GeV

H0: “Goodness of Fit” e.g. χ^2, p-values

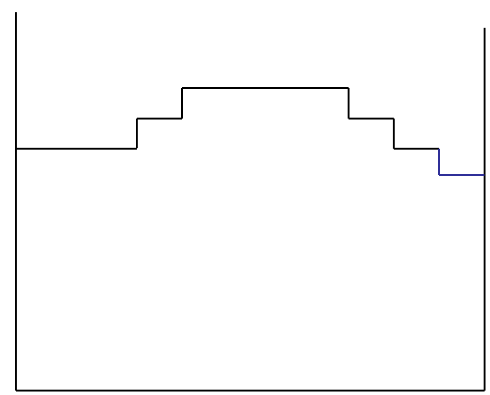
H0 v H1: “Hypothesis Testing” e.g. $\mathcal{L}$-ratio

Measures how much data favours one hypothesis wrt other

H0 v H1 likely to be more sensitive for H1



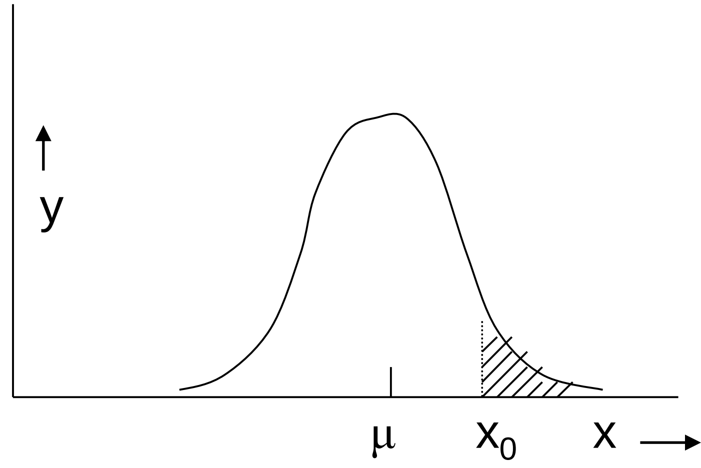
or



p-values

Concept of pdf

Example: Gaussian



y = probability density for measurement x

$y = 1/(\sqrt{(2\pi)}\sigma) \exp\{-0.5*(x-\mu)^2/\sigma^2\}$

p-value: probablity that $x \geq x_0$

Gives probability of "extreme" values of data (in interesting direction)

$(x_0-\mu)/\sigma$	1	2	3	4	5
p	16%	2.3%	0.13%	0. 003%	$0.3*10^{-6}$

i.e. Small p = unexpected

p-values, contd

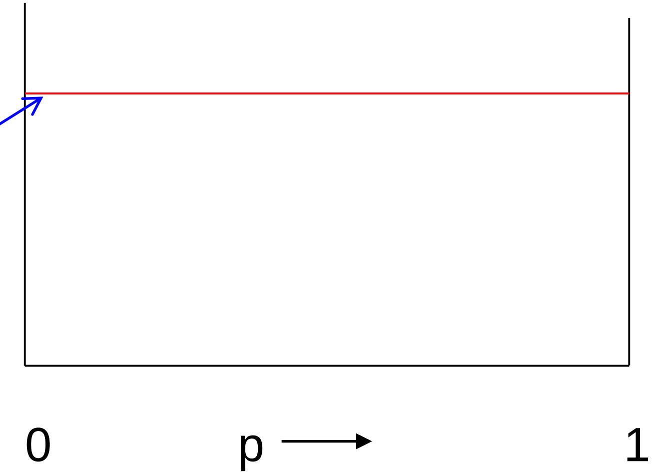
Assumes:

- Specific pdf for x (e.g. Gaussian, no long tails)
- Data is unbiassed
- σ is correct

If so, and x is from that pdf ⟹ uniform p-distribution

(Events at large x give small p)

Interesting region

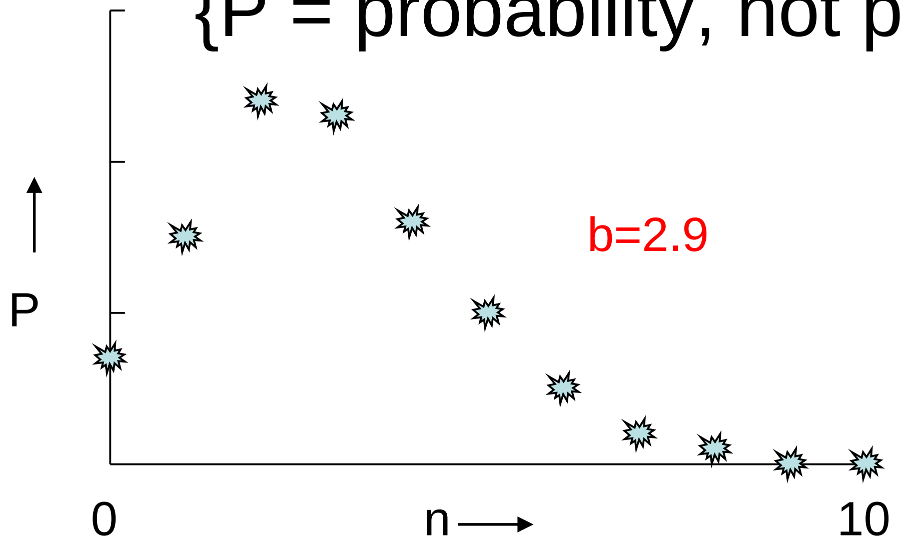


p-values for non-Gaussian distributions

e.g. Poisson counting experiment, bgd = b

$P(n) = e^{-b} * b^n/n!$

{P = probability, not prob density}



For n=7, p = Prob(at least 7 events) = P(7) + P(8) + P(9) +…….. = 0.03

p-values and σ

p-values often converted into equivalent Gaussian σ

e.g. $3*10^{-7}$ is "5σ" (one-sided Gaussian tail)

Does NOT imply that pdf = Gaussian

(Simply easier to remember number of σ, than p-value.)

What is p good for?

Used to test whether data is consistent with H0

Reject H0 if p is small : $p \leq \alpha$ (How small?)

Sometimes make wrong decision:

Reject H0 when H0 is true: **Error of 1st kind**

Should happen at rate α

OR

Fail to reject H0 when something else (H1,H2,…) is true: **Error of 2nd kind**

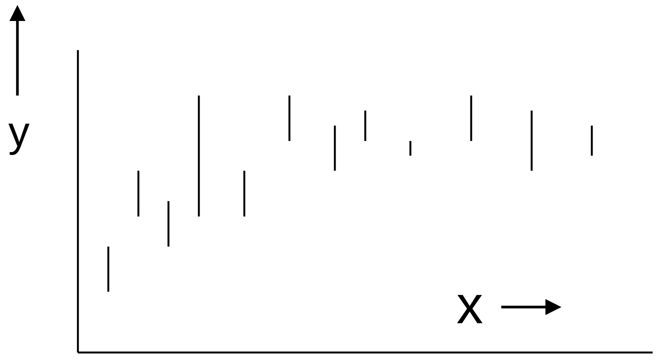
Rate at which this happens depends on……….

Errors of 2nd kind: How often?

e.g.1. Does data lie on straight line?

Calculate S_{min}

Reject if $S_{min} \geq 20$



Error of 1st kind: $S_{min} \geq 20$ Reject H0 when true

Error of 2nd kind: $S_{min} < 20$ Accept H0 when in fact quadratic or..

How often depends on:

Size of quadratic term

Magnitude of errors on data, spread in x-values,…….

How frequently quadratic term is present

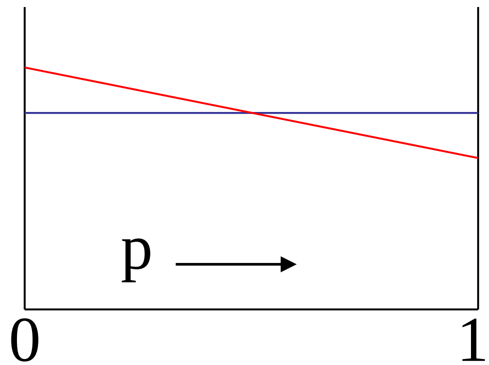
Compromise on error rates

Errors of 2nd kind: How often?

e.g. 2. Particle identification (TOF, dE/dx, Čerenkov,…….)

Particles are π or μ

Extract p-value for H0 = π from PID information



π and μ have similar masses

Of particles that have p ~ 1% ('reject H0'), fraction that are π is

a) ~ half, for equal mixture of π and μ

b) almost all, for "pure" π beam

c) very few, for "pure" μ beam

p-value is not ……..

Does **NOT** measure Prob(H0 is true)

i.e. It is **NOT** P(H0|data)

It is P(data|H0)

N.B. P(H0|data) $\neq$ P(data|H0)

P(theory|data) $\neq$ P(data|theory)

“Of all results with $p \leq 5\%$, half will turn out to be wrong”

N.B. Nothing wrong with this statement

e.g. 1000 tests of energy conservation

~50 should have $p \leq 5\%$, and so reject H0 = energy conservation

Of these 50 results, all are likely to be “wrong”

Combining different p-values

******* Better to combine data ************

Several results quote independent p-values for same effect:

p_1, p_2, p_3….. e.g. 0.9, 0.001, 0.3 ……..

What is combined significance? Not just $p_{1*}p_{2*}p_3$…..

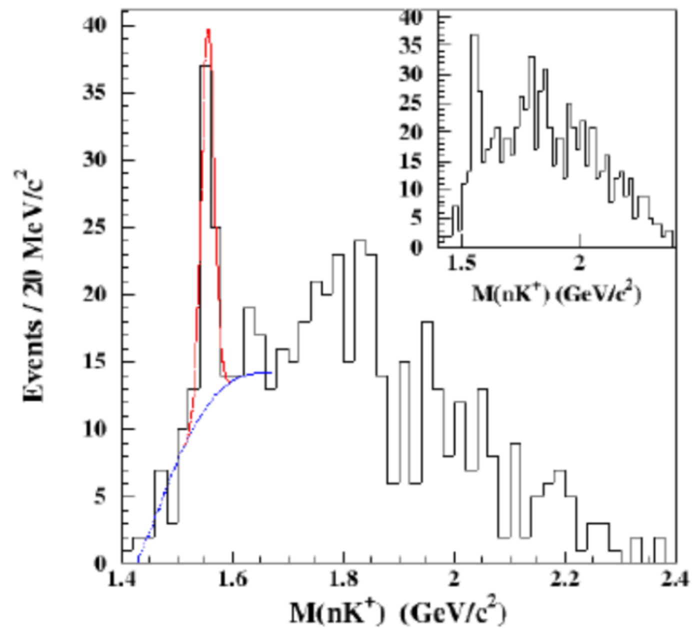
(If 10 expts each have p ~ 0.5, product ~ 0.001 and is clearly **NOT** correct combined p)

N.B. Problem does not have unique answer

$$S = z_* \sum_{j=0}^{n-1} (-\ln z)^j / j! \quad , \qquad z = p_1 p_2 p_3 \ldots\ldots.$$

(e.g. For 2 measurements, $S = z_* (1 - \ln z) \geq z$)

Significance



Significance = S/√B ? {or S/√(B+S)?}

Potential Problems:

•Uncertainty in B

•Non-Gaussian behaviour of Poisson, especially in tail

•Number of bins in histogram, no. of other histograms [LEE]

•Choice of cuts (Blind analyses)

•Choice of bins (……………….)

For future experiments:

• Optimising cuts: Could give S =0.1, B = 10^{-4}, S/√B =10

• N.B. S/ √(S+B) also has problems

• Best to use proper Poisson p

Look Elsewhere Effect

See ‘peak’ in bin of histogram

Assuming null hypothesis, p-value is chance of fluctuation at least as significant as observed ……….

1) at the position observed in the data; or
2) anywhere in that histogram; or
3) including other relevant histograms for your analysis; or
4) including other analyses in Collaboration; or
5) in any CERN experiment; or

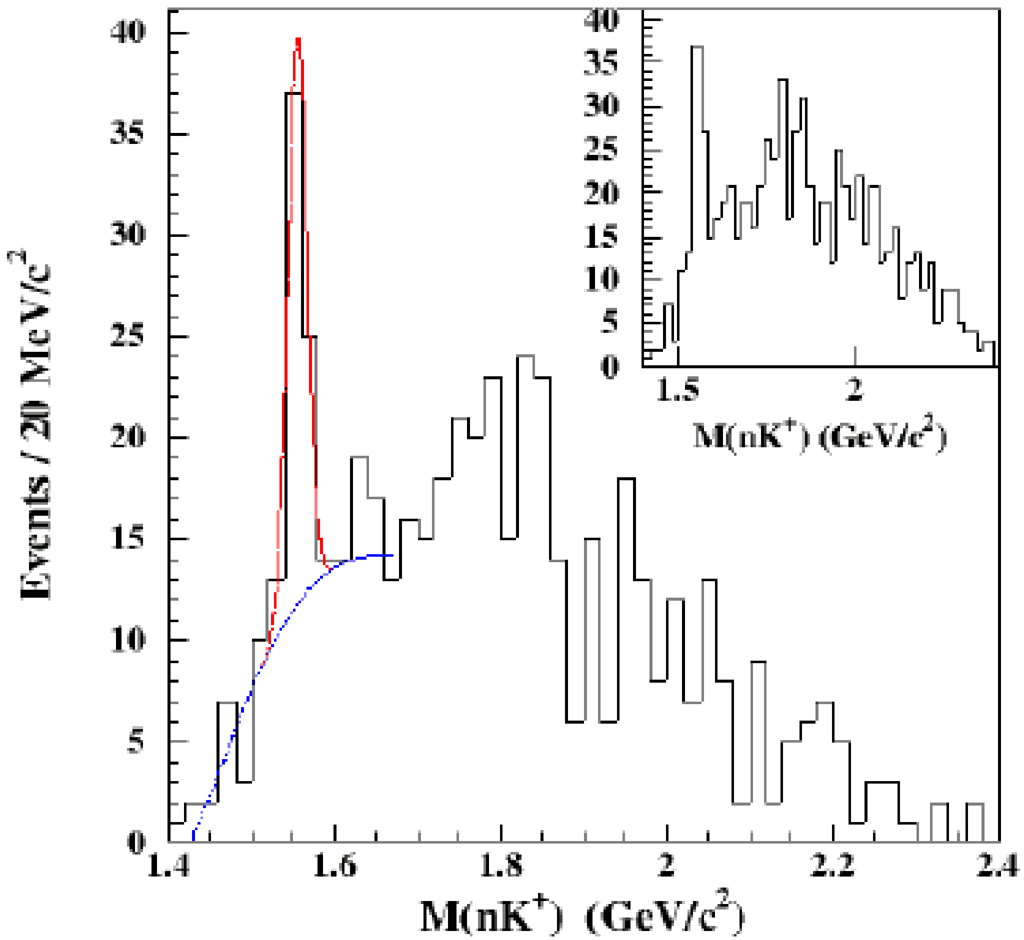
etc.

Contrast local p-value with ‘global’ p-value

Specify what is your ‘global’

Penta-quarks?

Hypothesis testing: New particle or statistical fluctuation?



Example of LEE: Stonehenge

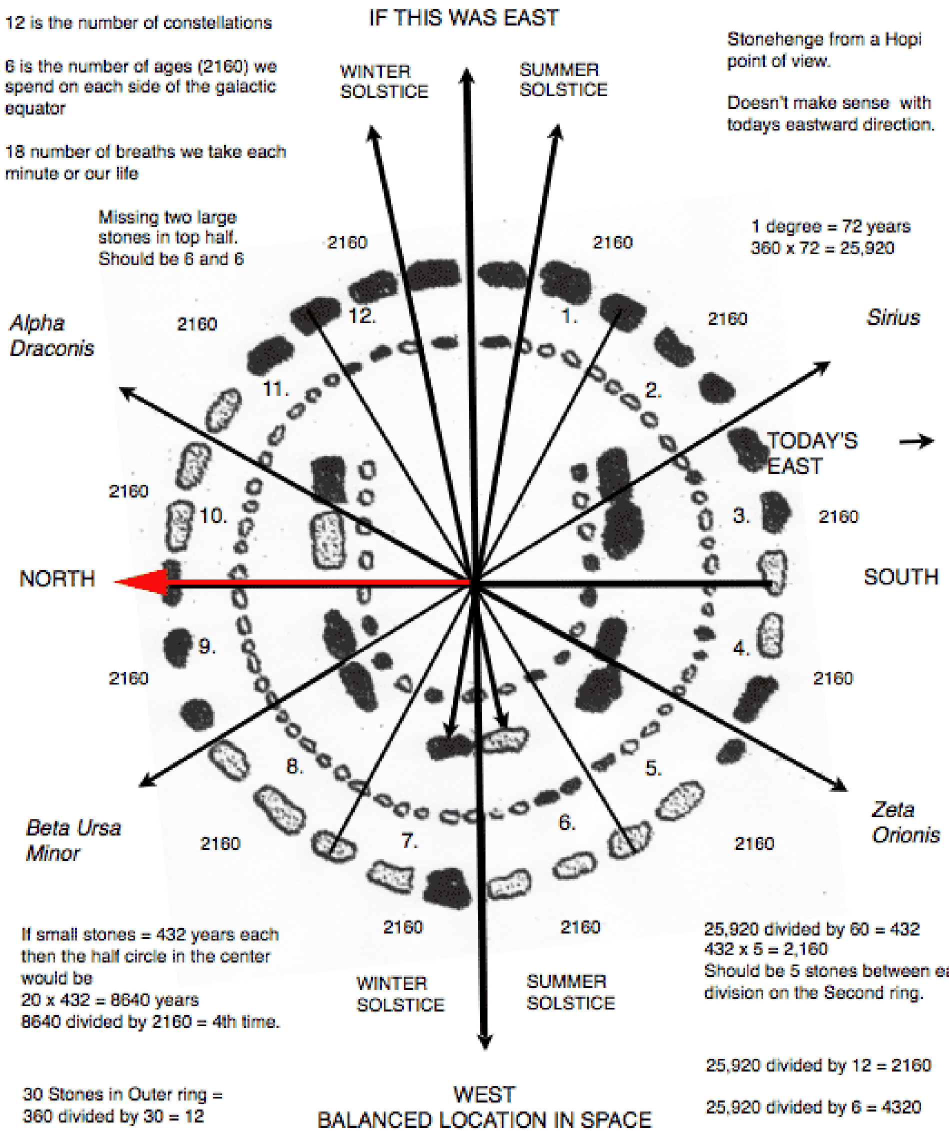
12 is the number of constellations

6 is the number of ages (2160) we spend on each side of the galactic equator

18 number of breaths we take each minute or our life

Stonehenge from a Hopi point of view.

Doesn't make sense with todays eastward direction.



If small stones = 432 years each then the half circle in the center would be
20 x 432 = 8640 years
8640 divided by 2160 = 4th time.

25,920 divided by 60 = 432
432 x 5 = 2,160
Should be 5 stones between each division on the Second ring.

30 Stones in Outer ring =
360 divided by 30 = 12

60 Stones in Second ring =
360 divided by 60 = 6

20 Stones in Center ring =
360 divided by 20 = 18

25,920 divided by 12 = 2160

25,920 divided by 6 = 4320

25,920 divided by 18 = 1440

Center Stone in Center Ring would be divided in half by sun rays when Earth in perfect balance.
Nine on each side + 2 = 20.

STONEHENGE

The Book of Truth
A New Perspective on the Hopi Creation Story
by Thomas O. Mills

Are alignments significant?

- Atkinson replied with his article "Moonshine on Stonehenge" in *Antiquity* in 1966, pointing out that some of the pits which ….. had used for his sight lines were more likely to have been natural depressions, and that he had allowed a margin of error of up to 2 degrees in his alignments. Atkinson found that the probability of so many alignments being visible from 165 points to be close to 0.5 rather that the "one in a million" possibility which ….. had claimed.

- ….. had been examining stone circles since the 1950s in search of astronomical alignments and the megalithic yard. It was not until 1973 that he turned his attention to Stonehenge. He chose to ignore alignments between features within the monument, considering them to be too close together to be reliable. He looked for landscape features that could have marked lunar and solar events. However, one of …..'s key sites, Peter's Mound, turned out to be a twentieth-century rubbish dump.

BLIND ANALYSES

Why blind analysis?

Selections, corrections, method

Methods of blinding

Add random number to result *

Study procedure with simulation only

Look at only first fraction of data

Keep the signal box closed

Keep MC parameters hidden

Keep unknown fraction visible for each bin

After analysis is unblinded, ……..

* Luis Alvarez suggestion re “discovery” of free quarks

Why 5σ?

- Past experience with 3σ, 4σ,… signals
- Look elsewhere effect:

  Different cuts to produce data

  Different bins (and binning) of this histogram

  Different distributions Collaboration did/could look at

  Other analyses in Physics subgroup, expt, CERN,…

. Worries about systematics (easily under-estimated?)

- Bayesian priors:

$$\frac{P(H0|data)}{P(H1|data)} = \frac{P(data|H0) * P(H0)}{P(data|H1) * P(H1)}$$

Bayes posteriors Likelihoods Priors

Prior for {H0 = S.M.} >>> Prior for {H1 = New Physics}

Why 5σ?

BEWARE of tails,

especially for nuisance parameters

Same criterion for all searches

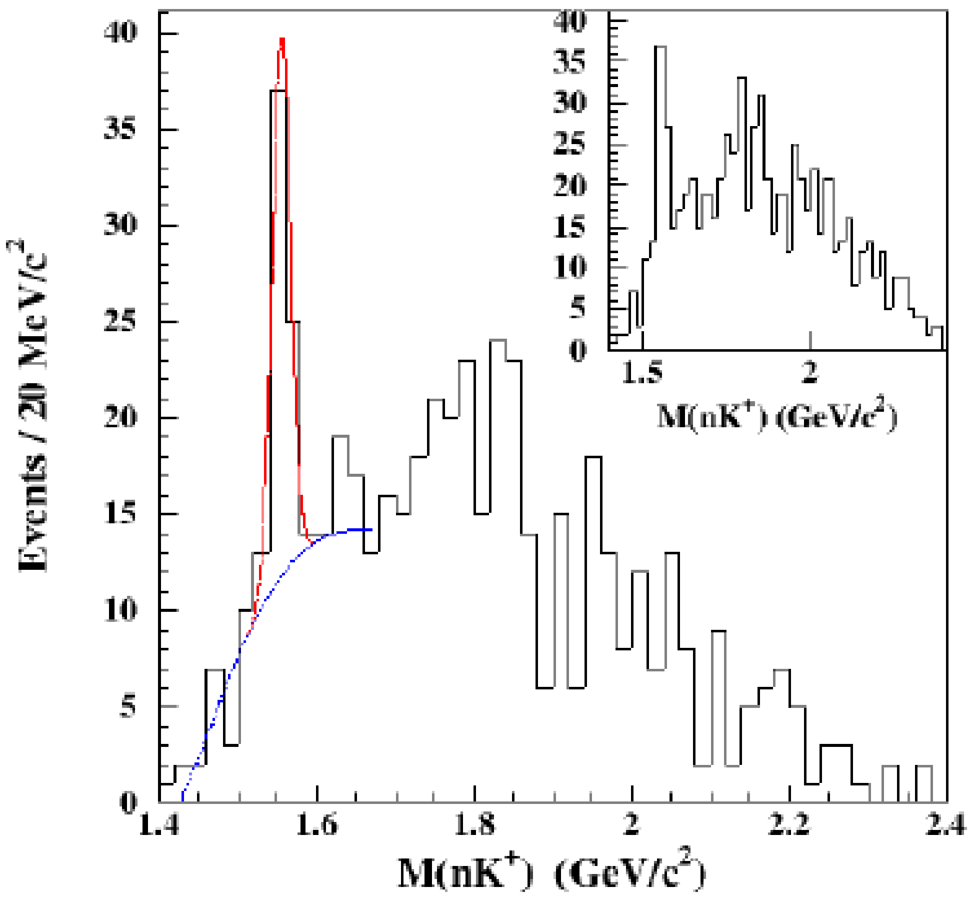
- Different LEE (contrast muon magnetic moment v. CMS)
- Different role of systematics
- Different Bayes priors, e.g.
  - Single top production
  - Higgs
  - Highly speculative particle
  - Energy non-conservation

Blind analysis helps

Choosing between 2 hypotheses

Hypothesis testing: New particle or statistical fluctuation?

H0 = b H1 = b + s



Choosing between 2 hypotheses

Possible methods:

$\Delta\chi^2$

p-value of statistic $\rightarrow$

$ln\mathcal{L}$–ratio

Bayesian:

Posterior odds

Bayes factor

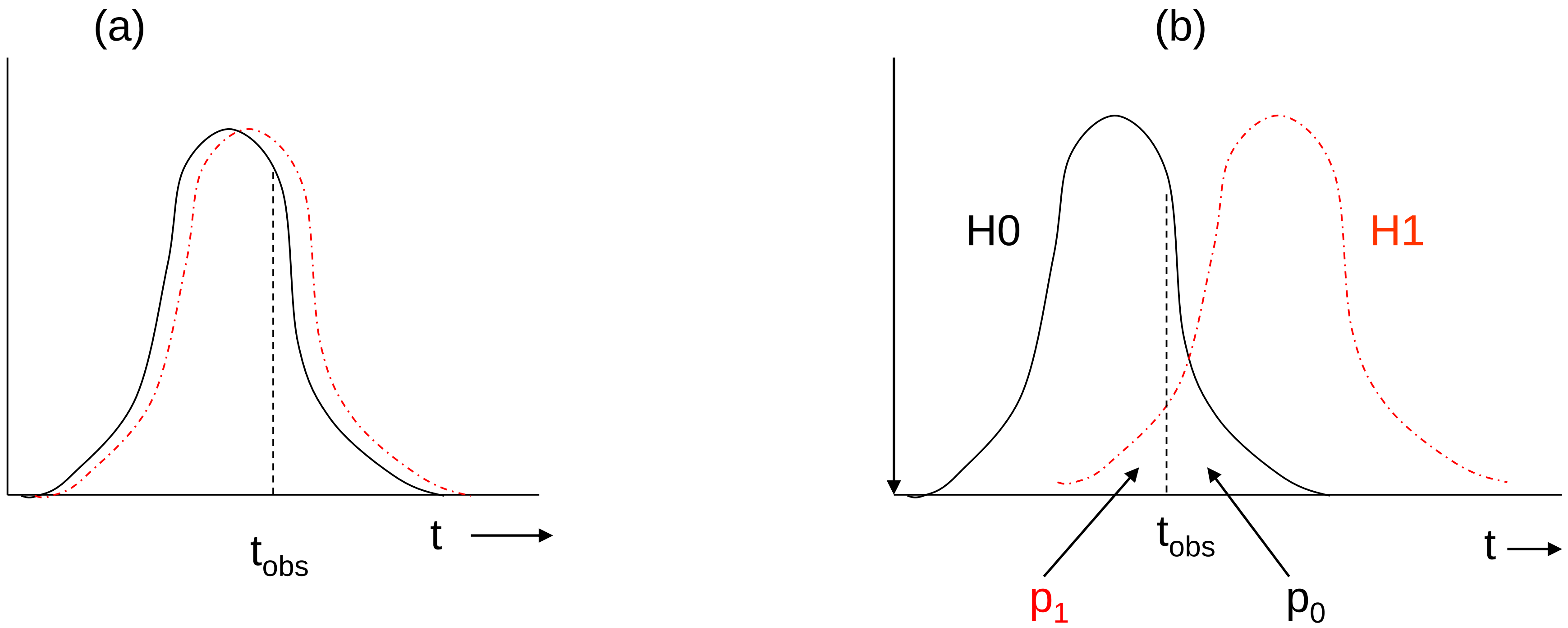
Bayes information criterion (BIC)

Akaike …….. (AIC)

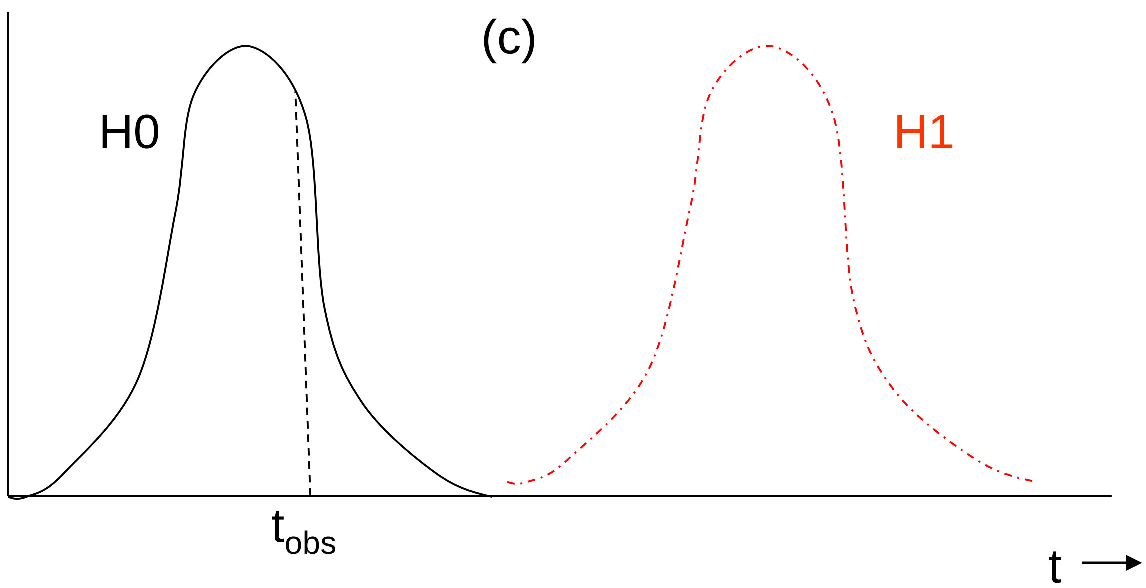
Minimise "cost"

See 'Comparing two hypotheses'

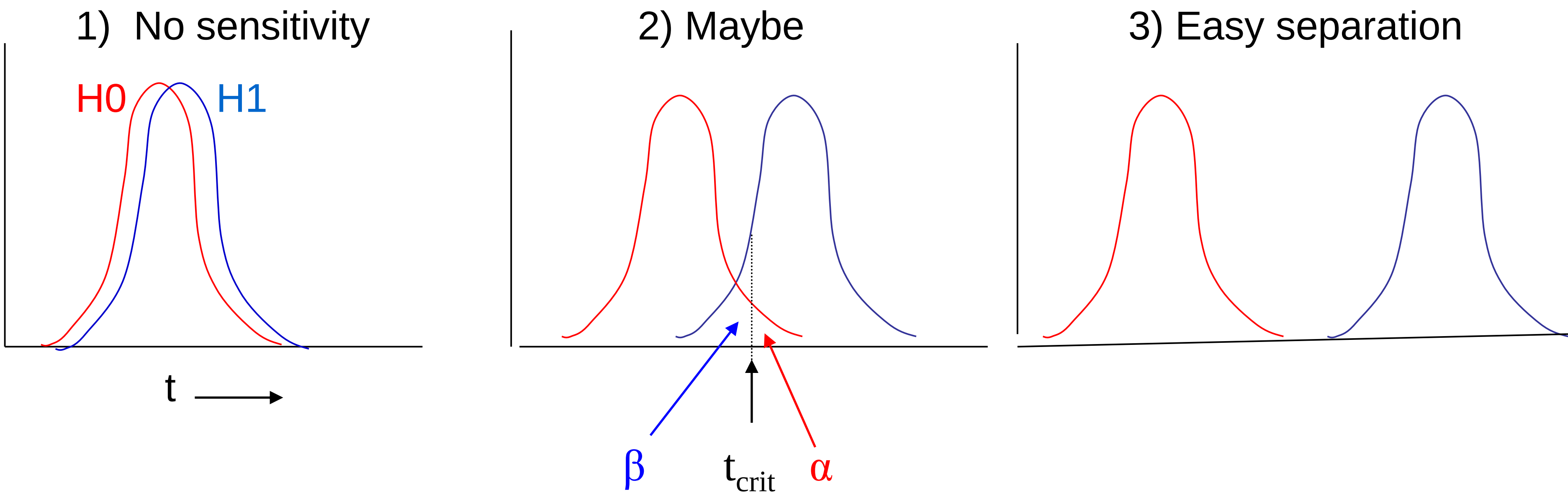
http://www.physics.ox.ac.uk/users/lyons/H0H1_A~1.pdf



With 2 hypotheses, each with own pdf, p-values are defined as tail areas, pointing in towards each other



Procedure for choosing between 2 hypotheses



Procedure: Choose α (e.g. 95%, 3σ, 5σ ?) and CL for β (e.g. 95%)

Given b, α determines t_{crit}

s defines β. For $s > s_{min}$, separation of curves → discovery or excln

1-β = Power of test

Now data: If $t_{obs} \geq t_{crit}$ (i.e. $p_0 \leq \alpha$), discovery at level α

If $t_{obs} < t_{crit}$, no discovery. If $p_1 <$ 1– CL, exclude H1

LIMITS

Look for New Physics s

See no effect. Set upper limit on s

If $s < s_{expected}$, exclude this sort of New Physics

HEP experiments: If UL on rate for new particle production < expected, exclude particle

Big industry in Particle Physics

Michelson-Morley experiment → death of aether

CERN CLW (Jan 2000)

FNAL CLW (March 2000)

Heinrich, PHYSTAT-LHC, "Review of Banff Challenge"

SIMPLE PROBLEM?

Gaussian

$\sim \exp\{-0.5*(x-\mu)^2/\sigma^2\}$, with data x_0

No restriction on param of interest μ; σ known exactly

$\mu \leq x_0 + k\,\sigma$

BUT Poisson $\{\mu = s\varepsilon + b\}$

$s \geq 0$

ε and b with uncertainties

Not like : $2 + 3 = ?$

N.B. Actual limit from experiment ≠ Expected (median) limit

Methods

Bayes (needs priors e.g. const, $1/\mu$, $1/\sqrt{\mu}$, μ, …..)

Frequentist (needs ordering rule,

possible empty intervals, F-C)

Likelihood (DON'T integrate your L)

χ^2 ($\sigma^2 = \mu$)

χ^2(σ^2 = n)

Also have to incorporate systematics

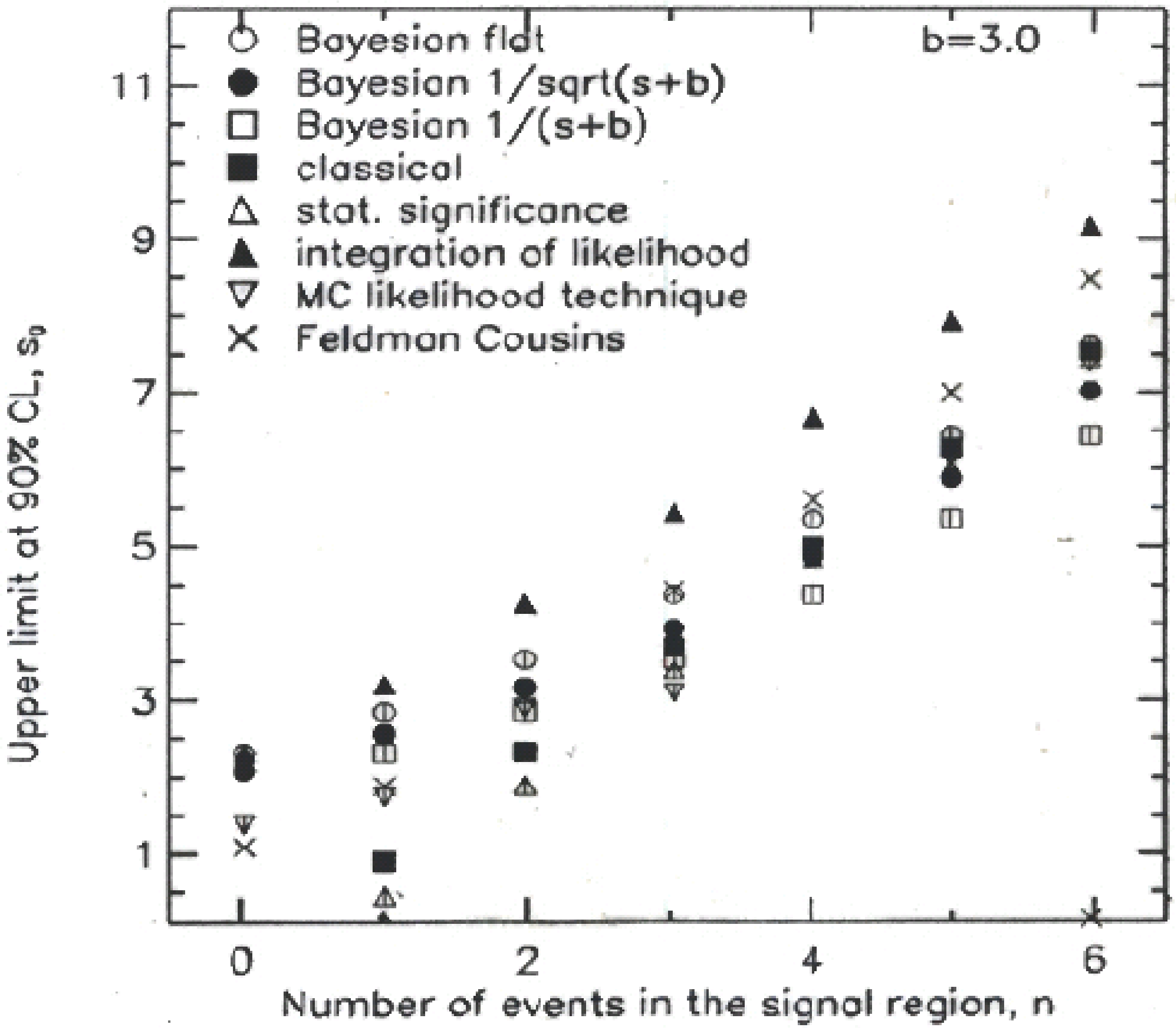
Recommendation 7 from CERN CLW (2000): "Show your **L**"

1) Not always practical

2) Not sufficient for frequentist methods

Ilya Narsky, FNAL CLW 2000

Poisson counting expt
Expected bgd = b
Observe n
Set UL for possible signal s



DESIRABLE PROPERTIES

- Coverage
- Interval length
- Behaviour when $n < b$
- Limit increases as σ_b increases
- Unified with discovery and interval estimation

INTERVAL LENGTH

Empty → Unhappy physicists

Very short→ False impression of sensitivity

Too long→ loss of power

(2-sided intervals are more complicated because 'shorter' is not metric-independent:

e.g. 0 → 9 or 4 → 16 for x^2

cf 0 → 3 or 2 → 4 for x)

Recommendations?

CDF note 7739 (May 2005)

Decide method and procedure in advance

No valid method is ruled out

Bayes is simplest for incorporating nuisance params

- Check robustness
- Quote coverage

Quote sensitivity

Use same method as other similar expts

Explain method used

Case study: Successful search for Higgs boson

(Meeting of statisticians, atomic physicists, astrophysicists and particle physicist:
"What is value of H0?")

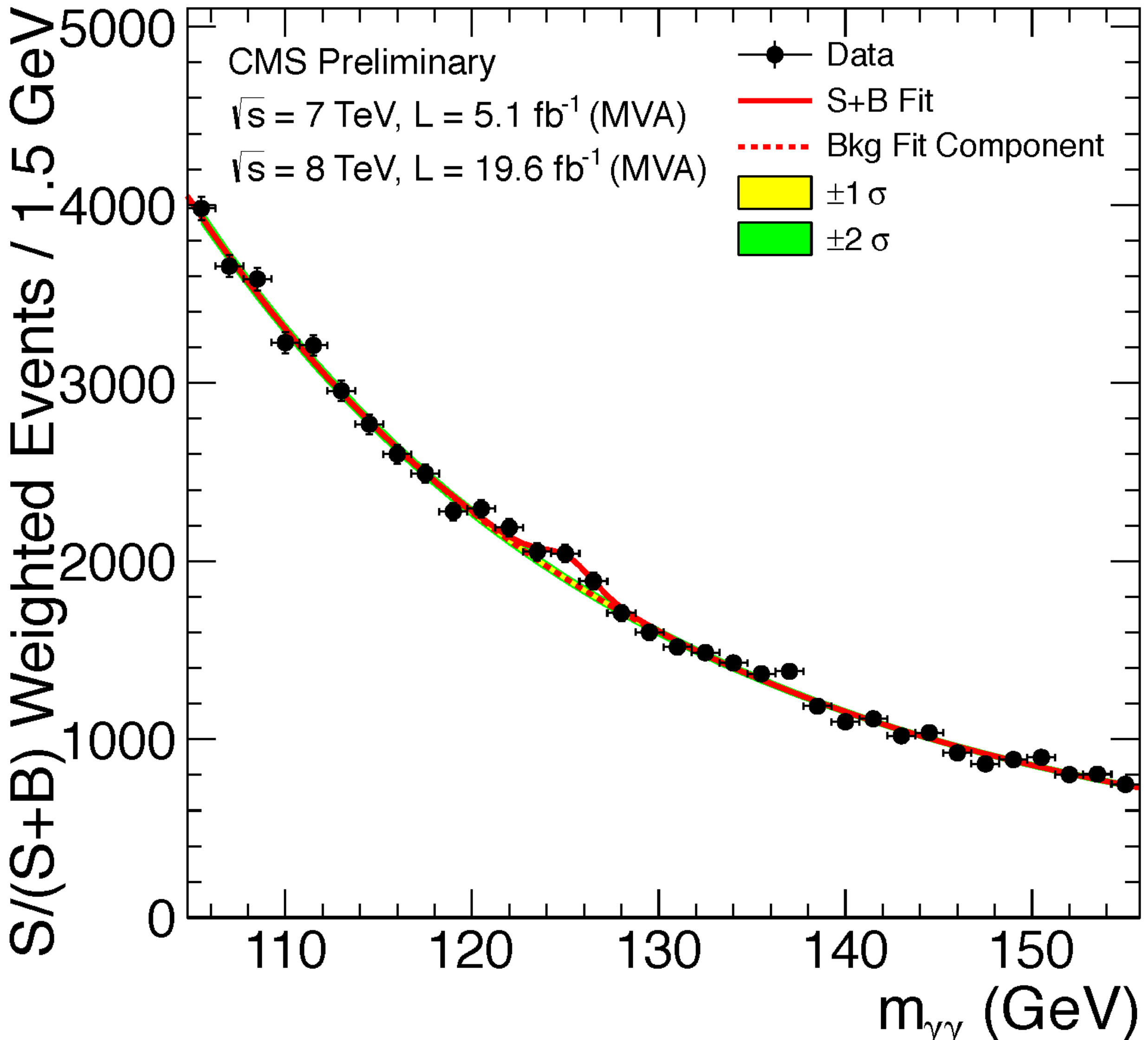
H^0 very fundamental

Wanted to discover Higgs,

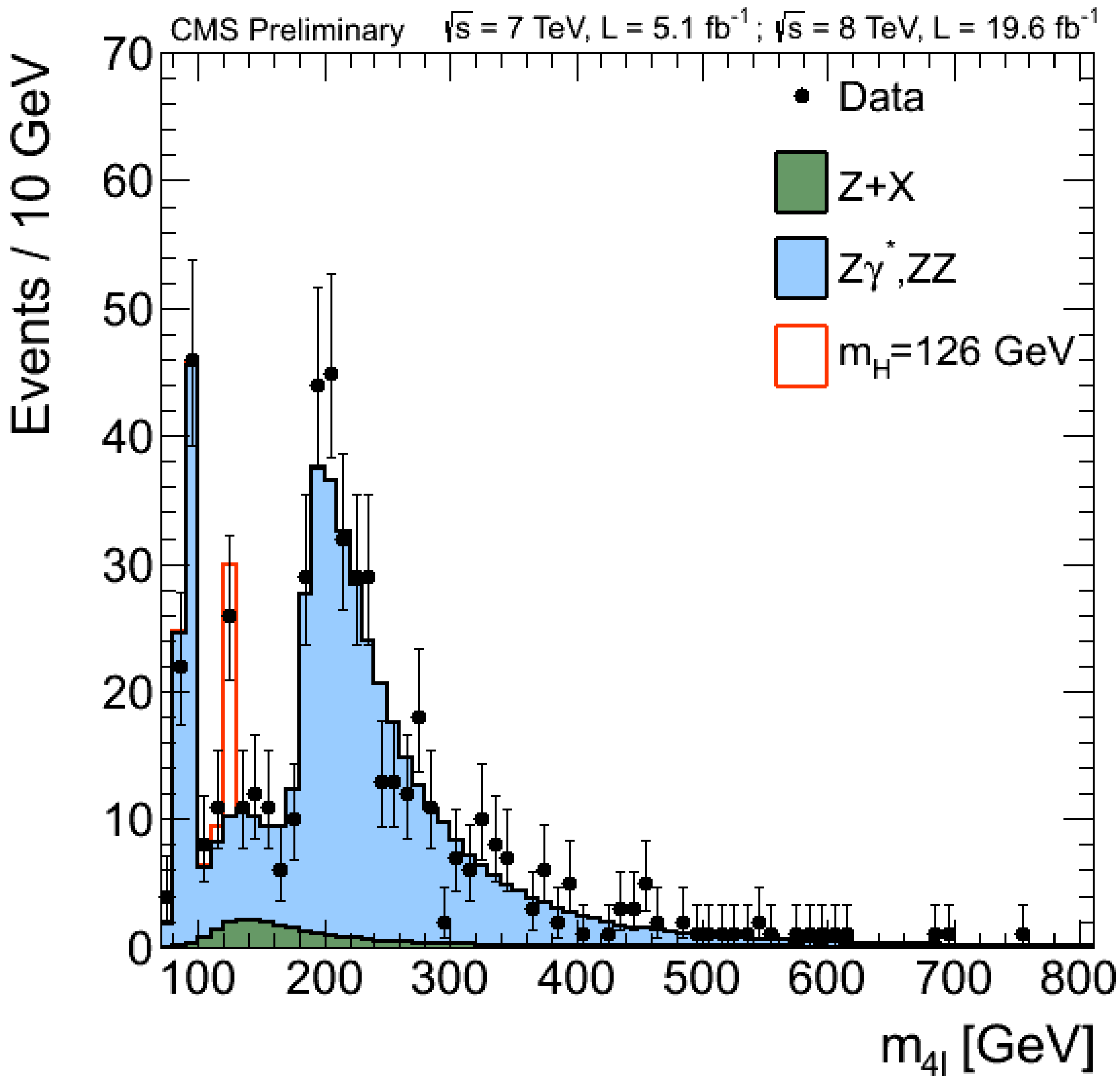
but otherwise exclude

{Other possibility is 'Not enough data to distinguish'}

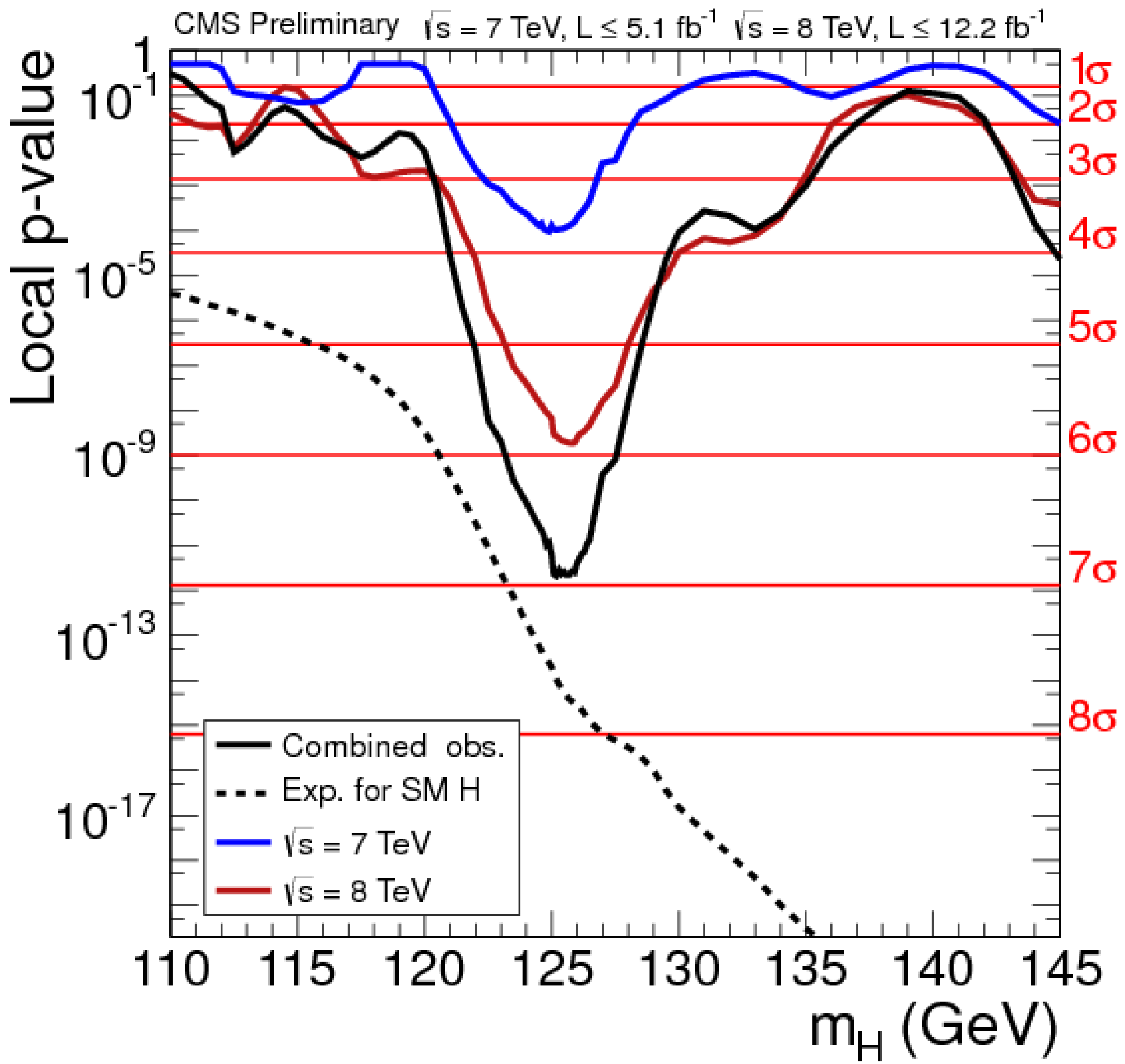
H→ γ γ: low S/B, high statistics



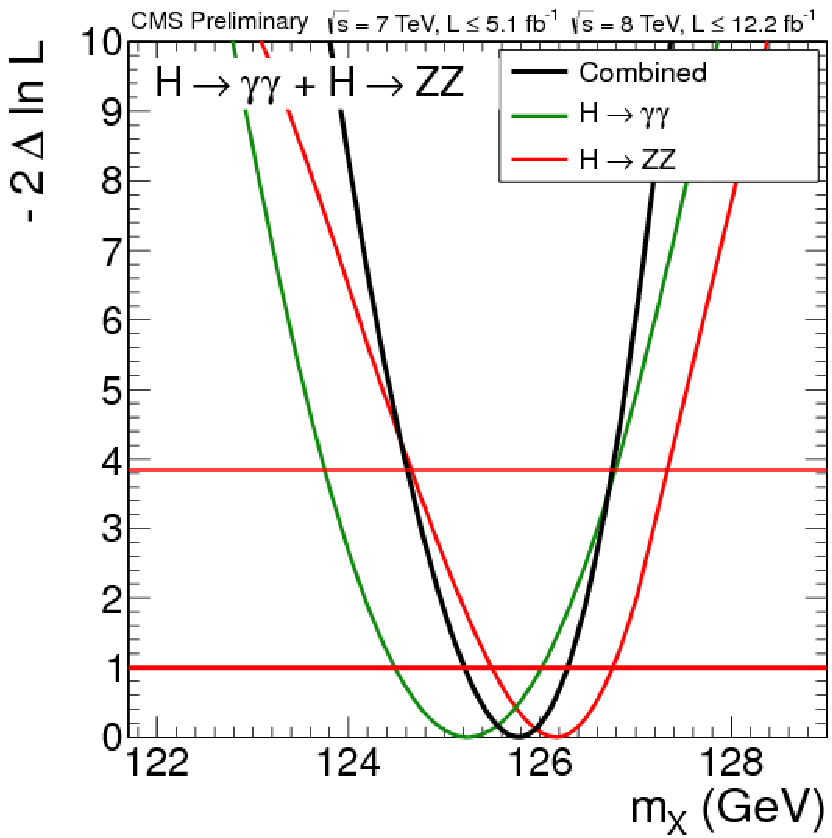
H→Z Z → 4 leptons: high S/B, low statistics

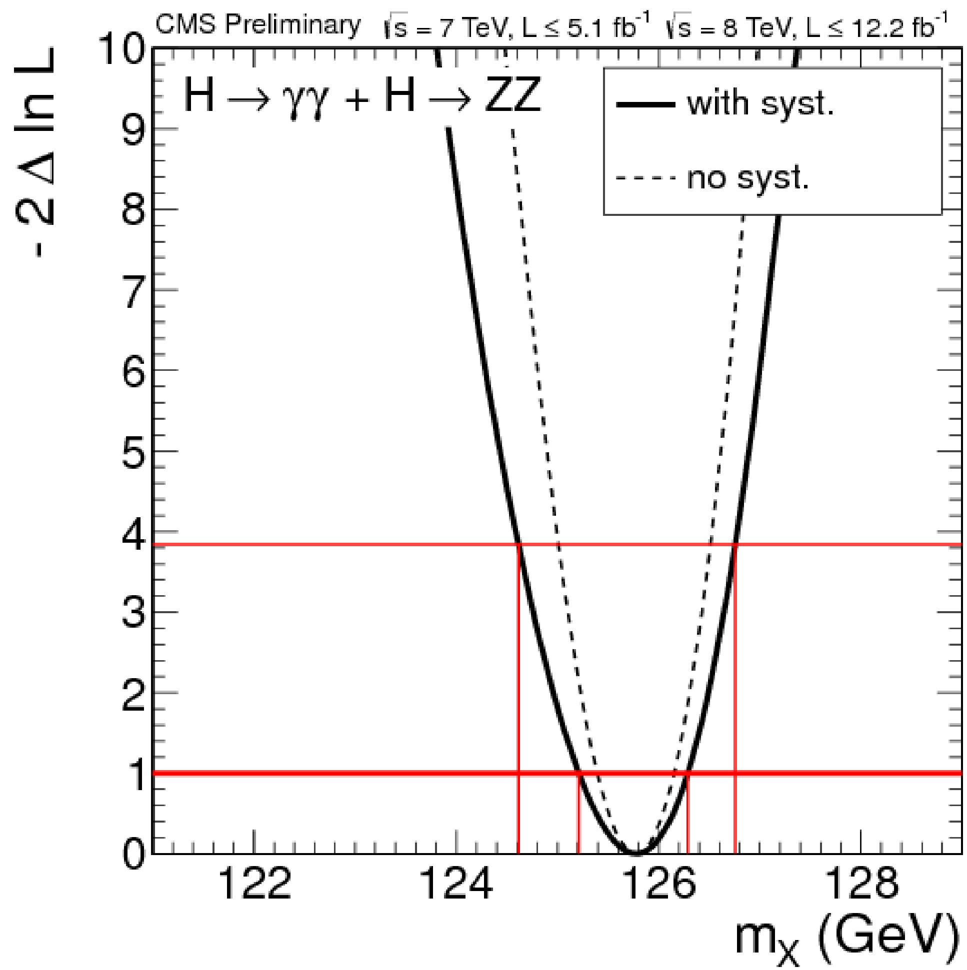


p-value for 'No Higgs' versus m_H

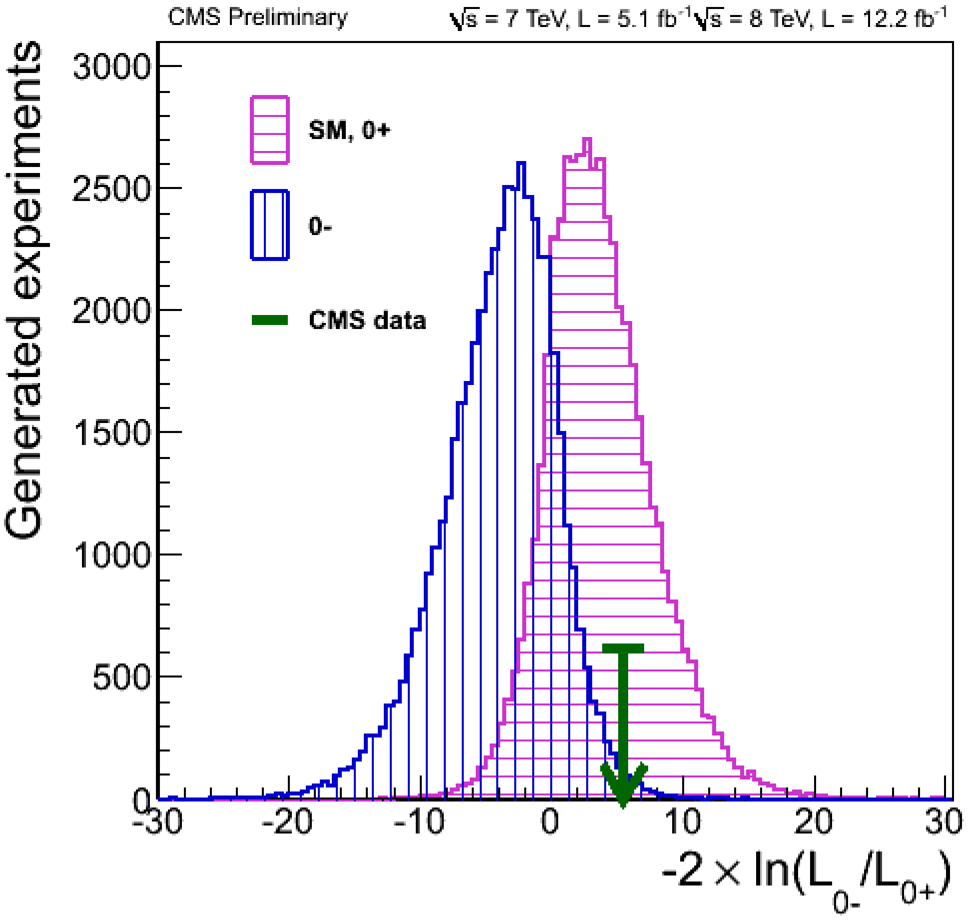


Likelihood versus mass





Comparing 0^+ versus 0^- for Higgs



http://cms.web.cern.ch/news/highlights-cms-results-presented-hcp

Summary

- P(H0|data) ≠ P(data|H0)
- p-value is NOT probability of hypothesis, given data
- Many different Goodness of Fit tests

  Most need MC for statistic → p-value
- For comparing hypotheses, $\Delta\chi^2$ is better than χ^2_1 and χ^2_2
- Blind analysis avoids personal choice issues
- Different definitions of sensitivity
- Worry about systematics
- H0 search provides practical example

PHYSTAT2011 Workshop at CERN, Jan 2011 (pre Higgs discovery)
"Statistical issues for search experiments"
Proceedings on website http://indico.cern.ch/conferenceDisplay.py?confId=107747

Overall Conclusions

1) Best of luck with your statistical analyses

2) Your statistical analysis should do justice to your data

3) Your problem has probably occurred before, and maybe has been solved

Consult text-books, and statistics information on the web, e.g.
CDF Statistics Committee
CMS Statistics Committee
Particle Data Group Statistics

Before re-inventing the wheel, try to see if Statisticians have already found a solution to your statistics analysis problem.

Don't use your own square wheel if a statistician's circular one already exists

4) Send me an e-mail (l.lyons@physics.ox.ac.uk)