Soc Indic Res (2013) 112:19–45
DOI 10.1007/s11205-012-0070-4

An Unbalance Adjustment Method for Development Indicators

Enrico Casadio Tarabusi · Giulio Guarini

Accepted: 16 April 2012 / Published online: 2 June 2012
© Springer Science+Business Media B.V. 2012

Abstract This paper analyzes some aggregation aspects of the procedure for constructing a composite index on a multidimensional socio-economic phenomenon such as development, the main focus being on the unbalance among individual dimensions. First a theoretical framework is set up for the unbalance adjustment of the index. Then an aggregation function is proposed that takes unbalance among development dimensions into account; a separate index is also introduced that measures the unbalance itself. Finally the dataset of the Index of African Governance for the year 2007 is used to test this method and compare it against the weighted arithmetic mean of variables with relation to the measured values of unbalance, yielding significantly different results for ratings and rankings, which in addition show negative correlations with the unbalance adjustment values. The changes ensuing from the adjustment are commented for some countries.

Keywords Development · Rating · Composite indicator · Aggregation method · Across-variables unbalance · Across-variables compensability · Index of African Governance

JEL Classification O15 · C43

This is a revised version of "An aggregation method for composite indicators with unbalance adjustment, with an application to the Index of African Governance".

E. Casadio Tarabusi (✉)
Dipartimento di Matematica "G. Castelnuovo", Sapienza Università di Roma,
Piazzale A. Moro 5, 00185 Rome, Italy
e-mail: casadio@mat.uniroma1.it

G. Guarini
Dipartimento per lo Sviluppo e la Coesione Economica, Ministero dello Sviluppo Economico,
Via Sicilia 162/C, 00187 Rome, Italy
e-mail: giulio.guarini@tesoro.it

1 Introduction

Many relevant socio-economic phenomena such as quality of life, standard of living, well-being, globalization, social capital, poverty, competitiveness, and development have a multidimensional nature. In particular, in the economics literature, development is usually measured by multidimensional statistical procedures. Important examples of composite indices are the United Nations' *Human Development Index* (*HDI*) and *Human Poverty Index*, Harvard University's *Index of African Governance*, and the World Economic Forum's *Global Competitiveness Index*.

Often, individual variables (or *dashboards*) are first grouped by general topic, or *pillar*, and pillar indicators are synthesized; then these undergo a second aggregation procedure to produce the final composite index (of course, further levels of grouping/aggregation are possible and used). Producing a unique composite indicator has advantages (such as simplicity) versus keeping the set of individual, or pillar, indicators, although the latter might in turn be preferable for other reasons (such as wealth of information). Historically, Sen initially criticized the composite indicator proposed by ul Haq to study human development because of the loss of complexity, but later (Sen 1999b) he was convinced by ul Haq that a single index is an appealing instrument to attract political attention, that is the necessary step for following insights. For the general public, a single composite indicator is more understandable than a set of several indicators in terms of synthesis of a phenomenon, although if a composite indicator is poorly built it can generate misinterpretation of reality (Organisation for Economic Co-operation and Development 2008, *Introduction*), (Saltelli 2007).

An unbalance is a disequilibrium among the variables (or the pillars) that are used to build a given composite index. For instance, in the case of only two variables X, Y whose (suitably normalized) values range between 0 and 1, there is perfect balance when $X = Y$, while the maximum unbalance occurs when $X = 1$ and $Y = 0$ or vice versa. Several composite indicators are built without considering unbalance, neither directly nor indirectly; in other words, they are neutral to unbalances. Most of them aggregate variables using the arithmetic mean (possibly with weights), which ignores unbalances; in the same example, the values $X = 0$ and $Y = 1$ yield the same final index value 0.5 (in case the weights are the same) that would be obtained with $X = Y = 0.5$, although with a much higher unbalance.

The reason to impute different factors to analyze a phenomenon also naturally leads to considering unbalances among factors. In any composite indicator each factor is introduced to represent a relevant aspect of the phenomenon considered, therefore the measure of unbalance among factors helps the overall knowledge. An adjustment of the composite index value that takes unbalance into account can be fruitful for various reasons. In general, the balance among factors of a multidimensional socio-economic phenomenon reinforces the importance of the conglomerative perspective. On the other hand, without considering unbalance adjustments, the analysis of a multidimensional phenomenon by several individual indicators instead of a single composite indicator appears to reinforce the multidimensionality of the phenomenon (Stiglitz et al. 2009, #134 in §I.3.2.1).

In the human development approach, the well-being of people needs a minimum level of income per capita, health status and education (United Nations Development Programme 1991, *Technical Notes*, §1). Furthermore, a harmonious growth of the various pillars can be an important condition for the sustainability of development (Soubbotina and Sheram 2000, Chapter 1, *Sustainable development*). Therefore sustainability of development reflects to a large extent in the balance among pillars (Soubbotina 2004, Chapter 1, *Sustainable development*).

The study of balance adjustment is useful for two more reasons. Firstly, this topic is not widespread, even in development studies; in fact mainly the equity is for gender or income but not among development dimensions (Palazzi and Lauri 1998, §2, Footnote 4]. Secondly, even when the unbalance problem is tackled, the normative aspects deriving from the different methods used are usually not explained satisfactorily. In general, this lack of clearness regards weighting procedures, but correspondingly it often concerns the entire aggregation methods linked to the balance problem. As pointed out by Stiglitz et al. (2009, #140 in §I.3.2.2), the principles that underlie the chosen procedures and their implications are often left unstated or unjustified.

In this paper we propose an unbalance-adjusted method for aggregating individual variables into a composite index, building at the same time an auxiliary index that measures the unbalance itself. The development analysis will be the main theoretical context to adapt this methodology, which will be tested on a specific development index. In the terminology of Stiglitz et al. (2009, #134 in §I.3.2.1), ours is a *strong* multidimensional approach, as opposed to those that do not consider unbalance, which are called *weak*: our perspective about unbalance adjustment is that compensability among variables can be possible, but its cost increases with unbalance.

We initially point out the various aspects of the measurement procedure for a multidimensional phenomenon by specifying all the steps. Next, by concentrating on aggregation functions, we build a theoretical framework in order to analyze some relevant aspects of the aggregation function and the unbalance adjustment. Then we survey some specific aggregation functions that incorporate an unbalance adjustment by underlining advantages and shortcomings with respect to some properties that we consider important. At this point we introduce a new aggregation function that enjoys all the previous properties. Finally we apply it to the dataset for the *Index of African Governance* (Rotberg and Gisselquist 2009). We compare our function to the weighted arithmetic mean in terms of rating and ranking. Moreover, we calculate the Unbalance Adjustment index as a measure of unbalance among dimensions.

In brief, our article focuses on the aggregation aspects regarding the indexing problem, by proposing a method that takes into account the relevance of the balance among dimensions of a composite indicator. Our proposal is to offer a contribution to extend scientific methods for better understanding the complex question of development balance. From the point of view of policy makers, considering the equilibrium among variables for a composite indicator means taking into account different political goals simultaneously (Soubbotina 2004, Chapter 1, *Sustainable development*). In this way, such a composite indicator can be used to successfully devise a complex political strategy.

2 The Measurement Procedure of a Multidimensional Phenomenon

Let us focus on the problem of cardinal measuring of a multidimensional phenomenon (such as human development) on a given set of units (such as world countries), which will be assumed labeled by $j = 1, \ldots, m$. We can summarize the measurement procedure, as developed in the existing literature (see, e.g., Organisation for Economic Co-operation and Development (2008)), in the following main steps (cf. Fig. 1):

(1) *Choice of theoretical framework*. The phenomenon is defined by analyzing its main theoretical aspects.
(2) *Choice of dimension approach*. A *unidimensional* or *multidimensional* approach is chosen.

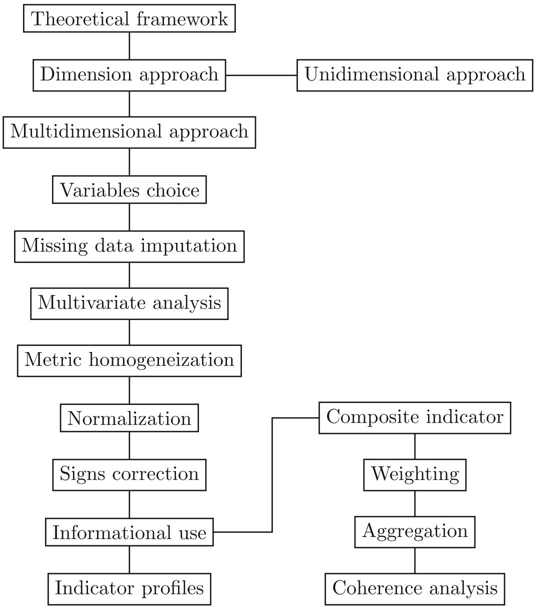


Fig. 1 Standard steps of a measurement procedure

In the former case, a single factor is deemed to be able to represent a complex phenomenon satisfactorily: for example, in development studies before the HDI, development was usually represented by the Gross Domestic Product.
Instead, many empirical studies take into account several factors in the measure of development, thereby enforcing a multidimensional approach (Baker et al. 2005). All factors are assumed to be of a cardinal nature, or previously converted into cardinal values.

We shall henceforth pursue the multidimensional approach.

(3) *Selection of variables*. The factors that significantly affect the phenomenon are chosen (Sen 1999a). The choice can be made according to a specific theoretical framework.
(4) *Imputation of missing data*. The possible (indeed frequent) incompleteness of the data set is addressed by means of suitable statistical tools.
(5) *Multivariate analysis*. Using statistical methods, relationships among variables are studied in order to retain and use only the most representative ones.
(6) *Metric homogeneity transformation*. The metric homogeneity of each variable should be improved in order for the same increment to have a similar significance for any starting value. For instance, the increase from a value of, say, 3 to a value of 5 must have a similar significance as the increase from 10 to 12. A suitable procedure is adopted in Casadio Tarabusi and Palazzi (2004, §2), where each variable is transformed as follows:

   (a) if the variable (e.g., the per capita income) can only take values greater than a fixed constant l (usually $l = 0$) but otherwise unrestricted, then we subtract l and take the decimal logarithm of the result.
   (b) if the variable can only take values between two fixed constants l and L (a percentage is usually of this kind, with $l = 0$ and $L = 100$), then we take the decimal logarithm of the ratio $(x_j - l)/(L - x_j)$.

(c) if the variable [...] can take unlimited positive and negative values, then we leave it untouched.

Other instances of transformation can be found for example in Annoni and Kozovska (2010, §4.3, *Data transformation*).

(7) *Normalization of data*. By a linear or non-linear approach, the values of all factors taken into consideration are normalized in order for the resulting individual indicators to have the same range and be possibly compared with one another.
Perhaps the simplest and most common normalization procedure is the *maximum-minimum* method, where each raw value x_j is linearly rescaled to $z_j = (x_j - \min x)/(\max x - \min x)$ so that the minimum value $\min x = \min(x_1, \ldots, x_m)$ of that factor among all units is mapped to 0 and the maximum value $\max x = \max(x_1, \ldots, x_m)$ to 1. In this method the difference between normalized values may be unduly compressed by outlying extremes. Furthermore, in such a way comparisons of different data sets (such as time comparisons) on the same set of units may be overly affected by variations of the extreme values. Setting a priori one or both of the extreme values would mitigate the second problem, but exacerbate the first one.
Among alternate normalization procedures we mention: *standardization* (in fact, by the Bienaymé-Chebychev theorem, the indicator values of at least 75 % of the units lie within the interval [−2,2], and, if the distribution is normal, about 95 % of them lie in such interval); the method of Casadio Tarabusi and Palazzi (2004), where part of the normalization is achieved by performing a linear regression.
With any normalization method, we shall also call (*individual*) *indicators* the normalized variables, which will be labeled by $i = 1, \ldots, n$.

(8) *Imputation of correct signs*. The sign of the normalized value is inverted (i.e., the value is multiplied by -1) if the variable is negatively correlated with the multidimensional phenomenon.

(9) *Informational use*. To analyze a multidimensional phenomenon it is possible to use normalized variables in two different ways: one is to evaluate units by the profile of their individual indicators, as in the approach of United Nations (2010) or by means of tools such as the World Bank development diamonds (Soubbotina 2004, Chapter 15, *"Development diamonds"*).
Another way is to construct a composite indicator, which can in turn be either *cardinal* (the outcome is a *rating* of each unit) or *ordinal* (the outcome is directly a *ranking* of the units, without rating information).
In all cases it is possible to use multivariate analysis to group information on individual indicators into thematic indicators (for example, by principal component analysis) or to group information into units (for example, by cluster analysis).

We shall henceforth concentrate on the construction of a composite cardinal indicator.

(10) *Weighting*. According to the assumptions made at the onset, if the factors are considered of the same importance, then all weights are taken to be equal; otherwise there are several more or less automatic methods (for example statistical techniques such as factor analysis) to establish the individual weight values.

(11) *Aggregation*. An aggregation procedure is chosen that will produce, for any given unit, the value of the composite indicator, or index, as a function of the vector $z = (z_1, \ldots, z_n)$ of individual ones. Given n individual indicators, we can call *thematic indicators* the k indicators that relate to a given aspect of the multidimensional phenomenon observed.

(12) *Coherence analysis*. It is verified whether the final composite indicator is coherent with the general theoretical framework chosen at the onset.

3 The Aggregation Framework

Compensability among variables is defined as the possibility of compensating any deficit in one dimension with a suitable surplus in another (Organisation for Economic Co-operation and Development 2008, §1.6). Thus we can define an aggregation approach *compensatory* (respectively, *non-compensatory*) depending on whether it permits compensability or not.

A common and simple non-compensatory approach uses the minimum $\min z = \min(z_1, \ldots, z_n)$ of variables independently of any weights. This function is non-compensable among indicators: for every i, j any excess value of z_i with respect to z_j does not increase the value of the index. In particular, this function realizes the maximum penalization of unbalances among indicators. Another instance of non-compensatory approach, applied to ranking analysis, is the multi-criteria approach developed in Munda and Nardo (2009). This approach in general does not reward outliers, but only retains ordinal information (Organisation for Economic Co-operation and Development 2008, §1.6).

We can divide compensatory approaches in two groups: those which adjust for all unbalances among factors and those which do not.

The simplest non-trivial and common compensatory, non-adjustment approach uses the *weighted arithmetic mean* (*WAM*)

$$\sum_{i=1}^{n} w_i z_i \tag{3.1}$$

with respect to fixed positive weights $w_1, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$. If $w_1 = \cdots = w_n$, this reduces to the *simple arithmetic mean* of variables. In the WAM, for every i, j there exists a trade-off constant c_{ij} (equal to w_i/w_j) such that any given amount A of indicator z_i may be replaced by the amount $B = c_{ij} A$ of indicator z_j without varying the resulting value of the index. Therefore there is no adjustment for unbalances of indicators. The most popular use of WAM is perhaps in the United Nations' Human Development Index, for which $n = 3$ and $w_1 = w_2 = w_3 = 1/3$.

4 Adjustment of Unbalances

In this section we state and describe some of the properties that ought to be expected of an aggregation scheme that adjusts for unbalances, adopting an axiomatic approach such as, e.g., in Chakravarty (2003). We assume that any composite indicator is obtained by applying to the (vector of) normalized variables $z = (z_1, \ldots, z_n)$ an aggregation function

$$I = F(z),$$

that we shall call *unbalance-adjusted function* (*UAF*). Such function may incorporate the weight values of variables.

In the first place we require that the value of the function increases (or, at least, it does not decrease) whenever any of the variables increases and the others are left unchanged:

Property (i) positive monotonicity. *If $t > 0$ we have $F(z_1, \ldots, z_j, \ldots, z_n) \leq F(z_1, \ldots, z_j + t, \ldots, z_n)$.The function F is weakly increasing with respect to each variable, that is, if the value of every individual indicator for unit A is greater than or equal to the value for unit B, then the value of the index at A is greater than or equal to that at B.*

If F is differentiable, an equivalent condition to Property (i) is that the gradient $(\partial F/\partial z_1, \ldots, \partial F/\partial z_n) \geq 0$ everywhere.

We shall henceforth assume that any unbalance among variables has a negative effect, or penalty, on the value of the final index, and that the penalty will be larger for larger unbalances. Whenever the index measures a "desirable" quantity, such as human development or governance, the penalty is a decrease. In case the index quantifies an "undesirable" quantity, such as human poverty, the penalty is an increase of the index value, reversing the situation above. For a function satisfying Property (i) this amounts to, respectively:

Property (ii.a) quasi-concavity. $F(\lambda z + (1 - \lambda)z') \geq \min(F(z), F(z'))$ *whenever* $0 < \lambda < 1$.

Property (ii.b) quasi-convexity. $F(\lambda z + (1 - \lambda)z') \leq \max(F(z), F(z'))$ *whenever* $0 < \lambda < 1$.

The latter case, though, can be reduced to the former simply by multiplying all variables by -1 and changing F to $\tilde{F}(z) = -F(-z)$, which will consequently change the sign of the resulting index. For a regular F, sufficient or necessary conditions for Property (ii.a) (respectively (ii.b)) can be expressed in terms of the sign distribution of principal minors of the bordered Hessian matrix of F; see for instance Takayama (1985, §1.E.c).

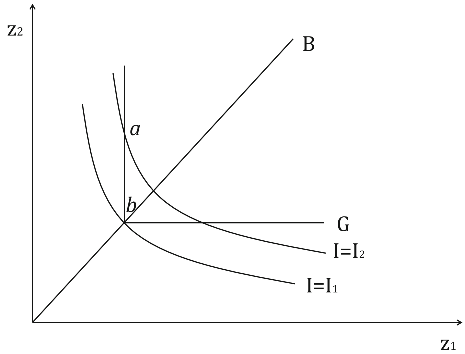
As indicated in Sect. 3, if the aggregation function is a WAM given in (3.1) a decrease of amount A in the variable z_i can always be compensated by an increase of amount $(w_i/w_j)A$ in the variable z_j (keeping the remaining variables unchanged), independently of the initial and final values of all the variables $z_1, \ldots, z_n$ and, in particular, independently of the amount of any unbalance.

In order to make the compensation rate dependent of the unbalance and, in general, of the values of variables, we need to consider non-linear functions. For a differentiable F and for each choice of i, j, the rate of compensation between z_i, z_j at the point z is given by the *Marginal Rate of Compensation* (*MRC*) between variables, defined by

$$\mathrm{MRC}_{ij}(z) = \frac{\partial F}{\partial z_i} \Big/ \frac{\partial F}{\partial z_j} = \left.\frac{dz_j}{dz_i}\right|_{\substack{z_k \text{ constant for } k \neq i,j, \\ F(z_1, \ldots, z_n) \text{ constant}}}$$

whenever $\partial F/\partial z_i$ and $\partial F/\partial z_j$ are not simultaneously zero. Thus, within the same index value, as long as the MRC remains positive and non-zero, a marginal increase (decrease) of z_i is compensated by a marginal decrease (increase) of z_j (keeping the remaining variables unchanged) in a proportion equal to the MRC. From the definition it follows immediately that $\mathrm{MRC}_{ij}\,\mathrm{MRC}_{jk} = \mathrm{MRC}_{ik}$ at every point and for every i, j, k such that at most one of $\partial F/\partial z_i, \partial F/\partial z_j$ and $\partial F/\partial z_k$ is zero (that is, the marginal trade-off rate from z_i to z_k is the product of that from z_i to z_j and that from z_j to z_k, as expected). For the WAM, the MRC is independent of z and equals the ratio of weights: $\mathrm{MRC}_{ij} = w_i/w_j$. Instead, for the UAF, Property (i) implies that $\mathrm{MRC}_{ij}(z)$ never takes a finite negative value for every z (in other words, a loss in one variable cannot be compensated by a loss in another). Property (ii.a) implies that, within the same index value, $\mathrm{MRC}_{ij}(z)$ never decreases with z_j if all variables different from z_i and z_j remain constant. In other words, under Property (ii.a) the trade-off

Fig. 2 Comparison of balanced and unbalanced points



rate of z_i versus z_j grows as z_i decreases; the cost of compensating for a loss in a variable with a gain in another variable grows progressively. The *balance line* will be the locus B of perfect balance, defined by the relation $z_1 = \cdots = z_n$.

In the present framework, the minimum function and the WAM are extreme cases of UAF. Indeed, the maximum penalization is given by the minimum function and the minimum (zero) penalization is given by the WAM; thereby, all aggregation functions of the compensatory approach with unbalance penalization are intermediate cases between these two extreme cases, that in the microeconomic literature correspond respectively to *perfect complementarity* and *perfect substitutability* among goods (or inputs). In the same literature, the MRC corresponds to the *Marginal Rate of Substitution* for goods and to the *Marginal Rate of Technical Substitution* for inputs.

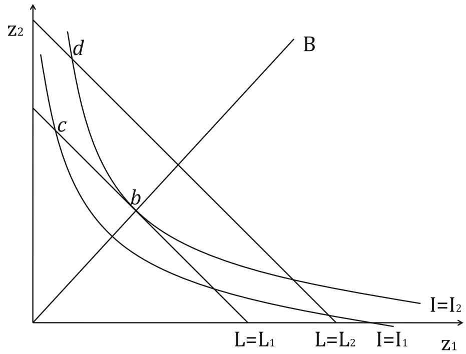
Let us compare intermediate with maximum penalization (or perfect complementarity). We shall assume only two variables, for simplicity of exposition. Figure 2 compares a perfectly balanced point b with an imbalanced point a having the same value of z_1 and a larger value of z_2. Points a, b lie on the same level curve (the broken line G) under the maximum penalization assumption, while they lie on different level curves (I_1, respectively I_2, with $I_1 < I_2$) under the UAF assumption. In this case the UAF favors the combination of z_1 and z_2 at point a, although unbalanced, over the perfectly balanced combination at point b, simply because one variable (namely z_2) is larger while the other is not smaller (and Property (i) is in force). For the maximum penalization function, the value of $\mathrm{MRC}_{12}(a)$ might be considered to be infinity, while $\mathrm{MRC}_{12}(b)$ is not defined.

Next, let us compare intermediate with zero penalization (or perfect substitutability). The weights for the WAM to be used for comparison are such that the MRC at b coincides with that for the UAF, namely w_i is proportional to $(\partial F/\partial z_i)(b)$ for each i. In the WAM case a decrease of z_1 can be compensated by a proportional increase of z_2 (the trade-off coefficient being w_1/w_2), regardless of the magnitude of the unbalance between z_1 and z_2; in Fig. 3 the unbalanced point c is obtained from b via this trade off, and therefore the WAM indices at b and c agree.

On the contrary, in the UAF assumption the decrease of z_1 can only be compensated by a more than proportional increase of z_2; the unbalanced point d is obtained by moving away from b along the same level curve. Therefore the WAM level L_2 at d is strictly greater than L_1 at b, whereas the UAF level I_1 of c is strictly smaller than I_2 at b, that is,

$$\begin{aligned} L_2 &= L(d) > L(b) = L(c) = L_1, \\ I_2 &= I(d) = I(b) > I(c) = I_1. \end{aligned}$$

Fig. 3 Comparison of penalizations



In the case of development studies, a UAF carries a growing development cost (implicit in the variation of MRC) for any worsening unbalance; a development policy that aims at achieving a harmonious growth of the variables is more practicable in the UAF approach than in the WAM approach because, in the first one, there is an incentive to move towards a balance position, due to the decreasing development costs.

Finally we want to define the amount $A(z)$ of adjustment at z. We shall compare a UAF with the WAM whose MRC at a point $\tilde{z}$ on the balance line coincides with that for the UAF at $\tilde{z}$; that is, the level curve $L(\tilde{z})$ of the WAM at $\tilde{z}$ and the level curve $I(\tilde{z})$ of the UAF are tangent (or else, the weights $w_1, \ldots, w_n$ are proportional to $(\partial F/\partial z_i)(\tilde{z}), \ldots, (\partial F/\partial z_n)(\tilde{z})$). For a given z, the point $\tilde{z}$ to be chosen is such that such WAM takes the same value at z and $\tilde{z}$, that is, $L(\tilde{z}) = L(z)$. Such a point $\tilde{z}$ exists and is unique under the following hypothesis on the function F, which will hold for all the examples we shall consider in the sequel.

Hypothesis (A). The gradient of F on the balance line is constant up to a proportionality factor.

In other words, if an aggregation function F satisfies Hypothesis (A) then there is a unique WAM to compare F with. Such a distinguished WAM is the only one whose trade-off constants among variables coincide with the respective marginal trade-off constants of the UAF along the balance line. Then, the Unbalance Adjustment of a UAF at a point $z = (z_1, \ldots, z_n)$ is defined as the difference of the values of I at $\tilde{z}$ and at z:

$$A(z) = I(\tilde{z}) - I(z).$$

Thus, the value of the unbalance adjustment at a point z is given by the difference between the (lower) value of the UAF at z and the (higher) value at the unique point $\tilde{z}$ on the balance line obtainable from z by compensations at the rates of the distinguished WAM. Hypothesis (A) means that the tangent hyperplanes to each level hypersurface of I at its unique balance point (given by the intersection of that level with the balance locus B) are parallel. Two of them are L_1 and L_2 in Fig. 3; these parallel lines are the level curves of the WAM function L described above. In order to obtain the value of adjustment of the UAF at a given unbalanced point c, say with z_2 strictly greater than z_1, take the unique balanced point b whose WAM value is $L(c)$ (graphically, among those parallel lines take the only one that contains c and determine its intersection b with the line B); the requested adjustment is the difference of the values I_2 (the level containing b) and I_1 (the level containing c). This difference $A(\tilde{z}) = I(b) - I(c)$ depends of course on the curvature of the levels of I, but also on the steepness of I on the balance line B. This latter feature translates graphically into how many levels for given equally-stepped values intersect a given segment of the balance line.

From these analytical comparisons, aspects emerge of the UAF approach that may be relevant for development studies. The assumption of strong interaction among development pillars where unbalances are relevant, see Ranis (2006) and Ranis et al. (2000), is incorporated in the UAF approach. Moreover, the UAF represents an intermediate position between two extreme development points of view concerning unbalances: perfect complementarity and perfect substitutability. On one hand, one could apply to the development the perfect complementarity approach according to which all kinds of unbalances bear a negative effect to development. However, many analytical and empirical studies argue that some unbalances due to an excess value of one crucial variable, like investments, may spur future growth effects unto other variables, thus generating virtuous cumulative development processes. For example, in Boozer et al. (2003), Ranis and Stewart (1997, 2000) it is affirmed that economic growth and human development can generate positive long virtuous circles, if, at the first stages, human development has the sequential priority. There is a virtuous circle when, given two factors A and B, factor A affects B positively, and B in turn reacts affecting A and generating a positive interaction. On the other side, one could apply to the development the perfect substitutability approach according to which all kinds of unbalances are indifferent from the development point of view. However, as many authors assert, too big unbalances among dimensions can create unsustainable development in social, economic and environmental terms. For example, in Bourguignon (2004) it is argued that too large values of a variable that measures income distribution spread can both increase poverty and reduce the positive effect of growth on poverty.

5 Some Unbalance-Adjusted Functions

Let us now survey a few known aggregation functions by taking into particular account the measurement of unbalance among factors.

The most used aggregation functions are special cases of the *generalized power mean of order* θ:

$$F(z_1, \ldots, z_n) = \left(\sum_{i=1}^{n} w_i z_i^{\theta} \right)^{1/\theta}. \tag{5.1}$$

This function yields:

(1) the *power mean of order three* if $\theta = 3$; this is used to build, for example, the United Nations' *Human Poverty Index* (*HPI*); in this particular case there is a positive adjustment for unbalances because of the convexity of the function (United Nations Development Programme 2009);

(2) the *geometric mean* if $\theta \to 0$; this one is used, for example, in the Germany *IFO Business Climate Index* by the Institute for Economic Research at the University of Munich (Broyer and Savry 2002) (see also http://www.cesifo-group.de), an *Index to Track Credit Card Debt and Predict Consumption* (but along with the arithmetic mean) (Dunn et al. 2004), the *Malmquist Productivity Index* (Zelenyuk 2006) and the *European Competitiveness Index* (Huggins and Davies 2006).

The minimum function and the WAM are two particular cases of the function in (5.1): the former is the limit case with $\theta \to -\infty$ and the last is the special case with $\theta = 1$.

Assuming that all variables considered are normalized, the purpose of the subsequent use of any specific mean is to calibrate the balance adjustment among factors and does not depend on the specific nature of the original variables.

In the *Mazziotta-Pareto approach* the aggregation function is

$$F(z_1, \ldots, z_n) = \bar{z} \pm \sigma_z \mathrm{cv} \tag{5.2}$$

where $\bar{z}$ is the arithmetic mean of z, σ_z is the standard deviation of $z, z_i = 100 \pm 10(x_i - \bar{x})/\sigma_z$ is the normalization formula with σ_z the standard deviation of $z, \mathrm{cv} = \sigma_z/\bar{z}$ is the coefficient of variation. The authors apply this approach to some Millennium Development Goals (De Muro et al. 2008).

The *Concave Average approach* has the following function

$$F(z_1, \ldots, z_n) = \sum_{i=1}^{n} w_i (z_i - a_i e^{-b_i z_i}) \tag{5.3}$$

where a_i, b_i are parameters related to the intensity of penalization of unbalance and of complementarity between factors. The authors use this function to build a new index of Sustainable Development (Casadio Tarabusi and Palazzi 2004); see also Corsi and Guarini (2011).

6 The Mean-Min Function

Let us list some more properties that an aggregation function ought to fulfill.

Property (iii) unrestricted domain. *The function F is defined on* $\mathbf{R}^n$. *In other words, the function must be defined on every possible n-tuple of variables, not only those with entries within a specific interval, such as [0,1]; indeed, depending on the normalization procedure, for each variable there may be no ex ante upper or lower bound on the possible values of outliers for that variable, although most of the transformed values do lie in a specific interval (namely [0,1] for the min-max normalization, or [− 3,3] for standardization).*

Property (iv) continuity. *The function F is continuous on its domain.*

Property (v) idempotence. *If* $z_1 = \cdots = z_n$*, then* $F(z_1, \ldots, z_n) = z_1$.

Property (vi) stability for translations. *For every* ϵ *we have* $F(z_1 + \epsilon, \ldots, z_n + \epsilon) = F(z_1, \ldots, z_n) + \epsilon$.

Property (iii) is relevant because then the choice of normalization procedure is not constrained by the domain, therefore it is possible to use the variable normalization that suits best the objectives of the specific analysis, including the characteristics of the data and the amount of emphasis on outliers, see Rotberg and Gisselquist (2009, Chapter I.III, *Normalizing the data*) or Organisation for Economic Co-operation and Development (2008, §1.5).

Property (iv) is the natural requirement that small variations of the index value follow from small enough variations of the values of variables.

Property (v) postulates that, in case all variables take the same value, then the value of the index is the same as well. Thanks to Property (v) we can compare the WAM approach with the UAF approach more directly, because in this case the unbalance adjustment equals the difference between WAM and UAF. Indeed, in the discussion following Hypothesis

(A) we have $L(\tilde{z}) = \tilde{z}_1 = \cdots = \tilde{z}_n = I(\tilde{z})$, so that $A(z) = L(z) - I(z)$, without reference to $\tilde{z}$.

Property (vi) means that if all variables increase (or decrease) by the same amount, then so does the index value. Property (vi) along with the assumption that $F(0) = 0$ implies Property (v). Property (vi) also implies that the unbalance adjustment does not vary by adding the same constant to all variables, that is, $A(z_1 + \epsilon, \ldots, z_n + \epsilon) = A(z_1, \ldots, z_n)$.

Properties (ii.a) and (vi) together imply the concavity of the function F.

None of the aggregation functions presented before fulfills all Properties (iii)–(vi). In fact, functions (5.1) and (5.2) enjoy only Properties (iv) and (v). Moreover, the function in formula (5.2), alternative "+", has a specific disadvantage: the function does not fulfil Property (i) in a large portion of its domain, namely, for $n = 2$, in the locus $\{0 < z_1 < (\sqrt{2} - 1)z_2 \text{ or } 0 < z_2 < (\sqrt{2} - 1)z_1\}$ (this can be readily checked by computing the partial derivatives of the aggregation function); for instance $F(52, 140) = 116.167 > 116.146 = F(54, 140.1)$, although the second point dominates the first in both coordinates. Finally, function (5.3) enjoys only Properties (iii) and (iv).

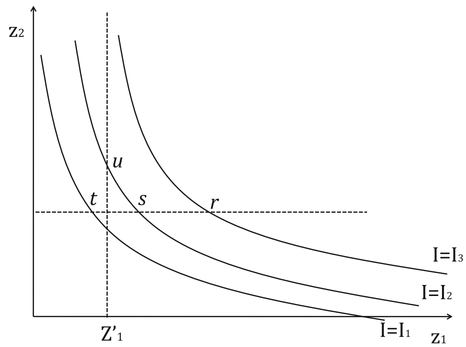
Before presenting our proposed aggregation function we encode in two more pairs of alternative properties the behavior of compensation among variables.

Property (vii.a) complete compensability. *For every j and every given c the set of values $F(z_1, \ldots, z_n)$ with $z_j = c$ has no upper bound.*

Property (vii.b) incomplete compensability. *For every j and every given c the set of values $F(z_1, \ldots, z_n)$ with $z_j = c$ has a finite upper bound.*

In other words, with complete compensability any decrease of any single variable can be compensated by suitable increases of the remaining variables; in the case of incomplete compensability, only decreases in one single variable that are smaller than a given amount are compensable with suitable increases of the remaining variables. In Fig. 4 the decrease of z_1 from point s to point t is compensable by the increase of z_2 from point t to point u (indeed $F(u) = I_2 = F(s)$); nevertheless, the decrease of z_1 from point r to point t cannot be compensated by any increase of z_2 from point t, because all the values $F(z_1',z_2)$ (where z_1' is the first coordinate of t) are smaller than I_3. Therefore, under incomplete compensability, starting from a given n-tuple of variables, for each single variable there exists an upper bound of its decrease beyond which the same index value cannot be restored by increases in the other variables.

Fig. 4 Incomplete compensability



The functions that enjoy Property (vii.a) are (5.1) (case (ii)), (5.2) alternative "$-$" and (5.3), while the functions (5.1) (case (i)) and (5.2) alternative "$+$" enjoy Property (vii.b).

The last pair of properties encodes a distinction made earlier on.

Property (viii.a) progressive compensability. *For any two variables i, j and any point z, the rate of compensation between z_i, z_j is increasing with the variable z_j, under the constraint that I and the remaining variables are kept constant.*

Property (viii.b) proportional compensability. *For any two variables i, j and any point z, the rate of compensation between z_i and z_j does not vary with the variable z_j, under the constraint that I and the remaining variables are kept constant.*

With progressive compensability, if one successively compensates the same decrease of one variable with the increase of another, this increase needs to be larger and larger. All instances of UAF previously presented fulfill Property (viii.a).

We now introduce a new aggregation function: the *mean-min* function. The theoretical starting point is to build a function that somehow incorporates the two extreme cases of penalization: the zero penalization represented by the WAM and the maximum penalization represented by the minimum function. All other possible cases are intermediate. That is, the interval of definition of the values of the mean-min aggregate function is

$$\min z \leq F(z) \leq \bar{z}.$$

The mean-min function is

$$I = F(z_1, \ldots, z_n) = \bar{z} - \alpha\left(\sqrt{(\bar{z} - \min z)^2 + \beta^2} - \beta\right), \tag{6.1}$$

where the parameters $0 \leq \alpha \leq 1$ and $\beta \geq 0$ are respectively related to the intensity of penalization of unbalance and intensity of complementarity between factors, as the parameters a, b in Casadio Tarabusi and Palazzi (2004). The function is a synthesis of the unbalance penalization framework by including various cases that can represent various aspects of this topic. This is a further property of this function: to provide a flexible framework for different kinds of analysis on the unbalances among factors affecting the phenomenon considered.

The function in (6.1) reduces to the arithmetic mean for $\alpha = 0$ (in this case β is irrelevant) and to the minimum function for $\alpha = 1$ and $\beta = 0$. Moreover, with $\alpha = 1$ the function has incomplete compensability; with $\beta = 0$ and $0 < \alpha < 1$ it has proportional compensability. Therefore, by choosing the values of parameters appropriately one can obtain the instance of this aggregation function that best suits the specific theoretical approach. Some level curves of the mean-min function in $n = 2$ variables for different combinations of values of α, β are plotted in Figs. 5, 6, 7, 8, and 9.

If standardization is the chosen normalization, reasonable values for the parameters seem to be $\alpha = \beta = 1$, an intermediate easy case of adjustment with incomplete and progressive compensability. We underline that performing standardization on variables that are already normalized by a linear transformation, such as the min-max normalization, yields the same results as performing standardization on raw variables directly; it is therefore usually not necessary to know the values of raw variables in order to obtain their standardization.

Fig. 5 Mean-min function for $\alpha = 0$ and $\beta = 0$

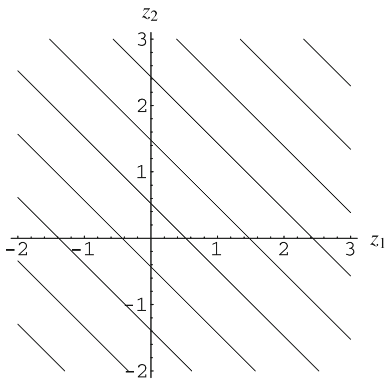


Fig. 6 Mean-min function for $\alpha = 0.5$ and $\beta = 0$

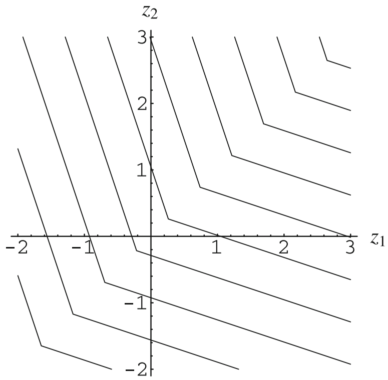
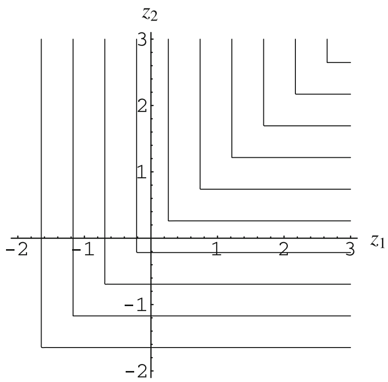


Fig. 7 Mean-min function for $\alpha = 1$ and $\beta = 0$



7 An Application of the Mean-Min Function to the Index of African Governance

In this section we apply the mean-min function on the *Index of African Governance* for the year 2007. This composite indicator has been devised and computed since 2007 by the *Program on Intrastate Conflict and Conflict Resolution* at Harvard's Kennedy School, under the direction of Robert I. Rotberg and Rachel M. Gisselquist (for free download and information see http://www.nber.org/data/iag.html). The declared purpose of the index is

Fig. 8 Mean-min function for $\alpha = 0.5$ and $\beta = 1$

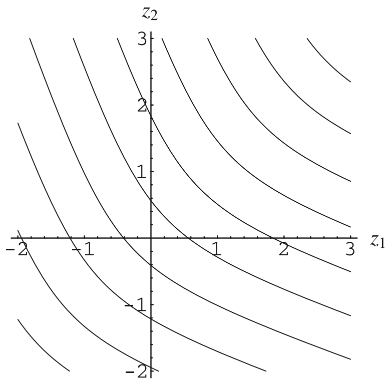
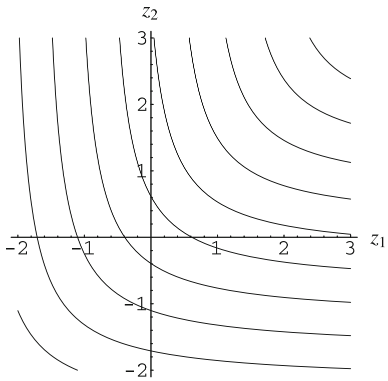


Fig. 9 Mean-min function for $\alpha = 1$ and $\beta = 1$



not so much ranking per se, but to offer a comparative diagnosis of the government of each of Africa's countries (Rotberg and Gisselquist 2009, *Acknowledgements*). This index is composed of the following 5 pillars representing, according to the authors, the essential political goods; the name of each pillar is preceded by the letter used here to denote it:

- S = *Safety and Security*;
- C = *Rule of Law*, *Transparency*, *and Corruption*;
- R = *Participation and Human Rights*;
- E = *Sustainable Economic Opportunity*;
- D = *Human Development*.

Each pillar is composed of subpillars that are, respectively:

- for S: *National Security*; *Public Safety*;
- for C: *Ratification of Critical Legal Norms*; *Judicial Independence and Efficiency*; *Corruption*;
- for R: *Participation in Elections*; *Respect for Civil and Political Rights*;
- for E: *Wealth Creation*; *Macroeconomic Stability and Financial Integrity*; *The Arteries of Commerce*;
- for D: *Poverty*; *Health and Sanitation*; *Education*.

Each subpillar is composed of different variables, whose complete list is in Rotberg and Gisselquist (2009, Table I.2). The analysis regarded 53 African countries, listed in the first column of Table 1. We start from the 13 subpillars (normalized by standardization) to

Table 1 Comparison on the *Index of African Governance* (G)

	Rating			Ranking			Rank difference
	G	G^m	A_G	G	G^m	A_G	$G - G^m$
Mauritius	1.55197	1.48485	0.06712	1	1	40	0
Seychelles	1.22967	1.10774	0.12193	2	2	26	0
Cape Verde	1.03439	0.91489	0.11950	3	3	28	0
Botswana	0.88830	0.64247	0.24583	4	4	13	0
Tunisia	0.88347	0.55428	0.32919	5	7	8	−2
Namibia	0.72951	0.64135	0.08816	6	5	36	1
Algeria	0.71169	0.40931	0.30238	7	9	10	−2
South Africa	0.68455	0.16984	0.51471	8	16	4	−8
Ghana	0.67514	0.60869	0.06644	9	6	41	3
Gabon	0.61209	0.49860	0.11349	10	8	30	2
Morocco	0.58636	0.40534	0.18102	11	10	21	1
Sao Tome and Principe	0.53967	0.34587	0.19381	12	11	20	1
Egypt	0.49425	−0.01732	0.51157	13	21	5	−8
Libya	0.35667	−0.15823	0.51491	14	27	3	−13
Lesotho	0.30412	−0.03361	0.33773	15	23	7	−8
Tanzania	0.28801	0.24940	0.03861	16	12	48	4
Malawi	0.27064	0.20949	0.06115	17	13	44	4
Senegal	0.26391	0.20788	0.05603	18	14	46	4
Gambia	0.24718	0.17816	0.06902	19	15	39	4
Benin	0.21705	0.15467	0.06239	20	17	42	3
Madagascar	0.19736	0.09636	0.10100	21	18	33	3
Burkina Faso	0.10230	−0.09514	0.19744	22	25	19	−3
Zambia	0.10116	0.01571	0.08545	23	19	37	4
Mauritania	0.05987	−0.11680	0.17667	24	26	22	−2
Kenya	0.02867	−0.03274	0.06141	25	22	43	3
Rwanda	0.02840	−0.20541	0.23380	26	31	15	−5
Uganda	0.01591	0.00894	0.00697	27	20	53	7
Comoros	0.01016	−0.07085	0.08101	28	24	38	4
Djibouti	−0.05940	−0.17411	0.11472	29	29	29	0
Mozambique	−0.11090	−0.23181	0.12091	30	32	27	−2
Cameroon	−0.11176	−0.17019	0.05843	31	28	45	3
Mali	−0.11424	−0.25561	0.14138	32	33	24	−1
Togo	−0.17141	−0.18619	0.01478	33	30	52	3
Swaziland	−0.19461	−0.40037	0.20575	34	36	17	−2
Niger	−0.23919	−0.47647	0.23728	35	38	14	−3
Congo	−0.24796	−0.27330	0.02534	36	34	50	2
Equatorial Guinea	−0.28996	−0.60428	0.31433	37	40	9	−3
Nigeria	−0.33458	−0.37074	0.03616	38	35	49	3
Burundi	−0.41271	−0.43670	0.02399	39	37	51	2
Sierra Leone	−0.45371	−0.65172	0.19800	40	44	18	−4
Ethiopia	−0.46465	−0.56582	0.10117	41	39	32	2

Table 1 continued

	Rating			Ranking			Rank difference
	G	G^m	A_G	G	G^m	A_G	$G - G^m$
Liberia	−0.48130	−0.69636	0.21506	42	45	16	−3
Guinea	−0.51532	−0.62635	0.11103	43	41	31	2
Angola	−0.54016	−0.64001	0.09985	44	42	34	2
Guinea Bissau	−0.55597	−1.51448	0.95851	45	51	1	−6
Côte d'Ivoire	−0.56055	−0.65121	0.09066	46	43	35	3
Zimbabwe	−0.57552	−1.22918	0.65366	47	49	2	−2
Eritrea	−0.64850	−0.89849	0.24999	48	47	12	1
Central African Republic	−0.77215	−0.89561	0.12346	49	46	25	3
Chad	−1.04888	−1.09064	0.04175	50	48	47	2
Sudan	−1.19088	−1.62421	0.43333	51	52	6	−1
Congo Democratic Republic	−1.22871	−1.37034	0.14163	52	50	23	2
Somalia	−1.91587	−2.17397	0.25809	53	53	11	0

calculate the 5 pillars and successively the composite index, by using, for aggregation, both the weighted arithmetic mean and the mean-min function. According to the original Index of African Governance, in every aggregation the weights are equal except for the aggregation of *National Security* and *Public Safety* whose weights are respectively 2/3 and 1/3.

Let us turn to the aggregation analysis. In all the tables that follow: X is the original index or pillar (for instance, $X = G$, the Index of African Governance itself), obtained applying the weighted arithmetic mean to its components; X^m is the same index or pillar obtained applying the mean-min function; A_X is the value of the corresponding unbalance adjustment. Table 1 regards the aggregation from pillars to final composite indicator, while Tables 2, 3, 4, 5, and 6 regard the aggregation from subpillars to pillars. Each table shows the rating and the ranking of WAM, the mean-min function with $\alpha = \beta = 1$, and the Unbalance Adjustment. Moreover these tables report the percentage of cases with rank difference ≤ -2 or ≥ 2 between WAM and the mean-min function to verify if the use of different approaches yields significantly different empirical results, and a ranking and rating correlation between WAM and the Unbalance Adjustment to verify how the amount of unbalance among dimensions is correlated with the level of the indicator. Table 7 synthesizes these statistics.

According to the empirical evidence, the use of the mean-min function instead of the WAM yields significantly different results. In fact, in general the percentage of cases with rank difference ≤ -2 or ≥ 2 is very high: for the final index it is 77.36 %. In particular for Libya, Egypt, Lesotho, South Africa, and Uganda the rank differences are very relevant, respectively: −13, −8, −8, −8, and 7. Moreover there are negative correlations (regarding both rating and ranking) between the WAM and the Unbalance Adjustment function: the maximum rating correlation, −0.37, regards the *Participation and Human Rights* pillar, while the maximum Spearman's ranking correlation, −0.38, regards the *Safety and Security* pillar.

According to the results, the change of method produces changes in terms of ranking and rating. Some cases are interesting. Lybia loses 13 positions because it ranks high in the pillars *Safety and Security*, *Sustainable Economic Opportunity*, and *Human Development*, but low in *Rule of Law, Transparency, and Corruption* and *Participation and Human*

Table 2 Comparison on the *Safety and Security* pillar (S)

	Rating			Ranking			Rank difference
	S	S^m	A_S	S	S^m	A_S	$S - S^m$
Cape Verde	0.97478	0.96671	0.00807	1	1	40	0
Mauritius	0.97476	0.96669	0.00807	2	2	41	0
Gabon	0.97445	0.96642	0.00803	3	3	42	0
Egypt	0.97440	0.96637	0.00802	4	4	43	0
Morocco	0.97405	0.96607	0.00798	5	5	44	0
Sao Tome and Principe	0.97343	0.96553	0.00790	6	6	45	0
Tunisia	0.97273	0.96491	0.00781	7	7	46	0
Libya	0.97133	0.96369	0.00764	8	8	47	0
Rwanda	0.81956	0.81946	0.00010	9	9	53	0
Tanzania	0.66256	0.41206	0.25051	10	20	8	−10
Namibia	0.65808	0.41026	0.24782	11	21	9	−10
Seychelles	0.65585	0.40936	0.24648	12	22	10	−10
Madagascar	0.46132	0.44285	0.01847	13	10	37	3
Mozambique	0.46105	0.44255	0.01849	14	11	36	3
Benin	0.46098	0.44248	0.01850	15	12	35	3
Burkina Faso	0.46091	0.44241	0.01851	16	13	34	3
Comoros	0.45926	0.44060	0.01866	17	14	33	3
Ghana	0.45861	0.43989	0.01873	18	15	32	3
Malawi	0.45553	0.43650	0.01902	19	16	31	3
Djibouti	0.45332	0.43409	0.01923	20	17	30	3
Equatorial Guinea	0.45285	0.43358	0.01928	21	18	29	3
Gambia	0.44822	0.42849	0.01973	22	19	28	3
Senegal	0.43020	0.40867	0.02153	23	23	26	0
Algeria	0.39680	0.37173	0.02507	24	24	24	0
Botswana	0.35130	−0.34915	0.70045	25	36	3	−11
Angola	0.20271	0.16123	0.04148	26	25	21	1
Zambia	0.14897	0.12122	0.02776	27	26	22	1
Cameroon	0.14174	0.11563	0.02611	28	27	23	1
Togo	0.11342	0.09328	0.02014	29	28	27	1
Niger	0.09184	0.02285	0.06899	30	29	18	1
Lesotho	0.03966	−1.19590	1.23557	31	47	1	−16
Eritrea	−0.00664	−0.09402	0.08737	32	31	17	1
Uganda	−0.03646	−0.03780	0.00134	33	30	52	3
Guinea Bissau	−0.06017	−0.15833	0.09816	34	32	15	2
Guinea	−0.11619	−0.22622	0.11003	35	34	13	1
Swaziland	−0.16315	−0.48266	0.31951	36	38	5	−2
Mali	−0.20989	−0.21173	0.00185	37	33	51	4
Congo	−0.24219	−0.51155	0.26936	38	40	7	−2
Zimbabwe	−0.26491	−0.26881	0.00389	39	35	49	4
Côte d'Ivoire	−0.38056	−0.56970	0.18913	40	42	11	−2
Sierra Leone	−0.42729	−0.44156	0.01426	41	37	39	4
Mauritania	−0.47364	−0.49204	0.01839	42	39	38	3

Table 2 continued

	Rating			Ranking			Rank difference
	S	S^m	A_S	S	S^m	A_S	$S - S^m$
South Africa	−0.47440	−1.26502	0.79062	43	48	2	−5
Kenya	−0.50672	−0.52837	0.02165	44	41	25	3
Burundi	−0.69315	−0.74646	0.05332	45	43	19	2
Nigeria	−0.69661	−0.74885	0.05223	46	44	20	2
Ethiopia	−0.85143	−1.16351	0.31208	47	46	6	1
Liberia	−0.93199	−0.93621	0.00423	48	45	48	3
Chad	−1.02568	−1.39408	0.36840	49	49	4	0
Central African Republic	−1.49607	−1.59835	0.10228	50	50	14	0
Congo Democratic Republic	−2.11982	−2.12302	0.00320	51	51	50	0
Sudan	−2.84682	−2.94076	0.09394	52	52	16	0
Somalia	−3.05089	−3.19003	0.13914	53	53	12	0

Table 3 Comparison on the *Rule of Law, Transparency, and Corruption* pillar (C)

	Rating			Ranking			Rank difference
	C	C^m	A_C	C	C^m	A_C	$C - C^m$
Cape Verde	1.87189	1.85944	0.01245	1	1	49	0
Mauritius	1.63118	1.52243	0.10876	2	2	30	0
Botswana	1.62883	1.40506	0.22377	3	3	14	0
Namibia	1.28530	0.83762	0.44768	4	6	4	−2
South Africa	1.24692	1.02407	0.22285	5	5	15	0
Ghana	1.19052	1.17095	0.01957	6	4	47	2
Seychelles	1.07452	0.75777	0.31675	7	8	10	−1
Tunisia	0.90006	0.77386	0.12620	8	7	26	1
Lesotho	0.77960	0.67968	0.09992	9	10	31	−1
Senegal	0.71924	0.69384	0.02540	10	9	45	1
Malawi	0.59597	0.39924	0.19673	11	13	18	−2
Mauritania	0.54772	0.37680	0.17092	12	15	22	−3
Burkina Faso	0.51411	0.50649	0.00761	13	11	52	2
Algeria	0.49817	0.48269	0.01548	14	12	48	2
Morocco	0.45787	0.38508	0.07279	15	14	35	1
Zambia	0.45644	0.33031	0.12613	16	16	27	0
Tanzania	0.33761	0.25920	0.07841	17	17	34	0
Swaziland	0.33004	−0.01006	0.34010	18	24	9	−6
Madagascar	0.32724	0.10299	0.22425	19	20	13	−1
Egypt	0.19904	0.12633	0.07271	20	18	36	2
Mali	0.19099	0.11875	0.07224	21	19	37	2
Uganda	0.09929	0.03908	0.06020	22	22	39	0
Benin	0.09033	0.05088	0.03945	23	21	42	2
Kenya	0.04566	−0.22088	0.26654	24	29	12	−5

Table 3 continued

	Rating			Ranking			Rank difference
	C	C^m	A_C	C	C^m	A_C	$C - C^m$
Gambia	0.04358	−0.34725	0.39084	25	32	6	−7
Gabon	0.03961	−0.17516	0.21477	26	28	16	−2
Niger	0.01391	0.00283	0.01108	27	23	50	4
Nigeria	−0.02288	−0.03185	0.00898	28	25	51	3
Comoros	−0.06388	−0.23598	0.17210	29	30	21	−1
Togo	−0.08405	−0.11469	0.03064	30	26	44	4
Guinea	−0.11044	−0.59694	0.48650	31	37	3	−6
Mozambique	−0.17188	−0.17516	0.00328	32	27	53	5
Ethiopia	−0.19234	−0.25870	0.06636	33	31	38	2
Rwanda	−0.23973	−1.17247	0.93274	34	48	1	−14
Sao Tome and Principe	−0.35590	−0.53461	0.17871	35	34	20	1
Eritrea	−0.36501	−0.41204	0.04703	36	33	41	3
Burundi	−0.42822	−0.54396	0.11574	37	35	28	2
Djibouti	−0.45524	−0.85151	0.39627	38	42	5	−4
Cameroon	−0.46867	−0.65364	0.18497	39	40	19	−1
Zimbabwe	−0.48054	−0.61621	0.13567	40	38	25	2
Congo	−0.50954	−0.59167	0.08213	41	36	33	5
Equatorial Guinea	−0.55403	−0.70041	0.14638	42	41	24	1
Central African Republic	−0.62640	−0.64639	0.01999	43	39	46	4
Libya	−0.63230	−1.00860	0.37630	44	45	7	−1
Chad	−0.78661	−0.87266	0.08605	45	43	32	2
Côte d'Ivoire	−0.80091	−1.07417	0.27326	46	46	11	0
Sierra Leone	−0.91395	−1.54166	0.62771	47	49	2	−2
Angola	−0.93234	−0.96719	0.03485	48	44	43	4
Guinea Bissau	−1.10849	−1.15664	0.04815	49	47	40	2
Sudan	−1.25529	−1.62977	0.37448	50	51	8	−1
Liberia	−1.42420	−1.62384	0.19964	51	50	17	1
Congo Democratic Republic	−1.59078	−1.76084	0.17005	52	52	23	0
Somalia	−2.54200	−2.65640	0.11440	53	53	29	0

Rights, thus its governance is strongly unbalanced. Considering the unbalance among variables makes Lybia do worse than Zambia. In fact, the ranks of Lybia and Zambia in the unadjusted index are respectively 14 and 23, which become 27 and 19 after the adjustment. Among the top ten countries in the ranking, South Africa has the largest worsening, its rank plunging from 8 to 16. Indeed social violence, a serious problem for South Africa, makes this country rank low in *Safety and Security*, while nevertheless ranking high in the remaining pillars.

These results have practical implications for countries. Those that gain positions show a more harmonious development than others and, in political terms, they appear to have a governance with a more comprehensive strategy, taking into account the overall

Table 4 Comparison on the *Participation and Human Rights* pillar (R)

	Rating			Ranking			Rank difference
	R	R^m	A_R	R	R^m	A_R	$R - R^m$
Mauritius	1.68249	1.60358	0.07891	1	1	18	0
Sao Tome and Principe	1.44747	1.43313	0.01435	2	2	40	0
Cape Verde	1.23486	1.18003	0.05482	3	4	24	−1
Liberia	1.19605	1.19274	0.00331	4	3	47	1
Botswana	1.03003	0.99987	0.03016	5	5	34	0
Benin	0.99522	0.95616	0.03907	6	7	29	−1
Ghana	0.98133	0.97796	0.00337	7	6	46	1
Namibia	0.91294	0.83887	0.07407	8	8	19	0
South Africa	0.89588	0.82555	0.07033	9	9	20	0
Niger	0.86013	0.77654	0.08359	10	12	17	−2
Madagascar	0.83605	0.78759	0.04846	11	10	26	1
Seychelles	0.82305	0.77842	0.04463	12	11	28	1
Lesotho	0.78993	0.75439	0.03554	13	13	32	0
Mali	0.78554	0.75114	0.03441	14	14	33	0
Guinea Bissau	0.73275	0.71057	0.02218	15	15	37	0
Burkina Faso	0.67820	0.66591	0.01228	16	16	42	0
Malawi	0.59527	0.59251	0.00276	17	17	48	0
Mauritania	0.51831	0.51831	0.00000	18	18	53	0
Sierra Leone	0.49698	0.49669	0.00029	19	19	52	0
Gambia	0.45152	0.44911	0.00241	20	20	50	0
Rwanda	0.45002	0.44750	0.00251	21	21	49	0
Tanzania	0.36682	0.35500	0.01181	22	22	43	0
Senegal	0.35698	0.33429	0.02269	23	23	36	0
Comoros	0.34343	0.32780	0.01564	24	24	39	0
Zambia	0.29994	0.27580	0.02414	25	25	35	0
Mozambique	0.24208	0.20392	0.03816	26	26	31	0
Uganda	0.14343	0.07454	0.06890	27	27	21	0
Gabon	0.08092	0.03205	0.04887	28	28	25	0
Central African Republic	0.03238	−0.08060	0.11298	29	29	12	0
Burundi	0.01438	−0.10661	0.12100	30	30	11	0
Algeria	0.00334	−0.12269	0.12603	31	31	10	0
Kenya	−0.08032	−0.24718	0.16686	32	32	8	0
Djibouti	−0.15965	−0.25792	0.09828	33	33	14	0
Cameroon	−0.22961	−0.29668	0.06707	34	34	22	0
Togo	−0.33380	−0.37225	0.03845	35	35	30	0
Congo	−0.35922	−0.54261	0.18339	36	37	6	−1
Congo Democratic Republic	−0.38091	−0.72754	0.34663	37	39	1	−2
Morocco	−0.42860	−0.44559	0.01700	38	36	38	2
Tunisia	−0.54609	−0.59321	0.04712	39	38	27	1
Nigeria	−0.66337	−0.75106	0.08769	40	40	16	0
Zimbabwe	−0.80996	−0.96321	0.15325	41	42	9	−1
Chad	−0.89162	−0.89665	0.00502	42	41	45	1

Table 4 continued

	Rating			Ranking			Rank difference
	R	R^m	A_R	R	R^m	A_R	$R - R^m$
Ethiopia	−0.96447	−1.20196	0.23749	43	43	3	0
Côte d'Ivoire	−1.01070	−1.25341	0.24271	44	44	2	0
Angola	−1.05518	−1.27201	0.21683	45	45	4	0
Guinea	−1.10198	−1.29276	0.19078	46	46	5	0
Swaziland	−1.13369	−1.30756	0.17387	47	47	7	0
Equatorial Guinea	−1.28382	−1.38651	0.10268	48	48	13	0
Egypt	−1.38318	−1.44781	0.06464	49	49	23	0
Libya	−1.57914	−1.59338	0.01424	50	50	41	0
Sudan	−1.79422	−1.79527	0.00105	51	51	51	0
Eritrea	−1.88869	−1.89847	0.00978	52	52	44	0
Somalia	−2.19952	−2.29654	0.09701	53	53	15	0

Table 5 Comparison on the *Sustainable Economic Opportunity* pillar (E)

	Rating			Ranking			Rank difference
	E	E^m	A_E	E	E^m	A_E	$E - E^m$
Seychelles	1.77093	0.73717	1.03377	1	6	4	−5
Mauritius	1.62636	1.51953	0.10683	2	1	26	1
Tunisia	1.23001	1.13103	0.09898	3	3	29	0
South Africa	1.20008	1.16924	0.03084	4	2	51	2
Libya	1.19078	0.63580	0.55497	5	8	7	−3
Gabon	1.11758	0.70922	0.40837	6	7	10	−1
Algeria	0.98165	0.81854	0.16311	7	5	20	2
Egypt	0.92802	0.82518	0.10283	8	4	27	4
Equatorial Guinea	0.89282	0.14762	0.74520	9	15	5	−6
Botswana	0.87189	0.62877	0.24312	10	9	16	1
Morocco	0.73667	0.41741	0.31925	11	11	12	0
Namibia	0.50949	0.44362	0.06587	12	10	39	2
Sao Tome and Principe	0.37239	0.22919	0.14320	13	12	23	1
Kenya	0.27231	0.17379	0.09852	14	14	30	0
Cape Verde	0.25569	0.22352	0.03217	15	13	50	2
Swaziland	0.17149	0.12606	0.04542	16	16	46	0
Ghana	0.15561	0.09316	0.06245	17	17	40	0
Gambia	0.03602	−0.01658	0.05260	18	18	45	0
Cameroon	0.02080	−0.06370	0.08450	19	20	35	−1
Tanzania	0.01295	−0.14357	0.15652	20	23	22	−3
Nigeria	−0.00688	-0.05986	0.05299	21	19	44	2

Table 5 continued

	Rating			Ranking			Rank difference
	E	E^m	A_E	E	E^m	A_E	$E - E^m$
Zambia	−0.01424	-0.10135	0.08711	22	22	32	0
Lesotho	−0.03462	-0.09006	0.05544	23	21	43	2
Congo	−0.05491	-0.28779	0.23288	24	29	17	−5
Ethiopia	−0.05940	−0.33009	0.27070	25	31	15	−6
Uganda	−0.06943	−0.15799	0.08857	26	24	31	2
Senegal	−0.11865	−0.16180	0.04315	27	25	47	2
Sudan	−0.12757	−0.16367	0.03609	28	26	49	2
Angola	−0.16221	−1.27074	1.10853	29	47	3	−18
Mozambique	−0.17013	−0.24365	0.07353	30	27	36	3
Malawi	−0.17129	−0.28508	0.11379	31	28	25	3
Côte d'Ivoire	−0.22059	−0.30634	0.08575	32	30	34	2
Rwanda	−0.23440	−0.33664	0.10224	33	32	28	1
Djibouti	−0.25731	−0.44873	0.19142	34	37	18	−3
Madagascar	−0.29188	−0.36251	0.07063	35	34	38	1
Togo	−0.29215	−0.42930	0.13715	36	36	24	0
Benin	−0.32847	−0.34488	0.01641	37	33	52	4
Somalia	−0.35755	−0.41955	0.06200	38	35	41	3
Liberia	−0.39841	−0.47008	0.07168	39	38	37	1
Comoros	−0.47467	−0.64009	0.16542	40	42	19	−2
Burundi	−0.49570	−0.55699	0.06129	41	39	42	2
Mauritania	−0.54073	−0.95743	0.41670	42	44	9	−2
Central African Republic	−0.54422	−0.58318	0.03896	43	40	48	3
Eritrea	−0.55179	−0.63773	0.08594	44	41	33	3
Sierra Leone	−0.62276	−1.29505	0.67229	45	48	6	−3
Guinea	−0.65040	−0.66559	0.01519	46	43	53	3
Burkina Faso	−0.65551	−0.98383	0.32832	47	45	11	2
Mali	−0.71683	−1.00761	0.29078	48	46	13	2
Niger	−0.96726	−1.45069	0.48343	49	51	8	−2
Zimbabwe	−1.01159	−2.84617	1.83458	50	52	1	−2
Congo Democratic Republic	−1.06159	−1.34995	0.28835	51	49	14	2
Chad	−1.22368	−1.38453	0.16085	52	50	21	2
Guinea Bissau	−1.64549	−3.47256	1.82706	53	53	2	0

development of the whole country. In terms of international institutions, ranking is a way to evaluate the soundness of national institutions, and considering balance among various dimensions of governance becomes fundamental. The original Ibrahim Index of African Governance is one of the main indices considered by international institutions and foreign investors; thus the changes of ranking and rating ensuing from the unbalance adjustment proposed here could be taken into account by economic and financial operators in their pursuit of adequate policies and profitable investments.

Table 6 Comparison on the *Human Development* pillar (D)

	Rating			Ranking			Rank difference
	D	D^m	A_D	D	D^m	A_D	$D - D^m$
Tunisia	1.86065	1.75637	0.10427	1	2	37	−1
Mauritius	1.84506	1.73143	0.11363	2	3	33	−1
Libya	1.83270	1.77626	0.05645	3	1	44	2
Seychelles	1.82402	1.64750	0.17652	4	4	24	0
Egypt	1.75298	1.48287	0.27011	5	6	12	−1
Algeria	1.67850	1.55138	0.12713	6	5	29	1
Morocco	1.19183	0.84702	0.34481	7	7	5	0
Gabon	0.84786	0.73154	0.11632	8	9	32	−1
Cape Verde	0.83471	0.75847	0.07624	9	8	41	1
Ghana	0.58962	0.57849	0.01113	10	10	53	0
Botswana	0.55948	0.35066	0.20881	11	11	16	0
South Africa	0.55427	0.25666	0.29760	12	13	7	−1
Kenya	0.41243	0.28033	0.13211	13	12	28	1
Namibia	0.28173	0.04752	0.23421	14	17	13	−3
Sao Tome and Principe	0.26098	−0.01723	0.27821	15	20	10	−5
Gambia	0.25655	0.14633	0.11022	16	15	35	1
Mauritania	0.24769	0.16033	0.08736	17	14	40	3
Djibouti	0.12189	−0.00486	0.12675	18	19	30	−1
Sudan	0.06949	0.05779	0.01170	19	16	51	3
Tanzania	0.06008	0.01258	0.04751	20	18	47	2
Cameroon	−0.02305	−0.12842	0.10537	21	23	36	−2
Lesotho	−0.05396	−0.24897	0.19501	22	25	20	−3
Uganda	−0.05728	−0.07209	0.01481	23	21	50	2
Senegal	−0.06820	−0.26555	0.19735	24	26	19	−2
Congo	−0.07394	−0.11429	0.04035	25	22	49	3
Malawi	−0.12228	−0.16430	0.04202	26	24	48	2
Benin	−0.13279	−0.33429	0.20149	27	27	18	0
Swaziland	−0.17775	−0.54232	0.36457	28	34	3	−6
Comoros	−0.21337	−0.42036	0.20699	29	30	17	−1
Ethiopia	−0.25561	−0.47532	0.21971	30	31	14	−1
Togo	−0.26049	−0.38501	0.12452	31	29	31	2
Nigeria	−0.28318	−0.33739	0.05420	32	28	45	4
Zimbabwe	−0.31061	−0.67177	0.36116	33	37	4	−4
Madagascar	−0.34591	−0.53925	0.19333	34	33	21	1
Zambia	−0.38533	−0.56893	0.18360	35	36	22	−1
Côte d'Ivoire	−0.38997	−0.67265	0.28267	36	38	9	−2
Eritrea	−0.43037	−0.49582	0.06545	37	32	42	5
Burundi	−0.46089	−0.55894	0.09805	38	35	38	3
Burkina Faso	−0.48620	−0.94674	0.46054	39	44	2	−5
Guinea	−0.59758	−0.68570	0.08813	40	39	39	1
Mali	−0.62100	−0.90727	0.28627	41	42	8	−1
Rwanda	−0.65345	−0.70300	0.04955	42	40	46	2

Table 6 continued

	Rating			Ranking			Rank difference
	D	D^m	A_D	D	D^m	A_D	$D - D^m$
Guinea Bissau	−0.69842	−0.97226	0.27384	43	45	11	−2
Angola	−0.75378	−0.89265	0.13888	44	41	27	3
Sierra Leone	−0.80154	−1.01797	0.21643	45	46	15	−1
Liberia	−0.84797	−1.02403	0.17607	46	47	25	−1
Mozambique	−0.91563	−0.92724	0.01160	47	43	52	4
Equatorial Guinea	−0.95760	−1.72558	0.76798	48	53	1	−5
Congo Democratic Republic	−0.99046	−1.16138	0.17092	49	48	26	1
Niger	−1.19456	−1.30711	0.11255	50	49	34	1
Central African Republic	−1.22641	−1.56759	0.34118	51	52	6	−1
Chad	−1.31683	−1.49931	0.18249	52	51	23	1
Somalia	−1.42941	−1.49416	0.06475	53	50	43	3

Table 7 Correlation of ratings and of rankings

Index or pillar symbol	G	S	C	R	E	D
Percentage with rank difference ≤ -2 or ≥ 2 between X and X^m	77.36	54.72	56.60	5.66	67.92	49.06
Rating correlation between X and A_X	−0.08	−0.12	−0.10	−0.37	−0.18	−0.15
Spearman rank correlation between X and A_X	−0.02	−0.38	−0.12	−0.35	−0.09	−0.12

8 Conclusions

This paper analyzes some aggregation aspects of the indexing procedure of a multidimensional socio-economic phenomenon, such as development. After illustrating a theoretical framework on unbalance adjustment, it proposes an aggregation function that takes into account the unbalance among dimensions and permits to measure its amount. This function, called *mean-min*, allows compensability among dimensions, although with a cost that increases with unbalance; this is an intermediate case between perfect substitutability among dimensions, represented by the weighted arithmetic mean, according to which no unbalance is penalized, and perfect complementarity among dimensions, represented by the min function, according to which the penalization is maximum. The mean-min function makes it possible to measure three aspects of the same phenomenon: an index that measures the average level, one that measures the amount of unbalance, and a composite indicator that combines level and unbalance. The mean-min function has some properties that other important unbalance-adjusted functions lack, such as an unrestricted domain that poses no constraint to the choice of the most appropriate normalization procedure. Furthermore, the mean-min function depends on parameters that allow the user to adapt it to different kinds of analysis (with progressive or proportional compensability, with complete or incomplete compensability). Finally, the dataset of the *Index of African Governance* has been used here to compare the mean-min function to the weighted arithmetic mean and to calculate the unbalance adjustment function both for every governance pillar (by aggregating its subpillars) and the final governance indicator (by aggregating the governance pillars). For all these indicators there is a significant difference of ranking between the mean-min function and the weighted arithmetic mean and there is a negative correlation, regarding both the rating and the ranking, between weighted arithmetic mean and unbalance adjustment function. We have explained the changes in the ranking of some countries in terms of the balance within their respective set of pillar values.

Acknowledgments The authors wish to thank Marcella Corsi, Marco Marini and Paolo Palazzi for their useful comments. Research partially supported by the Ateneo della Scienza e della Tecnologia 2007 project *Sviluppo economico e sviluppo civile secondo l'approccio di Paolo Sylos Labini*, coordinated by Marcella Corsi.

References

Annoni, P., & Kozovska, K. (2010). *EU regional competitiveness index (RCI) 2010*. Technical report EUR 24346 EN, Scientific and Technical Research Series, European Commission Joint Research Centre, Institute for the Protection and Security of the Citizen, Luxembourg.

Baker, D., Bhaduri, A., & Ocampo, J. A. (2005). The development imperative: Toward a people-centered approach. In E. Hershberg & C. Thornton (Eds.). New York: Social Science Research Council.

Boozer, M., Ranis, G., Stewart, F., & Suri, T. (2003). *Paths to success: The relationship between human development and economic growth*. Technical Report 874, Discussion Papers, Economic Growth Center, Yale University, December.

Bourguignon, F. (2004). *The poverty-growth-inequality triangle*. Technical report 125, working papers, Indian Council For Research on International Economic Relations, New Delhi, March.

Broyer, S., & Savry, G. (2002). German leading indicators: Which one should be monitored?. *CDC IXIS Capital Markets Flash, 38*, 1–10.

Casadio Tarabusi, E., & Palazzi, P. (2004). An index for sustainable development. *BNL Quarterly Review, 229*, 185–206; Italian transl., *Un indice per lo sviluppo sostenibile*, Moneta e Credito **226**, 123–149.

Chakravarty, S. R. (2003). A generalized human development index. *Review of Development Economics, 7*(1), 99–114.

Corsi, M., & Guarini, G. (2011). Measuring progress of Italian regions: A classical approach. *Ekonomiaz, Basque Economic Journal, 78*, 340–367.

De Muro, P., Mazziotta, M., & Pareto, A. (2008). *Composite indices for multidimensional development and poverty: An application to MDG indicators*. World Food Day Workshop, MDG1: Where do we stand with the implementation eight years after?

Dunn L. F., Ekici T. , Lavrakas P. J., & Stec J. A. (2004). *An index to track credit card debt and predict consumption*. OSU Economics Working Paper WP04-04.

Huggins, R., & Davies, W. (2006). *European Competitiveness Index 2006–07*, Pontypridd, Wales.

Munda, G., & Nardo, M. (2009). Non-compensatory/non-linear composite indicators for ranking countries: A defensible setting. *Applied Economics, 41*, 1513–1523. doi:10.1080/00036840601019364.

Organisation for Economic Co-operation and Development. (2008). *Handbook on constructing composite indicators: Methodology and user guide*.

Palazzi, P., & Lauri, A. (1998). The Human Development Index: Suggested corrections. *BNL Quarterly Review, 205*, 193–221.

Ranis, G. (2006). Human development and economic growth. *The Elgar companion to development studies*, 250–256.

Ranis, G., & Stewart, F. (1997). Growth and human development: Pakistan in comparative perspective. I. *The Pakistan Development Review, 36*(4), 333–352.

Ranis, G., & Stewart, F. (2000). Strategies for success in human development. *Journal of Human Development, 1*(1), 49–69.

Ranis, G., Stewart, F., & Ramirez, A. (2000). Economic growth and human development. *World Development, 28*(2), 197–219.

Rotberg, R. I., & Gisselquist, R. M. (2009). *Strengthening African governance: Index of African Governance results and rankings*. Cambridge: Harvard Kennedy School.

Saltelli, A. (2007). Composite indicators between analysis and advocacy. *Social Indicators Research, 81*, 65–77. doi:10.1007/s11205-006-0024-9.

Sen, A. (1999a). *Development as freedom*. Oxford: Oxford University Press.

Sen, A. (1999b). *Assessing human development*. Human Development Report 1999, p. 23. (Special contribution).

Soubbotina, T. P. (2004). *Beyond economic growth: An introduction to sustainable development, 2nd ed.*. Washington, D.C.: WBI Learning Resources Series, World Bank.

Soubbotina, T. P., & Sheram, K. A. (2000). *Beyond economic growth: Meeting the challenges of global development*. Washington, D.C.: WBI Learning Resources Series, World Bank.

Stiglitz, J. E., Sen, A., & Fitoussi, J.-P. (2009). *Report by the commission on the measurement of economic performance and social progress*.

Takayama, A. (1985). *Mathematical economics, 2nd ed.*. Cambridge: Cambridge University Press.

United Nations. (2010). *The Millennium Development Goals Report 2010*, New York.

United Nations Development Programme. (1991). *Human development report 1991*, New York.

United Nations Development Programme. (2009). *Human development report 2009*, New York.

Zelenyuk, V. (2006). Aggregation of Malmquist productivity indexes. *European Journal of Operational Research, 174*(2), 1076–1086. doi:10.1016/j.ejor.2005.02.061.