# Multi-Step Generalized Policy Improvement by Leveraging Approximate Models


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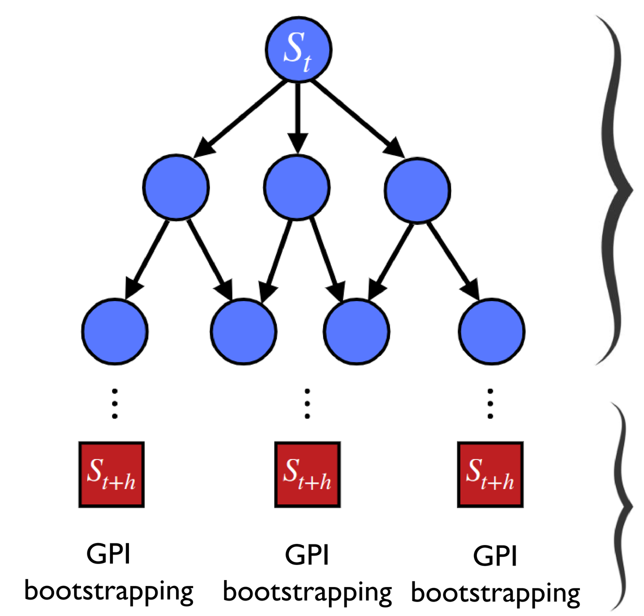
## Contribution

***h*-GPI**: Multi-Step Generalized Policy Improvement

- Interpolates between model-free GPI and fully model-based planning as a function of the planning horizon $h$
- Zero-shot policy transfer with performance guarantees by exploiting approximate, imperfect models




$h$ planning steps with approximate model

zero-shot GPI in states reachable in $h$ steps

## Successor Features (SFs)

Linear reward: $r_{\mathbf{w}}(s,a,s') = \phi(s,a,s') \cdot \mathbf{w}$

SFs: $\psi^{\pi}(s,a) \triangleq \mathbb{E}_{\pi}\left[\sum_{i=0}^{\infty} \gamma^i \phi_{t+i} \mid S_t = s, A_t = a\right]$

Generalized Policy Evaluation (GPE): $q^{\pi}_{\mathbf{w}}(s,a) = \psi^{\pi}(s,a) \cdot \mathbf{w}$

## Generalized Policy Improvement (GPI)

Determine a policy $\pi'$ that improves over a ***set** of policies* $\Pi = \{\pi_i\}_{i=1}^n$

$$\boxed{\pi^{GPI}(s;\mathbf{w}) = \arg\max_{a \in \mathcal{A}} \max_{\pi \in \Pi} q^{\pi}_{\mathbf{w}}(s,a)}$$

GPI Theorem

$$q^{GPI}_{\mathbf{w}}(s,a) \geq \max_{\pi \in \Pi} q^{\pi}_{\mathbf{w}}(s,a) \quad \textit{for any } \mathbf{w} \in \mathcal{W}$$

## $h$-GPI: Multi-Step Generalized Policy Improvement

- The ***h*-GPI** policy with planning horizon $h \geq 0$ is defined as:

$$\pi^{h\text{-}GPI}(s) \in \arg\max_{a \in \mathcal{A}} (\mathcal{T}^*_m)^h \max_{\pi \in \Pi} q^{\pi}(s,a)$$

$$=$$

$$\arg\max_{a \in \mathcal{A}} \max_{\mu_1 \dots \mu_{h-1}} \mathbb{E}_m \left[ \underbrace{\sum_{k=0}^{h-1} \gamma^k r(S_{t+k}, \mu_k(S_{t+k}))}_{\text{online planning}} + \underbrace{\gamma^h \max_{a' \in \mathcal{A}} \max_{\pi \in \Pi} q^{\pi}(S_{t+h}, a')}_{\text{GPI}} \mid \mu_0(S_t) = a \right]$$

where $\mu_k$ is any policy the agent could choose to deploy at time $k$.

$\Pi = \{\pi_i\}_{i=1}^n$ : set of policies $\qquad m = (p, r)$ : model

We characterize $h$-GPI's performance lower bound and optimality gap as a function of:

- $h$ (planning horizon)
- $\{w_i\}_{i=1}^n$ (reward weights for which policies in $\Pi$ are optimal)
- $\epsilon$ (action-value function error)
- $\epsilon_p, \epsilon_r,$ (model errors w.r.t. transition function $p$ and reward $r$)

Theorem 1 (lower bound):

$$q^{h-GPI}(s,a) \geq \max_{\pi \in \Pi} q^{\pi}(s,a) - \frac{2}{1-\gamma}\left(\gamma^h \epsilon + c(\epsilon_p, \epsilon_r, h)\right)$$

Theorem 2 (optimality gap):

$$q^*_{\mathbf{w}} - q^{h-GPI}_{\mathbf{w}}(s,a) \geq \frac{2}{1-\gamma}\left(\phi_{max} \min_i ||\mathbf{w} - \mathbf{w}_i|| + \gamma^h \epsilon + c(\epsilon_p, \epsilon_r, h)\right)$$

where $c(\epsilon_p, \epsilon_r, h)) = \frac{1-\gamma^h}{1-\gamma}(\epsilon_r + \gamma \epsilon_p v^*_{max})$

## Zero-Shot Transfer with $h$-GPI and SFs

**Goal:** Solve any task in $\mathcal{M}^{\phi} \triangleq \{ M = (\mathcal{S}, \mathcal{A}, p, r_{\mathbf{w}}, \gamma) \mid r_{\mathbf{w}} = \phi(s,a,s') \cdot \mathbf{w} \}$

**Algorithm 1:** $h$-GPI with Successor Features

**Input :** Model $\hat{m} = (\hat{p}, \hat{\phi})$, SFs $\{\hat{\psi}^{\pi_i}\}_{i=1}^n$, planning horizon $h \geq 0$, state $s$, reward weights $\mathbf{w}$

1 **for** *action* $a \in \mathcal{A}$ **do**

2 Let $S_t = s$, $\mu_0(s) = a$

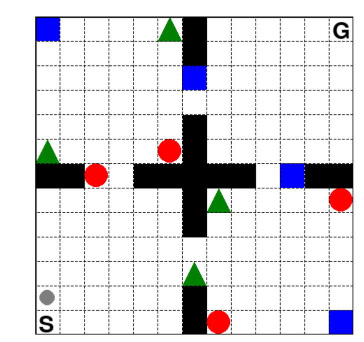
3 Compute $(\mathcal{T}^*_{\hat{m}})^h \max_{\pi \in \Pi} \hat{q}^{\pi}_{\mathbf{w}}(s,a) \leftarrow$

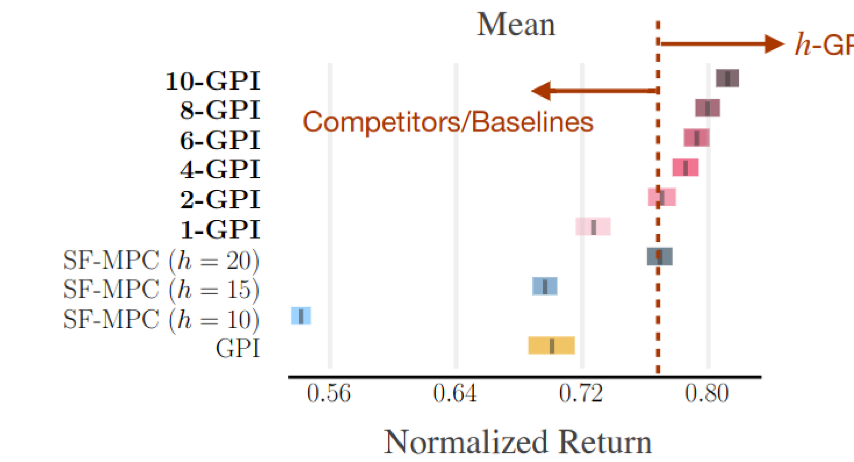
$$\max_{\mu_1 \dots \mu_{h-1}} \mathbb{E}_{\hat{m}}\left[\sum_{k=0}^{h-1} \gamma^k \hat{\phi}_{t+k}(\hat{S}_{t+k}, \mu_k(\hat{S}_{t+k})) \cdot \mathbf{w} + \gamma^h \max_{a' \in \mathcal{A}} \max_{\pi \in \Pi} \hat{\psi}^{\pi}(\hat{S}_{t+h}, a') \cdot \mathbf{w}\right]$$

4 **Return:** $\pi^{h\text{-GPI}}(s;\mathbf{w}) \in \arg\max_{a \in \mathcal{A}} (\mathcal{T}^*_{\hat{m}})^h \max_{\pi_i \in \Pi} \hat{q}^{\pi_i}_{\mathbf{w}}(s,a)$

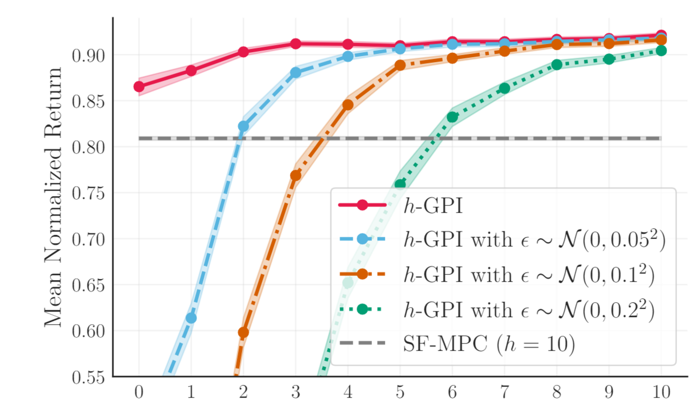
## Experiments & Results

Four Room



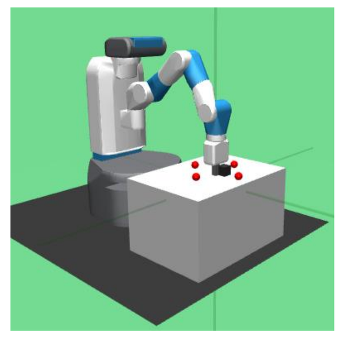


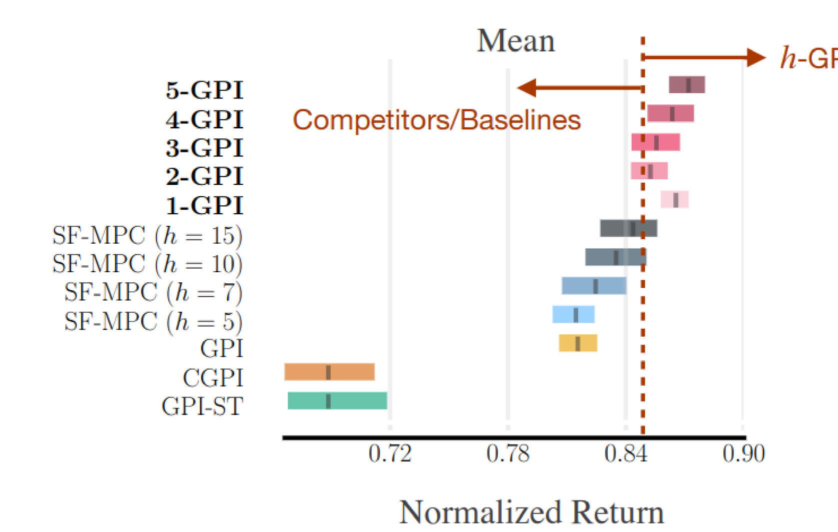

$h$-GPI outperforms SF-MPC baseline using *ten times* fewer planning steps




$h$-GPI is less susceptible to value function approximation errors as $h$ increases

Fetch Push






$h$-GPI outperforms competitors under all values of $h$ using a learned model

## Discussion & Conclusion

- $h$-GPI: multi-step extension of GPI
  - Interpolates between model-free GPI ($h = 0$) and fully model-based planning ($h \rightarrow \infty$)
  - Exploits approximate models
  - Solves tasks in a zero-shot manner
- $h$ *trades-off* approximation errors in the agent's:
  - Learned model
  - Action-value functions