Planejamento em IA - Lista 1

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1 Implementation

We did an object-oriented implementation of the problem. In the folder *src/Algorithms* there is an abstract *Algorithm* class, and a class implementation for each one of the five search algorithms. In the folder *src/NPuzzle* there is the implementation of a search *Node*, and the implementation of the *State* structure, which implements the N-Puzzle game logic.

To improve the memory and computation efficiency, the value of a State is represented as an unsigned integer of 64 bits. Because there are 16 different tiles in the 15-Puzzle, each tile is represented with 4 bits of the 64 bits. The generation of successor states is then calculated using bit-wise operations, which was very challenging to debug.

In addition, we implemented the Linear Conflict heuristic, as we found described in the paper *Finding Optimal Solutions to the Twenty-Four Puzzle* by R. E. Korf and L. A. Taylor. Adding it to the Manhattan distance was quite simple and improved the results significantly (see A* + LC in the tables below).

Machine Configuration   We ran the experiments in a machine with an Intel(R) Core(TM) i5-7500 CPU @ 3.40GHz processor and 16 GB of RAM. The program was compiled with g++ using -O3 optimization.

2 Results

Table 1: Average results for the 8-Puzzle instances

Algorithm	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
IDFS	2578290.56	22.16	0.127572	0.000000	13.88
BFS	81459.54	22.16	0.023579	0.000000	13.88
GBFS	392.42	140.52	0.000585	6.893046	13.88
IDA*	2373.03	22.16	0.000351	10.432847	13.88
A*	895.00	22.16	0.000646	10.027754	13.88
A* + LC	429.49	22.16	0.000364	11.061484	15.24

Table 2: Average results for the 15-Puzzle instances

Algorithm	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic	Solved Instances
A*	5.0400e+06	52.022727	4.997209	25.611379	36.636364	88
A* + LC	1.7723e+06	52.711340	3.044496	28.462806	39.103093	97

Table 3: BFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	70436	22	0.028509	0.0	12
1	98412	23	0.031114	0.0	15
2	11718	17	0.003386	0.0	13
3	134664	25	0.041216	0.0	17
4	5209	16	0.001377	0.0	10
5	59830	22	0.016505	0.0	12
6	71614	22	0.021173	0.0	14
7	42704	21	0.011437	0.0	11
8	10486	17	0.002889	0.0	7
9	55230	21	0.015199	0.0	13
10	86288	23	0.033060	0.0	15
11	87902	23	0.024838	0.0	11
12	24271	19	0.006737	0.0	13
13	67445	22	0.020553	0.0	14
14	21294	19	0.005866	0.0	13
15	67976	22	0.019460	0.0	10
16	30932	20	0.009319	0.0	10
17	51926	21	0.015745	0.0	19
18	74010	22	0.021577	0.0	18
19	57200	22	0.015302	0.0	16
20	6175	16	0.001627	0.0	14
21	4890	16	0.001295	0.0	10
22	128087	25	0.034855	0.0	15
23	59140	22	0.015603	0.0	14
24	86044	23	0.024720	0.0	13
25	124428	25	0.038601	0.0	15
26	55828	21	0.016006	0.0	15
27	1904	13	0.000524	0.0	7
28	89286	23	0.025972	0.0	13
29	96167	24	0.027382	0.0	16
30	108997	24	0.030253	0.0	14
31	6800	16	0.001954	0.0	12
32	12237	18	0.003283	0.0	16
33	91221	23	0.025714	0.0	13
34	146642	26	0.039294	0.0	14
35	166169	27	0.051233	0.0	17
36	61999	22	0.023433	0.0	14
37	149038	26	0.041785	0.0	14
38	66394	22	0.019935	0.0	12
39	134964	25	0.037607	0.0	13
40	3528	14	0.000977	0.0	8
41	141058	25	0.037610	0.0	19
42	51334	21	0.014190	0.0	11
43	147235	26	0.039897	0.0	16
44	96629	24	0.025970	0.0	14
45	117820	24	0.033011	0.0	20
46	12721	18	0.003379	0.0	14
47	82402	23	0.022765	0.0	15

Table 3: BFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
48	55950	21	0.015426	0.0	11
49	170358	28	0.053165	0.0	16
50	41346	20	0.011523	0.0	12
51	163708	27	0.055114	0.0	15
52	25948	19	0.007175	0.0	11
53	146431	26	0.040222	0.0	16
54	16644	18	0.004751	0.0	12
55	113398	24	0.031482	0.0	16
56	12763	18	0.003585	0.0	8
57	102477	24	0.028017	0.0	12
58	91593	23	0.027074	0.0	17
59	116022	24	0.032948	0.0	16
60	170975	27	0.051666	0.0	19
61	132375	25	0.035603	0.0	13
62	43631	21	0.011662	0.0	15
63	61343	22	0.016301	0.0	18
64	173107	28	0.052240	0.0	14
65	37323	20	0.010743	0.0	16
66	166627	27	0.050936	0.0	17
67	165544	27	0.051182	0.0	15
68	136365	25	0.036522	0.0	19
69	101428	24	0.027703	0.0	10
70	99337	24	0.026809	0.0	16
71	69657	22	0.020886	0.0	16
72	169153	27	0.052111	0.0	15
73	146614	26	0.038718	0.0	18
74	136769	25	0.037402	0.0	11
75	144168	26	0.038082	0.0	20
76	32178	20	0.009517	0.0	14
77	149505	26	0.040958	0.0	16
78	6810	16	0.002046	0.0	10
79	165286	27	0.049630	0.0	17
80	20088	19	0.005529	0.0	11
81	134133	25	0.036397	0.0	21
82	3471	15	0.000925	0.0	11
83	9452	17	0.002747	0.0	13
84	4001	15	0.001098	0.0	7
85	134421	25	0.036241	0.0	17
86	10951	17	0.003165	0.0	11
87	165164	27	0.048450	0.0	17
88	32194	20	0.009346	0.0	8
89	37083	20	0.010333	0.0	12
90	163614	27	0.053854	0.0	21
91	145000	26	0.038957	0.0	18
92	68791	22	0.020974	0.0	12
93	69185	22	0.021113	0.0	14
94	66051	22	0.019448	0.0	12
95	25603	19	0.007183	0.0	17
96	30499	20	0.008786	0.0	8

Table 3: BFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
97	45472	21	0.012578	0.0	9
98	106698	24	0.030674	0.0	14
99	130566	25	0.034728	0.0	13

Table 4: IDFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	809224	22	0.041842	0.0	12
1	1659716	23	0.085871	0.0	15
2	56332	17	0.002882	0.0	13
3	3832745	25	0.193167	0.0	17
4	20305	16	0.001096	0.0	10
5	572929	22	0.029549	0.0	12
6	840836	22	0.042891	0.0	14
7	336804	21	0.016719	0.0	11
8	48030	17	0.002519	0.0	7
9	523656	21	0.026272	0.0	13
10	1183127	23	0.058990	0.0	15
11	1229083	23	0.062712	0.0	11
12	144153	19	0.007345	0.0	13
13	726908	22	0.036265	0.0	14
14	119854	19	0.006141	0.0	13
15	746437	22	0.036923	0.0	10
16	201422	20	0.010049	0.0	10
17	463439	21	0.023382	0.0	19
18	880730	22	0.044072	0.0	18
19	529433	22	0.026344	0.0	16
20	25389	16	0.001359	0.0	14
21	18803	16	0.000991	0.0	10
22	3233556	25	0.160559	0.0	15
23	559966	22	0.027700	0.0	14
24	1161412	23	0.057642	0.0	13
25	3012022	25	0.148834	0.0	15
26	533665	21	0.026917	0.0	15
27	6472	13	0.000444	0.0	7
28	1258485	23	0.062886	0.0	13
29	1506581	24	0.074533	0.0	16
30	2082556	24	0.103740	0.0	14
31	28816	16	0.001538	0.0	12
32	57963	18	0.002965	0.0	16
33	1339032	23	0.067058	0.0	13
34	5227219	26	0.257895	0.0	14
35	10582097	27	0.522496	0.0	17
36	618550	22	0.030836	0.0	14
37	5647709	26	0.279502	0.0	14
38	707091	22	0.035207	0.0	12
39	3906491	25	0.193211	0.0	13
40	13166	14	0.000783	0.0	8

Table 4: IDFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
41	4826279	25	0.240285	0.0	19
42	445186	21	0.022230	0.0	11
43	5301125	26	0.261802	0.0	16
44	1521264	24	0.074936	0.0	14
45	2550556	24	0.126212	0.0	20
46	60744	18	0.003101	0.0	14
47	1071876	23	0.053283	0.0	15
48	542261	21	0.027088	0.0	11
49	13426501	28	0.657606	0.0	16
50	323493	20	0.016349	0.0	12
51	9617303	27	0.471732	0.0	15
52	160297	19	0.008079	0.0	11
53	5150682	26	0.253583	0.0	16
54	86884	18	0.004397	0.0	12
55	2296791	24	0.114427	0.0	16
56	60654	18	0.003075	0.0	8
57	1715624	24	0.085178	0.0	12
58	1336827	23	0.066551	0.0	17
59	2536222	24	0.126376	0.0	16
60	15171649	27	0.754167	0.0	19
61	3587527	25	0.177357	0.0	13
62	344582	21	0.017208	0.0	15
63	599226	22	0.029571	0.0	18
64	15073854	28	0.739847	0.0	14
65	277227	20	0.014075	0.0	16
66	10843669	27	0.538408	0.0	17
67	10295974	27	0.507446	0.0	15
68	4095213	25	0.202979	0.0	19
69	1665813	24	0.082706	0.0	10
70	1583030	24	0.078189	0.0	16
71	792904	22	0.039596	0.0	16
72	12496979	27	0.618993	0.0	15
73	5225955	26	0.257497	0.0	18
74	4088312	25	0.202607	0.0	11
75	4873453	26	0.239492	0.0	20
76	214527	20	0.010774	0.0	14
77	5735592	26	0.283498	0.0	16
78	28881	16	0.001548	0.0	10
79	10128430	27	0.499229	0.0	17
80	110829	19	0.005623	0.0	11
81	3745531	25	0.185348	0.0	21
82	12608	15	0.000752	0.0	11
83	42323	17	0.002128	0.0	13
84	15054	15	0.000806	0.0	7
85	3814064	25	0.188102	0.0	17
86	51080	17	0.002764	0.0	11
87	10083429	27	0.496073	0.0	17
88	216000	20	0.010934	0.0	8
89	269408	20	0.013487	0.0	12

Table 4: IDFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
90	9608157	27	0.471211	0.0	21
91	4963354	26	0.244994	0.0	18
92	761900	22	0.037736	0.0	12
93	779904	22	0.039032	0.0	14
94	713444	22	0.035590	0.0	12
95	156601	19	0.007853	0.0	17
96	197513	20	0.010165	0.0	8
97	365116	21	0.018259	0.0	9
98	1905883	24	0.094454	0.0	14
99	3411288	25	0.168256	0.0	13

Table 5: GBFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	579	156	0.001071	7.008772	12
1	437	187	0.000807	6.869509	15
2	430	189	0.000718	6.610747	13
3	466	207	0.000769	6.945455	17
4	16	16	0.000028	5.500000	10
5	407	152	0.000672	6.590846	12
6	422	168	0.000689	6.612601	14
7	96	27	0.000156	8.054217	11
8	576	87	0.000991	6.730019	7
9	634	119	0.001085	7.052538	13
10	90	57	0.000154	6.809816	15
11	560	141	0.001002	6.932392	11
12	83	47	0.000141	5.891156	13
13	502	146	0.000863	6.734305	14
14	66	39	0.000115	6.134454	13
15	501	146	0.000865	6.698768	10
16	81	44	0.000139	6.510638	10
17	46	37	0.000075	7.869048	19
18	386	160	0.000657	6.806405	18
19	429	190	0.000931	6.808201	16
20	540	132	0.000942	6.774428	14
21	453	166	0.000753	6.643750	10
22	464	181	0.000765	6.876074	15
23	651	198	0.001118	7.221930	14
24	406	189	0.000701	6.628453	13
25	423	187	0.000786	6.837366	15
26	453	203	0.000752	6.817164	15
27	17	15	0.000032	5.382353	7
28	405	165	0.000670	6.697772	13
29	542	140	0.000929	6.953271	16
30	135	38	0.000224	8.691304	14
31	67	34	0.000108	7.914530	12
32	452	194	0.000780	6.759051	16
33	432	195	0.000723	6.809211	13

Table 5: GBFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
34	521	162	0.000882	6.914502	14
35	111	79	0.000186	7.827411	17
36	590	98	0.001179	6.833492	14
37	549	148	0.000955	6.898773	14
38	447	184	0.000737	6.728192	12
39	460	211	0.000961	6.956790	13
40	394	140	0.000637	6.472103	8
41	65	45	0.000108	7.798319	19
42	510	155	0.000979	6.728982	11
43	525	222	0.000868	6.770652	16
44	533	130	0.000873	6.888771	14
45	416	184	0.000689	6.963464	20
46	19	18	0.000033	7.472222	14
47	428	181	0.000711	6.787599	15
48	552	149	0.000961	6.852342	11
49	461	198	0.000762	7.300371	16
50	496	174	0.000813	7.079128	12
51	88	49	0.000142	7.538961	15
52	601	101	0.001015	6.789916	11
53	452	206	0.000745	7.279040	16
54	560	132	0.000949	6.863911	12
55	43	36	0.000072	7.333333	16
56	480	180	0.000814	6.946746	8
57	459	216	0.000779	7.028290	12
58	80	55	0.000131	7.758865	17
59	68	46	0.000116	6.628099	16
60	525	219	0.000903	6.796518	19
61	401	153	0.000681	6.597475	13
62	632	195	0.001093	7.222924	15
63	416	188	0.000707	6.955224	18
64	496	210	0.000899	7.478857	14
65	20	20	0.000033	7.916667	16
66	659	129	0.001118	7.118803	17
67	474	197	0.000800	6.955847	15
68	62	41	0.000109	7.633929	19
69	423	170	0.000724	6.562249	10
70	514	158	0.000633	6.832969	16
71	394	166	0.000456	6.625000	16
72	420	181	0.000470	6.912634	15
73	410	178	0.000465	6.706207	18
74	386	175	0.000449	6.514577	11
75	448	176	0.000513	6.877525	20
76	389	164	0.000573	6.535456	14
77	466	194	0.000537	6.904126	16
78	453	194	0.000519	6.616149	10
79	442	193	0.000497	6.836551	17
80	590	129	0.000679	6.828820	11
81	454	197	0.000517	7.008696	21
82	57	39	0.000065	5.575472	11

Table 5: GBFS results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
83	18	17	0.000019	7.257143	13
84	387	149	0.000443	6.435860	7
85	572	175	0.000664	7.080357	17
86	417	179	0.000360	6.644898	11
87	596	175	0.000513	7.102927	17
88	382	168	0.000327	6.388807	8
89	522	150	0.000434	6.742734	12
90	580	151	0.000501	7.113592	21
91	458	208	0.000387	6.909988	18
92	485	176	0.000406	6.989449	12
93	580	152	0.000502	6.989279	14
94	409	152	0.000347	6.717241	12
95	403	161	0.000347	6.648951	17
96	636	186	0.000555	7.133752	8
97	48	29	0.000041	5.715909	9
98	613	104	0.000522	6.960514	14
99	450	203	0.000382	6.843750	13

Table 6: IDA* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	2631	22	0.000368	9.777978	12
1	3276	23	0.000477	10.598317	15
2	145	17	0.000020	9.987805	13
3	2720	25	0.000497	11.781368	17
4	165	16	0.000037	7.137809	10
5	1734	22	0.000267	9.245464	12
6	1741	22	0.000262	10.437286	14
7	439	21	0.000063	10.750000	11
8	464	17	0.000067	7.579949	7
9	611	21	0.000086	11.501490	13
10	724	23	0.000102	11.724138	15
11	1659	23	0.000278	10.909419	11
12	372	19	0.000051	9.075080	13
13	1420	22	0.000293	9.659368	14
14	465	19	0.000066	8.212312	13
15	2130	22	0.000335	9.635014	10
16	794	20	0.000123	9.229228	10
17	71	21	0.000010	12.972973	19
18	456	22	0.000068	11.548649	18
19	368	22	0.000055	10.911184	16
20	27	16	0.000004	9.636364	14
21	76	16	0.000011	8.563492	10
22	2805	25	0.000423	11.128685	15
23	805	22	0.000123	10.211965	14
24	1666	23	0.000255	10.285560	13
25	2470	25	0.000377	10.345441	15
26	1294	21	0.000196	9.684137	15

Table 6: IDA* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
27	100	13	0.000016	6.928177	7
28	1609	23	0.000243	11.262020	13
29	225	24	0.000033	14.002778	16
30	1820	24	0.000280	11.523000	14
31	129	16	0.000020	9.481651	12
32	37	18	0.000005	10.245902	16
33	1500	23	0.000231	10.929972	13
34	4075	26	0.000629	11.365988	14
35	5873	27	0.000886	12.113549	17
36	487	22	0.000074	11.715174	14
37	5296	26	0.000817	11.096193	14
38	2349	22	0.000335	9.770907	12
39	5296	25	0.000741	10.857709	13
40	125	14	0.000032	7.921659	8
41	3436	25	0.000491	11.446203	19
42	1193	21	0.000180	9.671308	11
43	9239	26	0.001520	10.244679	16
44	2256	24	0.000337	10.434656	14
45	626	24	0.000083	13.157214	20
46	92	18	0.000028	10.118421	14
47	1145	23	0.000198	11.042925	15
48	1201	21	0.000183	10.725062	11
49	5557	28	0.000783	12.401425	16
50	730	20	0.000104	9.972016	12
51	9355	27	0.001342	10.635501	15
52	661	19	0.000094	8.786607	11
53	2195	26	0.000306	12.091817	16
54	257	18	0.000037	9.306265	12
55	2440	24	0.000347	10.669042	16
56	339	18	0.000051	8.484483	8
57	2316	24	0.000328	10.876104	12
58	272	23	0.000037	13.210162	17
59	3401	24	0.000506	10.829942	16
60	9544	27	0.001330	11.501881	19
61	8845	25	0.001269	9.669585	13
62	360	21	0.000050	10.371859	15
63	103	22	0.000014	13.398773	18
64	11359	28	0.001584	11.551143	14
65	406	20	0.000059	9.686854	16
66	8302	27	0.001221	11.225802	17
67	6762	27	0.000941	11.858743	15
68	2421	25	0.000335	11.361035	19
69	3944	24	0.000588	9.623692	10
70	899	24	0.000123	11.516892	16
71	868	22	0.000122	10.504486	16
72	14476	27	0.002005	10.902896	15
73	3141	26	0.000441	11.723931	18
74	8979	25	0.001327	9.914988	11
75	1631	26	0.000269	11.602260	20

Table 6: IDA* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
76	413	20	0.000055	9.506589	14
77	8936	26	0.001380	10.443036	16
78	166	16	0.000024	8.810345	10
79	7986	27	0.001147	11.354104	17
80	490	19	0.000067	8.291415	11
81	324	25	0.000041	14.170635	21
82	56	15	0.000007	8.010870	11
83	83	17	0.000011	9.753623	13
84	186	15	0.000026	7.257862	7
85	2713	25	0.000433	11.475498	17
86	206	17	0.000029	9.260623	11
87	5691	27	0.000823	11.795929	17
88	1408	20	0.000253	8.118487	8
89	573	20	0.000078	9.433022	12
90	2764	27	0.000430	11.330114	21
91	2791	26	0.000397	10.976108	18
92	2006	22	0.000304	9.715939	12
93	1238	22	0.000182	10.963160	14
94	1016	22	0.000151	10.976387	12
95	244	19	0.000046	9.935802	17
96	722	20	0.000117	9.396411	8
97	2213	21	0.000377	8.091271	9
98	2265	24	0.000339	11.277571	14
99	3613	25	0.000516	10.676061	13

Table 7: A* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	835	22	0.001664	9.082437	12
1	1024	23	0.002011	10.066706	15
2	72	17	0.000133	9.150794	13
3	953	25	0.001809	11.678322	17
4	91	16	0.000169	6.725610	10
5	652	22	0.001234	9.371818	12
6	350	22	0.000682	10.439523	14
7	612	21	0.001397	9.811357	11
8	232	17	0.000443	7.862500	7
9	177	21	0.000322	10.827119	13
10	511	23	0.000937	9.976553	15
11	1037	23	0.001988	10.386350	11
12	197	19	0.000362	8.389381	13
13	697	22	0.001285	9.280956	14
14	239	19	0.000431	7.905707	13
15	1622	22	0.002087	9.725907	10
16	315	20	0.000389	8.873394	10
17	30	21	0.000035	10.326923	19
18	373	22	0.000465	10.561798	18
19	550	22	0.000700	10.553030	16

Table 7: A* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
20	34	16	0.000041	8.229508	14
21	118	16	0.000135	8.475490	10
22	1449	25	0.001820	10.864540	15
23	873	22	0.000813	10.083902	14
24	808	23	0.000745	9.784357	13
25	1167	25	0.001100	10.505647	15
26	308	21	0.000285	9.424762	15
27	41	13	0.000045	6.080000	7
28	533	23	0.000497	11.037079	13
29	346	24	0.000339	11.500000	16
30	929	24	0.000893	10.288535	14
31	117	16	0.000105	8.699507	12
32	26	18	0.000023	9.020833	16
33	666	23	0.000623	10.135993	13
34	1734	26	0.001367	10.785144	14
35	1824	27	0.001462	12.141566	17
36	231	22	0.000168	11.418182	14
37	1807	26	0.001440	10.707728	14
38	655	22	0.000505	9.455455	12
39	2298	25	0.001884	10.224357	13
40	76	14	0.000046	7.451852	8
41	786	25	0.000489	11.269619	19
42	609	21	0.000407	9.228036	11
43	2231	26	0.001543	10.650431	16
44	1757	24	0.001159	9.851374	14
45	299	24	0.000185	11.503006	20
46	41	18	0.000029	9.959459	14
47	543	23	0.000338	10.689503	15
48	438	21	0.000282	10.062078	11
49	2864	28	0.001912	12.255917	16
50	221	20	0.000135	9.784974	12
51	2849	27	0.001604	11.071413	15
52	179	19	0.000095	8.827922	11
53	1075	26	0.000643	11.579865	16
54	177	18	0.000109	8.581169	12
55	752	24	0.000415	10.091700	16
56	182	18	0.000110	8.186495	8
57	1355	24	0.000741	10.085323	12
58	205	23	0.000104	11.060345	17
59	712	24	0.000377	10.585833	16
60	1421	27	0.000766	12.022786	19
61	2186	25	0.001160	9.916120	13
62	168	21	0.000087	10.681818	15
63	39	22	0.000019	12.014706	18
64	4038	28	0.002152	11.154132	14
65	90	20	0.000039	9.616883	16
66	2905	27	0.001370	10.862562	17
67	2301	27	0.001091	11.877161	15
68	698	25	0.000393	10.871050	19

Table 7: A* results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
69	2190	24	0.001057	9.485266	10
70	936	24	0.000430	11.838397	16
71	238	22	0.000108	9.926290	16
72	3038	27	0.001553	10.956962	15
73	1632	26	0.000744	10.818418	18
74	4306	25	0.001878	10.300277	11
75	739	26	0.000302	11.903624	20
76	382	20	0.000161	9.364353	14
77	2084	26	0.000890	10.702233	16
78	99	16	0.000041	7.528409	10
79	2764	27	0.001189	11.431719	17
80	428	19	0.000171	8.660714	11
81	715	25	0.000293	13.085198	21
82	68	15	0.000035	7.352459	11
83	34	17	0.000012	9.274194	13
84	121	15	0.000046	7.377990	7
85	993	25	0.000408	10.744873	17
86	163	17	0.000066	8.925532	11
87	2389	27	0.001016	11.712006	17
88	369	20	0.000147	8.338118	8
89	356	20	0.000148	8.593023	12
90	1461	27	0.000619	11.142620	21
91	854	26	0.000465	11.481377	18
92	730	22	0.000286	9.600164	12
93	257	22	0.000104	11.321759	14
94	414	22	0.000168	9.852482	12
95	181	19	0.000076	9.896440	17
96	336	20	0.000135	9.108997	8
97	707	21	0.000286	8.577593	9
98	1467	24	0.000590	11.038823	14
99	1319	25	0.000582	10.782807	13

Table 8: A* with linear conflicts heuristic results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	475	22	0.000467	10.135970	12
1	318	23	0.000290	11.459048	15
2	43	17	0.000045	9.512500	17
3	484	25	0.000432	13.165217	17
4	44	16	0.000052	7.573171	10
5	247	22	0.000228	11.304455	16
6	208	22	0.000194	10.583569	14
7	310	21	0.000336	10.405959	11
8	125	17	0.000141	8.328638	11
9	118	21	0.000119	11.064677	13
10	260	23	0.000267	11.245413	17
11	445	23	0.000441	11.627049	15
12	132	19	0.000148	8.865801	13

Table 8: A* with linear conflicts heuristic results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
13	227	22	0.000229	10.868984	16
14	135	19	0.000143	9.013043	13
15	478	22	0.000482	11.747793	18
16	183	20	0.000193	9.605769	10
17	24	21	0.000024	10.261905	19
18	161	22	0.000157	11.677778	20
19	219	22	0.000218	11.906417	16
20	23	16	0.000026	8.465116	14
21	58	16	0.000061	9.405941	10
22	407	25	0.000400	12.371387	17
23	405	22	0.000406	11.443001	14
24	472	23	0.000451	10.733417	13
25	656	25	0.000609	11.791099	15
26	154	21	0.000125	10.576336	15
27	33	13	0.000030	6.216667	7
28	207	23	0.000161	12.099707	19
29	160	24	0.000127	12.974453	18
30	445	24	0.000360	11.341238	16
31	46	16	0.000037	8.426829	14
32	24	18	0.000021	8.888889	16
33	229	23	0.000188	12.184896	15
34	660	26	0.000508	12.798701	18
35	857	27	0.000656	14.045944	17
36	93	22	0.000083	12.229299	16
37	572	26	0.000446	13.525696	16
38	345	22	0.000293	10.691638	12
39	952	25	0.000795	11.374381	15
40	39	14	0.000036	7.753425	10
41	293	25	0.000215	13.932632	19
42	206	21	0.000169	10.412979	13
43	1068	26	0.000865	11.989456	16
44	700	24	0.000574	10.886033	16
45	174	24	0.000134	12.300341	20
46	29	18	0.000025	9.433962	14
47	282	23	0.000235	11.451143	17
48	173	21	0.000143	10.295918	15
49	1528	28	0.001178	13.774721	18
50	138	20	0.000111	10.448718	14
51	1190	27	0.000947	12.991201	19
52	131	19	0.000113	9.375000	11
53	567	26	0.000442	12.669133	18
54	108	18	0.000091	8.983871	12
55	464	24	0.000389	10.866083	18
56	107	18	0.000094	9.048387	10
57	592	24	0.000497	11.178465	16
58	51	23	0.000040	13.306818	19
59	404	24	0.000329	11.623358	16
60	704	27	0.000531	13.854561	21
61	1244	25	0.001067	10.857831	13

Table 8: A* with linear conflicts heuristic results for the 8-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
62	126	21	0.000102	11.167421	15
63	37	22	0.000029	11.815385	18
64	2238	28	0.001827	12.211564	22
65	46	20	0.000036	10.705128	16
66	1468	27	0.001189	12.234723	17
67	1221	27	0.000984	13.189375	19
68	493	25	0.000398	11.507729	19
69	958	24	0.000818	10.667508	12
70	242	24	0.000191	13.423267	16
71	146	22	0.000122	11.118367	16
72	1466	27	0.001182	12.210157	17
73	775	26	0.000622	12.256231	18
74	2069	25	0.001732	11.340628	15
75	396	26	0.000298	13.253499	20
76	221	20	0.000189	10.303523	14
77	1011	26	0.000816	12.014775	16
78	62	16	0.000057	7.895652	10
79	1431	27	0.001331	12.689626	17
80	215	19	0.000223	9.500000	11
81	457	25	0.000341	14.342742	21
82	26	15	0.000023	8.520833	11
83	27	17	0.000023	9.100000	13
84	63	15	0.000056	8.280374	9
85	463	25	0.000364	12.444588	17
86	84	17	0.000093	9.718121	13
87	1173	27	0.001024	13.025038	17
88	169	20	0.000143	9.943060	12
89	212	20	0.000181	9.719008	14
90	788	27	0.000649	12.506033	21
91	542	26	0.000425	12.382675	18
92	315	22	0.000261	10.667946	14
93	157	22	0.000129	11.856604	14
94	236	22	0.000199	10.958333	16
95	135	19	0.000114	10.365546	17
96	143	20	0.000117	10.599190	10
97	458	21	0.000431	9.233850	11
98	530	24	0.000431	11.676768	18
99	724	25	0.000603	11.931316	15

Table 9: A* results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	14028344.0	57.0	14.511616	27.186900	41.0
1	4415279.0	55.0	4.124220	23.394823	43.0
2	-	-	-	-	-
3	5253685.0	56.0	4.893337	26.620789	42.0
4	2021382.0	56.0	1.794775	28.153198	42.0
5	969356.0	52.0	0.834599	27.416488	36.0

Table 9: A* results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
6	7170013.0	52.0	7.190005	23.383736	30.0
7	2618503.0	50.0	2.372958	24.424523	32.0
8	313211.0	46.0	0.234823	21.503714	32.0
9	17805045.0	59.0	17.903045	27.734989	43.0
10	3525006.0	57.0	3.331819	28.750544	43.0
11	32334.0	45.0	0.019354	26.236208	35.0
12	831333.0	46.0	0.724653	19.507479	36.0
13	26088053.0	59.0	28.664199	28.499028	41.0
14	25792756.0	62.0	27.853632	31.454643	44.0
15	909219.0	42.0	0.823109	18.651622	24.0
16	-	-	-	-	-
17	1267588.0	55.0	1.103690	27.259433	43.0
18	154092.0	46.0	0.110455	22.987150	36.0
19	1995813.0	52.0	1.840264	24.572979	36.0
20	5668025.0	54.0	5.339204	26.403395	34.0
21	13338796.0	59.0	13.879372	28.247415	41.0
22	1398857.0	49.0	1.244144	24.277053	33.0
23	4691710.0	54.0	4.406190	26.312234	34.0
24	6227205.0	52.0	6.375423	24.700272	32.0
25	20161235.0	58.0	20.180084	27.585266	40.0
26	11278428.0	53.0	11.074760	25.459157	33.0
27	909442.0	52.0	0.790016	27.872710	36.0
28	3917549.0	54.0	3.741804	26.119066	38.0
29	215990.0	47.0	0.160137	24.840169	35.0
30	192534.0	50.0	0.139601	28.164810	38.0
31	17025056.0	59.0	17.105740	29.273736	43.0
32	18535767.0	60.0	18.399434	30.110062	42.0
33	2063117.0	52.0	1.908374	25.982463	36.0
34	4626661.0	55.0	4.296182	26.974809	39.0
35	2309697.0	52.0	2.107343	25.791491	36.0
36	20889986.0	58.0	21.072038	27.636038	40.0
37	827864.0	53.0	0.728854	28.710055	41.0
38	1118025.0	49.0	0.979180	24.280758	35.0
39	6540243.0	54.0	6.626199	26.380557	36.0
40	5727374.0	54.0	5.415196	26.121909	36.0
41	48447.0	42.0	0.030237	23.300039	30.0
42	7332466.0	64.0	7.013604	33.052826	48.0
43	2731989.0	50.0	2.512646	24.607501	32.0
44	539196.0	51.0	0.432881	27.316122	39.0
45	1430455.0	49.0	1.267093	22.917919	35.0
46	179689.0	47.0	0.134497	25.475309	35.0
47	119535.0	49.0	0.084243	25.168521	39.0
48	-	-	-	-	-
49	4399444.0	53.0	4.189249	25.099392	39.0
50	3031922.0	56.0	2.980969	28.074825	44.0
51	11641228.0	56.0	11.398982	26.803445	38.0
52	-	-	-	-	-
53	7221615.0	56.0	7.194406	27.820720	40.0
54	152123.0	41.0	0.107984	20.696139	29.0

Table 9: A* results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
55	-	-	-	-	-
56	484048.0	50.0	0.398043	26.117385	36.0
57	506538.0	51.0	0.406125	25.545294	37.0
58	-	-	-	-	-
59	-	-	-	-	-
60	508957.0	45.0	0.423315	21.727099	31.0
61	1828847.0	57.0	1.686502	29.084569	43.0
62	21862510.0	56.0	22.525062	25.547554	40.0
63	5622335.0	51.0	5.340898	23.618298	31.0
64	920623.0	47.0	0.815182	22.622819	31.0
65	-	-	-	-	-
66	8430894.0	50.0	8.412781	23.921401	28.0
67	5471060.0	51.0	5.227797	25.117996	31.0
68	4579005.0	53.0	4.331187	25.342840	37.0
69	8179227.0	52.0	8.136434	24.664403	30.0
70	557862.0	44.0	0.453416	20.630122	30.0
71	-	-	-	-	-
72	188717.0	49.0	0.154051	23.873066	37.0
73	1373939.0	56.0	1.204893	28.771618	46.0
74	3440513.0	48.0	3.406849	22.060243	30.0
75	7622602.0	57.0	7.613999	27.729741	41.0
76	2184110.0	54.0	1.985791	27.597980	34.0
77	619299.0	53.0	0.501606	27.380697	41.0
78	68633.0	42.0	0.044895	21.781103	28.0
79	3951356.0	57.0	3.787402	26.913583	43.0
80	744209.0	53.0	0.657967	28.051743	39.0
81	-	-	-	-	-
82	1900358.0	49.0	1.795206	24.127775	31.0
83	10972804.0	55.0	10.629059	26.658957	37.0
84	158355.0	44.0	0.118864	20.888586	32.0
85	195222.0	45.0	0.145773	22.290463	35.0
86	4681483.0	52.0	4.401522	25.962040	34.0
87	-	-	-	-	-
88	3559259.0	54.0	3.458699	26.461863	38.0
89	820923.0	50.0	0.721287	26.079187	36.0
90	18150875.0	57.0	18.212592	26.873361	41.0
91	-	-	-	-	-
92	438960.0	46.0	0.367072	20.965217	34.0
93	276325.0	53.0	0.201408	27.766560	45.0
94	751475.0	50.0	0.665775	25.063400	34.0
95	1263033.0	49.0	1.112401	22.922221	35.0
96	191601.0	44.0	0.141911	21.267765	32.0
97	9499694.0	54.0	9.422438	25.127289	34.0
98	5342754.0	57.0	4.969159	28.762484	39.0
99	4484857.0	54.0	4.230395	25.170228	38.0

Table 10: A* with linear conflicts heuristic results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
0	1008716.0	57.0	1.650546	29.299446	43.0
1	1092263.0	55.0	1.844149	25.562991	43.0
2	9187949.0	59.0	16.046418	29.813407	41.0
3	887333.0	56.0	1.448838	30.597423	42.0
4	526731.0	56.0	0.819769	29.274386	44.0
5	156707.0	52.0	0.228973	29.795242	40.0
6	2940235.0	52.0	5.467507	24.690952	30.0
7	538097.0	50.0	0.889736	27.115030	36.0
8	24752.0	46.0	0.038044	25.152667	36.0
9	5036413.0	59.0	8.431558	30.553457	45.0
10	729647.0	57.0	1.191329	31.379149	45.0
11	11459.0	45.0	0.016676	28.116675	35.0
12	80845.0	46.0	0.122729	23.221488	38.0
13	1309372.0	59.0	1.985366	33.640993	43.0
14	7767470.0	62.0	13.014778	33.542406	46.0
15	310381.0	42.0	0.515667	22.162580	26.0
16	13564375.0	66.0	23.582254	34.035984	46.0
17	284346.0	55.0	0.422590	30.668205	43.0
18	49942.0	46.0	0.077007	24.339306	38.0
19	548667.0	52.0	0.904230	26.105480	36.0
20	1206756.0	54.0	2.067972	29.567055	36.0
21	1996328.0	59.0	3.244696	30.741612	45.0
22	82805.0	49.0	0.128945	26.723677	37.0
23	1640567.0	54.0	2.950206	27.214693	40.0
24	1108401.0	52.0	1.940663	26.837777	36.0
25	5268707.0	58.0	8.787885	29.596255	44.0
26	1343999.0	53.0	2.258928	28.446785	37.0
27	271923.0	52.0	0.417983	30.022161	36.0
28	455454.0	54.0	0.718904	30.032340	40.0
29	67363.0	47.0	0.103713	27.604188	35.0
30	47988.0	50.0	0.067841	31.167088	40.0
31	4794120.0	59.0	8.014120	30.640356	45.0
32	2117852.0	60.0	3.374324	34.089841	42.0
33	182966.0	52.0	0.275471	29.470206	42.0
34	1037787.0	55.0	1.665640	29.687659	39.0
35	289175.0	52.0	0.432257	29.997768	38.0
36	2658675.0	58.0	4.337260	32.368176	42.0
37	172867.0	53.0	0.243890	30.873189	43.0
38	305924.0	49.0	0.502954	26.756767	35.0
39	1610986.0	54.0	2.856209	29.004803	38.0
40	1491664.0	54.0	2.635698	27.856531	42.0
41	13365.0	42.0	0.019005	25.333519	32.0
42	1317019.0	64.0	1.914664	36.175455	56.0
43	166115.0	50.0	0.241281	28.887483	38.0
44	81478.0	51.0	0.114973	30.895656	39.0
45	125725.0	49.0	0.176320	27.248962	41.0
46	32711.0	47.0	0.045345	28.211681	35.0
47	15705.0	49.0	0.020663	28.687829	39.0

Table 10: A* with linear conflicts heuristic results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
48	12115883.0	59.0	21.334278	31.361820	37.0
49	398080.0	53.0	0.663475	27.679156	41.0
50	604705.0	56.0	0.931614	31.393673	44.0
51	1729025.0	56.0	2.874196	30.155368	42.0
52	12119505.0	64.0	21.482982	32.964799	50.0
53	1157523.0	56.0	1.903755	30.481629	42.0
54	23487.0	41.0	0.037641	22.606199	31.0
55	9930761.0	55.0	18.042428	27.904238	33.0
56	216962.0	50.0	0.334195	27.400629	36.0
57	133032.0	51.0	0.205127	26.922856	41.0
58	4754047.0	57.0	8.198372	30.482821	39.0
59	-	-	-	-	-
60	57126.0	45.0	0.082853	24.191563	35.0
61	681646.0	57.0	1.092744	30.446158	45.0
62	718361.0	56.0	1.228913	30.201772	44.0
63	1246819.0	51.0	2.277783	25.415214	35.0
64	154359.0	47.0	0.235213	24.633122	35.0
65	-	-	-	-	-
66	1575028.0	50.0	2.973330	25.809011	32.0
67	807460.0	51.0	1.443339	27.937001	35.0
68	289464.0	53.0	0.440129	30.274768	39.0
69	2785541.0	52.0	4.963534	26.894679	32.0
70	37611.0	44.0	0.058518	23.839078	34.0
71	3842300.0	56.0	6.689455	29.751017	40.0
72	68792.0	49.0	0.097426	26.755062	39.0
73	242302.0	56.0	0.351539	31.554884	46.0
74	685897.0	48.0	1.220427	24.261016	34.0
75	2010446.0	57.0	3.420868	30.158937	43.0
76	653815.0	54.0	1.074857	29.653775	38.0
77	164940.0	53.0	0.236397	29.701219	41.0
78	23010.0	42.0	0.034446	22.743868	30.0
79	2195739.0	57.0	3.818732	27.747242	43.0
80	102712.0	53.0	0.146165	31.654942	41.0
81	14781502.0	62.0	25.999458	31.137335	44.0
82	99928.0	49.0	0.153610	27.451894	37.0
83	2404525.0	55.0	4.058588	28.528995	43.0
84	57309.0	44.0	0.090196	22.787388	32.0
85	24768.0	45.0	0.034794	25.218728	37.0
86	867794.0	52.0	1.400675	28.664328	36.0
87	-	-	-	-	-
88	745312.0	54.0	1.291613	28.205200	40.0
89	93159.0	50.0	0.132566	28.938132	40.0
90	3999739.0	57.0	6.603807	29.718498	41.0
91	5369187.0	57.0	9.030130	29.616615	39.0
92	88425.0	46.0	0.138819	23.357158	34.0
93	73629.0	53.0	0.102134	29.179973	45.0
94	161874.0	50.0	0.243701	27.501468	36.0
95	135902.0	49.0	0.203034	26.735106	37.0
96	34716.0	44.0	0.051857	23.488393	34.0

Table 10: A* with linear conflicts heuristic results for the 15-Puzzle

Instance	Expanded Nodes	Solution Cost	Total Time (s)	Mean Heuristic	Initial Heuristic
97	2570910.0	54.0	4.556437	27.286979	34.0
98	1514416.0	57.0	2.602551	30.120961	39.0
99	1404153.0	54.0	2.471428	26.770716	40.0