1. Let $\boldsymbol{r} = [r_1 \quad r_2 \quad \cdots \quad r_m]$ and $\boldsymbol{a}_s = \begin{bmatrix} a_{1s} \\ a_{2s} \\ \vdots \\ a_{ms} \end{bmatrix}$, thus $r_s = \boldsymbol{r}^\top \boldsymbol{a}_s$

   Reduced cost:

$$r_s - \boldsymbol{\alpha}^\top \boldsymbol{a}_s - \beta = (\boldsymbol{r} - \boldsymbol{\alpha})^\top \boldsymbol{a}_s - \beta$$

   Assume that you are using column generation for solving the problem, so we have a restricted master problem with fewer columns.

   We solve this RMP, then we want to check if the solution can be further improved:

$$(\boldsymbol{r} - \boldsymbol{\alpha})^\top \boldsymbol{a}_s - \beta \le 0$$

   If not, we need to add a new pattern to the problem, but we should add a feasible pattern. So, the pricing problem would be as follows:

$$\max_{\boldsymbol{a}_s} (\boldsymbol{r} - \boldsymbol{\alpha})^\top \boldsymbol{a}_s - \beta$$

   Subject to

$$\boldsymbol{c}^\top \boldsymbol{a}_s \le \frac{1}{2} \boldsymbol{r}^\top \boldsymbol{a}_s$$

$$\boldsymbol{a}_s \ge \boldsymbol{0}$$

   Add new columns:

$$\begin{bmatrix} a_{1s} \\ a_{2s} \\ \vdots \\ 1 \end{bmatrix}$$