First-arrival traveltime tomography based on the adjoint-state method

September 16, 2021

Outline

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Motivation

Gradient of the misfit function

- Misfit function for a single-shot

$$J(c) = \frac{1}{2} \int_{\partial\Omega} d\boldsymbol{r} |T(c, \boldsymbol{r}) - T_{obs}(\boldsymbol{r})|^2 \tag{1}$$

- Gradient method

$$c_{n+1} = c_n - \alpha_n \nabla J(c_n), \tag{2}$$

where α is a positive scalar.

Gradient of the misfit function

- Extended misfit function

$$L(c,t,\lambda) = \frac{1}{2}\int_{\partial\Omega} d\boldsymbol{r}|t(\boldsymbol{r}) - T_{obs}(\boldsymbol{r})|^2 - \frac{1}{2}\int_{\Omega} d\boldsymbol{x}\lambda(\boldsymbol{x})\left(|\nabla t(\boldsymbol{x})|^2 - \frac{1}{c(\boldsymbol{x})^2}\right) \quad (3)$$

- for $t(\boldsymbol{x}) = T(\boldsymbol{x})$

$$L(c,T,\lambda) = J(c) \quad (4)$$

Outline

Fast-sweeping method

- Eikonal equation

$$|\nabla T(\boldsymbol{x})|^2 = \frac{1}{c^2(\boldsymbol{x})} \tag{5}$$

$$T(\boldsymbol{x}_s) = 0 \tag{6}$$

- Adjoint-state system

$$\Lambda(\boldsymbol{r})\boldsymbol{n}(\boldsymbol{r}) \cdot \nabla T(\boldsymbol{r}) = T(\boldsymbol{r}) - T_{obs}(\boldsymbol{r}) \tag{7}$$

$$\nabla \cdot \Lambda(\boldsymbol{x}) \nabla T(\boldsymbol{x}) = 0 \tag{8}$$

Fast-sweeping method

- Godunov upwind FD scheme

$$\left[\frac{(T(\boldsymbol{x}) - T(\boldsymbol{x})^{xmin})^+}{\triangle x}\right]^2 + \left[\frac{(T(\boldsymbol{x}) - T(\boldsymbol{x})^{zmin})^+}{\triangle z}\right]^2 = \frac{1}{c^2(\boldsymbol{x})}$$

Fast-sweeping method

- Initialize $T(\boldsymbol{x}_s) = 0$ and assing very large positive values to the rest of the grid points.
- Update grid points with Gauss-Seidel and keep the smallest value between the old and the calculated. $min(T_{old}, T^*)$
- Check the convergence $||T^{n+1} - T^n||_{L1} \leq \varepsilon$ for $\varepsilon > 0$

Fast-sweeping method

- Adjoint-state

$$\frac{\partial}{\partial x}(a\lambda) + \frac{\partial}{\partial z}(b\lambda) = 0 \tag{9}$$

where $a = \partial t(x,z)/\partial x$ and $b = \partial t(x,z)/\partial z$

- a and b values

$$a_{i+\frac{1}{2},j} = \frac{t_{i+1,j} - t_{i,j}}{\triangle x}, \qquad a_{i-\frac{1}{2},j} = \frac{t_{i,j} - t_{i-1,j}}{\triangle x} \tag{10}$$

$$b_{i,j+\frac{1}{2}} = \frac{t_{i,j+1} - t_{i,j}}{\triangle z}, \qquad b_{i,j-\frac{1}{2}} = \frac{t_{i,j} - t_{i,j-1}}{\triangle z} \tag{11}$$

- Introducing the notations:

$$a^{\pm}_{i+\frac{1}{2},j} = \frac{a_{i+\frac{1}{2},j} \pm |a_{i+\frac{1}{2},j}|}{2} \tag{12}$$

$$a^{\pm}_{i-\frac{1}{2},j} = \frac{a_{i-\frac{1}{2},j} \pm |a_{i-\frac{1}{2},j}|}{2} \tag{13}$$

$$b^{\pm}_{i,j+\frac{1}{2}} = \frac{b_{i,j+\frac{1}{2}} \pm |b_{i,j+\frac{1}{2}}|}{2} \tag{14}$$

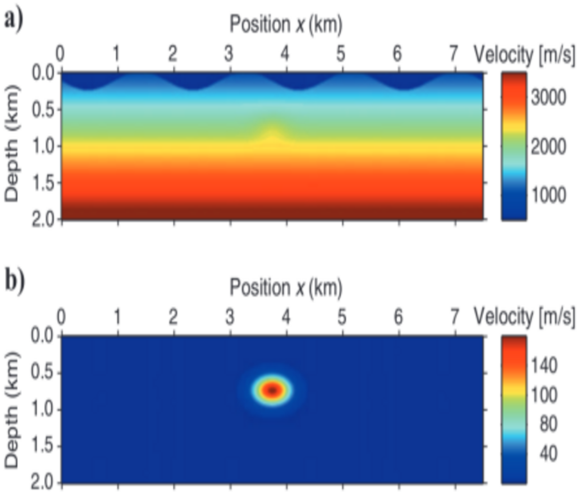
$$b^{\pm}_{i,j-\frac{1}{2}} = \frac{b_{i,j-\frac{1}{2}} \pm |b_{i,j-\frac{1}{2}}|}{2} \tag{15}$$

Outline

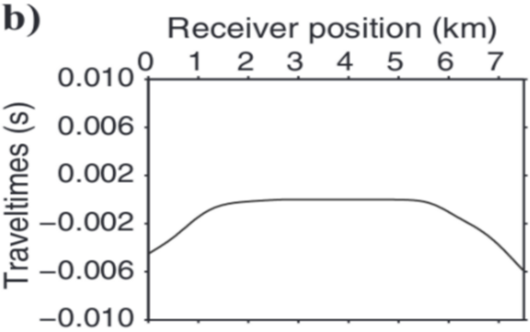
Gradient calculation

- Constant vertical-gradient velocity model with a velocity anomaly.
- 2km in depth and 7.5km laterally. 12.5m by 12.5 grid size.
- 600 sources and 600 receivers for each source.

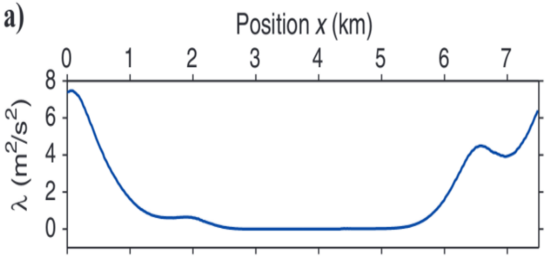
Fast Sweeping Method



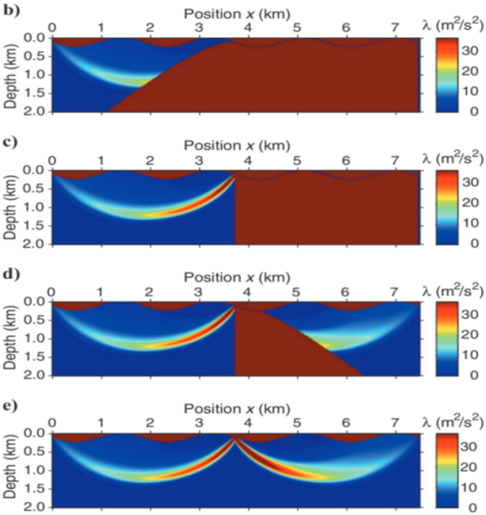
Fast Sweeping Method



Fast Sweeping Method



Fast Sweeping Method



Fast Sweeping Method