First-arrival traveltime tomography based on the adjoint-state method

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Outline

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Motivation

Gradient of the misfit function

- Misfit function for a single-shot

$$J(c) = \frac{1}{2} \int_{\partial\Omega} d\boldsymbol{r} |T(c, \boldsymbol{r}) - T_{obs}(\boldsymbol{r})|^2 \tag{1}$$

- Gradient method

$$c_{n+1} = c_n - \alpha_n \nabla J(c_n), \tag{2}$$

where α is a positive scalar.

Gradient of the misfit function

- Extended misfit function

$$
\begin{aligned}
L(c,t,\lambda) = \quad & \frac{1}{2}\int_{\partial\Omega} d\boldsymbol{r}|t(\boldsymbol{r}) - T_{obs}(\boldsymbol{r})|^2 \\
- \quad & \frac{1}{2}\int_{\Omega} d\boldsymbol{x}\lambda(\boldsymbol{x})\left(|\nabla t(\boldsymbol{x})|^2 - \frac{1}{c(\boldsymbol{x})^2}\right)
\end{aligned} \tag{3}
$$

- for $t(\boldsymbol{x}) = T(\boldsymbol{x})$

$$
L(c,T,\lambda) = J(c) \tag{4}
$$

$$\frac{\partial L}{\partial c} = -\int_{\Omega} dx \frac{\lambda(\boldsymbol{x})}{c^3(\boldsymbol{x})} = \frac{\partial J}{\partial c} \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = 0 \tag{6}$$

$$\frac{\partial L}{\partial t} = 0 \tag{7}$$

$$\frac{\partial L}{\partial t} = \int_{\partial\Omega} d\boldsymbol{r}(t(\boldsymbol{r}) - T_{obs}(\boldsymbol{r})) - \int_{\Omega} d\boldsymbol{x}\lambda(\boldsymbol{x})\nabla t(\boldsymbol{x}) \cdot \frac{\partial \nabla t}{\partial t} \tag{8}$$

$$\begin{aligned}\frac{\partial L}{\partial t} \quad &= \quad \int_{\partial\Omega} d\boldsymbol{r}(t(\boldsymbol{r}) - T_{obs}(\boldsymbol{r}) - \lambda \boldsymbol{n} \cdot \nabla t) \\ &\quad + \int_{\Omega} d\boldsymbol{x}\nabla \cdot \lambda(\boldsymbol{x})\nabla T(\boldsymbol{x})\end{aligned} \tag{9}$$

$$\Delta(\boldsymbol{r})\boldsymbol{n}(\boldsymbol{r}) \cdot \nabla T(\boldsymbol{r}) = T(\boldsymbol{r}) - T_{obs}(\boldsymbol{r}) \tag{10}$$

$$\nabla \cdot \Delta(\boldsymbol{x})\nabla T(\boldsymbol{x}) = 0 \tag{11}$$