Lesson 2 Brief Analysis of Forward Kinematics

1. DH Parameter Introduction

The DH parameter is a mechanical arm mathematical model and coordinate system determination system that uses four parameters to express the position and angle relationship between two pairs of joint links. As we will see below, it artificially reduces two degrees of freedom by limiting the position of the origin and the direction of the X axis, so it only needs four parameters to define a coordinate system with six degrees of freedom.

The four parameters selected by DH have very clear physical meanings, as follows:

① link length : The length of the common normal between the axes of the two joints (Rotation axis of rotation joint, translation axis of translation joint)

② link twist: The angle at which the axis of one joint rotates around their common normal relative to the axis of the other joint

③ link offset: The common normal of one joint and the next joint and the distance between the common normal of one joint and the previous joint along this joint axis

④ joint angle: The common normal of one joint and the next joint and the angle of rotation around the joint axis with the common normal of the previous joint

The above definition is very complicated, but it will be much clearer when combined with the coordinate system.

First of all you should pay attention to the two most important "lines": the joint axis, and the common normal between the axis joint and the adjacent joint.

In the DH parameter system, we set axis as the z axis; common normal as the x axis, and the direction of the x axis is: from this joint to the next joint.

Of course, these two rules alone are not enough to completely determine the coordinate system of each joint. Let's talk about the steps to determine the coordinate system in detail below.

In applications such as the simulation of the robotic arm, we often adopt other methods to establish the coordinate system, but mastering the methods mentioned here is necessary for you to understand the mathematical expression of the robotic arm and understand our subsequent analysis.

The figure below shows two typical robot joints. Although such joints and links are not necessarily similar to the joints and links of any actual robot, they are very common and can easily represent any joint of the actual robot.
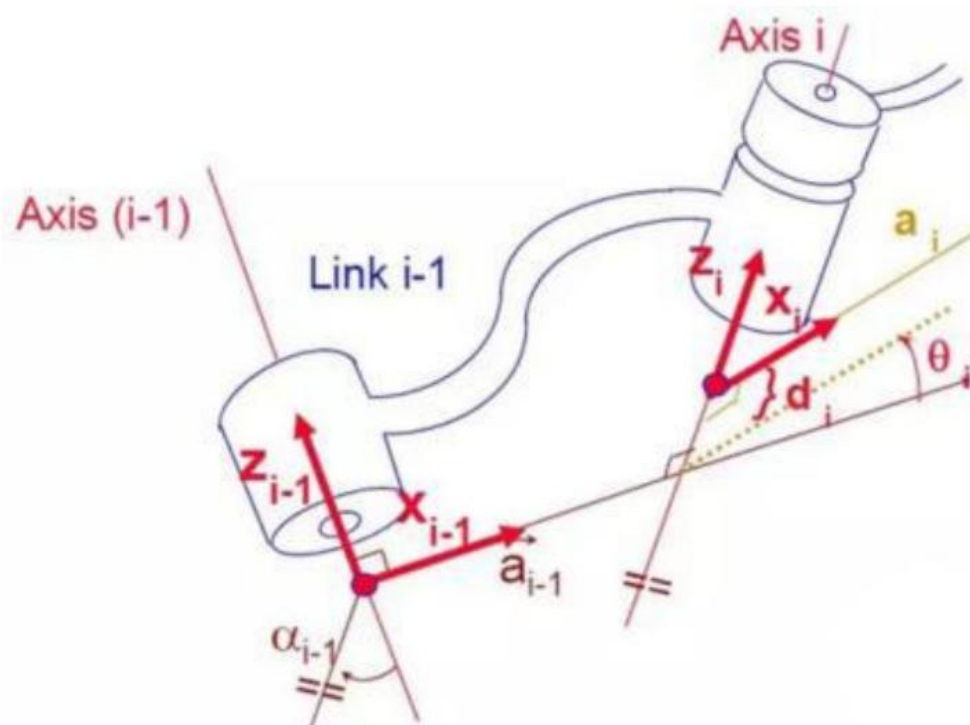
2.Determine the Coordinate System

To determine the coordinate system, there are generally the following steps:

In order to model the robot with DH notation, the first thing is to specify a local ground reference coordinate system for each joint, so a Z axis and an X axis must be specified for each joint.

Specify the Z axis. If the joint is rotating, the Z axis is in the direction of rotation according to the right-hand rule. The rotation angle around the Z axis is a variable of the joint; if the joint is a sliding joint, the Z axis is the direction of movement along a straight line. The link length d along the Z axis is the joint variable.

Specify the X axis.When the two joints are not parallel or intersect, the Z axis is usually a diagonal line, but there is always a common vertical line with the shortest distance, which is orthogonal to any two diagonal lines. Define the X axis of the local reference coordinate system in the direction of the common perpendicular. If an represents the common perpendicular between Zn1, the direction of Xn will be along an.

Of course there are special circumstances. When the Z axes of the two joints are parallel, there will be countless common perpendiculars. At this time, you can select the one that is collinear with the common perpendicular of the previous joint, which can simplify the model; if two joints intersect, there is no common perpendicular between them. In this case, the line perpendicular to the plane formed by the two axes can be defined as X Shaft can simplify the model.

After attaching the corresponding coordinate system to each joint, as shown in the following figure:
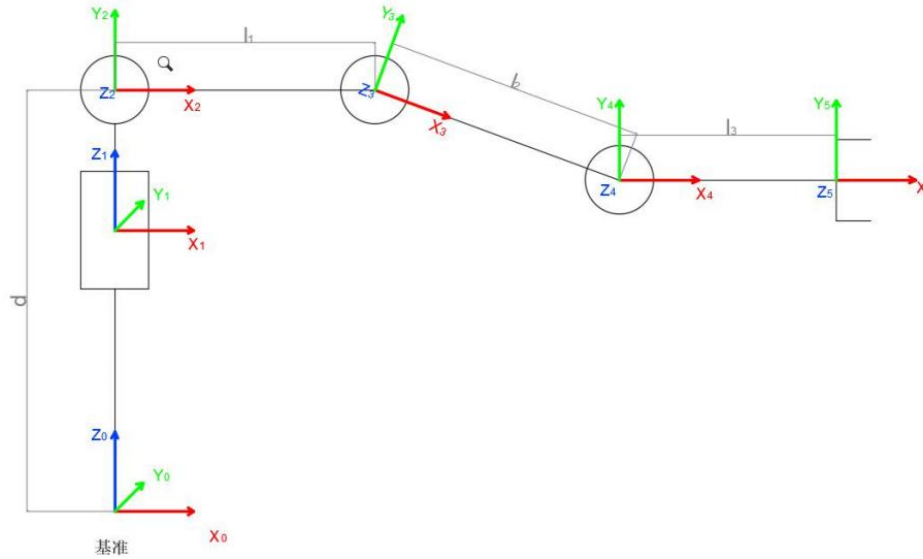
After determining the coordinate system, we can express the above four parameters in a more concise way:

link length $\alpha_{i-1:}$ the distance from Z_{i-1} to Z_i along X_{i-1}

link twist α_{i-1} : Z_i the angle of Z_i relative to Z_{i-1} to rotate around X_{i-1}

link offset $_id$: the distance from X_{i-1} to X_{i-1} along Z_i

joint angle θ_i : X_i relative to X_{i-1} around Z_i

Next we can write the DH parameter table of the robotic arm:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d	θ_0
2	90	0	0	θ_1
3	0	l_1	0	θ_2
4	0	l_2	0	θ_3
5	0	l_3	0	0

According to the formula:

$$
{}^{i-1}_{i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

We can calculate each joint at once, and finally get the positive kinematics formula of the robotic arm:

$$
{}^{0}_{1}T = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

After obtaining the rotation matrix of each joint, the coordinates of the end can be obtained according to the following formula:

$$
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$