Ci421b - Modeling asteroid population in the solar system
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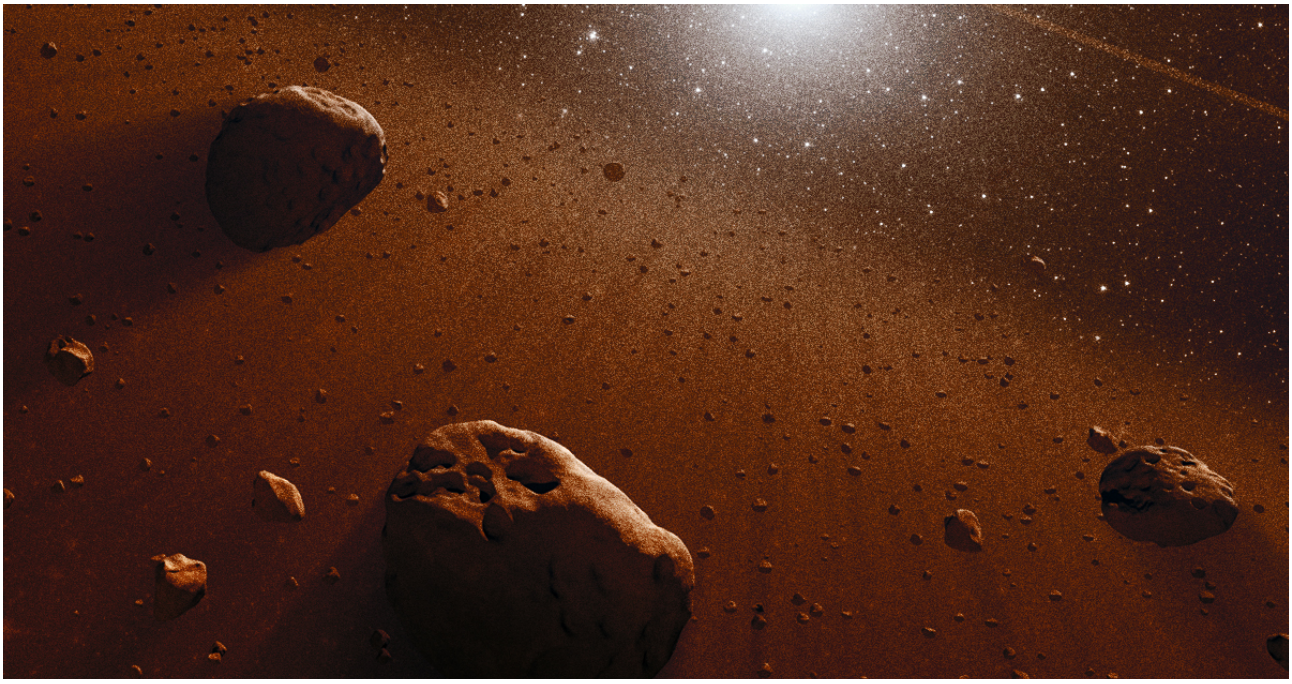
Dynamical simulation of the asteroid population



This project presents the numerical simulation of the asteroid 2007 VW_{266}.

1 Context

This paper will study the asteroid 2007 VW_{266}. It is based on the article A retrograde object near Jupiter's orbit. Here, the researchers study the asteroid 2007 VW_{266} because it holds a special place in our solar system. It is one of the rare object with a heliocentric retrograde orbit, and it is in 13/14 retrograde mean motion resonance with Jupiter. This orbit is only temporarily stable, and the study estimate that the asteroid will leave it in about 8000 years. This temporary stable state is due to an elliptic orbit with high inclination. In this paper, we try to reproduce the results found by the authors by performing a numerical simulation.

2 Aims

This paper has a few objectives. The first one is to model the asteroid, Jupiter and the Sun to simulate the trajectory of the asteroid for 10 000 years. We will compare those results with what the authors found in the study.

3 Resolution methods

3.1 Equations of motion with perturbation

Using Newton's Second law and Newton's law of universal gravitation, the motion of a celestial body around the Sun, its central body can be modelled. This equation is a second order ordinary differential equations, such as:

$$\begin{aligned}
\ddot{\mathbf{r}} &= \mathbf{a}_\odot + \sum_i \mathbf{a}_i + \mathbf{a}_e \qquad (1)\\
&= -\frac{Gm_\odot}{r^3}\mathbf{r} - \sum \frac{Gm_i}{\Delta_i^3}(\mathbf{r}-\mathbf{r}_i) - \frac{Gm}{r^3}\mathbf{r} - \sum \frac{Gm_i}{r_i^3}\mathbf{r}_i\\
\ddot{\mathbf{r}} &= -\frac{G(m_\odot + m)}{r^3}\mathbf{r} - G\sum_i m_i\left(\frac{\mathbf{r}-\mathbf{r}_i}{\Delta_i^3} + \frac{\mathbf{r}_i}{r_i^3}\right)
\end{aligned}$$

For the asteroid under study, only the perturbation of the Sun and of Jupiter are considered. Therefore, the equation of motion becomes:

$$\ddot{\mathbf{r}} = -\frac{G(m_\odot + m)}{r^3}\mathbf{r} - Gm_J\left(\frac{\mathbf{r}-\mathbf{r}_J}{\Delta_J^3} + \frac{\mathbf{r}_J}{r_J^3}\right) \qquad (2)$$

To solve Equation (2), the Runge-Kutta 4 method will be used.

3.2 Runge-Kutta 4 Method

The Runge-Kutta methods are a family of implicit and explicit iterative methods of resolution of ordinary differential equations (ODE). Because of its accuracy, the Runge-Kutta 4 (order 4) will be used in this project. It follows the subsequent steps:

$$\begin{cases} k_1 = & f(y_i, t_i) \\ k_2 = & f(y_i + \frac{h}{2}k_1, t_i + \frac{h}{2}) \\ k_3 = & f(y_i + \frac{h}{2}k_2, t_i + \frac{h}{2}) \\ k_4 = & f(y_i + \frac{h}{2}k_3, t_i + h) \\ y_{i+1} = & y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{cases}$$

where f is the studied function.
For this simulation, let $\mathbf{u} = (\mathbf{r}, \dot{\mathbf{r}})$, its derivative is $\dot{\mathbf{u}} = (\dot{\mathbf{r}}, \ddot{\mathbf{r}})$, such that the Runge-Kutta 4 method is applied to $f(\dot{\mathbf{u}}, t)$, where f is defined by Equation (2). This allows to get the position $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ at any time t, but also, the eccentricity, semi-major axis and inclination through the formulas:

$$\begin{cases} a = \left(\dfrac{2}{r} - \dfrac{\dot{\mathbf{r}}^2}{\mu}\right)^{-1} \\ \\ e = \left\| \dfrac{\dot{\mathbf{r}} \wedge (\mathbf{r} \wedge \dot{\mathbf{r}})}{\mu} - \dfrac{\mathbf{r}}{r} \right\| \\ \\ \cos i = k_z \quad \text{where} \quad (k_x, k_y, k_z) = \dfrac{r \wedge \dot{\mathbf{r}}}{r\dot{\mathbf{r}}} \end{cases}$$

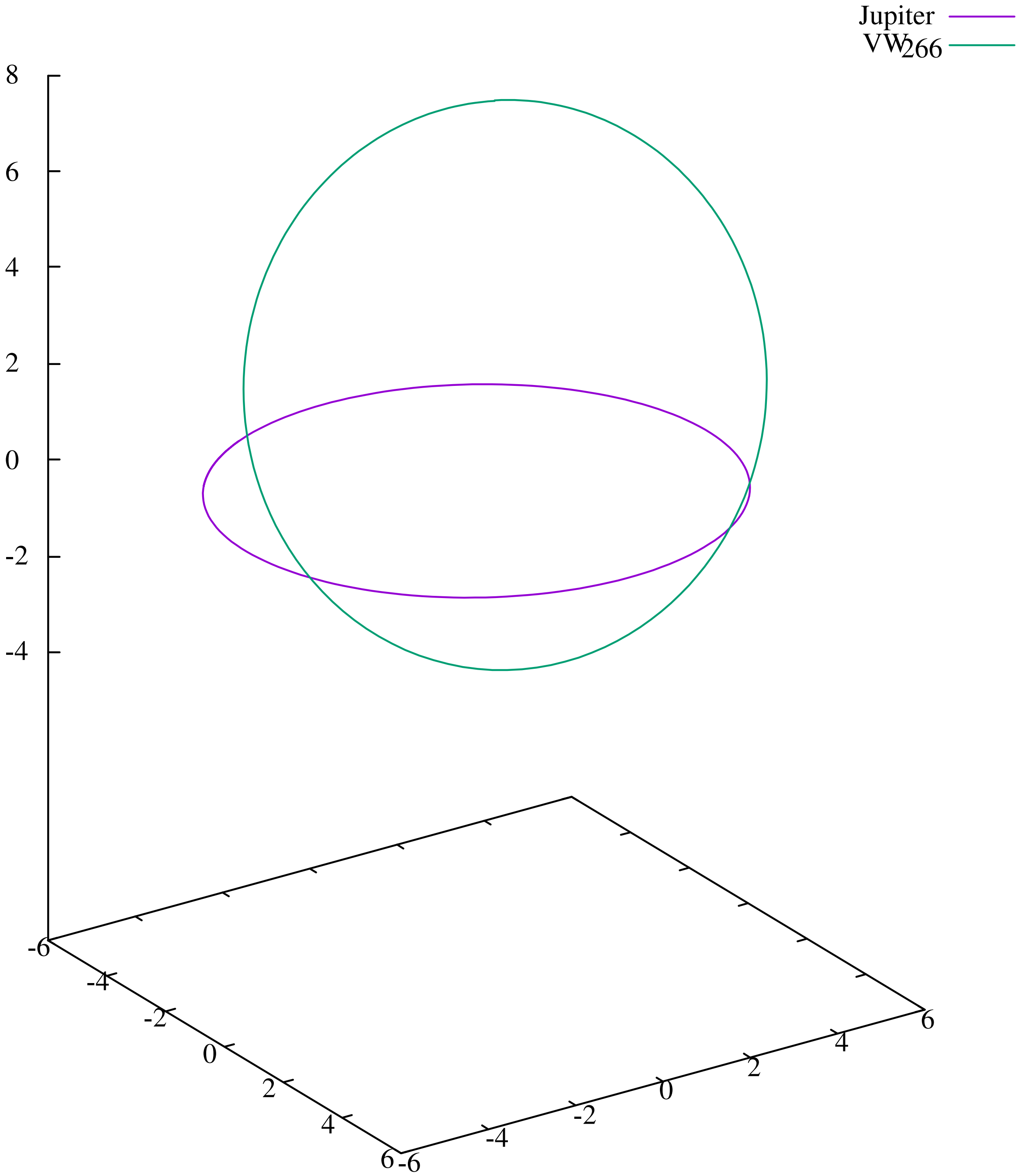
The solution obtained through the numerical resolution will be plotted and compared to the graphs provided in the article.

3.3 Choices of development

As the aim is to run the simulation for a duration of 10 000 years, we expect the calculations to be quite long if we want to use a precise time step. For this reason, we choose to develop a first prototype using python, but quickly swapped for C++ after validating the results obtained with the python code. Both code are available on GitHub at the following link: Asteroids. Only the C++ code was commented, since it was the one used for the final simulation.

4 Results

To ensure that we are heading in the right direction, we plotted the orbits of the asteroid and Jupiter. As we can see on this image, the result we get is close to what is expected with the reference paper.



Then for the results of the computation of three physical parameters over 10 000 years: the semi-major axis, the eccentricity and the inclination.

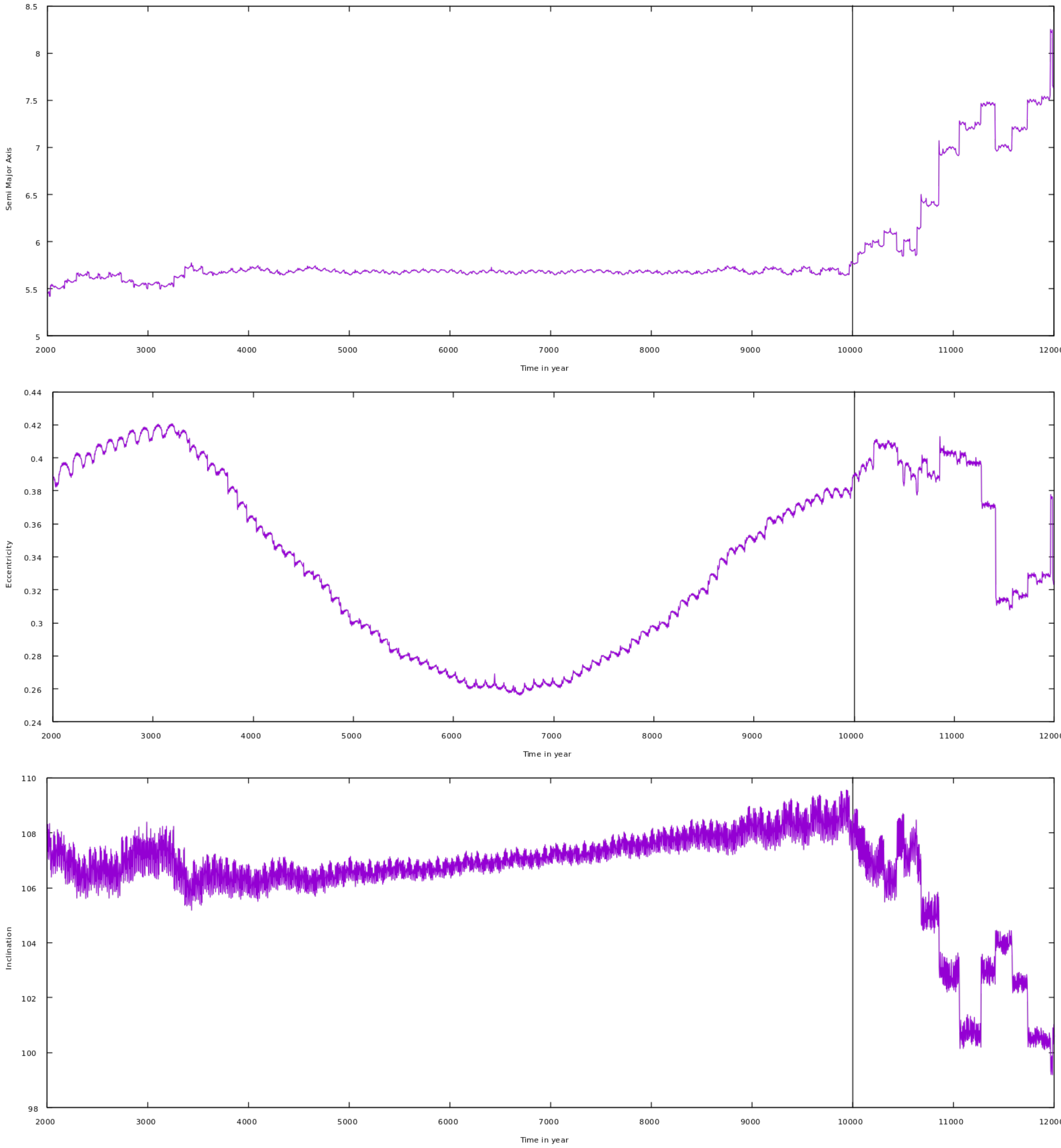


Figure 1: Physical results of the simulation

5 Discussion

As we saw in the previous section, the results found using our calculations seem to follow the general idea of the results found by the researchers. Obviously, a lot of difference can also be seen.

The good points of our simulation seem to be the fact that the system does not remain in a stable state as it was expected. After being stable for a few thousand years, the system explodes. This is seen by the big step taken at the same time by the three parameters of the asteroid 2007 VW_{266}. Its eccentricity, semi-major axis and inclination all have a discontinuity around the year 10 000. This date is also encouraging, since in the paper, the authors also found that the stability ended around the year 10 000.

The general look of the curves we found is also quite similar to the ones in the paper. The semi-major axis is staying in a straight line, the eccentricity is going down before moving up again, while the inclination is steadily augmenting.

However, our results need to be criticized. The eccentricity values do not correspond to the values in the paper. In the paper, the value is included between 0.35 and 0.45 (in the time interval we studied) while our eccentricity varies between 0.26 and 0.42. The inclination curve also looks quite noisy.

We can try to explain those differences with a few hypotheses. In the paper, the simulation took all the planets (and Pluto) of the solar system into account. Even if Jupiter and the Sun take up to 99.9% of the solar system mass and the fact that the high inclination of 2007 VW_{266} limits the influence of other planets, we can expect some small variations in the simulation. If these dissimilarities may not be seen on small timescale, adding these small perturbations for 10 000 years can make a big difference at the end. This can explain why the stable state ends not at the same time in our simulation.

Moreover, the researchers used a well known and proven integrator: *RADAU*. In opposition, we used a basic Runge-Kutta method without any safe check concerning the stability of the method. Even if we tested multiples time step, we cannot ensure that the integration is not the source of many numerical errors.