Estruturas de Dados II

Heap e Heap-Sort

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Sumário

Heap-Sort

O algoritmo de ordenação **heap-sort** foi inventado por J. Williams em 1964. Este algoritmo é considerado ótimo, com tempo de execução de $\mathcal{O}(n \log n)$ no pior caso. O algoritmo é **in-place** (requer um número constante de elementos armazenados fora do arranjo de entrada em qualquer instante) e **não estável** (não mantém a ordem de elementos iguais).

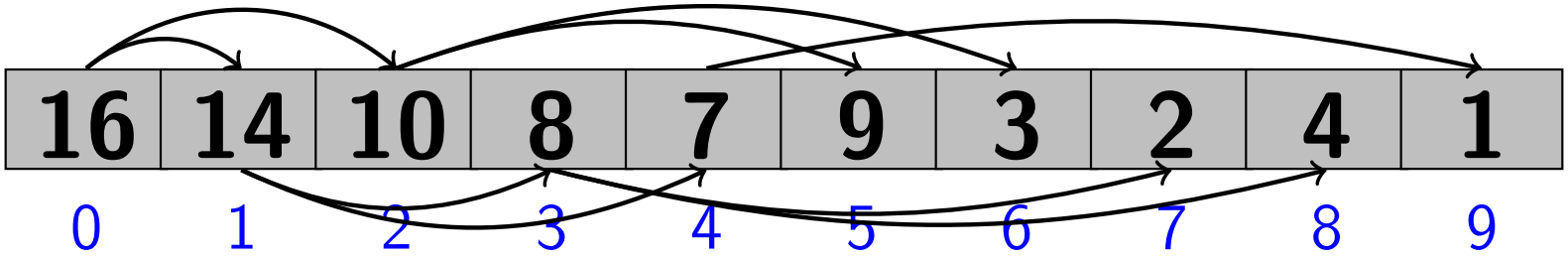
Heap-Sort

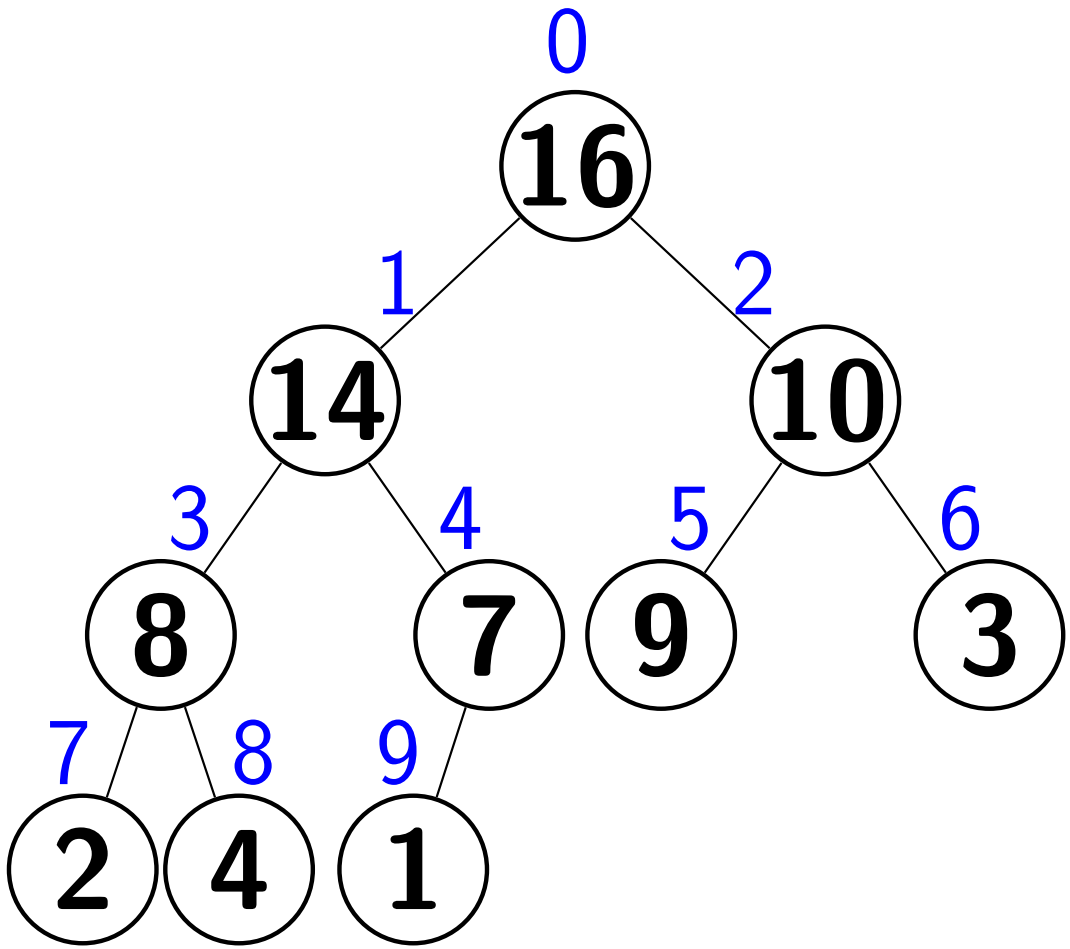
O algoritmo **heap-sort** introduziu outra técnica de projetos de algoritmos: o uso de um TAD conhecido como **heap** (ou "monte") para gerenciar os dados durante a execução do algoritmo. A estrutura do **heap** não é apenas importante para o heap-sort mas ela é essencial para criar uma eficiente **fila de prioridades**.

Heap

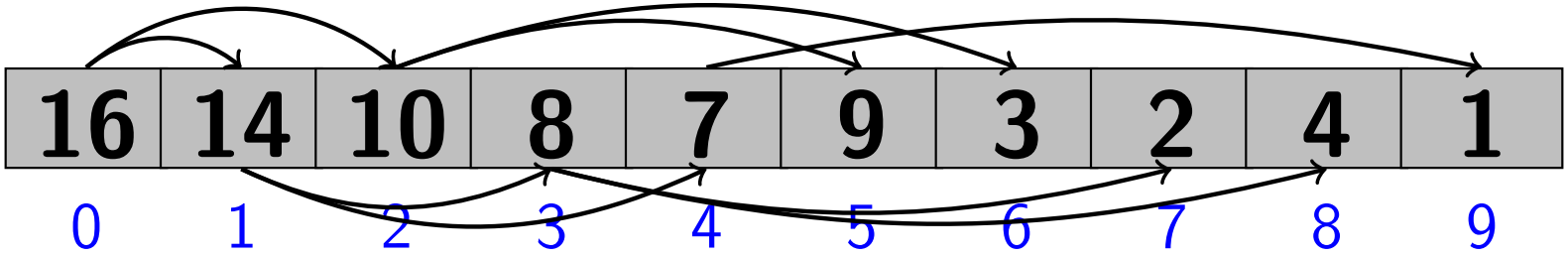
A estrutura de dados **heap** (**binário**) é um arranjo que pode ser visualizado como uma árvore binária praticamente completa (a exceção ocorre no último nível que é preenchido da esquerda para direita enquanto existirem elementos). Cada nó da árvore corresponde a um elemento do arranjo.

Heap





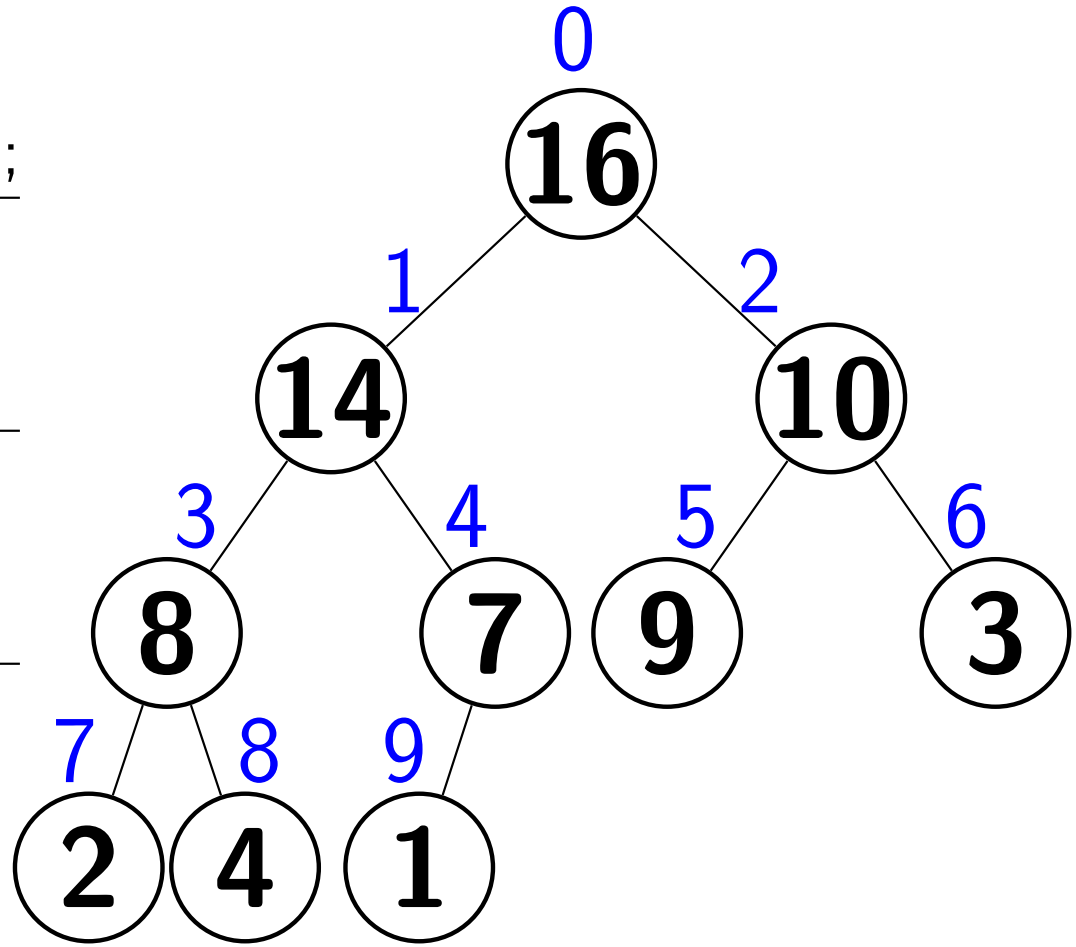
Heap



PAI (**i**)
    Return $\lfloor (\mathbf{i} - 1)/2 \rfloor$;

ESQUERDA (**i**)
    Return $\mathbf{i} * 2 + 1$;

DIREITA (**i**)
    Return $\mathbf{i} * 2 + 2$;



Heap

A raiz da árvore para um vetor V sempre se encontra no elemento $V[0]$. Existem dois tipos de heaps: **heaps máximos** e **heaps mínimos**. Em um **heap máximo** temos a seguinte propriedade:

$$V[\textsc{Pai}(i)] \geq V[i]$$

isto é, o valor de um nó é no máximo o valor de seu pai. Desse modo, o maior elemento em um heap máximo é armazenado na raiz.

Heap

Um **heap mínimo** é organizado de forma oposta, ele possui a seguinte propriedade:

$$V[\textsc{Pai}(i)] \leq V[i]$$

isto é, o menor valor em um heap mínimo está na raiz. O algoritmo *heap-sort* usa um *heap-máximo*. Heaps mínimos são comumente utilizados em filas de prioridades.

Heap

Visualizando um **heap** como uma **árvore**, definimos a **altura** de um nó em um heap como o número de arestas no caminho descendente mais longo da raiz até uma folha. Tendo em vista que um **heap** com n elementos é baseado em uma árvore binária completa, sua altura é $\Theta(\log n)$. As operações básicas sobre um **heap** têm complexidade proporpocional à altura da árvore e assim demoram $\mathcal{O}(\log n)$.

Heap

(Cormem 6.1-1) Quais são os números mínimo e máximo de elementos em um heap de altura h?

(Cormem 6.1-2) Mostre que um heap de n elementos tem altura $\log_2 n$.

Heap

(Cormem 6.1-1) Quais são os números mínimo e máximo de elementos em um heap de altura h?

- Para $h = 1$: mínimo 2 e máximo 3.
- Para $h = 2$: mínimo 4 e máximo 7.
- Para $h = 3$: mínimo 8 e máximo 15.
- Equivalente a $2^h \leq n \leq 2^{h+1} - 1$

Heap

(Cormem 6.1-2) Mostre que um heap de n elementos tem altura $\log_2 n$.

- Vimos que $2^h \leq n \leq 2^{h+1} - 1$
- Logo, $2^h \leq n \leq 2^{h+1} - 1 < 2^{h+1}$
- Aplicando $\log_2$ em cada um dos termos
- $\log_2 2^h \leq \log_2 n < \log_2 2^{h+1}$
- $h \leq \log_2 n < h + 1$
- Assim, $h \leq \log_2 n$.

Sumário

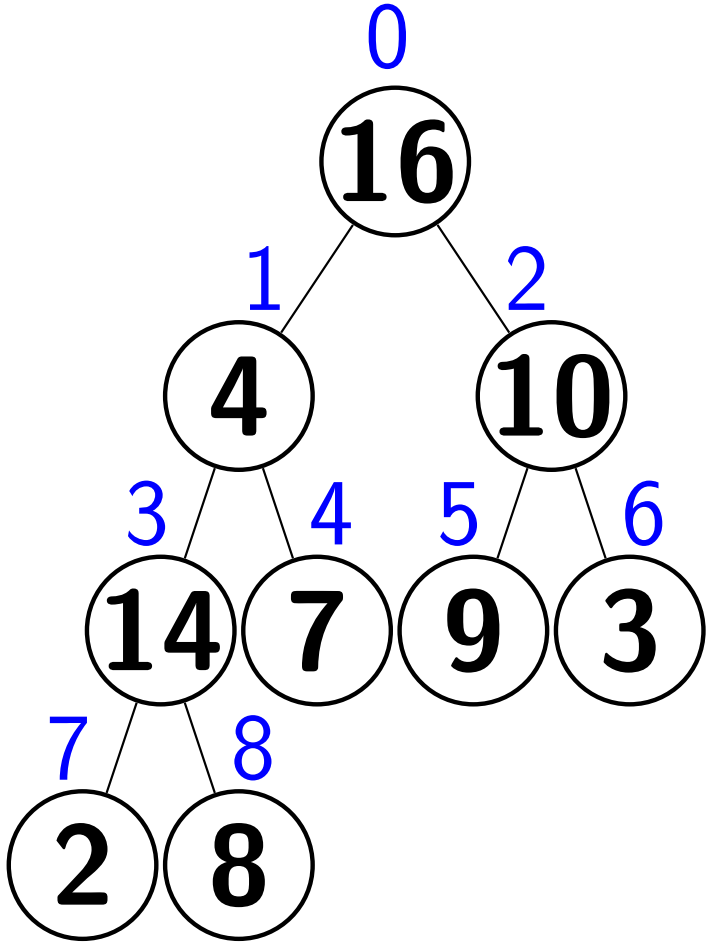
Heap máximo

A operação MAX-HEAPIFY é útil para manipular **heaps máximos**. Quando MAX-HEAPIFY é invocada ela supõe que as árvores binárias com raízes em ESQUERDA(i) e DIREITA(i) são heaps máximos, mas que $V[i]$ pode ser menor que seus filhos, violando assim a propriedade de heap máximo. Assim, $V[i]$ deve "descer" no heap máximo, de tal forma que a sub-árvore com raiz em i se torne um heap-máximo.

Heap máximo

MAX-HEAPIFY (V, SIZE, i)

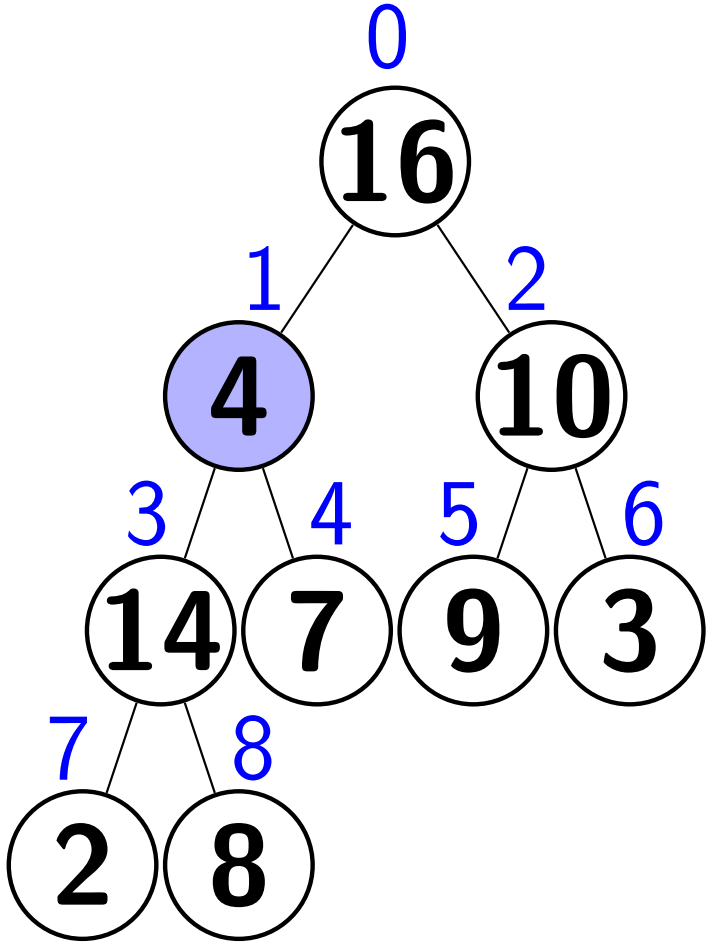
```
e = ESQUERDA(i);
d = DIREITA(i);
If e < SIZE and V[e] > V[i]
    maior = e;
Else
    maior = i;
If d < SIZE and V[d] > V[maior]
    maior = d;
If maior ≠ i
    V[i] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, SIZE, 1)

```
e = ESQUERDA(1);
d = DIREITA(1);
If e < SIZE and V[e] > V[1]
    maior = e;
Else
    maior = 1;
If d < SIZE and V[d] > V[maior]
    maior = d;
If maior ≠ 1
    V[1] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, SIZE, 1)

```
3 = ESQUERDA(1);
d = DIREITA(1);
If 3 < SIZE and V[3] > V[1]
    maior = 3;
Else
    maior = 1;
If d < SIZE and V[d] > V[maior]
    maior = d;
If maior ≠ 1
    V[1] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
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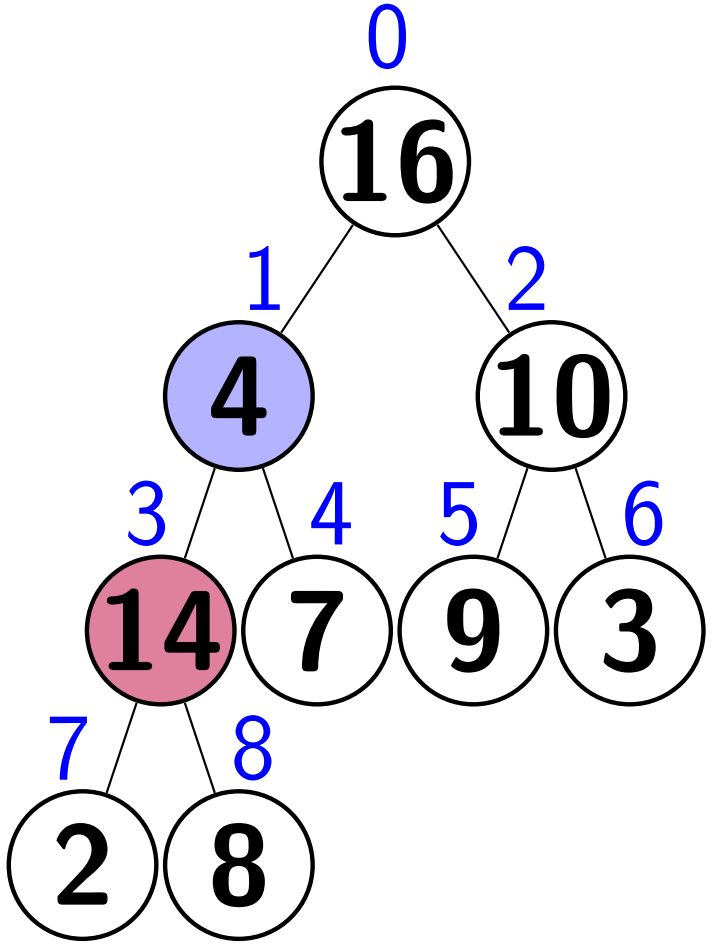

Heap máximo

MAX-HEAPIFY (V, SIZE, 1)

```
3 = ESQUERDA(1);
4 = DIREITA(1);
If 3 < SIZE and V[3] > V[1]
    maior = 3;
Else
    maior = 1;
If 4 < SIZE and V[4] > V[maior]
    maior = 4;
If maior ≠ 1
    V[1] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
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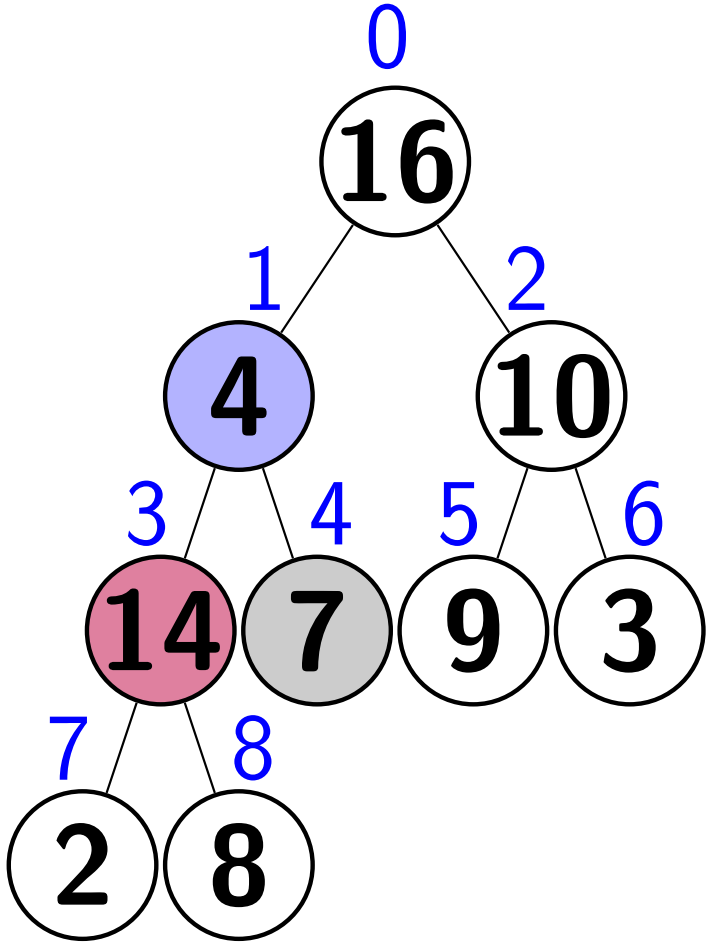
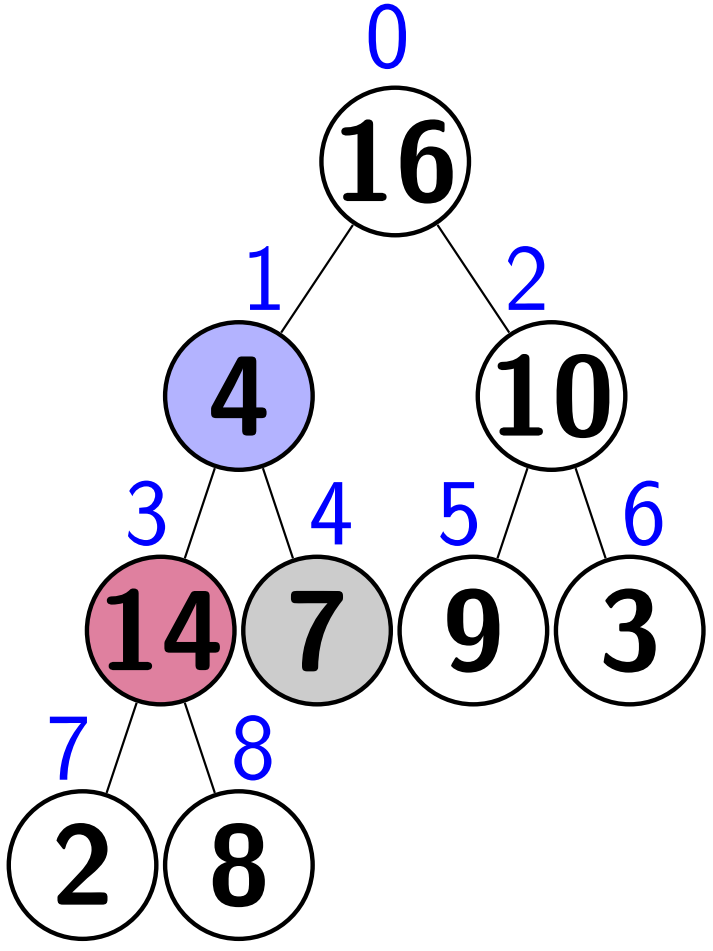


Heap máximo

MAX-HEAPIFY (V, 9, 1)

```
3 = ESQUERDA(1);
4 = DIREITA(1);
If 3 < 9 and 14 > 4
    maior = 3;
Else
    maior = 1;
If 4 < SIZE and V[4] > V[maior]
    maior = 4;
If maior ≠ 1
    V[1] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



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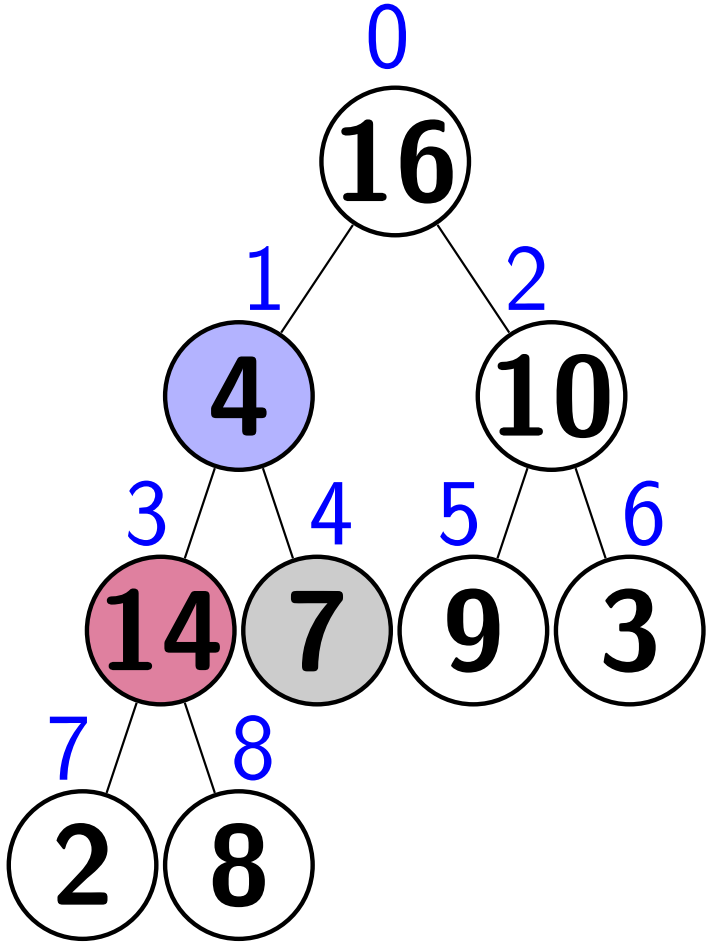

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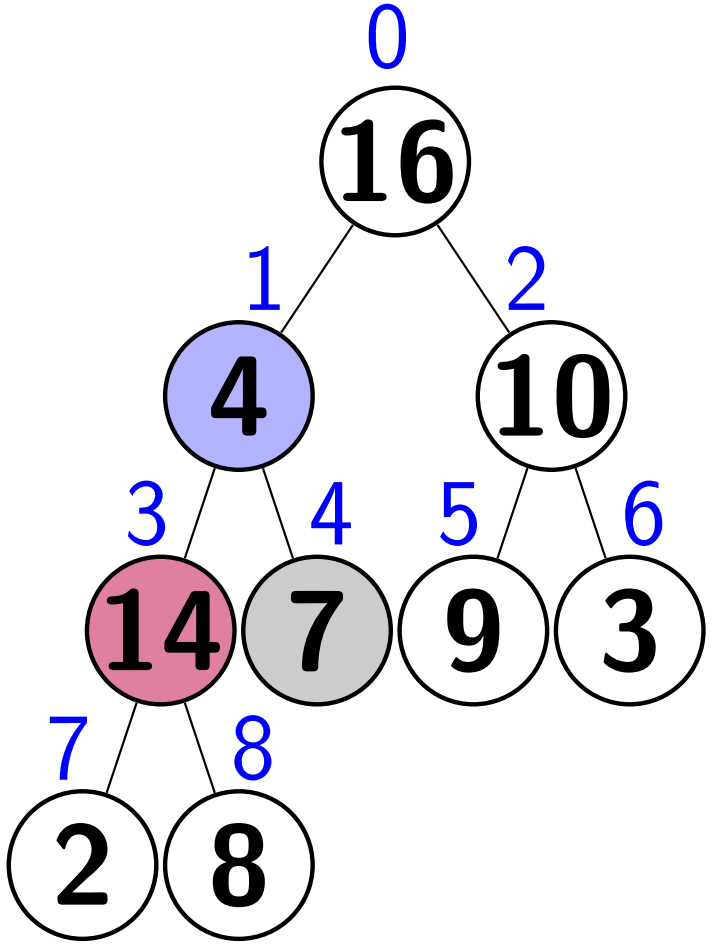
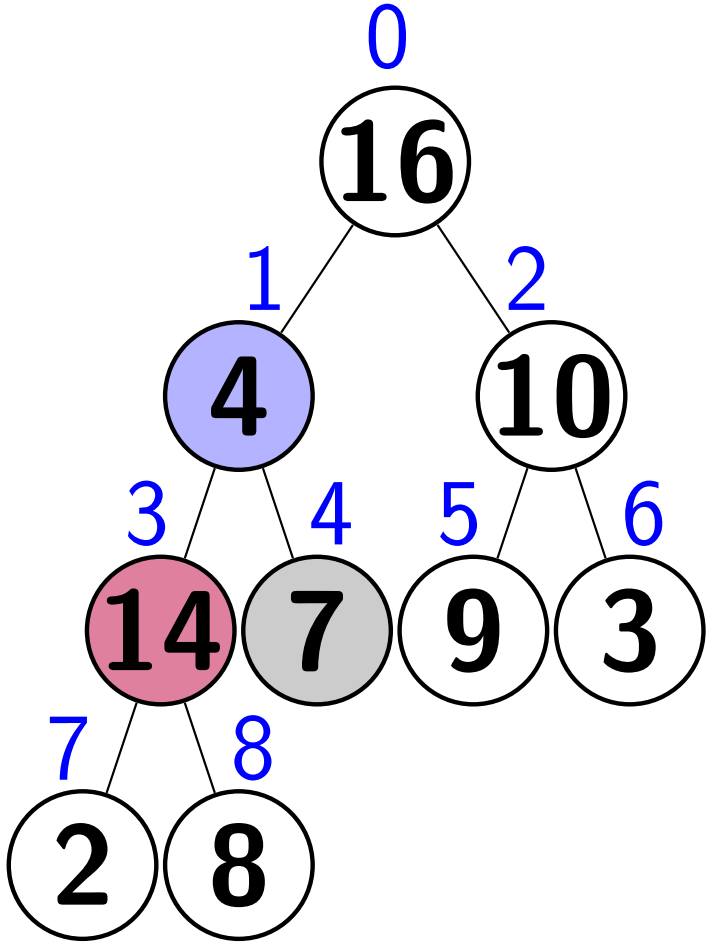


Heap máximo

MAX-HEAPIFY (V, 9, 1)

3 = ESQUERDA(1);
4 = DIREITA(1);
If 3 < 9 **and** 14 > 4
    ▷**maior** = 3;
Else
    maior = 1;
If 4 < 9 **and** 7 > 14
    maior = 4;
If **maior** ≠ 1
    V[1] ↔ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



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    maior = 4;
If 3 ≠ 1
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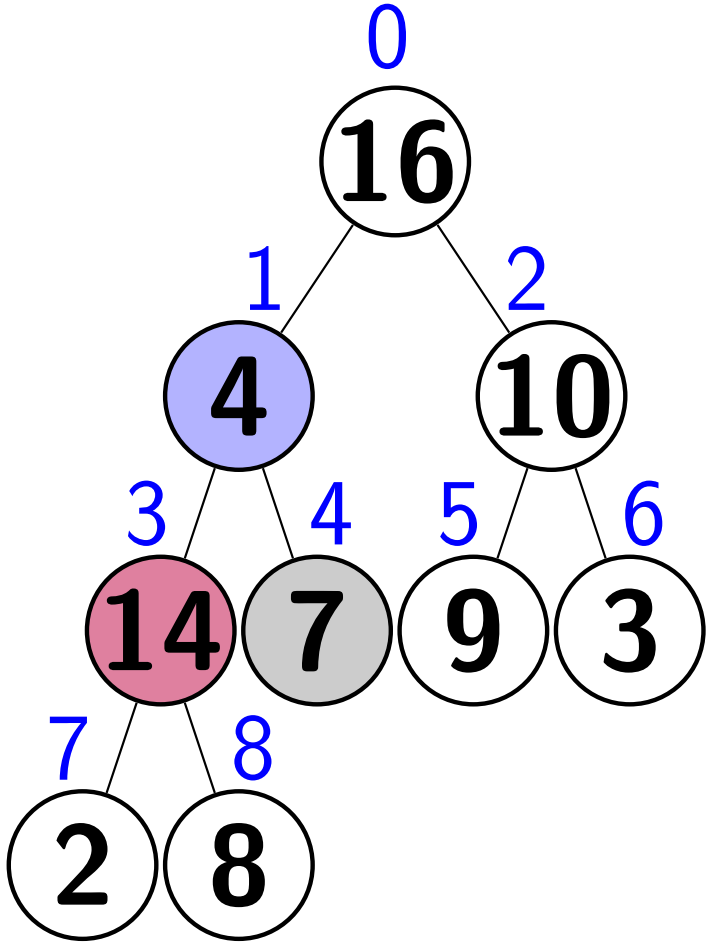
    MAX-HEAPIFY (V, SIZE, **maior**);



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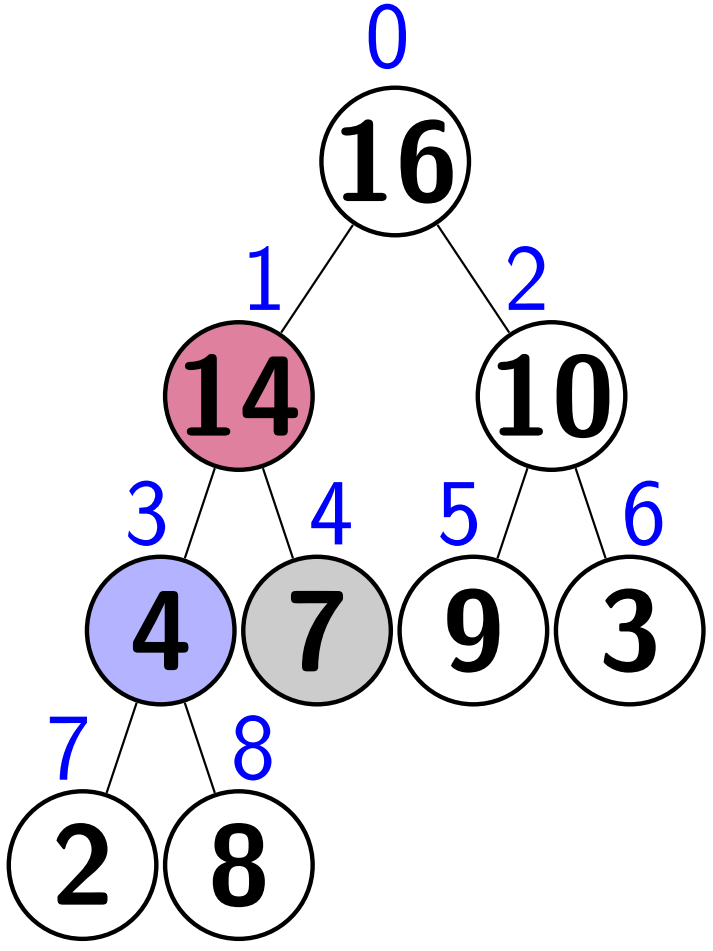

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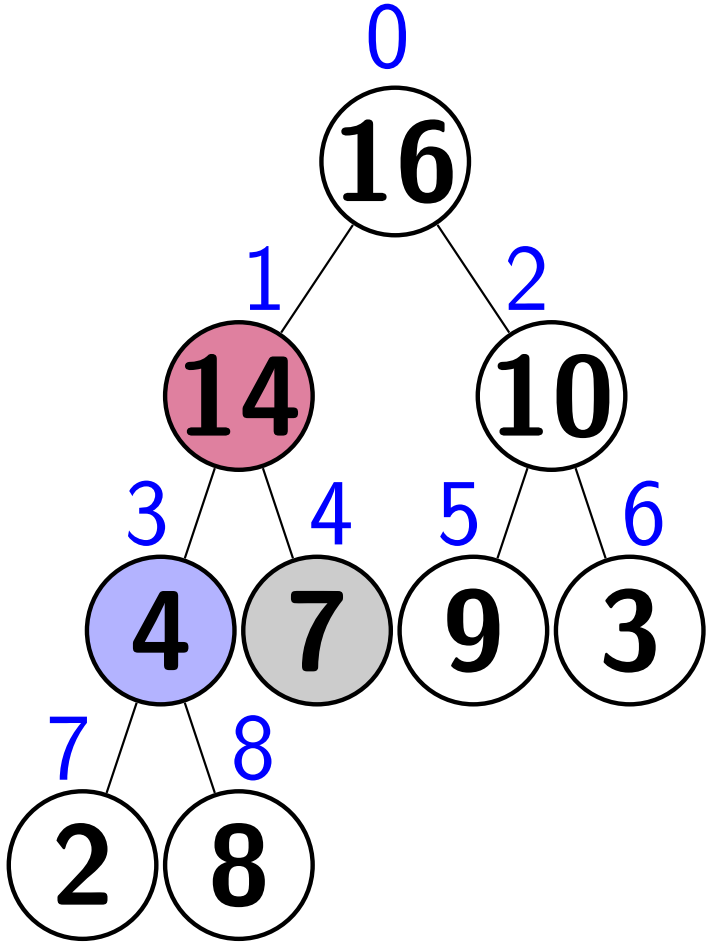
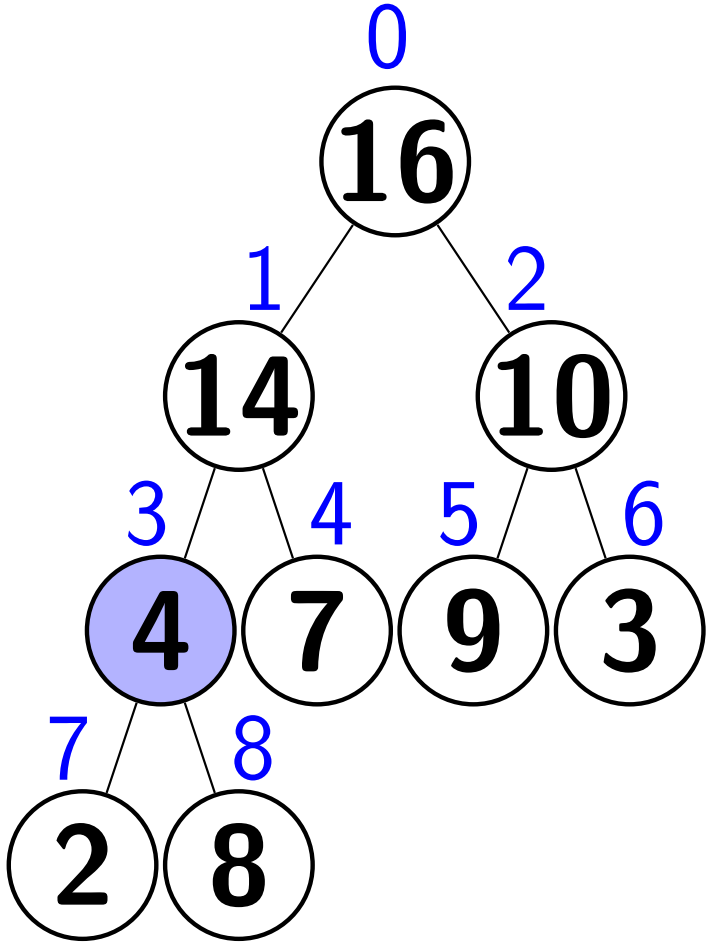
    ▷**maior** = 3;
Else
    maior = 1;
If 4 < 9 **and** 7 > 14
    maior = 4;
If 3 ≠ 1
    V[1] ↔ V[3];
    MAX-HEAPIFY (V, 9, 3);



Heap máximo

MAX-HEAPIFY (V, SIZE, 3)

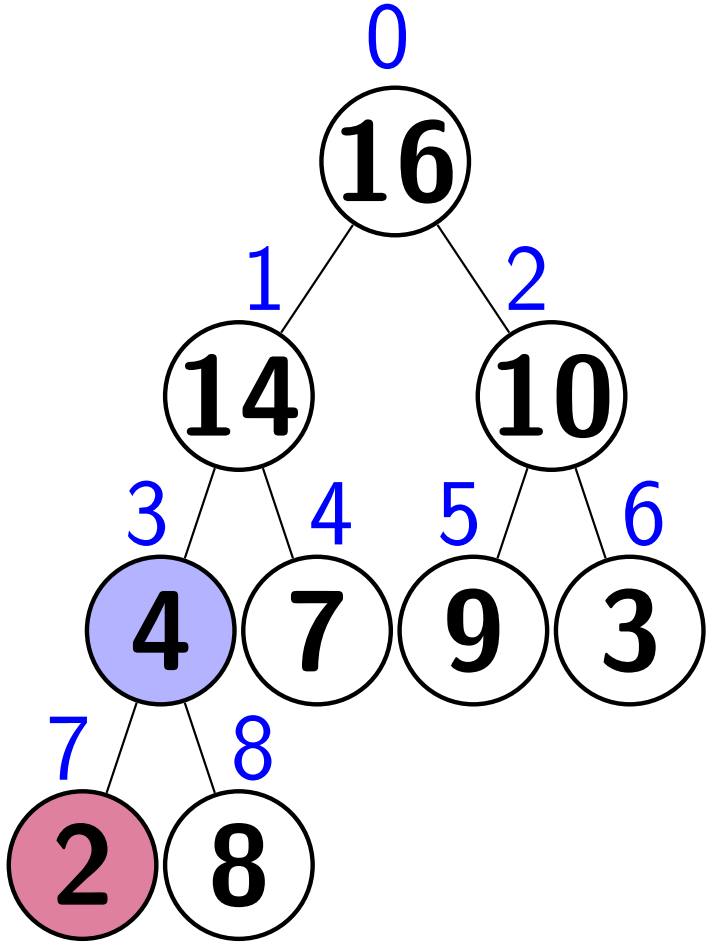
```
e = ESQUERDA(3);
d = DIREITA(3);
If e < SIZE and V[e] > V[3]
    maior = e;
Else
    maior = 3;
If d < SIZE and V[d] > V[maior]
    maior = d;
If maior ≠ 3
    V[3] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, SIZE, 3)

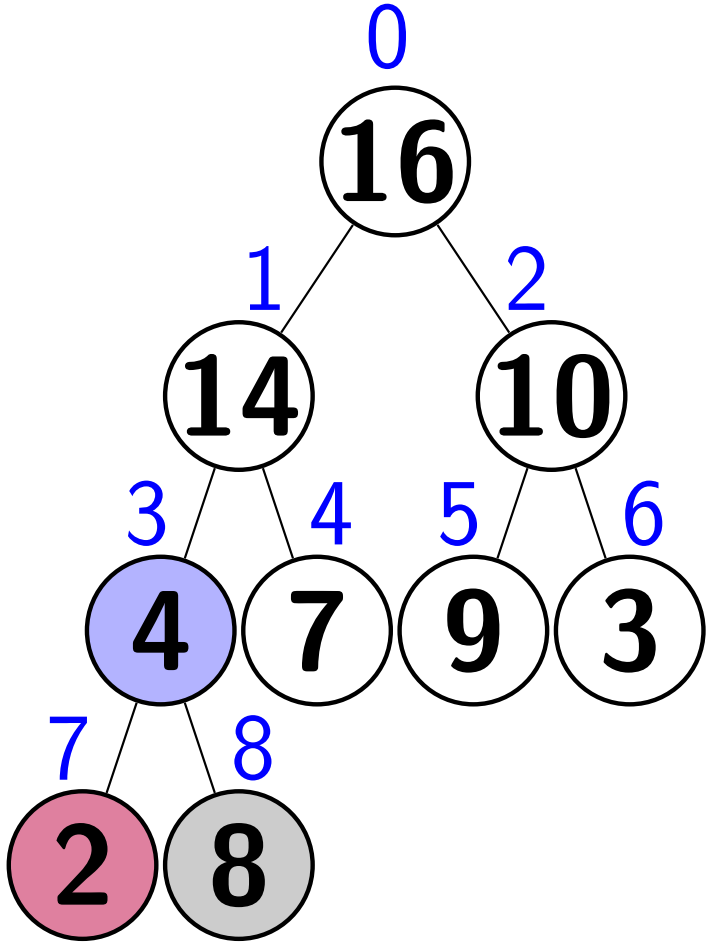
```
7 = ESQUERDA(3);
d = DIREITA(3);
If 7 < SIZE and V[7] > V[3]
    maior = 7;
Else
    maior = 3;
If d < SIZE and V[d] > V[maior]
    maior = d;
If maior ≠ 3
    V[3] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, SIZE, 3)

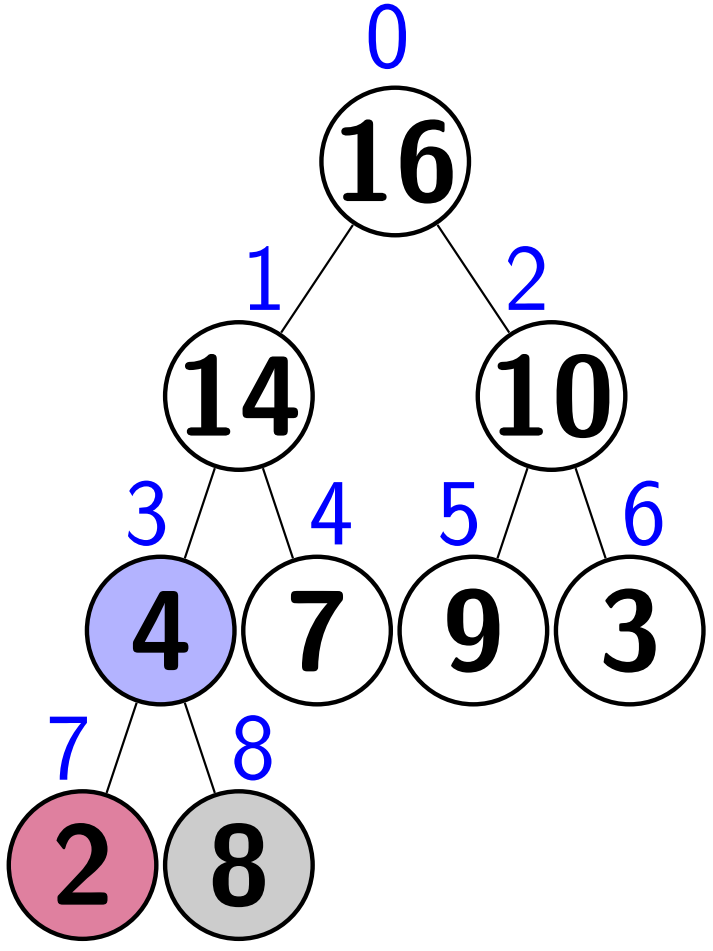
```
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < SIZE and V[7] > V[3]
    maior = 7;
Else
    maior = 3;
If 8 < SIZE and V[8] > V[maior]
    maior = 8;
If maior ≠ 3
    V[3] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, 9, 3)

7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    maior = 3;
If 8 < SIZE **and** V[8] > V[**maior**]
    maior = 8;
If **maior** ≠ 3
    V[3] ↔ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



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    maior = 8;
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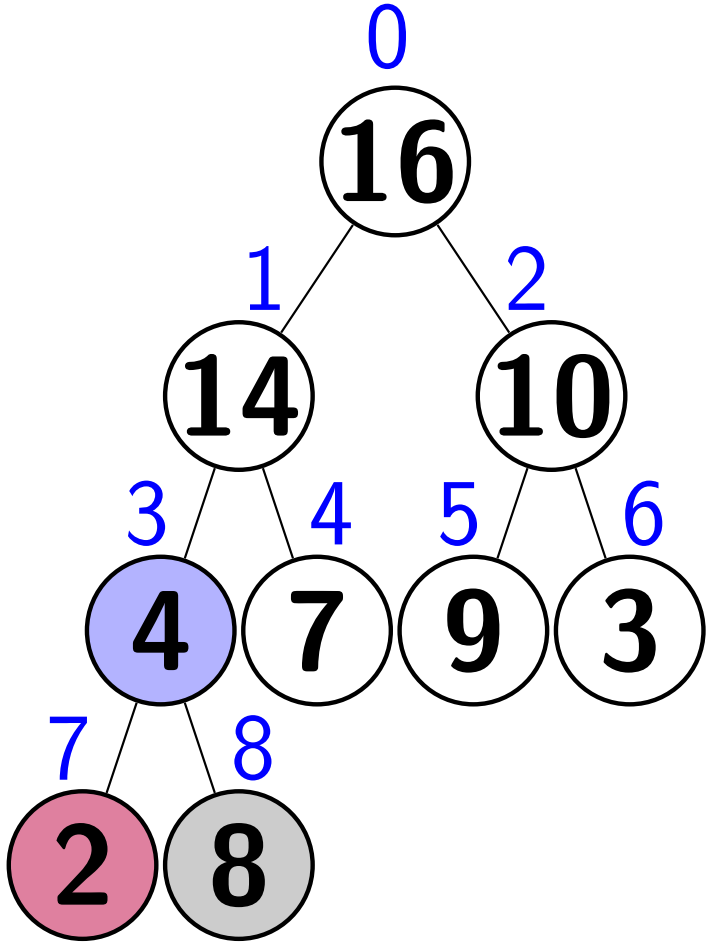

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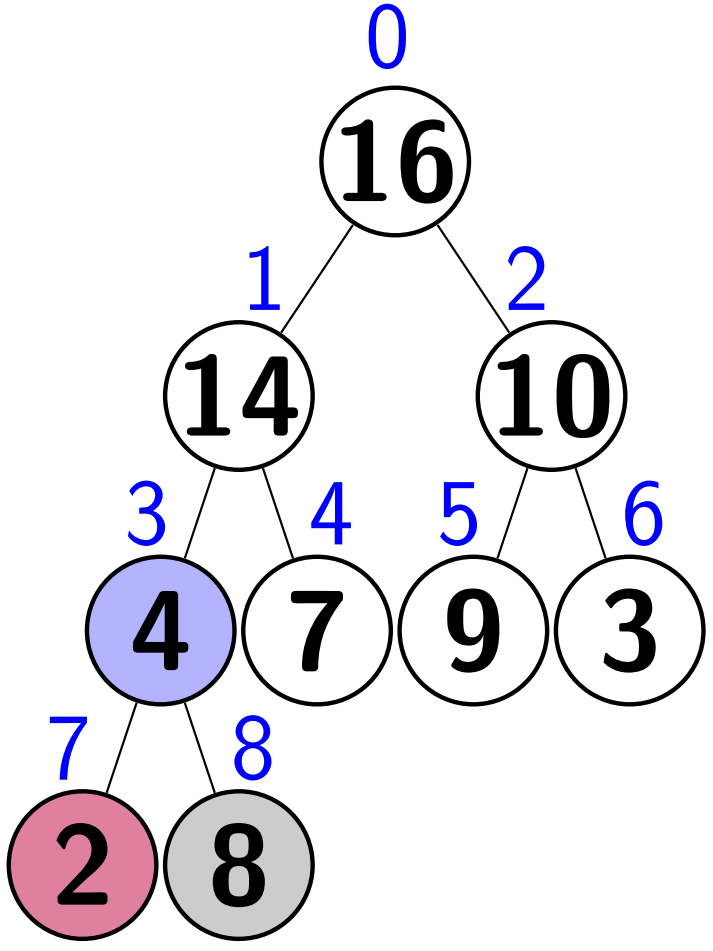
MAX-HEAPIFY (V, 9, 3)

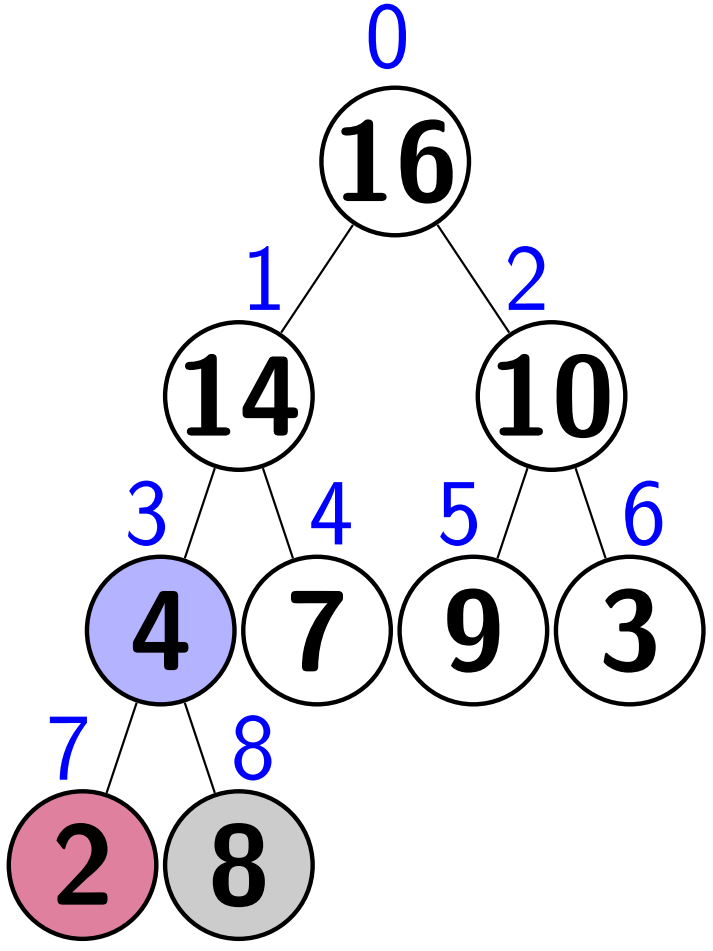
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    ▷**maior** = 3;
If 8 < SIZE **and** V[8] > V[3]
    maior = 8;
If **maior** ≠ 3
    V[3] ↔ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, 3)

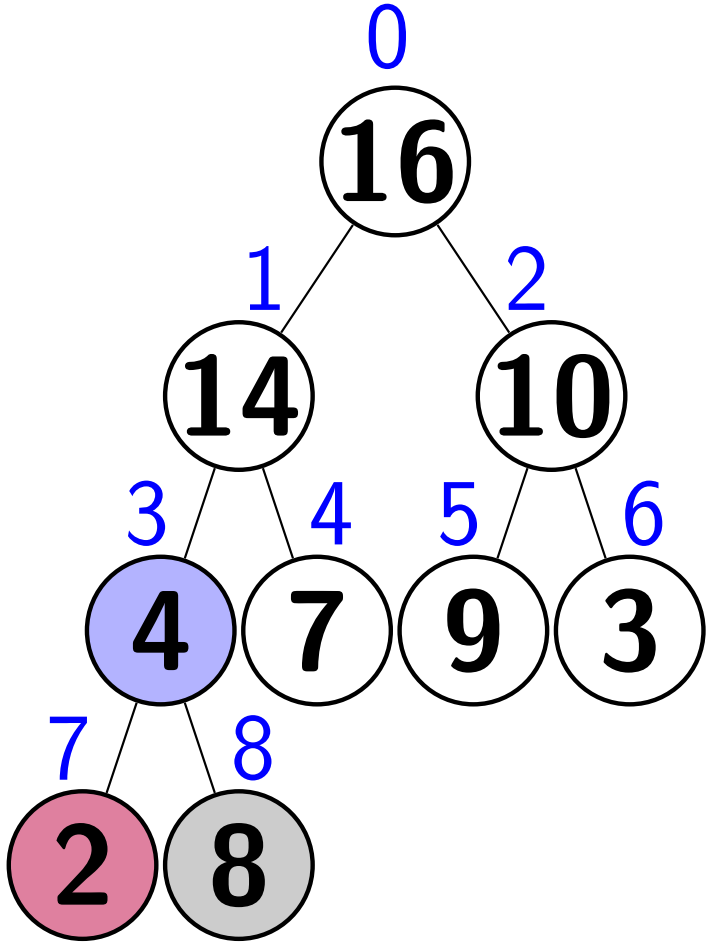
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 $<$ 9 **and** 2 $>$ 4
    maior = 7;
Else
    ▷**maior** = 3;
If 8 $<$ 9 **and** 8 $>$ 4
    maior = 8;
If **maior** $\neq$ 3
    V[3] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, 3)

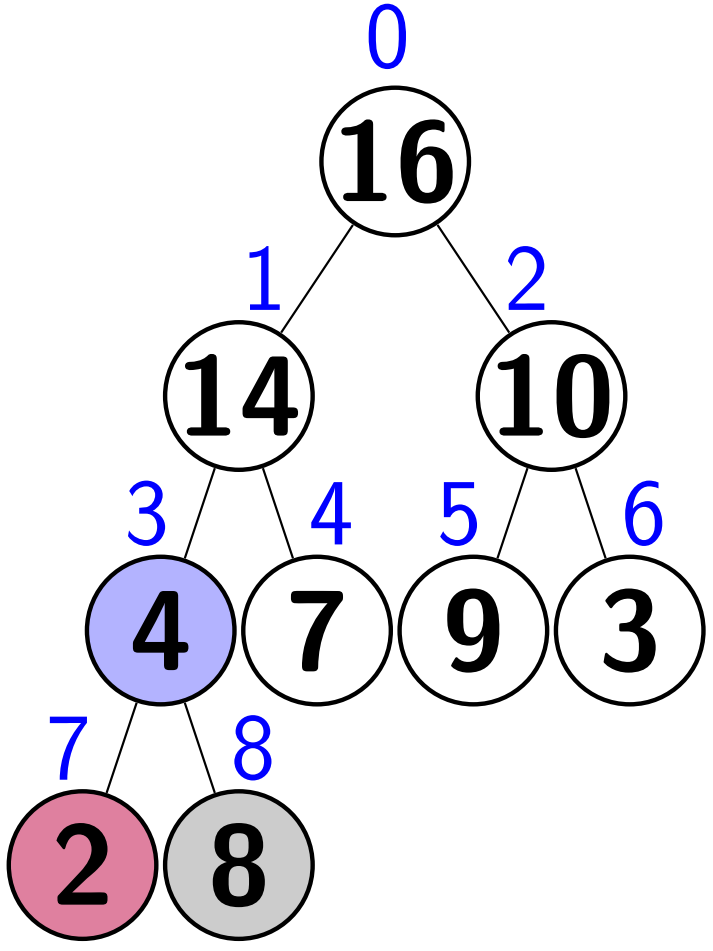
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    maior = 3;
If 8 < 9 **and** 8 > 4
    ▷ **maior** = 8;
If **maior** ≠ 3
    V[3] ↔ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, 3)

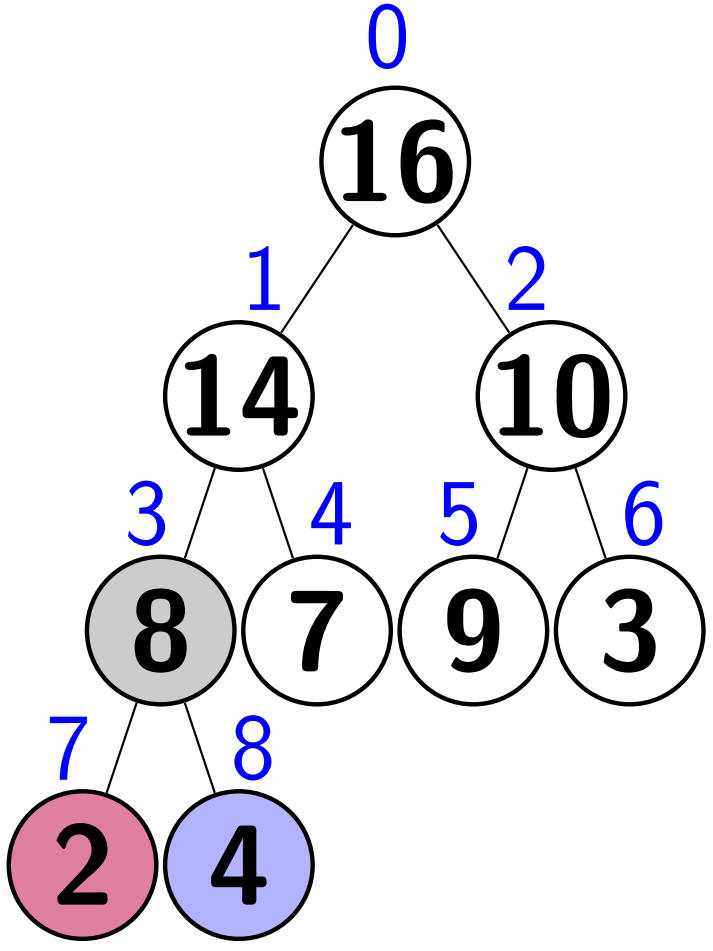
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    maior = 3;
If 8 < 9 **and** 8 > 4
    ▷ **maior** = 8;
If 8 ≠ 3
    V[3] ↔ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, 3)

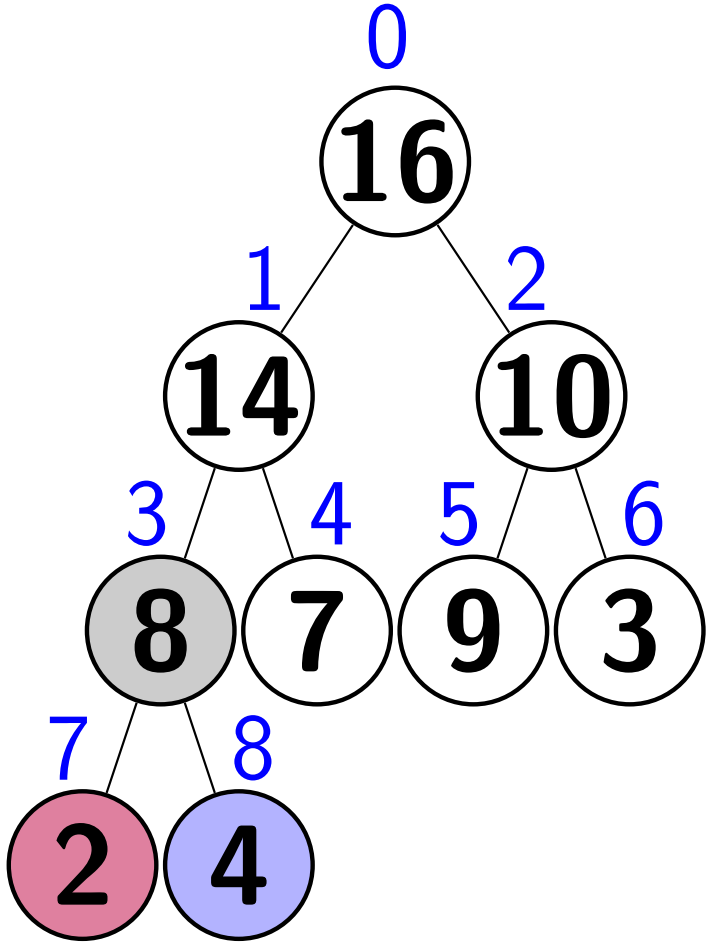
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    maior = 3;
If 8 < 9 **and** 8 > 4
    ▷ **maior** = 8;
If 8 ≠ 3
    V[3] ↔ V[8];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, 3)

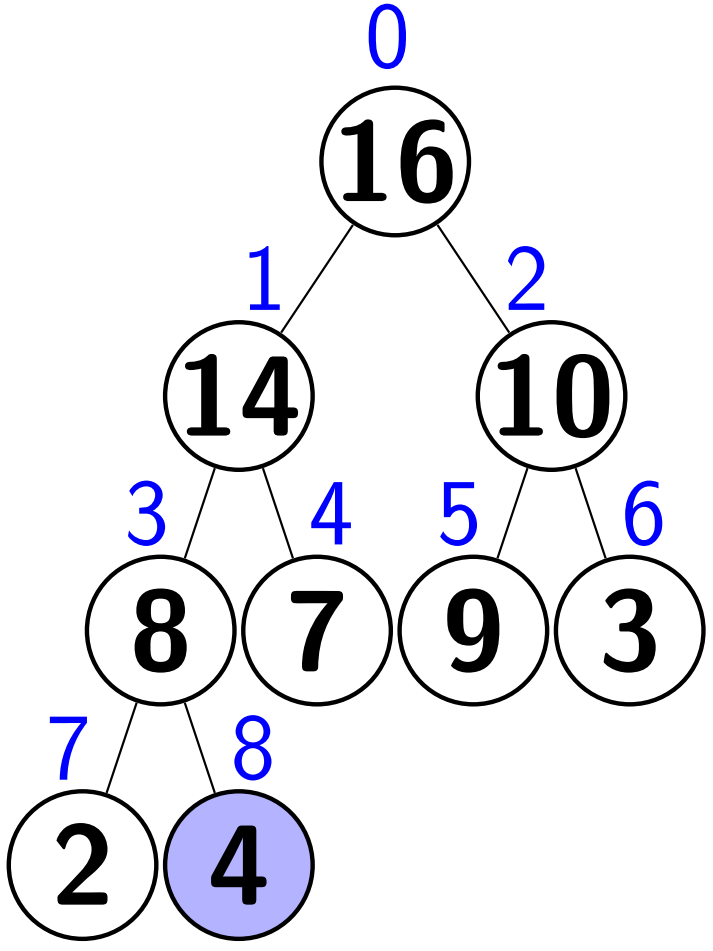
7 = ESQUERDA(3);
8 = DIREITA(3);
If 7 < 9 **and** 2 > 4
    maior = 7;
Else
    maior = 3;
If 8 < 9 **and** 8 > 4
    ▷ **maior** = 8;
If 8 ≠ 3
    V[3] ↔ V[8];
    MAX-HEAPIFY (V, 9, 8);



Heap máximo

MAX-HEAPIFY (V, SIZE, 8)

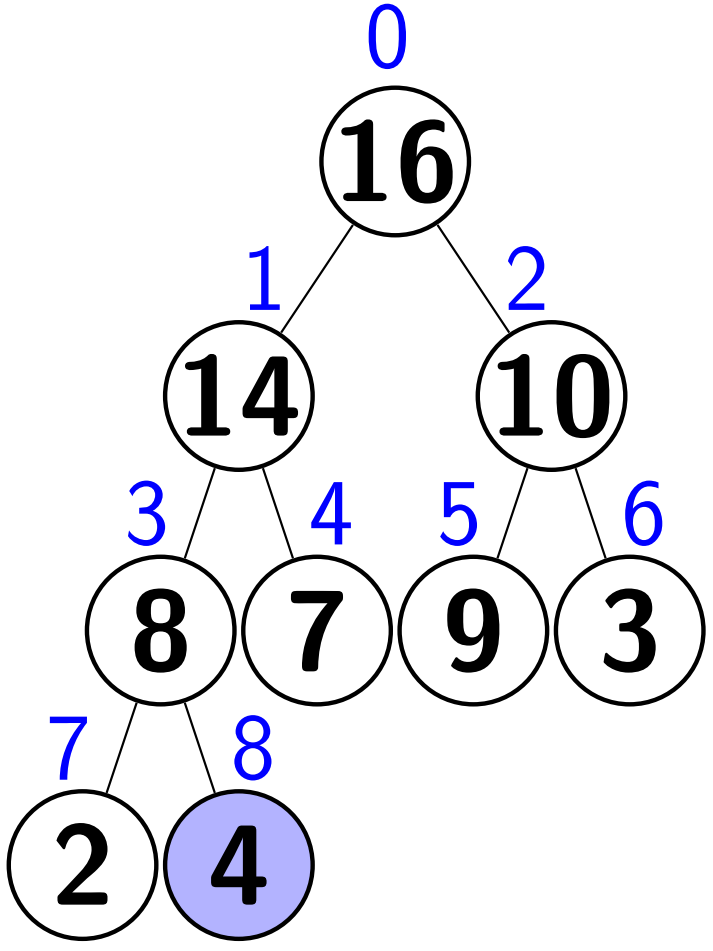
```
e = ESQUERDA(8);
d = DIREITA(8);
If e < SIZE and V[e] > V[8]
    maior = e;
Else
    maior = 8;
If d < SIZE  and V[d] > V[maior]
    maior = d;
If maior ≠ 8
    V[8] ↔ V[maior];
    MAX-HEAPIFY (V, SIZE, maior);
```



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

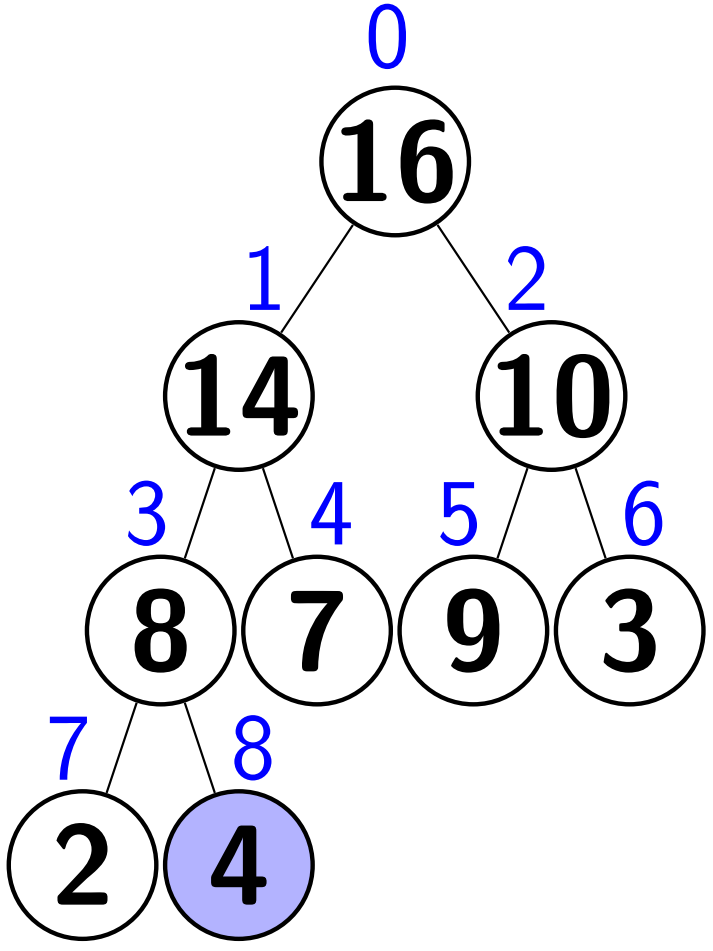
17 = ESQUERDA(**8**);
d = DIREITA(**8**);
If **17** $<$ 9 **and** V[**17**] $>$ V[**8**]
    maior = **17**;
Else
    maior = **8**;
If **d** $<$ SIZE **and** V[**d**] $>$ V[**maior**]
    maior = **d**;
If **maior** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

17 = ESQUERDA(**8**);
18 = DIREITA(**8**);
If **17** $<$ 9 **and** V[**17**] $>$ V[**8**]
    maior = **17**;
Else
    maior = **8**;
If **18** $<$ 9 **and** V[**18**] $>$ V[**maior**]
    maior = **18**;
If **maior** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

17 = ESQUERDA(**8**);
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If **17** $<$ 9
    maior = **17**;
Else
    maior = **8**;
If **18** $<$ 9 **and** V[**18**] $>$ V[**maior**]
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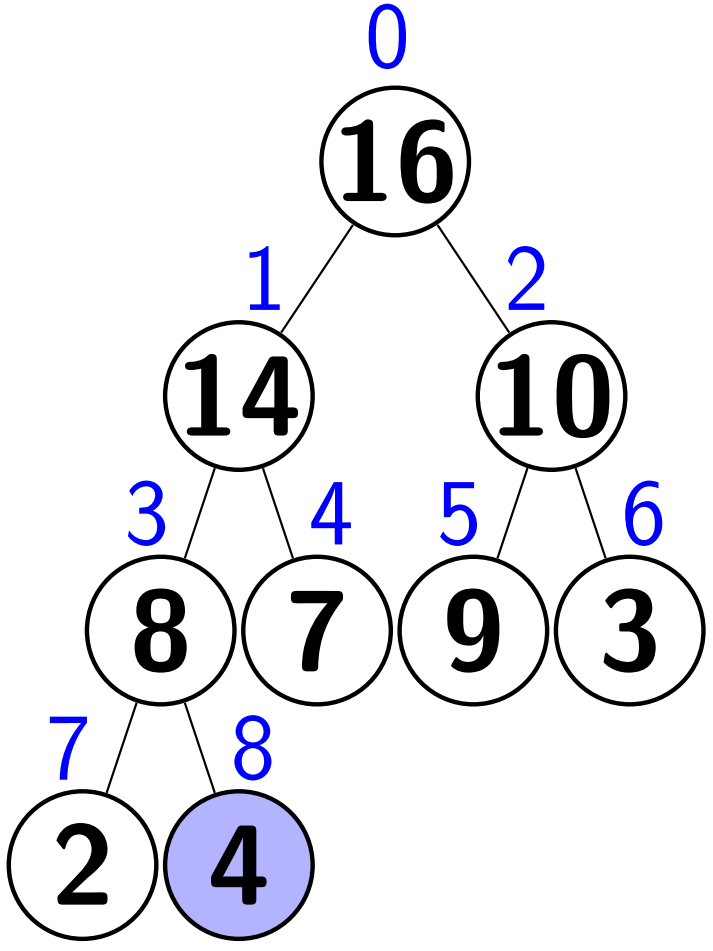
If **maior** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

17 = ESQUERDA(**8**);
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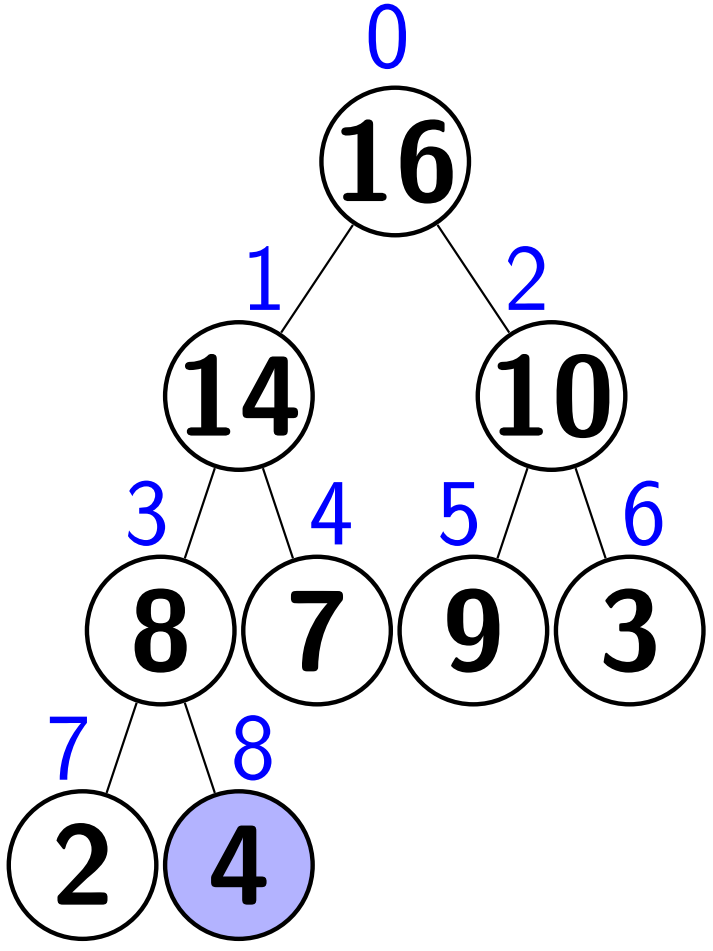
    maior = **18**;
If **maior** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

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    maior = **18**;
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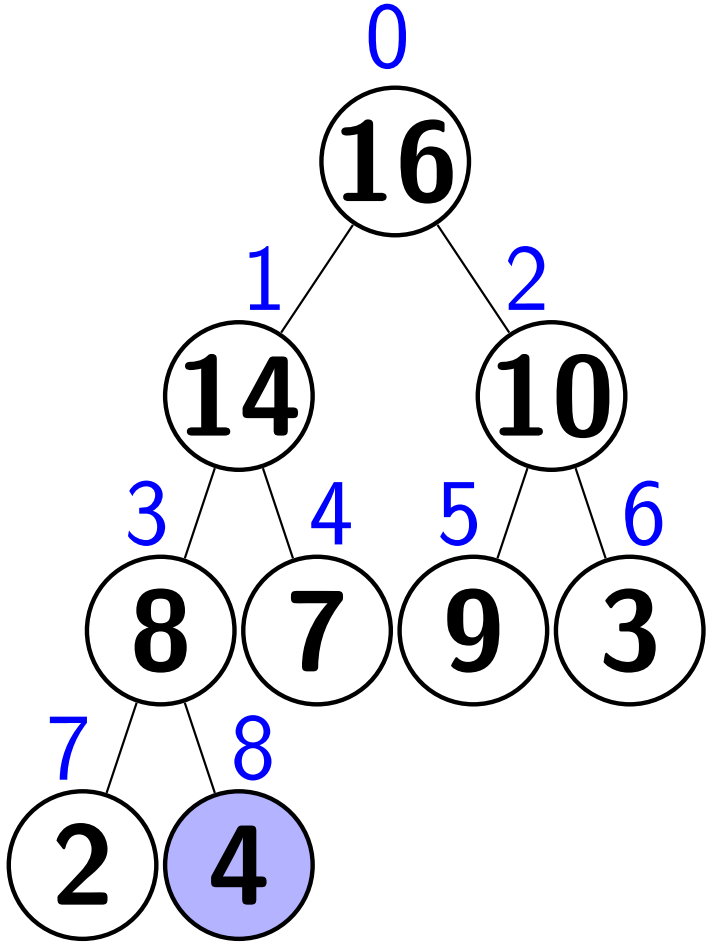

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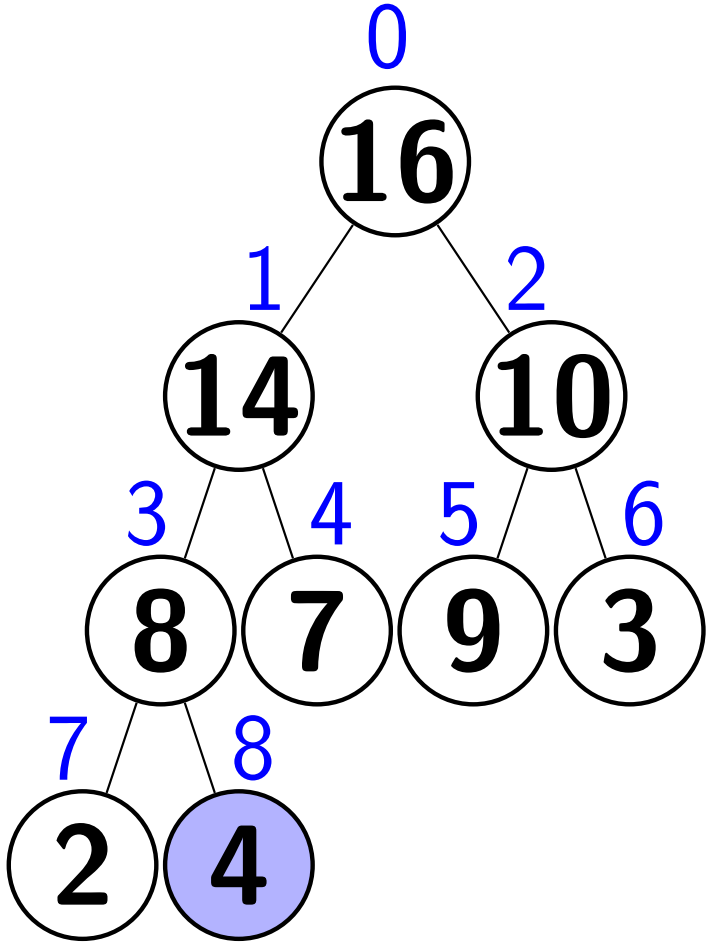
    maior = **18**;
If **maior** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

17 = ESQUERDA(**8**);
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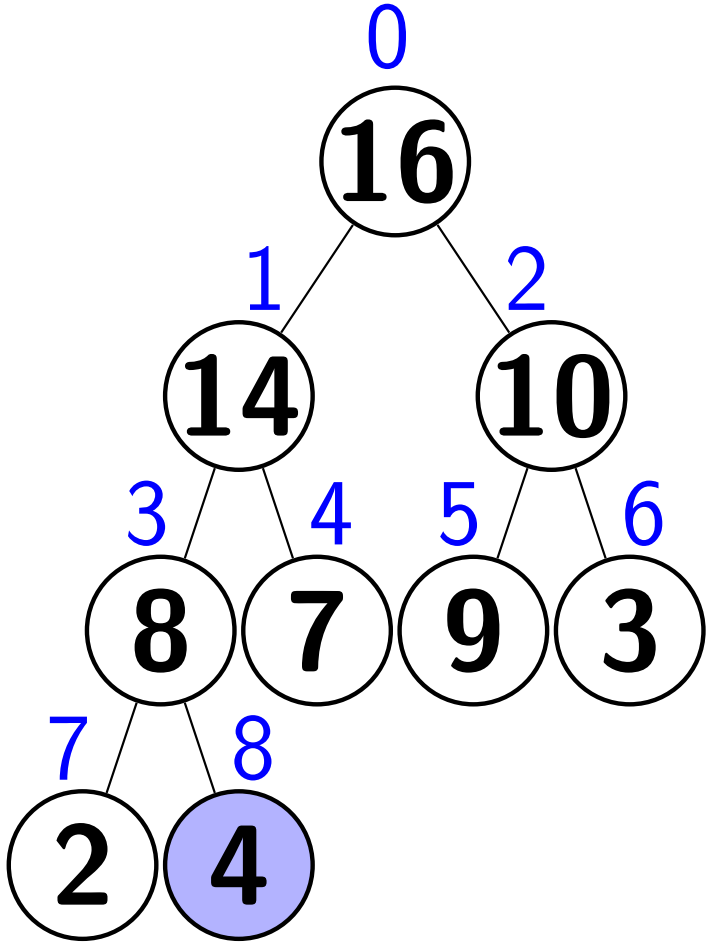
If **17** $<$ 9
    maior = **17**;
Else
    $\triangleright$**maior** = **8**;
If **18** $<$ 9
    maior = **18**;
If **8** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);



Heap máximo

MAX-HEAPIFY (V, 9, **8**)

17 = ESQUERDA(**8**);
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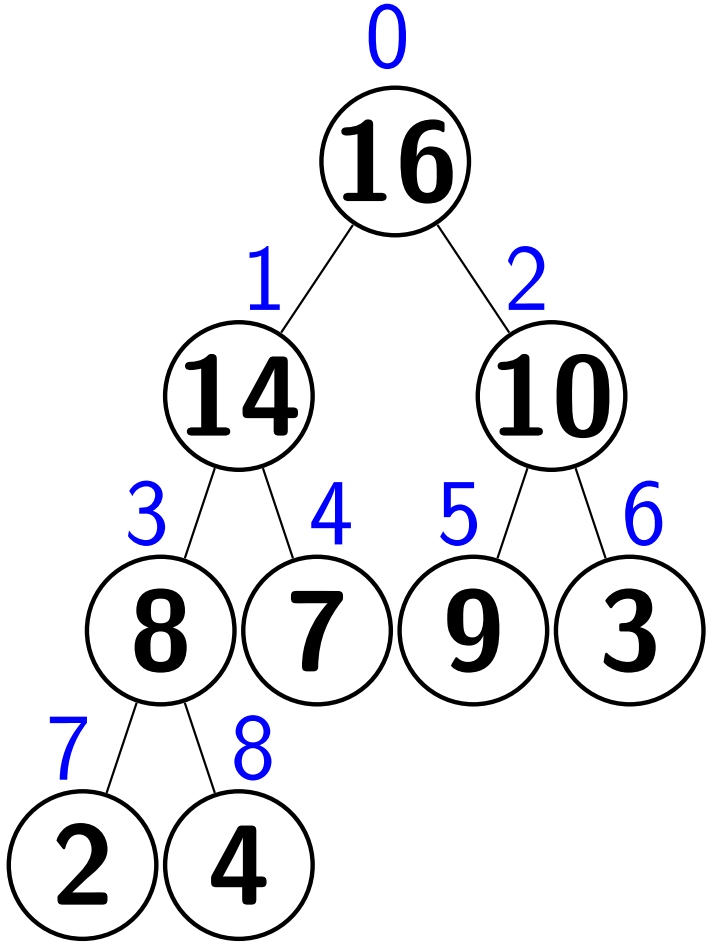
If **17** $<$ 9
    maior = **17**;
Else
    $\triangleright$**maior** = **8**;
If **18** $<$ 9
    maior = **18**;
If **8** $\neq$ **8**
    V[**8**] $\leftrightarrow$ V[**maior**];
    MAX-HEAPIFY (V, SIZE, **maior**);

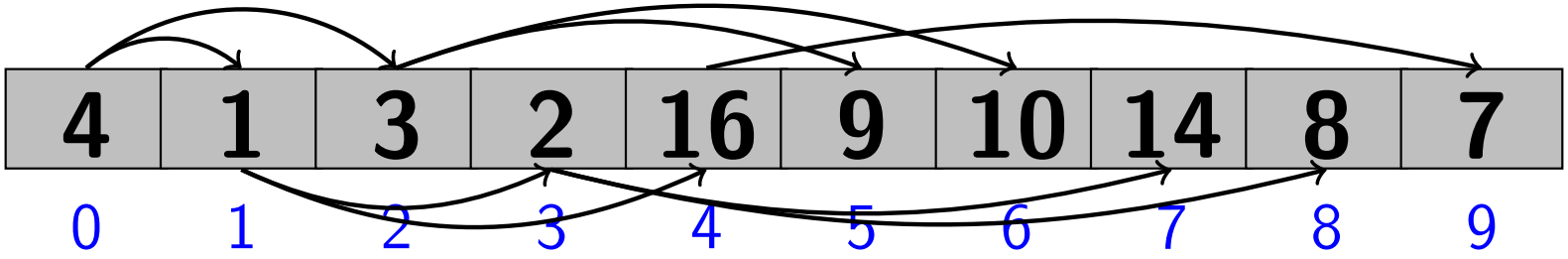


Sumário

Heap máximo

Podemos usar o procedimento MAX-HEAPIFY de baixo para cima, com o objetivo de converter um arranjo $V[1 \ldots n]$ em um **heap máximo**. Note que em um **heap** os elementos $V[(\lfloor n/2 \rfloor + 1) \ldots n]$ são todos folhas da árvore, e assim não precisam ser organizados (são heaps máximos de 1 elemento).

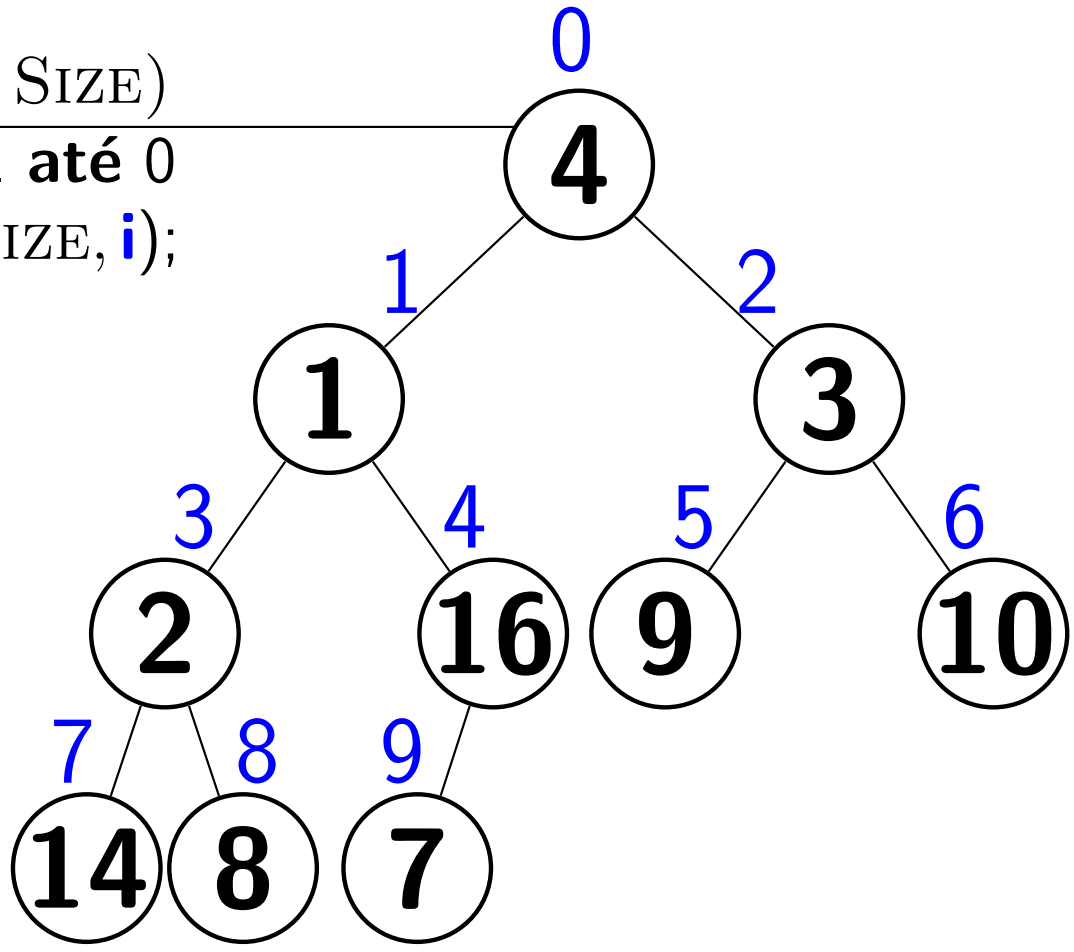
Construindo um Heap Máximo



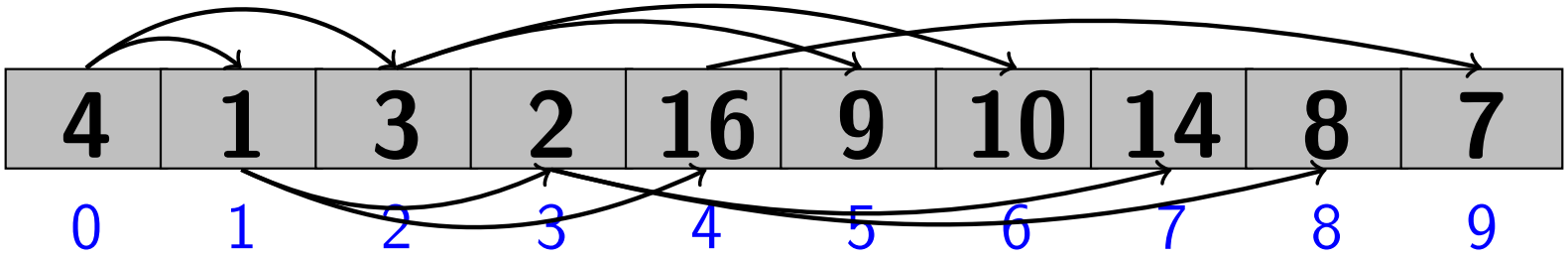
BUILD-MAX-HEAP $(V,$ SIZE$)$

Para **i** de $\lfloor$(SIZE/2)$\rfloor - 1$ **até** 0

    MAX-HEAPIFY $(V,$ SIZE$,$ **i**$)$;



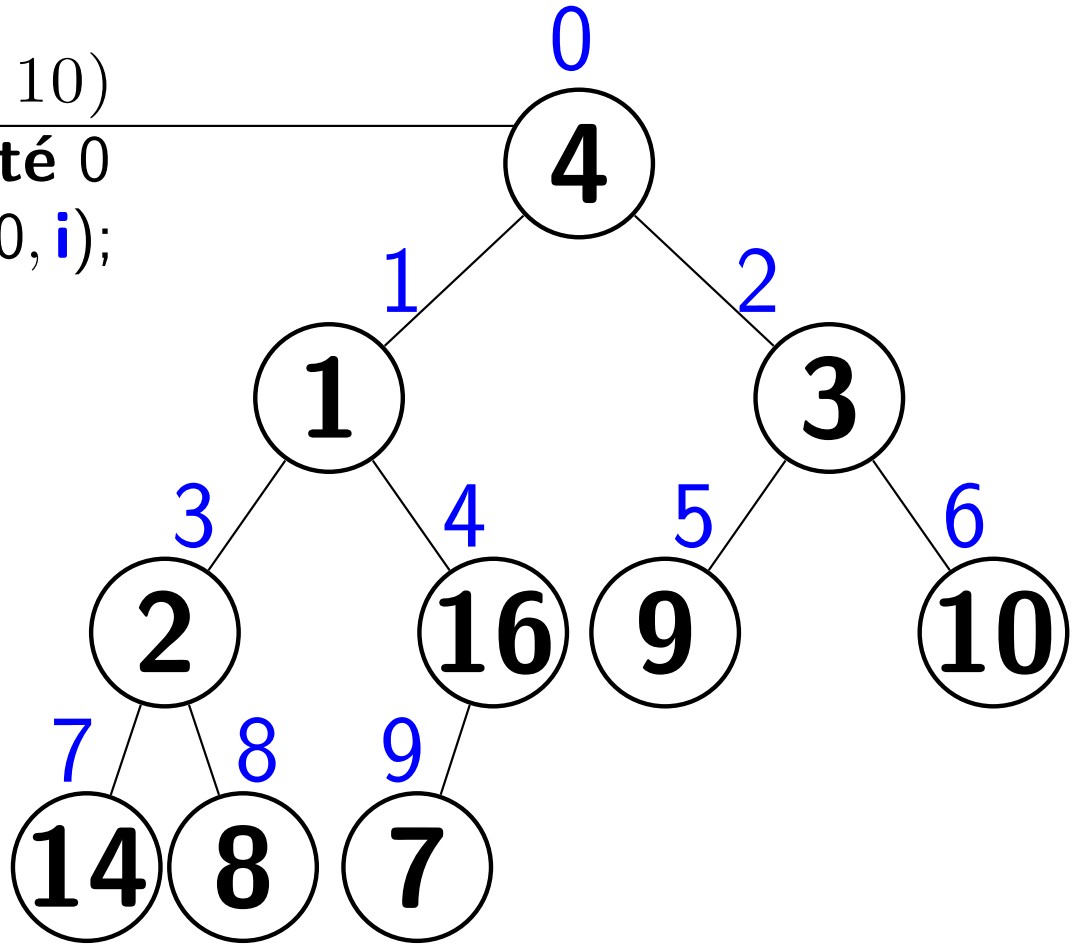
Construindo um Heap Máximo



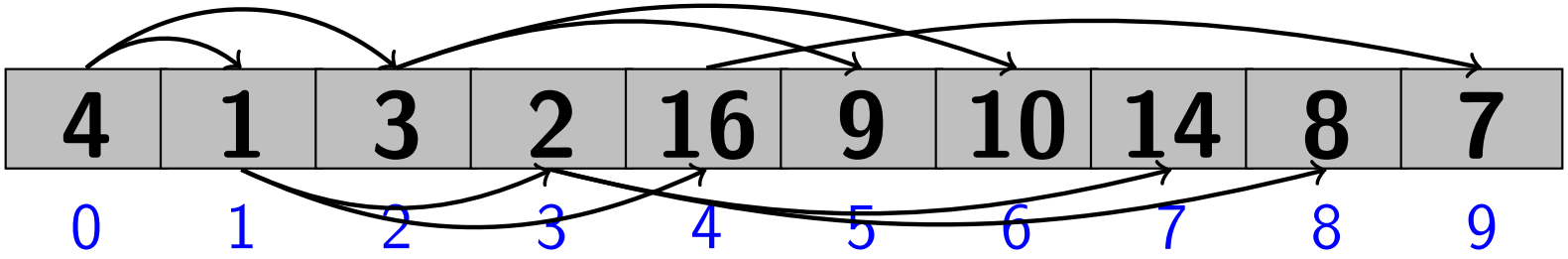
BUILD-MAX-HEAP $(V, 10)$

Para **i** de $\lfloor(10/2)\rfloor - 1$ **até** 0

    MAX-HEAPIFY $(V, 10,$ **i**$)$;



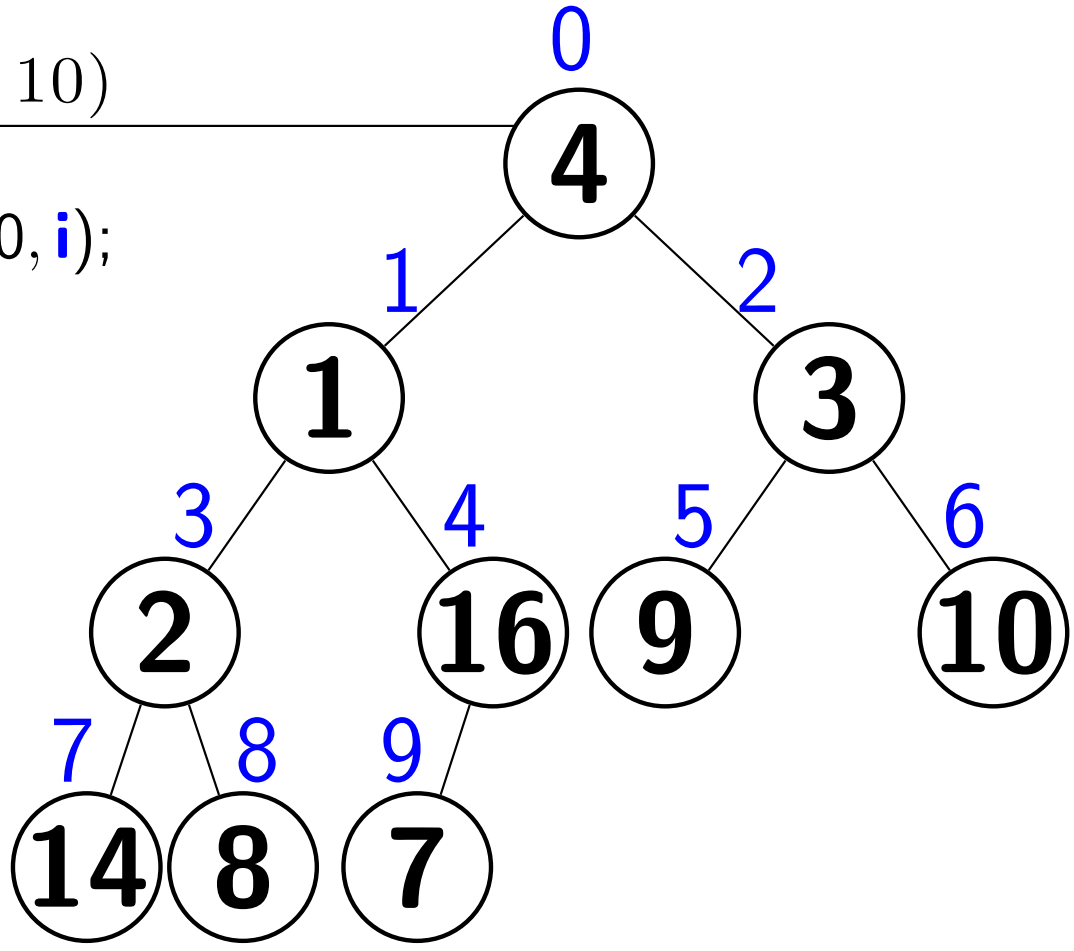
Construindo um Heap Máximo



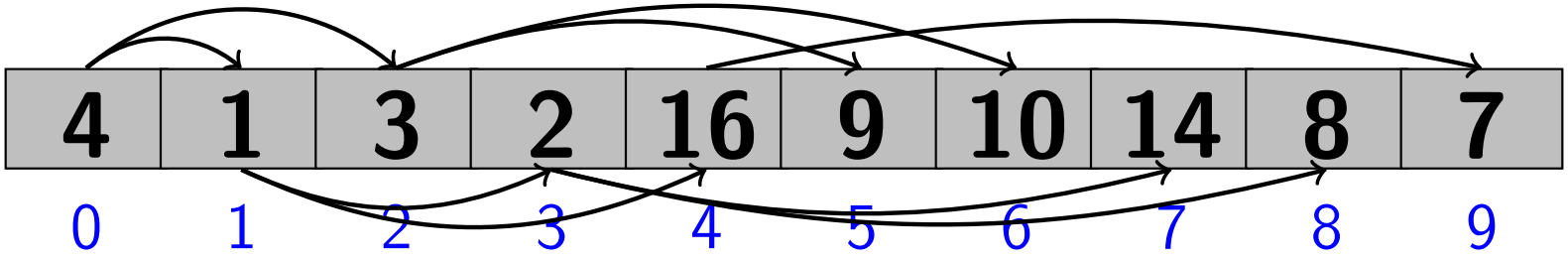
BUILD-MAX-HEAP $(V, 10)$

Para **i** de $\lfloor 5 \rfloor$-1 **até** 0

    MAX-HEAPIFY $(V, 10,$ **i**$)$;



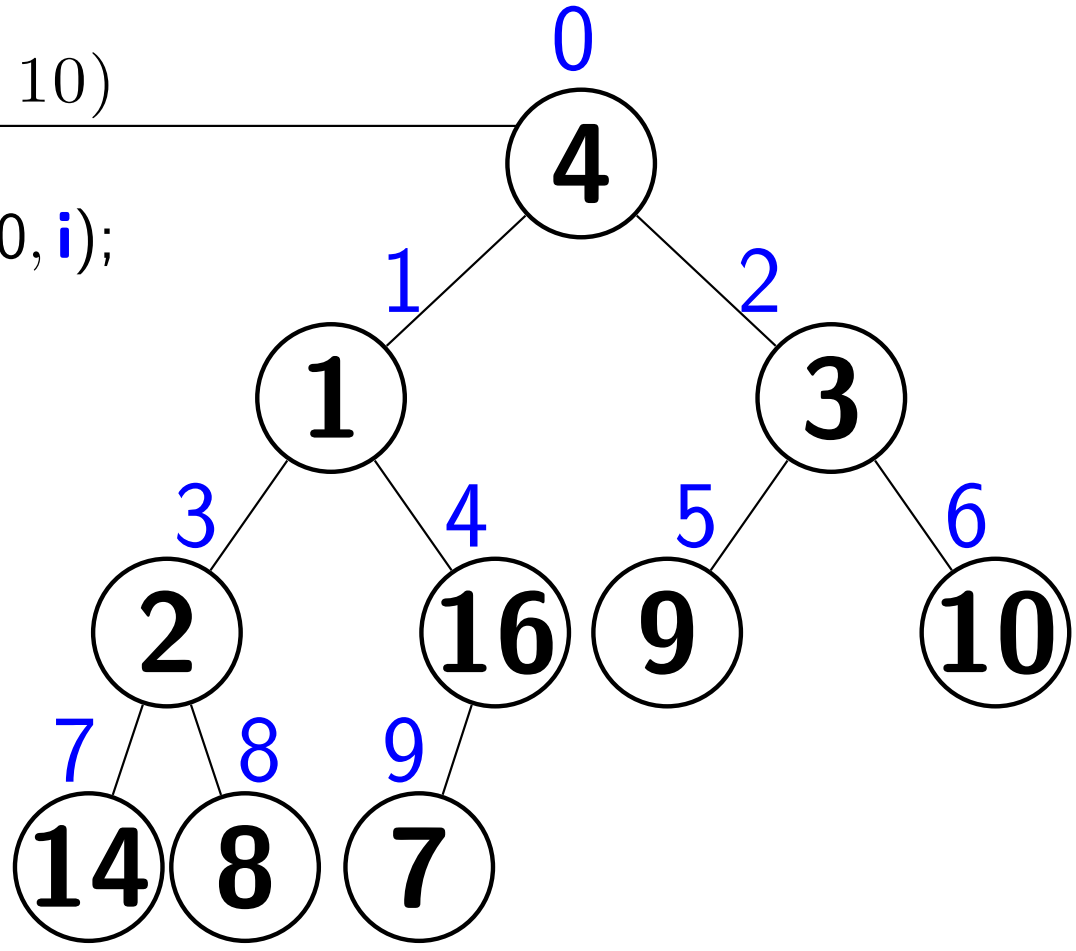
Construindo um Heap Máximo



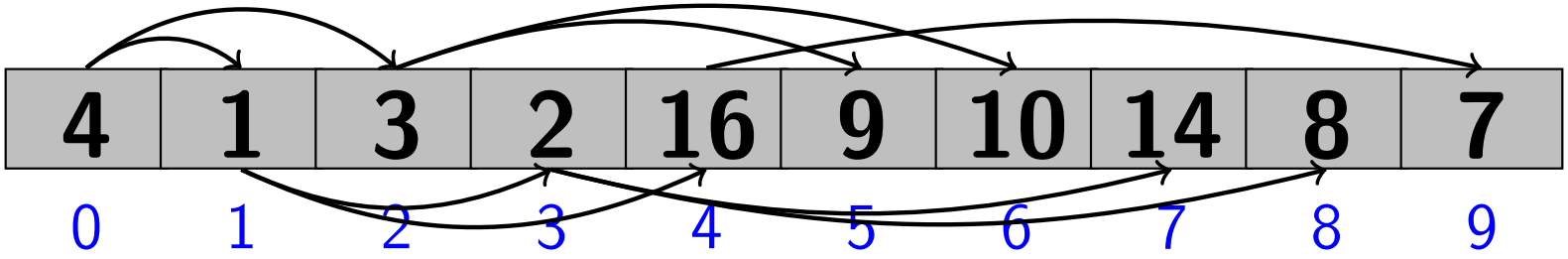
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10,$ **i**$)$;



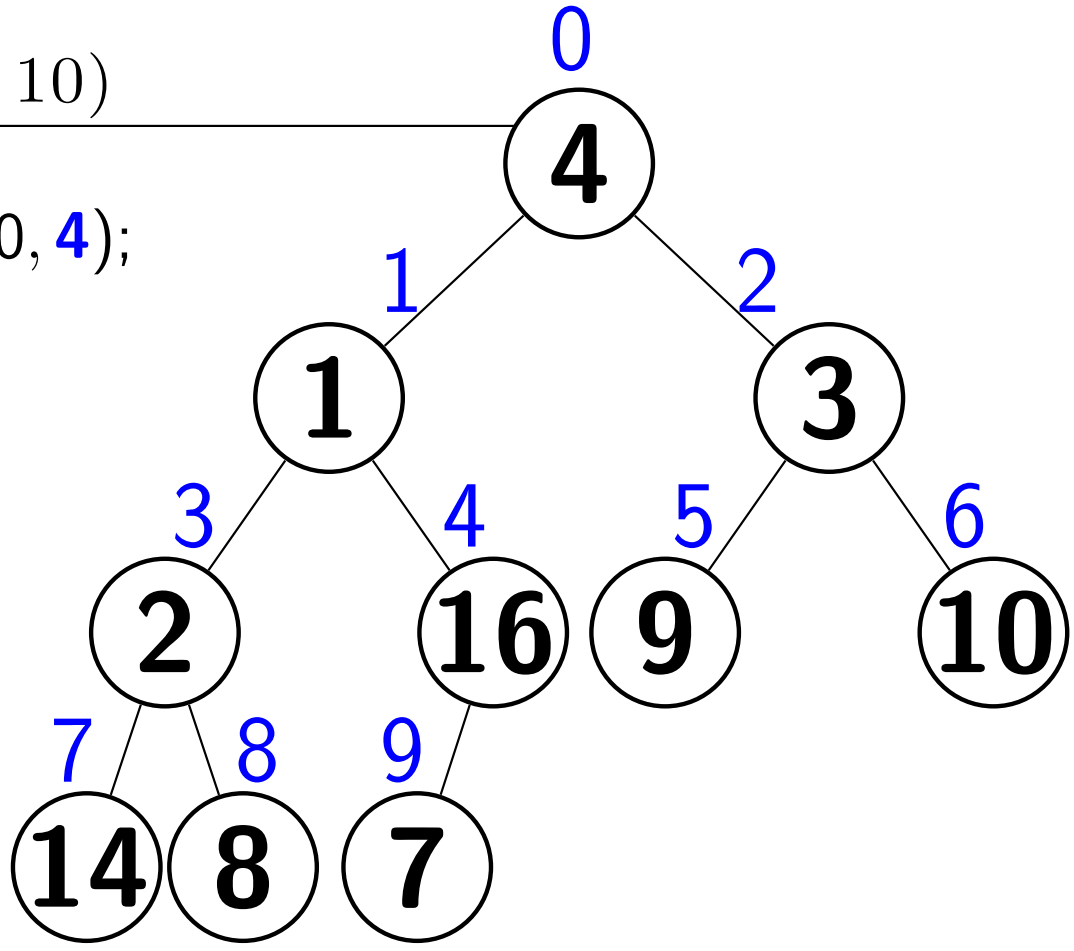
Construindo um Heap Máximo



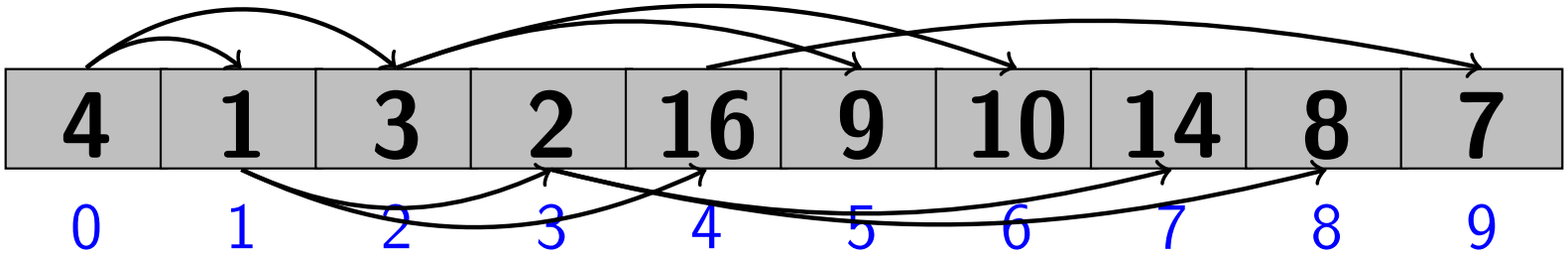
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{4})$;



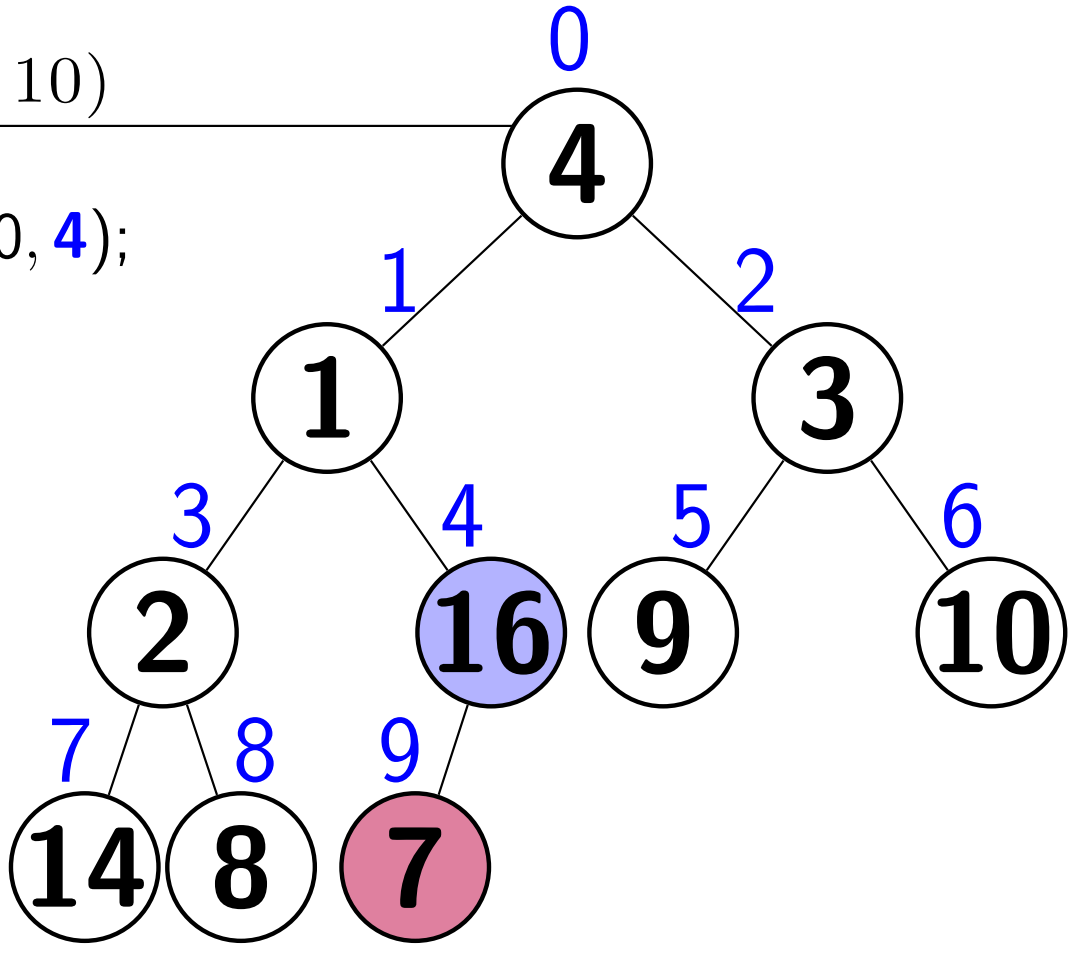
Construindo um Heap Máximo



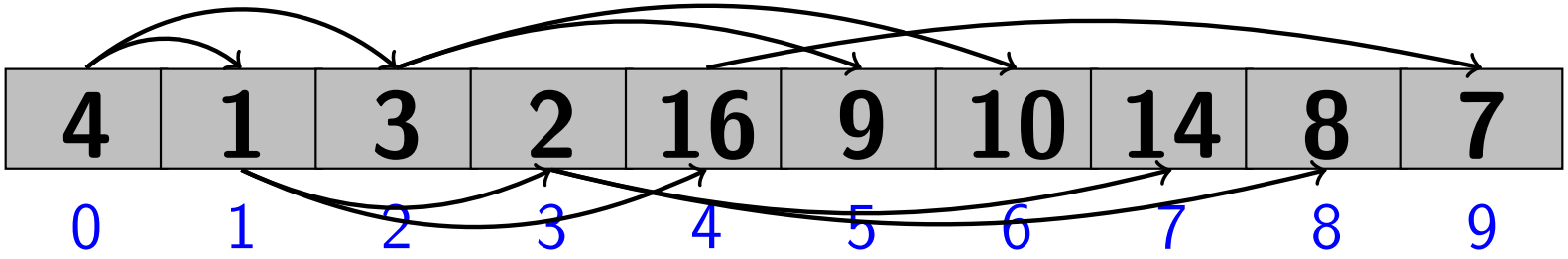
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{4})$;



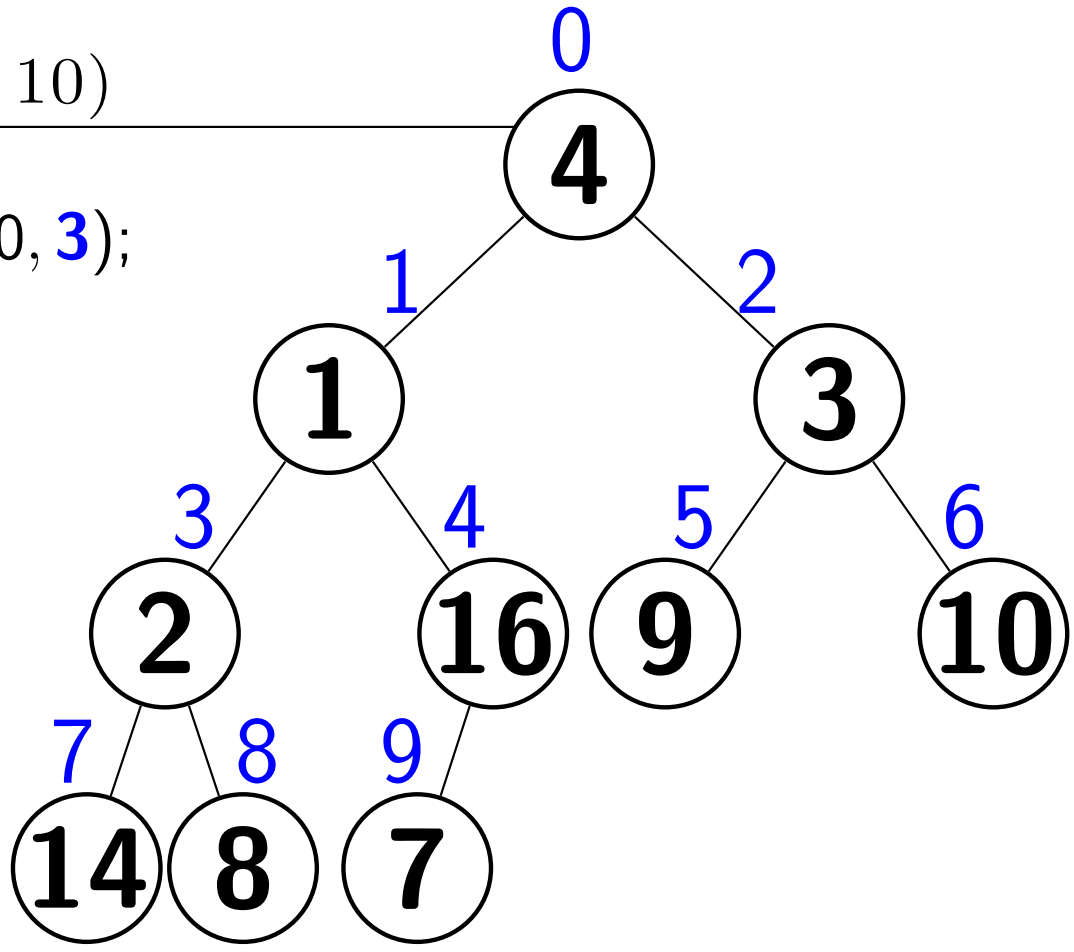
Construindo um Heap Máximo



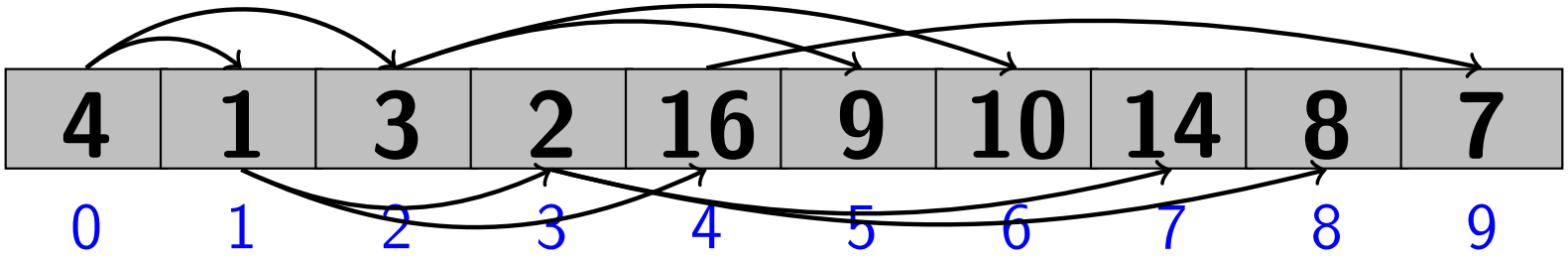
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{3})$;



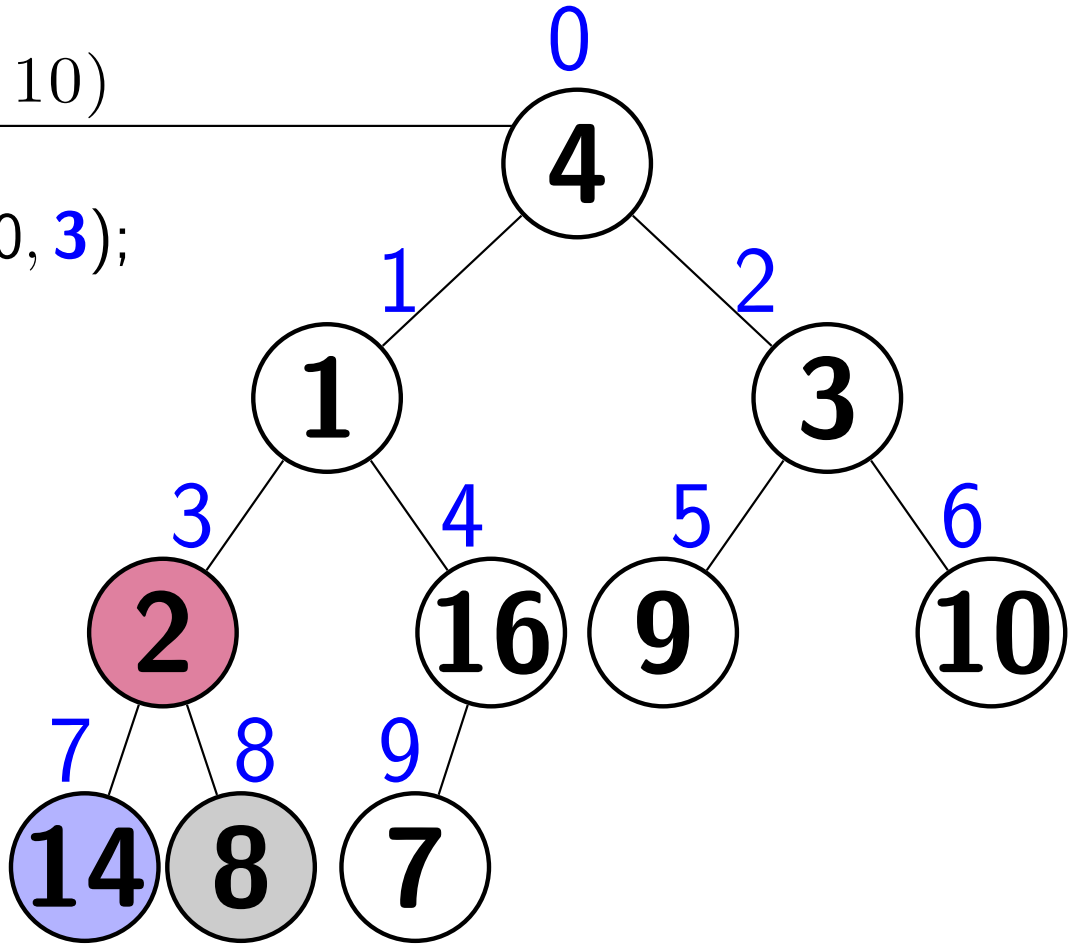
Construindo um Heap Máximo



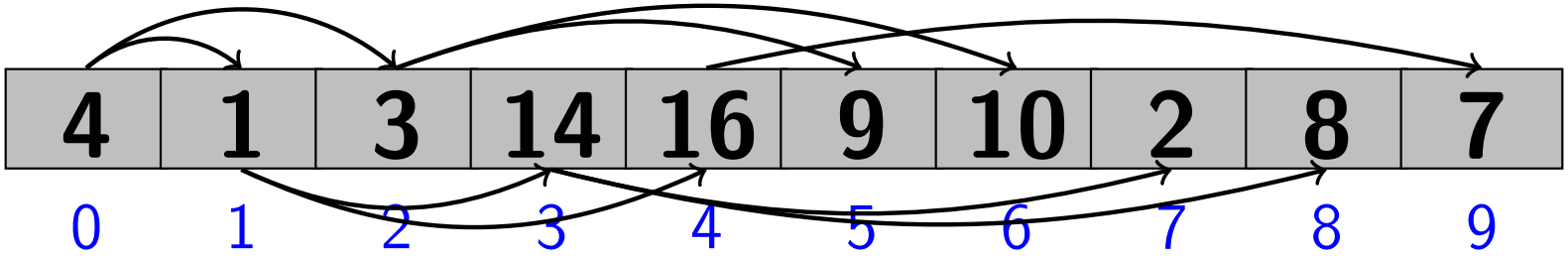
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{3})$;



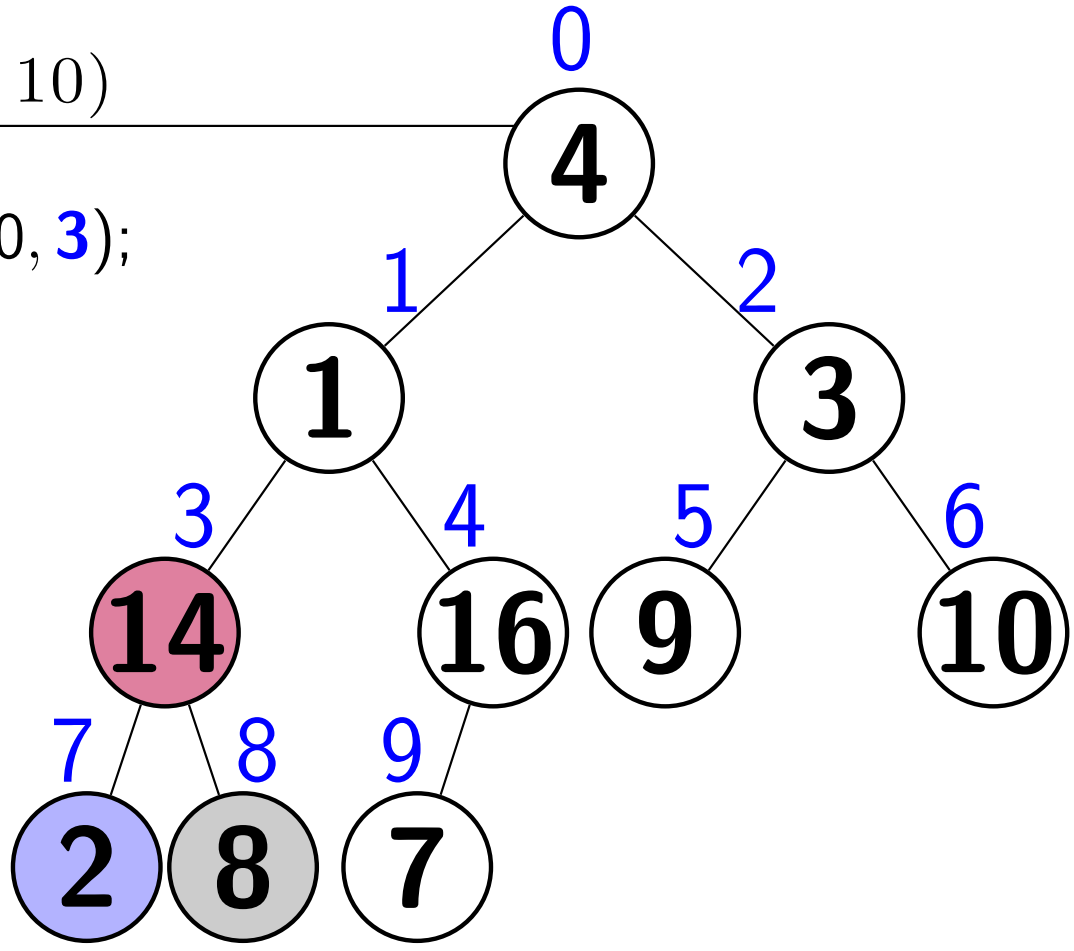
Construindo um Heap Máximo



BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{3})$;



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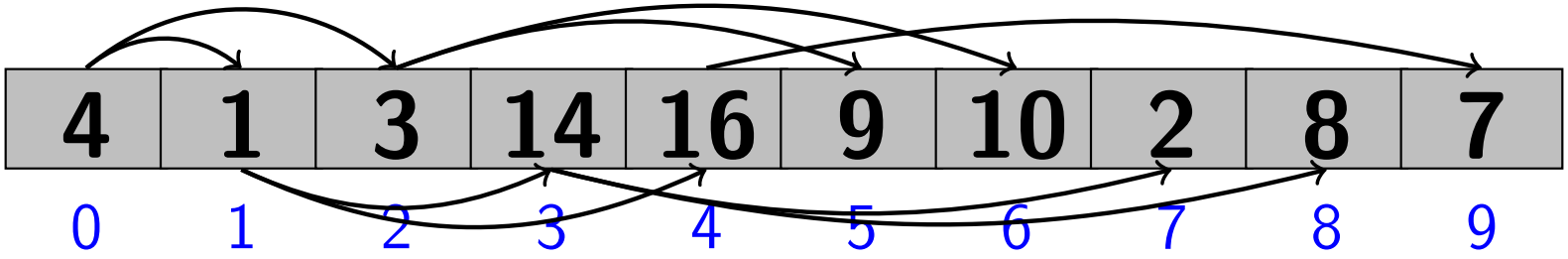
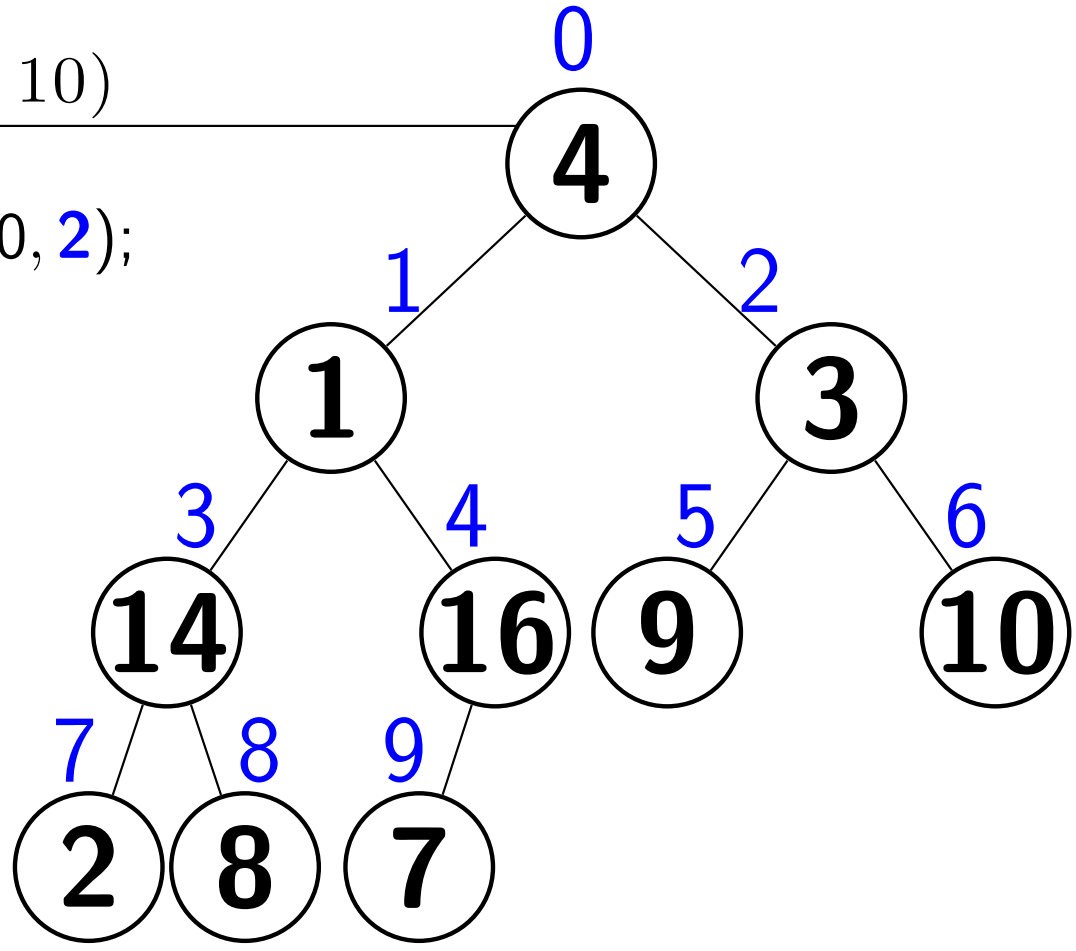


BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{2})$;



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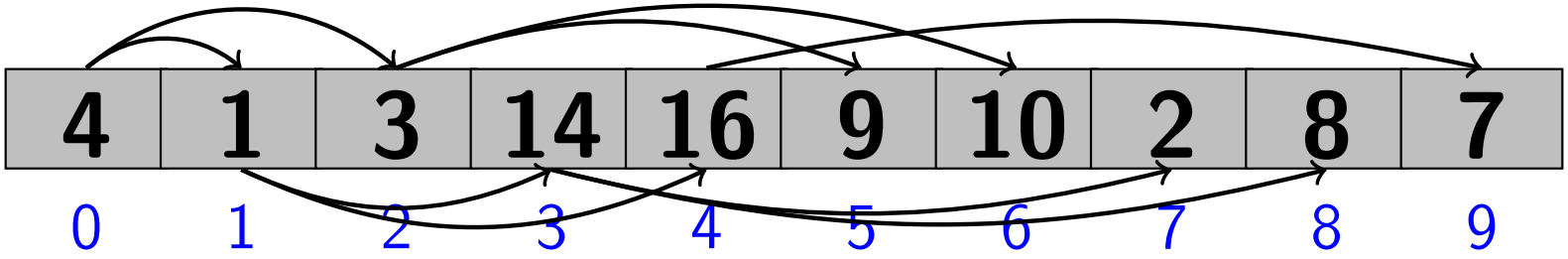
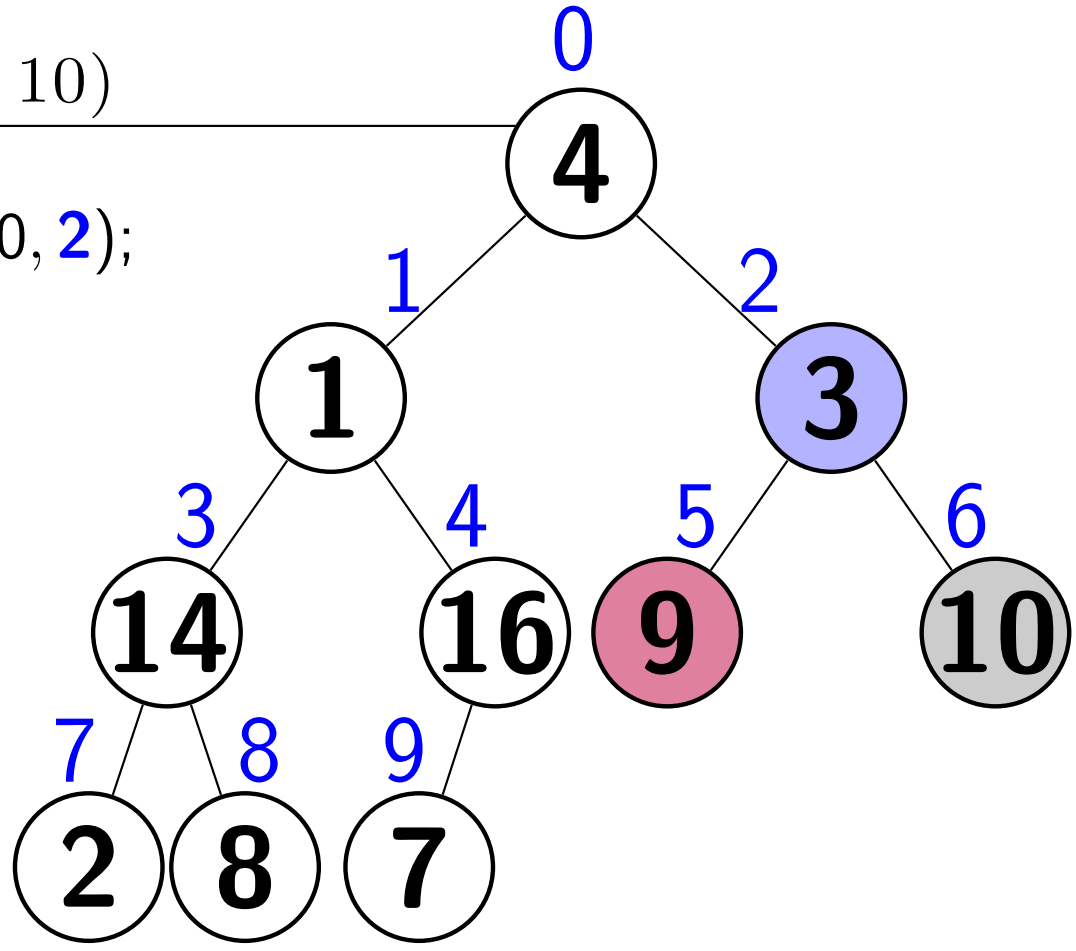


BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{2})$;



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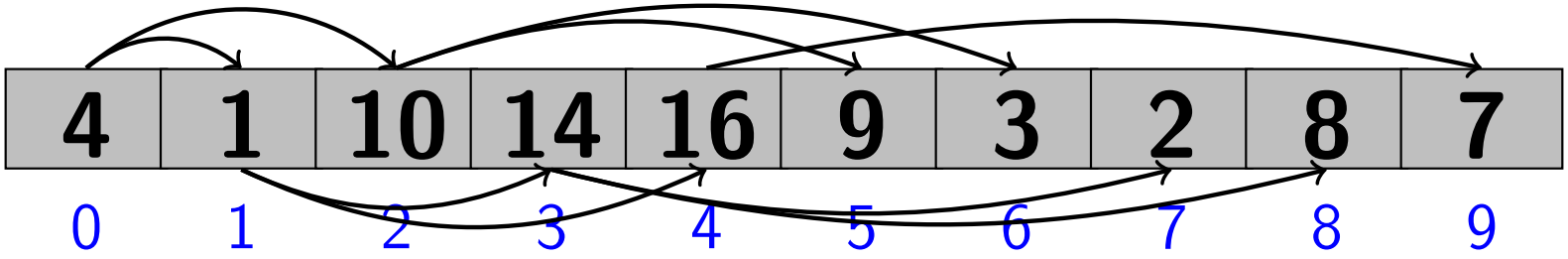
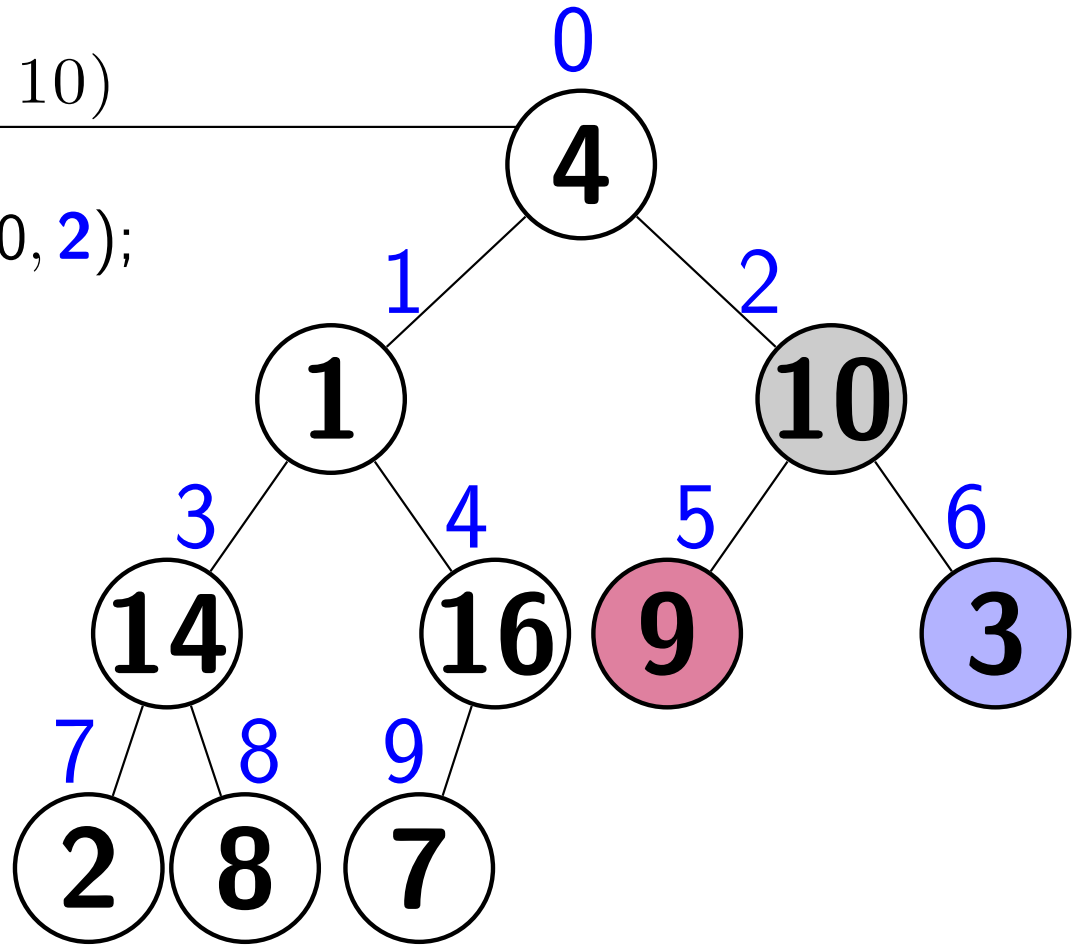


BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{2})$;



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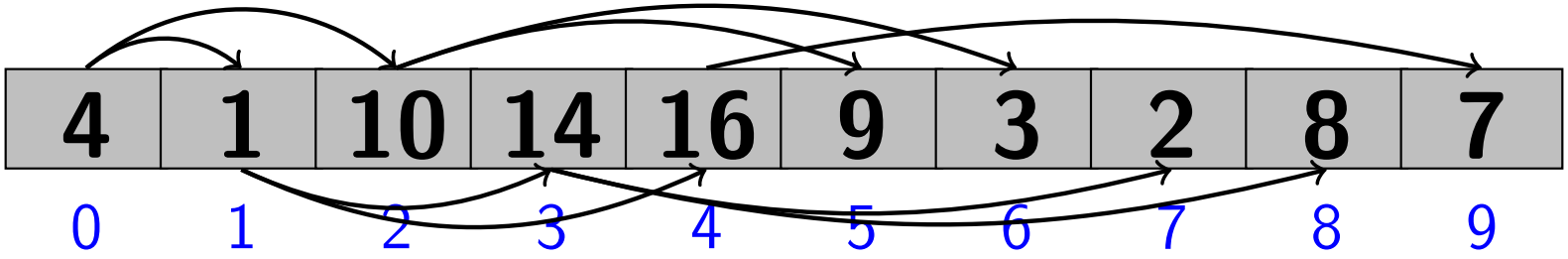
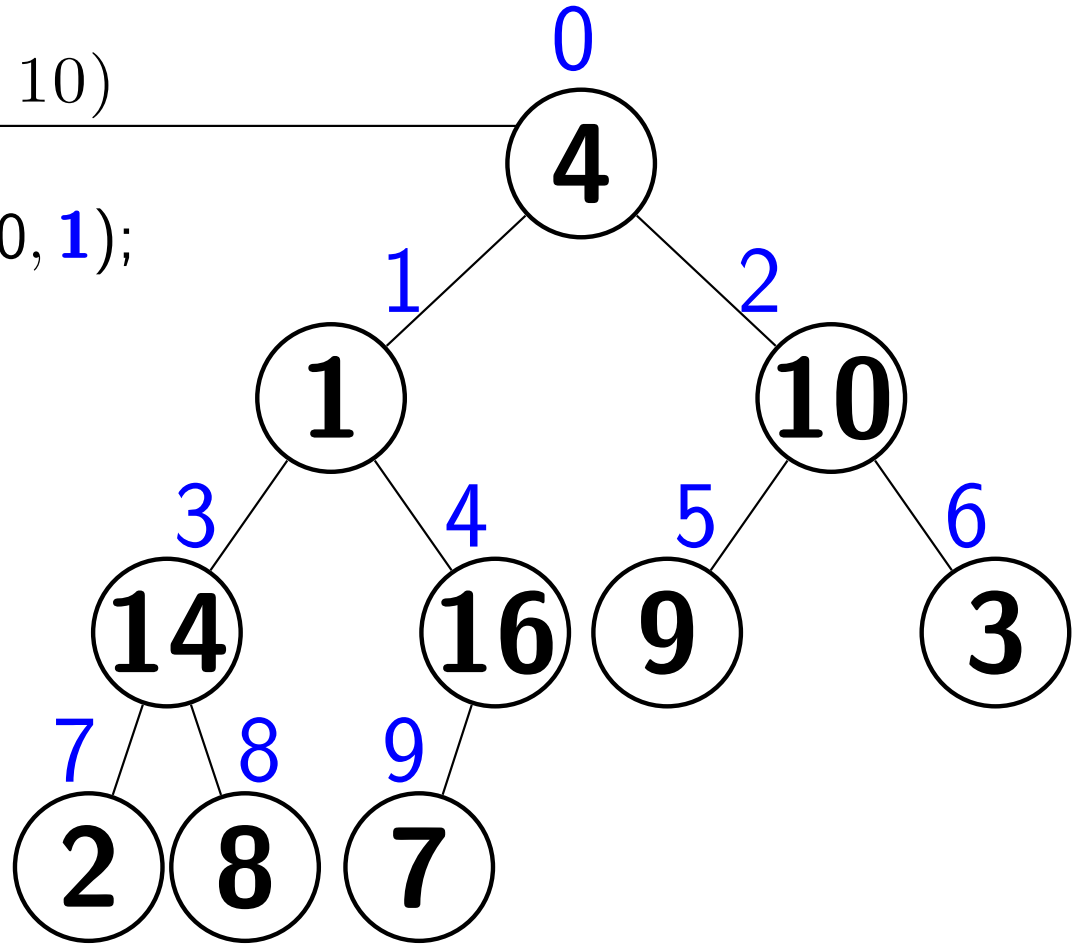


BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{1})$;



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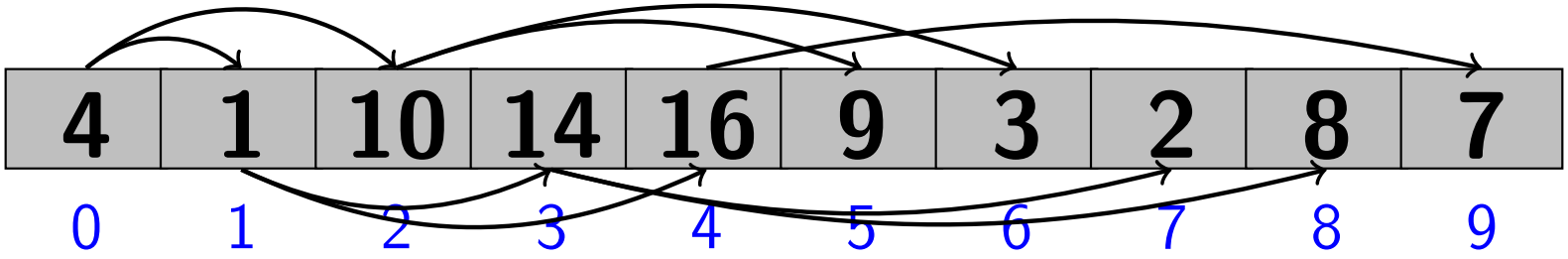

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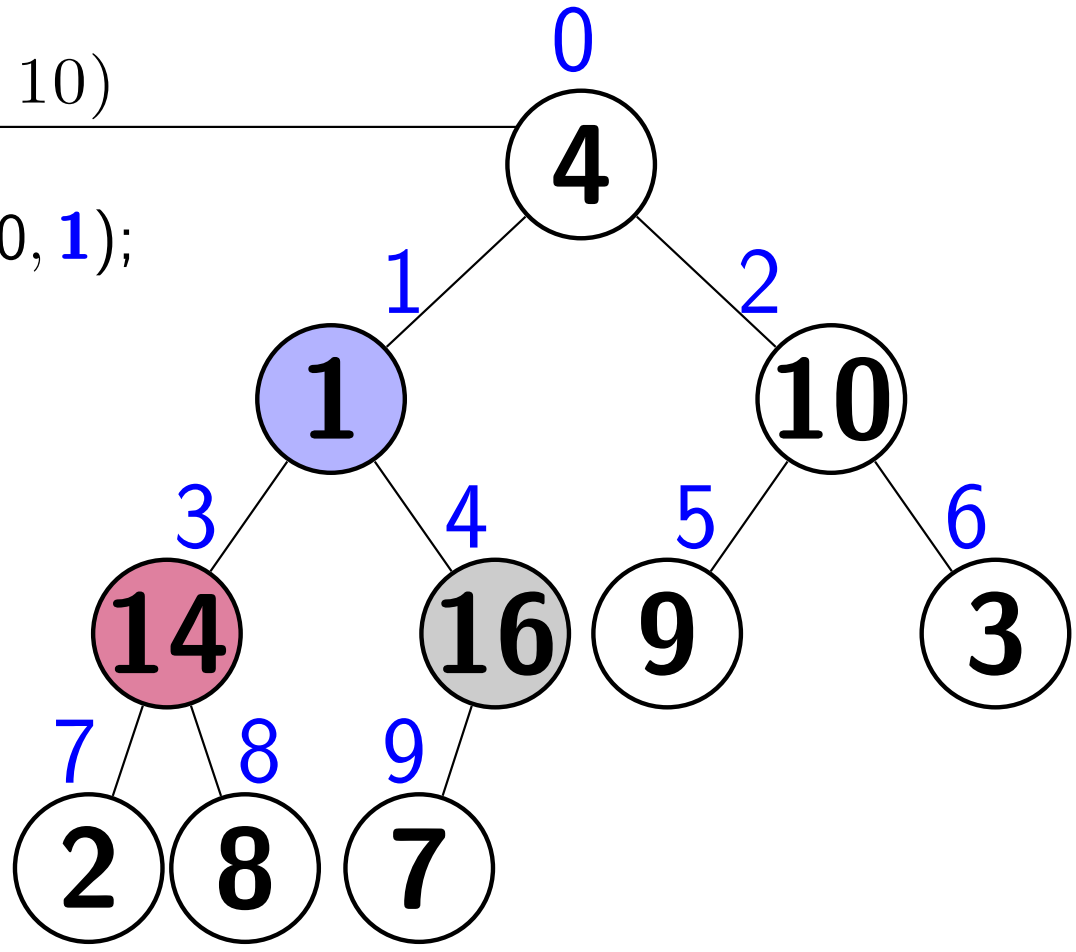

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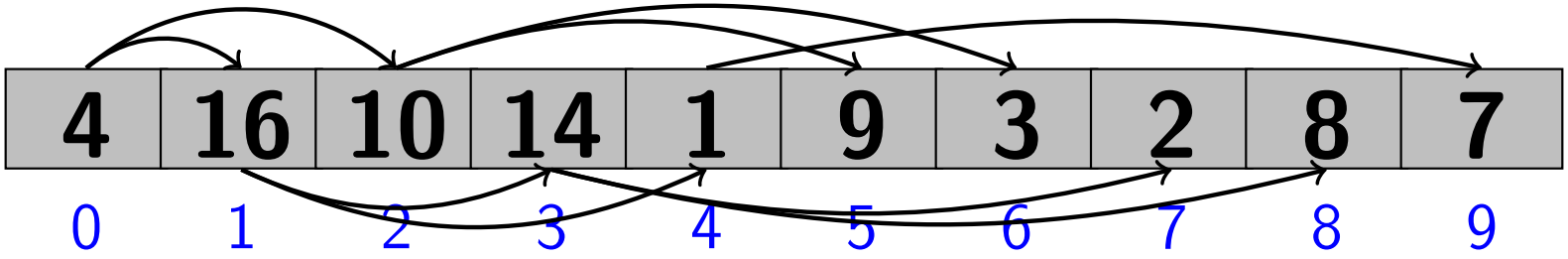

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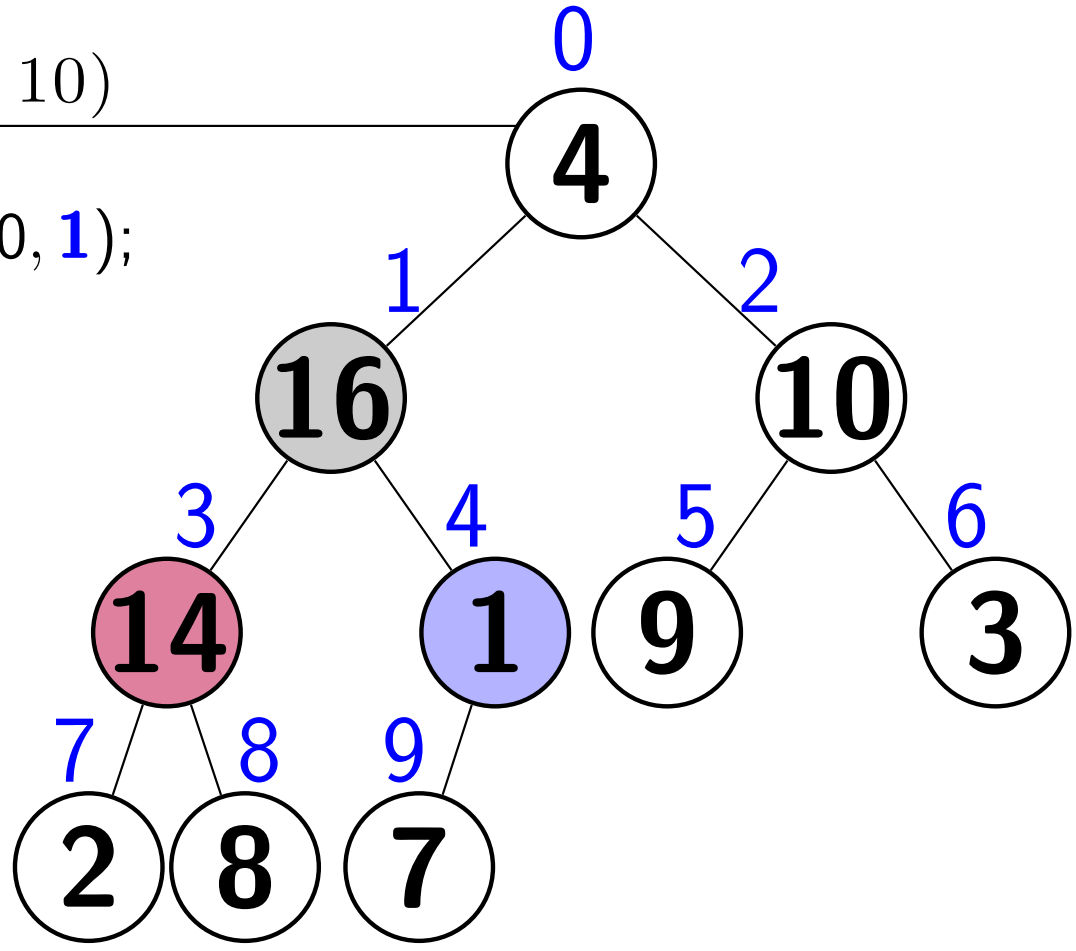

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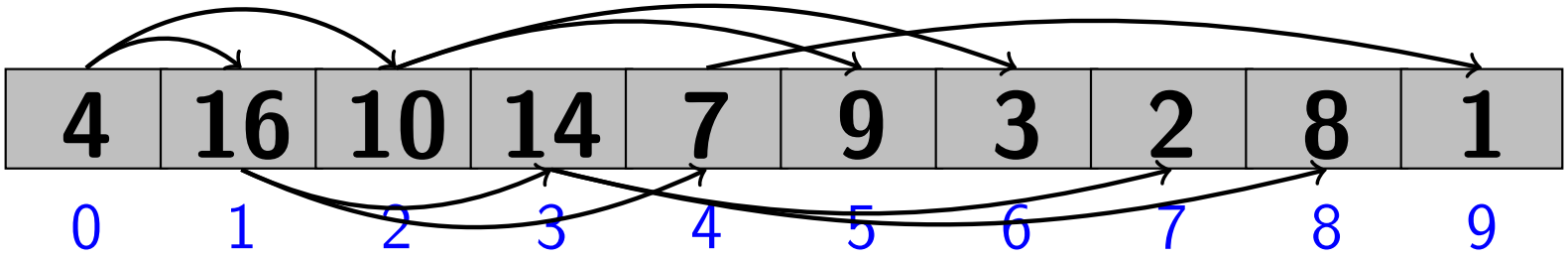

BUILD-MAX-HEAP $(V, 10)$

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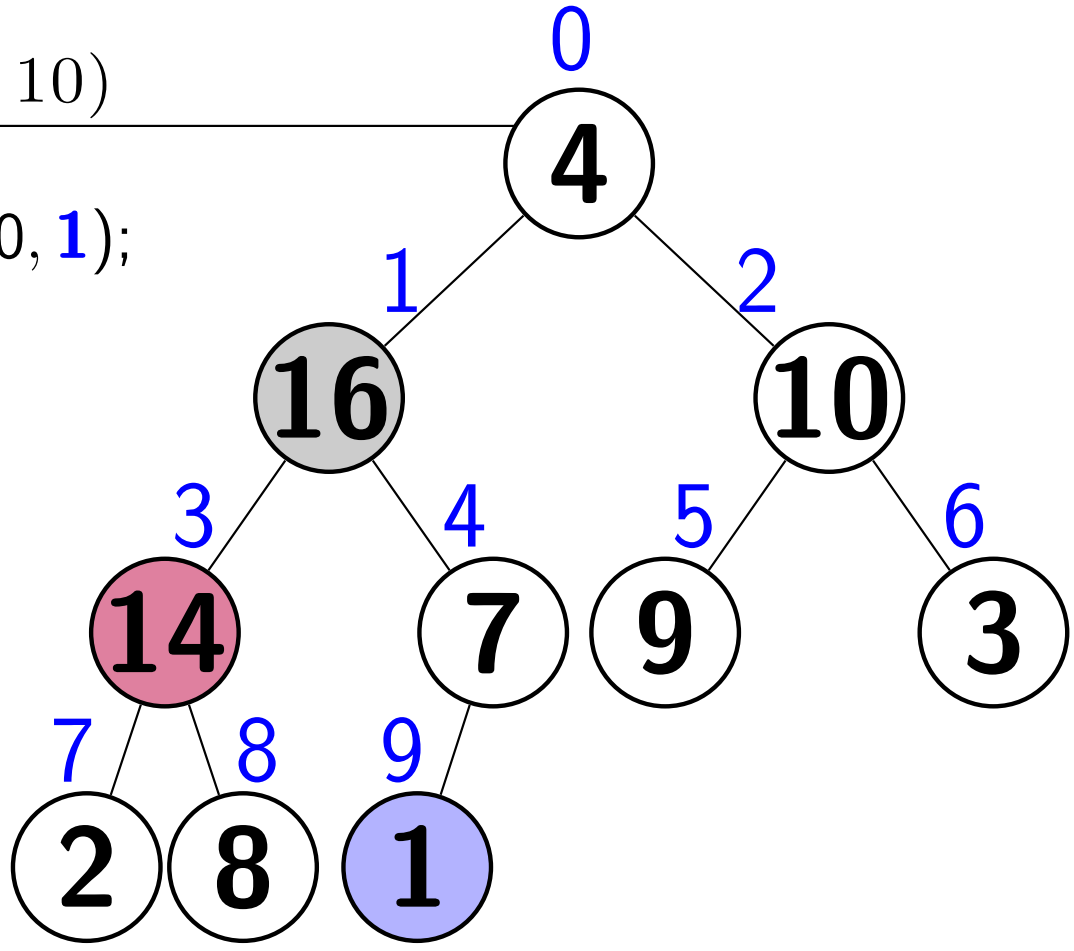
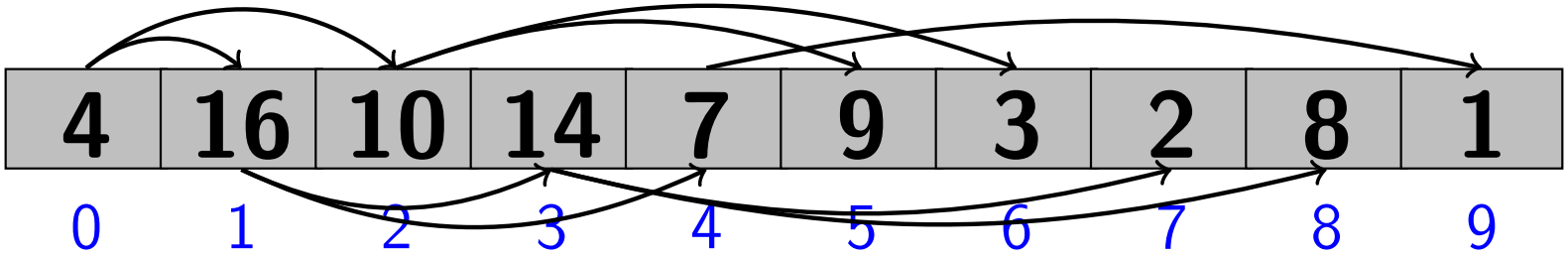
    MAX-HEAPIFY $(V, 10, \mathbf{1})$;



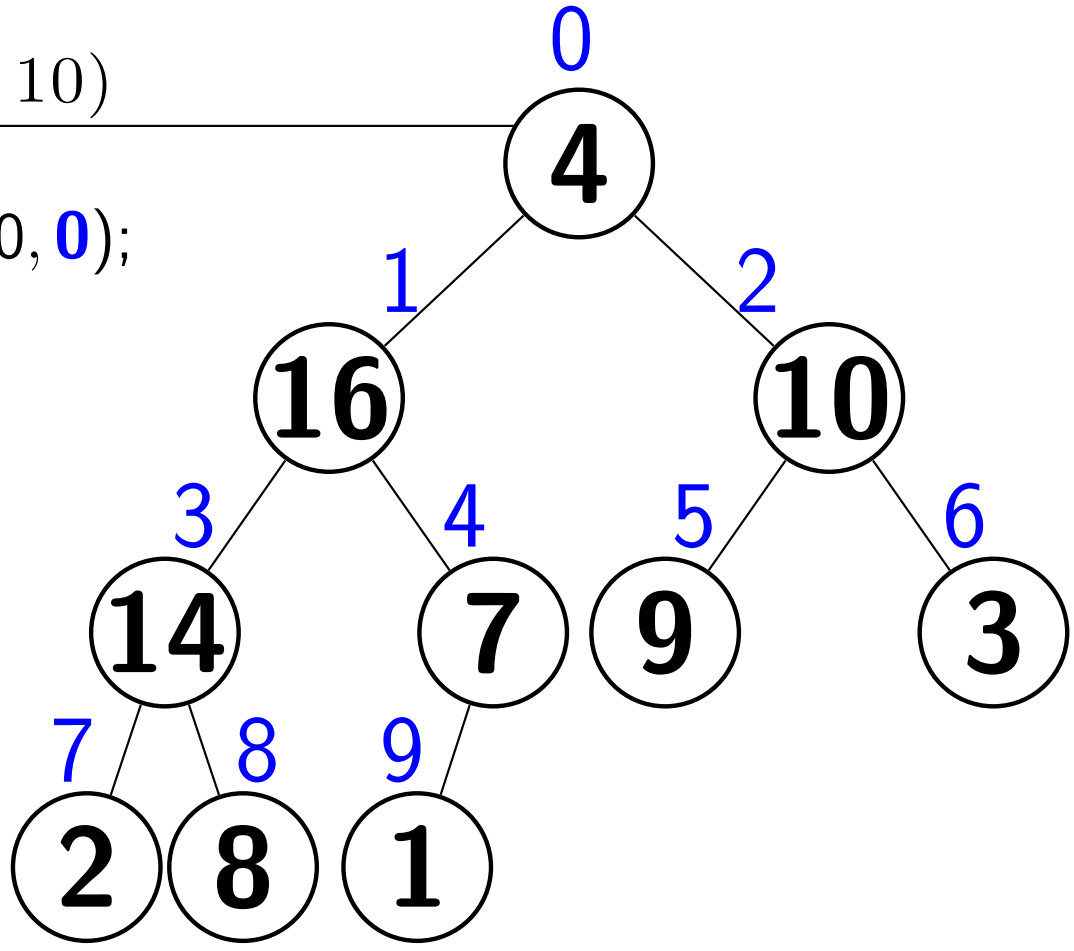
Construindo um Heap Máximo



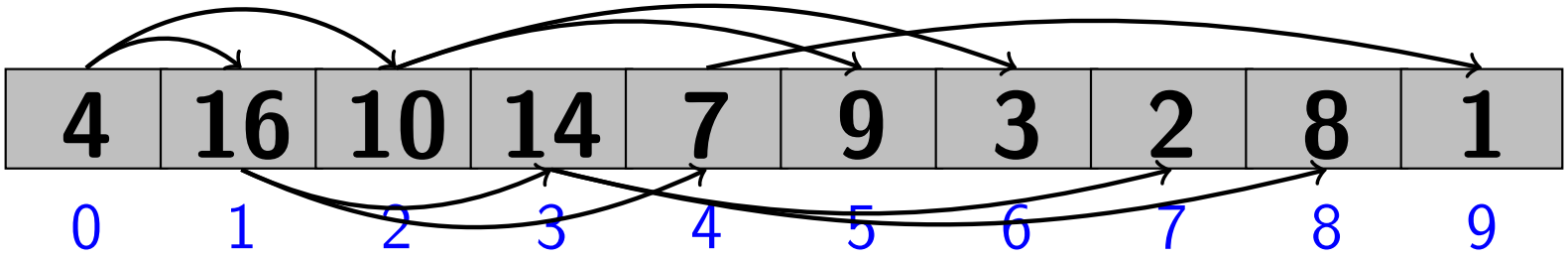
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;



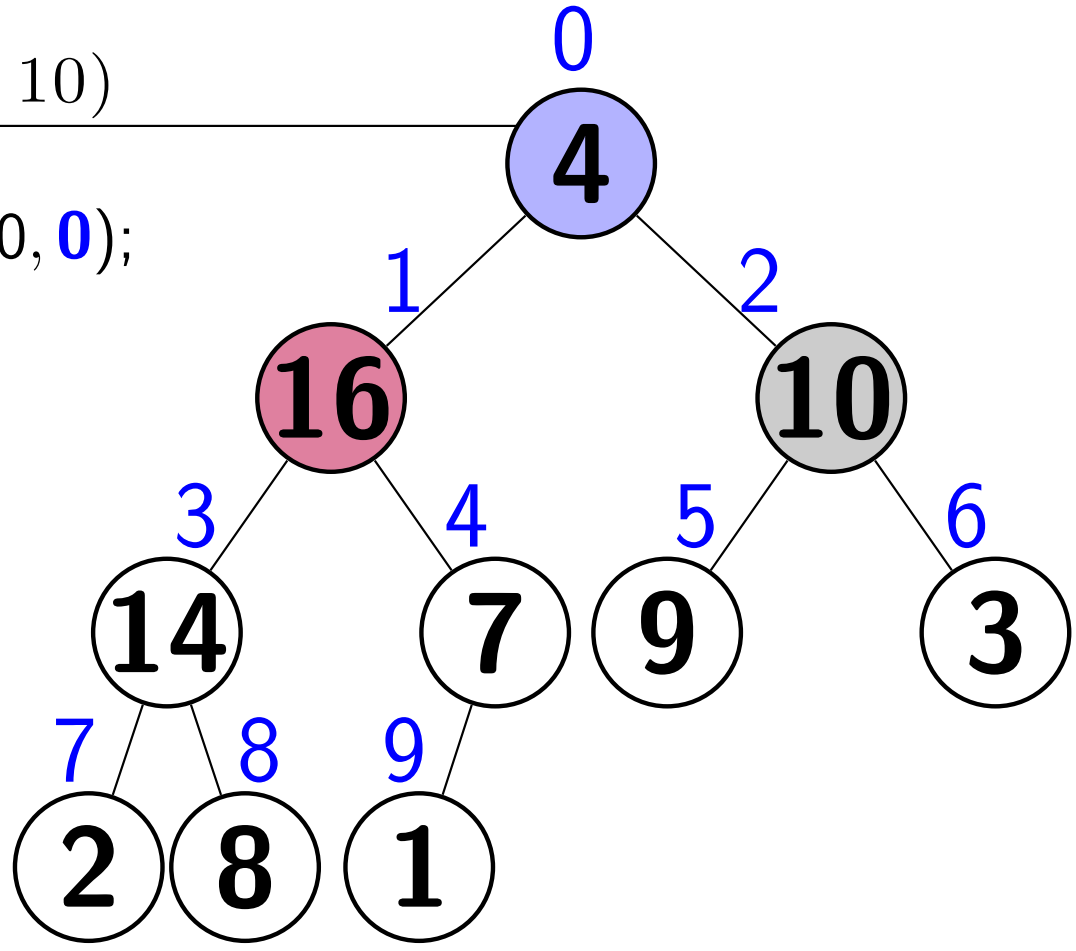
Construindo um Heap Máximo



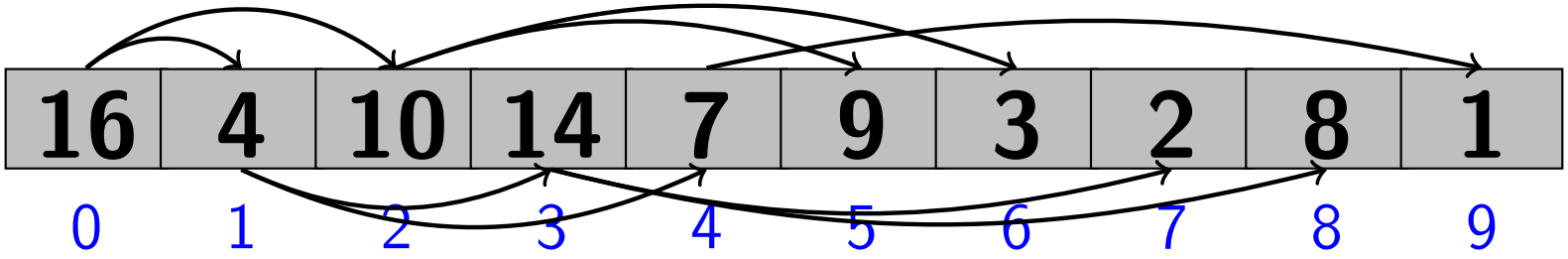
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;



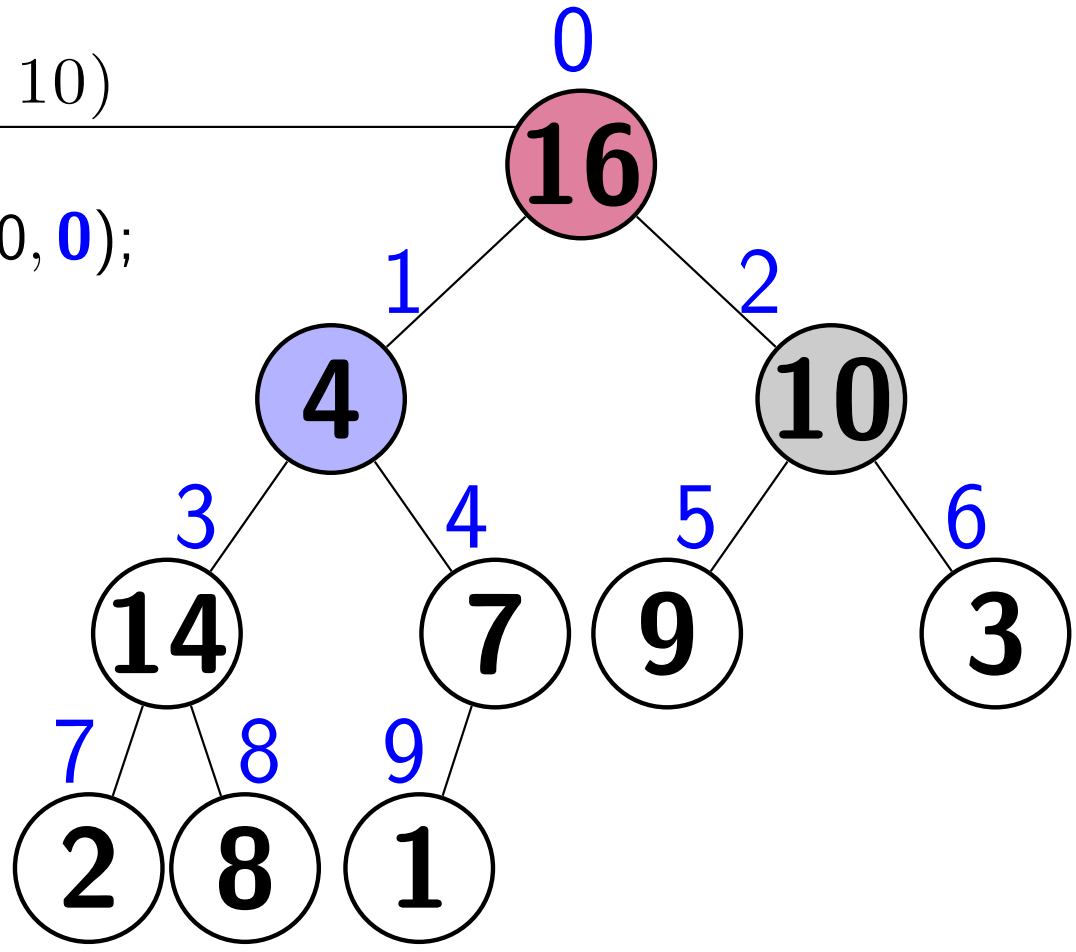
Construindo um Heap Máximo



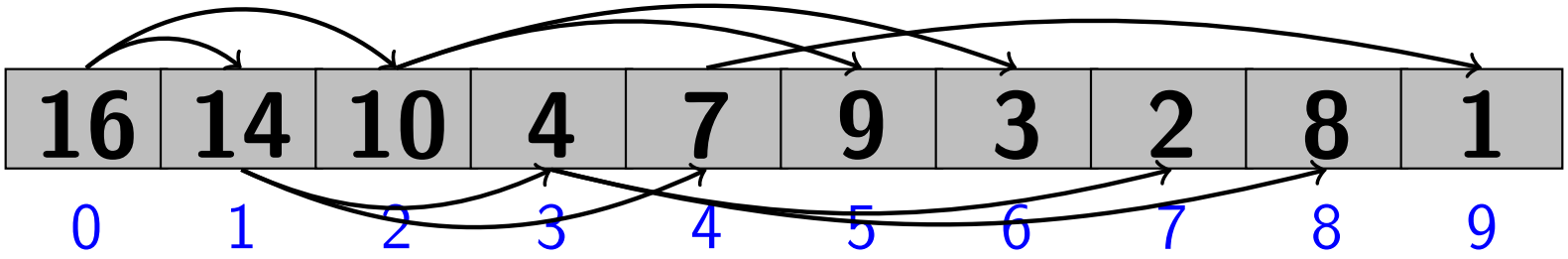
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;



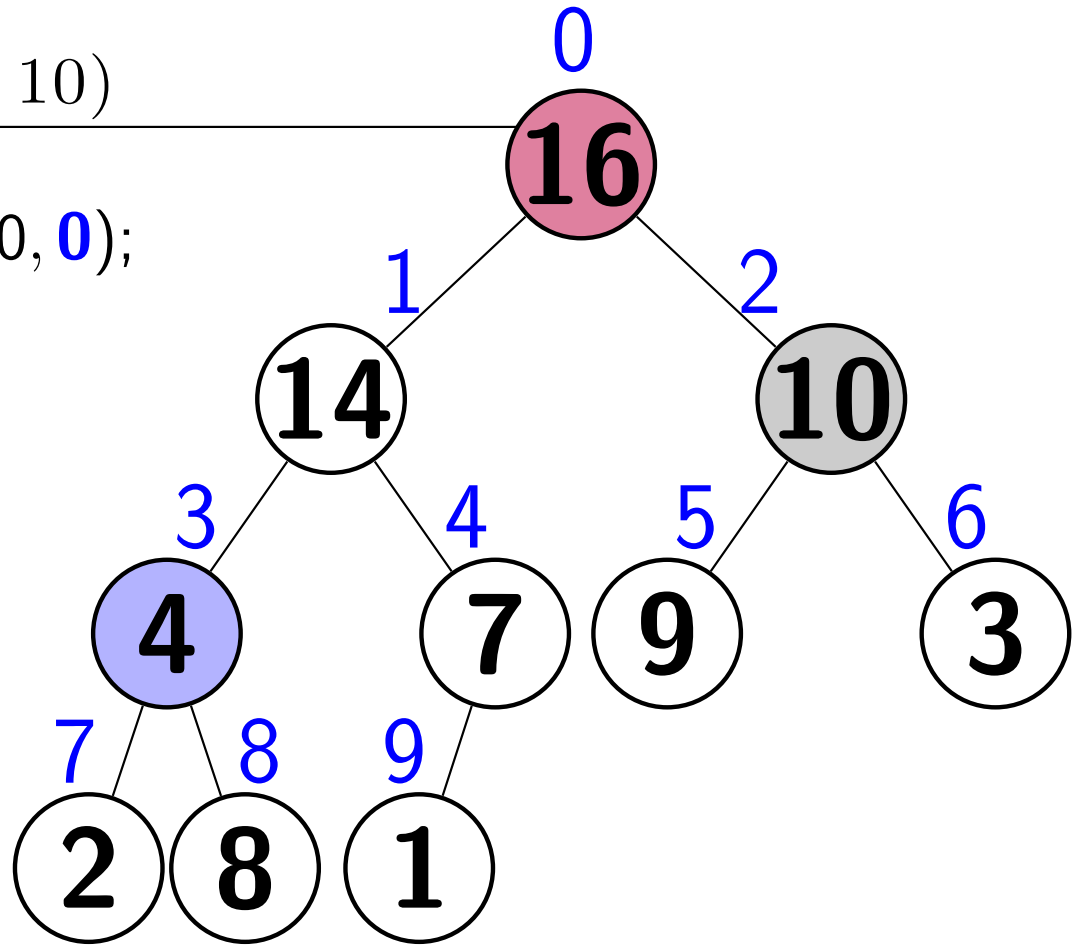
Construindo um Heap Máximo



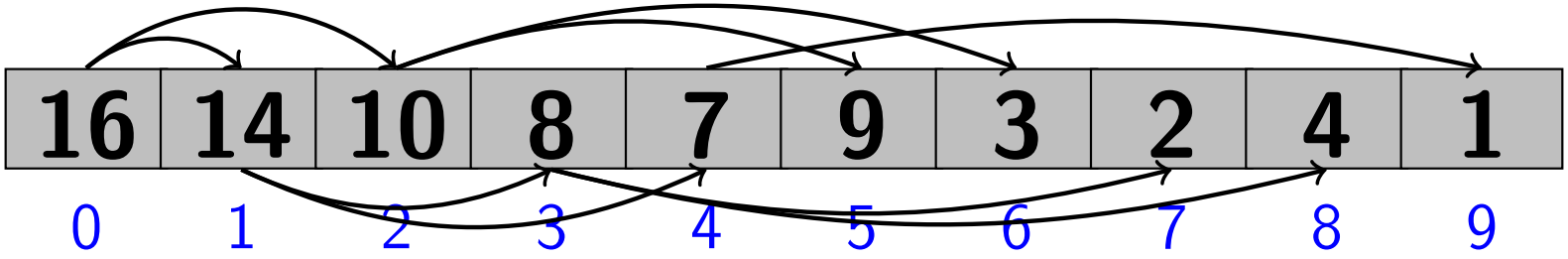
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;



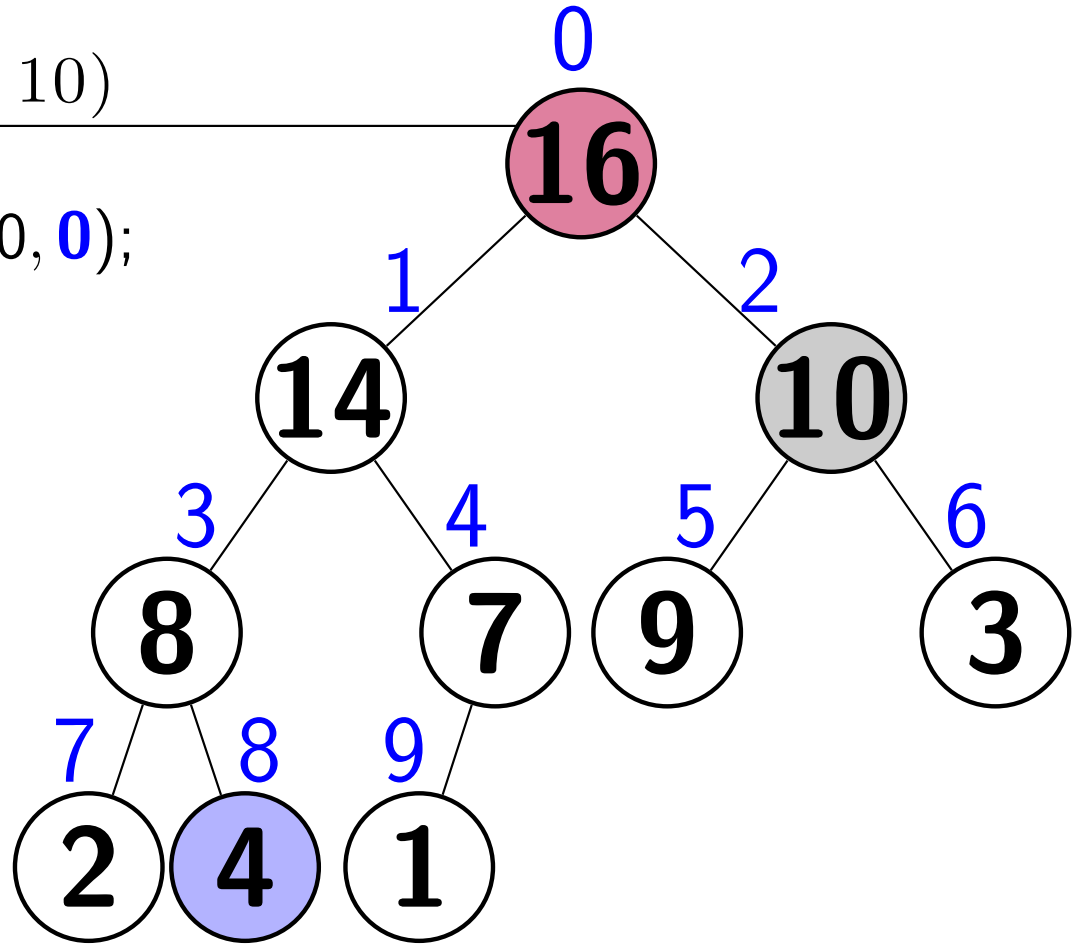
Construindo um Heap Máximo



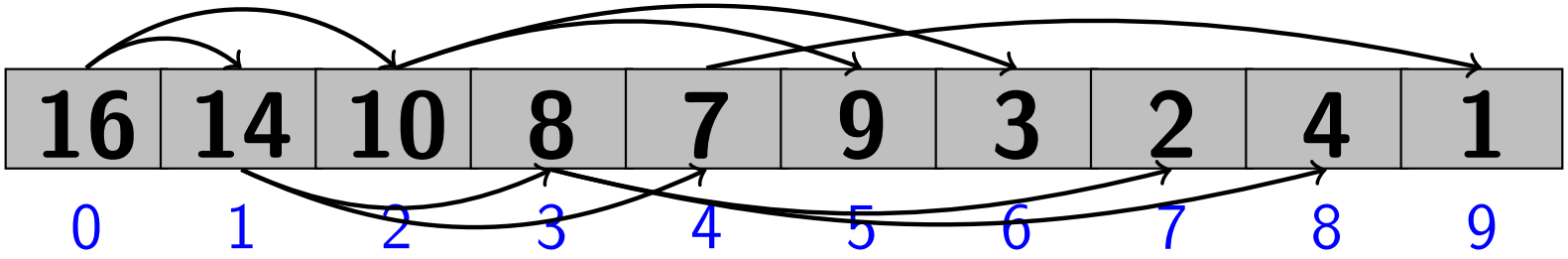
BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;



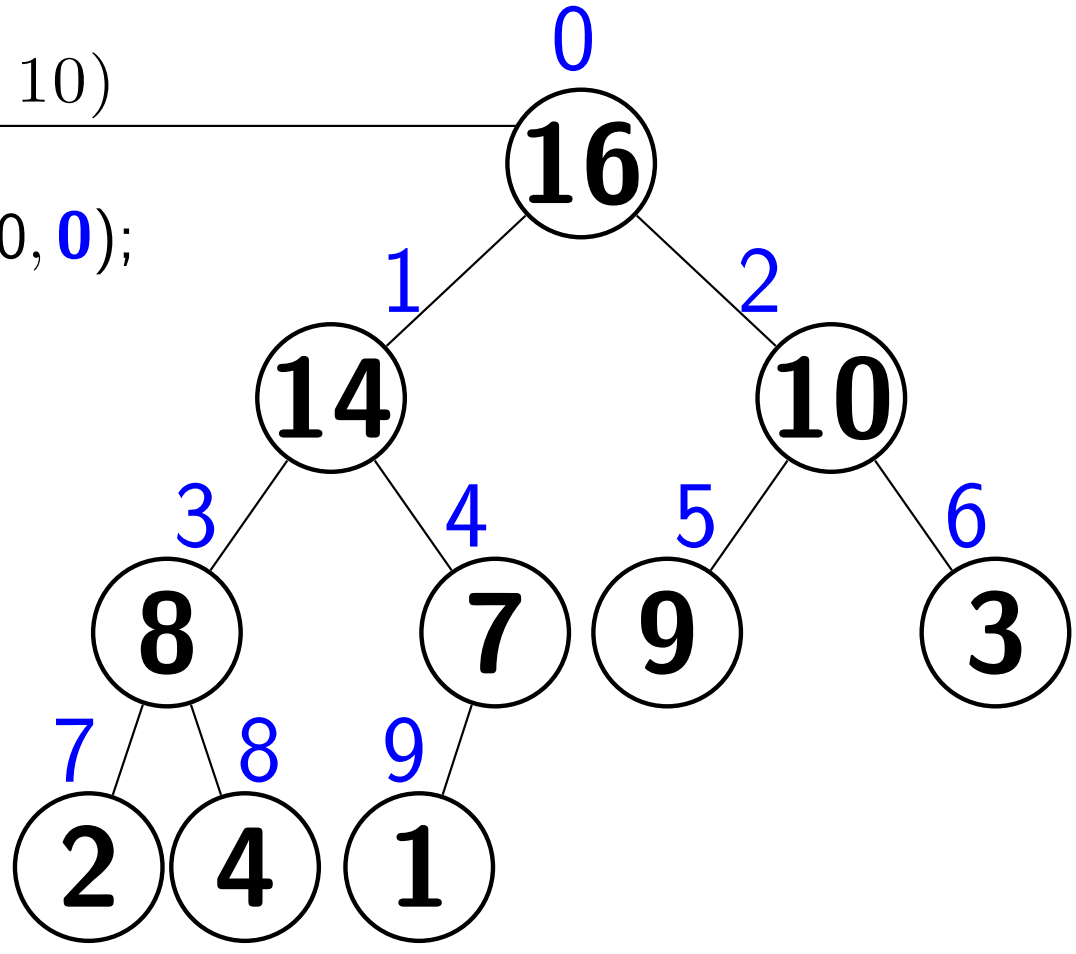
Construindo um Heap Máximo



BUILD-MAX-HEAP $(V, 10)$

Para **i** de 4 **até** 0

    MAX-HEAPIFY $(V, 10, \mathbf{0})$;

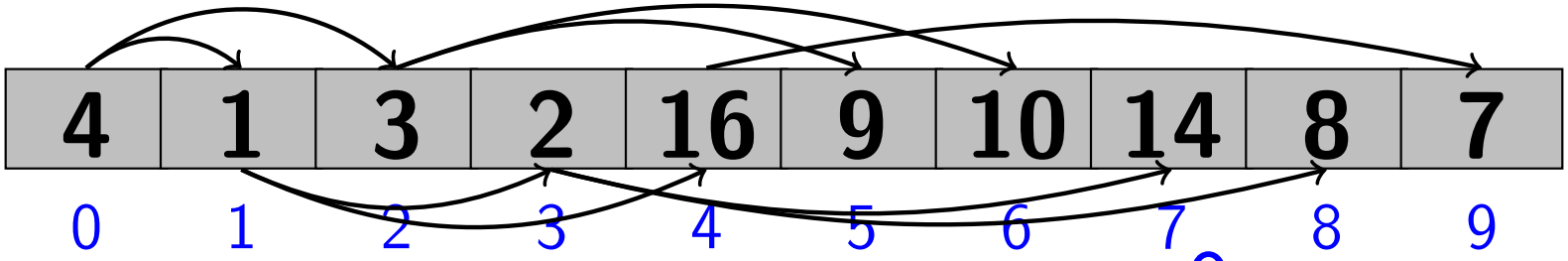


Construindo um Heap Máximo

Complexidade: um limite superior simples para o procedimento BUILD-MAX-HEAP pode ser obtido através do número de $\mathcal{O}(n)$ chamadas do procedimento MAX-HEAPIFY que por sua vez tem um custo de $\mathcal{O}(\log n)$. Deste modo, o tempo de execução do BUILD-MAX-HEAP é $\mathcal{O}(n \log n)$. Embora esse limite seja correto, ele não é restrito, veja em Cormen um limite mais exato.

Sumário

Heap-Sort



HEAP-SORT (V, SIZE)

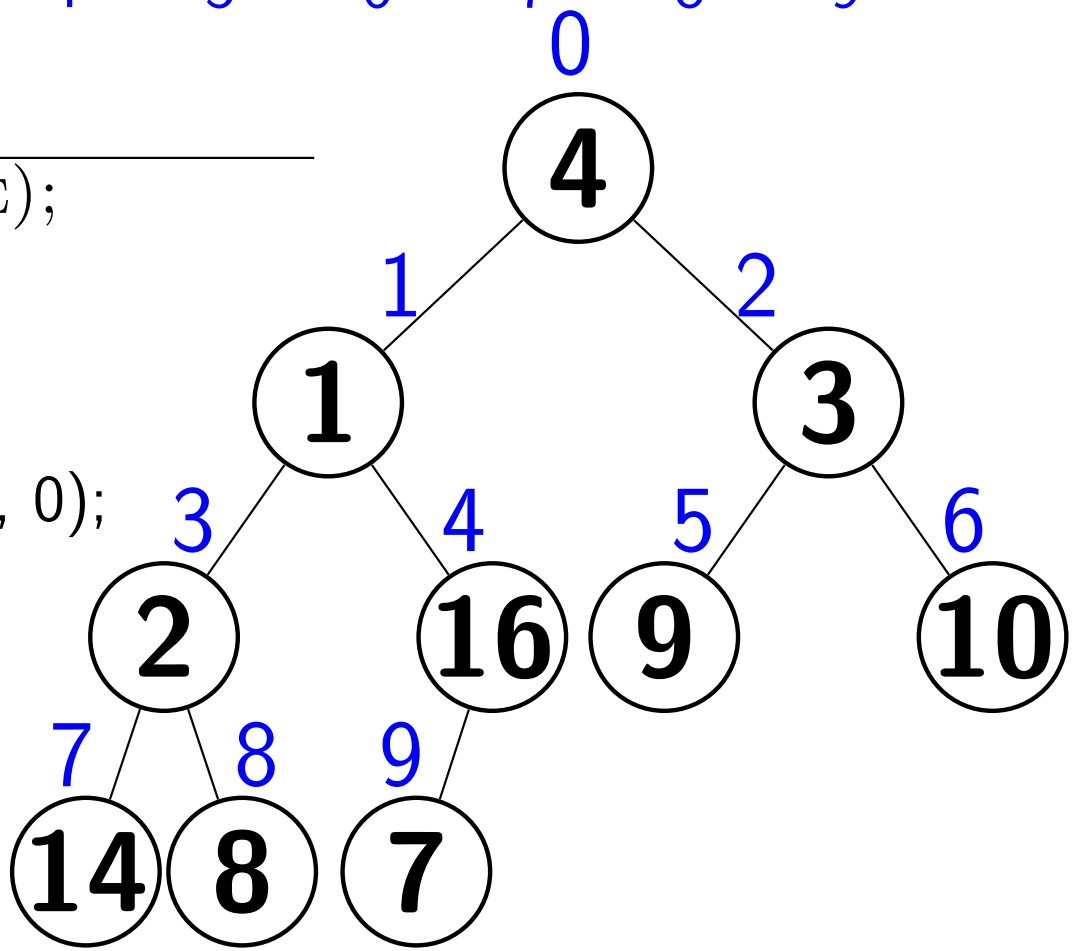
BUILD-MAX-HEAP (V,SIZE);

Para **i** de SIZE-1 **até** 1 **faça**

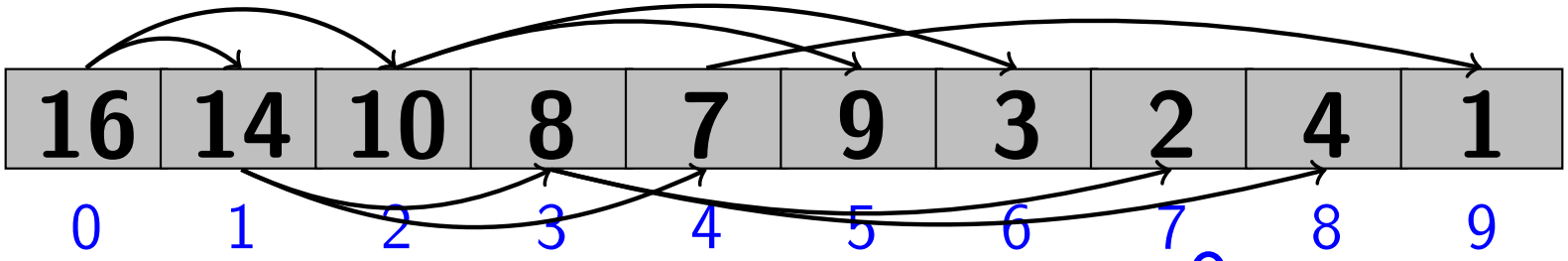
    $V[0] \leftrightarrow V[\mathbf{i}]$;

    SIZE = i;

    MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

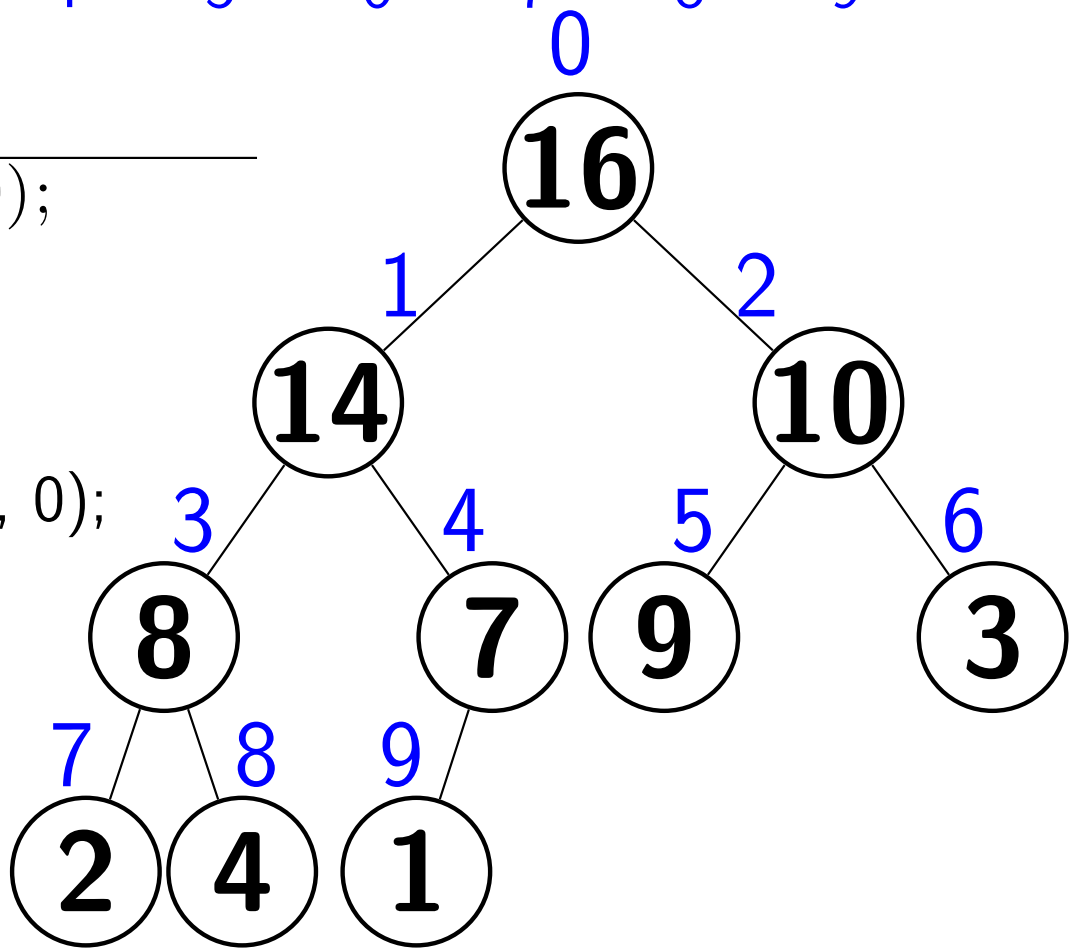
▷BUILD-MAX-HEAP $(V,10)$;

Para i de SIZE-1 **até** 1 **faça**

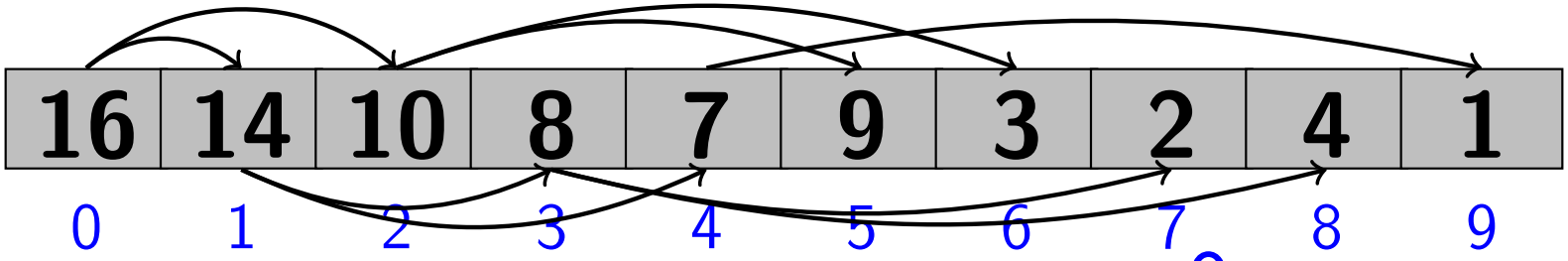
$V[0] \leftrightarrow V[\mathbf{i}]$;

SIZE $=$ i;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT (V, 10)

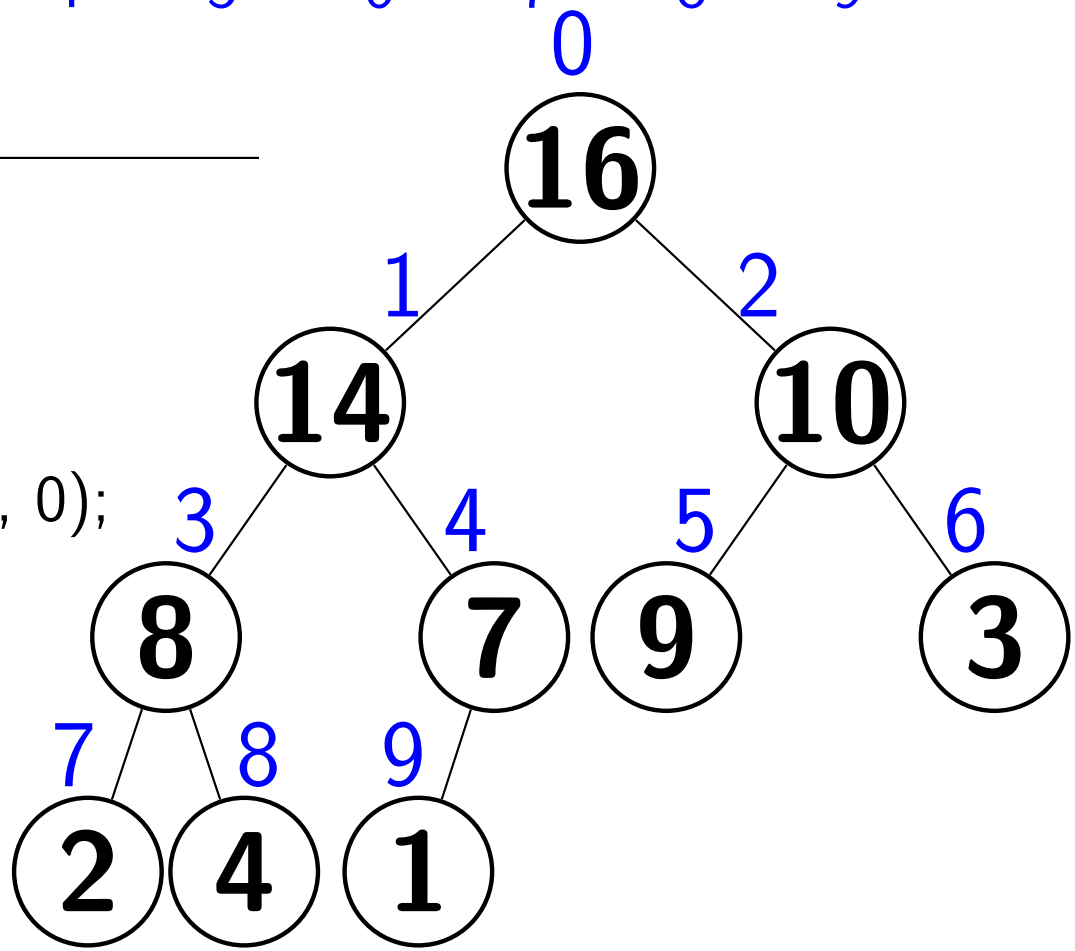
BUILD-MAX-HEAP (V,10);

▷**Para i** de 9 **até** 1 **faça**

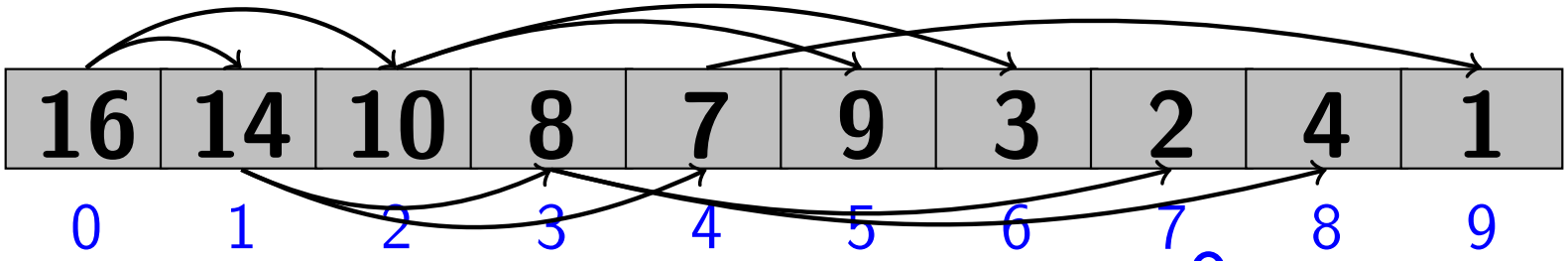
$V[0] \leftrightarrow V[\mathbf{i}]$;

SIZE = i;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT (V, 10)

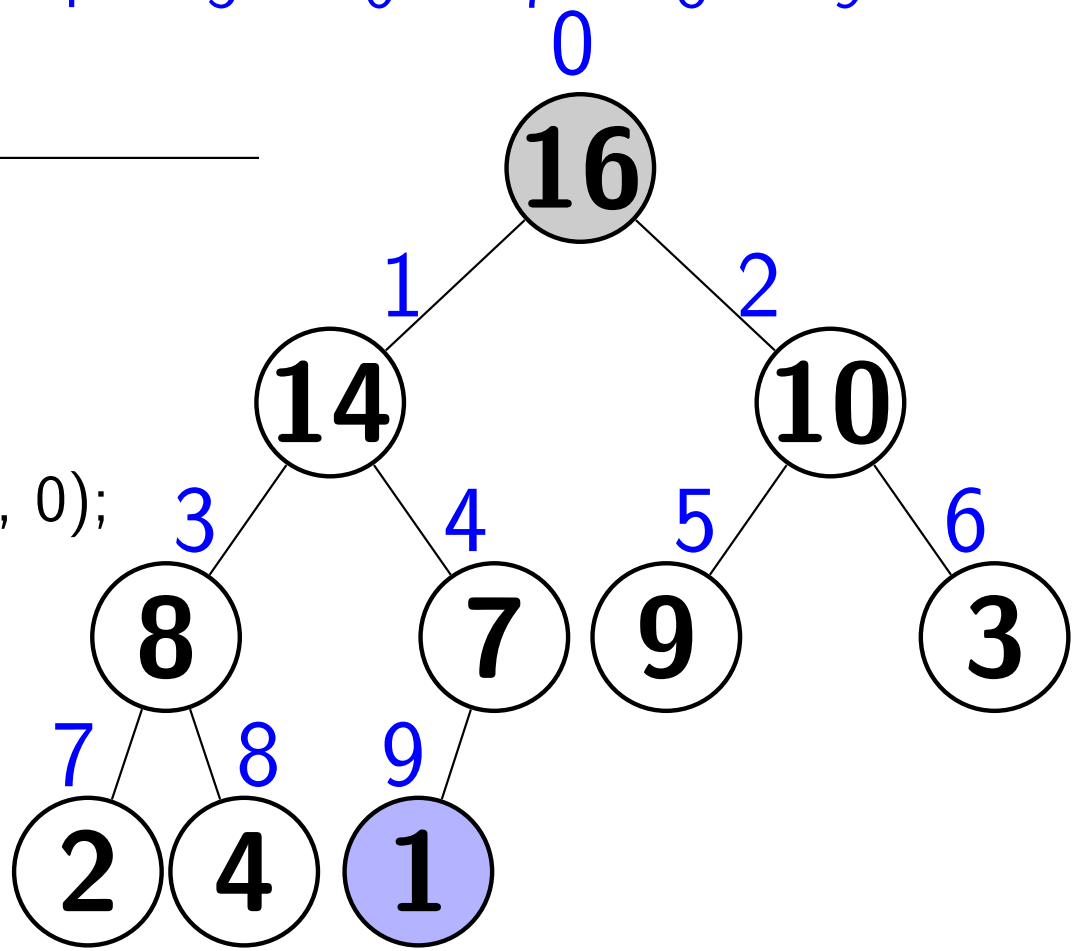
BUILD-MAX-HEAP (V,10);

Para i de 9 **até** 1 **faça**

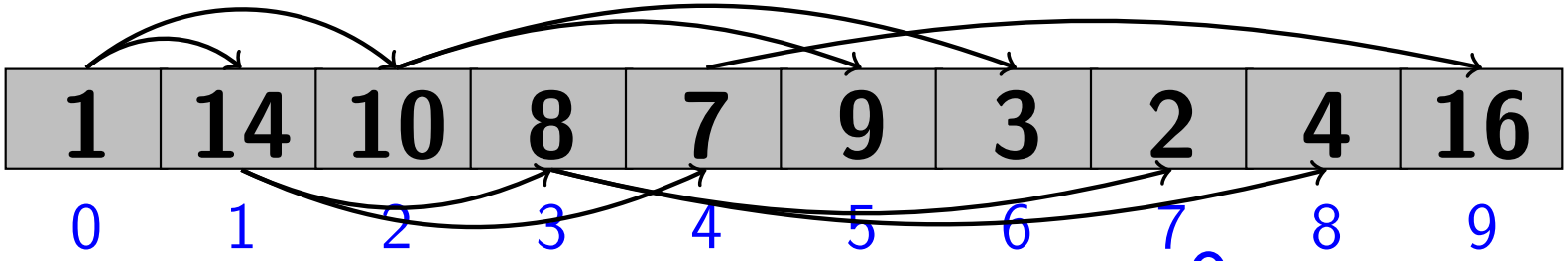
$\triangleright V[0] \leftrightarrow V[\mathbf{9}]$;

SIZE = i;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

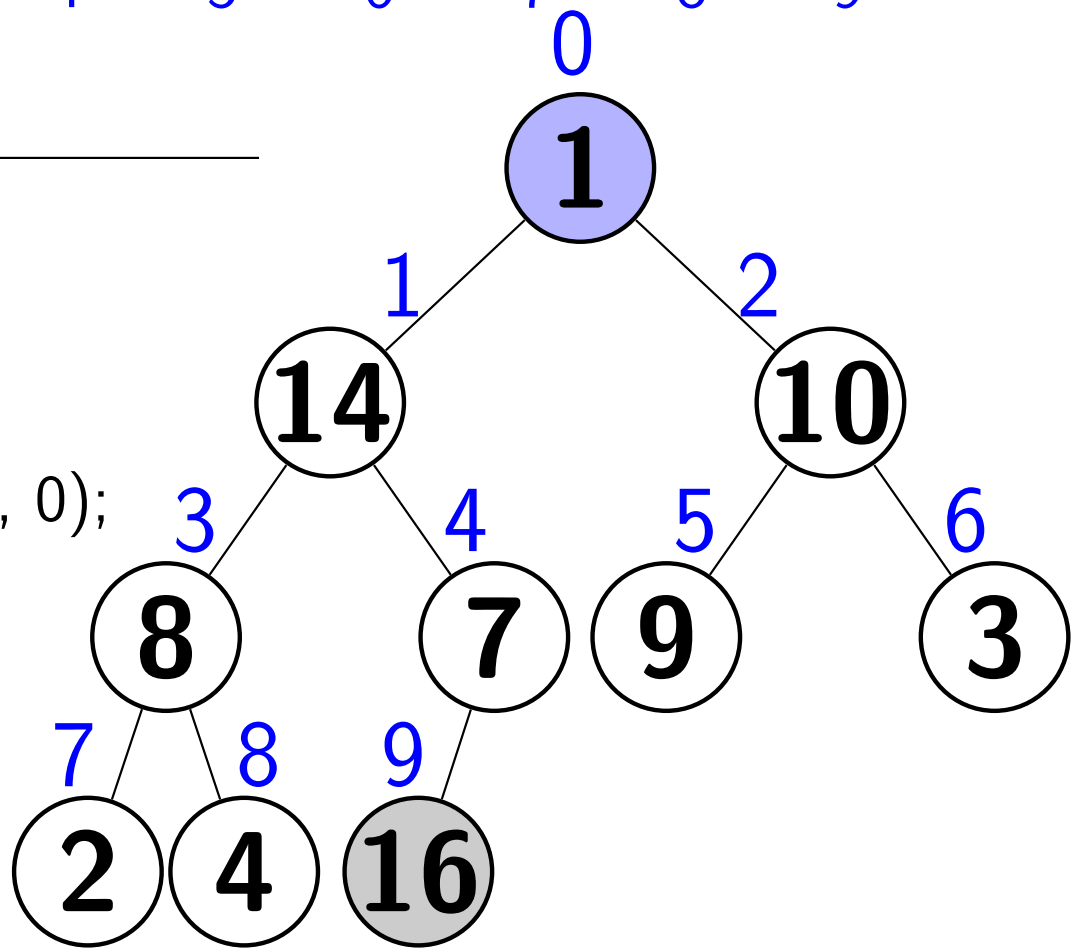
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

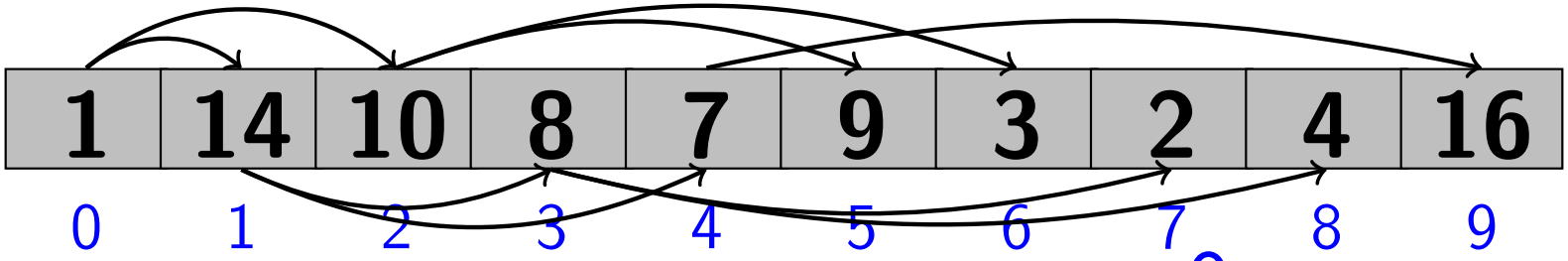
$\triangleright V[0] \leftrightarrow V[9]$;

SIZE $=$ i;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

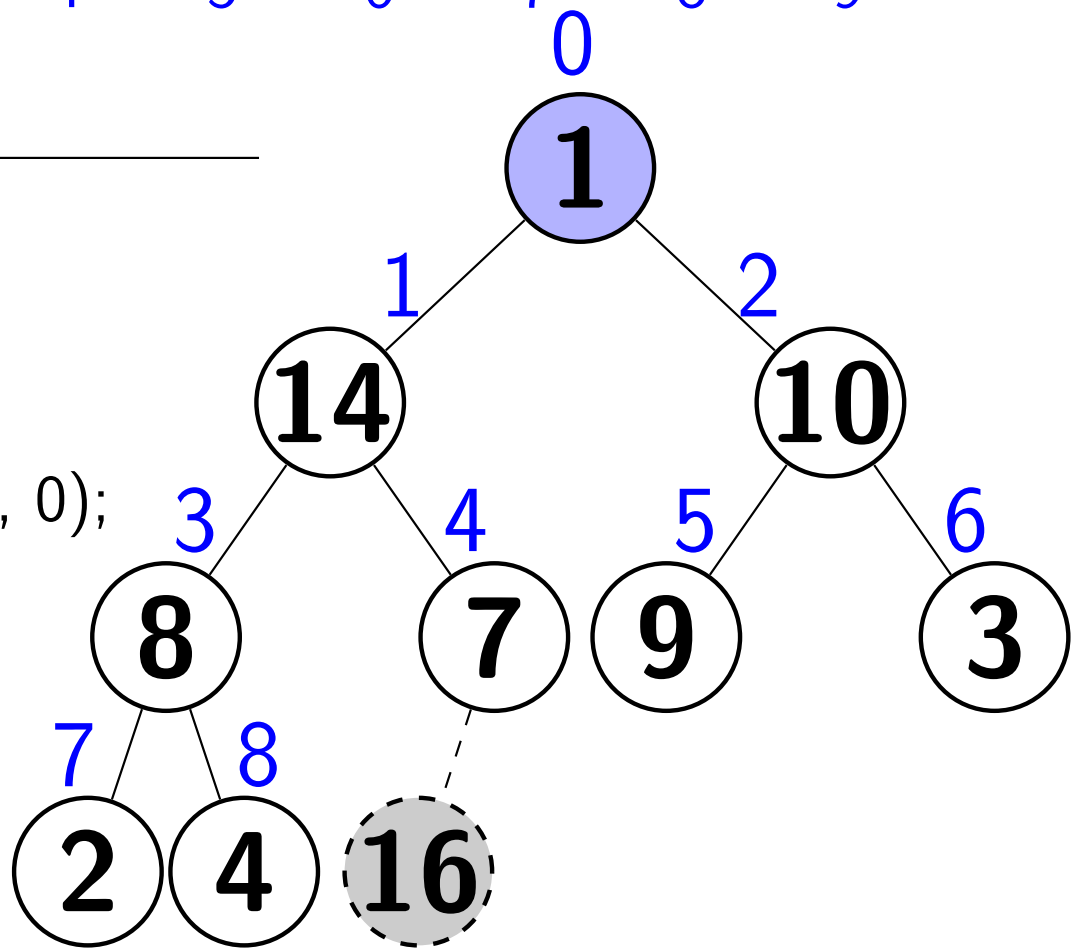
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

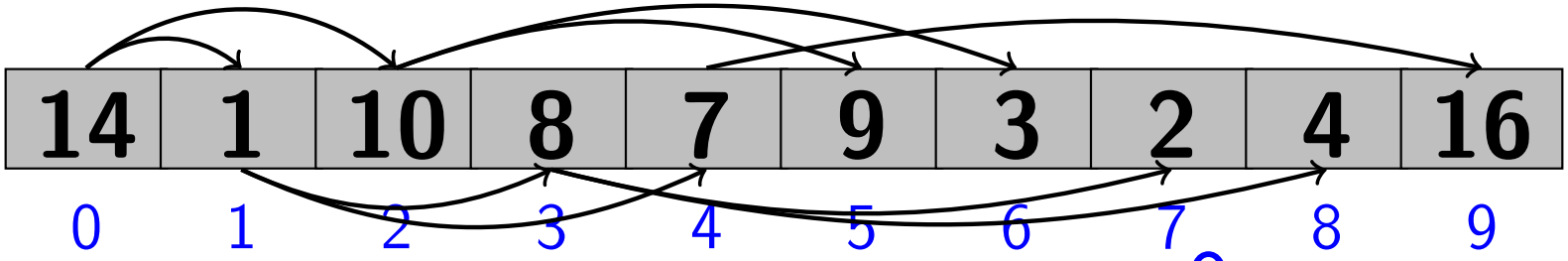
$V[0] \leftrightarrow V[\mathbf{9}]$;

$\triangleright$SIZE $= 9$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

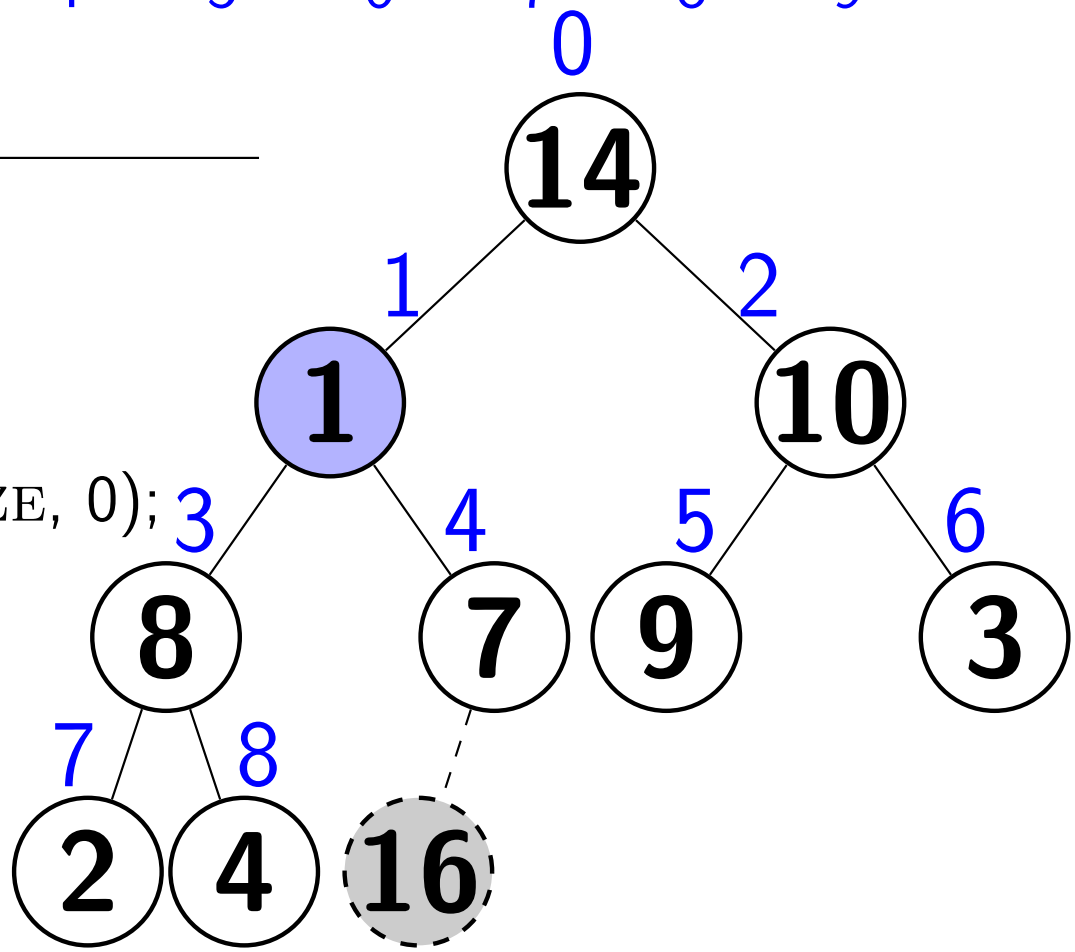
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

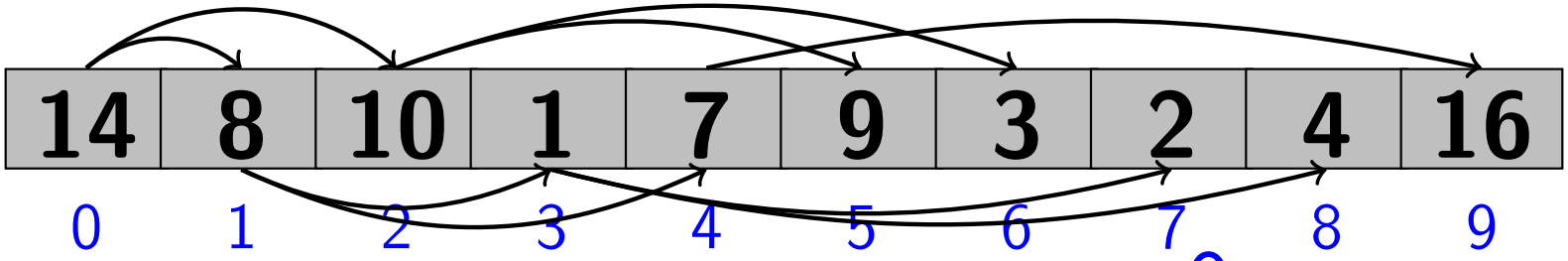
$V[0] \leftrightarrow V[9]$;

SIZE $= 9$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

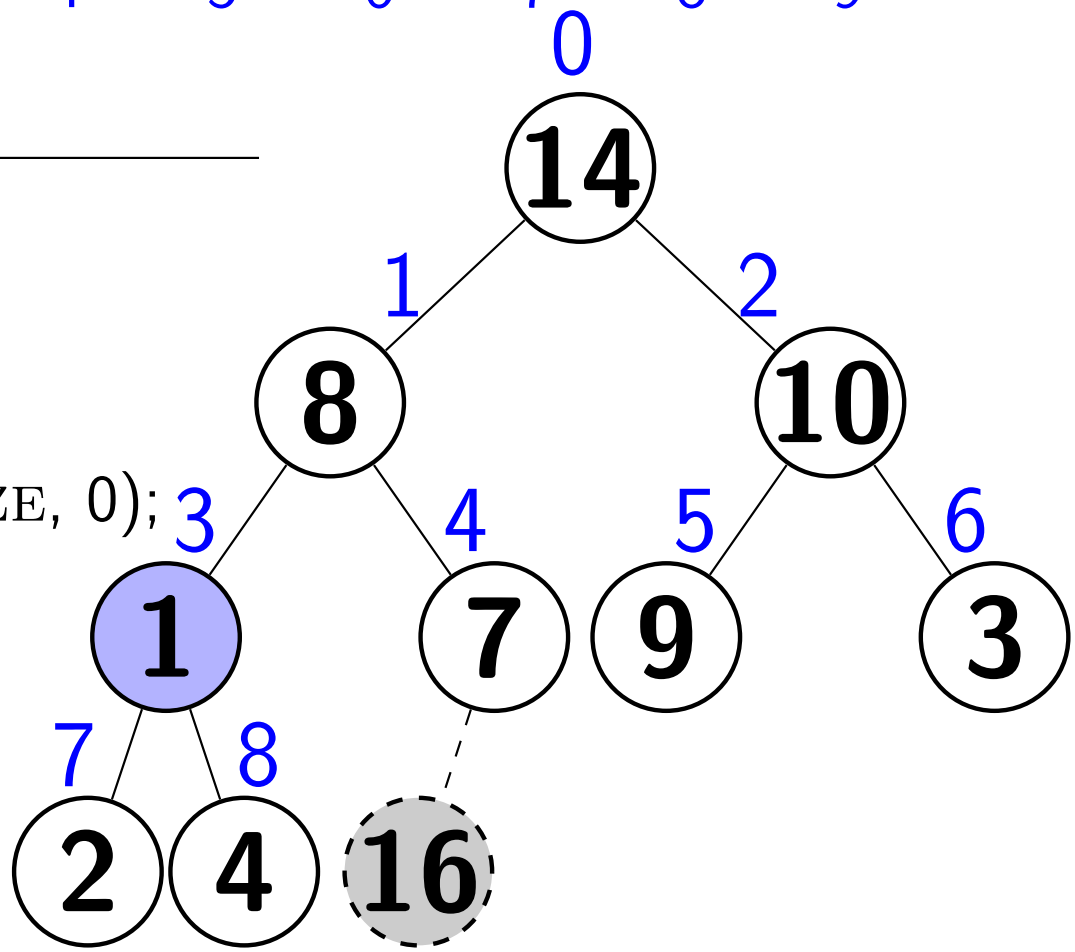
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[\mathbf{9}]$;

SIZE $= 9$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



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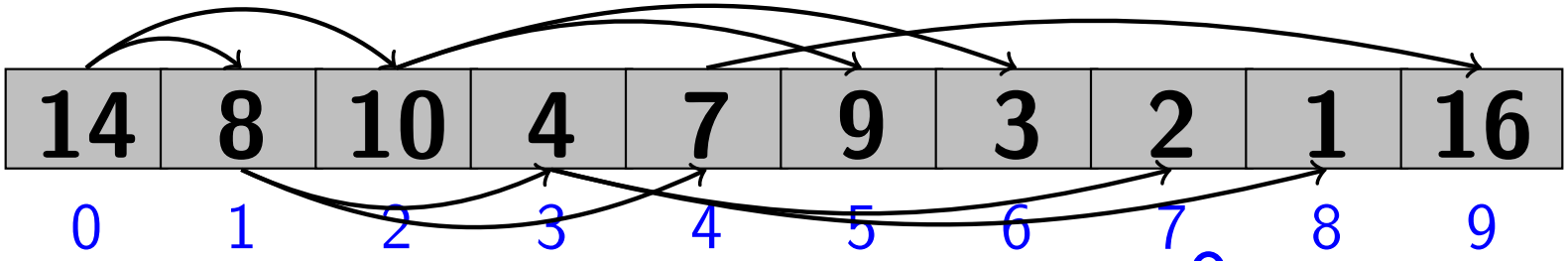


HEAP-SORT $(V, 10)$

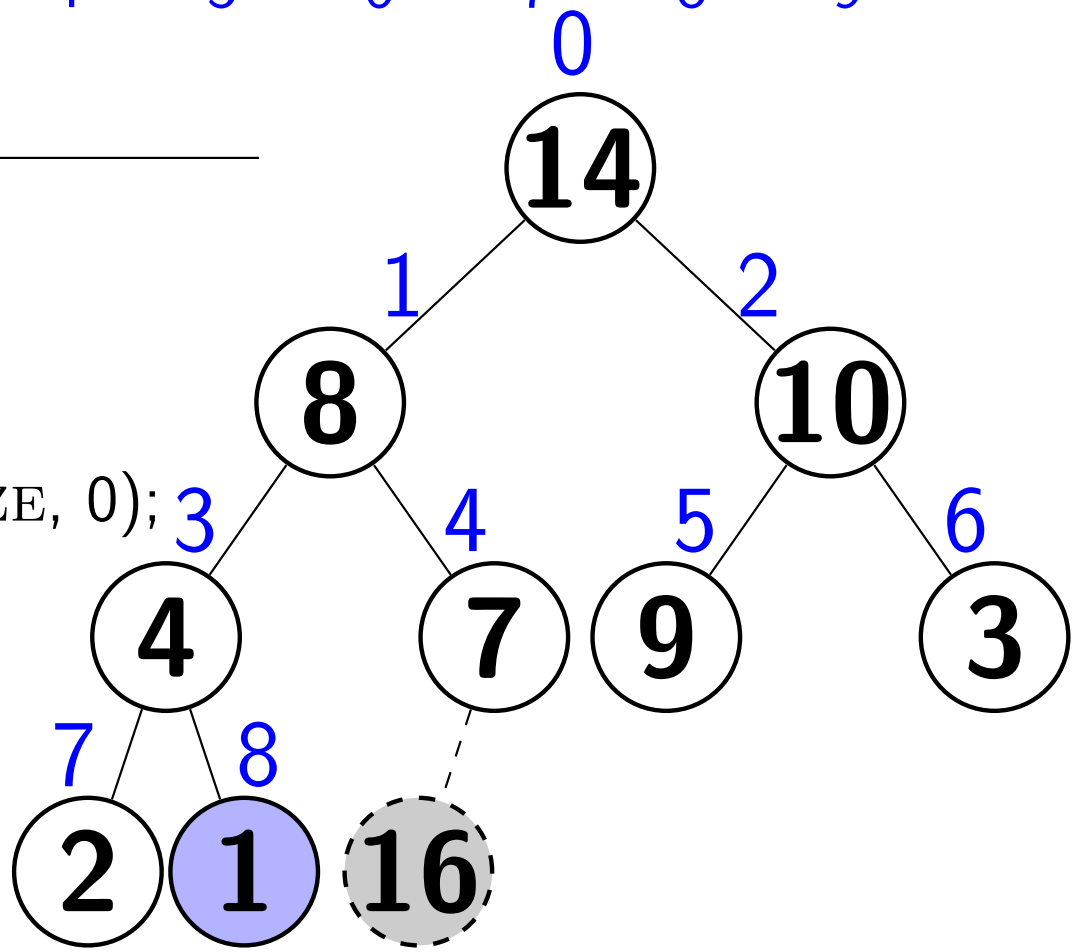
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

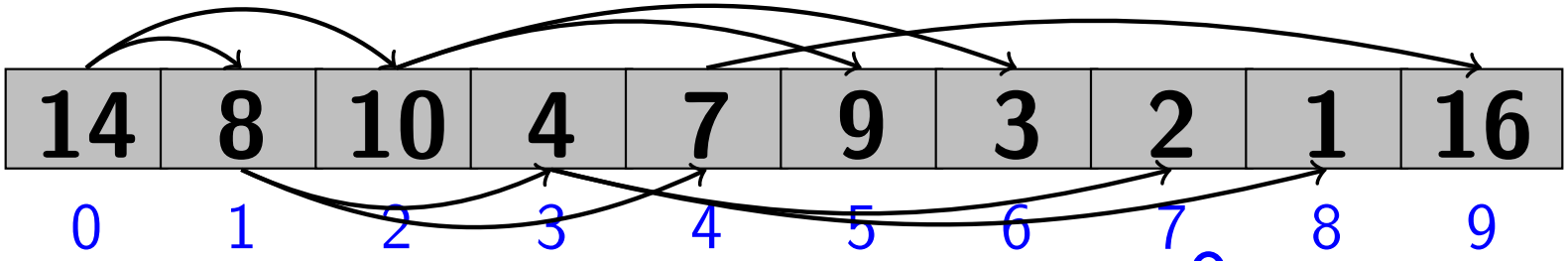
$V[0] \leftrightarrow V[9]$;

SIZE $= 9$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

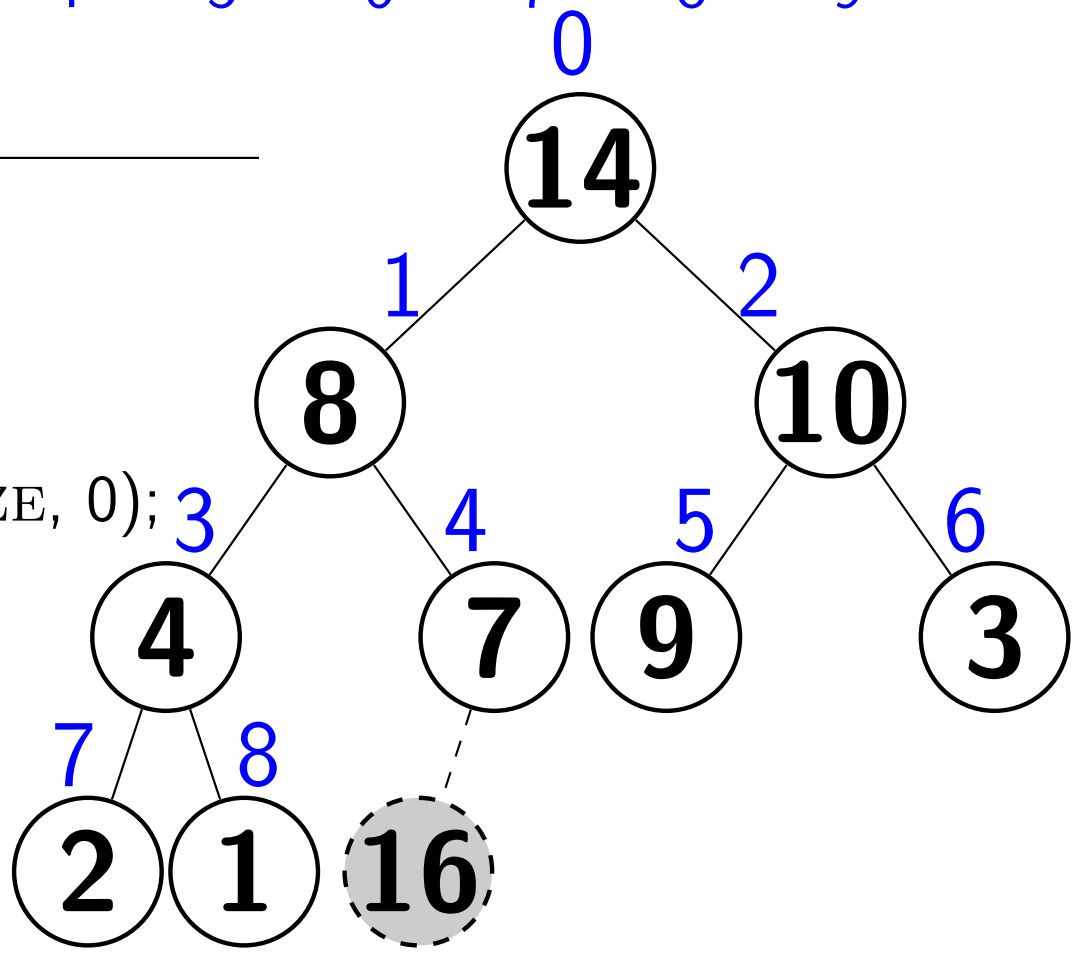
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[9]$;

SIZE $= 9$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



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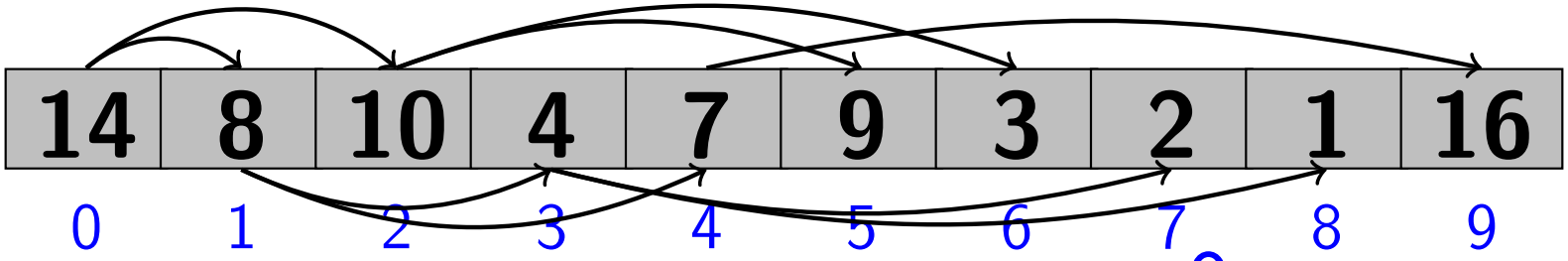


HEAP-SORT $(V, 10)$

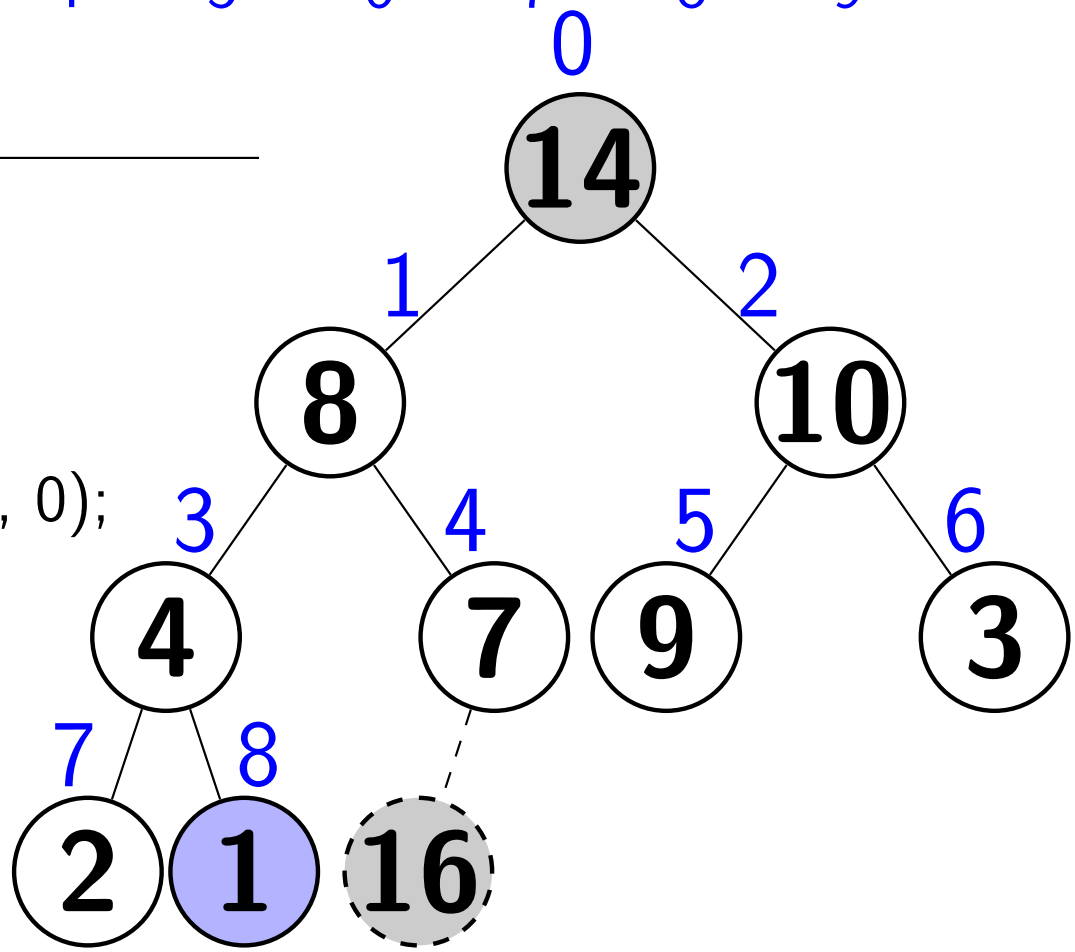
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

$\triangleright V[0] \leftrightarrow V[\mathbf{8}]$;

SIZE $= 9$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



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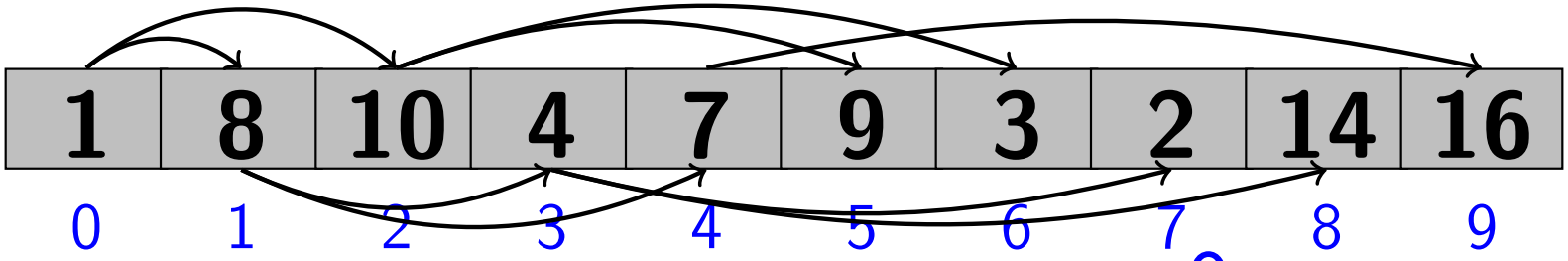


HEAP-SORT $(V, 10)$

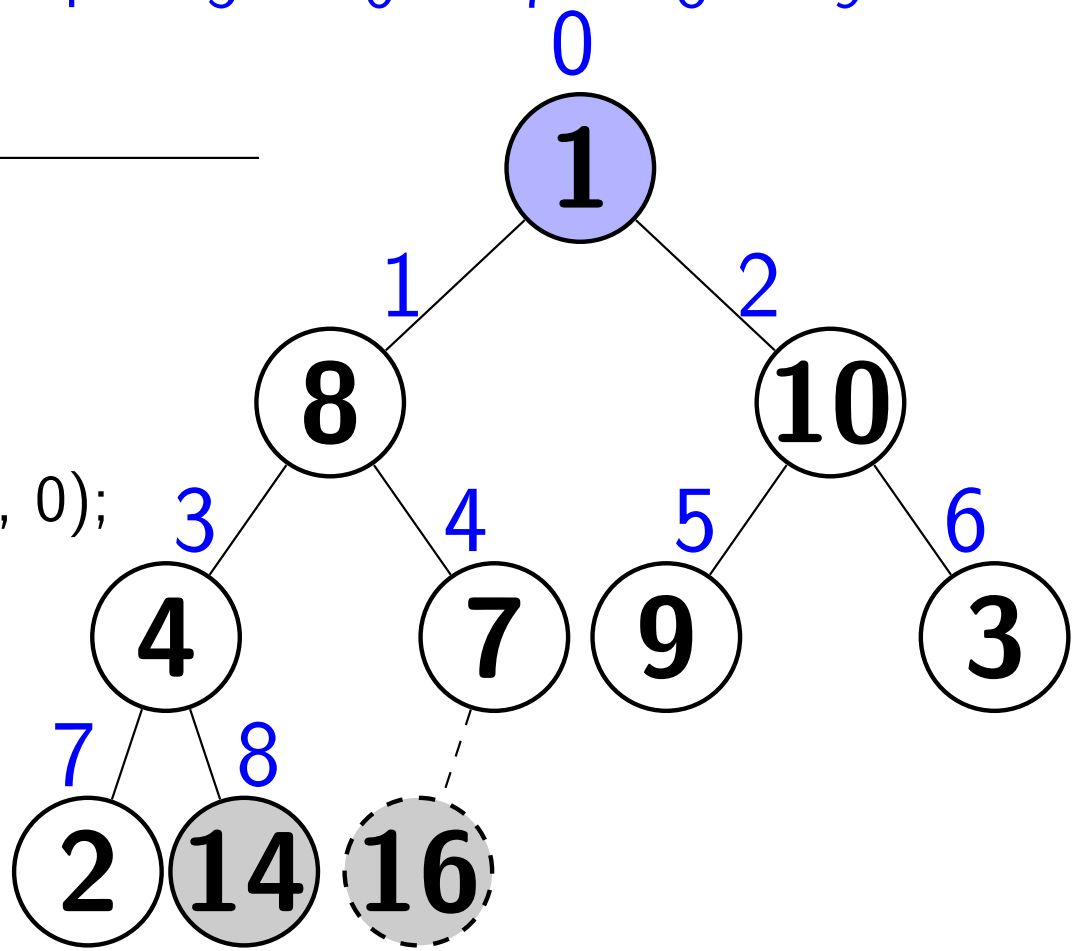
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

$\triangleright V[0] \leftrightarrow V[\mathbf{8}]$;

SIZE $= 9$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



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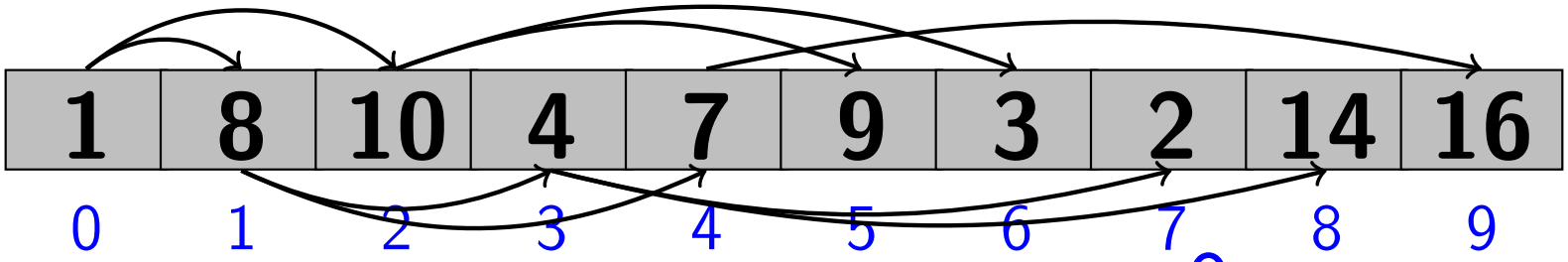


HEAP-SORT $(V, 10)$

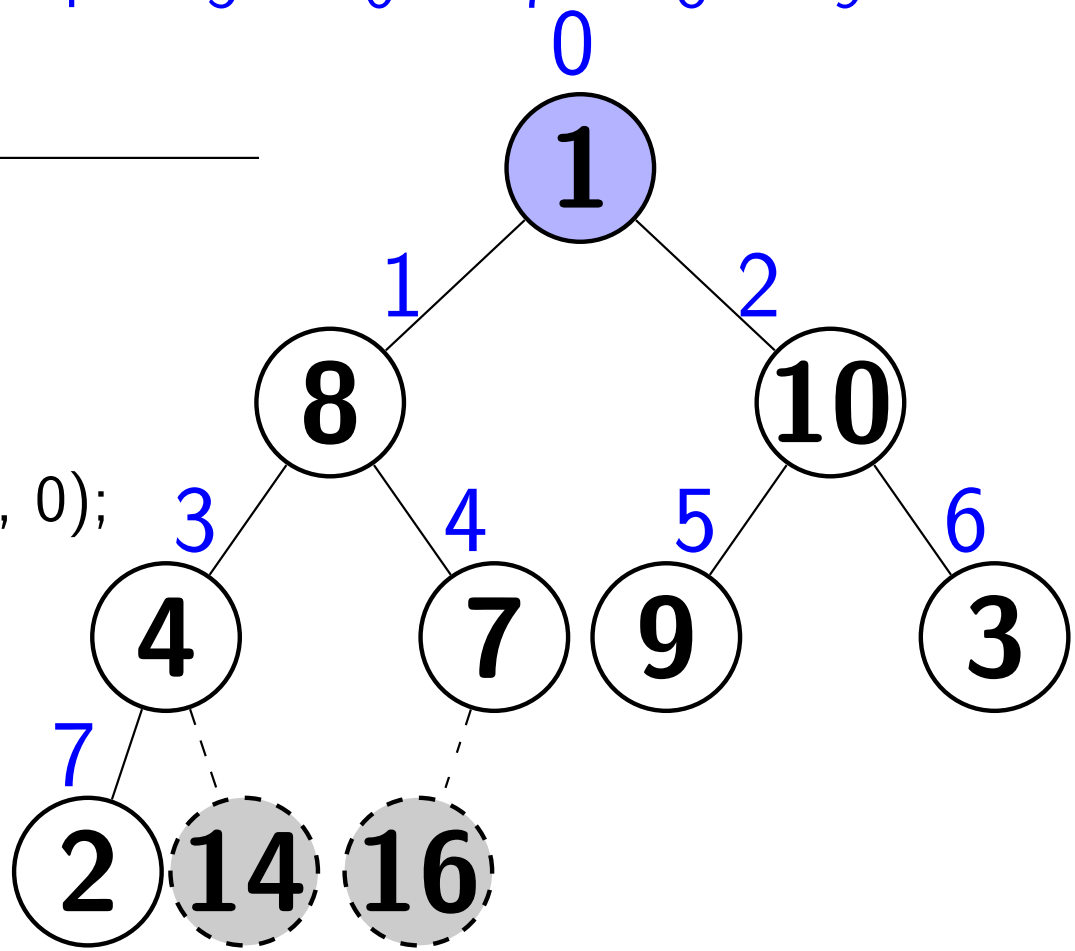
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[8]$;

$\triangleright$SIZE $= 8$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



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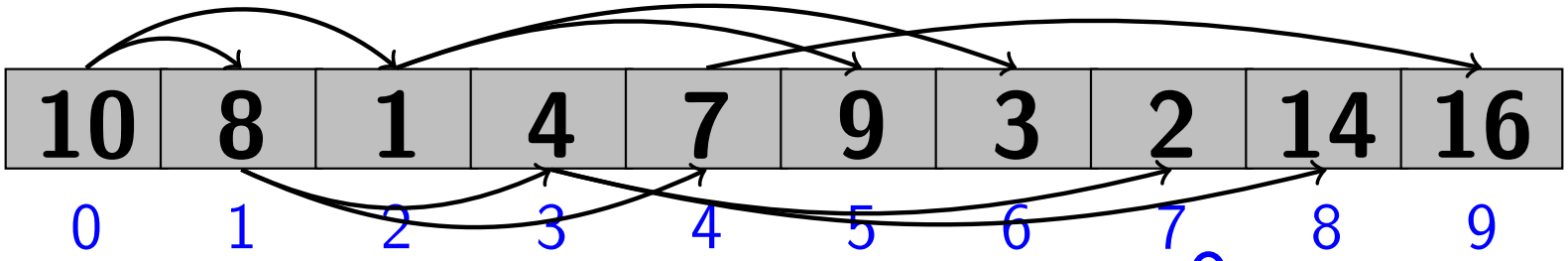


HEAP-SORT $(V, 10)$

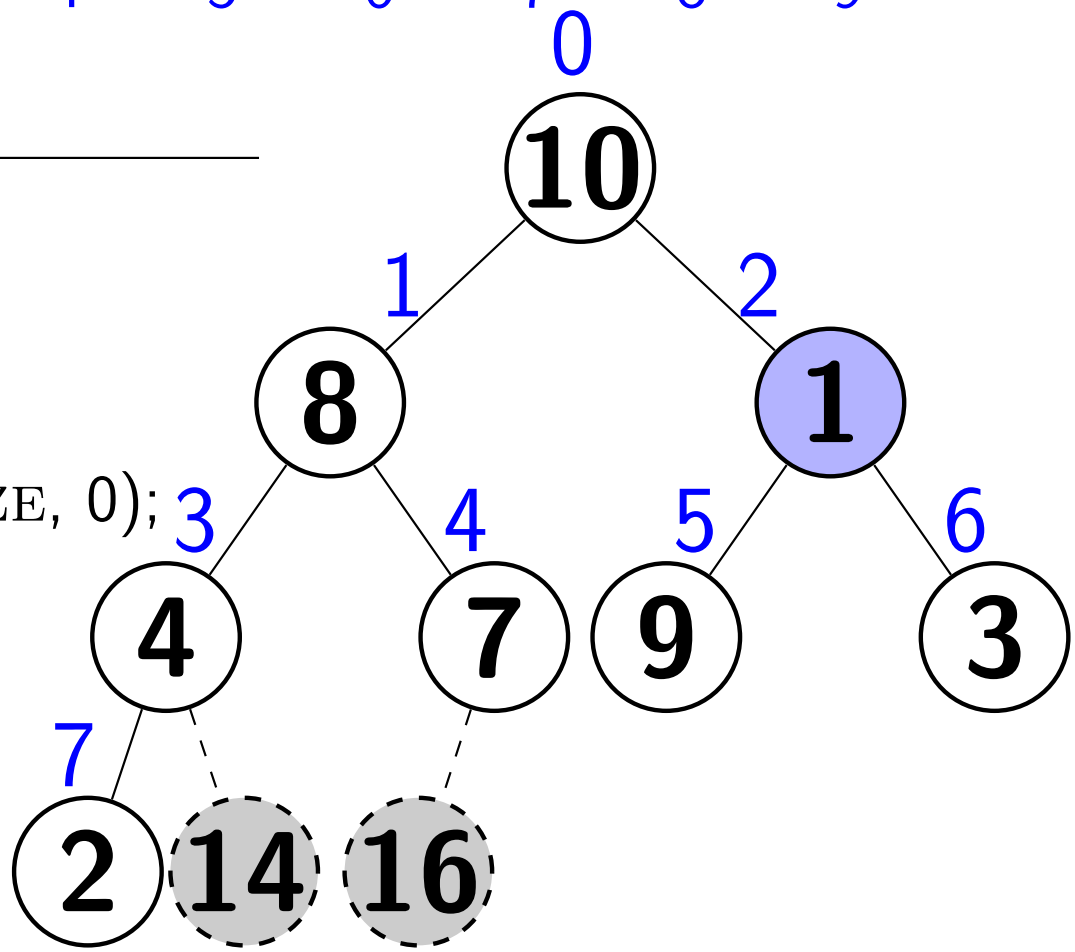
BUILD-MAX-HEAP $(V,10)$;

Para i de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[8]$;

SIZE $= 8$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



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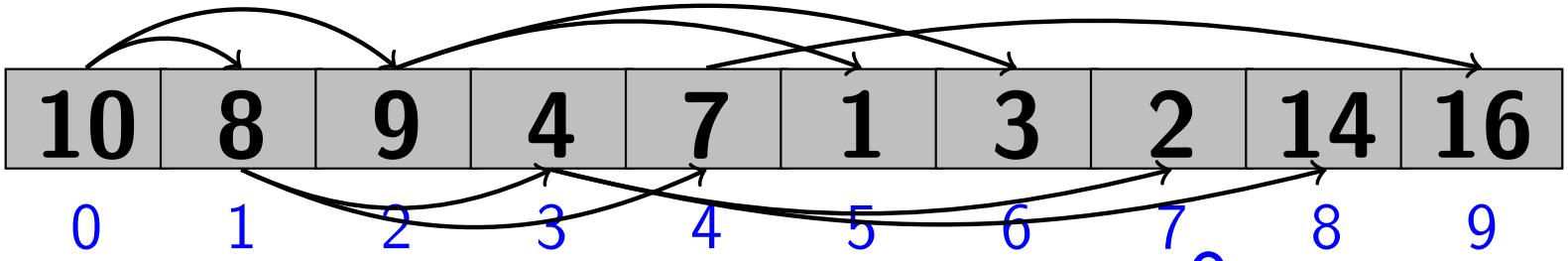


HEAP-SORT $(V, 10)$

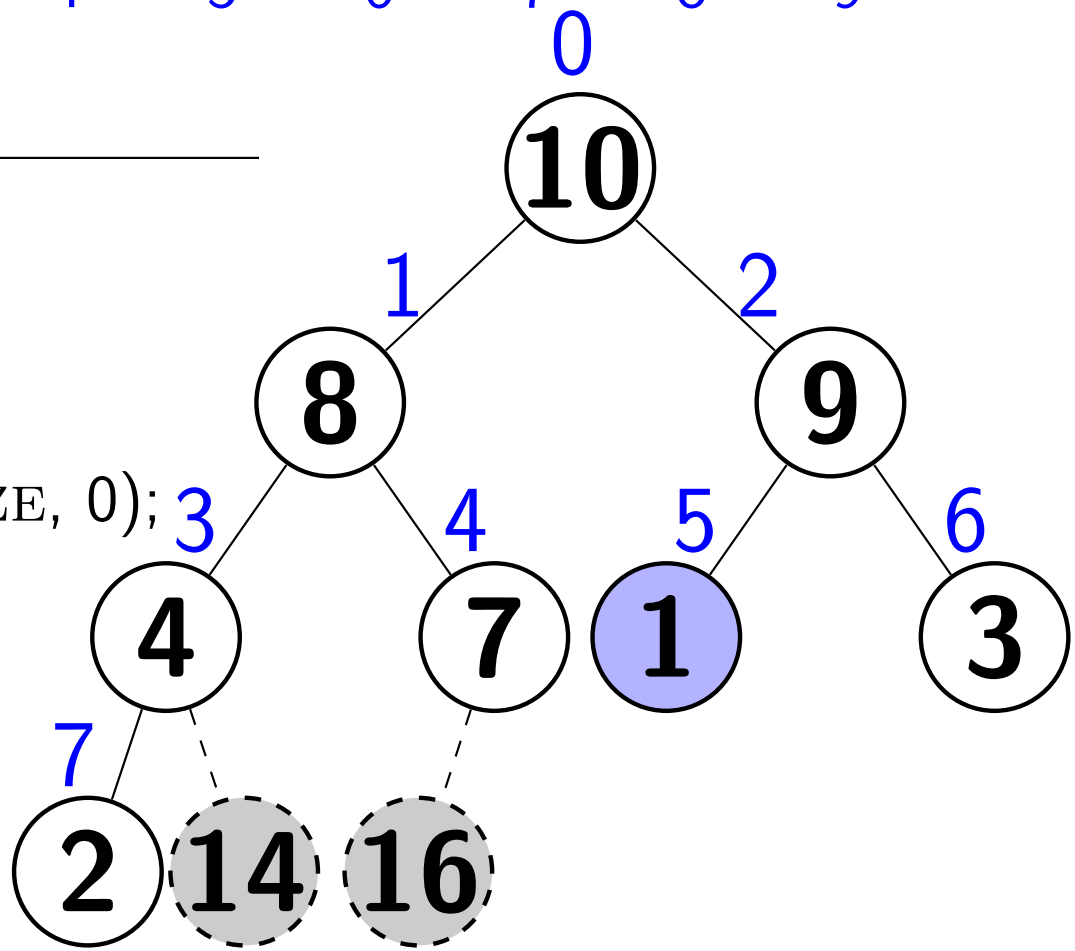
BUILD-MAX-HEAP $(V, 10)$;

Para i de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[\mathbf{8}]$;

SIZE $= 8$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



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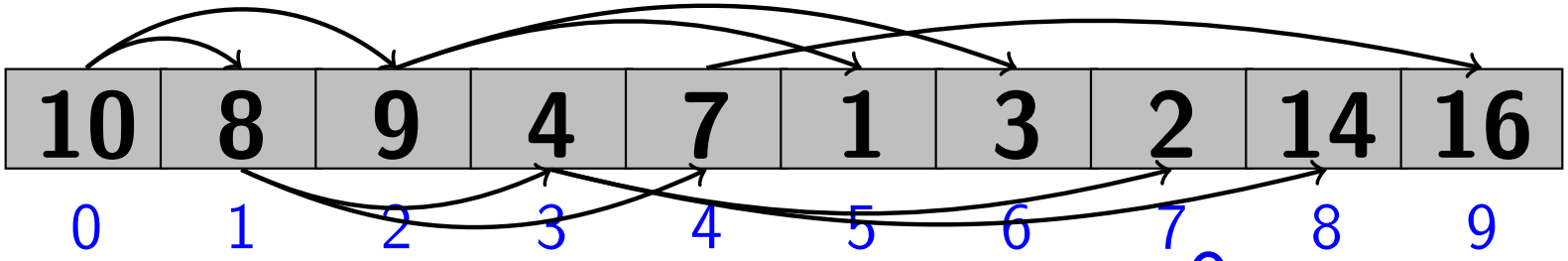


HEAP-SORT $(V, 10)$

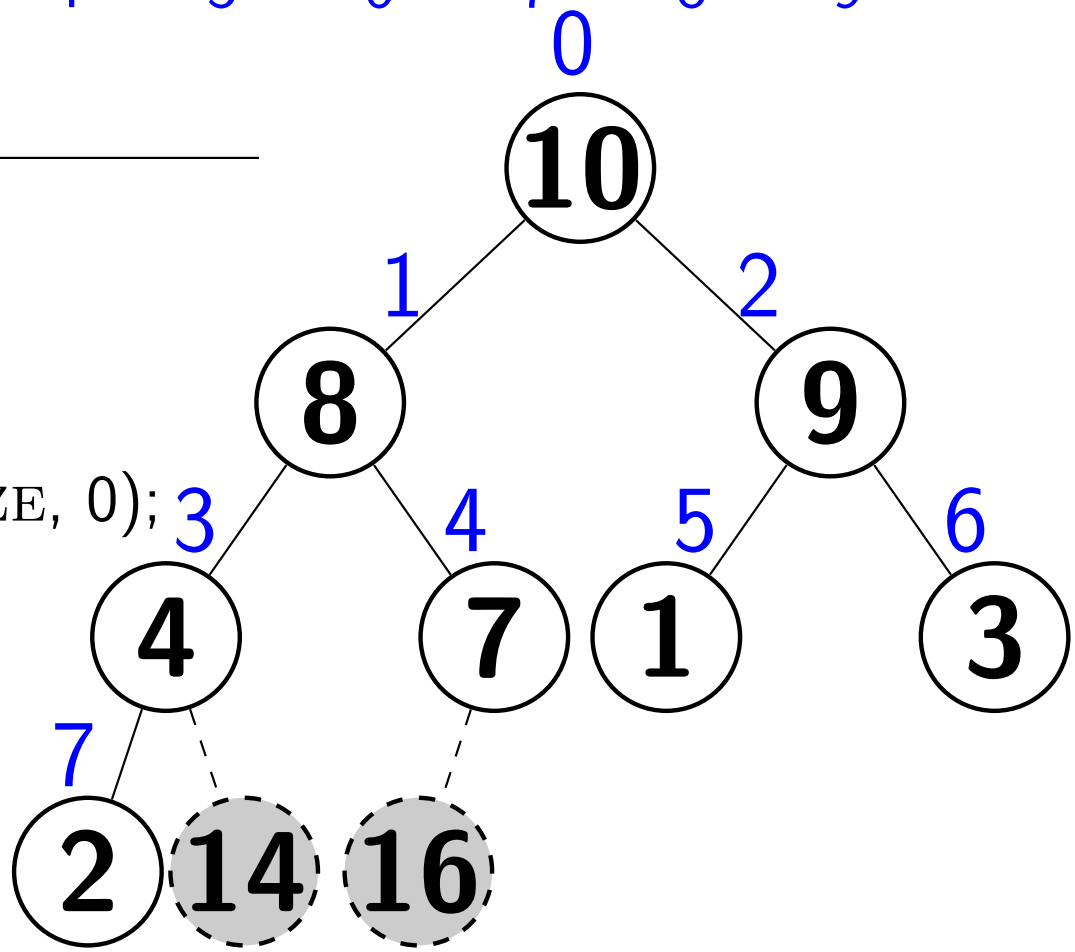
BUILD-MAX-HEAP $(V, 10)$;

Para i de 9 **até** 1 **faça**

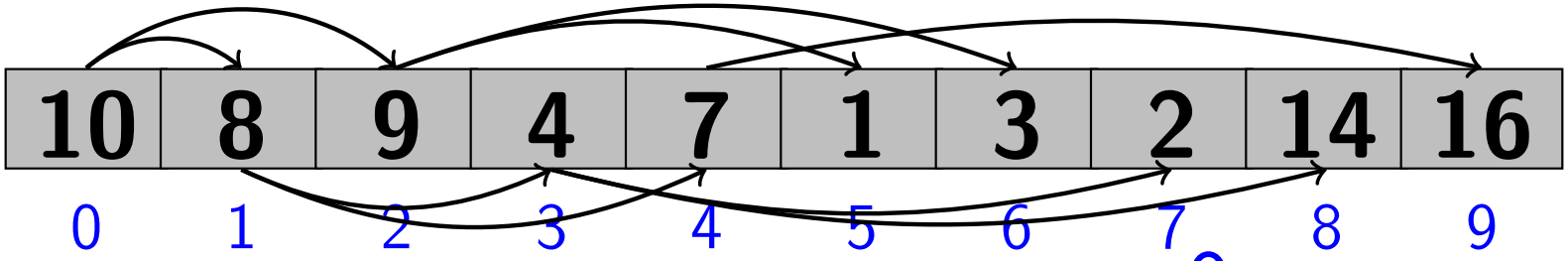
$V[0] \leftrightarrow V[\mathbf{8}]$;

SIZE $= 8$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

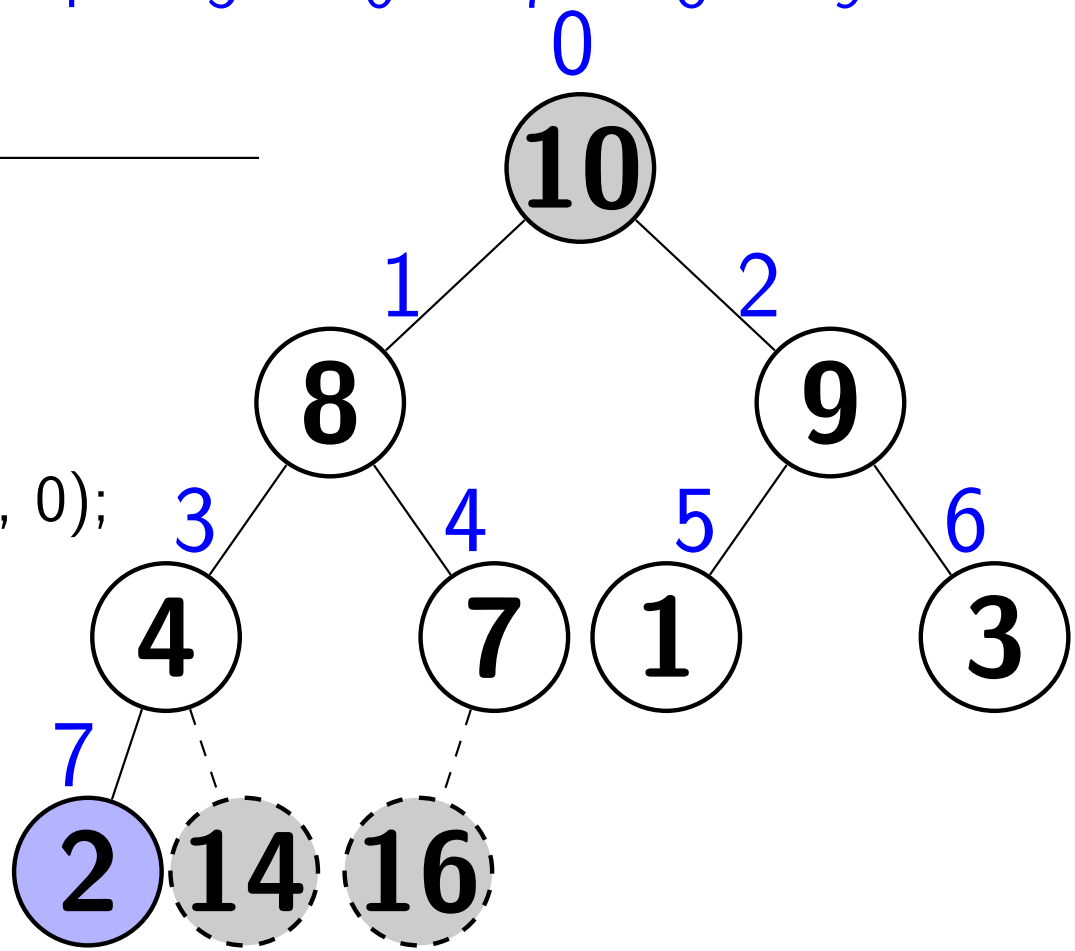
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

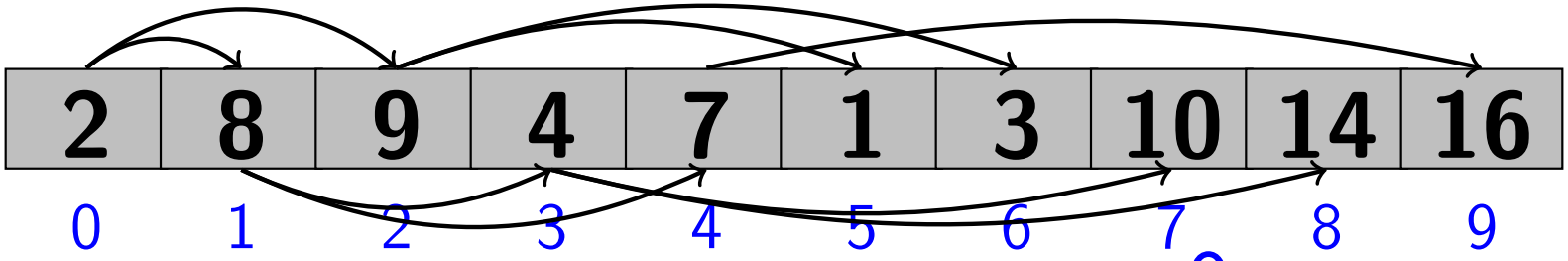
$\triangleright V[0] \leftrightarrow V[7]$;

SIZE $= 8$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

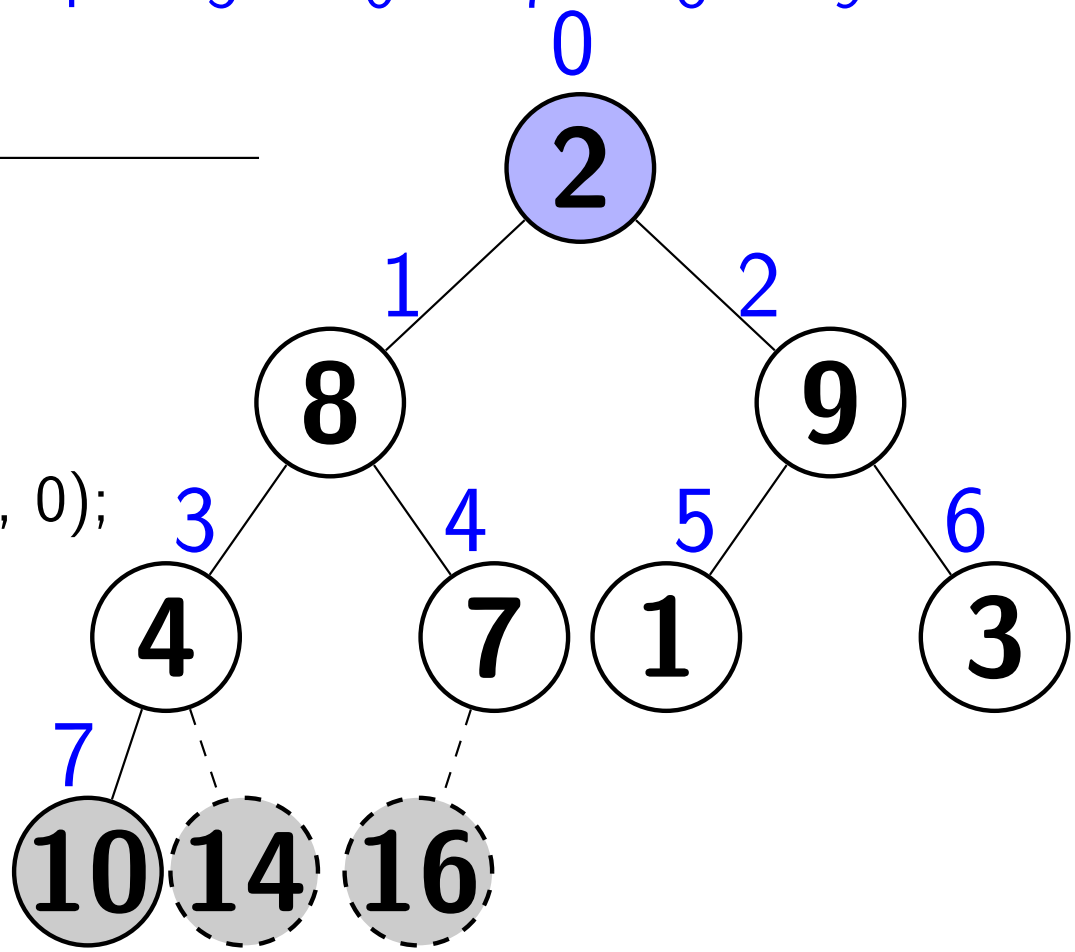
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

$\triangleright V[0] \leftrightarrow V[7]$;

SIZE $= 8$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



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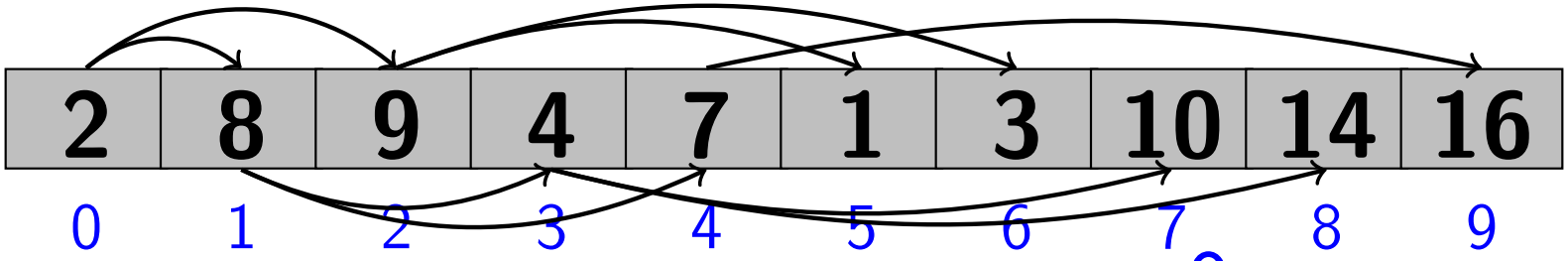


HEAP-SORT $(V, 10)$

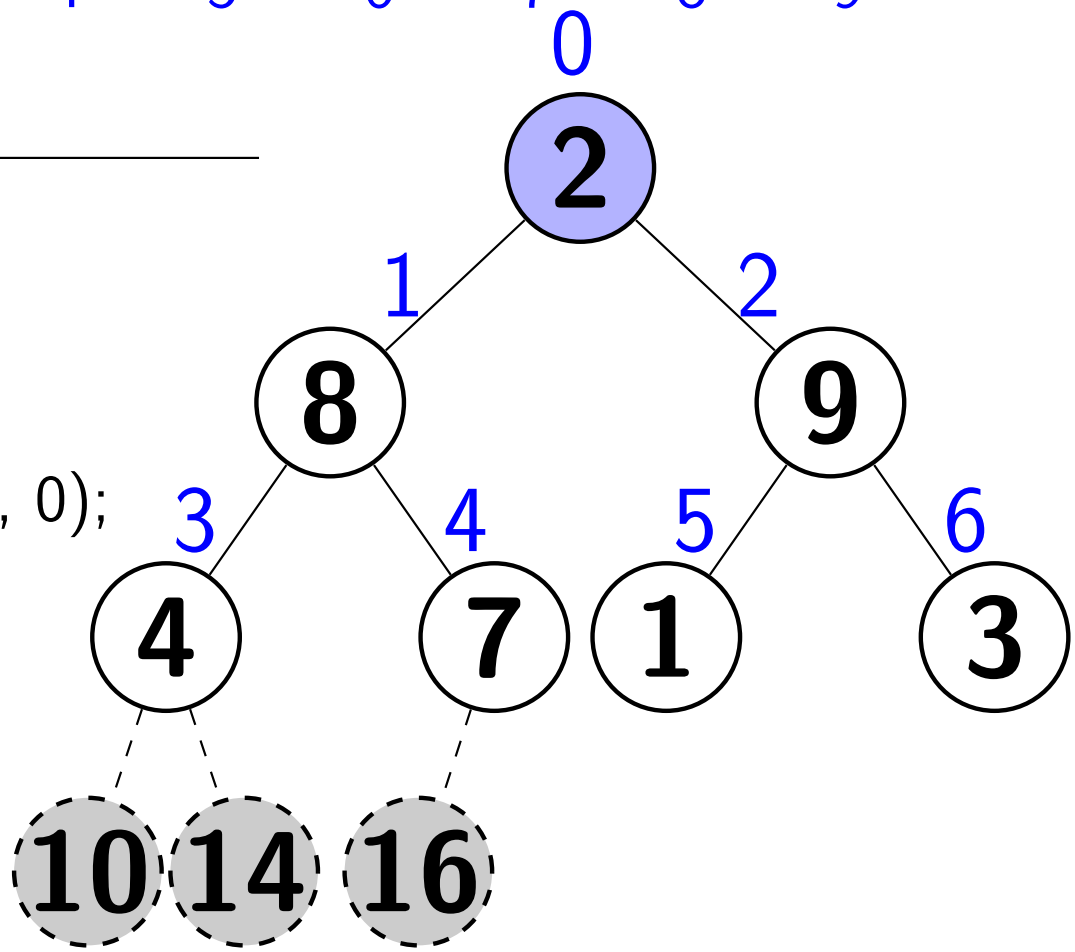
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

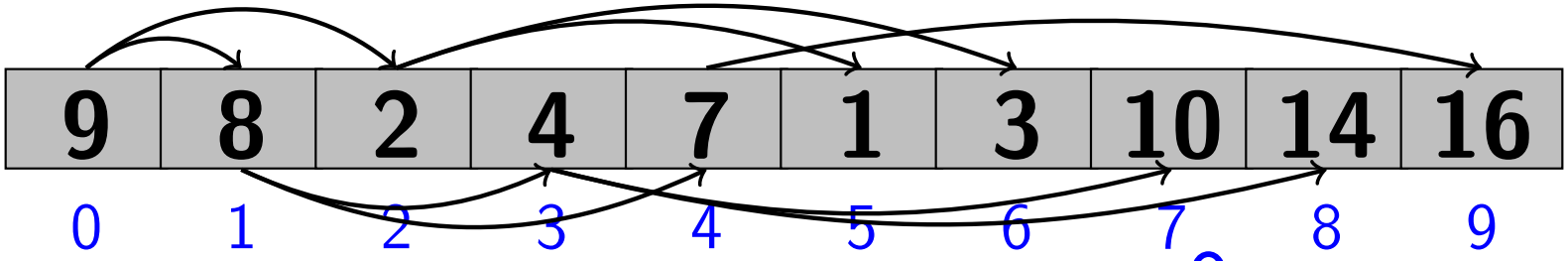
$V[0] \leftrightarrow V[7]$;

$\triangleright$SIZE $= 7$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

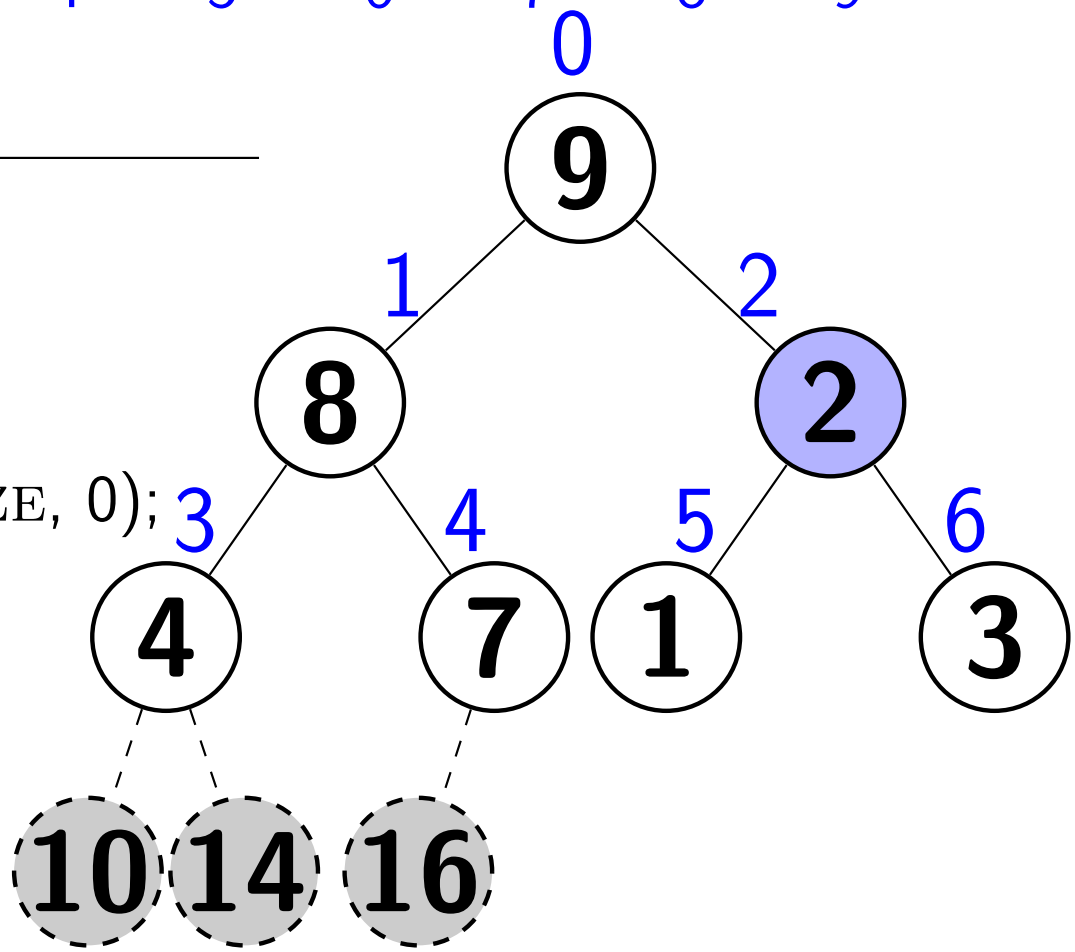
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

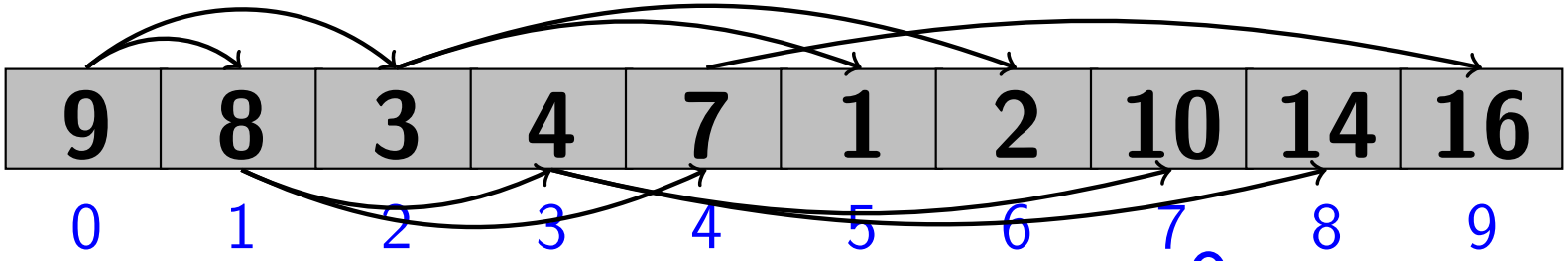
$V[0] \leftrightarrow V[7]$;

SIZE $= 7$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

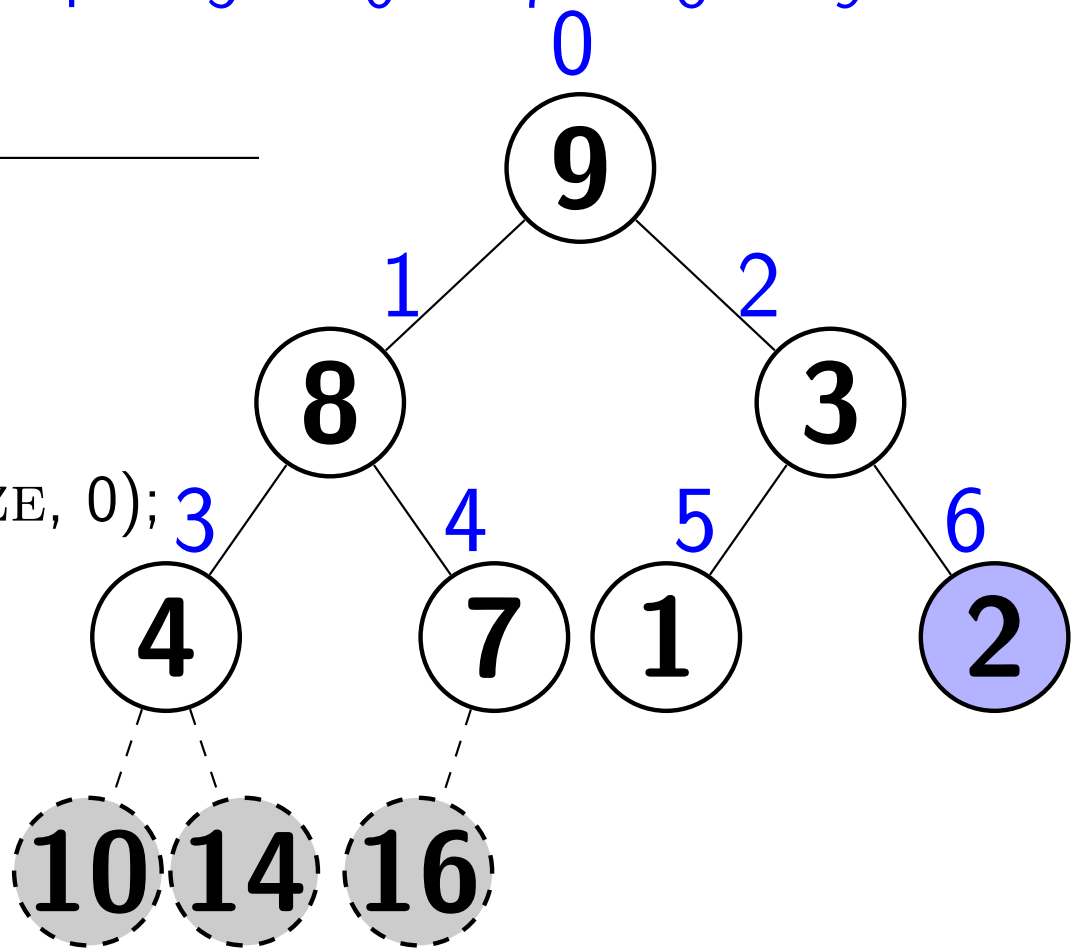
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

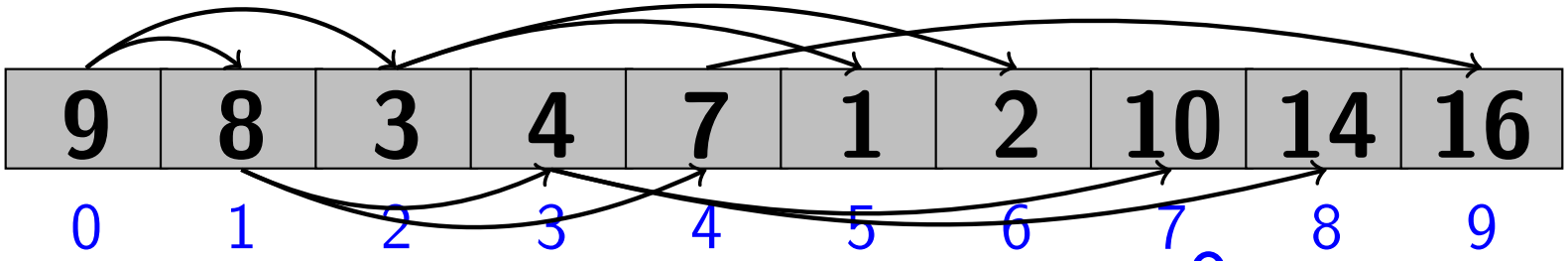
$V[0] \leftrightarrow V[7]$;

SIZE $= 7$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

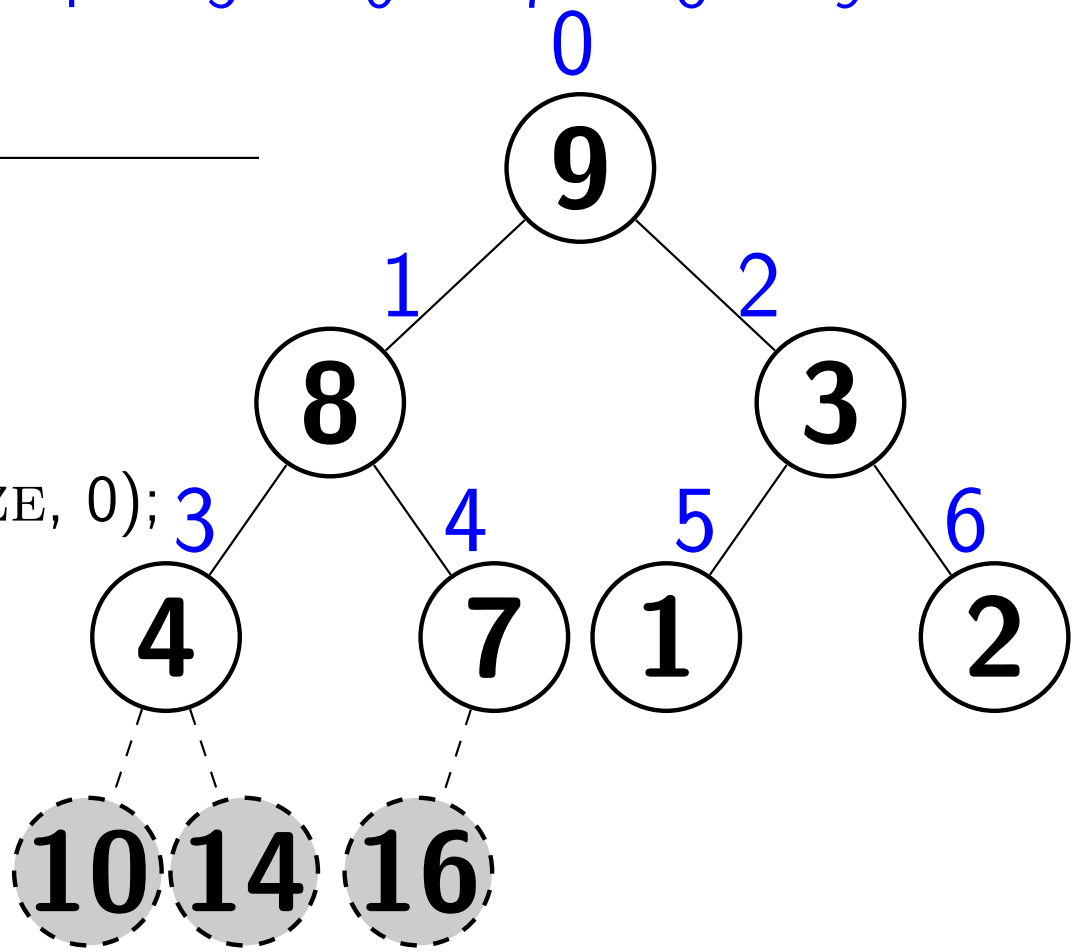
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

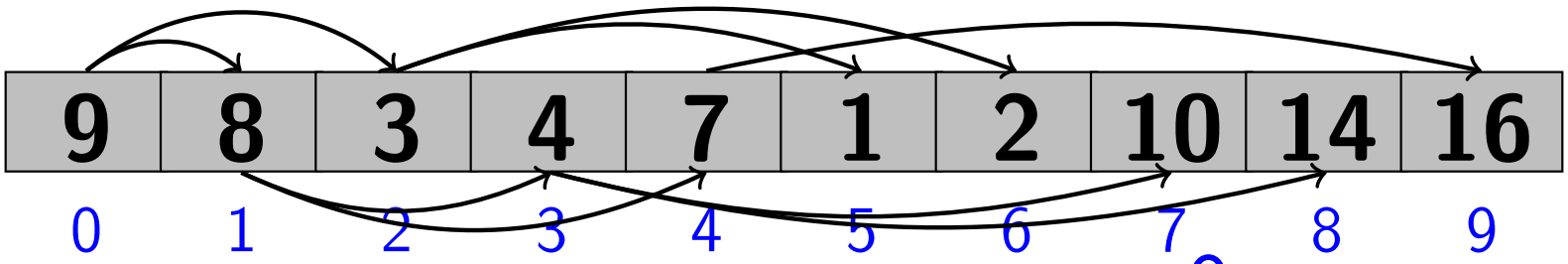
$V[0] \leftrightarrow V[7]$;

SIZE $= 7$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

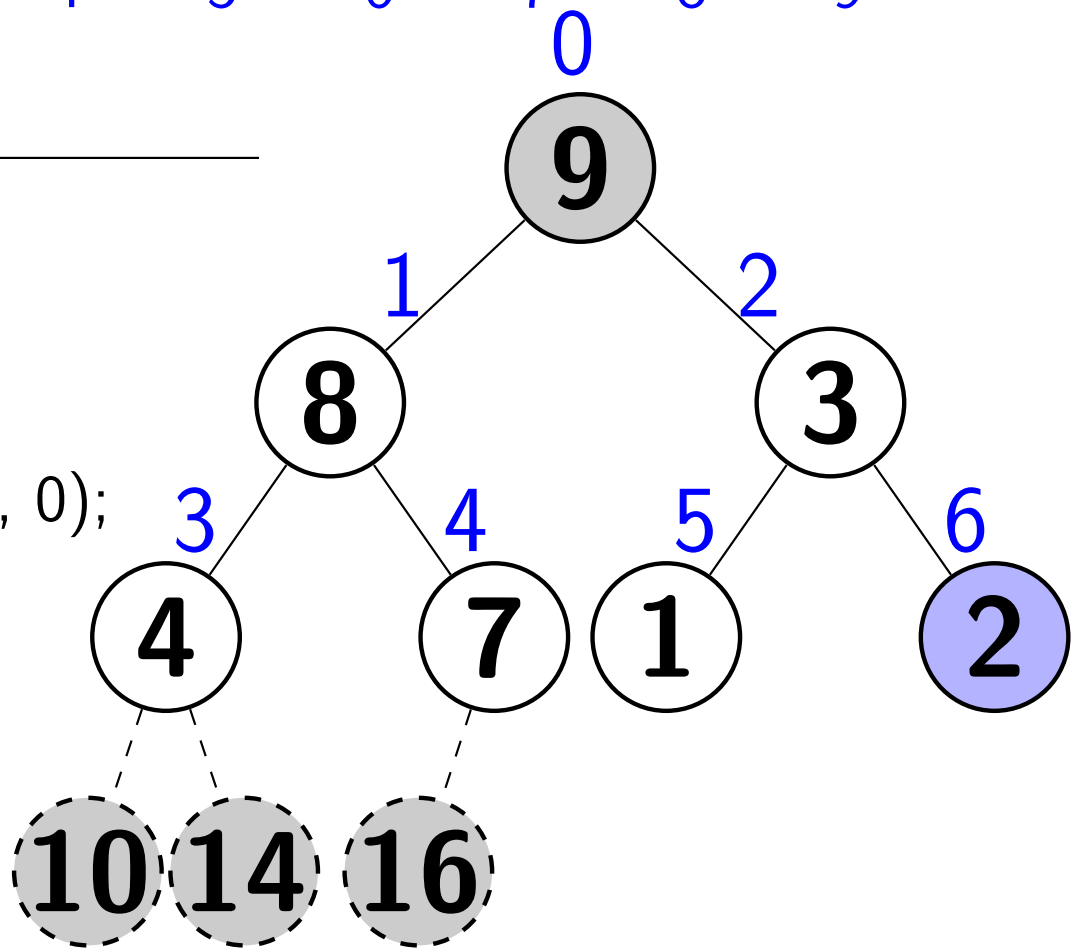
BUILD-MAX-HEAP $(V, 10)$;

Para i de 9 **até** 1 **faça**

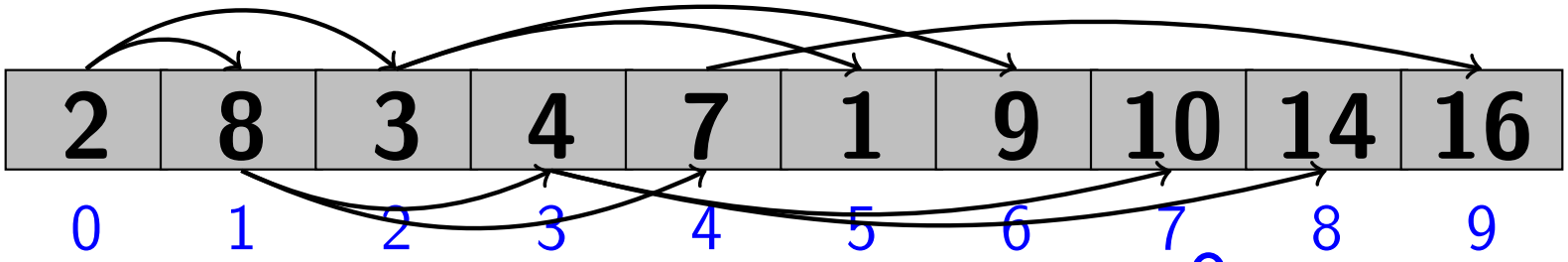
$\triangleright V[0] \leftrightarrow V[\mathbf{6}]$;

SIZE $= 7$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

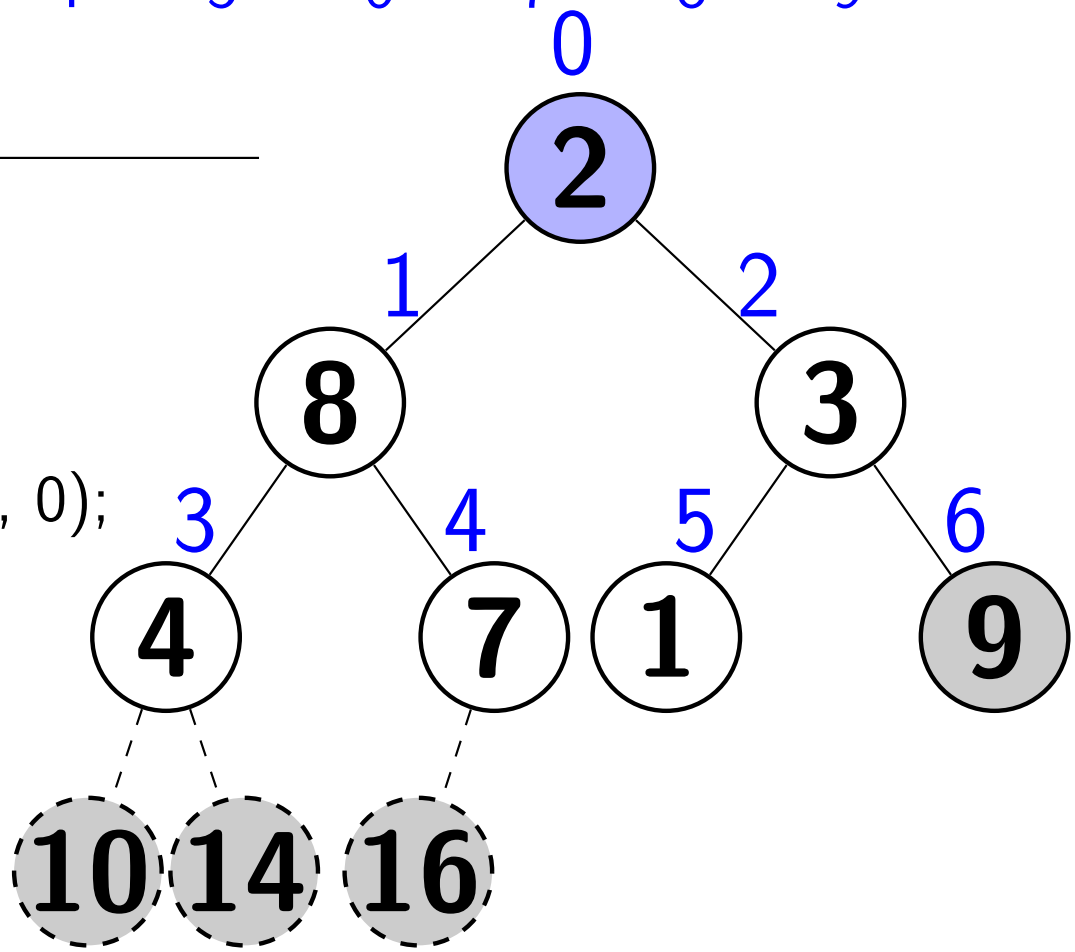
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

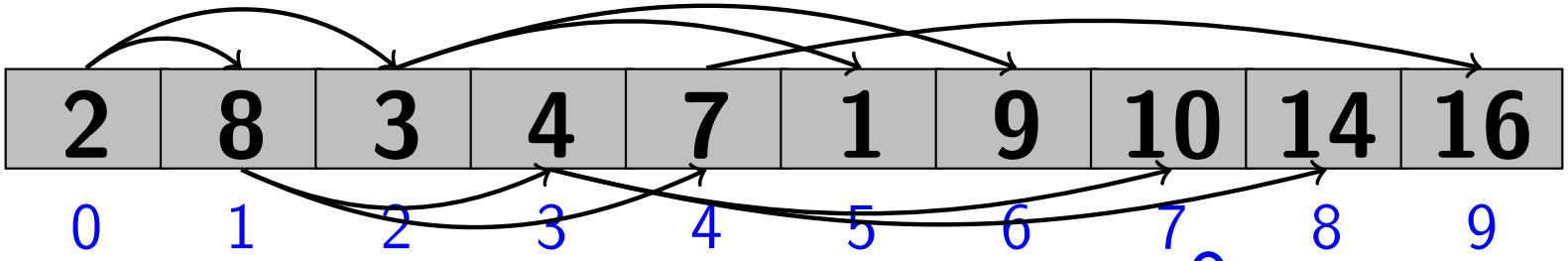
$\triangleright V[0] \leftrightarrow V[\mathbf{6}]$;

SIZE $= 7$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

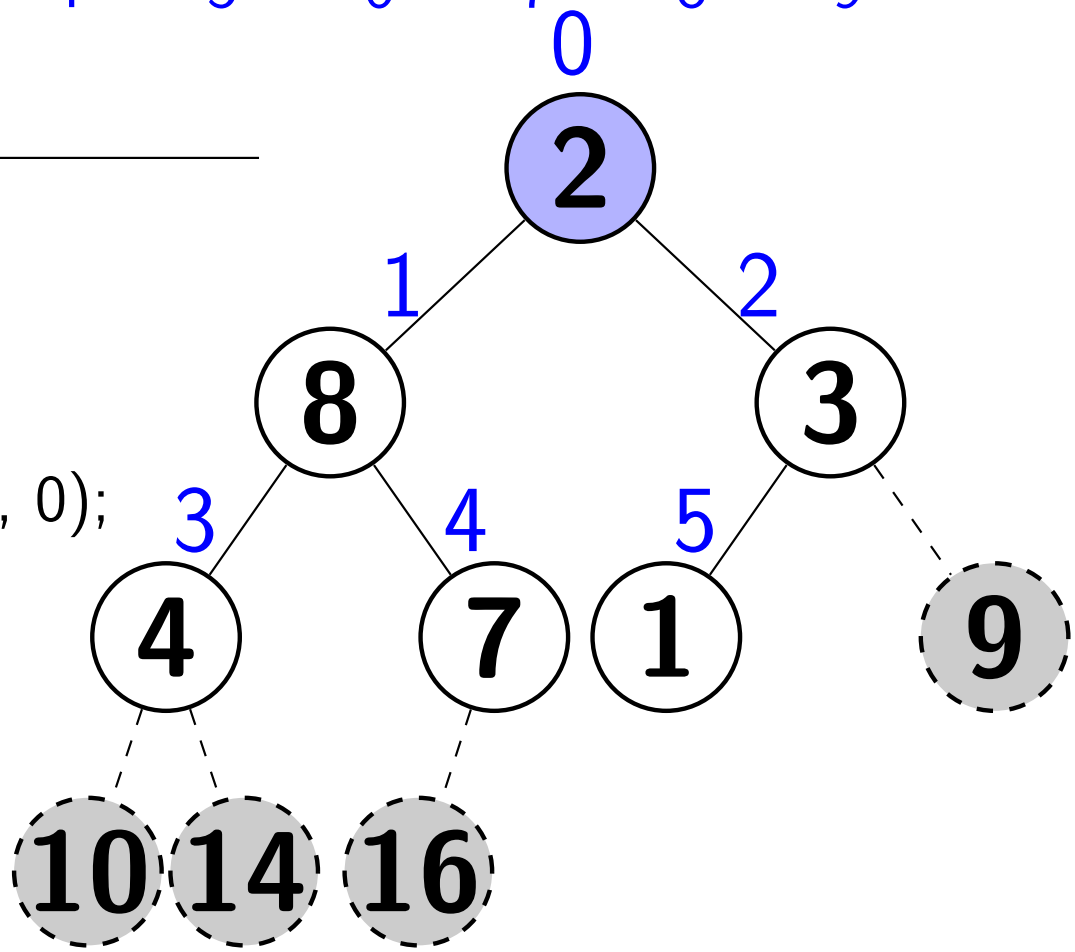
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

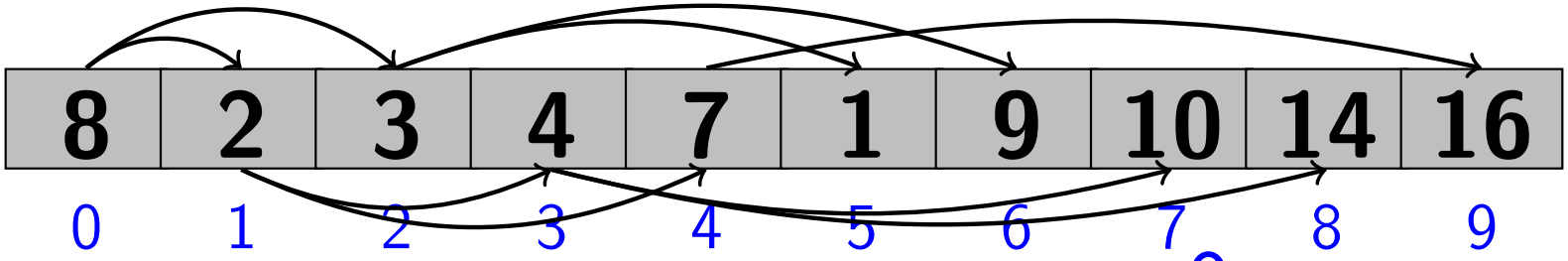
$V[0] \leftrightarrow V[\mathbf{6}]$;

$\triangleright$SIZE $= 6$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT (V, 10)

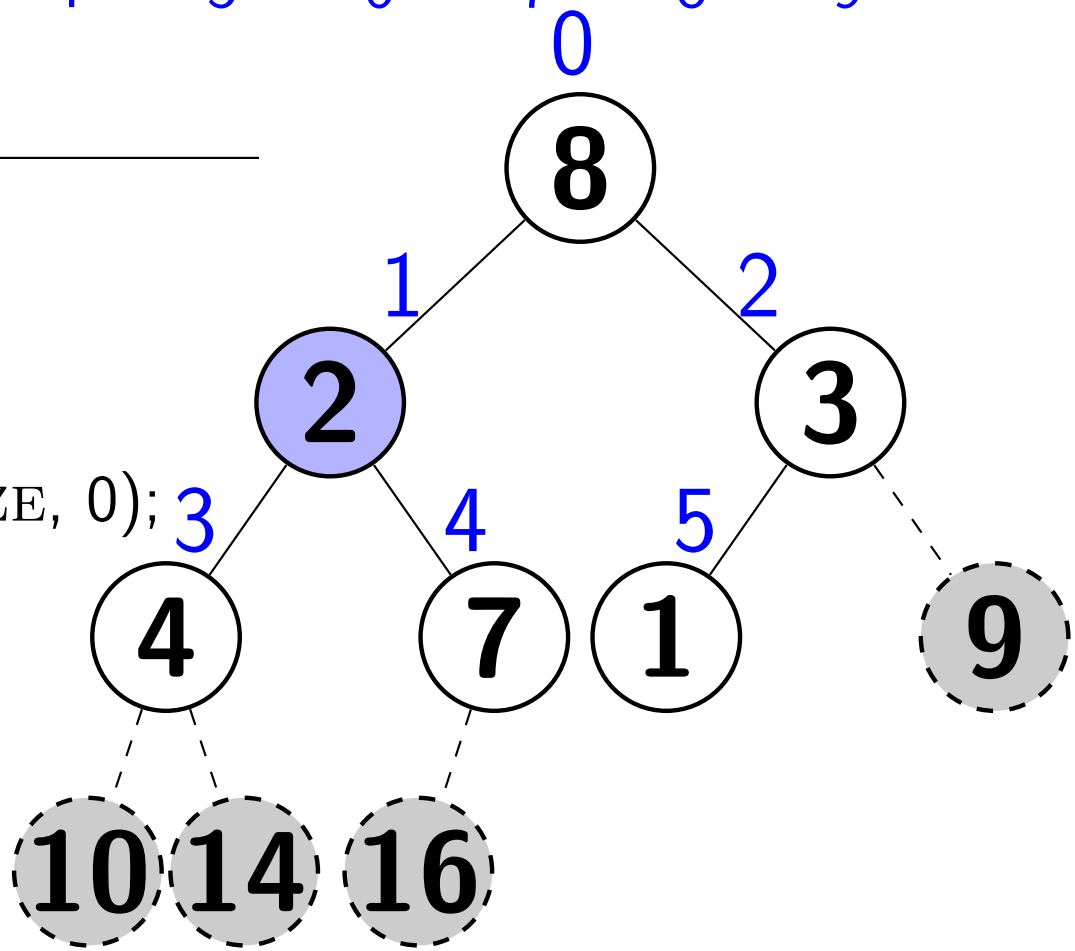
BUILD-MAX-HEAP (V,10);

Para **i** de 9 **até** 1 **faça**

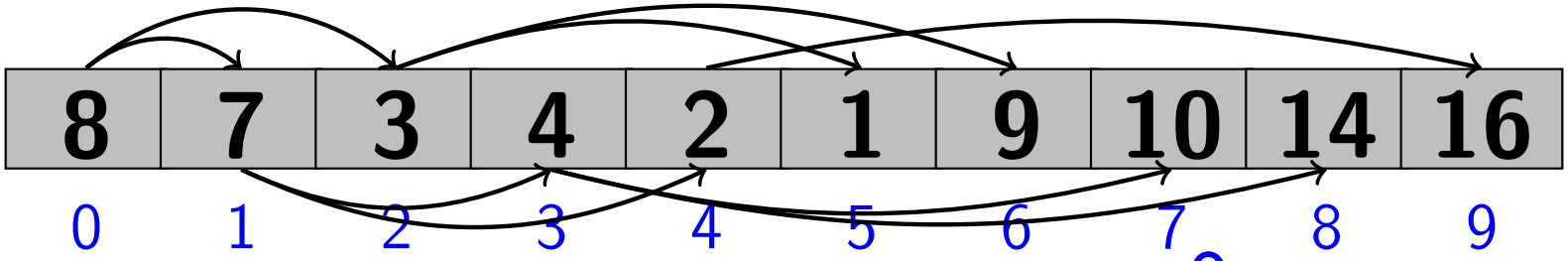
$V[0] \leftrightarrow V[\mathbf{6}]$;

SIZE $= 6$;

$\triangleright$MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

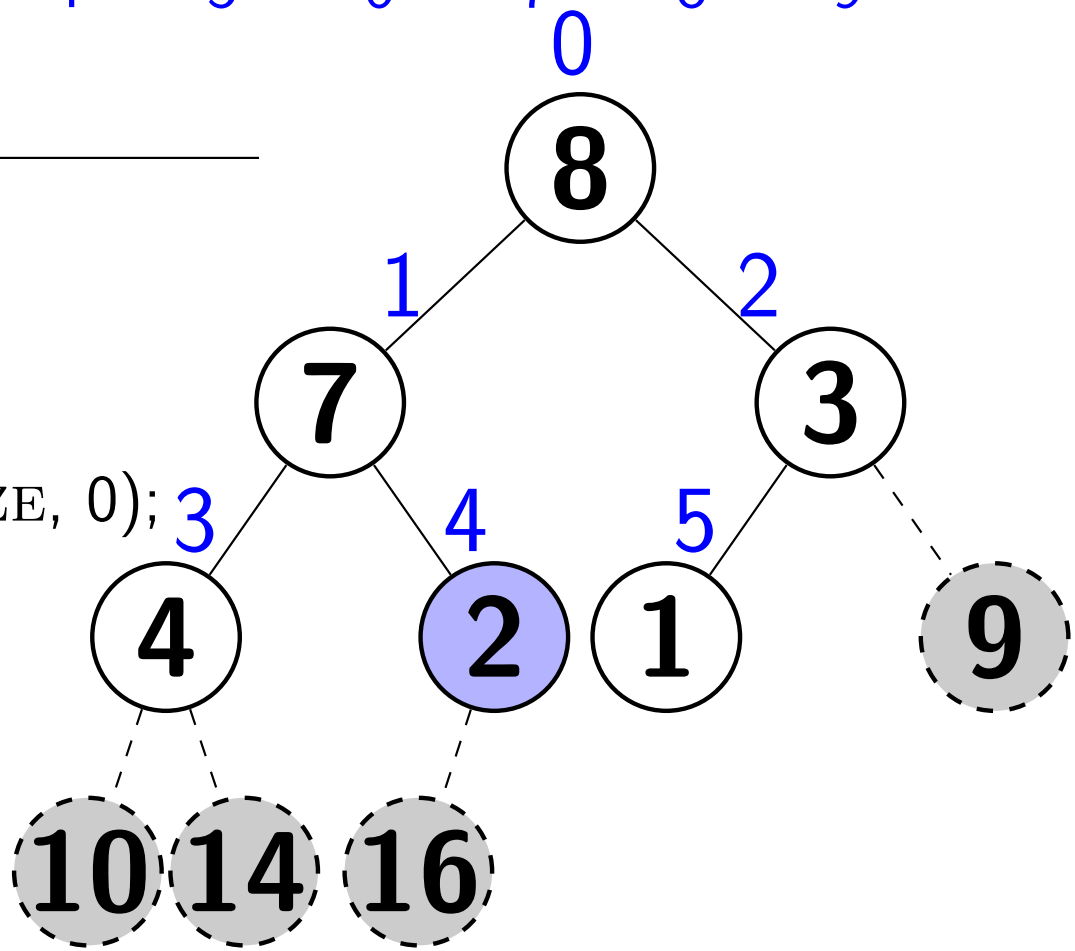
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

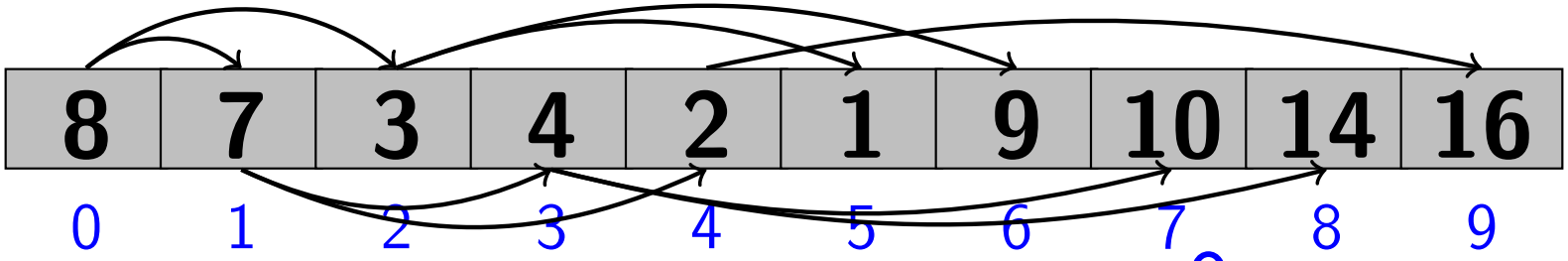
$V[0] \leftrightarrow V[\mathbf{6}]$;

SIZE $= 6$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

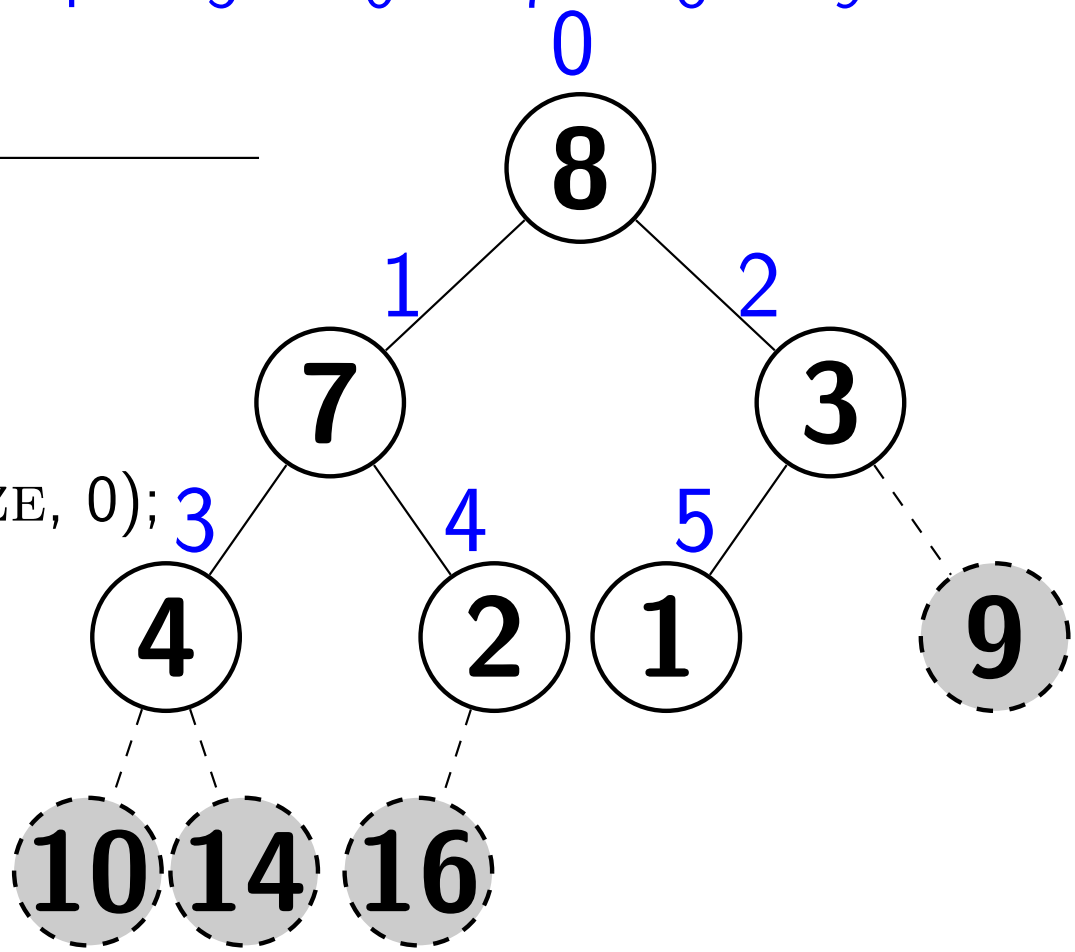
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

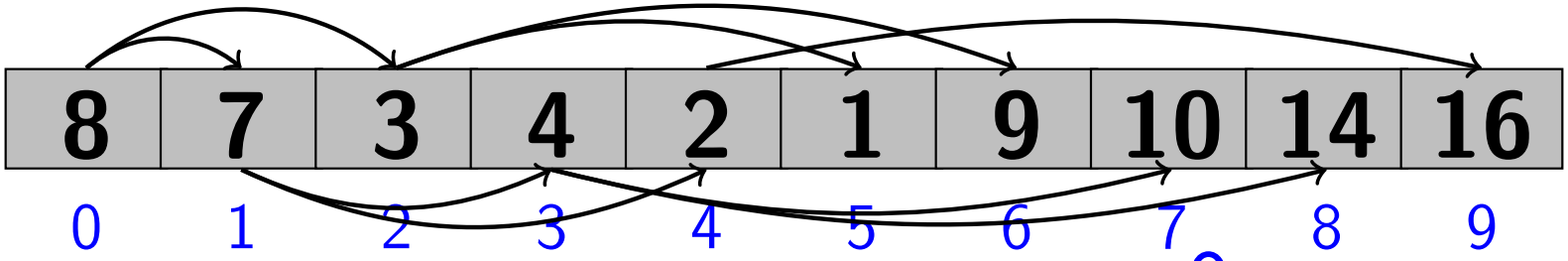
$V[0] \leftrightarrow V[\mathbf{6}]$;

SIZE $= 6$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

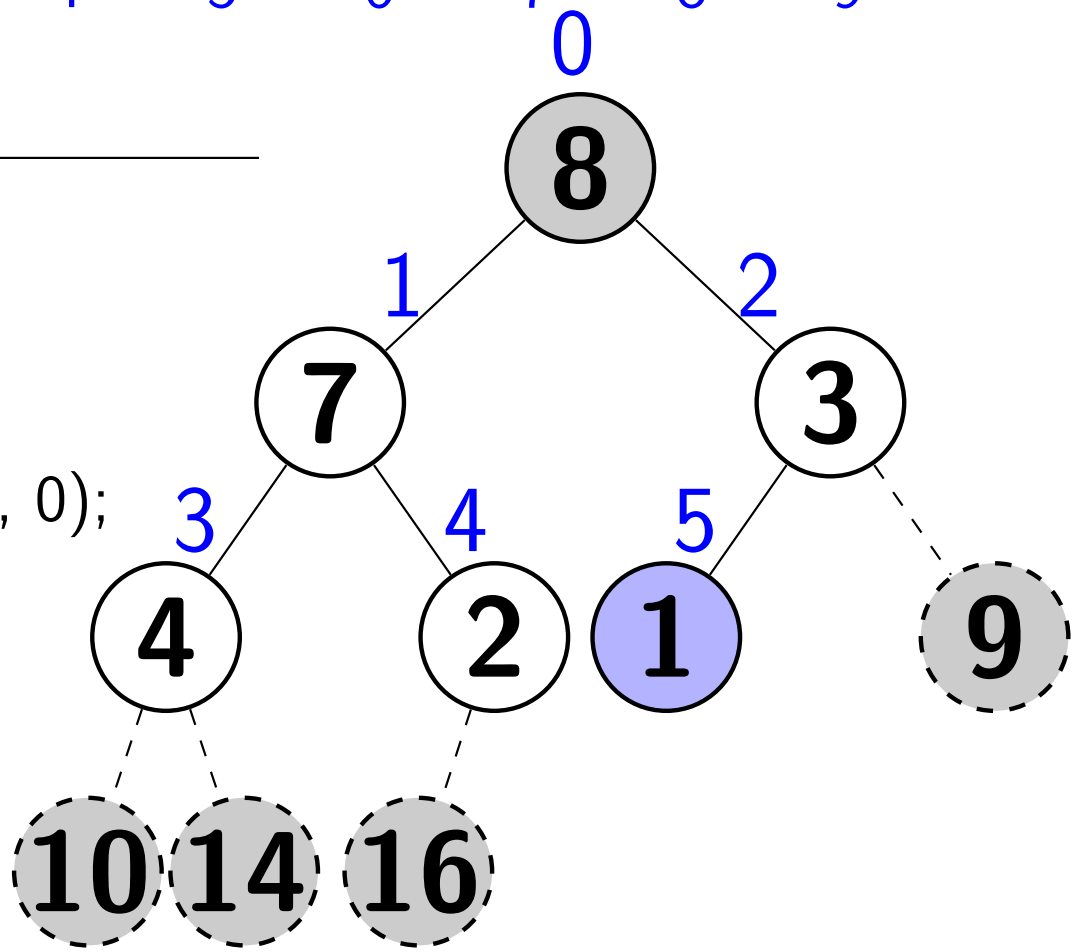
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

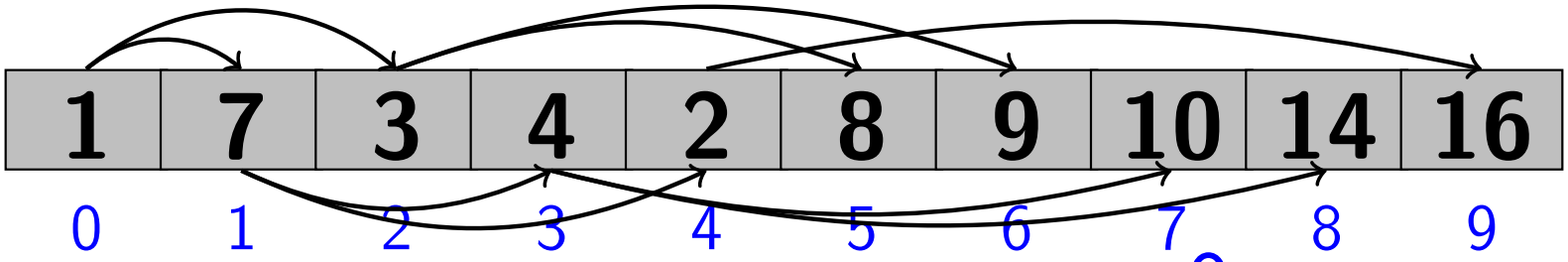
$\triangleright V[0] \leftrightarrow V[\mathbf{5}]$;

SIZE $= 6$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

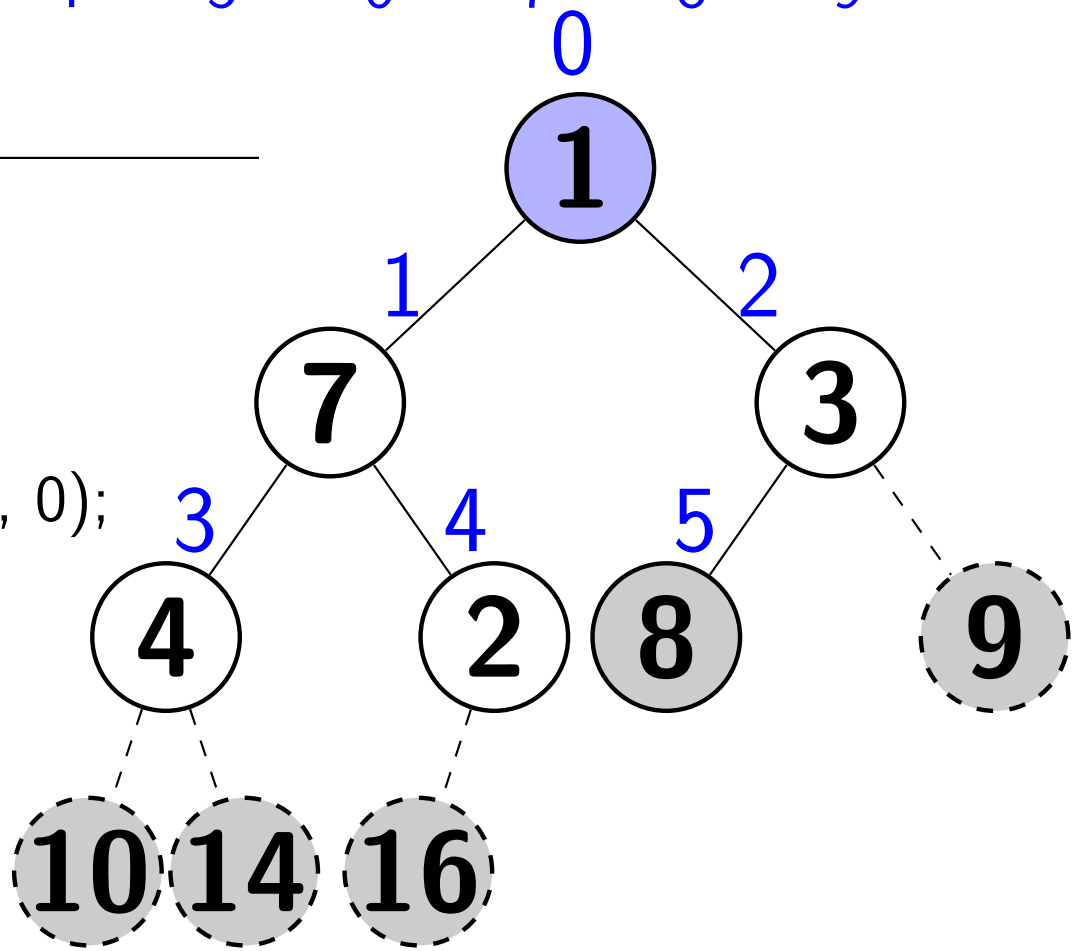
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

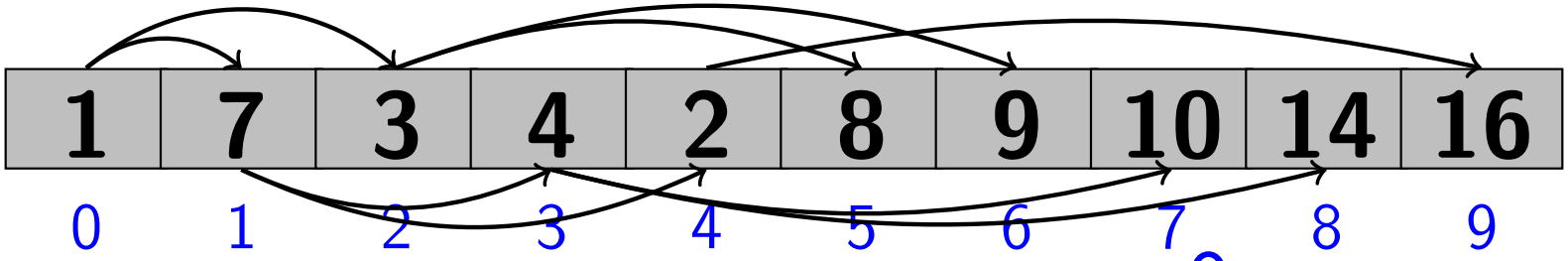
$\triangleright V[0] \leftrightarrow V[\mathbf{5}]$;

SIZE $= 6$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

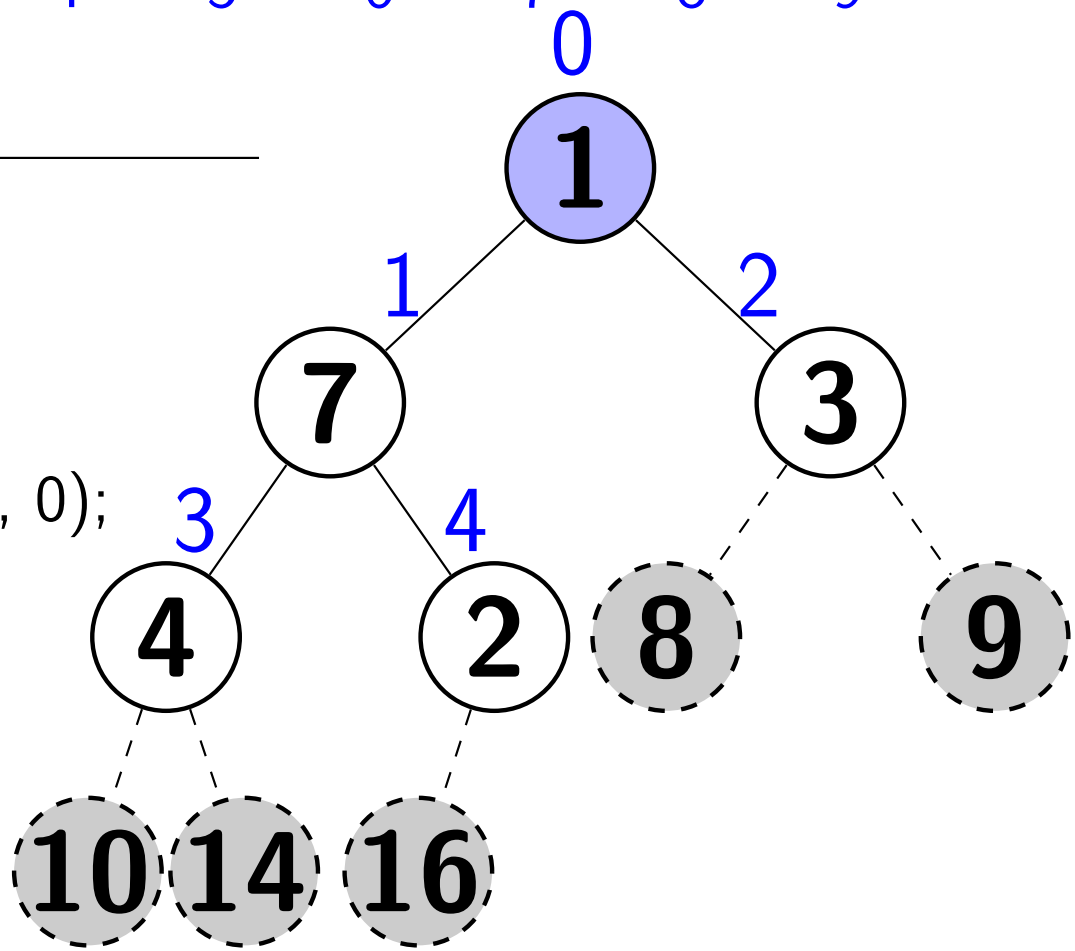
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

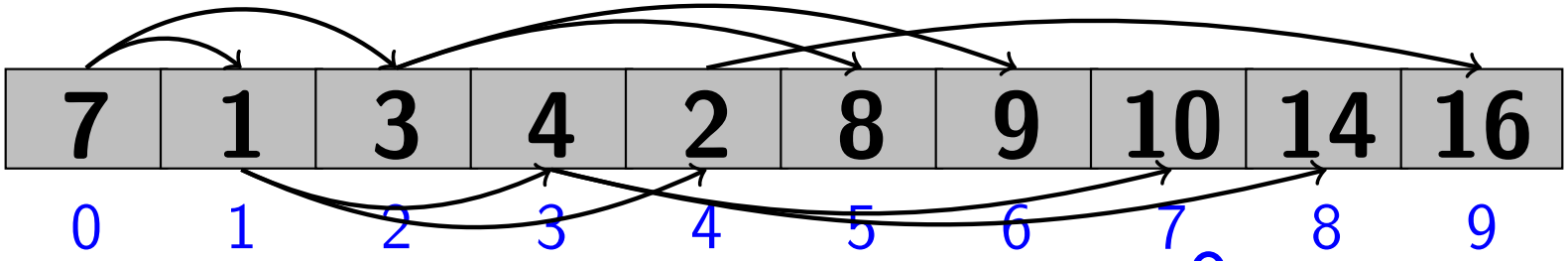
$V[0] \leftrightarrow V[\mathbf{5}]$;

$\triangleright$SIZE $= 5$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

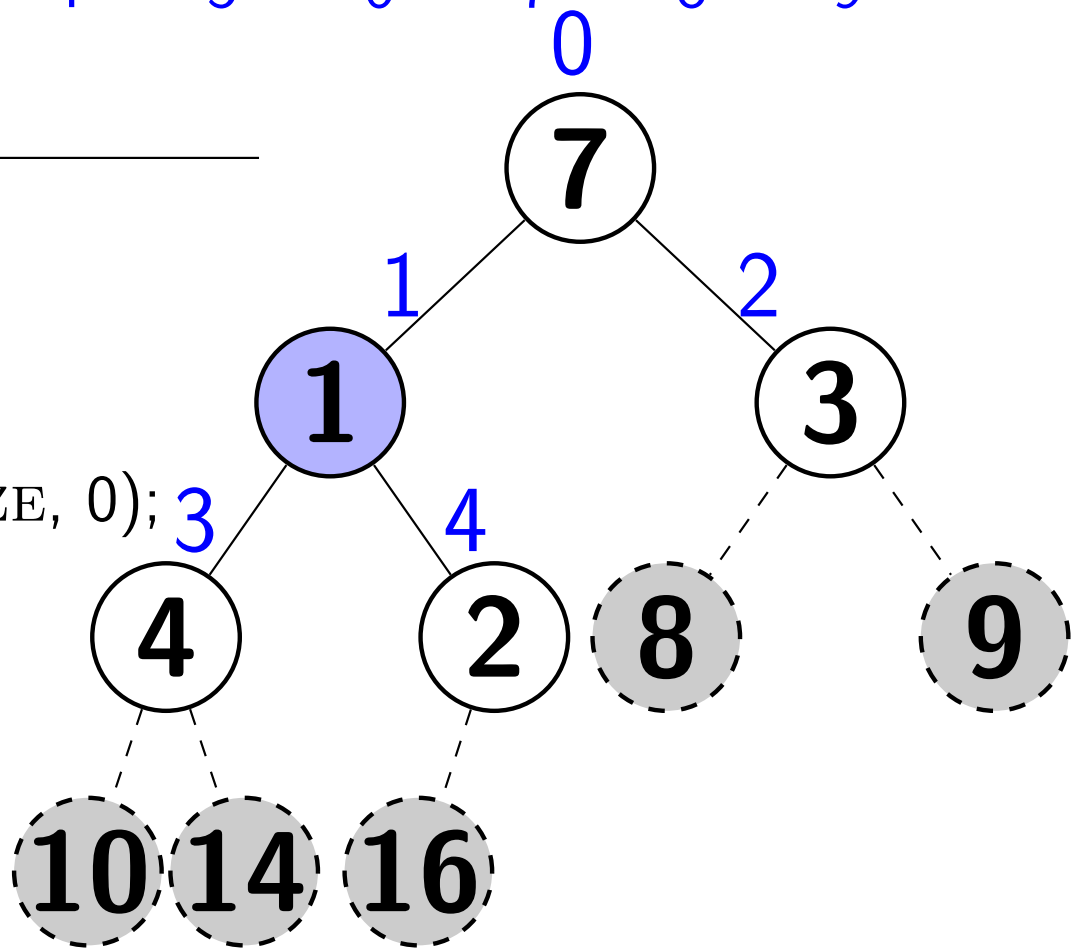
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

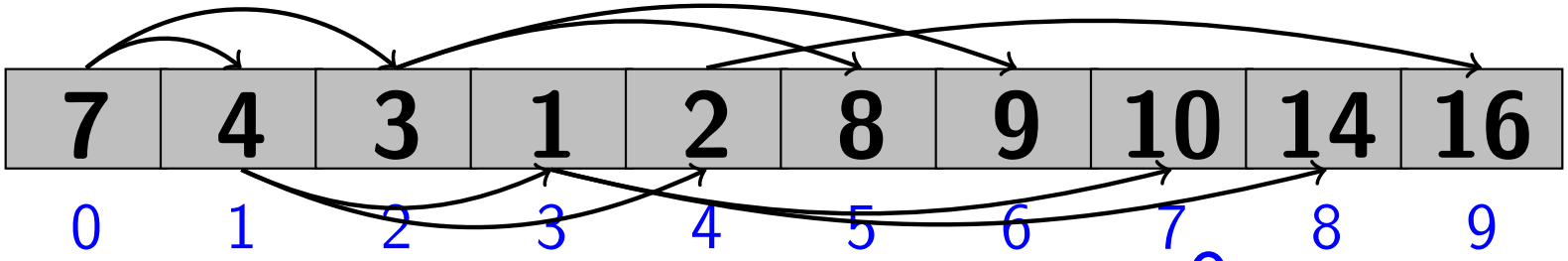
$V[0] \leftrightarrow V[\mathbf{5}]$;

SIZE $= 5$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

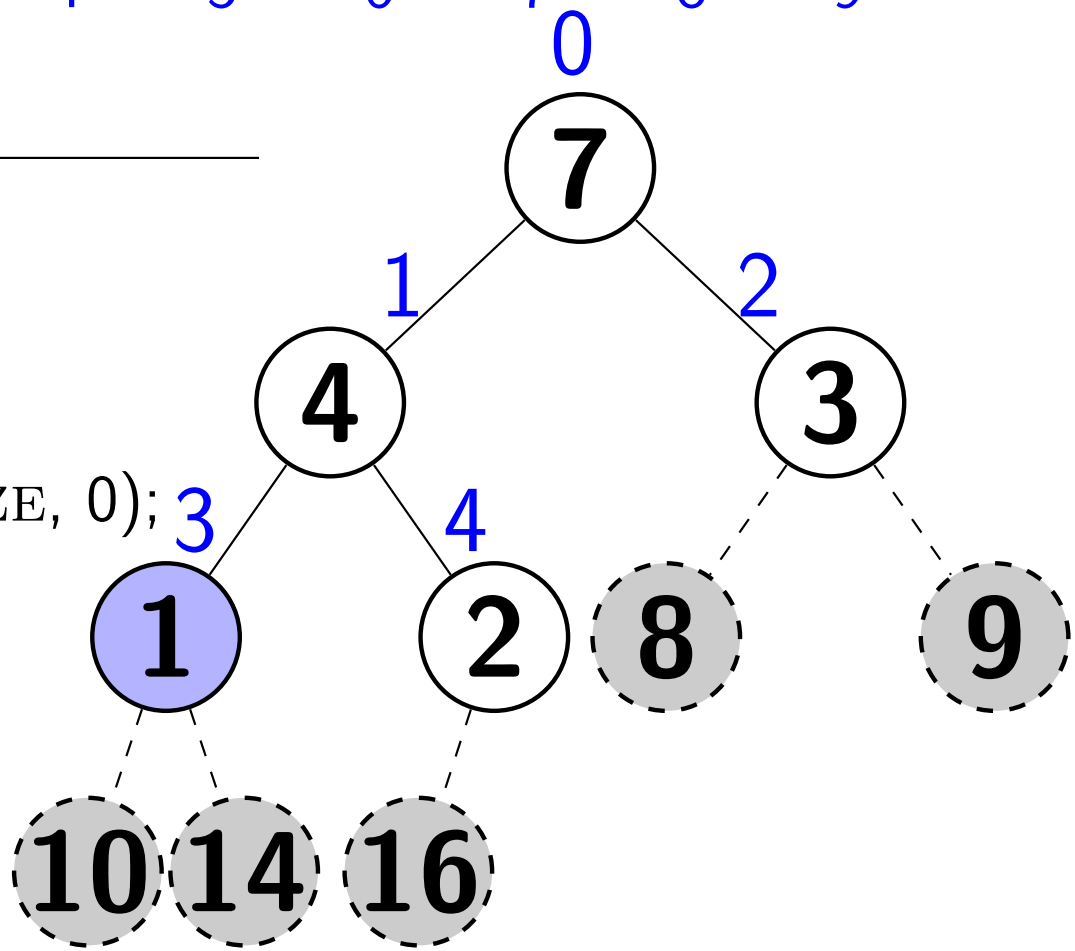
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

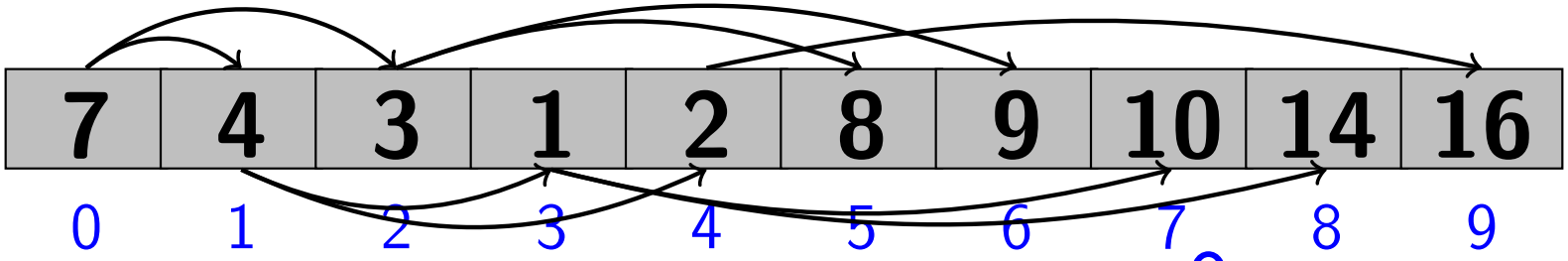
$V[0] \leftrightarrow V[\mathbf{5}]$;

SIZE $= 5$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

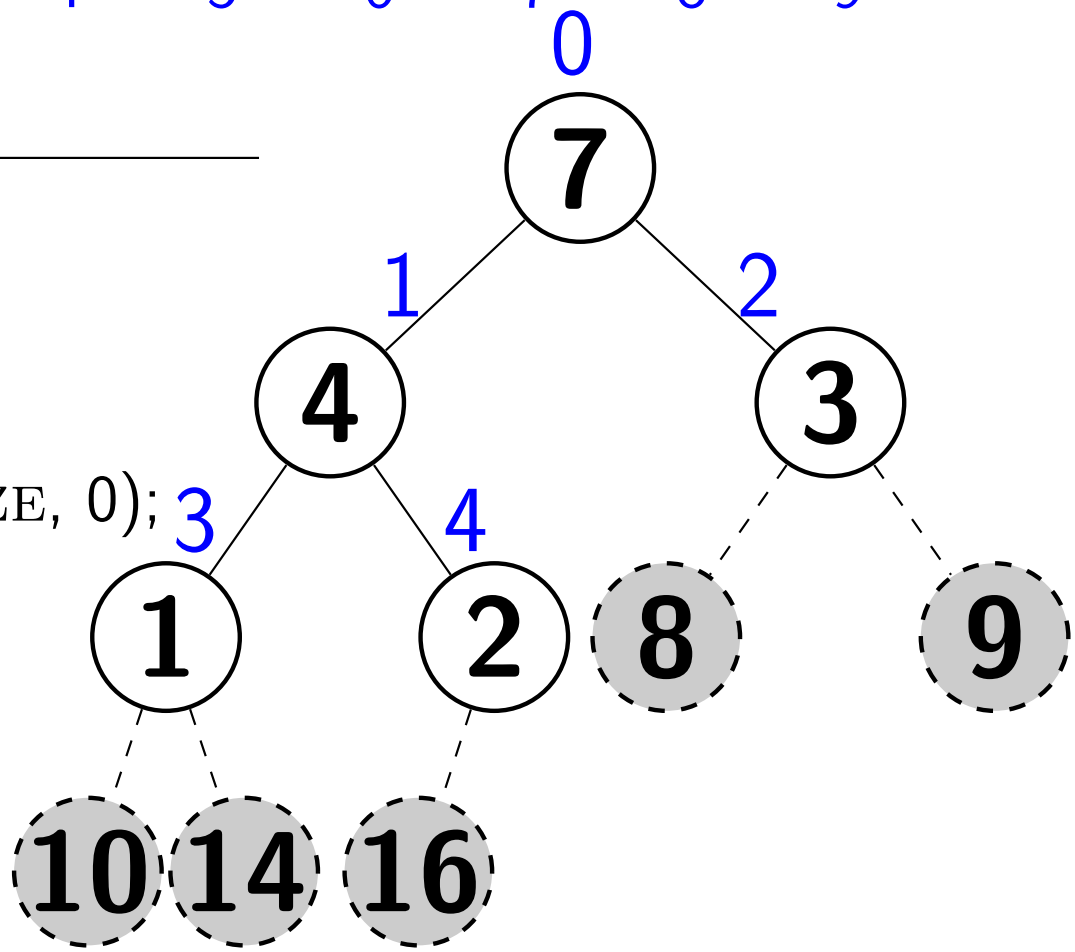
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

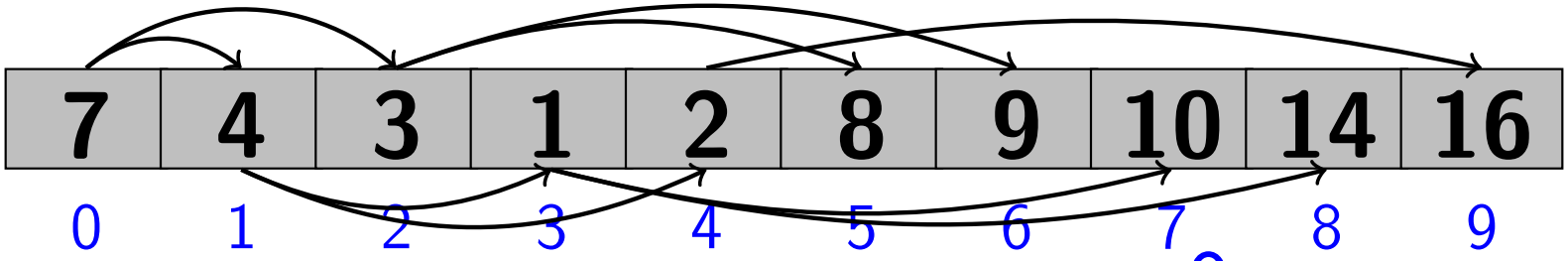
$V[0] \leftrightarrow V[\mathbf{5}]$;

SIZE $= 5$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

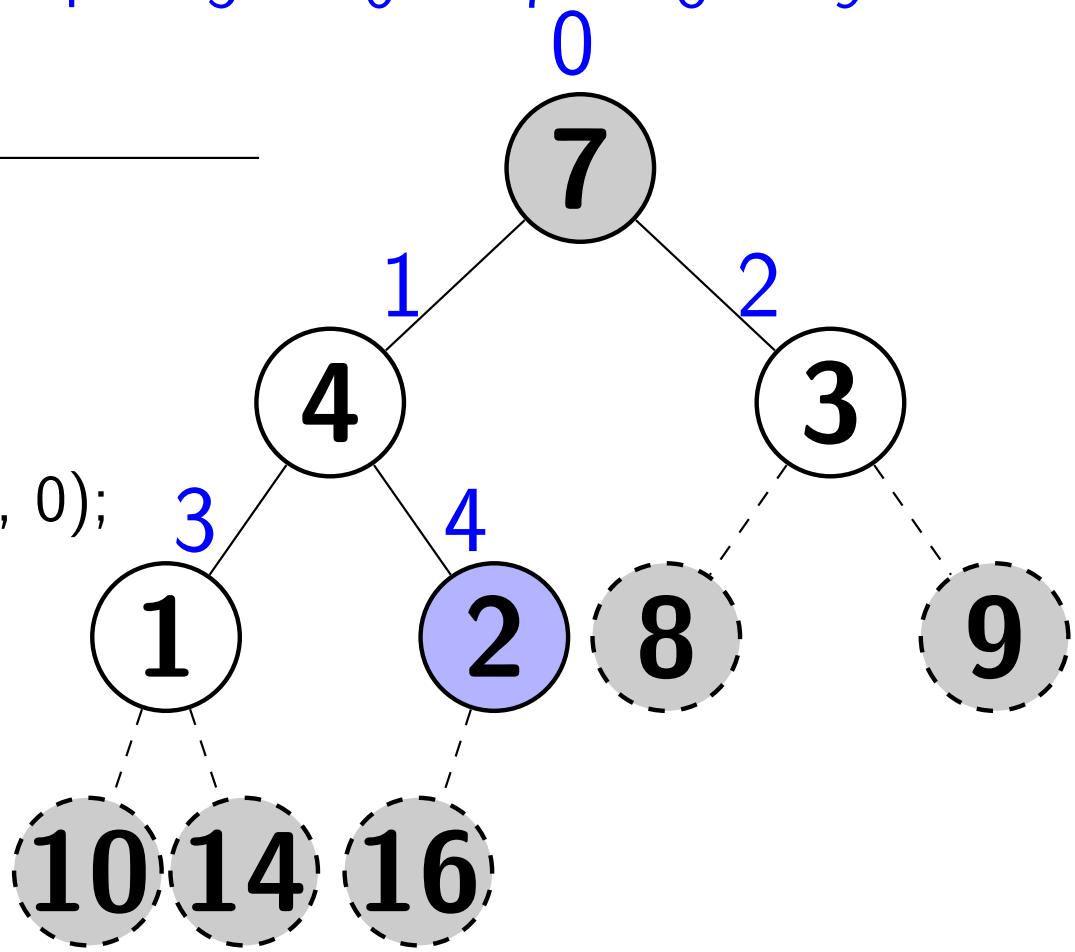
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

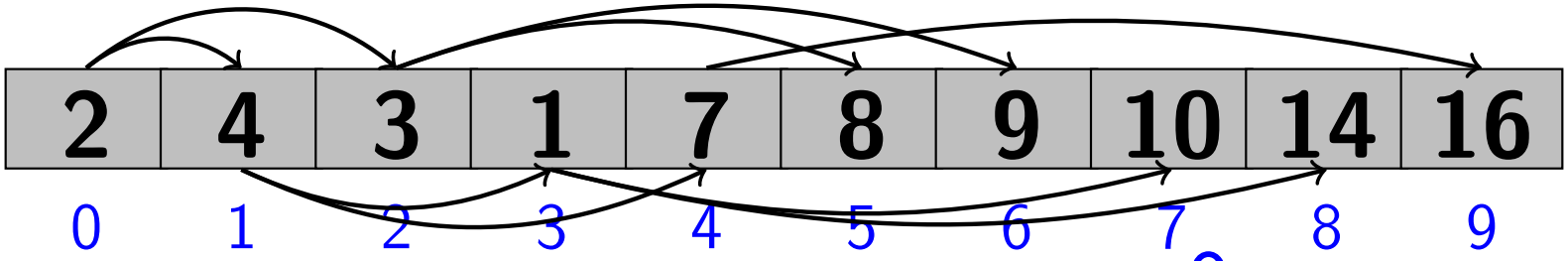
$\triangleright V[0] \leftrightarrow V[\mathbf{4}]$;

SIZE $= 5$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

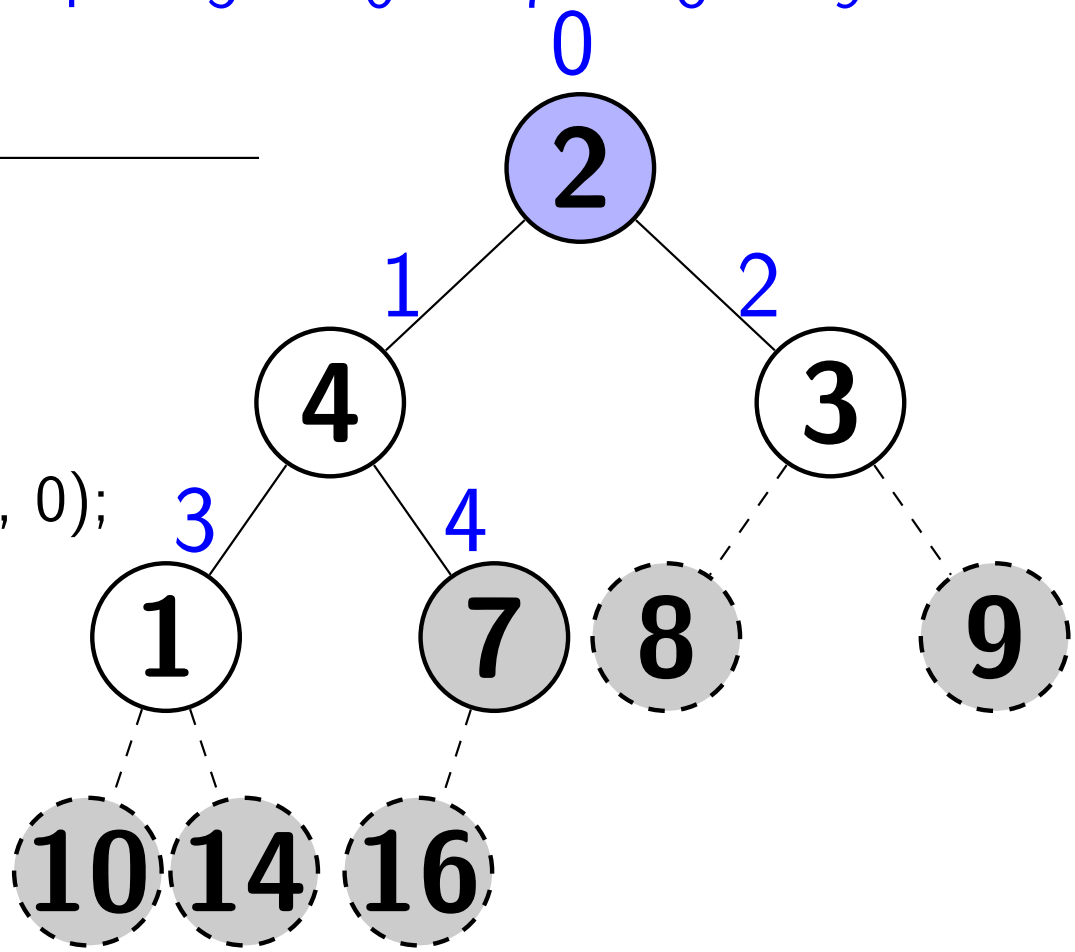
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

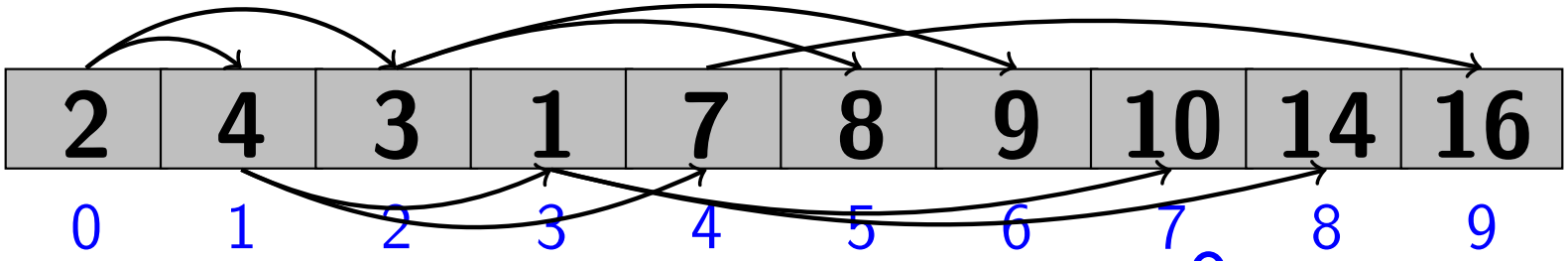
$\triangleright V[0] \leftrightarrow V[\mathbf{4}]$;

SIZE $= 5$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

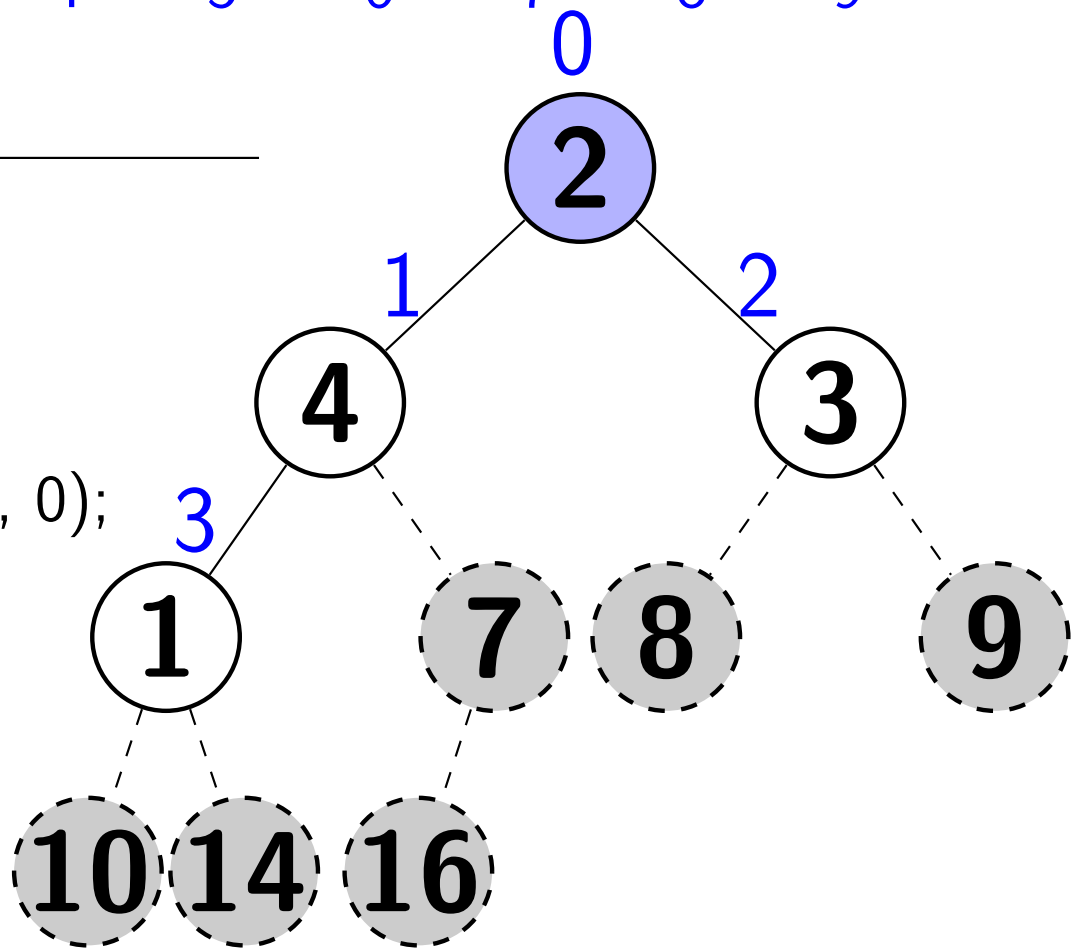
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

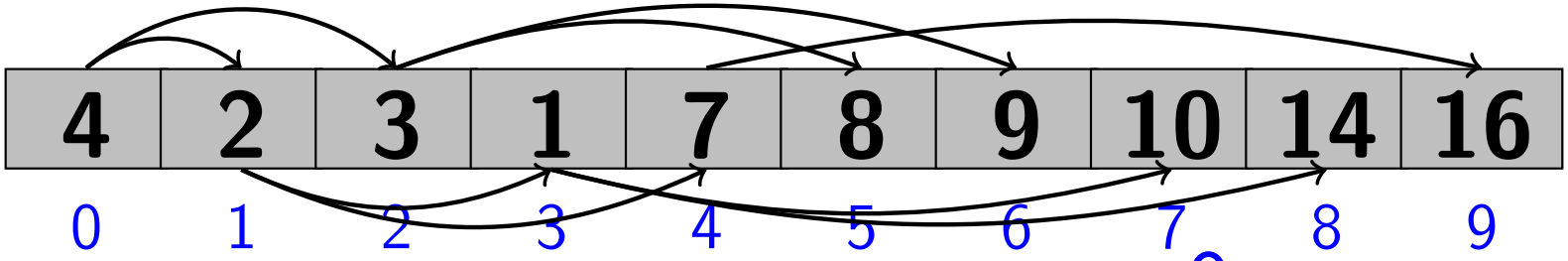
$V[0] \leftrightarrow V[\mathbf{4}]$;

$\triangleright$SIZE $= 4$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

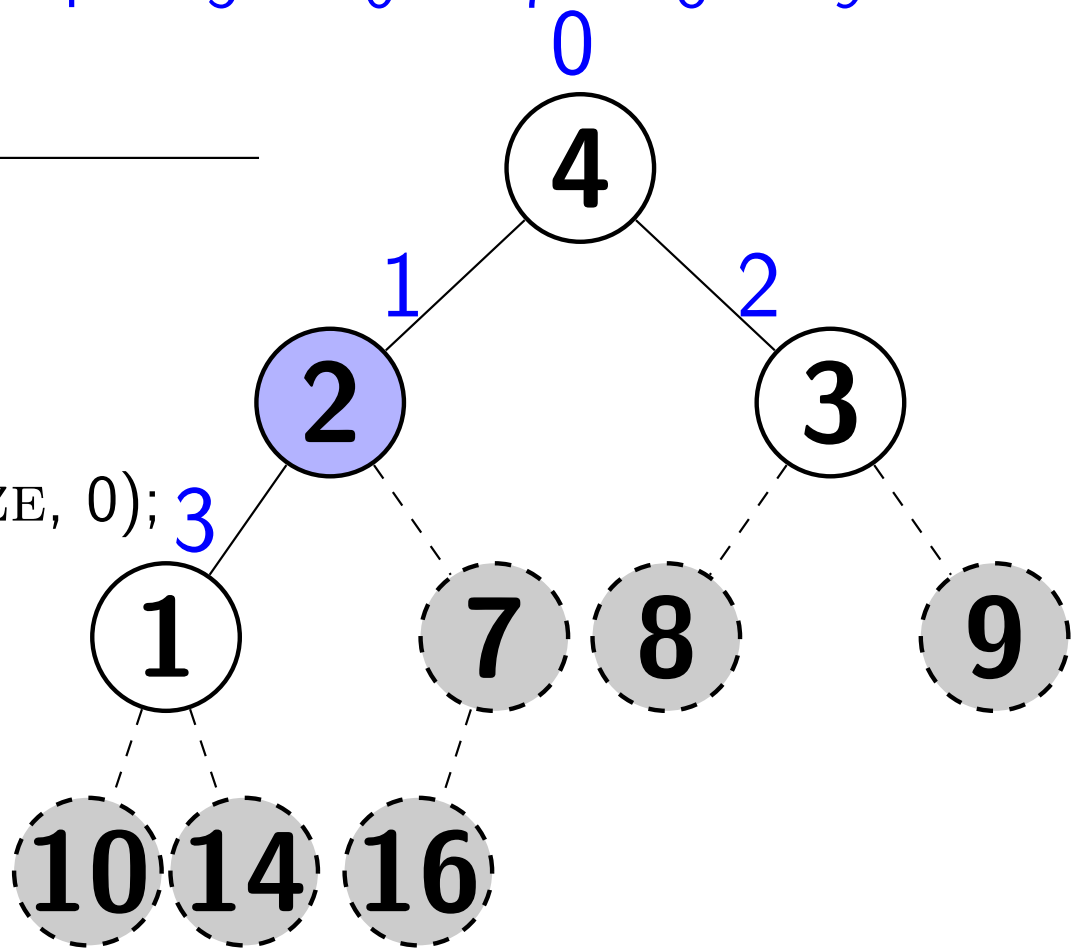
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

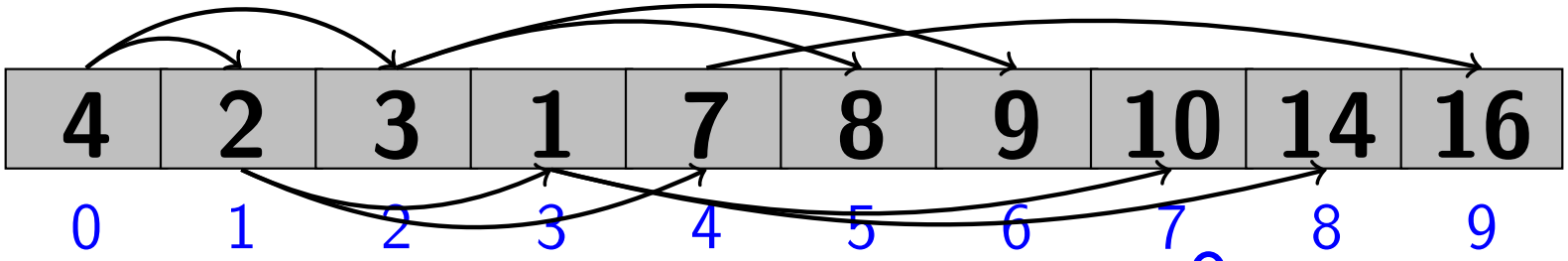
$V[0] \leftrightarrow V[\mathbf{4}]$;

SIZE $= 4$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

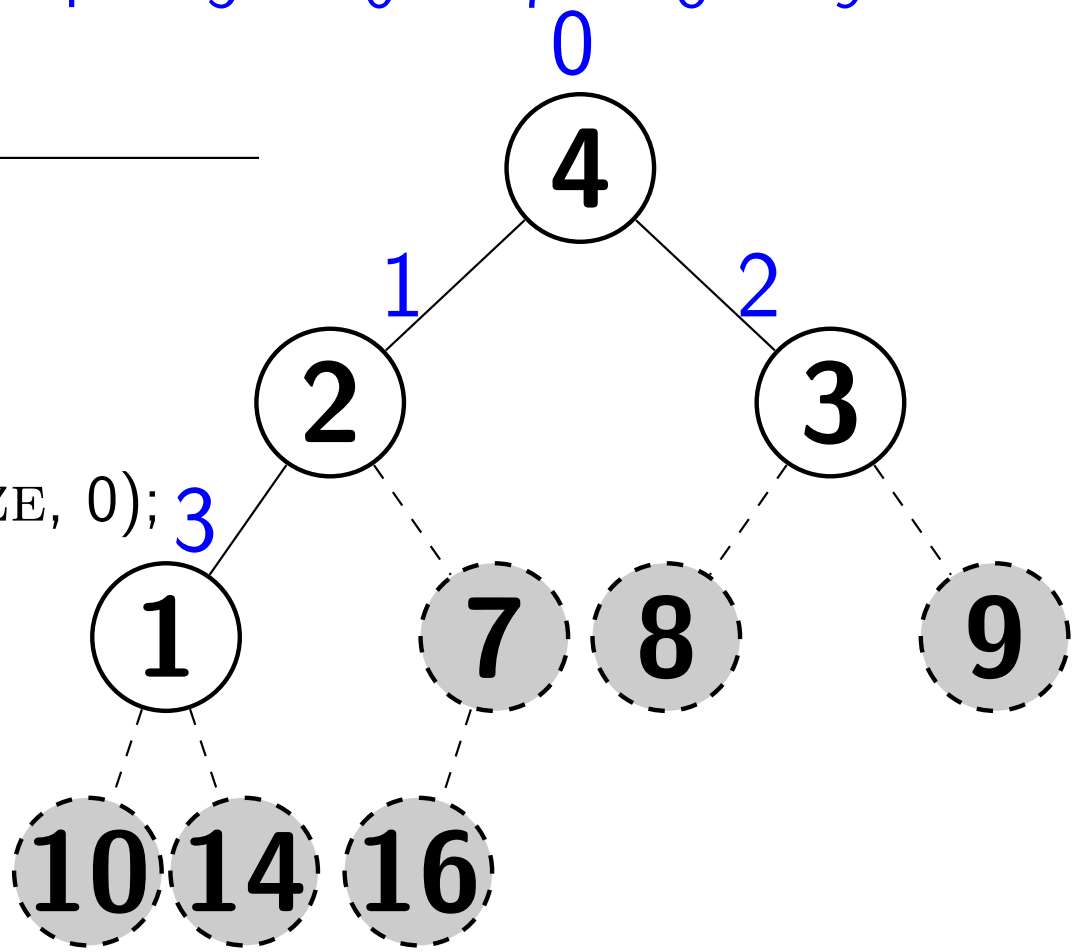
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

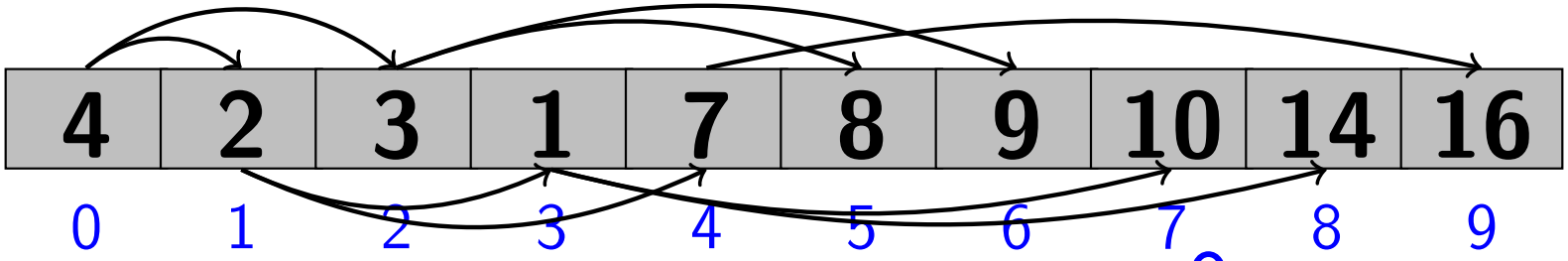
$V[0] \leftrightarrow V[\mathbf{4}]$;

SIZE $= 4$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

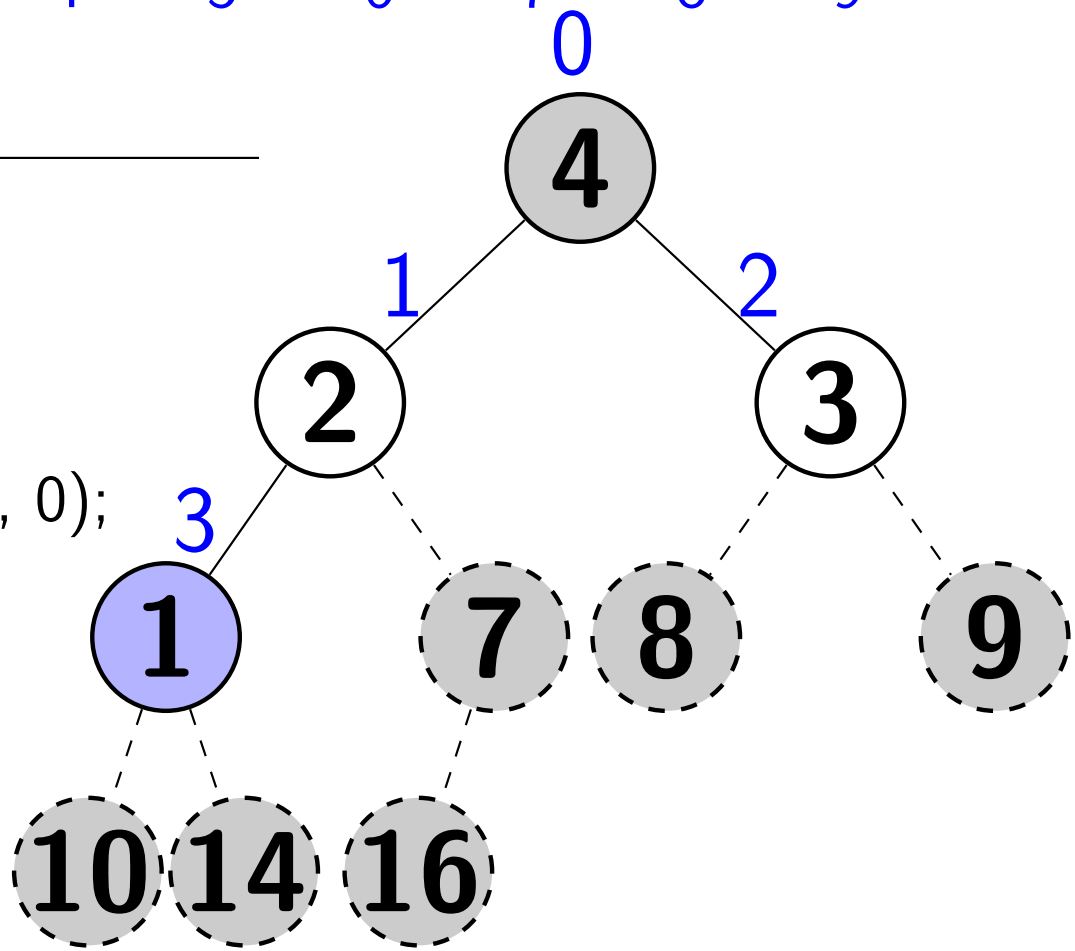
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

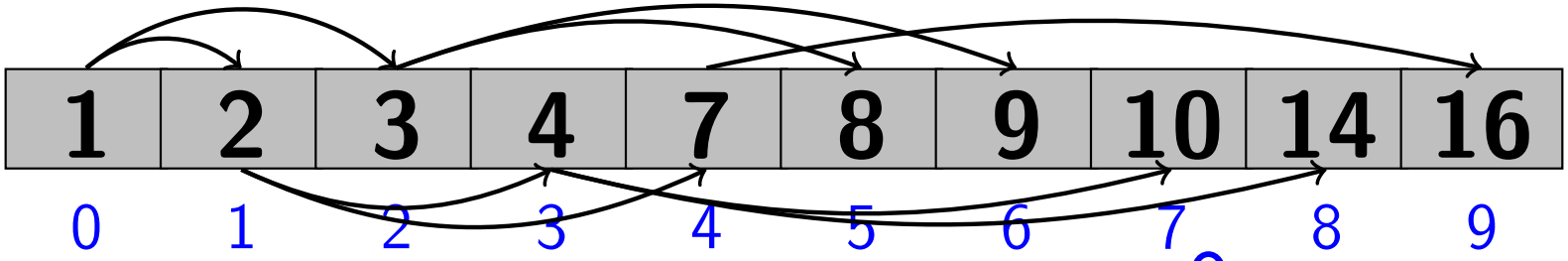
$\triangleright V[0] \leftrightarrow V[\mathbf{3}]$;

SIZE $= 4$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT (V, 10)

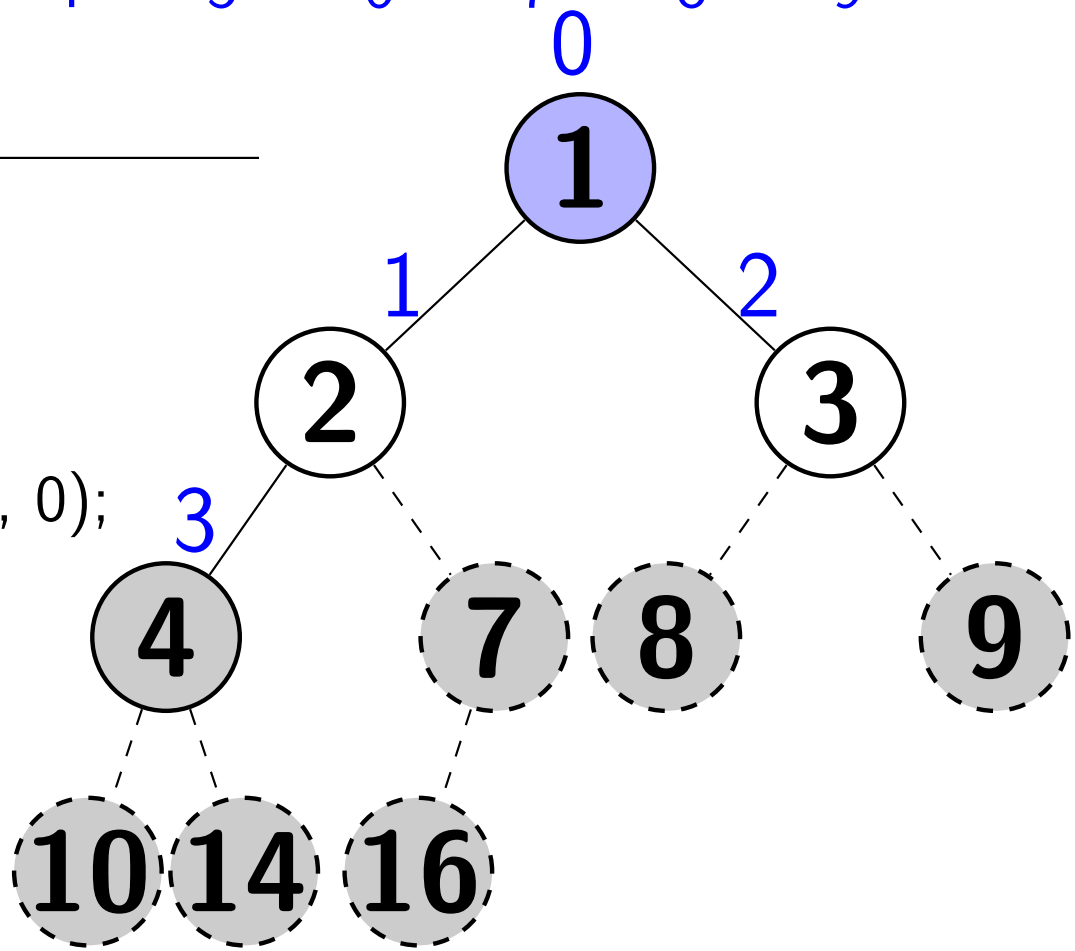
BUILD-MAX-HEAP (V,10);

Para **i** de 9 **até** 1 **faça**

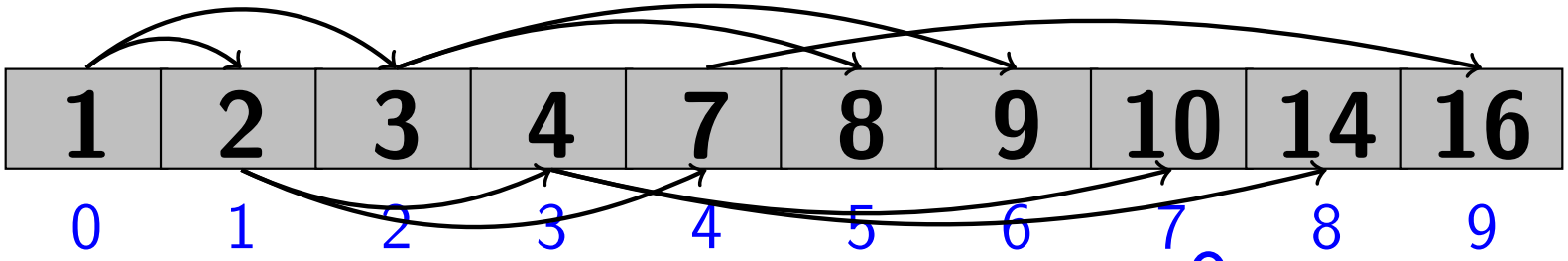
$\triangleright V[0] \leftrightarrow V[3]$;

SIZE $= 4$;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

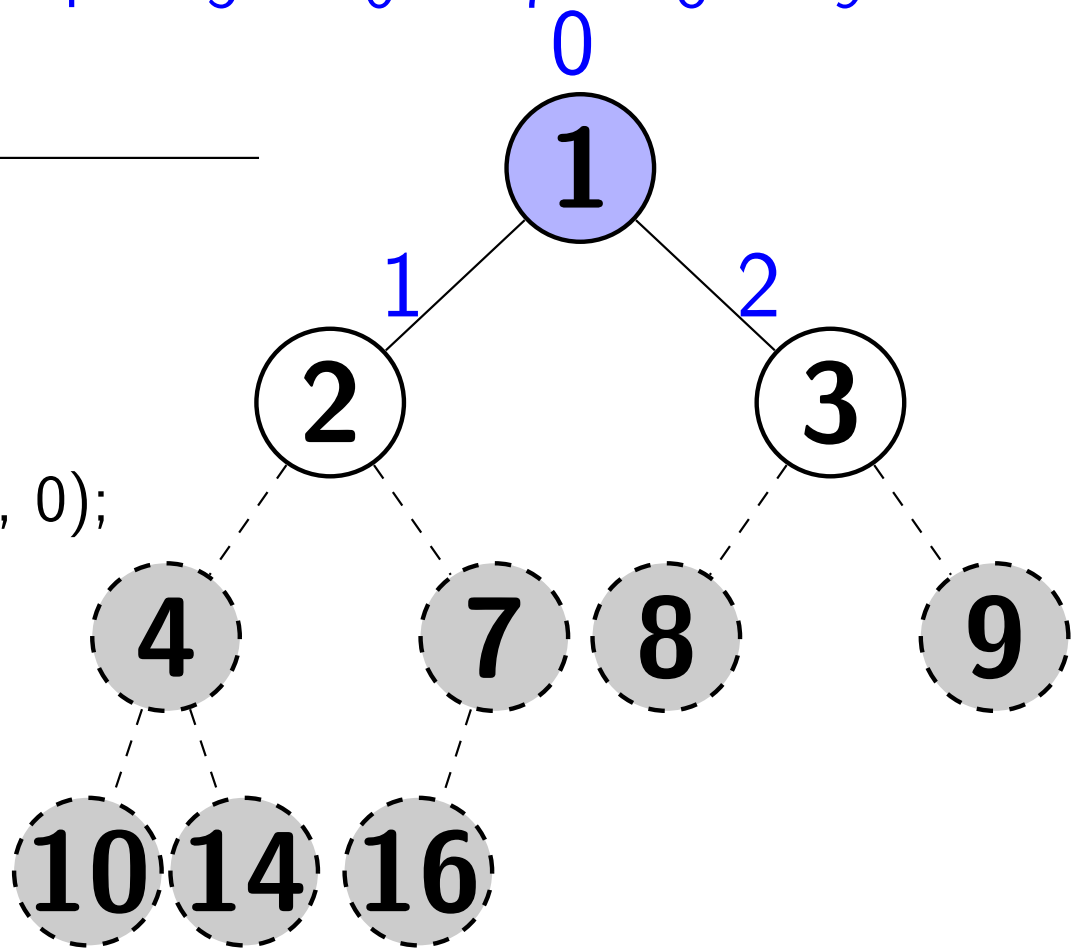
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

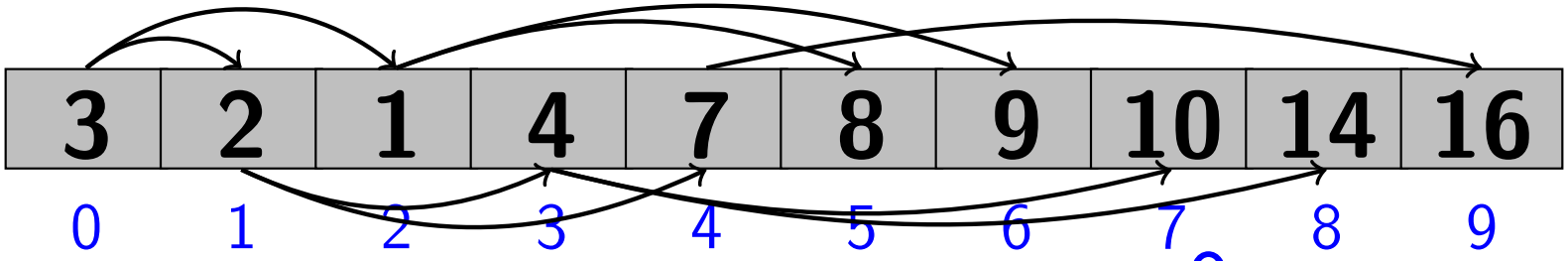
$V[0] \leftrightarrow V[3]$;

$\triangleright$SIZE $= 3$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

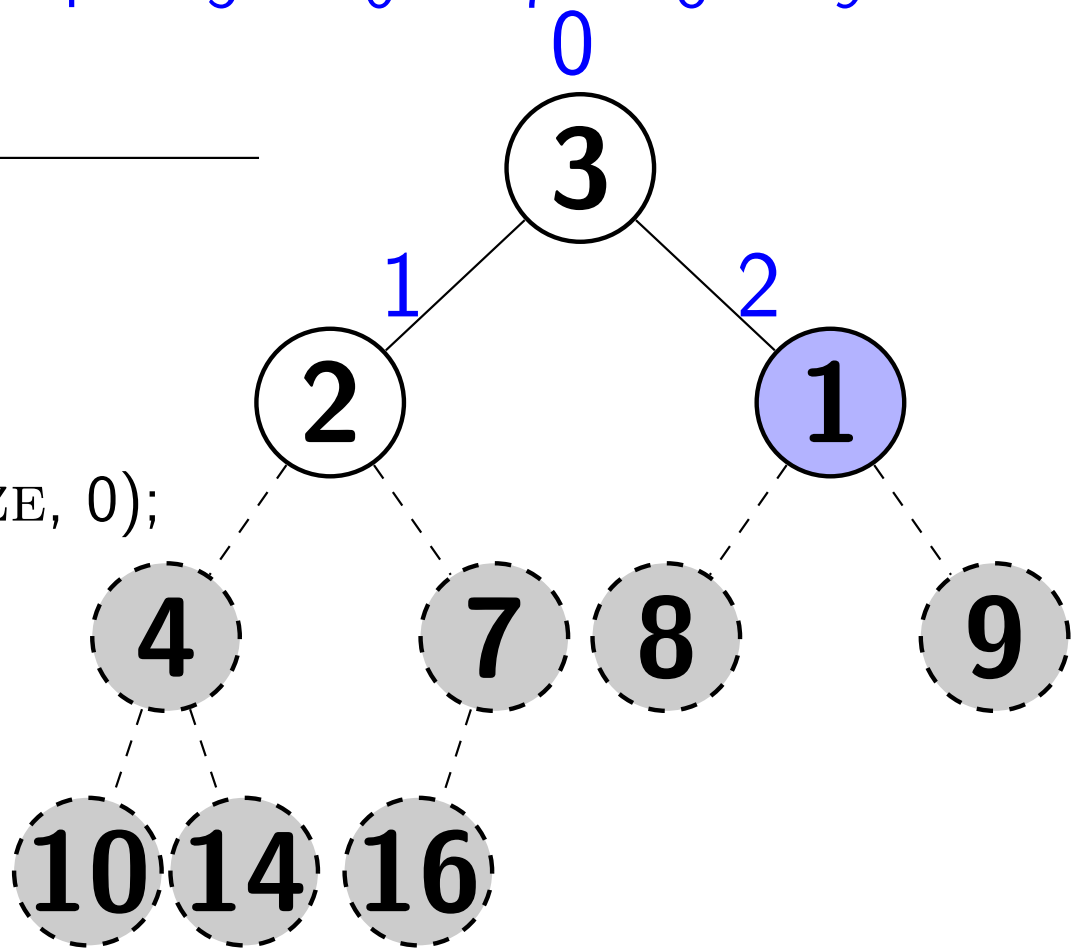
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

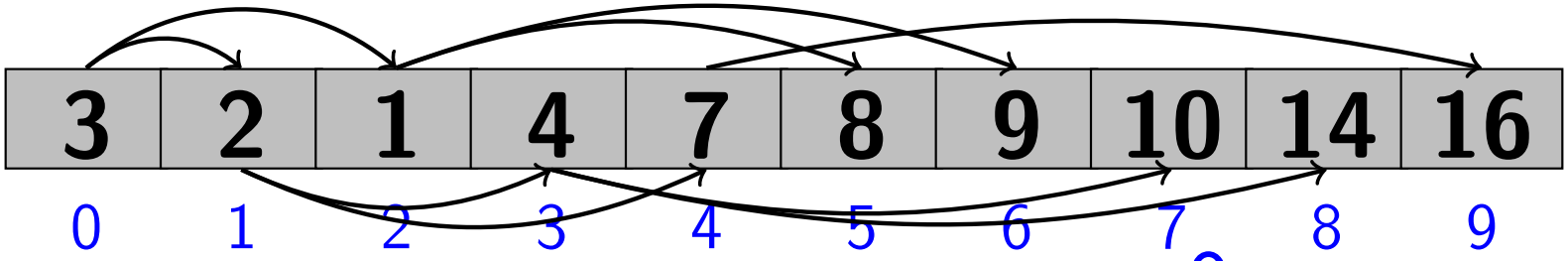
$V[0] \leftrightarrow V[3]$;

SIZE $= 3$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

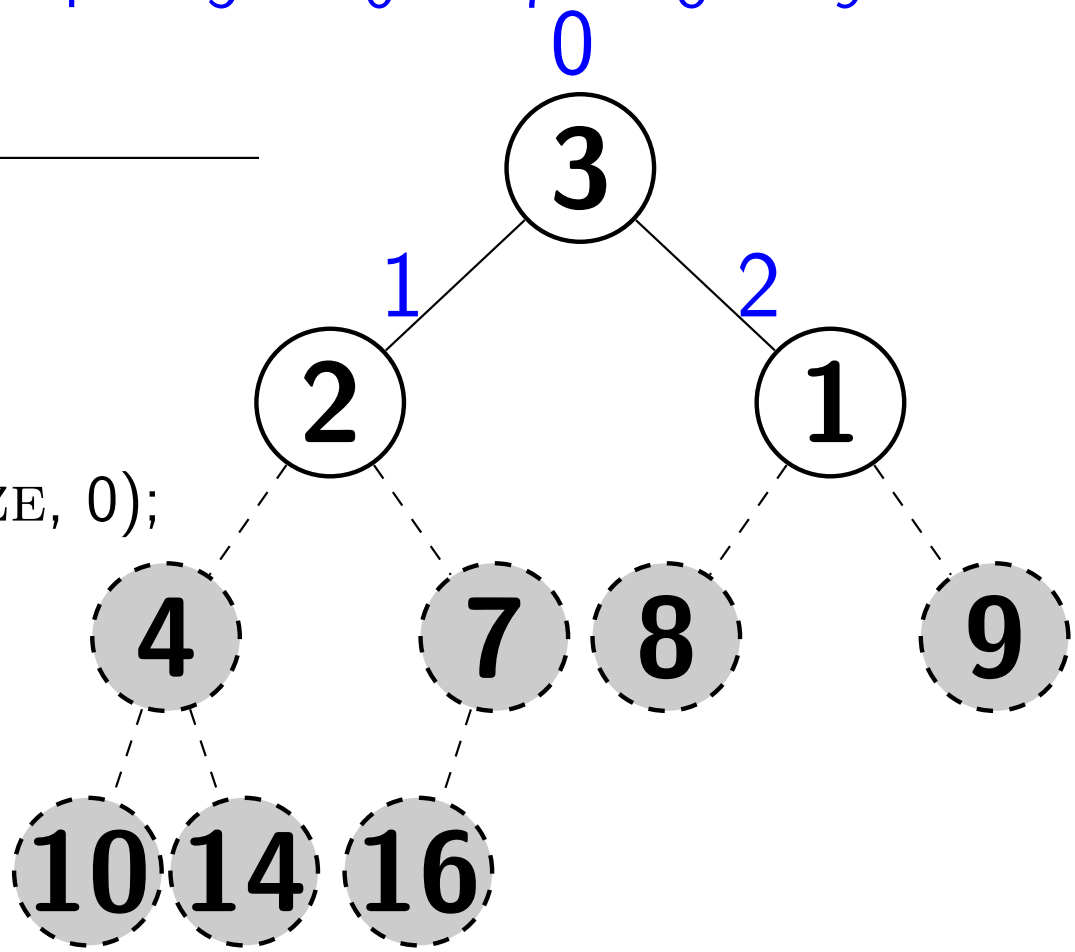
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

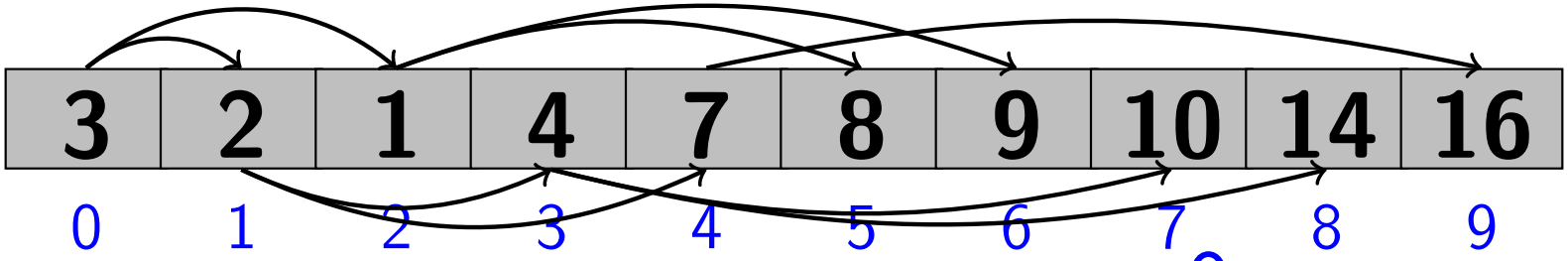
$V[0] \leftrightarrow V[\mathbf{3}]$;

SIZE $= 3$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

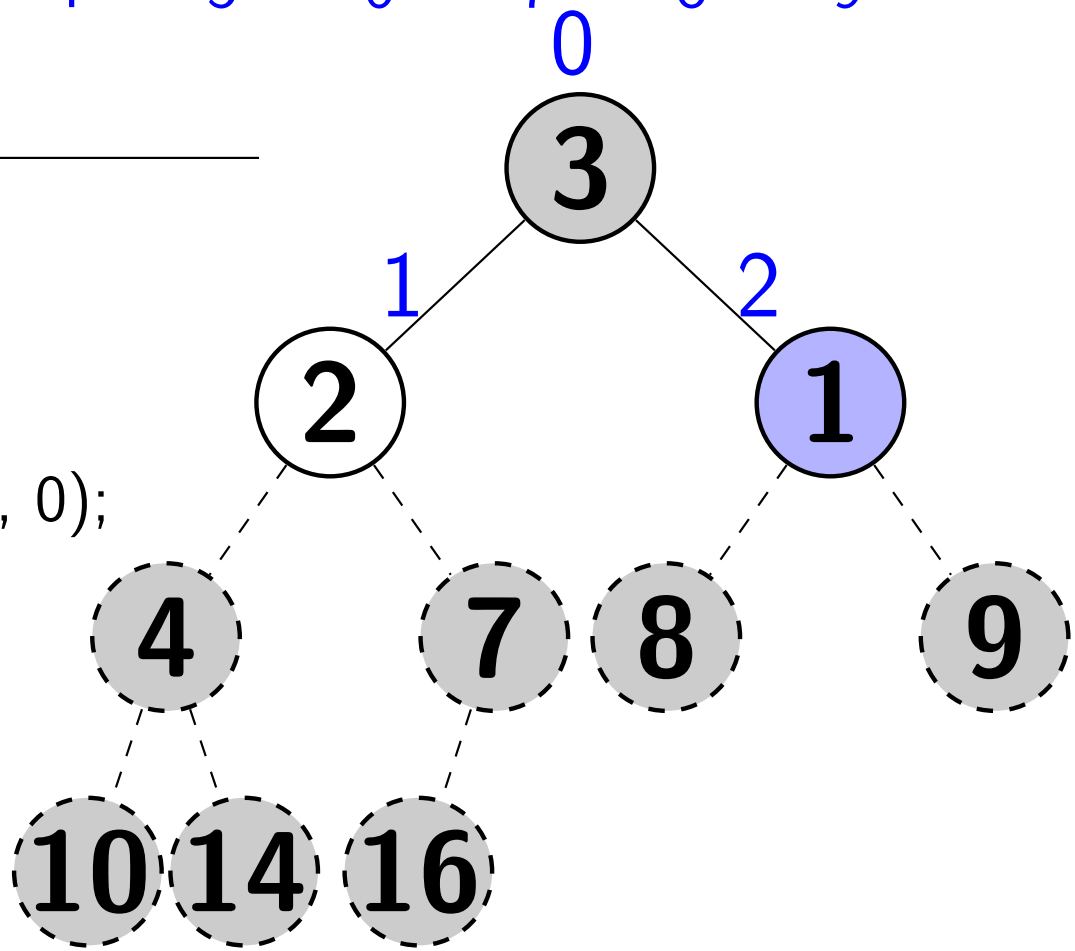
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

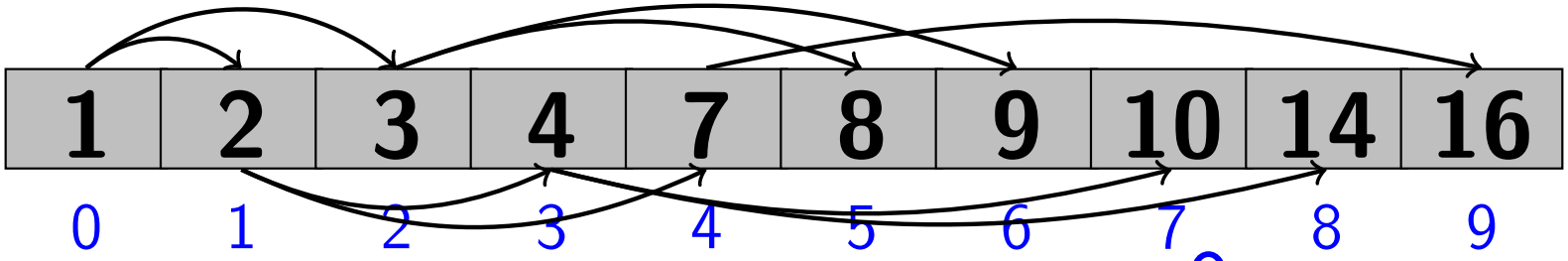
$\triangleright V[0] \leftrightarrow V[\mathbf{2}]$;

SIZE $= 3$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

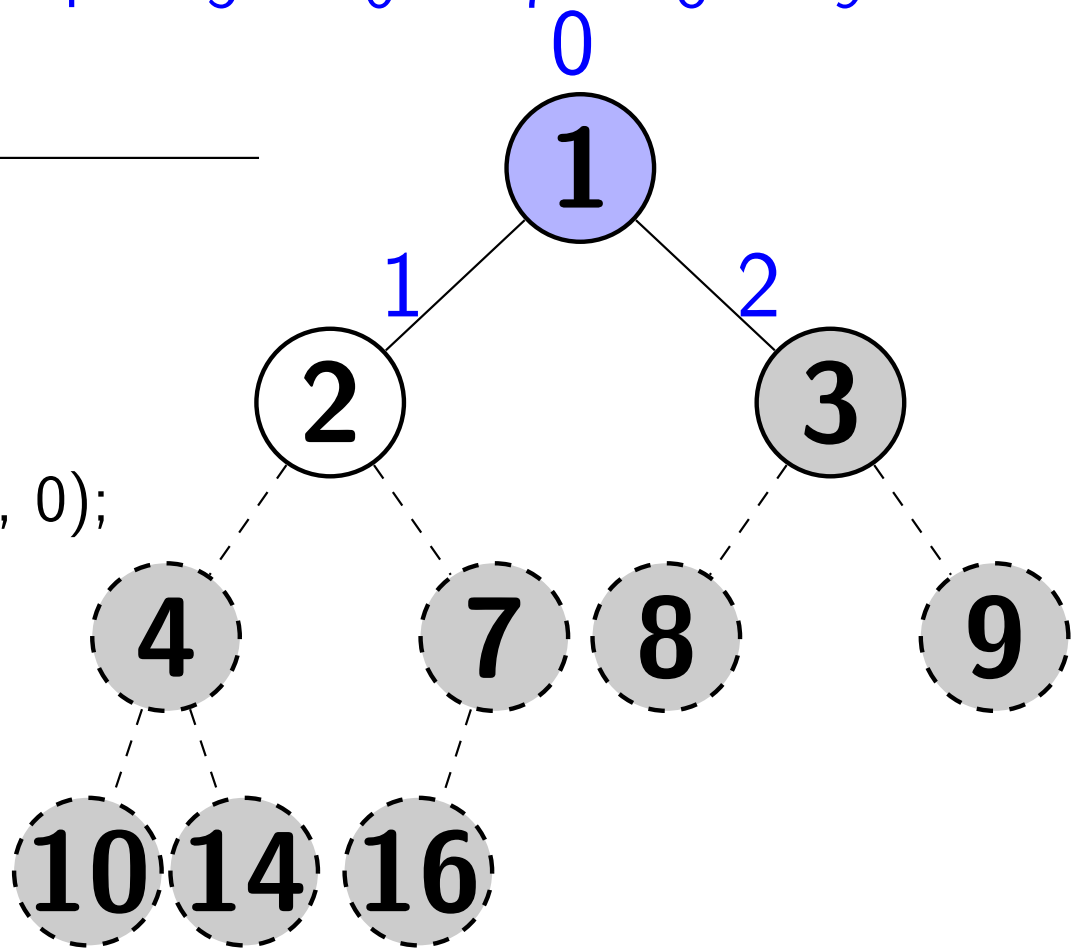
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

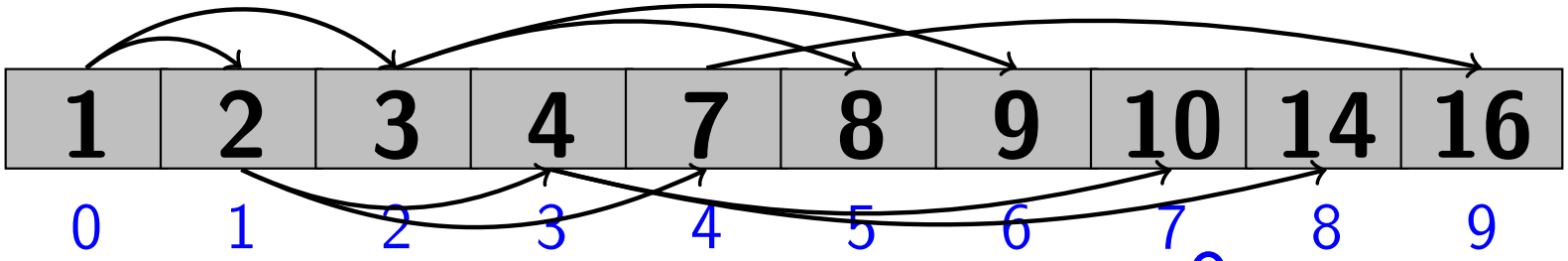
$\triangleright V[0] \leftrightarrow V[\mathbf{2}]$;

SIZE $= 3$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT (V, 10)

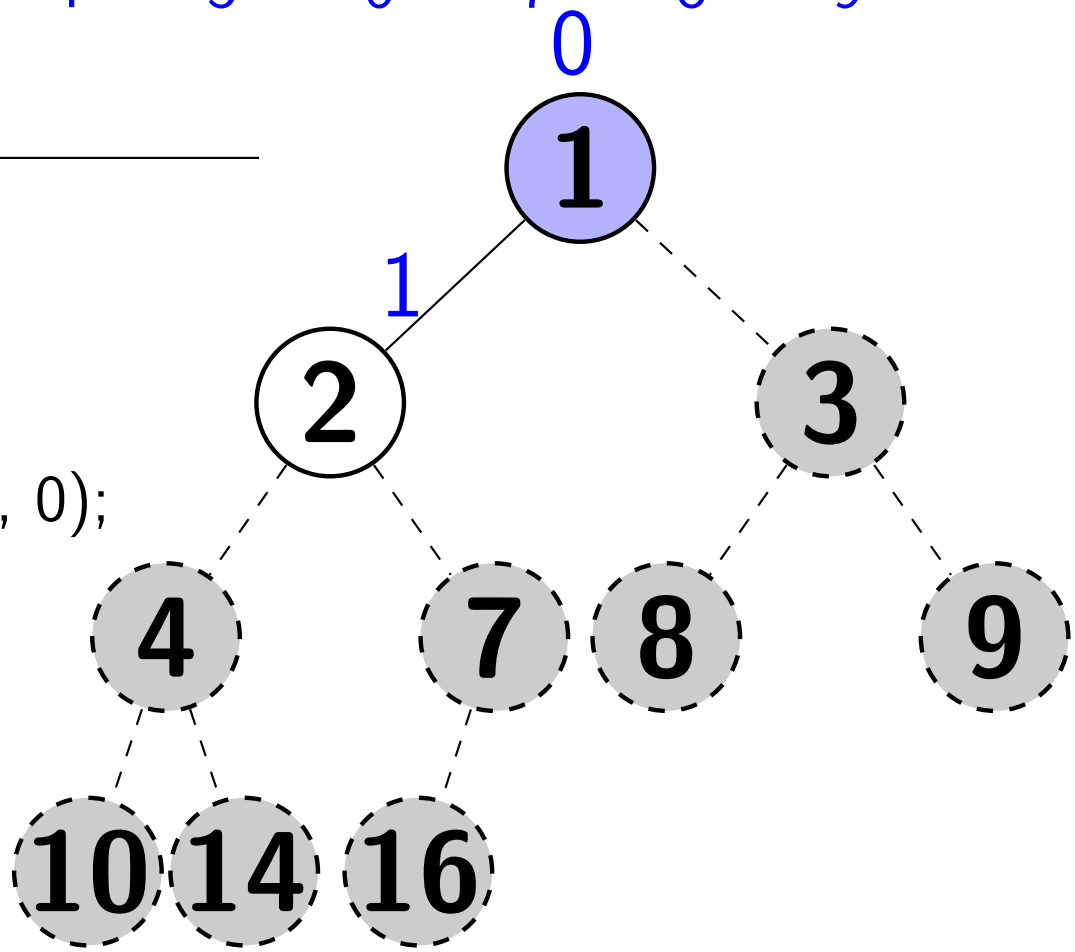
BUILD-MAX-HEAP (V,10);

Para **i** de 9 **até** 1 **faça**

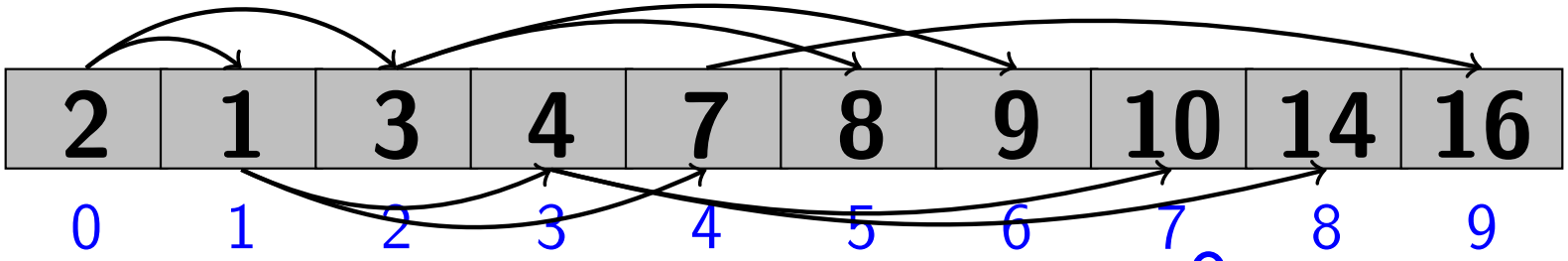
$V[0] \leftrightarrow V[\mathbf{2}]$;

$\triangleright$SIZE $= 2$;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

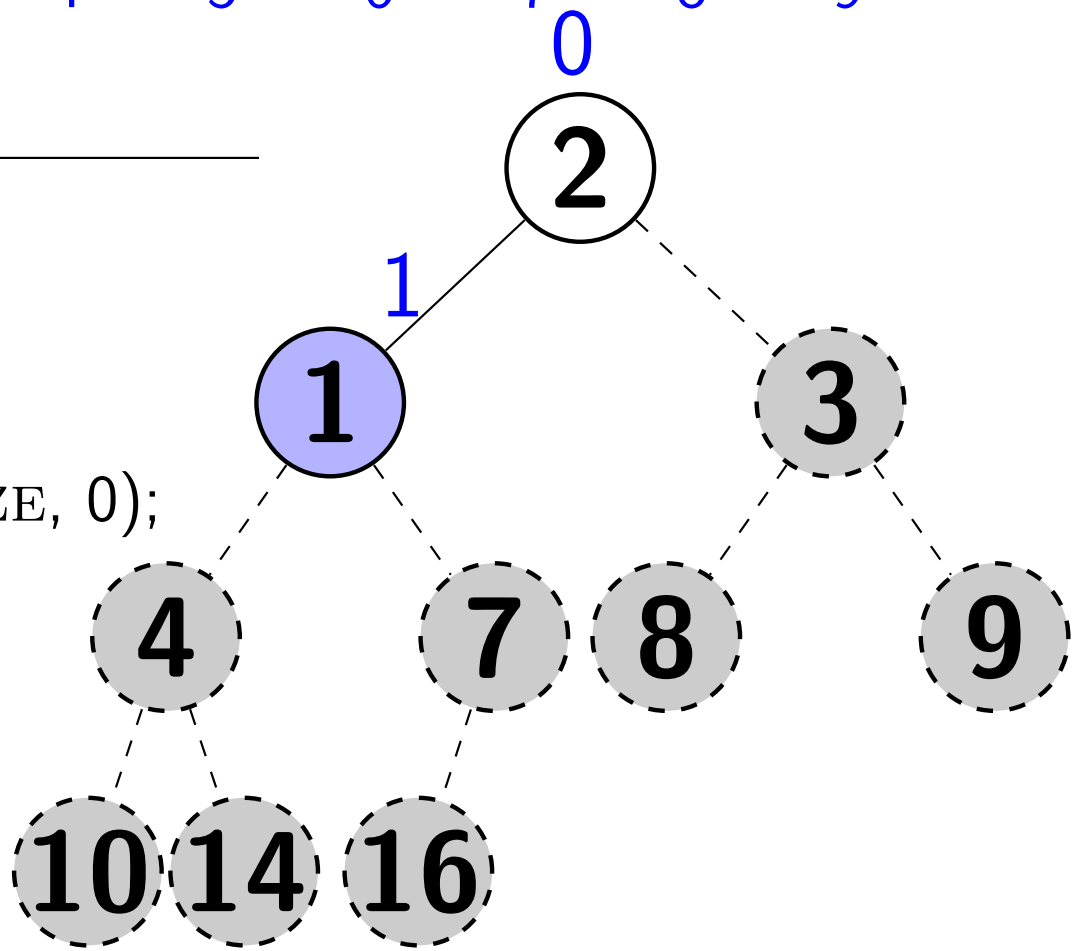
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

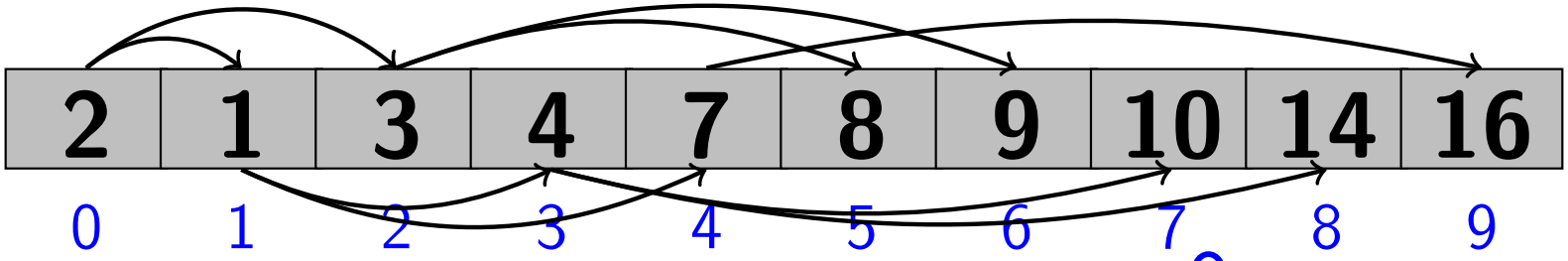
$V[0] \leftrightarrow V[\mathbf{2}]$;

SIZE $= 2$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

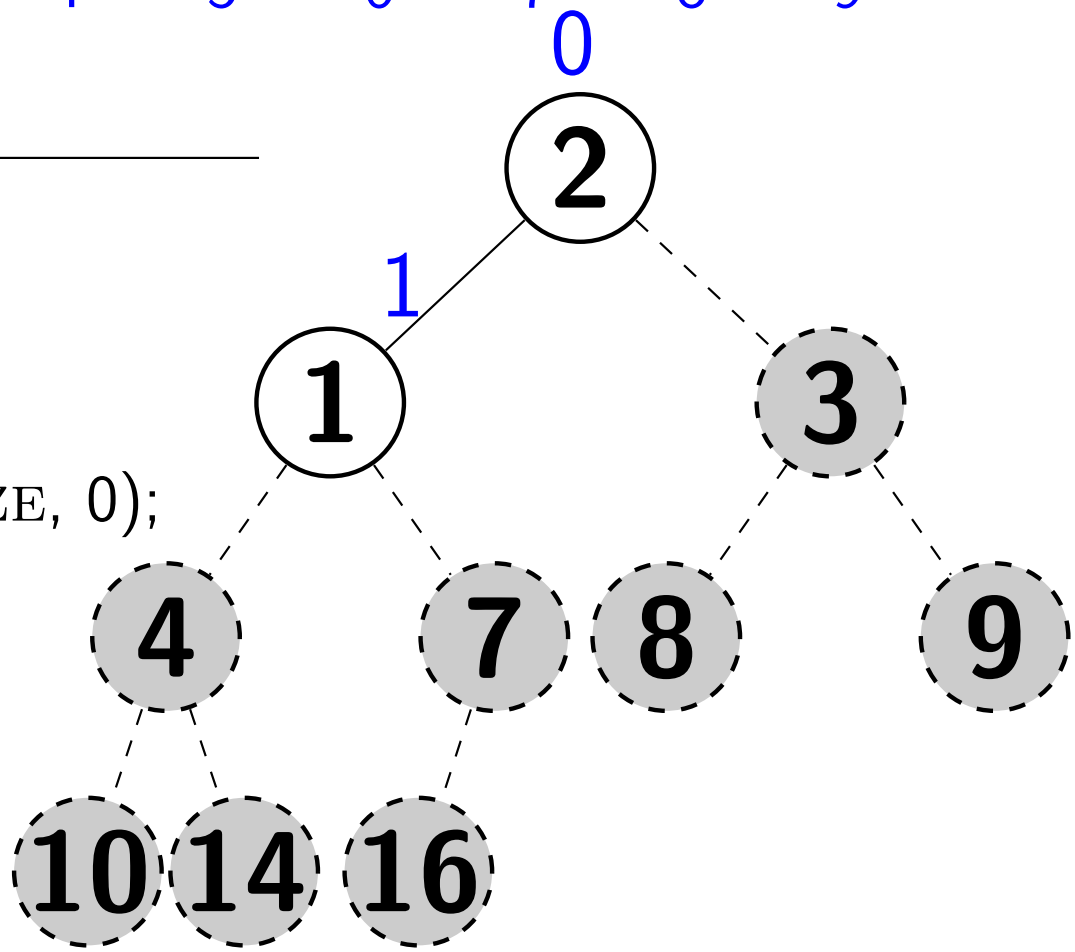
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

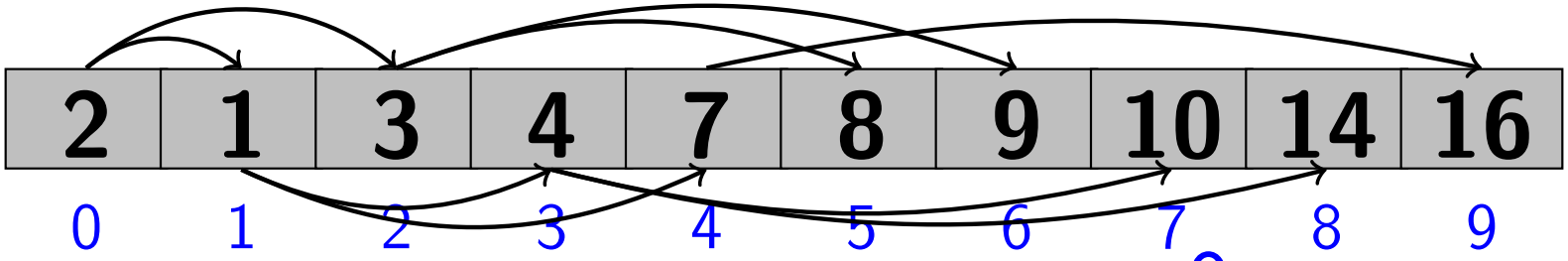
$V[0] \leftrightarrow V[\mathbf{2}]$;

SIZE $= 2$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT (V, 10)

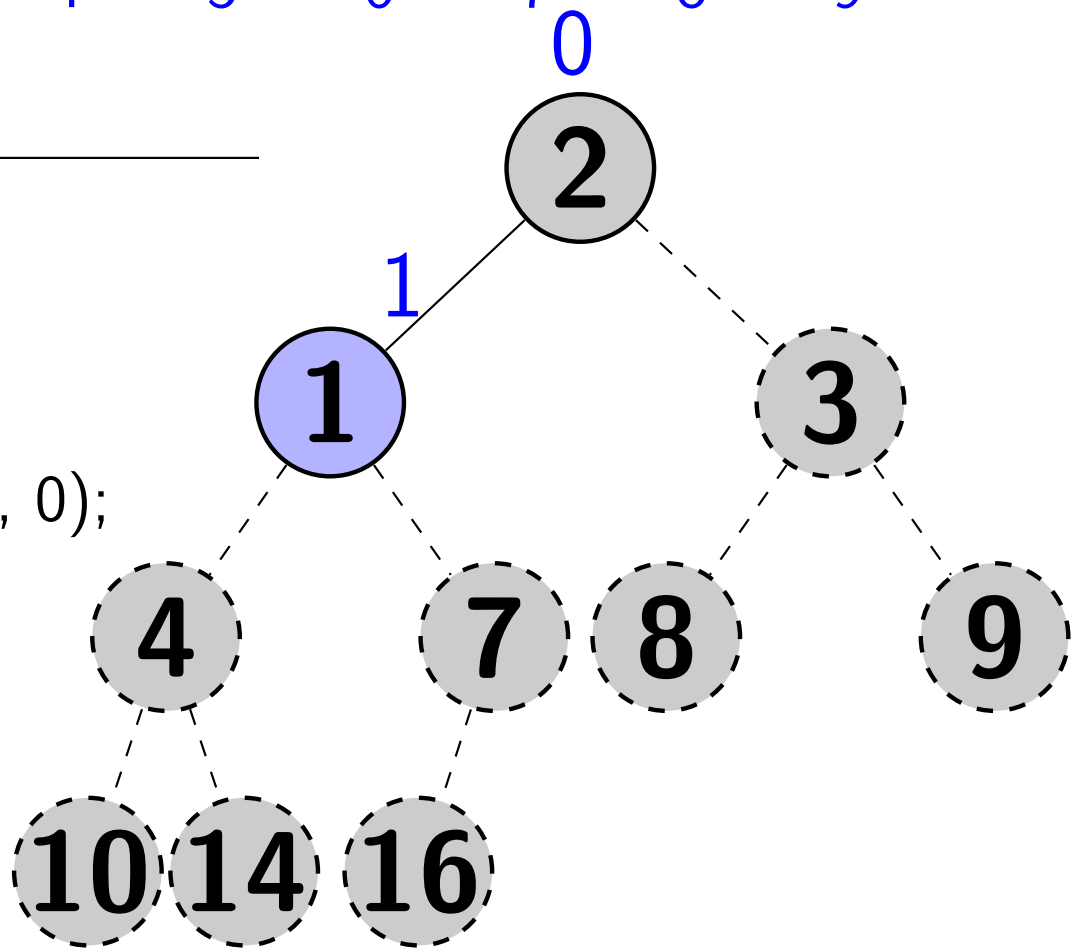
BUILD-MAX-HEAP (V,10);

Para **i** de 9 **até** 1 **faça**

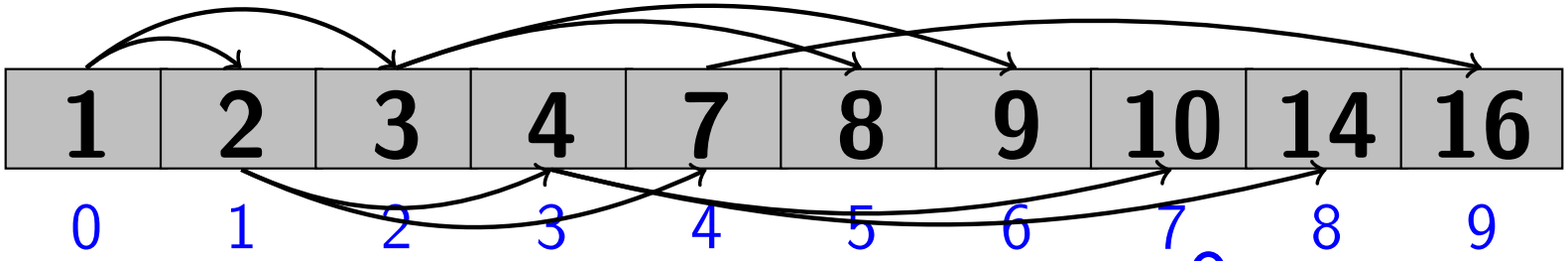
$\triangleright V[0] \leftrightarrow V[1]$;

SIZE $= 2$;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT (V, 10)

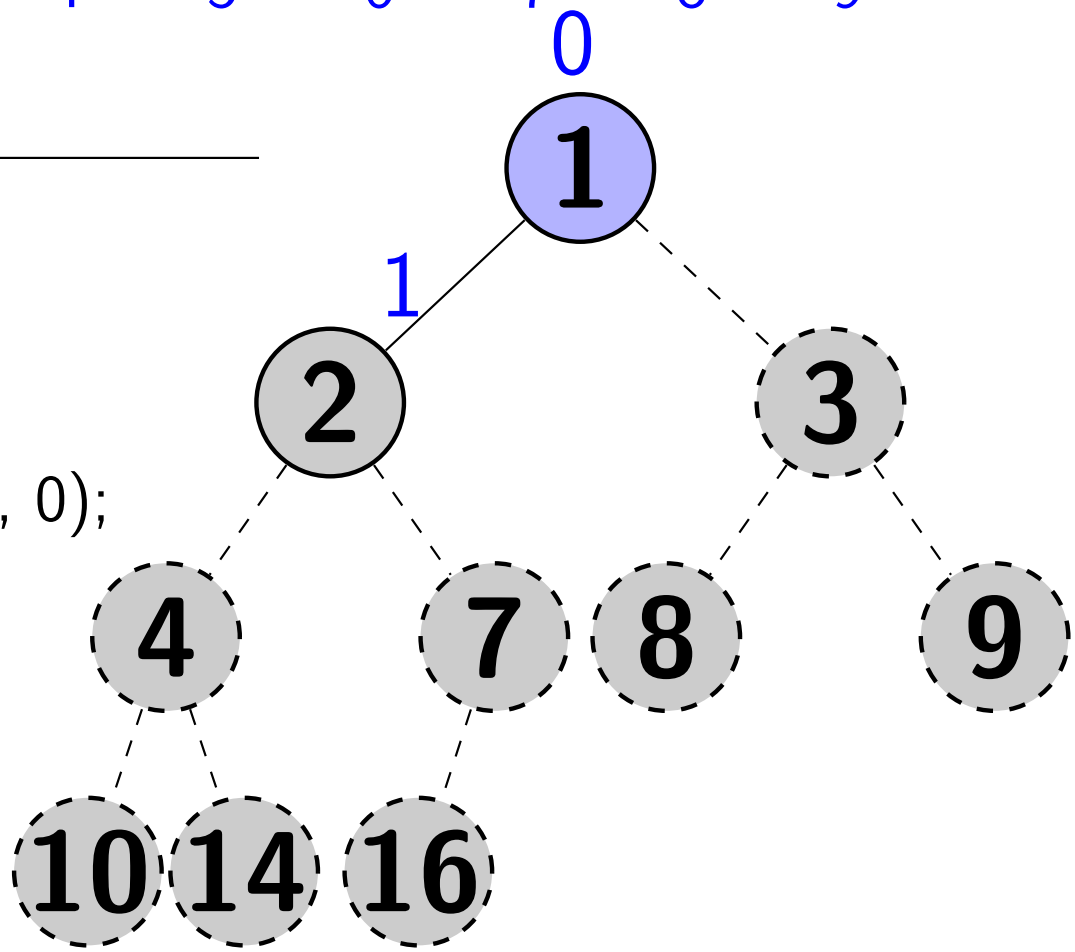
BUILD-MAX-HEAP (V,10);

Para **i** de 9 **até** 1 **faça**

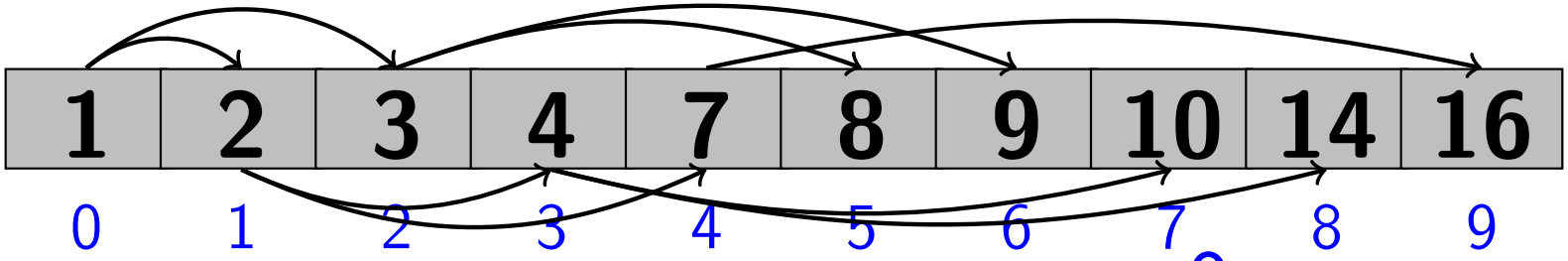
$\triangleright V[0] \leftrightarrow V[1]$;

SIZE $= 2$;

MAX-HEAPIFY (V, SIZE, 0);



Heap-Sort



HEAP-SORT $(V, 10)$

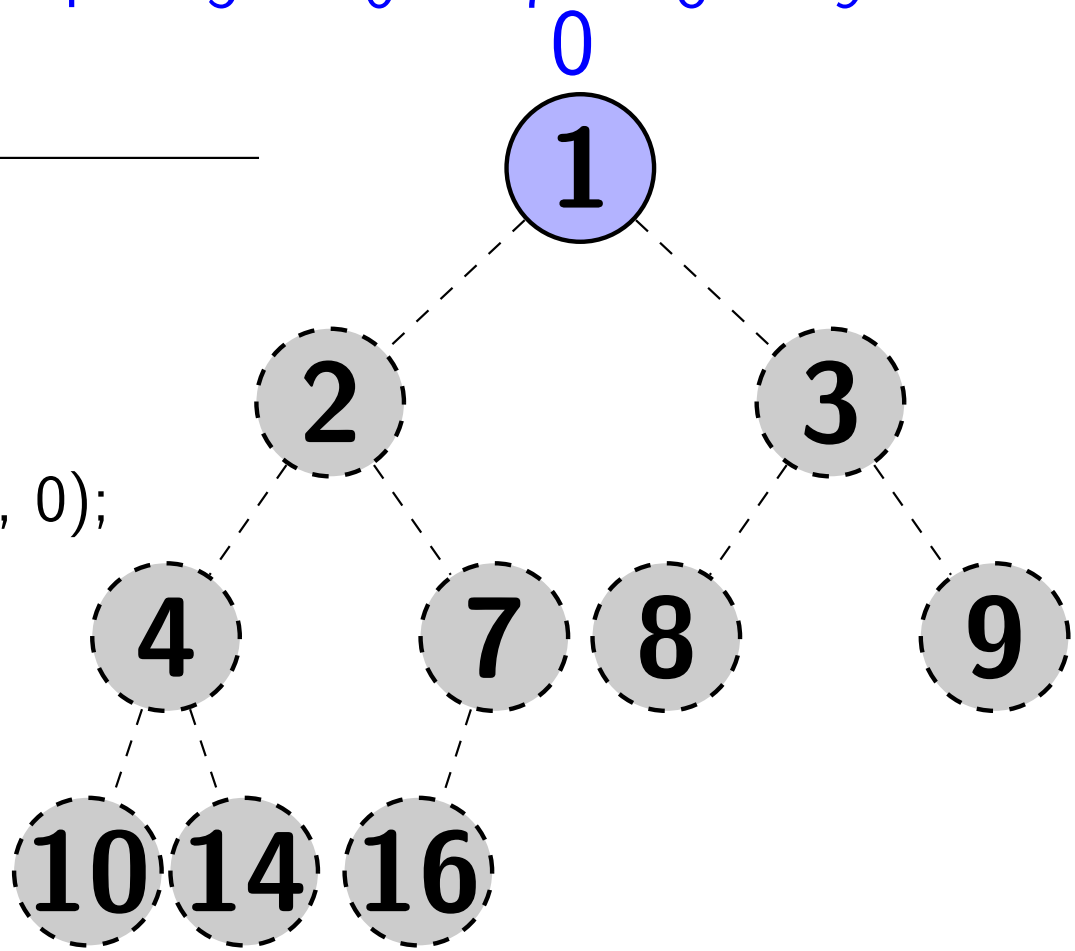
BUILD-MAX-HEAP $(V,10)$;

Para **i** de 9 **até** 1 **faça**

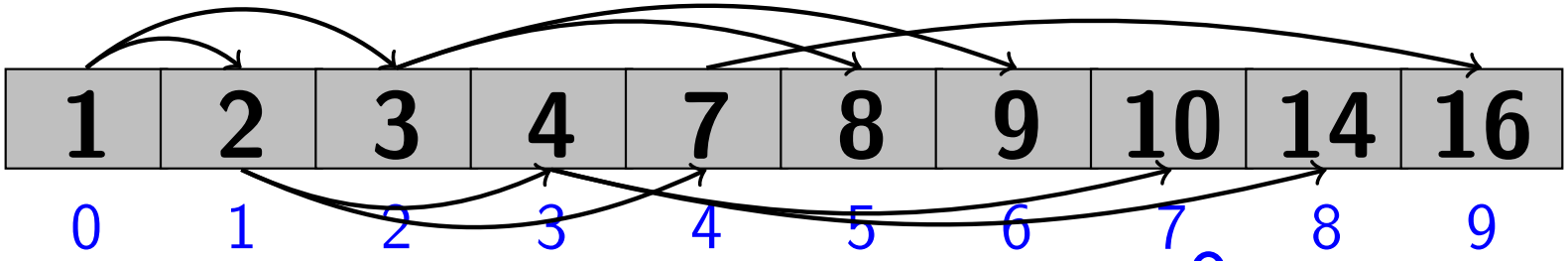
$V[0] \leftrightarrow V[1]$;

$\triangleright$SIZE $= 1$;

MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort



HEAP-SORT $(V, 10)$

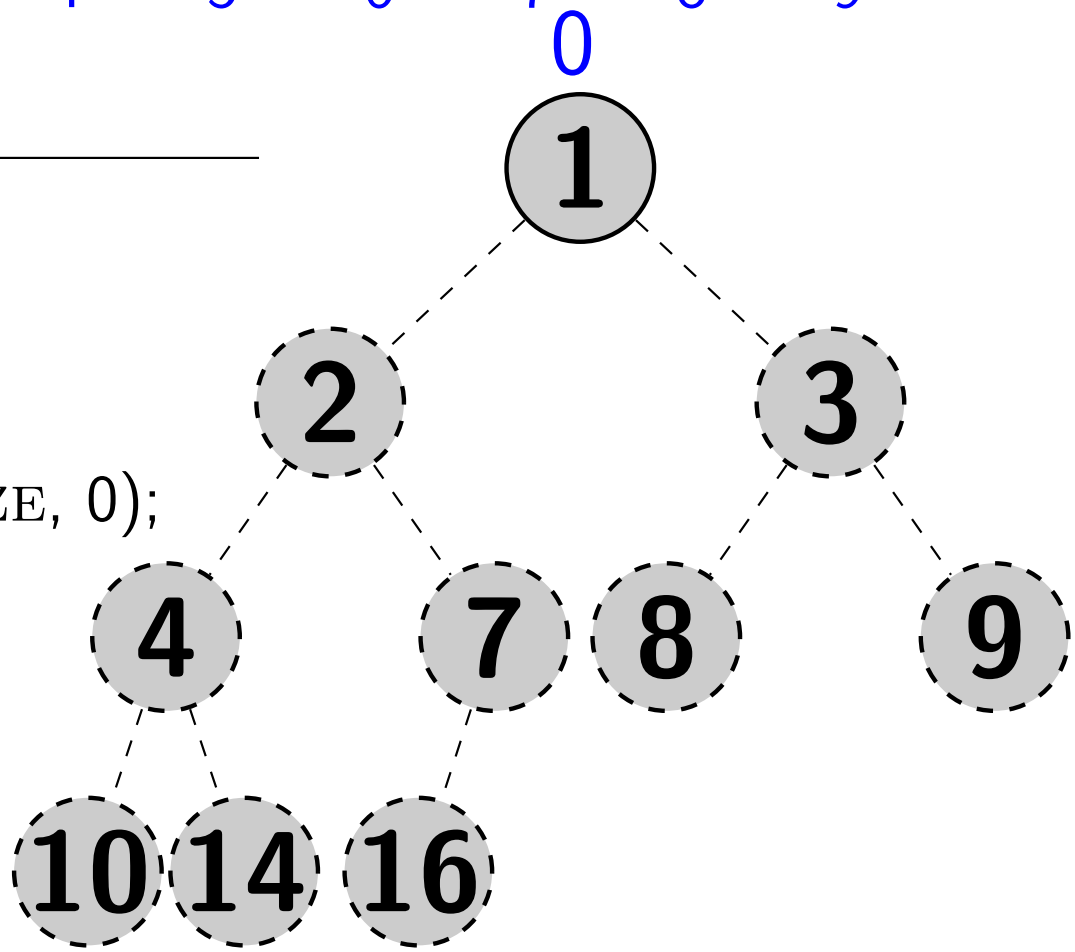
BUILD-MAX-HEAP $(V, 10)$;

Para **i** de 9 **até** 1 **faça**

$V[0] \leftrightarrow V[1]$;

SIZE $= 1$;

$\triangleright$MAX-HEAPIFY $(V,$ SIZE, $0)$;



Heap-Sort

Complexidade: o algoritmo **Heap-Sort** possui complexidade $\mathcal{O}(n \log n)$, pois a chamada a BUILD-MAX-HEAP demora tempo $\mathcal{O}(n)$, e cada uma das $n - 1$ chamadas a MAX-HEAPIFY demora tempo $\mathcal{O}(\log n)$.

Referências

Algoritmos: Teoria e prática. Cormen et al.