Estruturas de Dados II

Filas de prioridades

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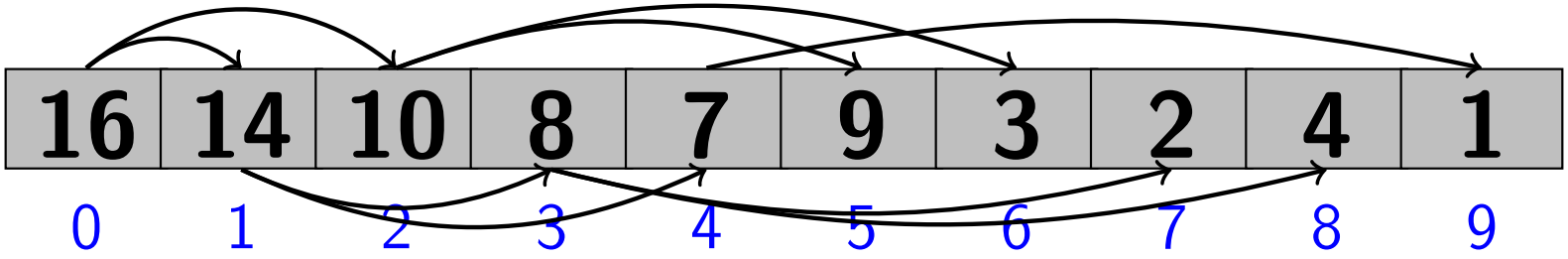
Sumário

Heap

Relembrando: a estrutura de dados **heap** (**binário**) é um arranjo que pode ser visualizado como uma árvore binária praticamente completa (a exceção ocorre no último nível que é preenchido da esquerda para direita enquanto existirem elementos). Cada nó da árvore corresponde a um elemento do arranjo.

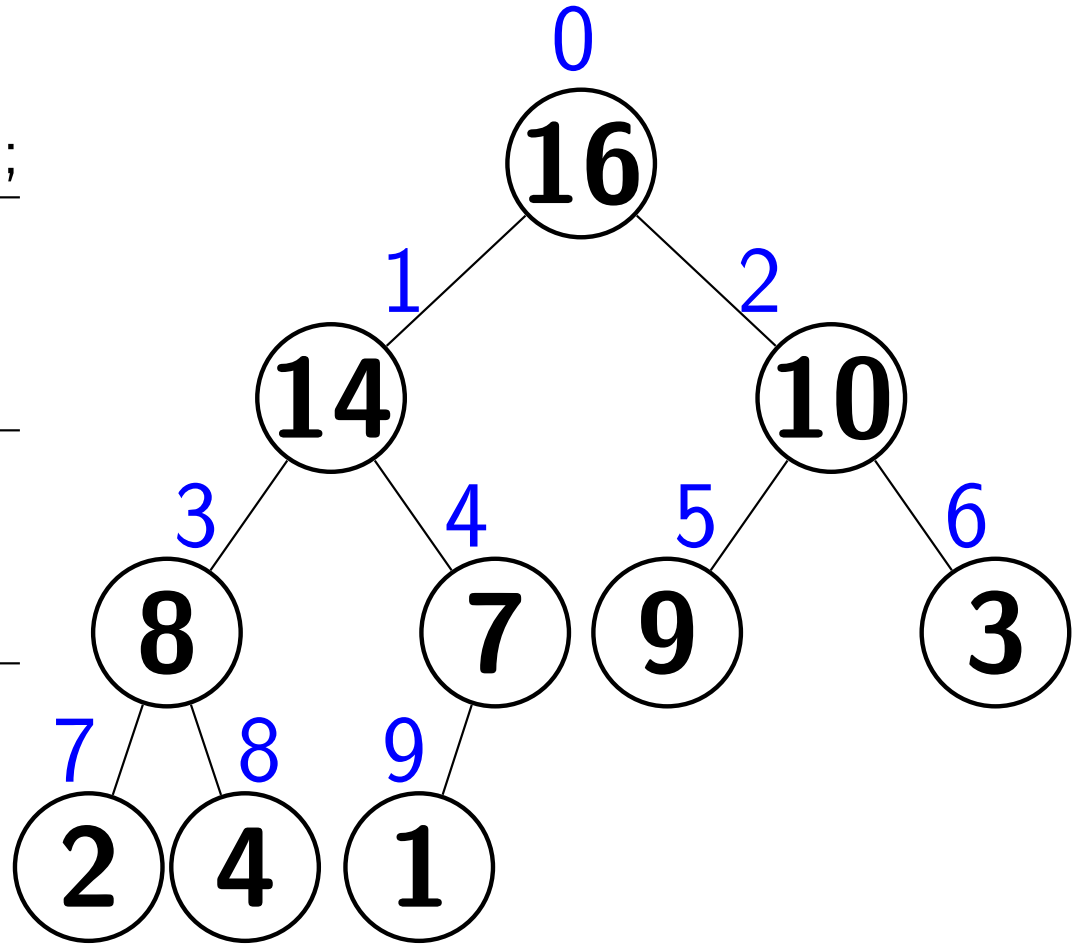
Heap



PAI (**i**)
    Return $\lfloor (\mathbf{i} - 1)/2 \rfloor$;

ESQUERDA (**i**)
    Return $\mathbf{i} * 2 + 1$;

DIREITA (**i**)
    Return $\mathbf{i} * 2 + 2$;



Heap

A raiz da árvore para um vetor V sempre se encontra no elemento $V[0]$. Existem dois tipos de heaps: **heaps máximos** e **heaps mínimos**:

$$V[\textsc{Pai}(i)] \geq V[i] \quad (\text{máximo})$$

$$V[\textsc{Pai}(i)] \leq V[i] \quad (\text{mínimo})$$

Em um heap máximo o maior valor está armazenado na raiz (em um heap mínimo o menor). O algoritmo *heap-sort* usa um *heap-máximo*.

Sumário

Filas de prioridades

O tipo abstrato de dados **heap** tem uma utilidade enorme: seu uso como uma **fila de prioridades**. Os elementos de um vetor V são chamados de **chaves** ou **prioridades**, podendo existir outros dados associados a cada elemento. As operações básicas em uma fila de prioridade são: **encontrar** o elemento máximo (ou mínimo), **inserir**, **extrair** ou **modificar** uma chave.

Filas de prioridades

Uma **aplicação** comum **de filas de prioridade máxima** é o seu uso por **sistemas operacionais** para escalonar processos em uma CPU compartilhada. Quando um processo é finalizado ou interrompido, o processo de prioridade mais alta é selecionado para executar dentre àqueles processos pendentes. Filas de prioridade também são usadas em algoritmos famosos na computação como os algoritmos de Dijkstra, Prim e Huffman.

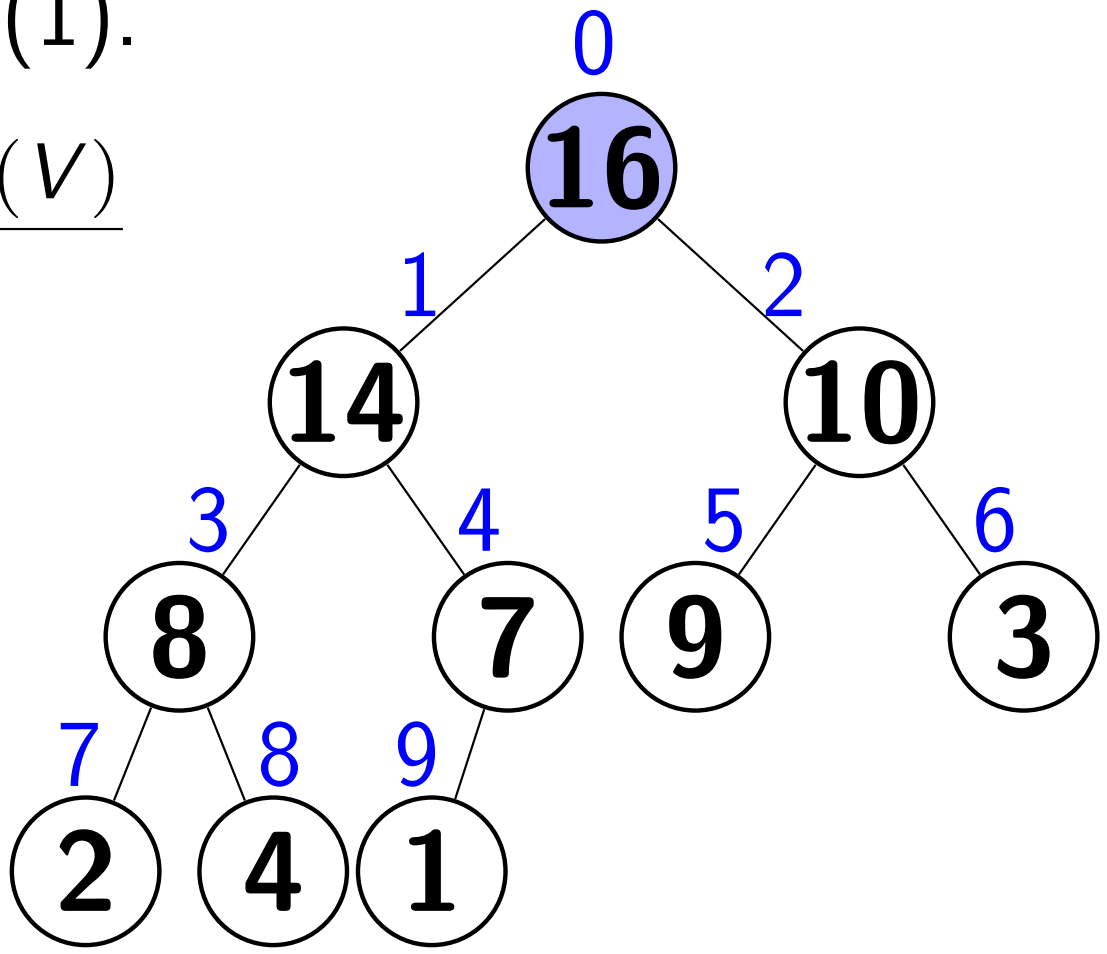
Sumário

Heap-Maximum

A operação Heap-Maximum (V) **retorna** o elemento de V com **maior chave (prioridade)**.
Complexidade: $\Theta(1)$.

HEAP-MAXIMUM (V)

return $V[0]$;



Sumário

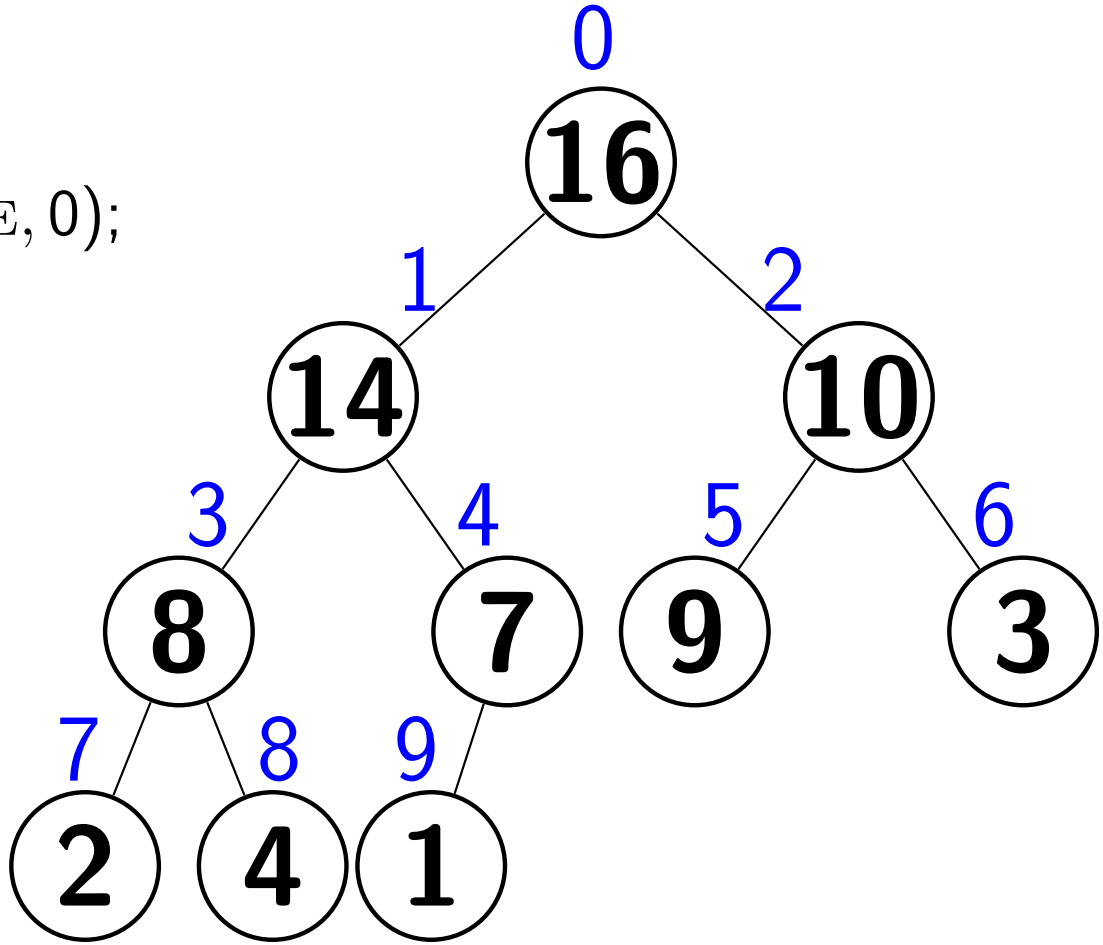
Heap-Extract-Max

A operação Heap-Extract-Max (V) **remove** e **retorna** o elemento de V com a **maior chave (prioridade)**. Ele é semelhante a extração da maior chave pelo algoritmo Heap-Sort. Note que uma vez que a maior chave é retirada é **necessário refazer o heap com** o procedimento **Max-Heapify**.

Heap-Extract-Max

HEAP-EXTRACT-MAX $(V,$ SIZE$)$

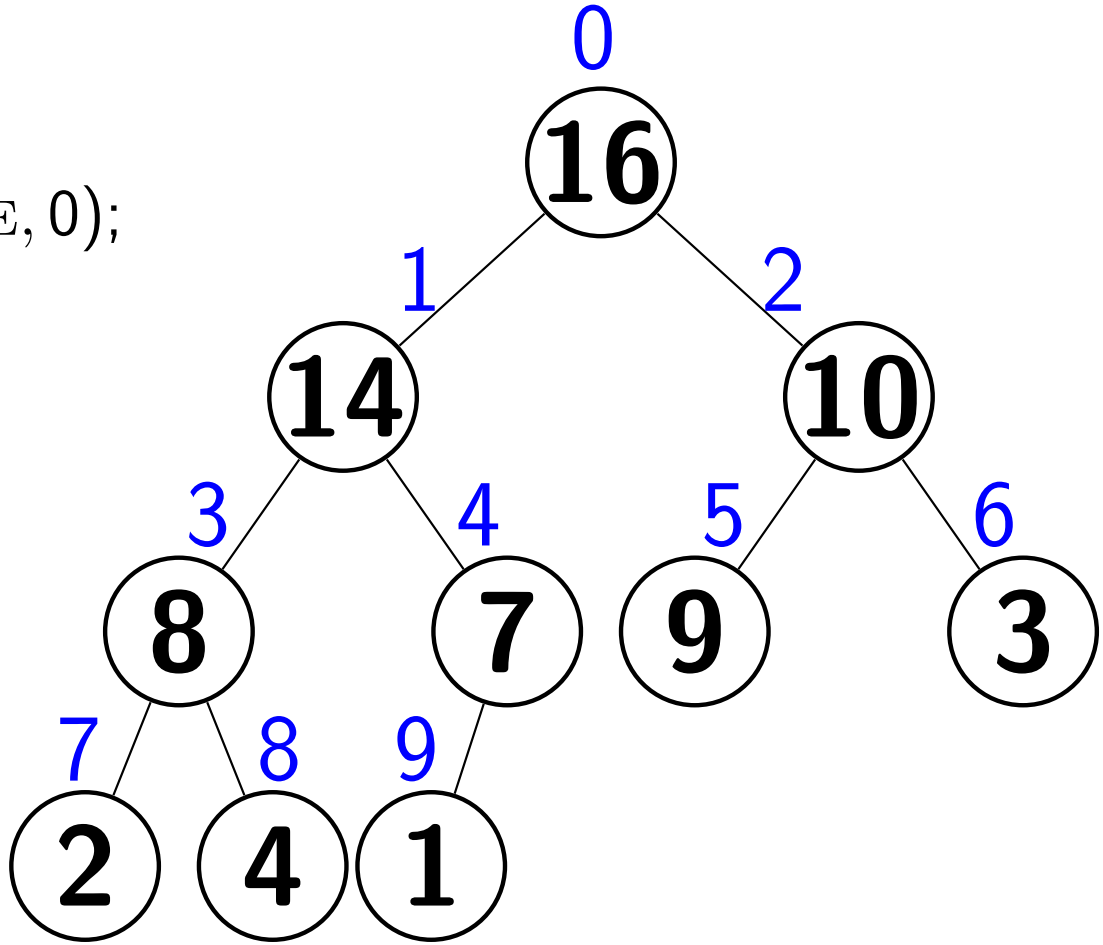
if SIZE < 1 **then**
    printf ("erro: heap underflow");
max $= V[0]$;
$V[0] = V[$SIZE $-1]$;
SIZE $=$ SIZE -1;
MAX-HEAPIFY $(V,$ SIZE$, 0)$;
return max;



Heap-Extract-Max

HEAP-EXTRACT-MAX (V, SIZE)

▷**if** SIZE < 1 **then**
    printf ("erro: heap underflow");
max $= V[0]$;
$V[0] = V[\text{SIZE} - 1]$;
SIZE $=$ SIZE $- 1$;
MAX-HEAPIFY $(V, \text{SIZE}, 0)$;
return max;



Heap-Extract-Max

HEAP-EXTRACT-MAX (V, SIZE)

▷**if** $10 < 1$ **then**
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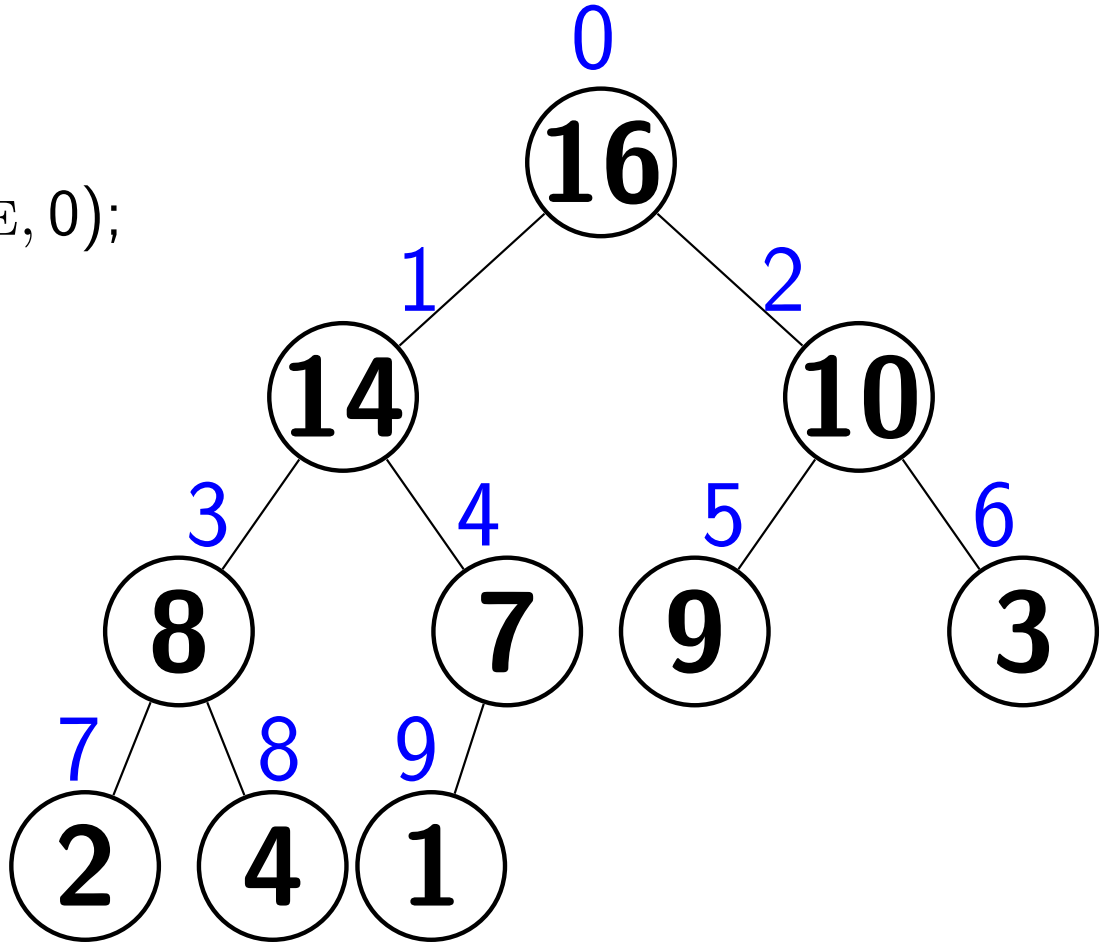

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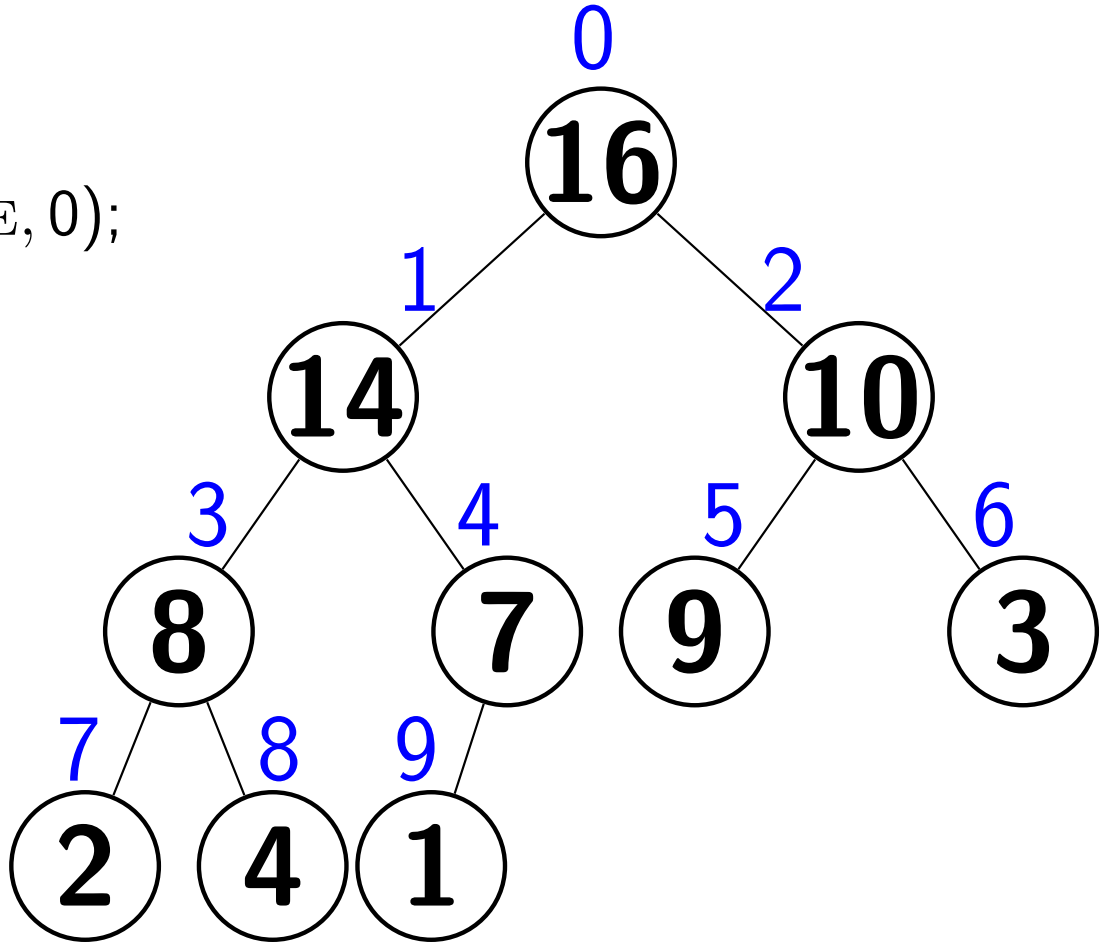


Heap-Extract-Max

HEAP-EXTRACT-MAX (V, SIZE)

if $10 < 1$ **then**

    printf ("erro: heap underflow");

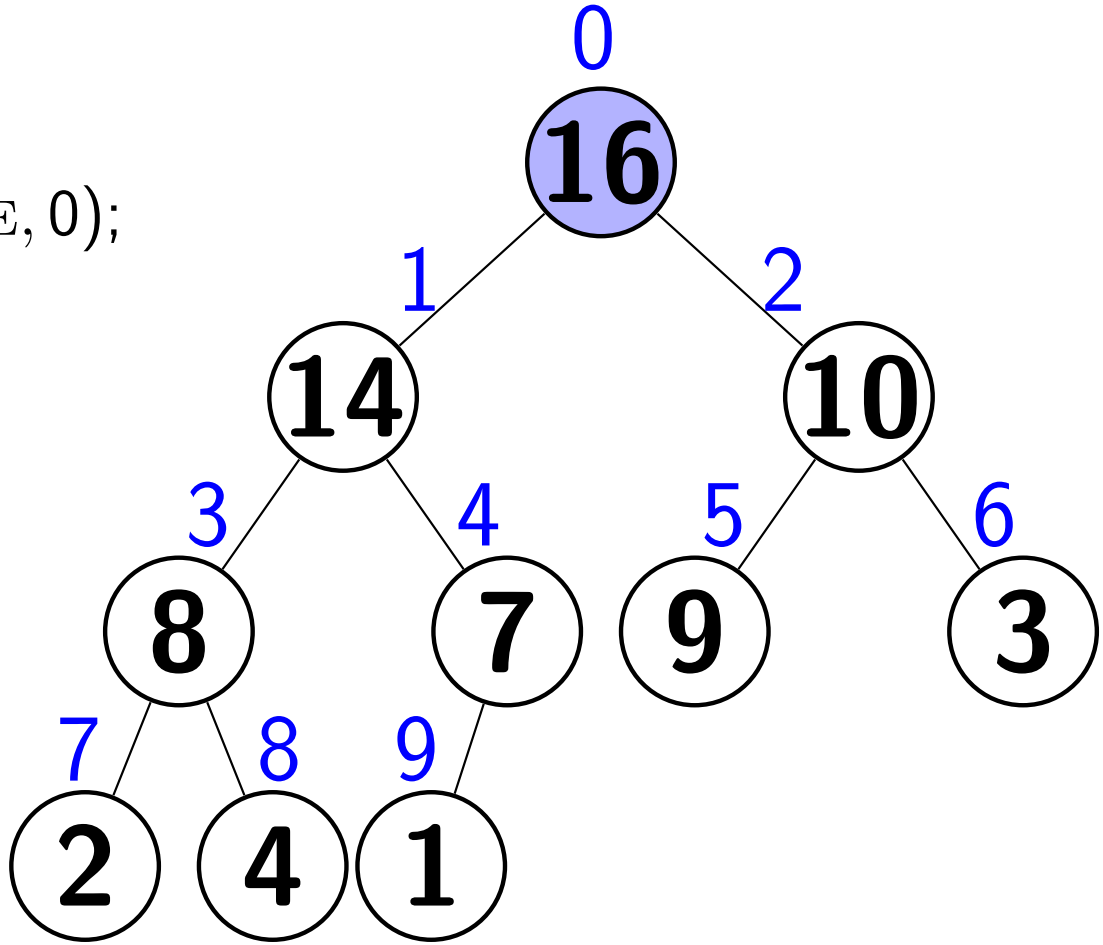
$\triangleright$max $=$ **16**;

$V[0] = V[\text{SIZE} - 1]$;

$\text{SIZE} = \text{SIZE} - 1$;

MAX-HEAPIFY $(V, \text{SIZE}, 0)$;

return max;



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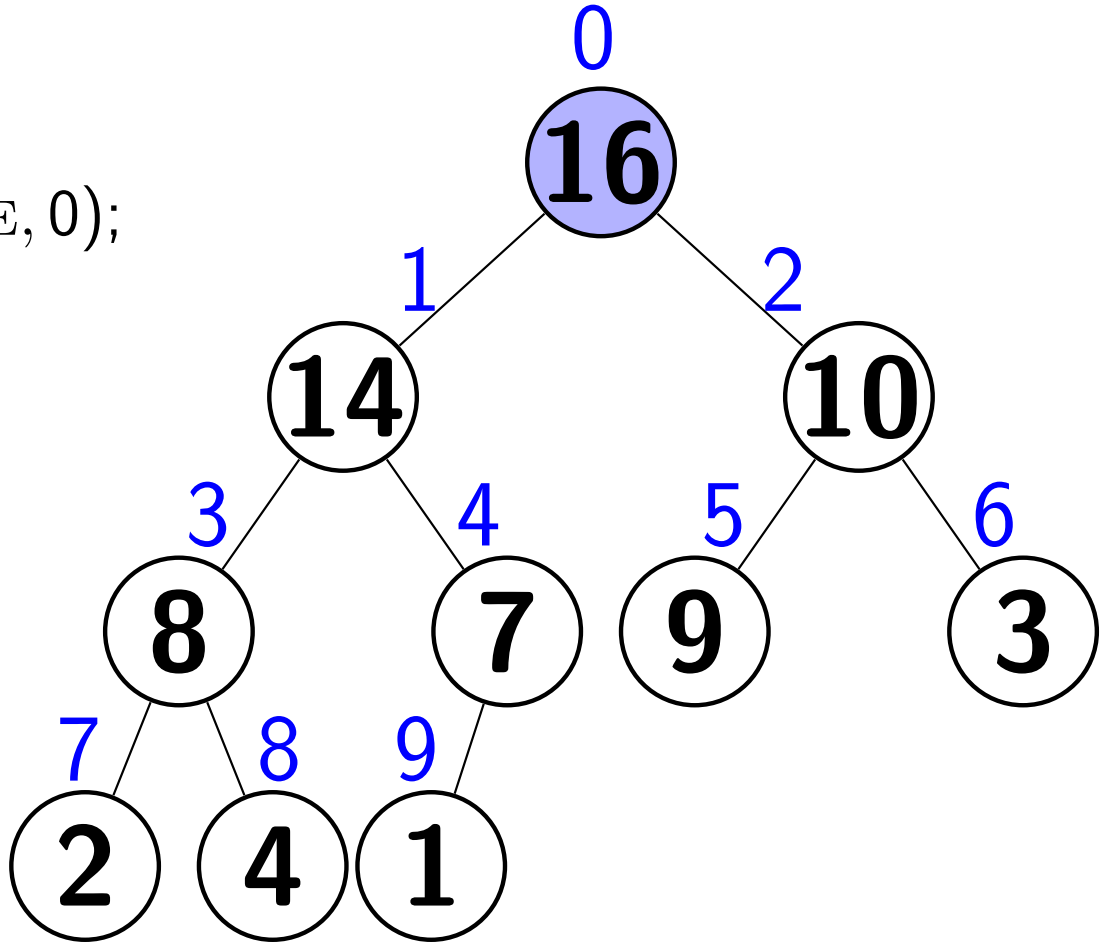

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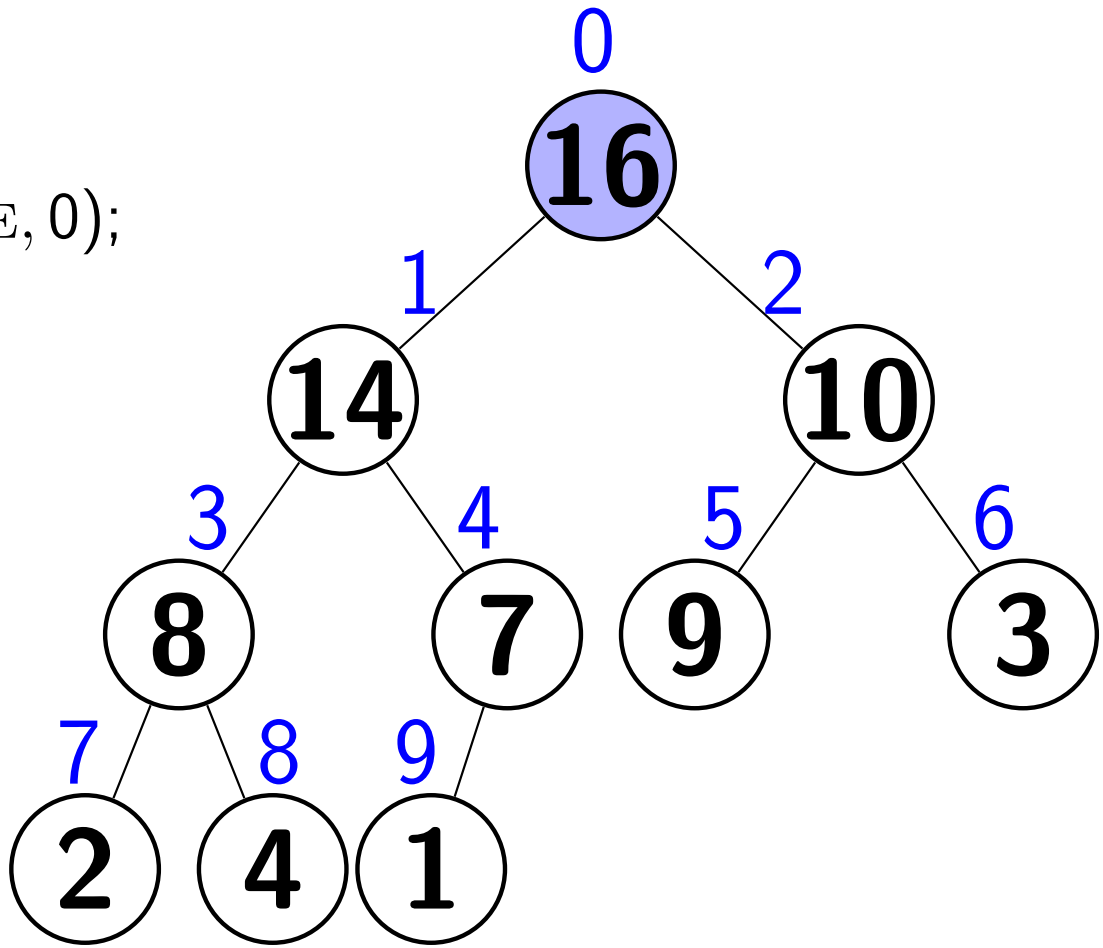
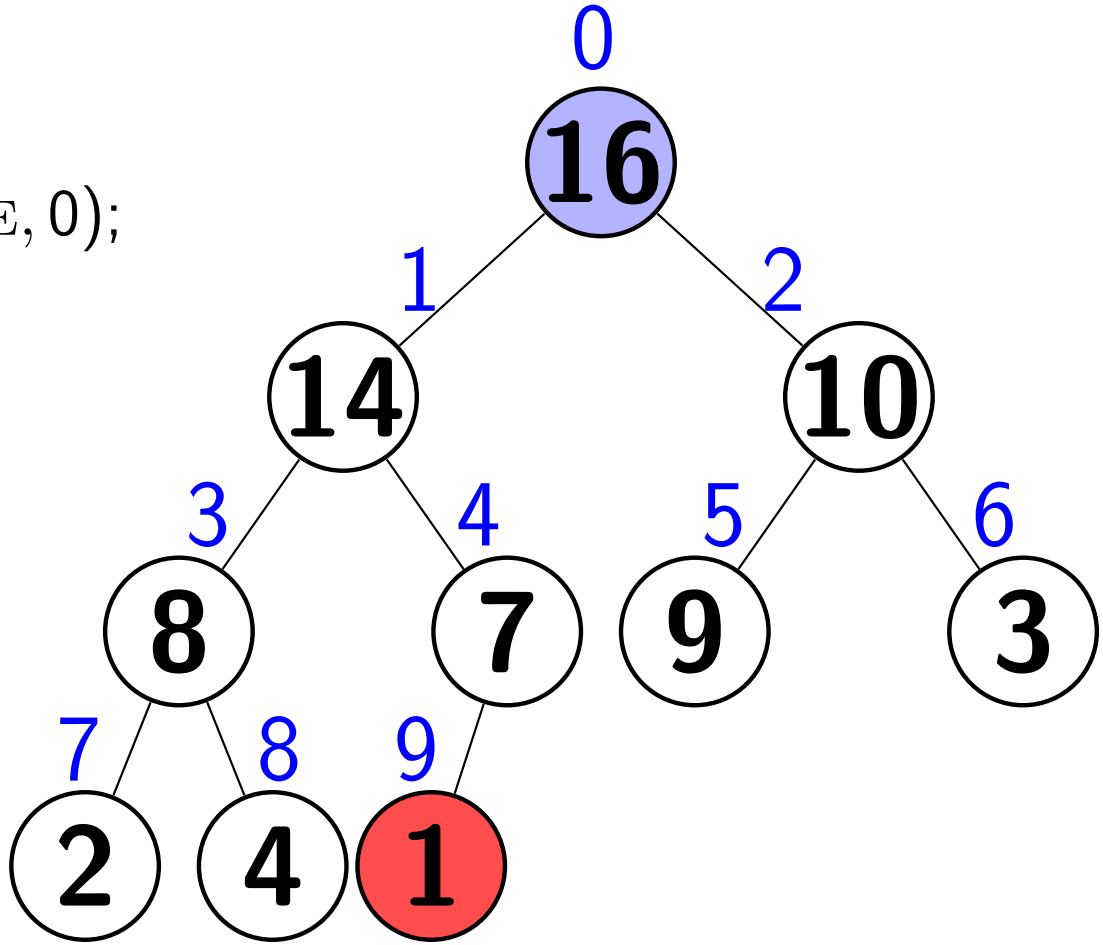


Heap-Extract-Max

HEAP-EXTRACT-MAX (V, SIZE)

if $10 < 1$ **then**
    printf ("erro: heap underflow");
max $= 16$;
$\triangleright V[0] = V[9]$;
$\text{SIZE} = \text{SIZE} - 1$;
MAX-HEAPIFY $(V, \text{SIZE}, 0)$;
return max;



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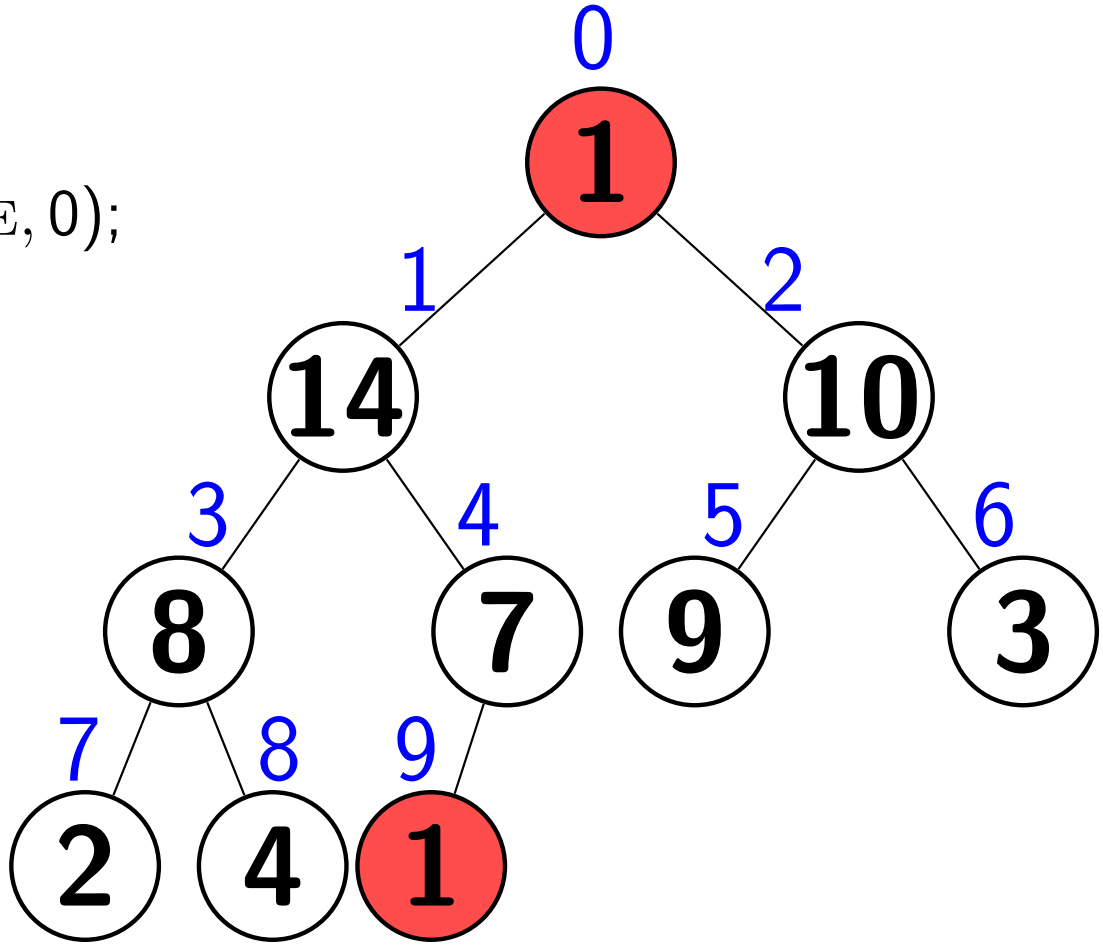

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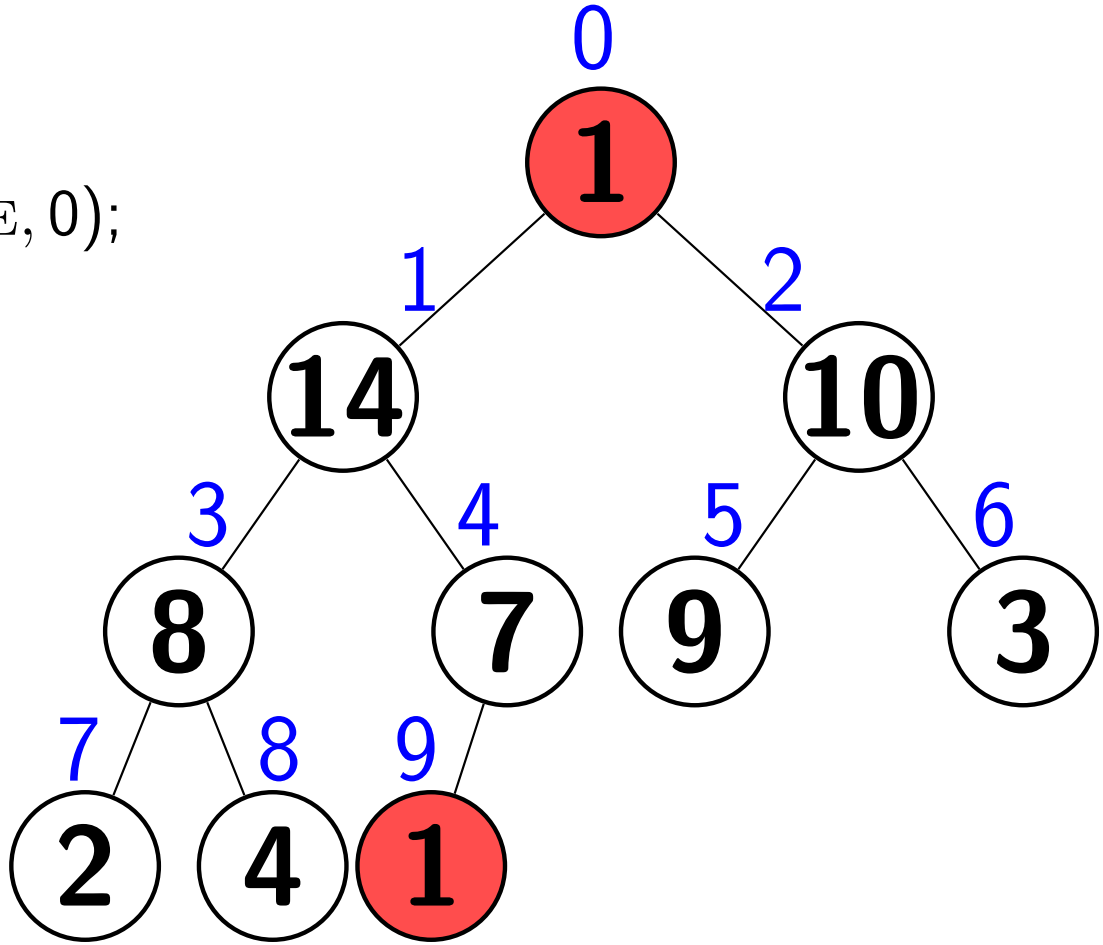

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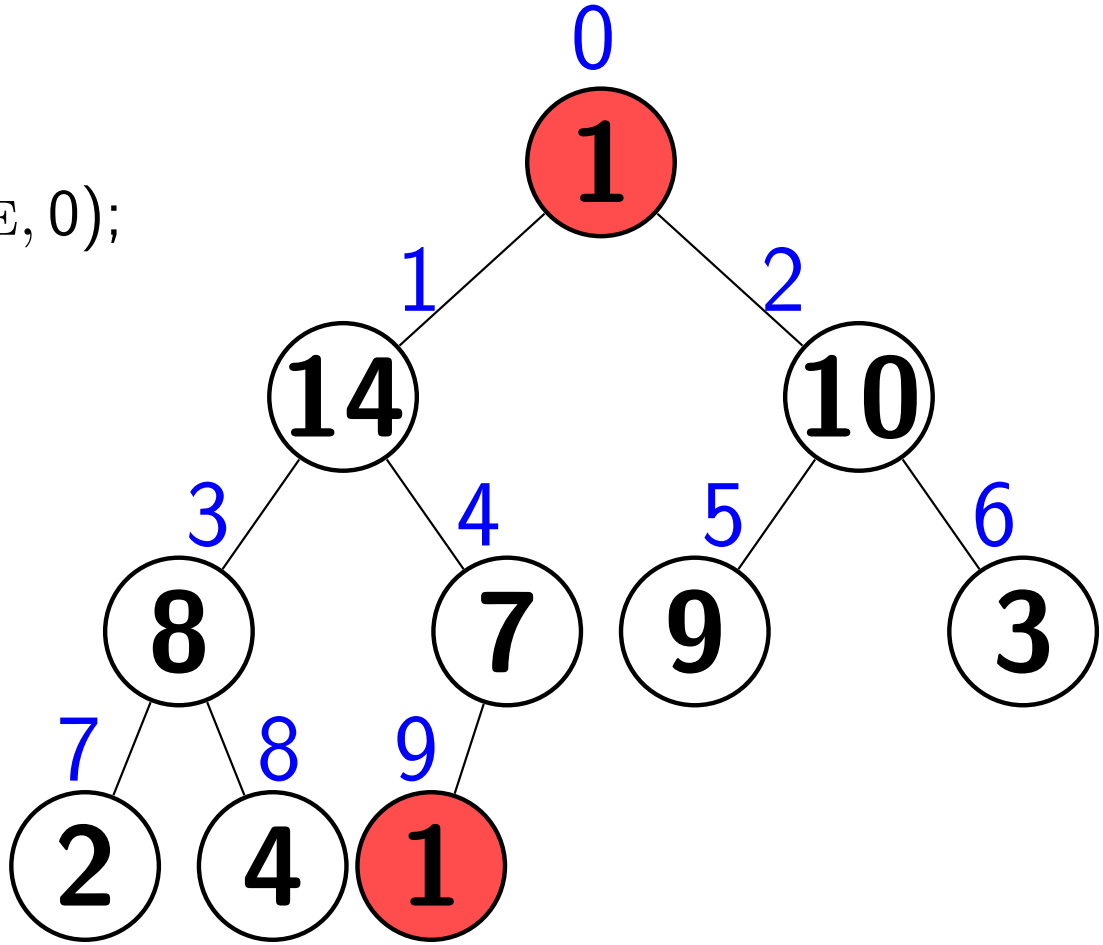

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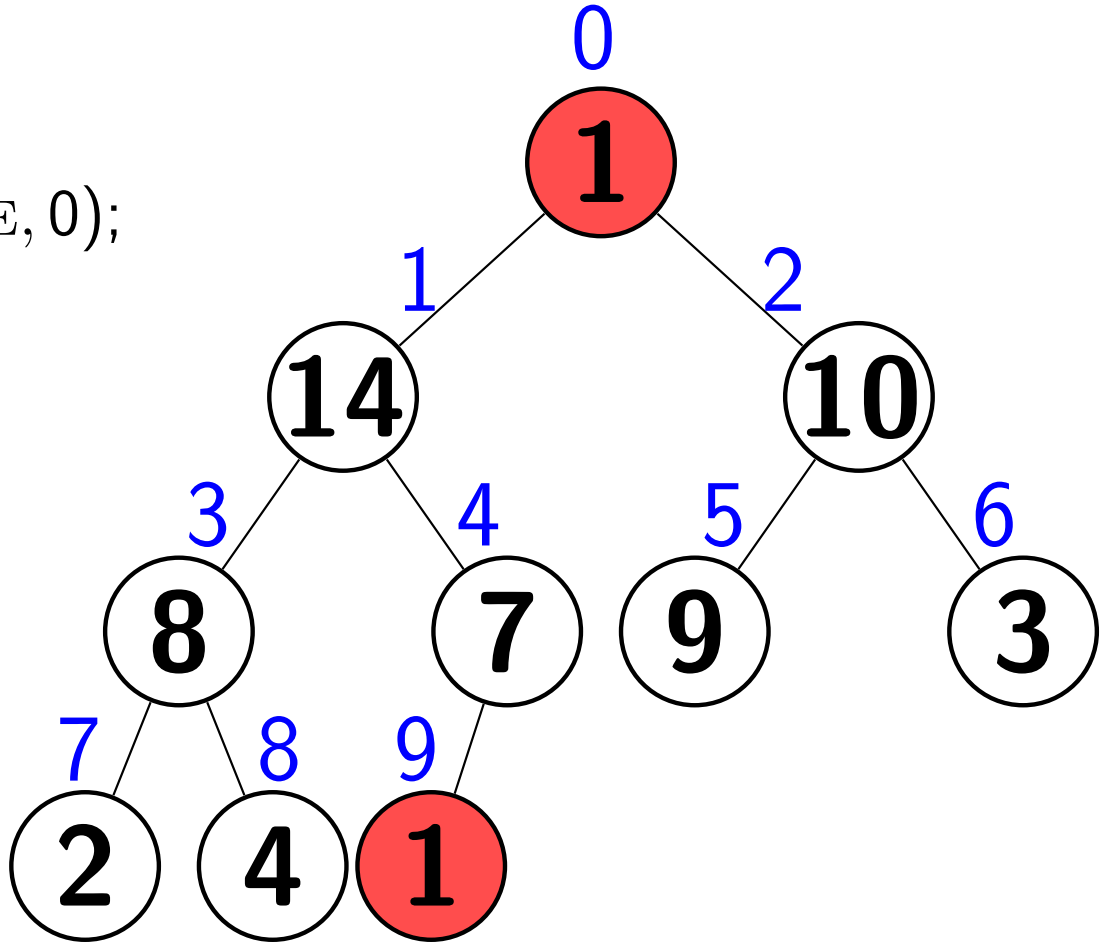

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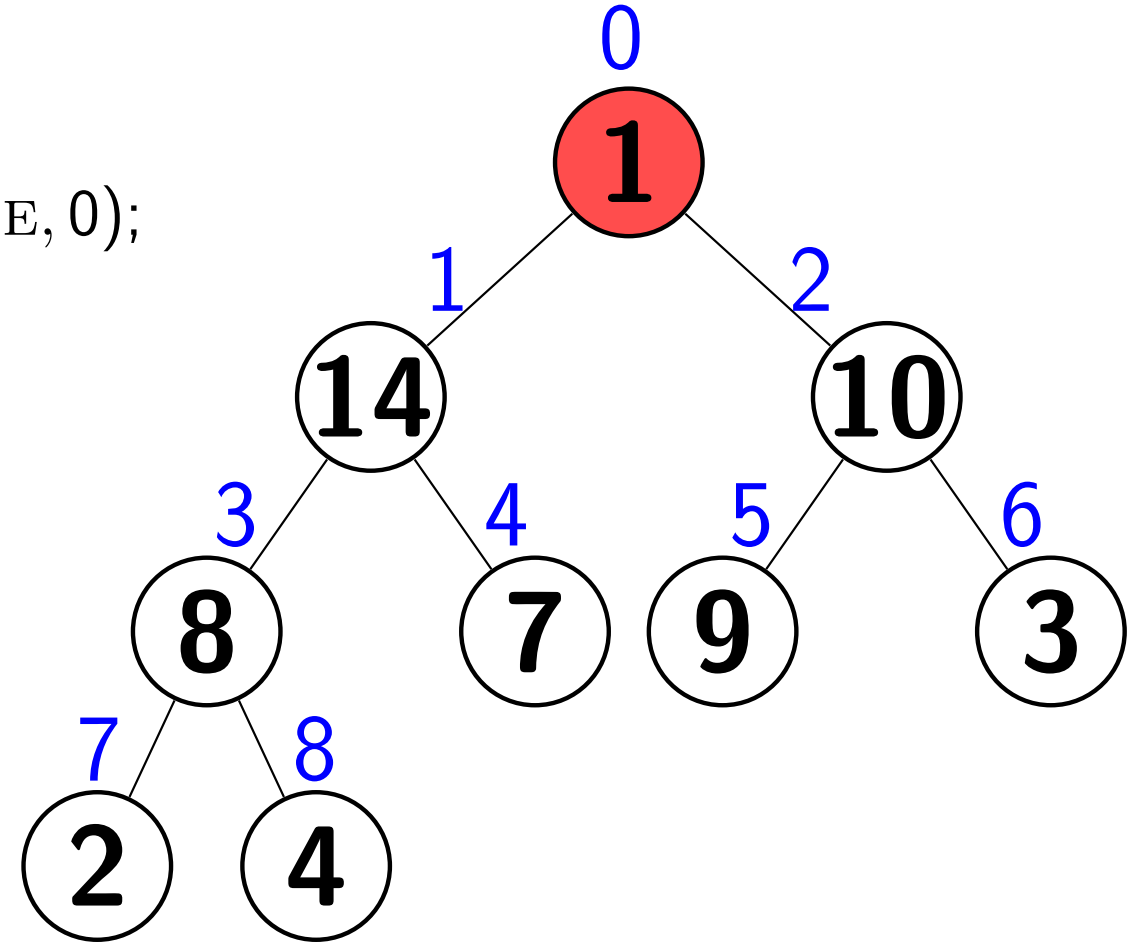

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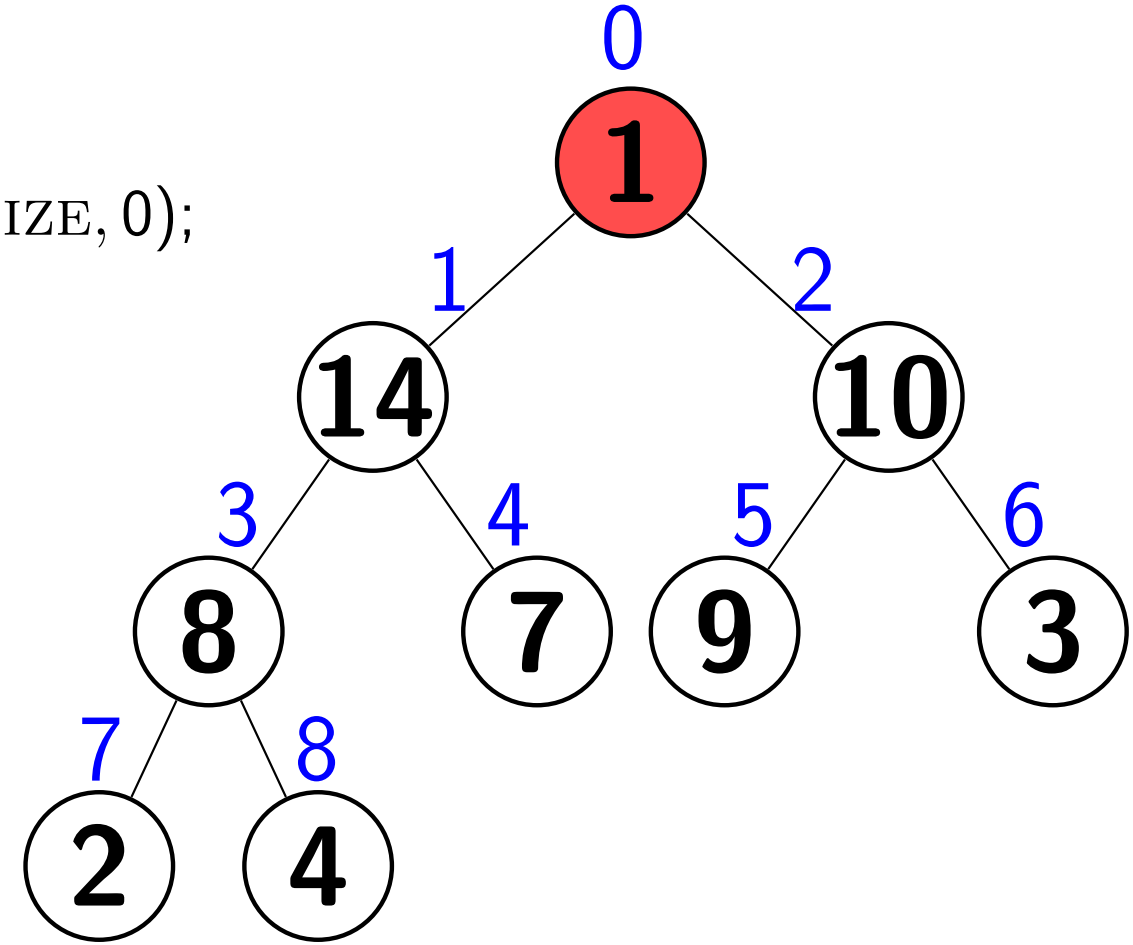

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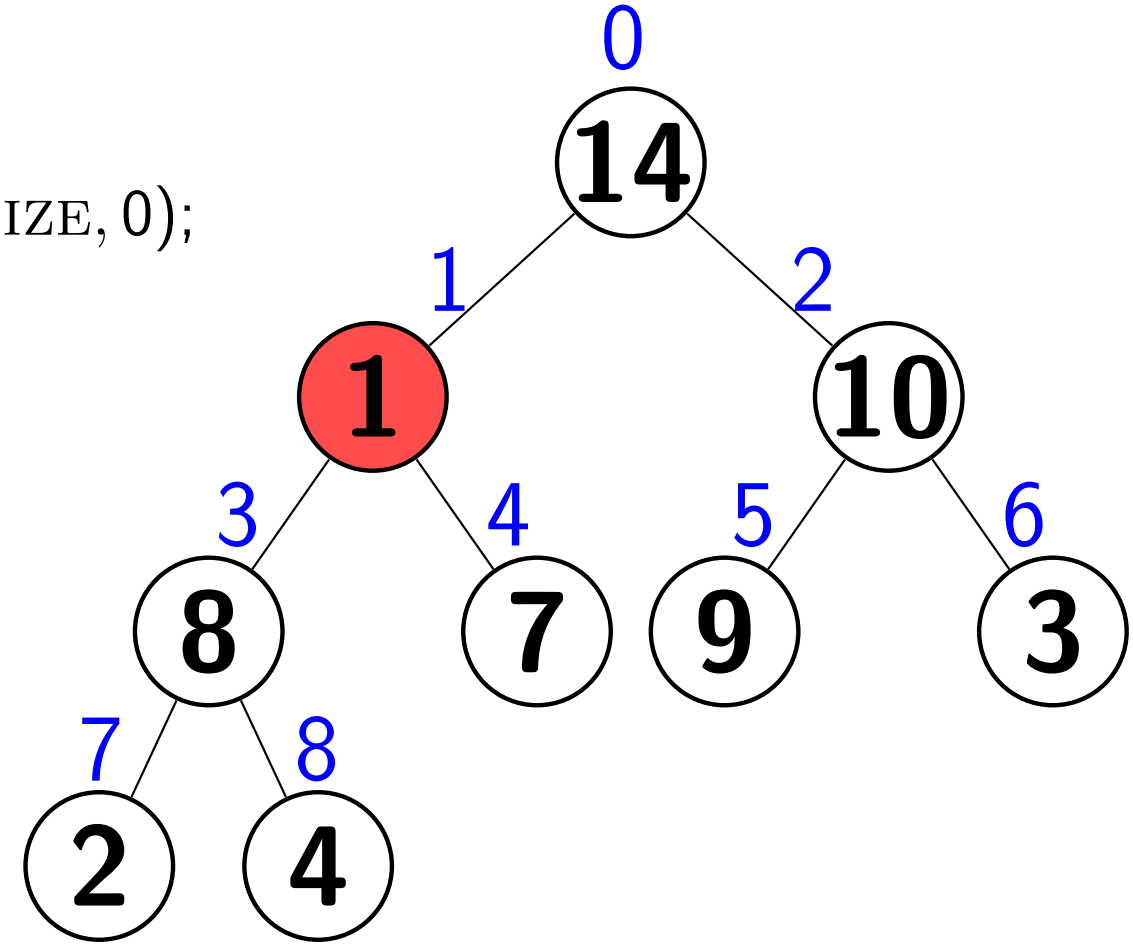

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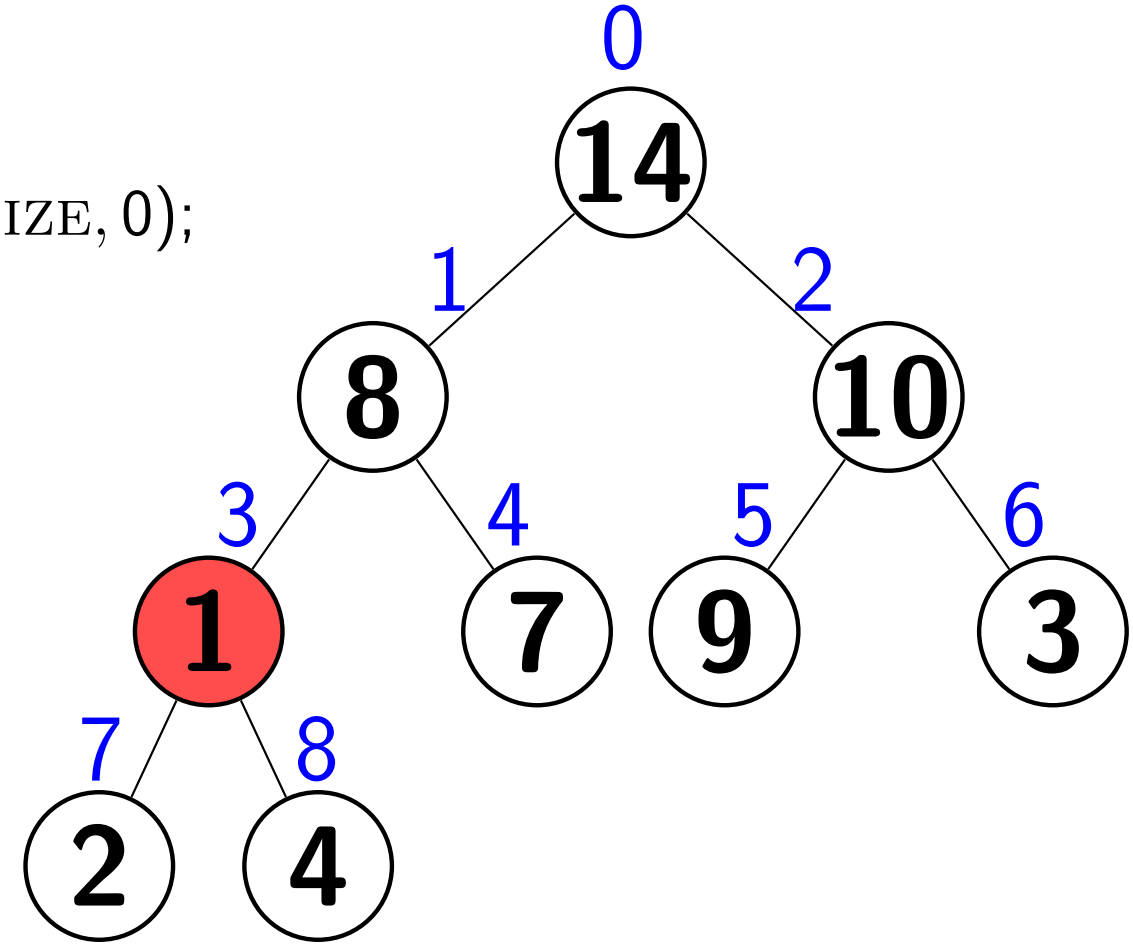

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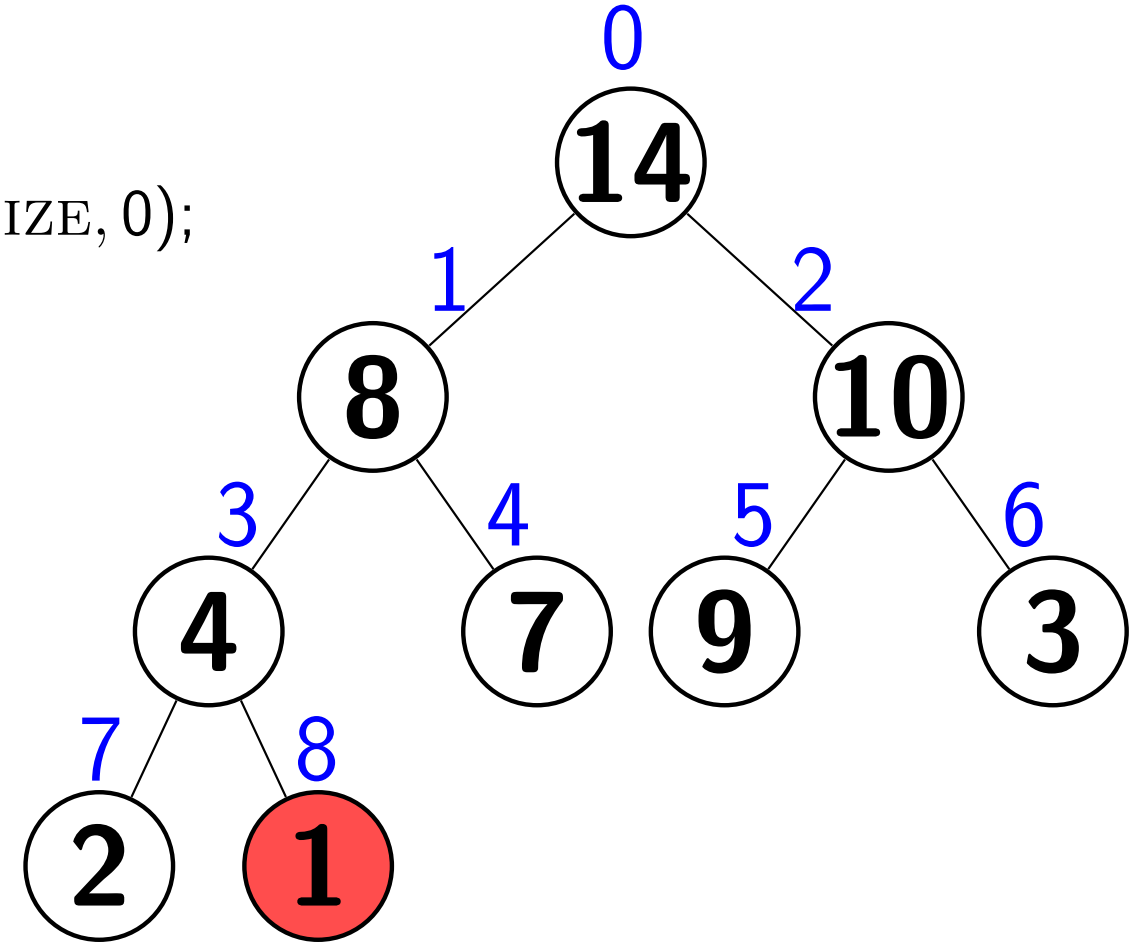

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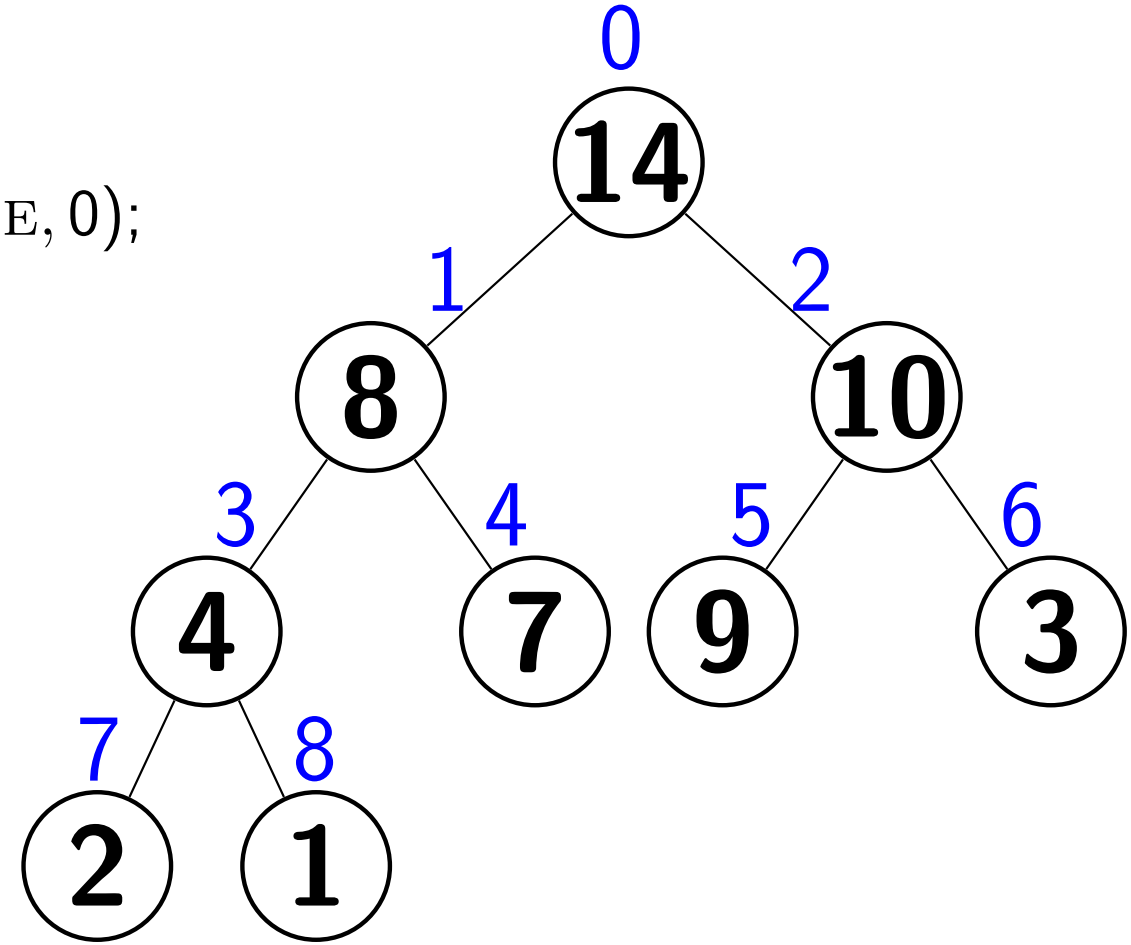
▷**return** max;



Heap-Extract-Max

Complexidade: o procedimento Heap-Extract-Max possui complexidade $\mathcal{O}(\log n)$ para um heap com n elementos, pois o procedimento Max-Heapify possui complexidade $\mathcal{O}(\log n)$ (comprimento máximo da raiz até um nó folha). Os outros passos do procedimento Heap-Extract-Max possuem complexidade $\mathcal{O}(1)$.

Sumário

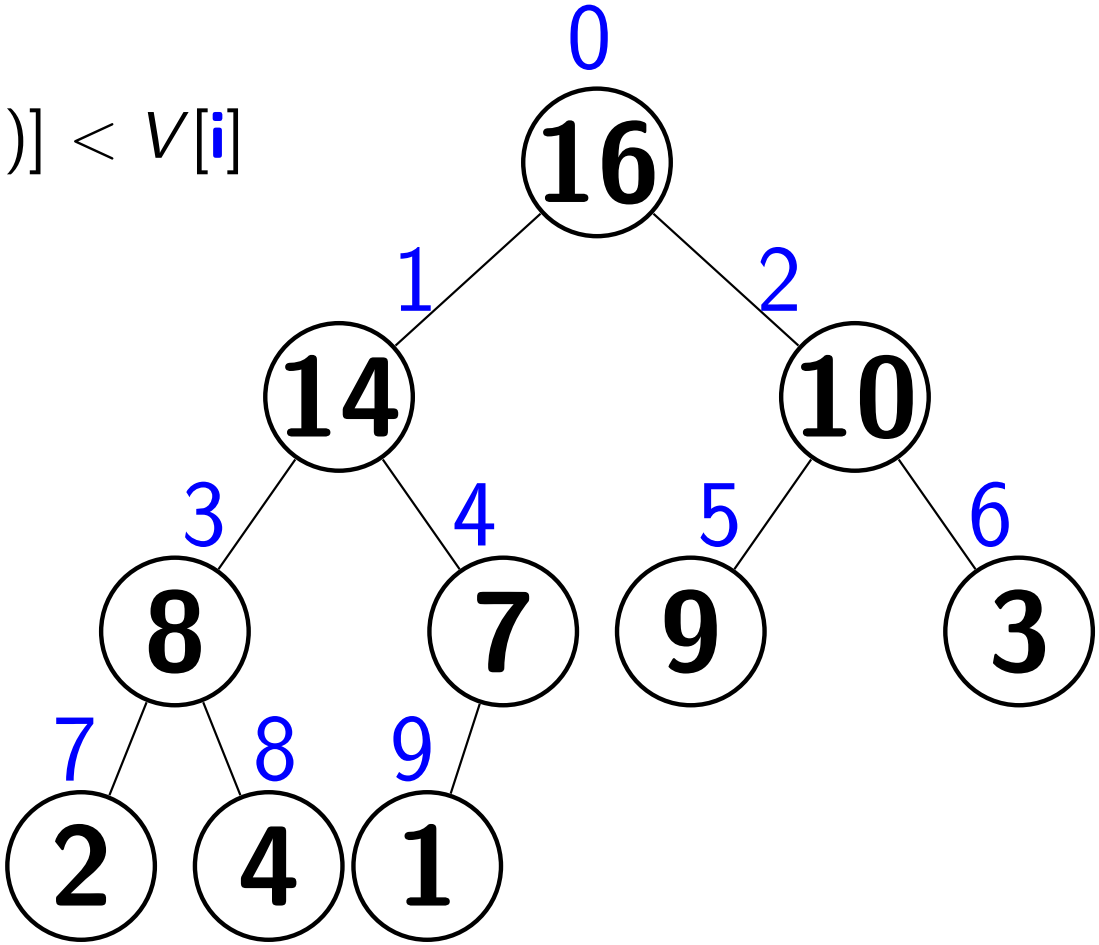
Heap-Increase-Key

A operação Heap-Increase-Key $(V, \mathbf{x}, \mathbf{k})$ **aumenta** o valor da chave (prioridade) do elemento **x** para um novo valor **k**, que se presume ser pelo menos tão grande quanto o valor da chave atual **x**. Note que essa operação **pode violar** a propriedade de **heap máximo**, assim, é necessário encontrar a posição apropriada para o elemento recém atualizado.

Heap-Increase-Key

HEAP-INCREASE-KEY (V, i, chave)

```
if chave < V[i] then
    printf ("erro: chave menor que atual");
    return;
V[i] = chave;
while i > 0 and V[PAI(i)] < V[i]
   V[i] ↔ V[PAI(i)];
   i = PAI(i);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

```
if chave < V[i] then
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V[i] = chave;
while i > 0 and V[PAI(i)] < V[i]
   V[i] ↔ V[PAI(i)];
   i = PAI(i);
```
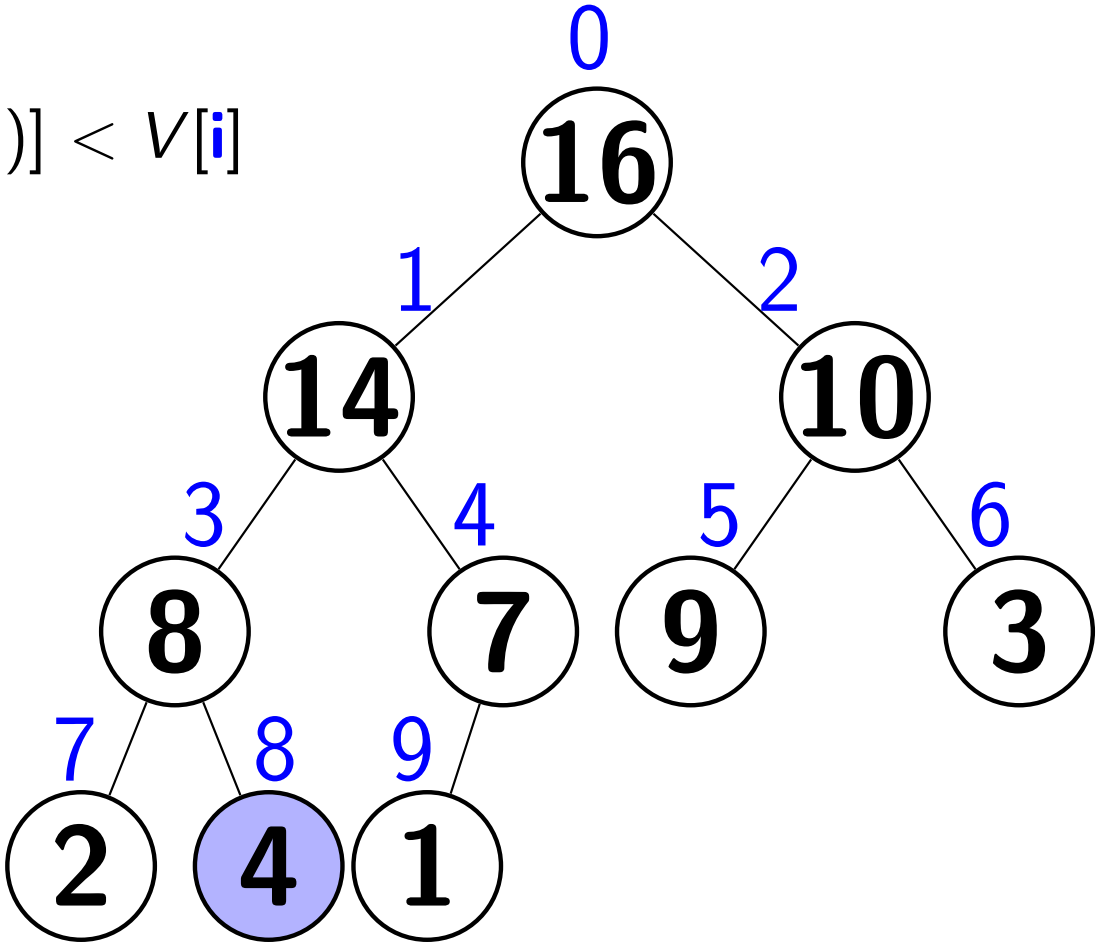
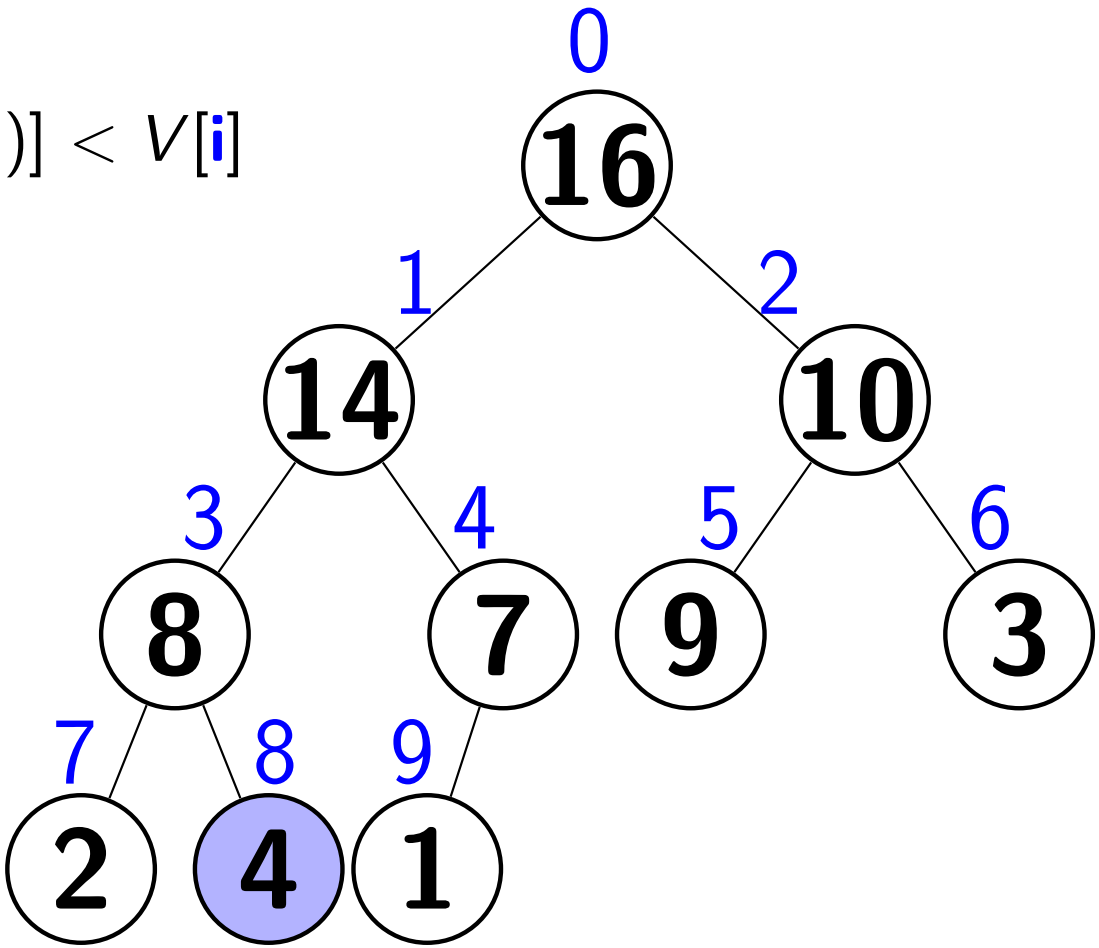



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

▷**if** 15 $<$ 4 **then**
    printf ("erro: chave menor que atual");
    return;
V[i] $=$ **chave**;
while i $>$ 0 **and** V[PAI(i)] $<$ V[i]
    V[i] $\leftrightarrow$ V[PAI(i)];
    i $=$ PAI(i);



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

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if 15 < 4 then
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   V[i] ↔ V[PAI(i)];
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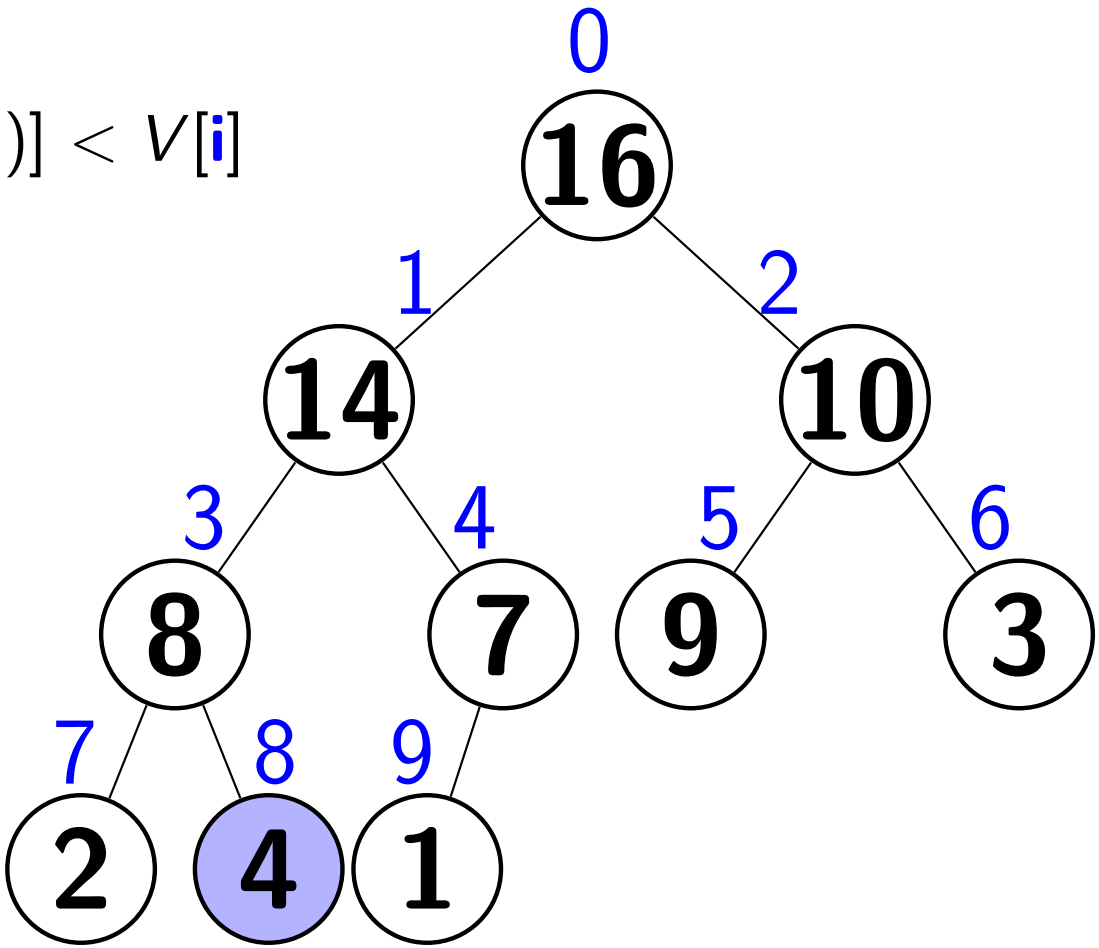
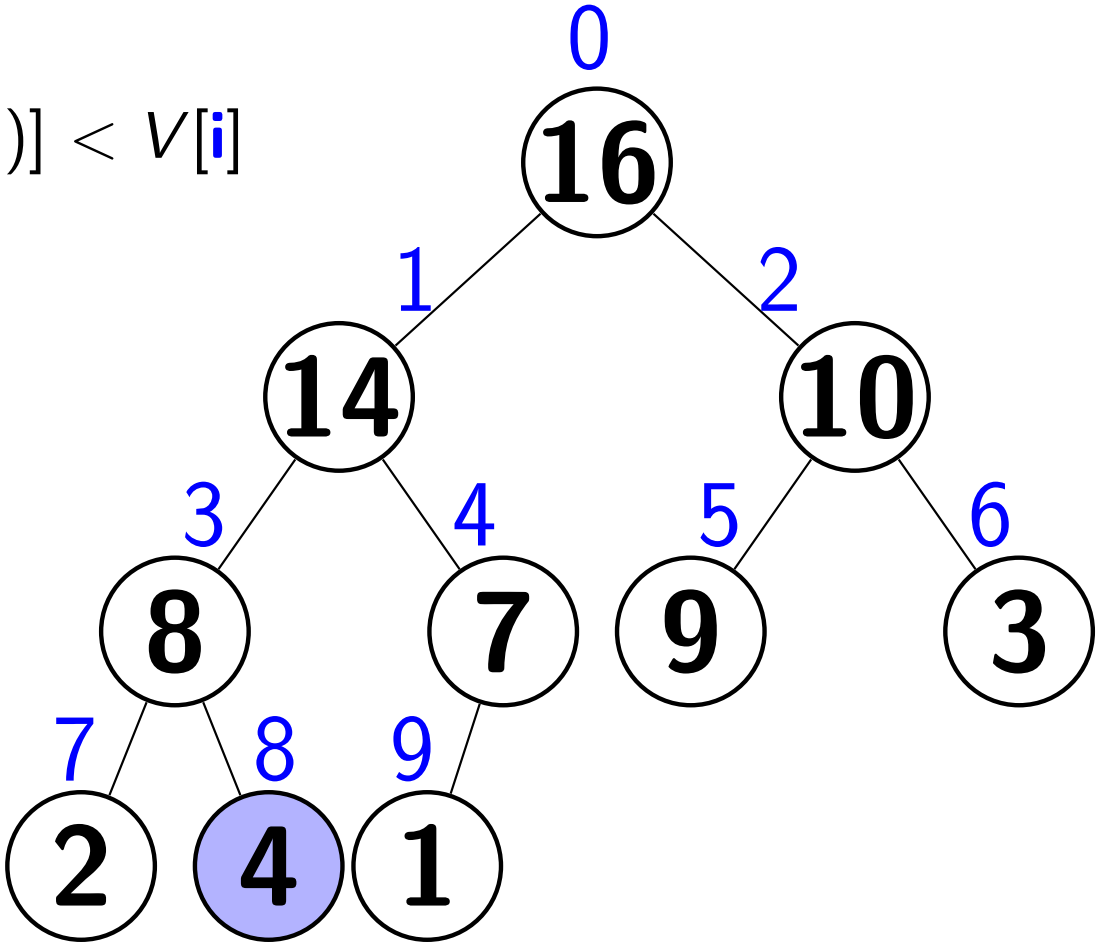
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
▷ V[8] = 15;
while i > 0 and V[PAI(i)] < V[i]
   V[i] ↔ V[PAI(i)];
   i = PAI(i);
```



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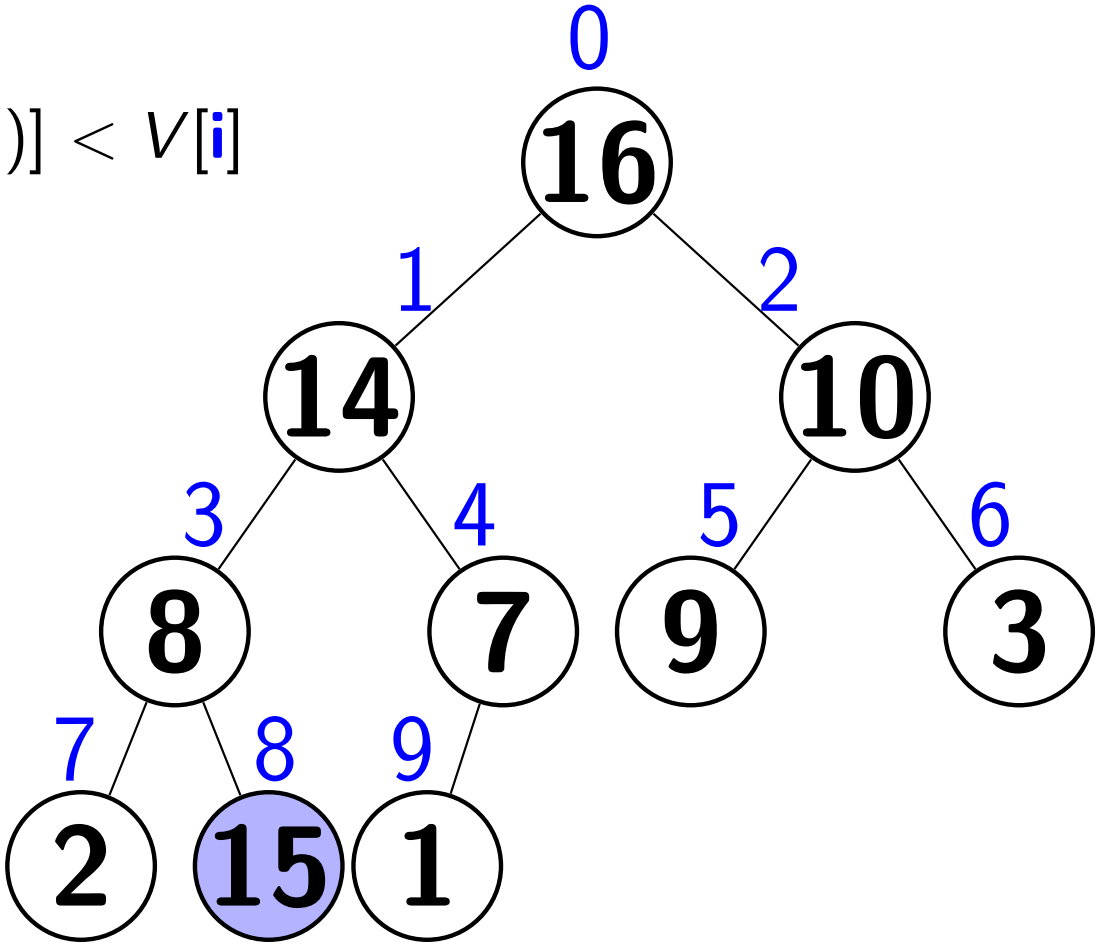

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V[8] = 15;
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   V[i] ↔ V[PAI(i)];
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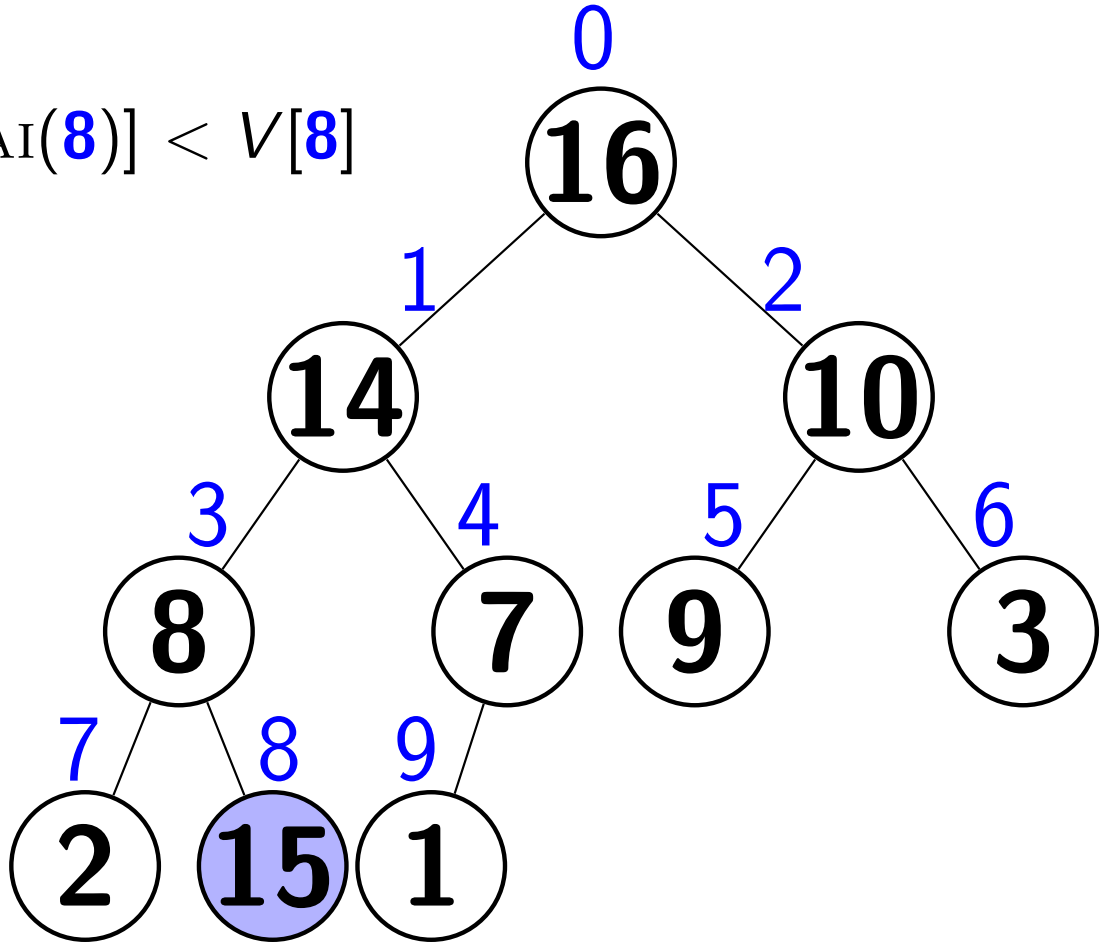

Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
▷while 8 > 0 and V[3] < V[8]
   V[i] ↔ V[PAI(i)];
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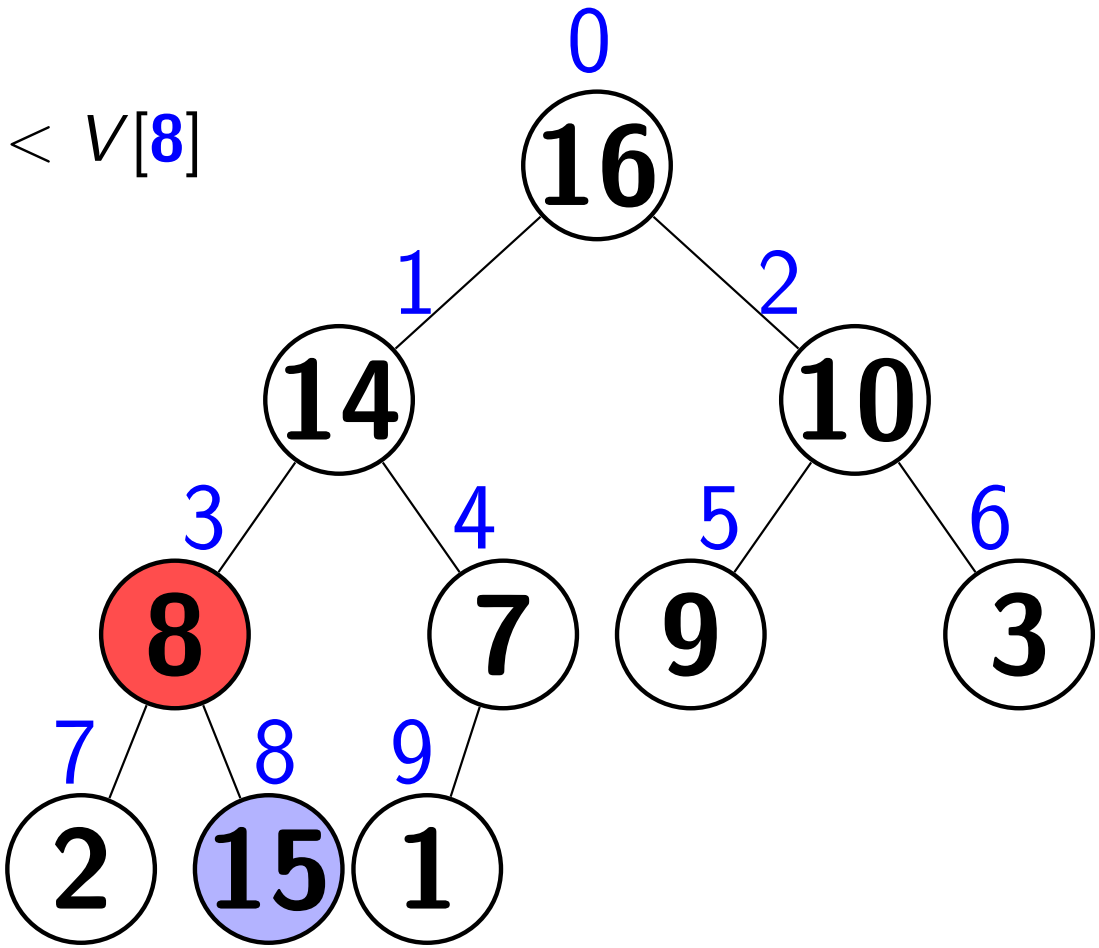
```



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   i = PAI(i);
```
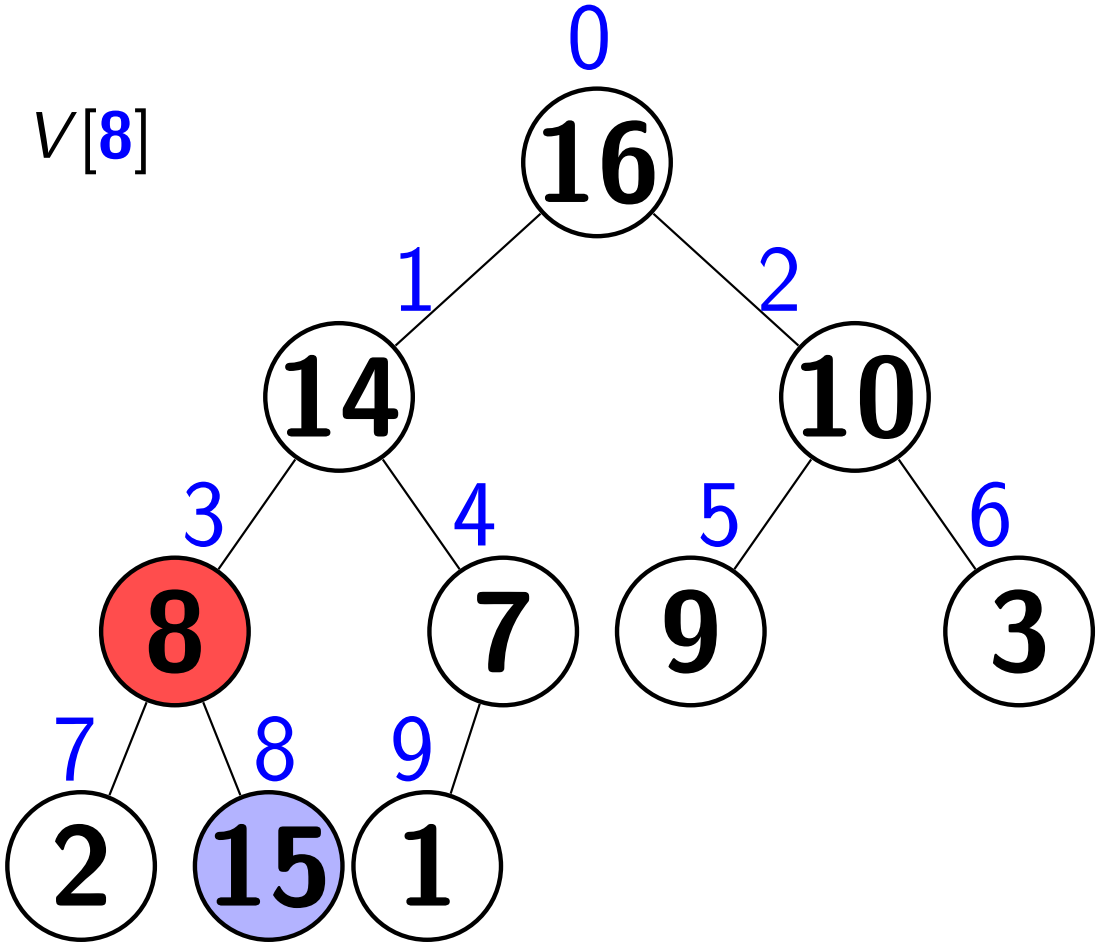
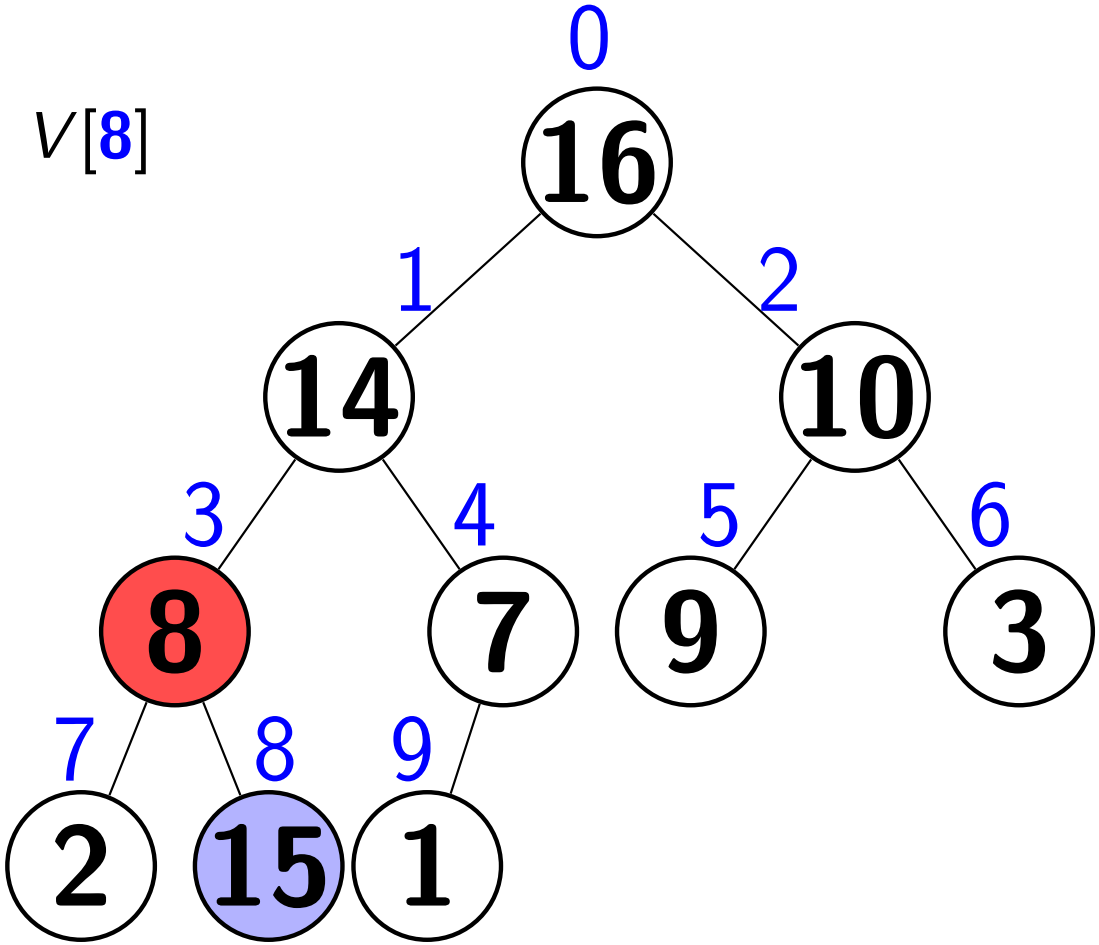



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 8 > 0 and V[3] < V[8]
   ▷ V[8] ↔ V[3];
   i = PAI(i);
```



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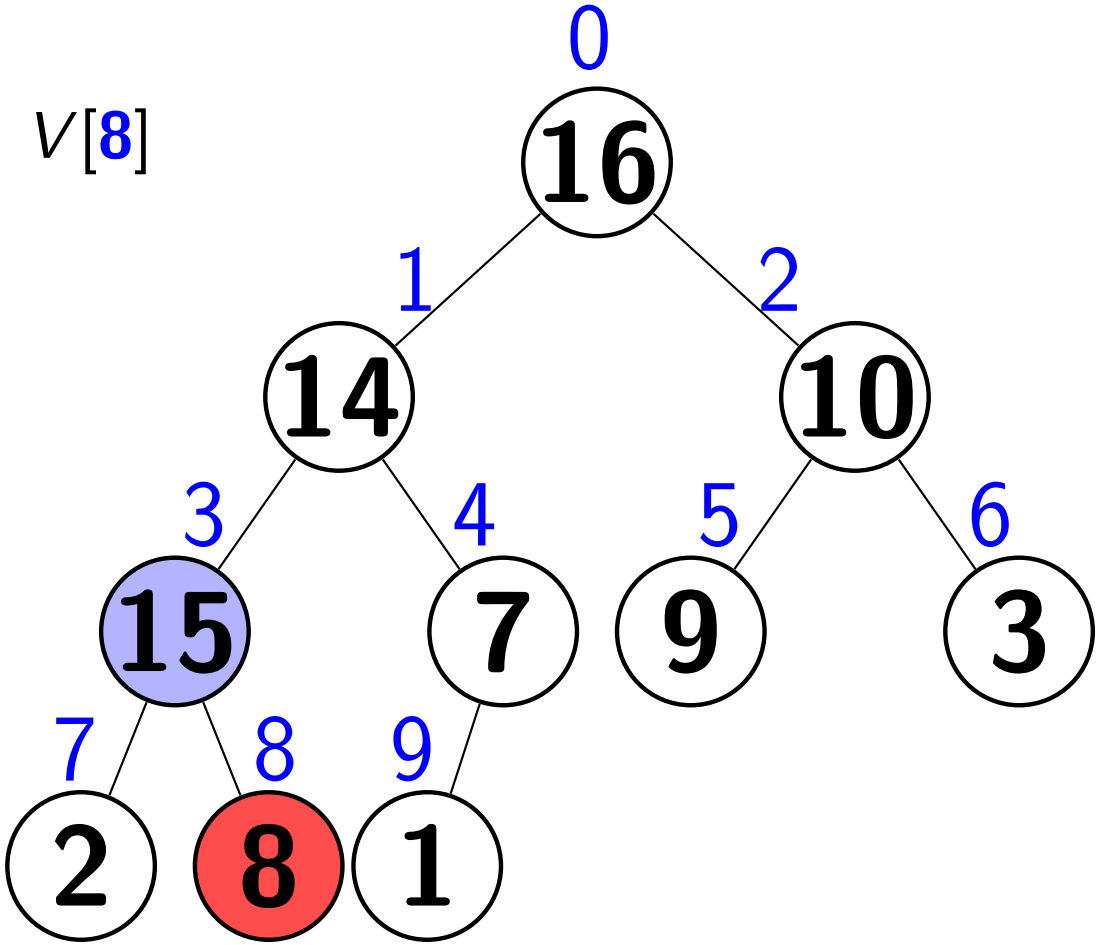

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   V[8] ↔ V[3];
   ▷i = PAI(i);
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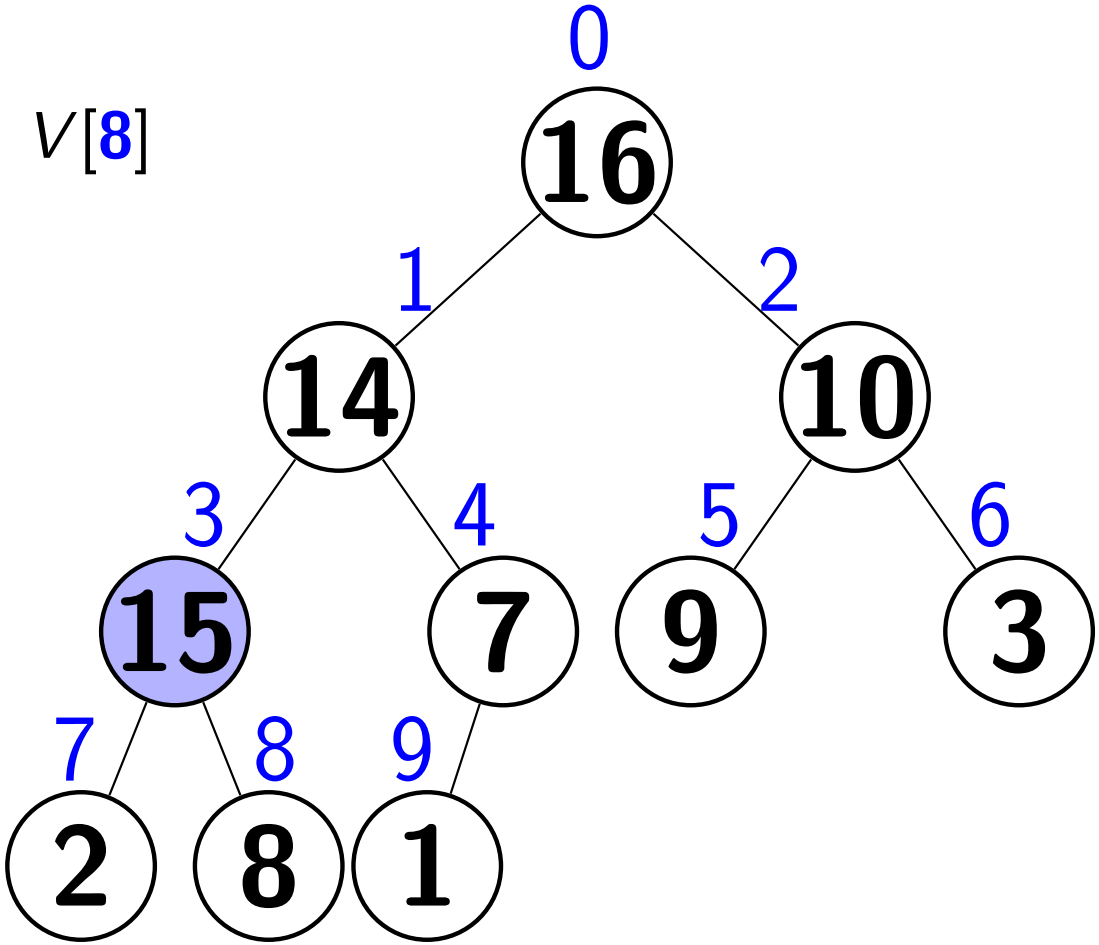

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if 15 < 4 then
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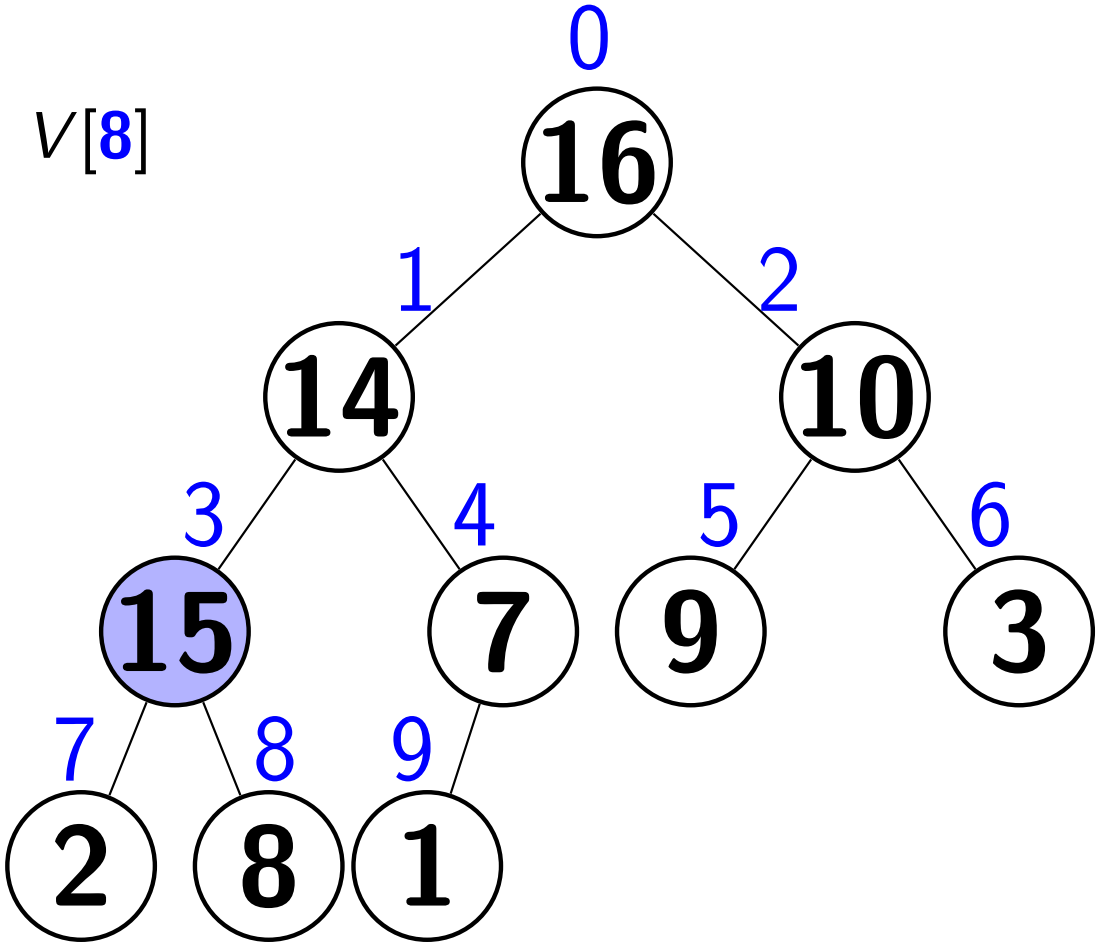
    return;
V[8] = 15;
while 8 > 0 and V[3] < V[8]
   V[8] ↔ V[3];
   ▷i = PAI(8);
```



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if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 8 > 0 and V[3] < V[8]
   V[8] ↔ V[3];
   ▷i = 3;
```

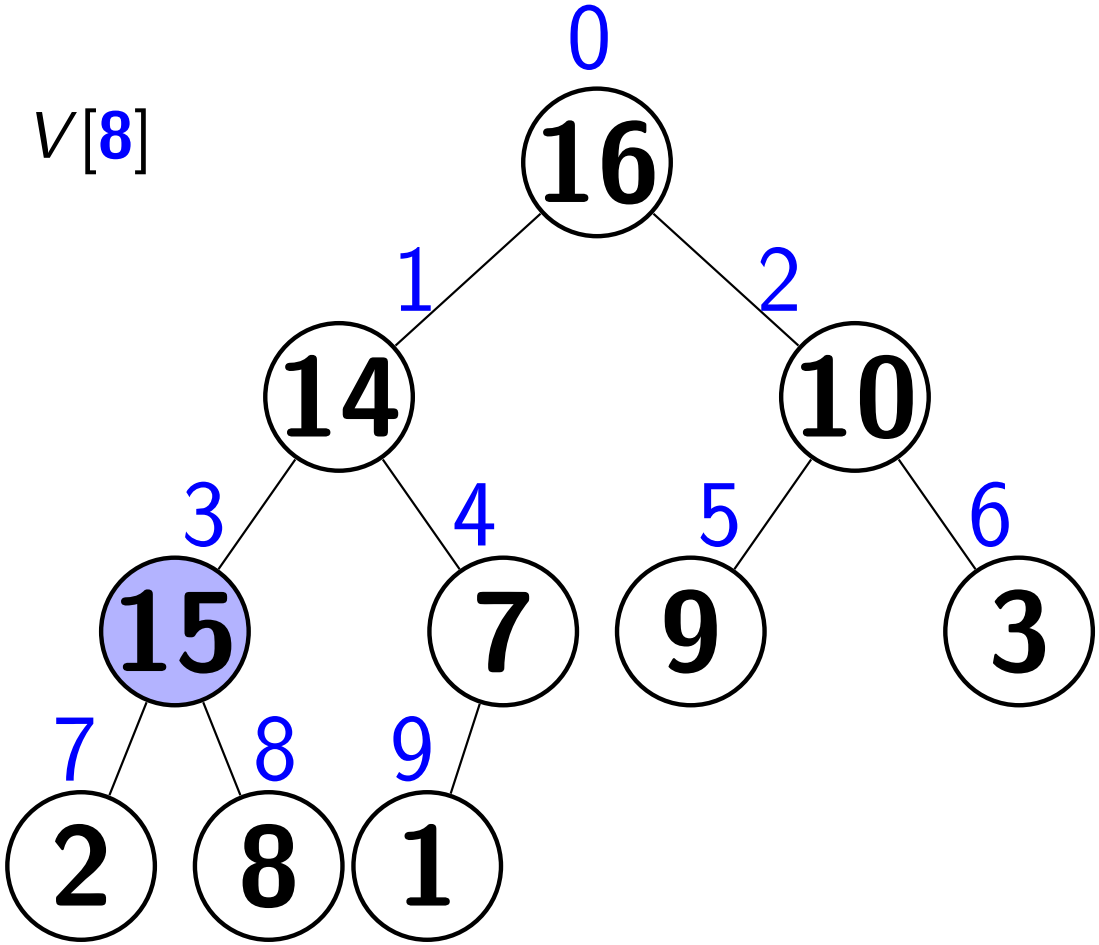

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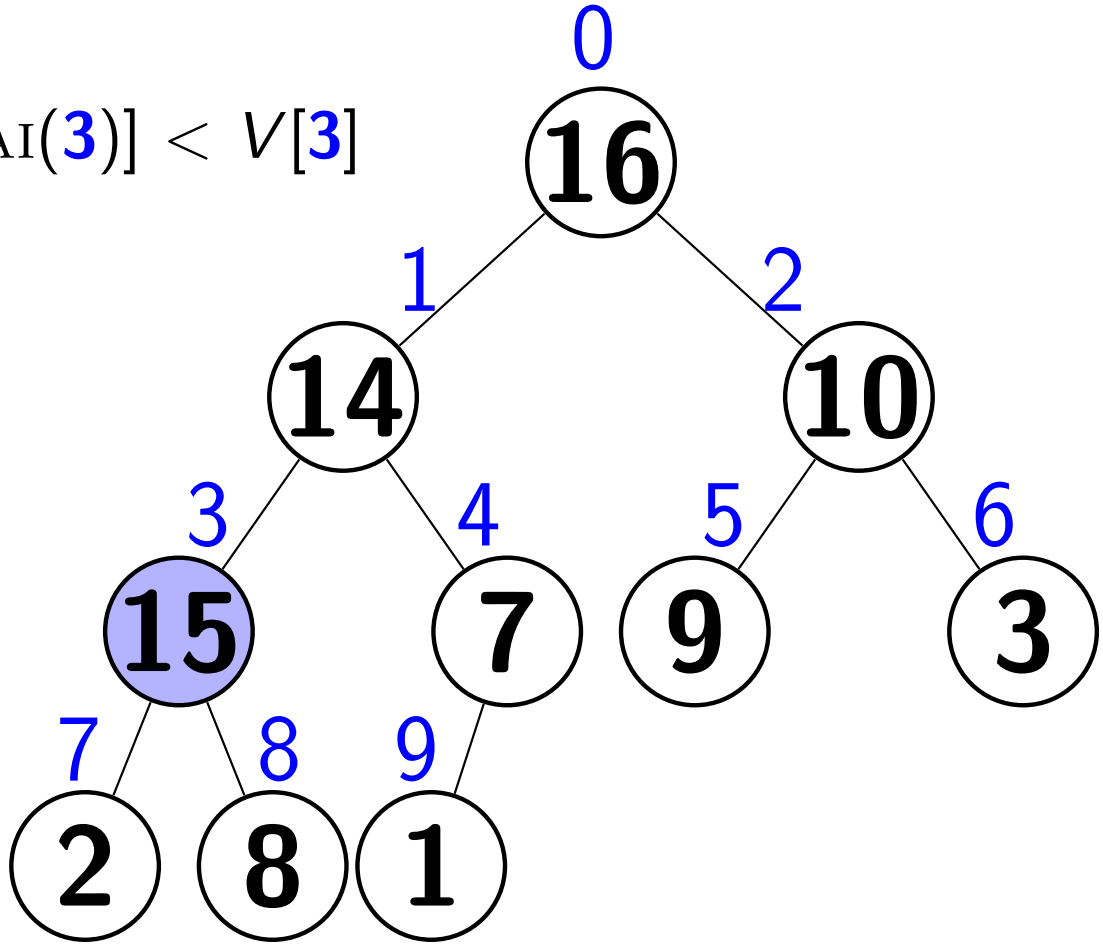
```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
▷while 3 > 0 and V[PAI(3)] < V[3]
   V[i] ↔ V[PAI(3)];
   i = PAI(i);
```



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```
if 15 < 4 then
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    return;
V[8] = 15;
▷while 3 > 0 and V[1] < V[3]
   V[i] ↔ V[PAI(3)];
   i = PAI(i);
```

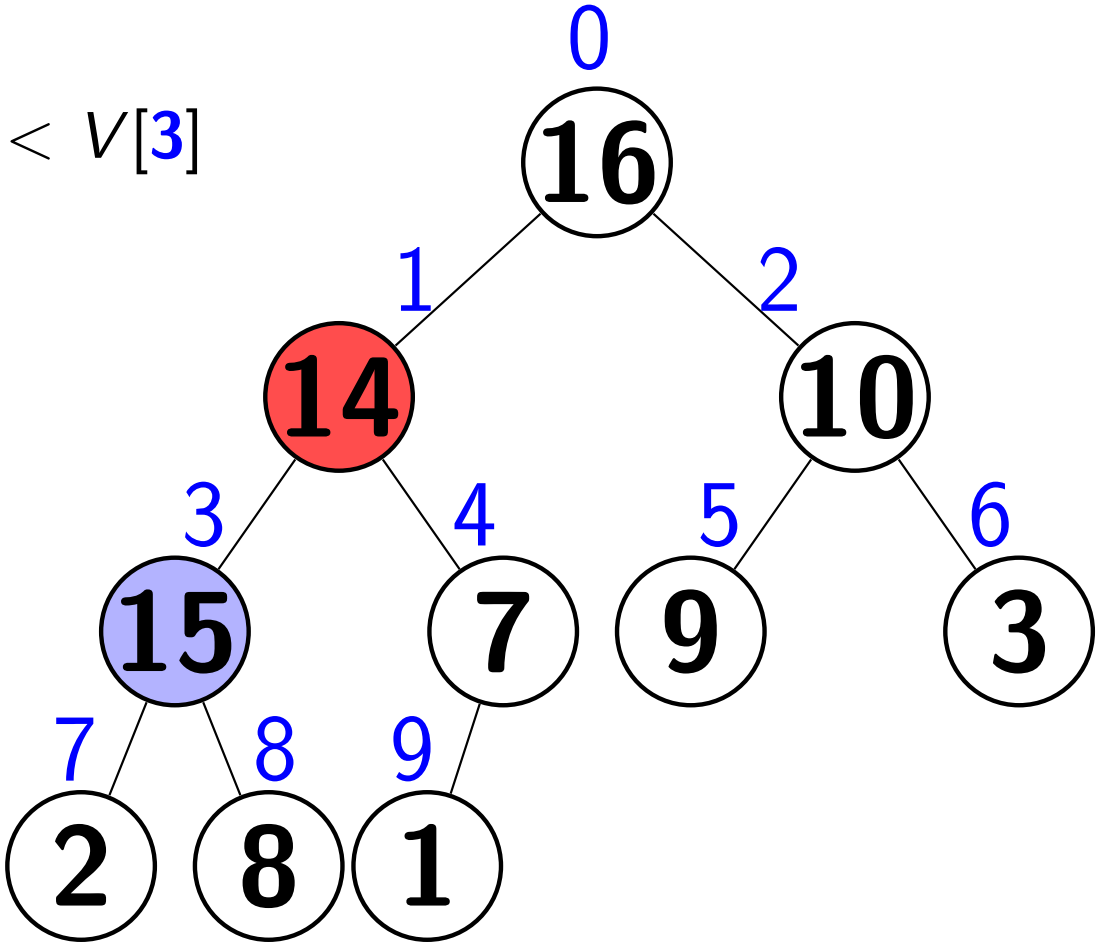

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if 15 < 4 then
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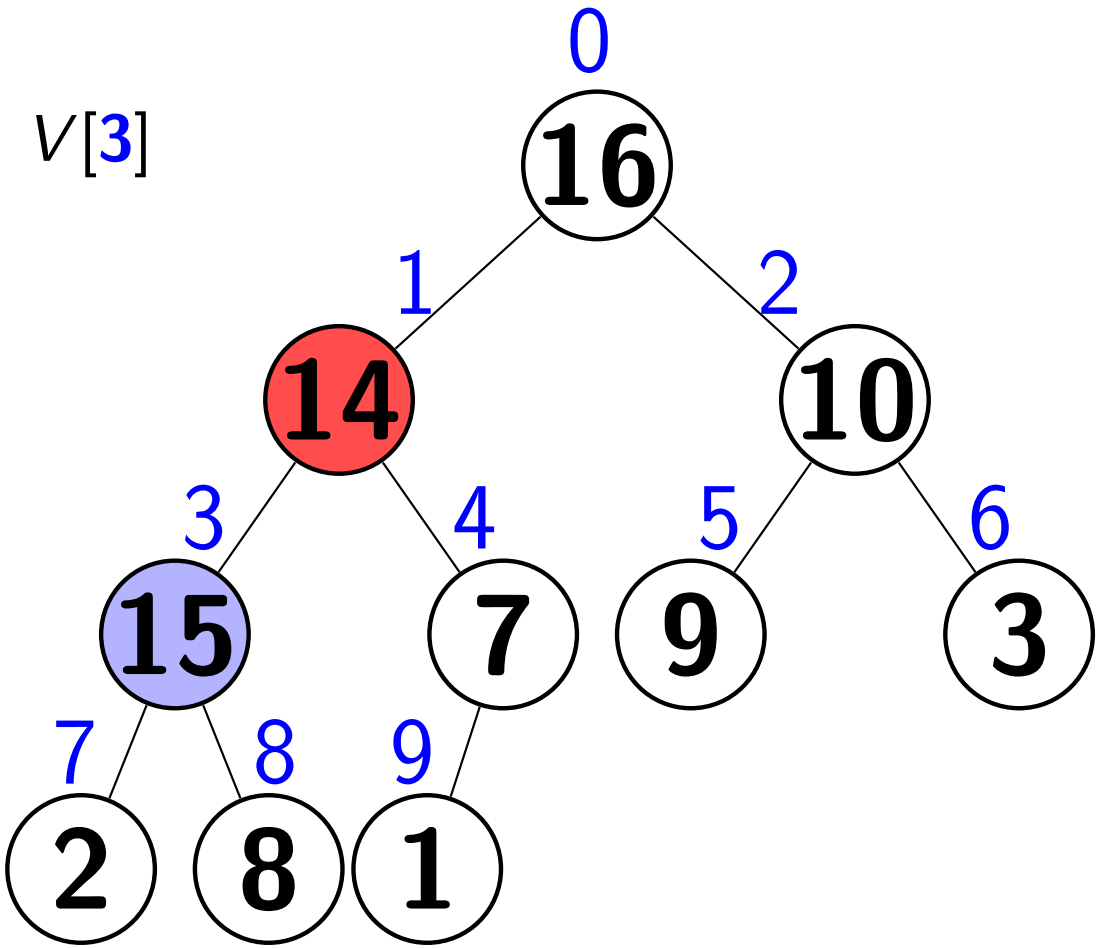
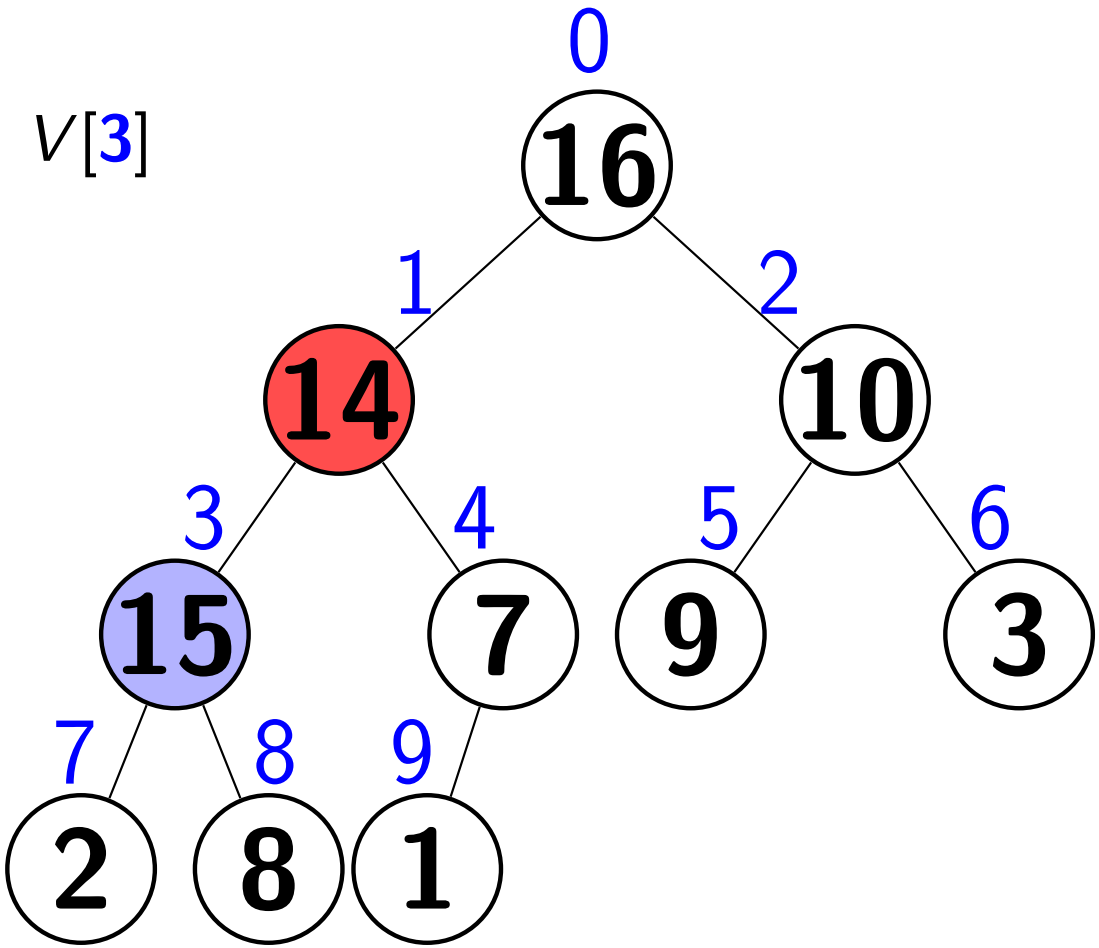
    return;
V[8] = 15;
while 3 > 0 and V[1] < V[3]
   ▷ V[i] ↔ V[PAI(3)];
   i = PAI(i);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

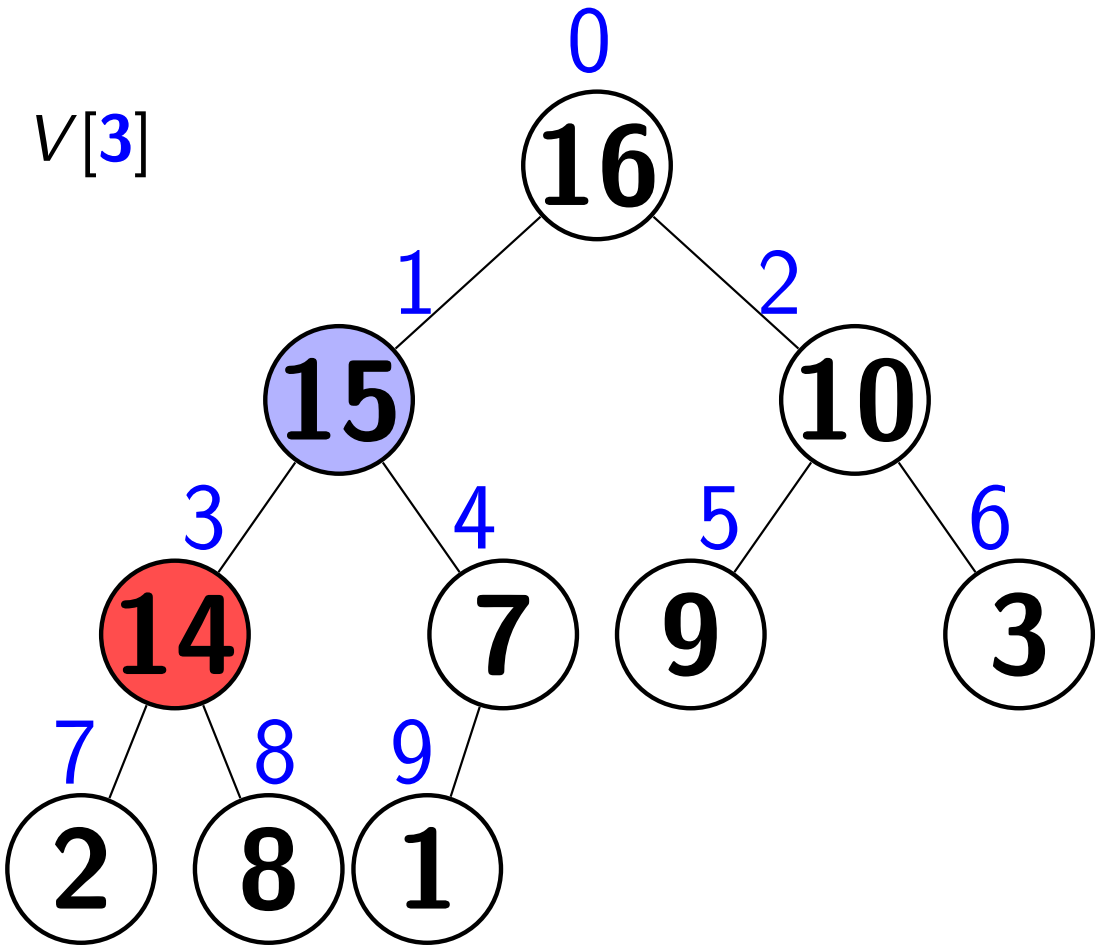
```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 3 > 0 and V[1] < V[3]
   ▷ V[3] ↔ V[1];
   i = PAI(i);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

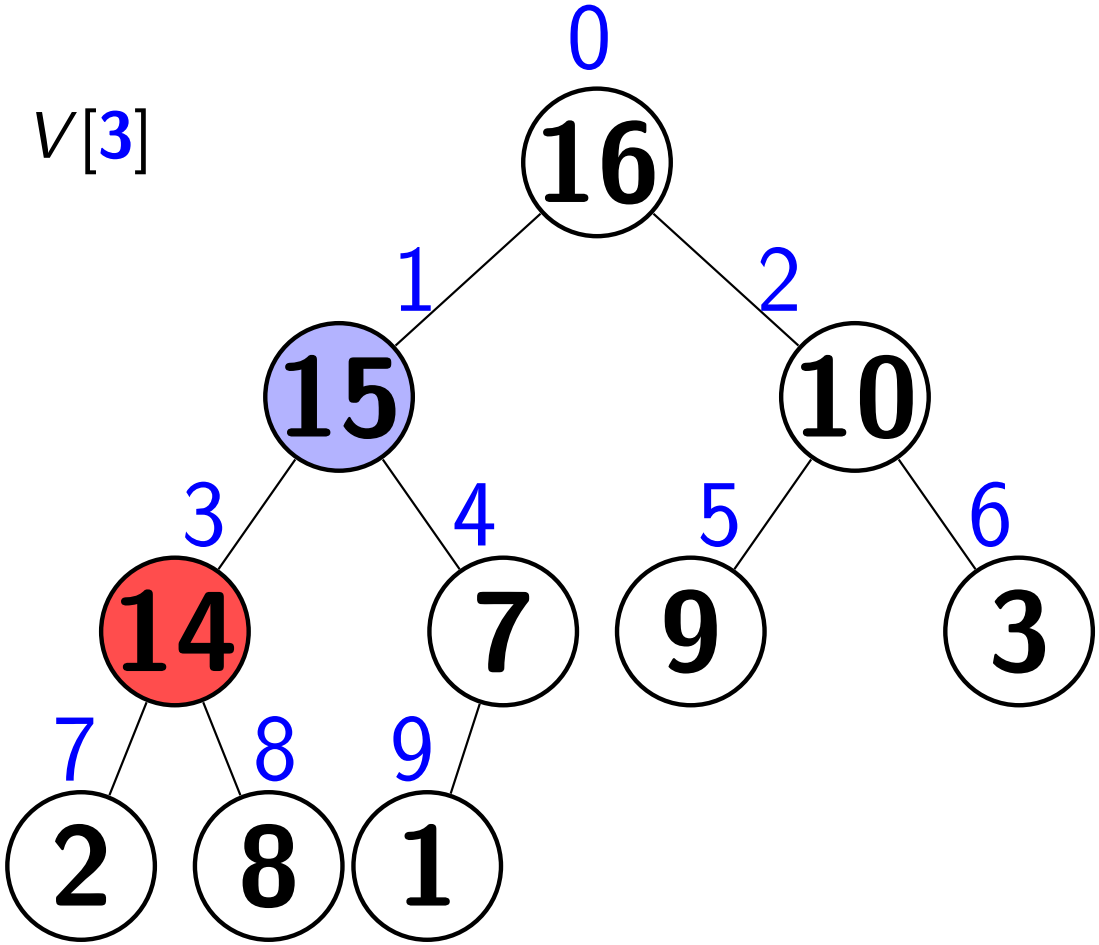
```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 3 > 0 and V[1] < V[3]
   ▷ V[3] ↔ V[1];
   i = PAI(i);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

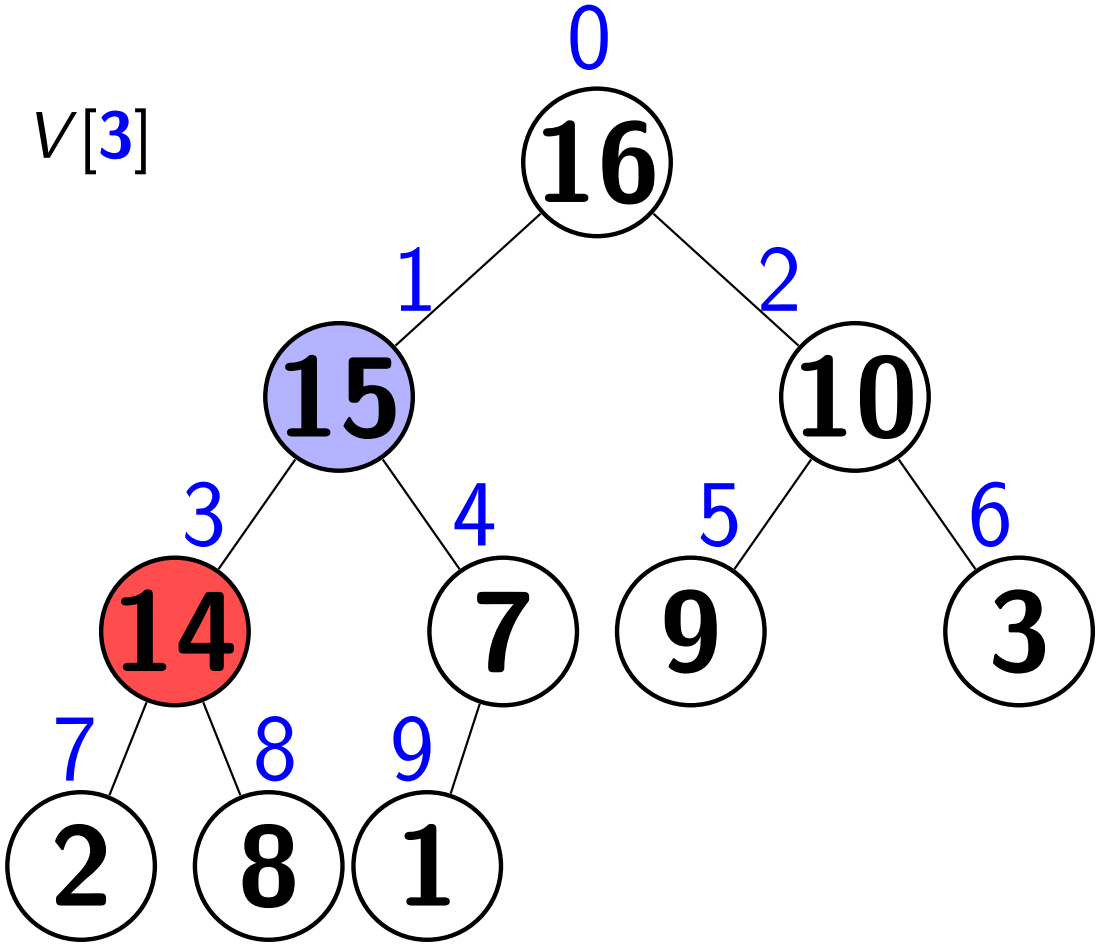
```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 3 > 0 and V[1] < V[3]
   V[3] ↔ V[1];
   ▷i = PAI(3);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

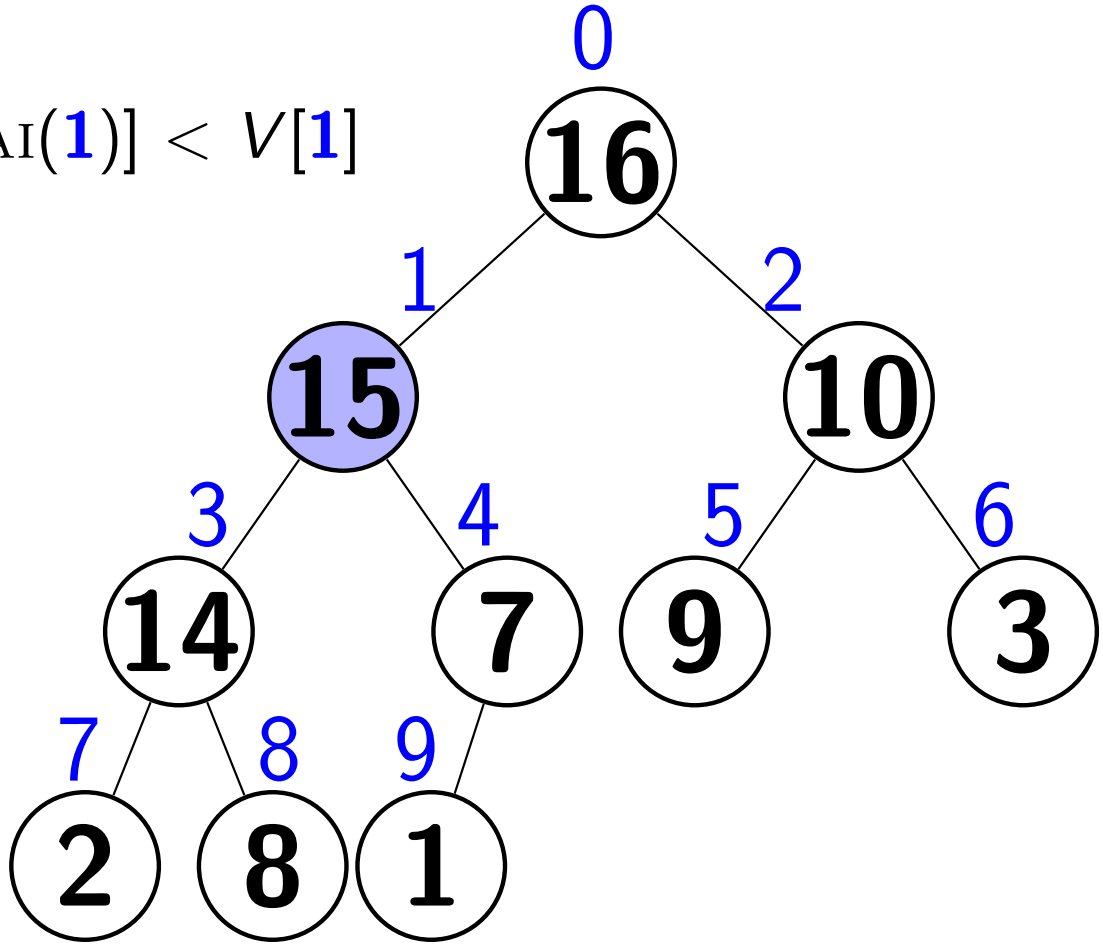
```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
while 3 > 0 and V[1] < V[3]
   V[3] ↔ V[1];
   ▷i = 1;
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

```
if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
▷while 1 > 0 and V[PAI(1)] < V[1]
   V[i] ↔ V[PAI(1)];
   i = PAI(i);
```



Heap-Increase-Key

HEAP-INCREASE-KEY (V, 8, 15)

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if 15 < 4 then
    printf ("erro: chave menor que atual");
    return;
V[8] = 15;
▷while 1 > 0 and V[0] < V[1]
   V[i] ↔ V[PAI(1)];
   i = PAI(i);
```

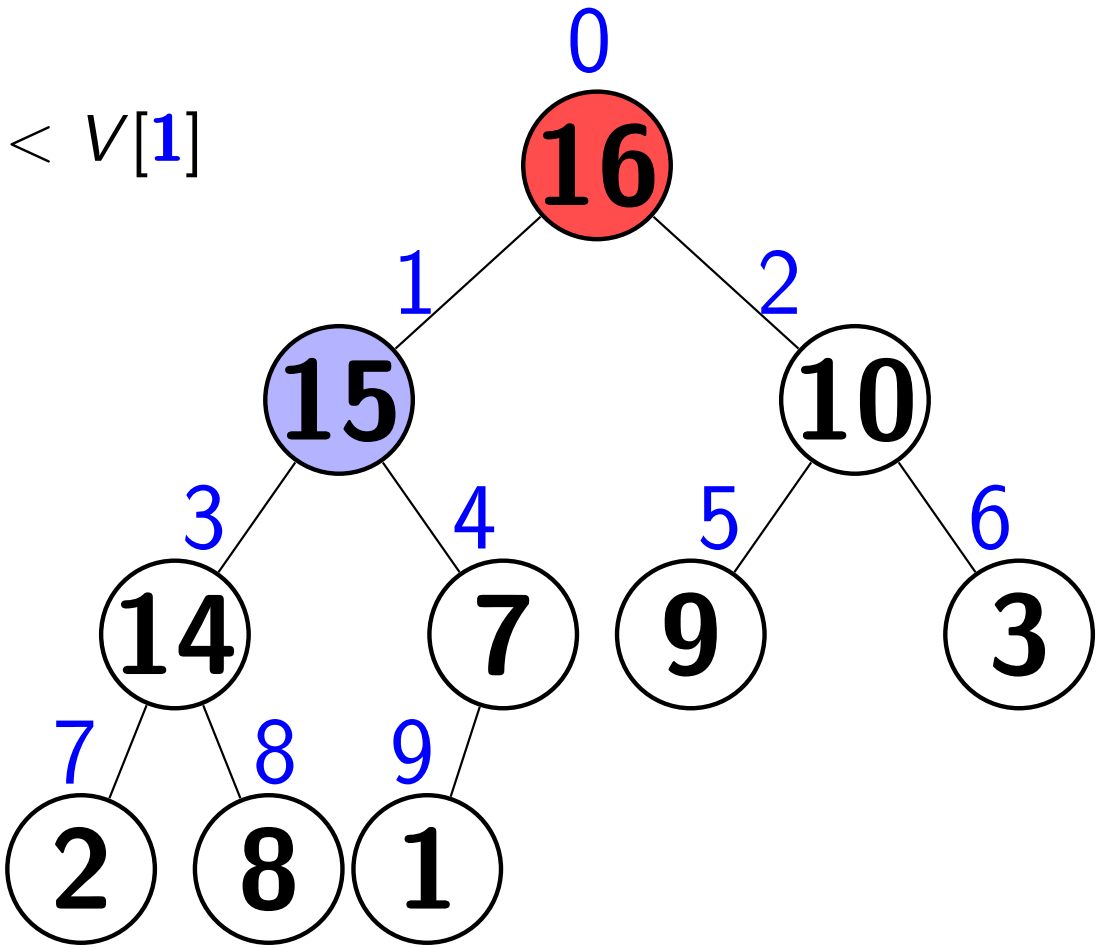

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if 15 < 4 then
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   V[i] ↔ V[PAI(1)];
   i = PAI(i);
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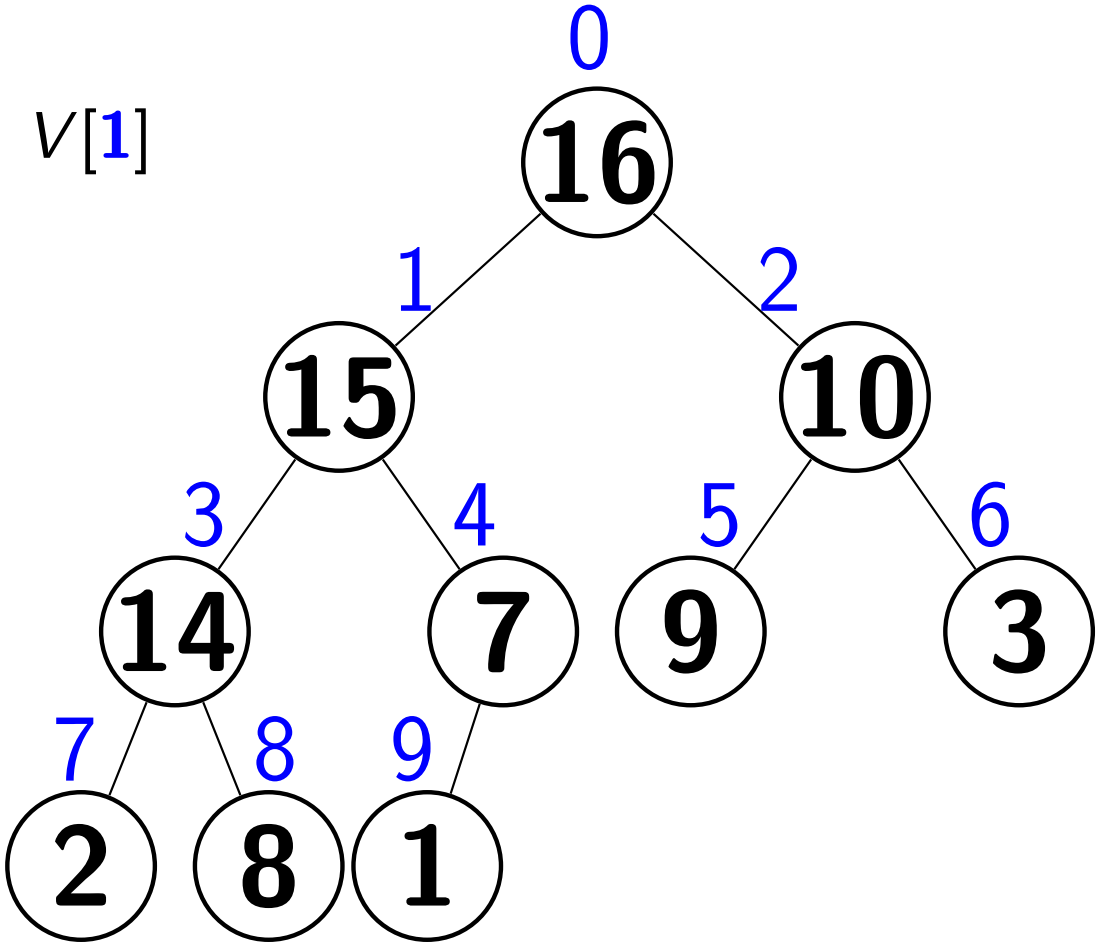


Heap-Increase-Key

Complexidade: o procedimento Heap-Increase-Key possui complexidade $\mathcal{O}(\log n)$ para um heap com n elementos, pois o caminho traçado do nó atualizado até a raiz tem comprimento máximo de $\mathcal{O}(\log n)$.

Sumário

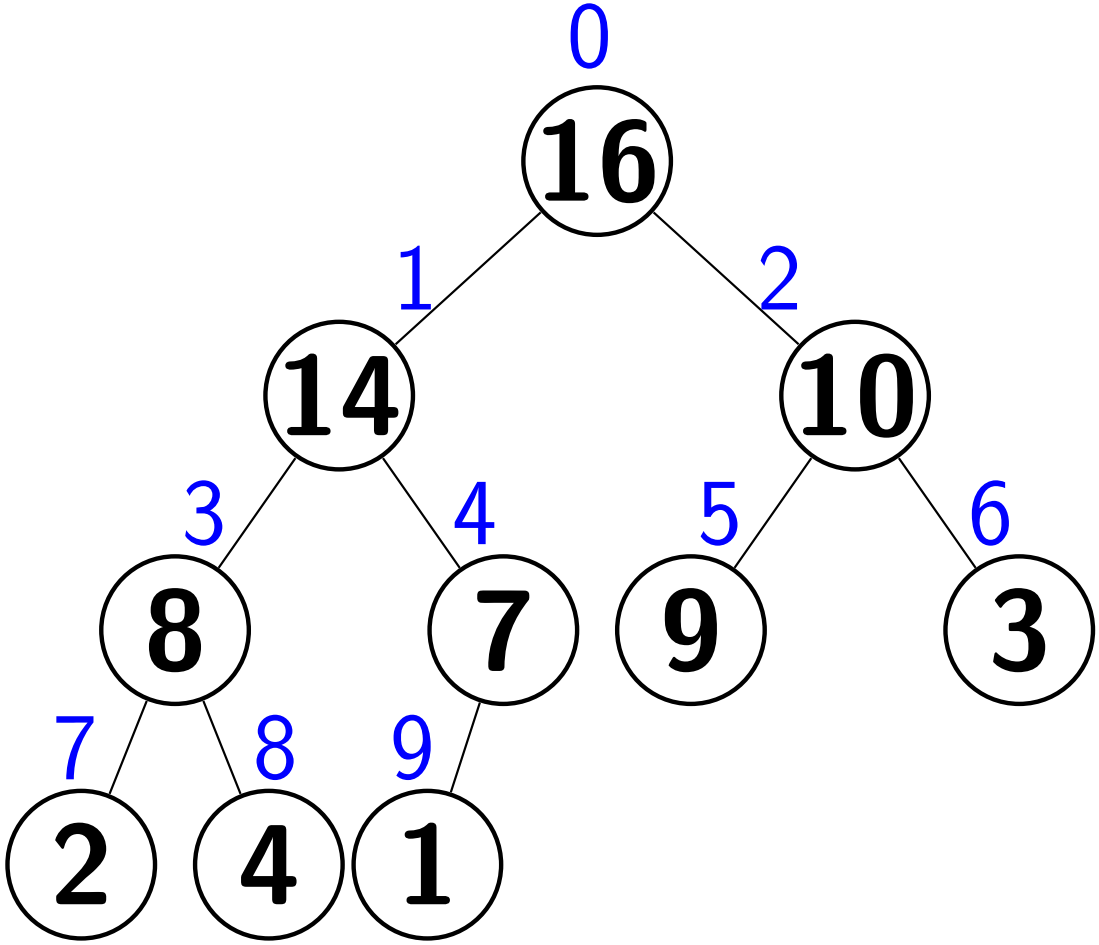
Max-Heap-Insert

A operação Max-Heap-Insert ($\mathbf{V}, \mathbf{x}$) insere o elemento $\mathbf{x}$ no conjunto V. Essa operação poderia ser escrita como $V \leftarrow V \cup \{\mathbf{x}\}$. Essa operação recebe como entrada a **chave** a ser inserida e **expande o heap com a nova chave**. Note que nessa operação é necessário encontrar a posição apropriada para o elemento recém adicionado.

Max-Heap-Insert

MAX-HEAP-INSERT (V, Size, **chave**)

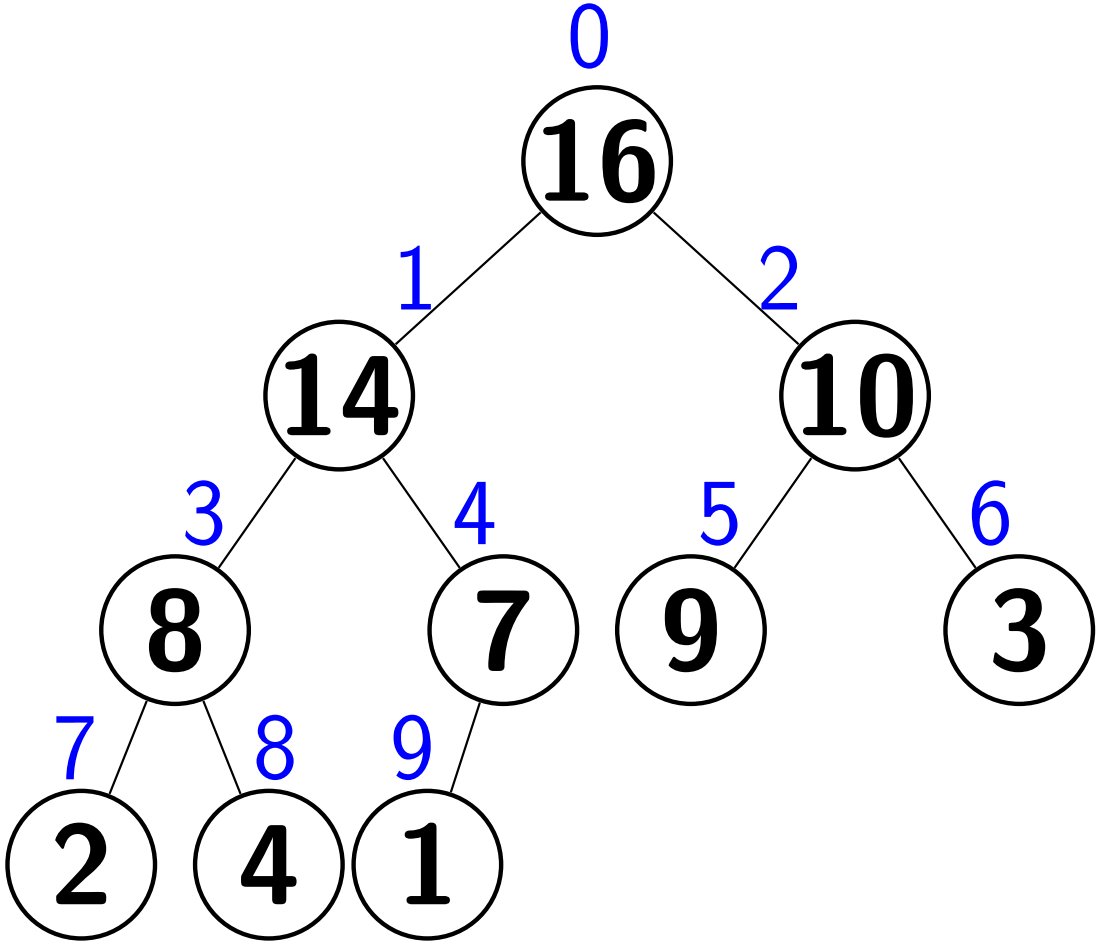
Size = Size+1;
V[Size−1] = $-\infty$;
HEAP-INCREASE-KEY (V, Size−1, **chave**);



Max-Heap-Insert

MAX-HEAP-INSERT (V, Size, **20**)

Size = Size+1;
V[Size−1] = $-\infty$;
HEAP-INCREASE-KEY (V, Size−1, **chave**);



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MAX-HEAP-INSERT (V, Size, **20**)

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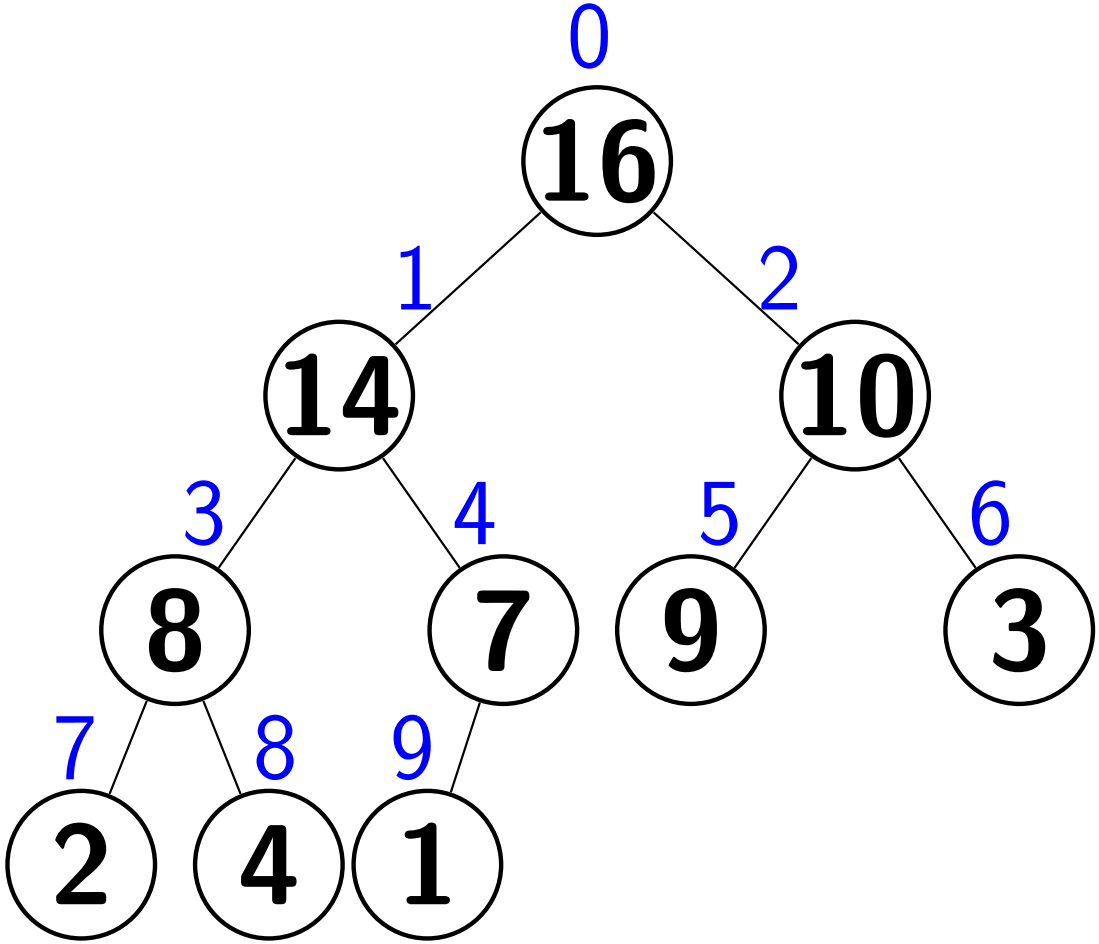
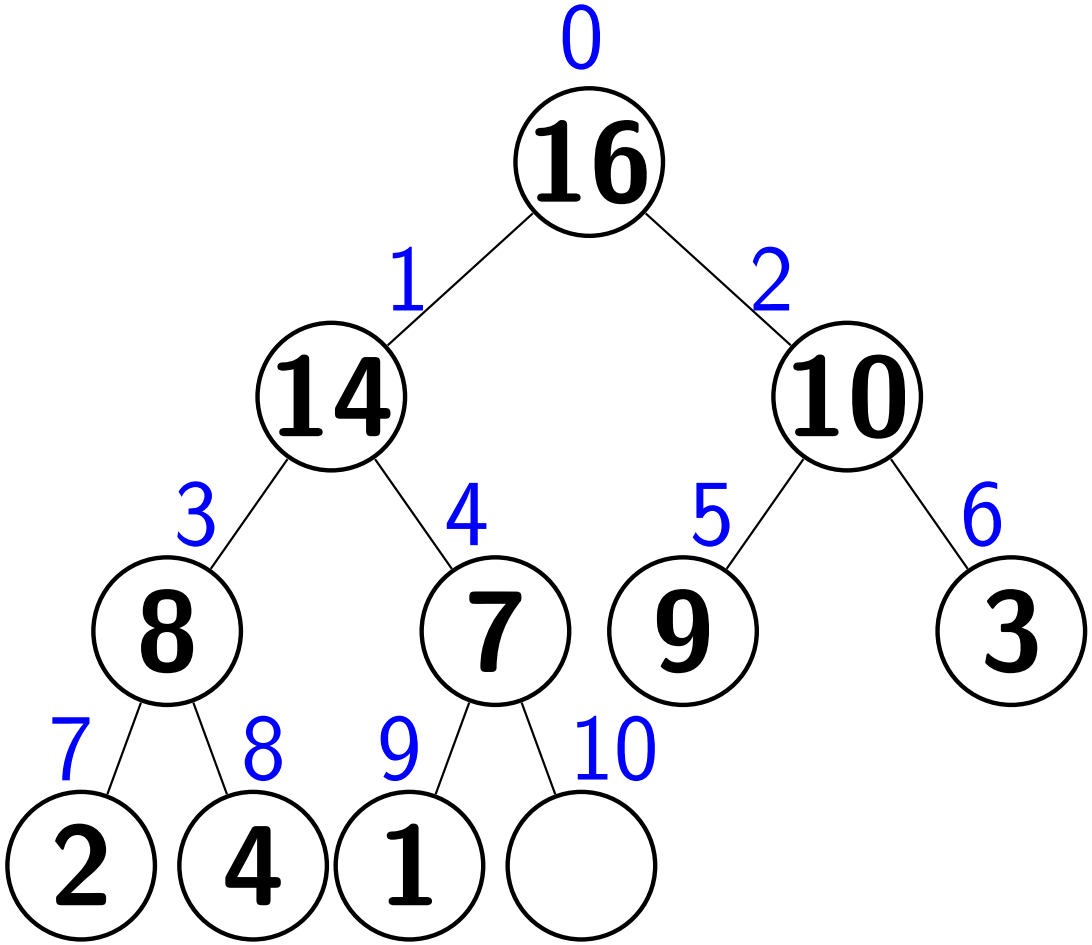


Max-Heap-Insert

MAX-HEAP-INSERT (V, Size, **20**)

▷Size = 10+1;
$V[Size - 1] = -\infty$;
HEAP-INCREASE-KEY (V, Size−1, **chave**);



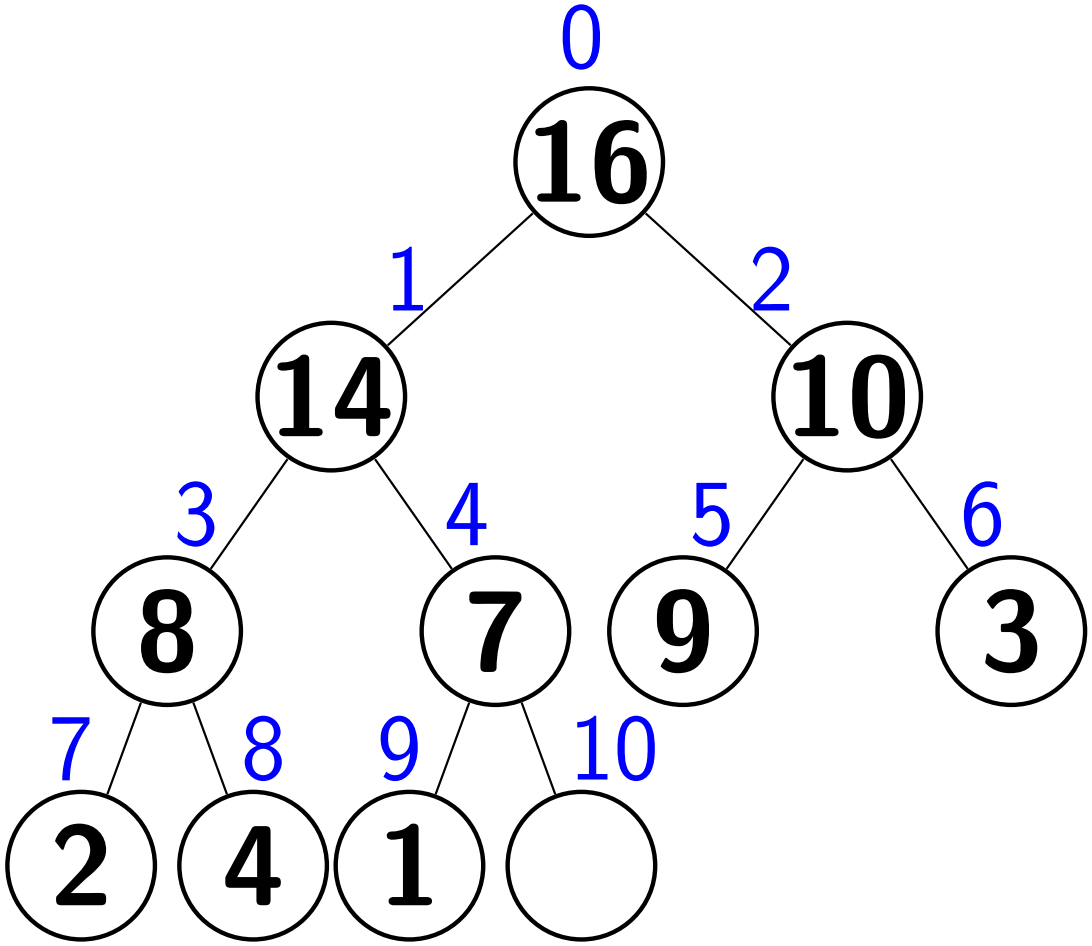
Max-Heap-Insert

MAX-HEAP-INSERT (V, Size, **20**)

Size $= 11$;

$\triangleright V[Size - 1] = -\infty$;

HEAP-INCREASE-KEY (V, Size-1, **chave**);



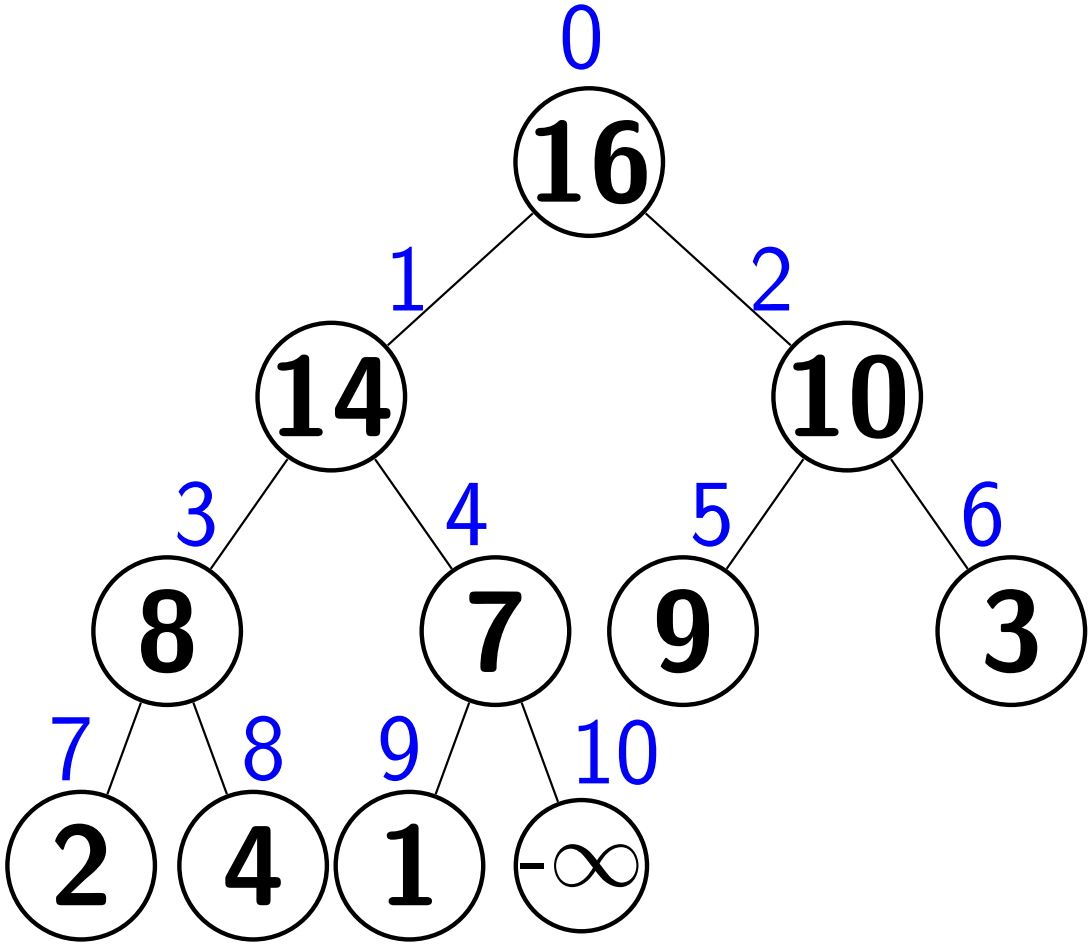
Max-Heap-Insert

MAX-HEAP-INSERT (V, Size, **20**)

Size $= 11$;

$\triangleright V[10] = -\infty$;

HEAP-INCREASE-KEY (V, Size-1, **chave**);



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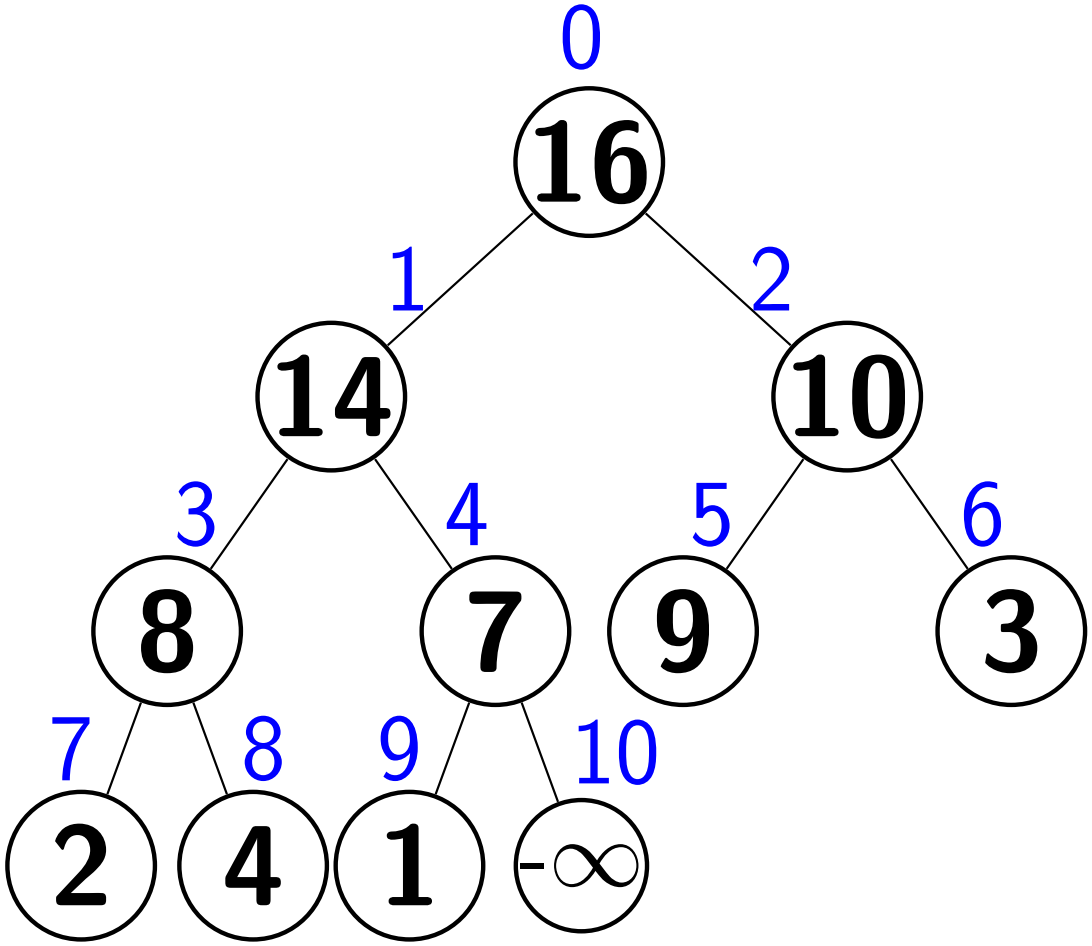

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Size $= 11$;

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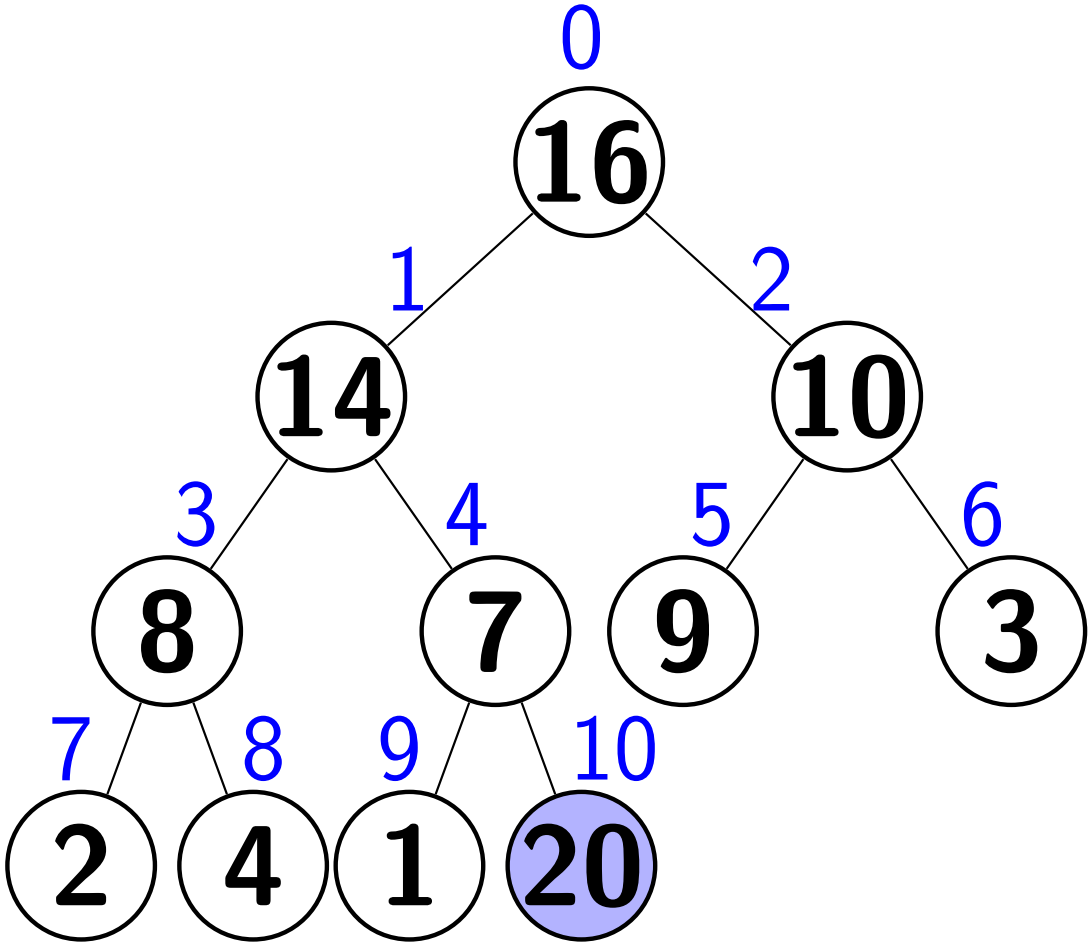

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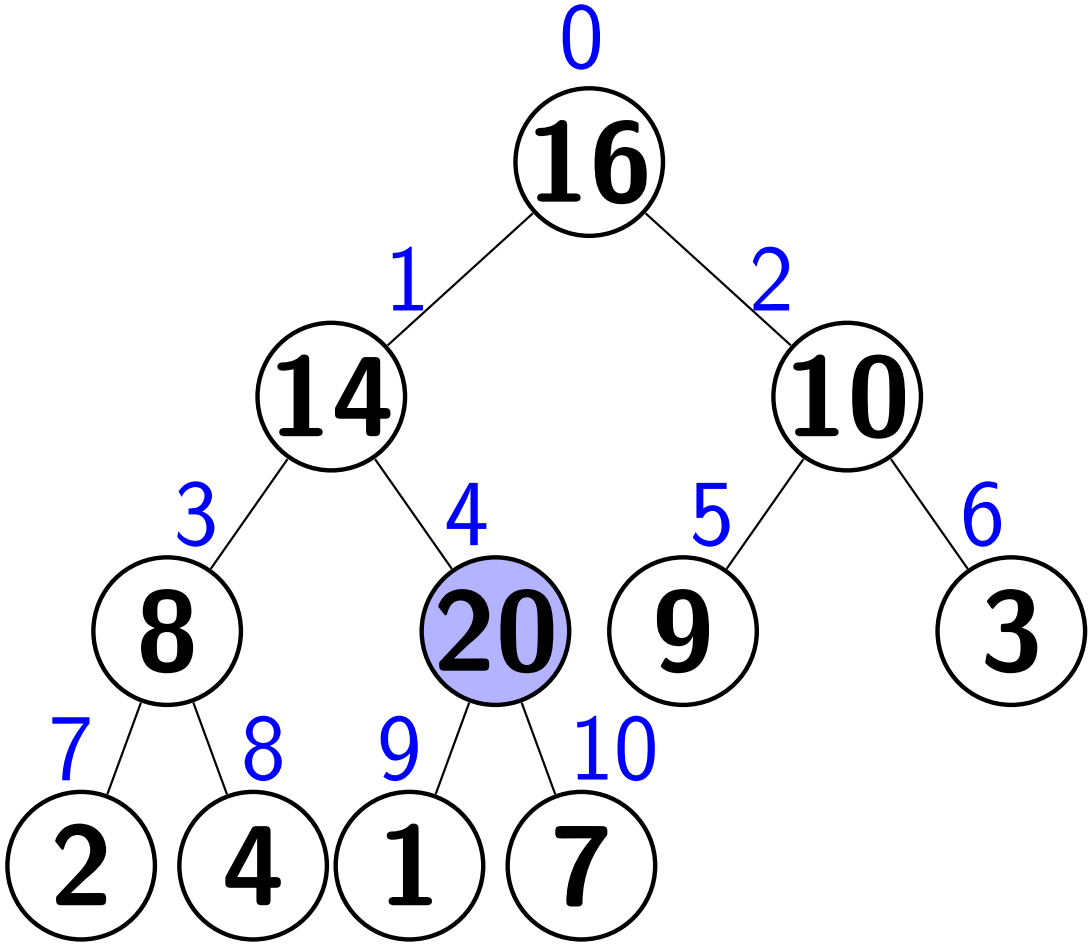

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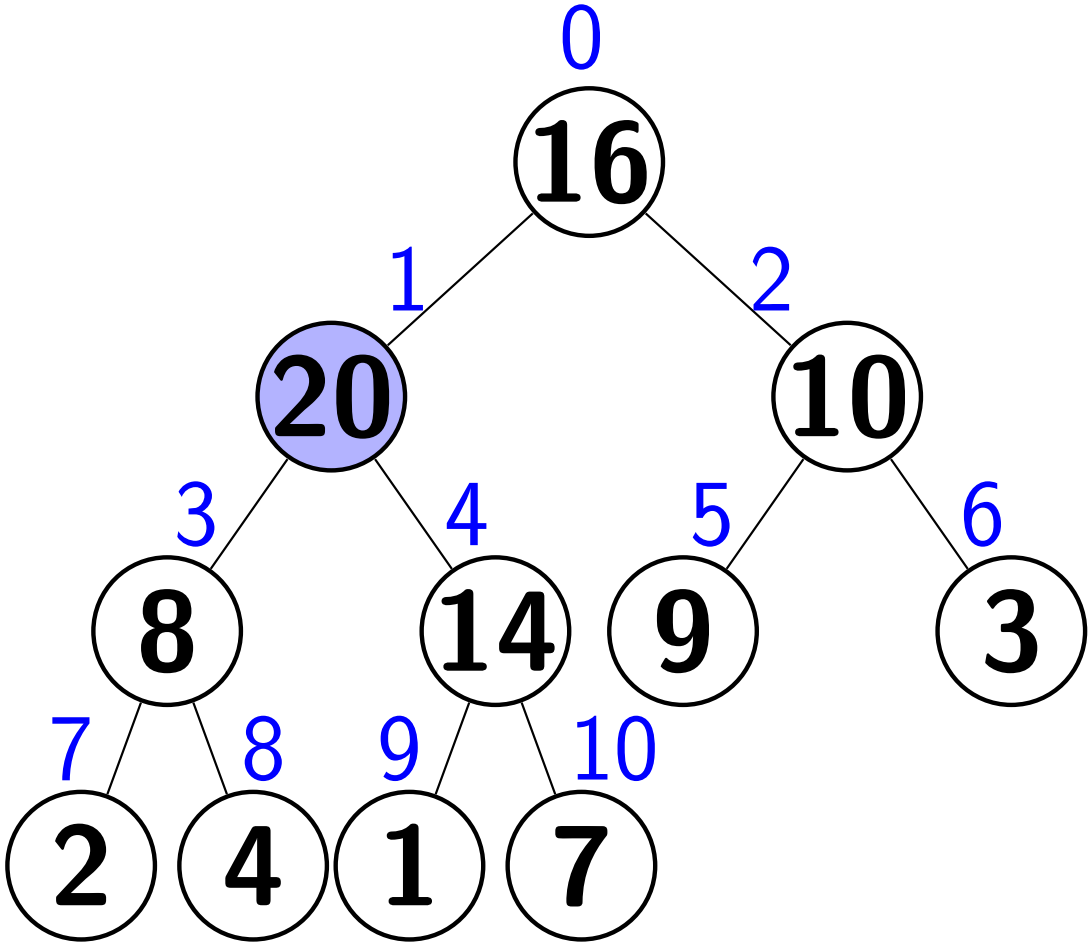

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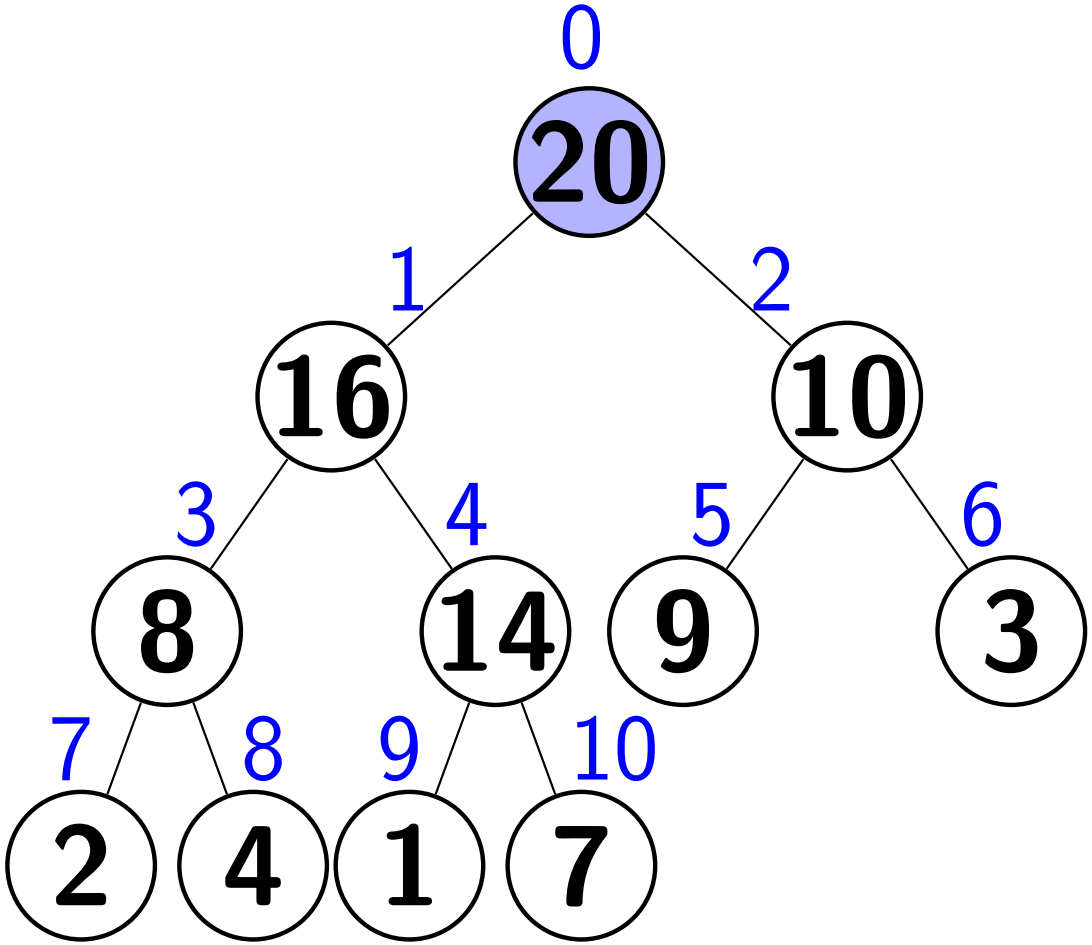

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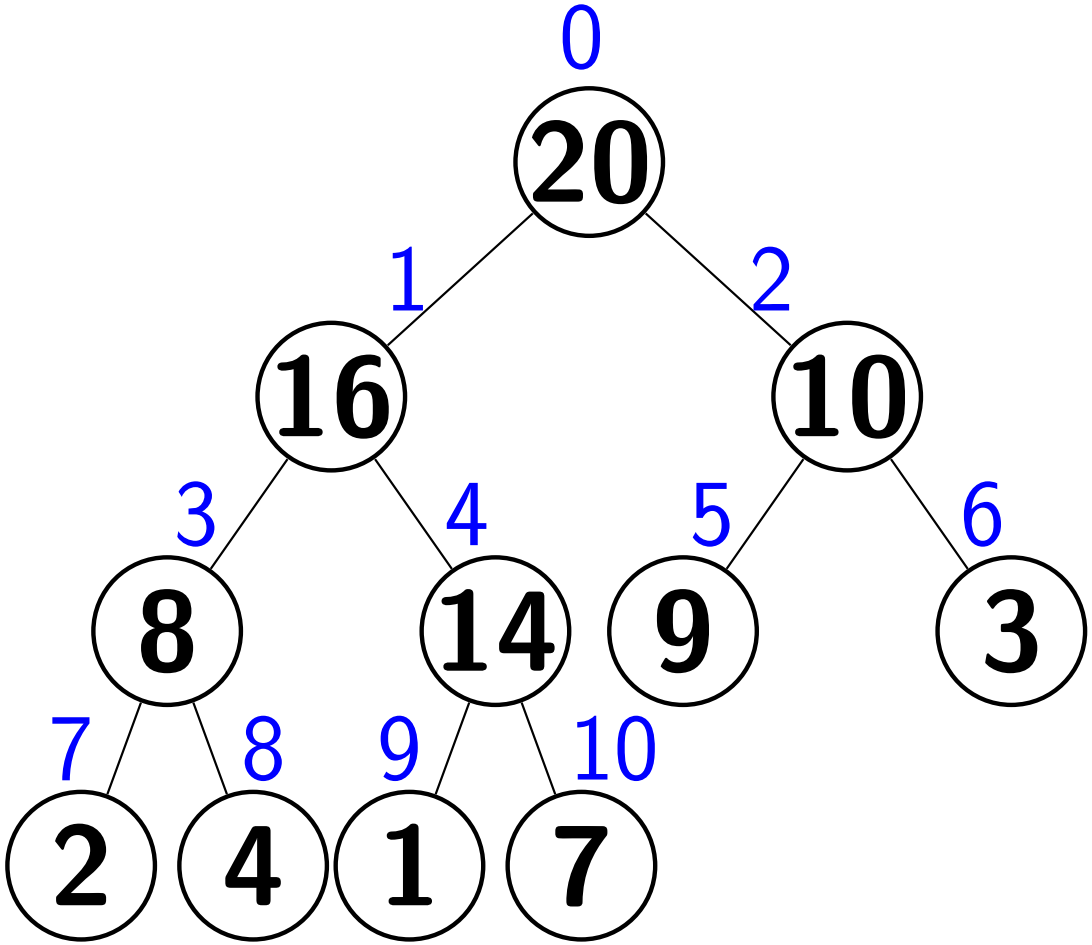

Max-Heap-Insert

Complexidade: o procedimento Max-Heap-Insert possui complexidade $\mathcal{O}(\log n)$ para um heap com n elementos, pois a complexidade do procedimento Heap-Increase-Key é de $\mathcal{O}(\log n)$. Em resumo, um heap pode admitir qualquer operação de fila de prioridades em um conjunto de tamanho n no tempo $\mathcal{O}(\log n)$.

Referências

Algoritmos: Teoria e prática. Cormen et al.