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Resistance and Trim Predictions for the NPL High Speed Round Bilge Displacement Hull Series

by

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SUMMARY

Mathematical representation of calm water resistance and trim of the systematic NPL series, which is often used for high speed pilot boats, work boats, patrol craft, etc. is presented. A predictive technique is established by regression analysis. Dependent variables are the resistance-displacement ratio (R_T/Δ for $\Delta_{standard}$ = 100,000 lb) and dynamic trim (τ), while independent variables are length-displacement ratio ($L/\nabla^{1/3}$), the ratio of length to beam (L/B) and the ratio of beam to draught (B/T), as well as their cross-products and their different powers multiplied by powers from 0 to 8 of the displacement Froude number ($F_{n\nabla}$).

Similar mathematical models published previously are based on the resistance data of NPL series combined with the data of other series. This paper analyses broader speed range ($F_{n\nabla}$=0.8-3.0) and is based on the NPL series only, resulting in a more reliable resistance prediction method. The mathematical models are suitable for implementation in software and can replace the "manual" power prediction calculations for the NPL series.

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1. INTRODUCTION

The NPL systematic high speed round bilge displacement (for higher speeds semidisplacement) hull series, published more than 20 years ago [1], is well known and is still rated as the most useful series for work boats, patrol craft, pilot boats, etc. It is designed for operation in the Froude number range F_{nL}= 0.3-1.2 ($F_{n\nabla}$ = 0.6-3.0). The series covers :

- length-beam ratio L/B = 3.33-7.50
- length-displacement ratio (M) = 4.5-8.3
- beam-draft ratio B/T = 1.75-10.77

while constant values are taken for :

- position of longitudinal center of buoyancy LCB = 6.4%L aft amidships
- block coefficient C_B = 0.397
- ratio of transom area to maximum section area A_T/A_X = 0.52.

Besides calm water resistance and dynamic trim, Reference [1] enables calculation of maneuvering and seakeeping characteristics, stability underway, propulsion coefficients, rise and fall of CG, as well as the influence of transom wedge and spray rail.

Other published relevant round bilge series are SKLAD, Series 63, SSPA series etc. The advantages and reliability of the NPL series has made it a standard for small craft. Many authors have used the NPL body lines and characteristics to develop new series, as in for example Reference [2].

It is surprising that although it is so widely used, a literature search did not turn up any evidence of a mathematical representation of the resistance and trim data specific to the NPL series. That is, mathematical models based on the resistance data of the NPL series combined with the data of other series are available. In the Mercier and Savitsky [3] model, regression analysis was applied to the resistance data of 7 transom-stern hull series (NPL, Nordstrom, DeGroot, SSPA, Series 64, Series 63 and Series 62), which included 118 separate hull forms. In the Finnish VTT [2] model, resistance prediction equation was developed using the regression analysis based on the NPL, SSPA and VTT series, as well as on some other suitable models from the records; 65 models in all. Both resistance equations were developed for total resistance-displacement weight ratio R_T/Δ for standard displacement of 100,000 lb (45.36 t). The Mercier and Savitsky model is suggested for use with lower speeds ($F_{n\nabla}$ between 1 and 2) while the Finnish VTT model is suitable for higher speeds ($F_{n\nabla}$ between 1.8 and 3.2). In both cases, the NPL data are one among many other data sets, so that the resistance prediction equations are not very reliable when used specifically for the NPL hull forms.

Consequently, this paper is directed towards mathematical representation of the resistance and dynamic trim specifically for the NPL series. Polynomials derived to describe $(R_T/\Delta)_{100000}$ and τ enable application of existing computer routines, thereby replacing manual calculations. Representation of dynamic trim is necessary for oblique flow propulsor (propeller) evaluation. This enables feasibility studies with power as an objective function (min P_d), rather than having to perform separate minimization of resistance and maximization of propulsor efficiency, as is usually done. This approach (min P_d) and its advantages are given in the Reference [4].

2. REGRESSION ANALYSIS TECHNIQUES - AN OVERVIEW

Regression analysis has been successfully used to analyze the resistance data for both, random hull forms, and methodical series. The equations based on random hull forms have broader applicability but are often not reliable. The reliability aspect of the model is very important, particularly for application in everyday problems when the correct value is not known in advance. The individual characteristics of each series, i.e. the secondary hull form parameters, can not be taken into consideration when several random hull forms are treated simultaneously. Secondary hull form parameters may be lost among primary parameters, even if many independent (explanatory) variables are introduced into the mathematical model. Therefore, it is often better to narrow the applicability to particular methodical series, and so to increase the reliability of the model. With this approach only the primary hull form parameters are considered, but the user, having in mind the subject hull, should be familiar with the other characteristics of the series which are not explicitly included in this mathematical model. The disadvantage of this approach is that multiple models are required, each for given methodical series or a group of very similar hull forms, instead of having only one model, or few models needed for random hull form approach.

Various resistance models were used, from those based on the wave theory to the ones using brute force regression analysis with exponential, logarithmic, polynomial or some other functions. It has been found that, when the number of terms is relatively high and when the cross-products and different powers of the independent variables are used, there is no need for function transformations. That is, the original polynomial form sufficed.

Reference [5] gives a very good summary of the history of use of regression equations for resistance evaluation. Two general types of regression equations for resistance evaluation have evolved:

- Speed-independent models, when separate equations are generated for a series of discrete speeds, since the speed is not included as an independent variable.
- Speed-dependent models, with vessel's speed included as an independent variable.

The advocates of speed-dependent models claim that the predicted resistance often does not vary properly with speed, since the resistance computed at one speed is not directly linked to that at another speed. This is because the speed variable is not explicitly included in the regression, as explained in, for example, References [5] and [6]. The accuracy of the speed-independent models is, believed to be, somewhat better since independent equations are developed for each speed.

Reference [7] offers a compromise solution that has been used by the first author on several occasions. Note that this method is similar to the approach given in Reference [8]. Essentially, with this method the speed-independent equations (with the same variables for whole speed range) are first developed. A second regression analysis is then performed with the regression coefficients cross-faired against speed (or Froude number). Thus, accurate speed-independent equations are obtained for discrete Froude numbers through the first step; the second step provides speed-dependent equation. Of course, either of equations may be used independently to estimate resistance.

An important and delicate part of this method is the formation of the "best subsets" from the initial (for all speeds the same) equation. The usual statistics (coefficient of determination, t-test or F-test, standard deviation, significance test for each variable etc.) were found to be insufficient and in fact were sometimes misleading. Therefore, trial and error technique was also used to define the best subsets for the whole speed range. Some variables, judged to be less significant were rejected deliberately, although a stepwise method was used throughout the analysis. The accuracy of the model, a primary goal of this analysis, was checked after every incremental step in the process was undertaken. It should be pointed out, however, that several very good, although dissimilar, models could be derived.

3. SURVEY OF THE MATHEMATICAL MODELS APPLIED TO HIGH SPEED CRAFT

The aforementioned Mercier and Savitsky [3] is the speed-independent mathematical model, while the one for the VTT series [2] is the speed-dependent model. Jin's speed-independent model, described in Reference [9], for calculation of residuary resistance of the round bilge displacement hulls over a speed range of F_{nL} = 0.4-1.0, should also be mentioned. Speed-independent and speed-dependent models are given in References [5] and [6], respectively for the high speed transom stern ships (F_{nL} = 0.15-0.9).

Reference [10] gives information about high speed displacement hull series AMECRC (which is an extension of MARIN series) and is in many respects similar to the NPL series. A nonlinear speed-independent method was used there, since it was proven to be better than the least square minimization used by multiple regression analysis. Unfortunately, mathematical model itself was not given.

Resistance and trim of planing hulls for $F_{n\nabla}$ = 1.0-3.0 can be estimated from the speed-independent and speed-dependent mathematical models given in Reference [11], while speed-independent model for resistance prediction for the hard chine catamaran hull series '89 ($F_{n\nabla}$ = 1.0-3.5) is given in the Reference [12].

For the sake of completeness, two more references concerning the NPL series should be mentioned. Reference [13] deals with the propulsion coefficients of the NPL series, which are necessary for power estimation. Reference [14] deals with the method for the determination of hydrostatic data and form stability characteristics for the NPL series, but doesn't have any connections to the regression analysis.

An excellent background and guidance to designers regarding the application of numerical methods through the use of commercially or individually developed computer software may be found in Reference [15].

4. DEVELOPMENT OF THE PREDICTION EQUATIONS

4.1. INITIAL MATHEMATICAL MODEL

Residuary resistance-displacement ratio (R_R/Δ [kN/t]) and running trim (τ [°]) data, drawn against (M) = $L/\nabla^{1/3}$ and $F_{n\nabla}$ are presented graphically in Reference [1] for five L/B groups of models (L/B = 3.33, 4.55, 5.41, 6.25 and 7.5). Model-size wetted area (L_m = 2.54 m) at rest is drawn as a function of (M) and L/B only. These diagrams were scanned and transformed to numerical form forming between 120 and 160 data points for each group of the L/B models. Breadth-draft ratio could now be calculated, since $B/T = (C_B \cdot (M)^3) / (L/B)^2$, ($C_B$ = 0.397 = const.).

Three principal hull form and loading parameters were chosen for further evaluation, covering the following range:

$$3.33 < L/B < 7.50 \qquad 4.50 < (M) < 8.30 \qquad 1.76 < B/T < 10.77.$$

These parameters were transformed into another set of variables with a range from -1 to +1. The new variables, which were subsequently used throughout the regression analysis, are:

$$x_1 = (L/B - 5.415) / 2.085$$

$$x_2 = ((M) - 6.4) / 1.9$$

$$x_3 = (B/T - 6.2615) / 4.5063.$$

$(R_T/\Delta)_{100000}$ (for Δ = 100,000 lb = 45.36 t), τ° and (S) = $S/\nabla^{2/3}$ were chosen as the dependent variables. Transformation from given residuary resistance and model-size wetted area format to the new format, that was used throughout the analysis, was done according to the flow chart given in Figure 1.

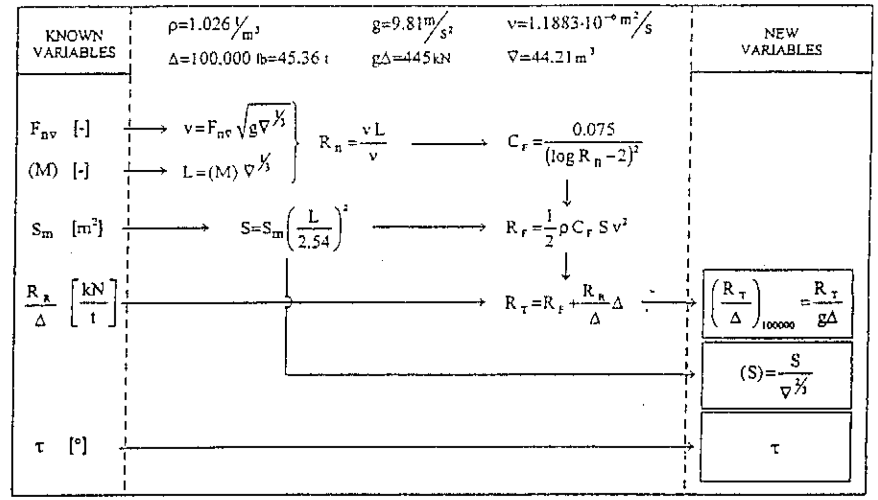


Fig. 1 Transformation of resistance and wetted area from one format to another

Polynomial form was chosen for all three mathematical models. The initial polynomial equation, which was subsequently used for the speed-independent least square curve fitting, had 27 terms, i.e.

$$(R_T/\Delta)_{100000},\ \tau^\circ,\ (S) = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_1^2 + a_5x_2^2 + a_6x_3^2 + a_7x_1x_2 + a_8x_1x_3 + a_9x_2x_3 + a_{10}x_1^2x_2 +$$

$$+ a_{11}x_1^2x_3 + a_{12}x_2^2x_1 + a_{13}x_2^2x_3 + a_{14}x_3^2x_1 + a_{15}x_3^2x_2 + a_{16}x_1^3 + a_{17}x_2^3 + a_{18}x_3^3 + a_{19}x_1^3x_2 +$$

$$+ a_{20}x_1^3x_3 + a_{21}x_2^3x_1 + a_{22}x_2^3x_3 + a_{23}x_3^3x_1 + a_{24}x_3^3x_2 + a_{25}x_1^2x_2^2 + a_{26}x_1^2x_3^2 + a_{27}x_2^2x_3^2.$$

From now on $x_4 = x_1^2$, $x_5 = x_2^2$ ... $x_{15} = x_3^2x_2$ ... $x_{27} = x_2^2x_3^2$.

4.2. SPEED-INDEPENDENT MATHEMATICAL MODELS

The 27 term polynomial equation, defined above, was the starting point for all calculations presented in this paper, i.e. for the 12 speed-independent resistance and dynamic trim equations (one for each $F_{n\nabla}$, $F_{n\nabla}$ = 0.8, 1.0, 1.2 ... 3.0), and 1 wetted surface coefficient equation (which doesn't depend on speed). All further statistical calculations were done with a program Statistica 5.0 (StatSoft Inc.) and specifically the subroutine for forward stepwise regression.

Several models, actually more than 600 instances concerning the resistance alone, were obtained following the procedures which have been briefly described above. However, representation for the lower Froude numbers was always relatively poor, so $F_{n\nabla}$ = 0.6 was rejected from further consideration. Therefore, speed-independent models are valid for $F_{n\nabla}$ range between 0.8 and 3.0, while speed-dependent models are valid for $F_{n\nabla}$ = 1.0-3.0, due to instability between $F_{n\nabla}$ = 0.8-1.0.

The final speed-independent resistance, dynamic trim and wetted surface polynomial terms, regression coefficients and control information are given in the Appendix 1, Tables 1, 2 and 3, respectively for $(R_T/\Delta)_{100000}$ ("a" regression coefficients), τ^{o} ("b" regression coefficients) and (S) ("c" regression coefficients).

4.3. SPEED-DEPENDENT MATHEMATICAL MODELS

The speed-dependent models for resistance and dynamic trim were developed through cross-faring of resistance and dynamic trim regression coefficients against $F_{n\nabla}$, i.e. through an evaluation of $a_i = f\ (F_{n\nabla})$ and $b_i = f\ (F_{n\nabla})$. Different powers of $F_{n\nabla}$ were tried, and satisfactory results were obtained when $F_{n\nabla}$ was raised to sixth power for lower $F_{n\nabla}$ and eight power for higher values of $F_{n\nabla}$. It was felt that higher Froude numbers are more important, so both $a_i = f\ (F_{n\nabla})$ and $b_i = f\ (F_{n\nabla})$ are eight order polynomial equations.

Of course, transformation of $F_{n\nabla}$ was introduced as well, so the new $\varphi_{n\nabla}$ variable, ranging from -1 to +1, was subsequently used throughout the speed-dependent mathematical models

$$\varphi_{n\nabla} = (F_{n\nabla} - 1.8) / 1.2.$$

New regression coefficients together with their control information are given in the Appendix 2, Tables 4 and 5 respectively for resistance ("α" regression coefficients) and dynamic trim ("β" regression coefficients). The final speed-dependent models for resistance and dynamic trim have the following form:

$$\left(\frac{R_T}{\Delta}\right)_{100000} = \sum_{i=0}^{8}\alpha_i\,\varphi_{n\nabla_i} + \left(\sum_{i=0}^{8}\alpha_{i+9}\,\varphi_{n\nabla_i}\right)x_1 + \quad \cdots \quad + \left(\sum_{i=0}^{8}\alpha_{i+117}\,\varphi_{n\nabla_i}\right)x_{27}$$

$$\tau^{o} = \sum_{i=0}^{8}\beta_i\,\varphi_{n\nabla_i} + \left(\sum_{i=0}^{8}\beta_{i+9}\,\varphi_{n\nabla_i}\right)x_2 + \quad \cdots \quad + \left(\sum_{i=0}^{8}\beta_{i+126}\,\varphi_{n\nabla_i}\right)x_{27}.$$

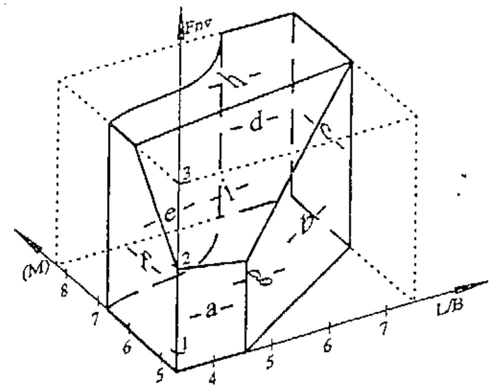
If the displacement of the new design (subject hull) is approximately 45 t, than there is no need for wetted area evaluation. Otherwise the wetted area coefficient (obtained in the previous step, Table 3) can be evaluated from the following equation:

$$(S) = c_0 + c_1 x_1 + \cdots + c_{26} x_{26}.$$

The step-by-step calculation procedure for arbitrary size of the subject hull is given in the Reference [11], for instance.

4.4. BOUNDARIES OF APPLICABILITY

Boundaries of applicability are very important part of the mathematical model, since "diligent but stupid" computers are intended to be used. The mathematical model presented should only be used to predict the performance of a new hull whose characteristics (secondary hull form parameters as well) are similar to the data underlying its derivation, i.e. to the NPL series. The distribution of principal parameters should be as shown in Figure 2 (feasible parameters should be inside the odd-shaped box). All nine boundaries (surfaces) are relatively simple and are equations of inequality type. This approach enables application of nonlinear constrained optimization routines, as is discussed in the Reference [4].



-a-	$(M) \geq 4.5$	-f-	$(L/B) \geq 3.33$
-b-	$(M) \geq 0.6757\cdot(L/B) + 1.4324$	-g-	$F_{n\nabla} \geq 0.8$
-c-	$(L/B) \leq 7.5$	-h-	$F_{n\nabla} \leq 3.0$
-d-	$(M) \leq 8.3$	-i-	$F_{n\nabla} \leq 0.714\cdot(M) - 0.103\cdot(L/B) - 0.872$
-e-	$(M) \leq 0.1534\cdot(L/B)^3 - 1.9906\cdot(L/B)^2 + 8.732\cdot(L/B) - 5.9683$		

Fig. 2 Boundaries of applicability of mathematical models

5. ACCURACY OF THE PREDICTED MATHEMATICAL MODELS

The quality of the models, i.e. important statistics like standard error, coefficient of determination, F-test etc. are given for each regression fitted equation and are shown in the Tables 1 to 5. These statistics are not important for the end user, but are good measures of the model validity. The degree of agreement for resistance and dynamic trim, between the NPL series and the results obtained by the proposed models, are illustrated in the Figures 3 to 8 in the Appendix 3. The comparison is satisfactory even for speed-dependent models.

The discrepancies for all speed-independent and speed-dependent models are summarized in the Table 6, Appendix 3. For instance, for towing power and $\Delta = 45.36$ t out of 678 data points used in the model derivation, only 6 low-Froude-number-points ($F_{n\nabla} = 0.8$ and 1.0) "jumped over" 5% error barrier, none of them above 7%.

A very important part of the accuracy checking is identification of any instabilities which may occur between points used in model derivation. Therefore, an examination of the intermediate values of principal parameters, which were not employed in the development of the mathematical model, is necessary; see for instance Figures 9 to 12, Appendix 4.

6. CONCLUDING REMARKS

For the well-known NPL series, three groups of mathematical models were derived, speed-independent and speed-dependent models for $(R_T/\Delta)_{100000}$ and τ, and one for (S) evaluation. They can be used together with other available performance prediction methods as given in, for instance, References [2] through [6] and [9] through [12]. The validity of the model across a relatively wide speed range should be pointed out ($F_{n\nabla}$ = 0.8-3.0), especially in the relation to the currently available models presented in the References [3] and [2], valid for $F_{n\nabla}$ = 1.0-2.0 and $F_{n\nabla}$ = 1.8-3.2, respectively.

Mathematical models are presented in the usual format which facilitates the application of existing computer routines. This was one of the original goals of this study. Resistance and trim equations enable power performance evaluation as given in Reference [4].

Moreover, it should be noted that tank testing is rarely used in design of the small craft due to the high price of the tests in relation to the cost of the vessel. However, numerical towing tank performance predictions, whose mathematical models for resistance and trim evaluations for the NPL series are presented here, are often used in all design phases. The mathematical models used for small craft performance evaluations, therefore, need to be both reliable and accurate. Both, speed-independent and speed-dependent models presented here meet these criteria.

NOMENCLATURE

A_T	m^2	Transom area
A_X	m^2	Maximum section area
B	m	Breadth (beam) of hull on DWL
C_B	-	Block coefficient
C_F	-	Frictional resistance coefficient
C_R	-	Residuary resistance coefficient
DWL	-	Designed waterline
F_{nL}	-	Froude number $(v/\sqrt{gL})$
$F_{n\nabla}$	-	Volumetric Froude number $(v/\sqrt{g\nabla^{1/3}})$
g	m/s^2	Acceleration of gravity (9.81)
L	m	Length on DWL
LCB	%L	Longitudinal centre of buoyancy
(M)	-	Length-displacement ratio $(L/\nabla^{1/3})$
P_d	kW	Power delivered to the propeller
R_n	-	Reynolds number (vL/ν)
R_F	kN	Frictional resistance
R_R	kN	Residuary resistance
R_T	kN	Total resistance of bare hull
S	m^2	Wetted surface
(S)	-	Wetted surface coefficient $(S/\nabla^{2/3})$
T	m	Draught at DWL
v	m/s	Speed
Δ	t	Displacement mass
∇	m^3	Displacement volume
ρ	t/m^3	Mass density of salt water (1.026)
ν	m^2/s	Kinematic viscosity of salt water ($1.1883\cdot10^{-6}$ at 15°C)
τ	°	Dynamic (running) trim angle

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APPENDIX 1 - SPEED-INDEPENDENT MODELS

$$\left(\frac{R_T}{\Delta}\right)_{100000} = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_5x_5 + a_{12}x_{12} + a_{13}x_{13} + a_{16}x_{16} + a_{19}x_{19} + a_{20}x_{20} + a_{22}x_{22} + a_{23}x_{23} + a_{26}x_{26} + a_{27}x_{27}$$

	$F_{n\nabla}$											
a_i	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
a_0	0.012677	0.031092	0.057789	0.070863	0.080384	0.092692	0.105658	0.113350	0.118892	0.123105	0.120859	0.115058
a_1	-0.008102	0.000000	0.008445	0.004134	0.000962	0.005733	0.010769	0.007299	-0.001703	-0.014977	-0.041899	-0.077104
a_2	0.000061	-0.014184	-0.042841	-0.044967	-0.041259	-0.043200	-0.046379	-0.042913	-0.039950	-0.034003	-0.017179	0.005255
a_3	-0.002725	0.007983	0.015331	0.013614	0.014880	0.022656	0.031878	0.031266	0.023441	0.008990	-0.022888	-0.067378
a_5	-0.001185	0.002904	0.010337	0.013687	0.009803	0.008203	0.007646	0.013292	0.026694	0.043975	0.057718	0.071086
a_{12}	0.004254	-0.003601	0.000000	0.005111	0.012468	0.012205	0.010466	0.000922	-0.018860	-0.043354	-0.046133	-0.048406
a_{13}	-0.025702	0.000000	-0.000655	-0.013032	-0.038512	-0.043373	-0.050962	-0.050318	-0.036356	0.000000	0.051148	0.057769
a_{16}	0.001395	0.000062	0.000000	0.002578	0.004118	0.005143	0.006964	0.008068	0.009177	0.009824	0.009804	0.008532
a_{19}	0.004913	0.005626	0.004361	0.004098	0.004899	0.004763	0.010023	0.012955	0.012276	0.006514	-0.008769	-0.034102
a_{20}	-0.005901	0.002059	0.010917	0.001183	0.000667	0.002649	0.002493	-0.003659	-0.018867	-0.033644	-0.057179	-0.101116
a_{22}	0.015476	0.000000	-0.003984	0.011616	0.030670	0.033934	0.044572	0.047574	0.025806	-0.015288	-0.047272	-0.062657
a_{23}	-0.008887	0.002059	0.006095	0.001874	-0.004834	-0.006093	-0.009346	-0.010929	-0.015162	-0.020789	-0.019478	-0.051664
a_{26}	-0.008427	0.005375	0.016608	0.006989	0.001094	-0.001804	-0.010127	-0.017531	-0.033868	-0.050023	-0.060285	-0.118198
a_{27}	-0.033521	0.010811	0.008090	-0.023857	-0.057101	-0.064610	-0.081529	-0.080688	-0.063470	-0.014102	0.040157	0.073258
R^2	0.9847	0.9950	0.9977	0.9966	0.9952	0.9944	0.9931	0.9946	0.9952	0.9902	0.9952	0.9985
F-test	232.5	1001.5	1963.0	1054.9	741.9	647.8	519.1	636.2	665.3	319.9	525.9	1209.7
Std. err.	0.00045	0.00078	0.00109	0.00136	0.00154	0.00153	0.00168	0.00151	0.00140	0.00209	0.00142	0.00084
Number of cases	61	61	61	61	61	61	61	59	56	51	47	38

Table 1 Resistance polynomial terms, regression coefficients and important statistics

$$\tau^\circ = b_0 + b_2x_2 + b_7x_7 + b_9x_9 + b_{10}x_{10} + b_{11}x_{11} + b_{14}x_{14} + b_{18}x_{18} + b_{19}x_{19} + b_{20}x_{20} + b_{23}x_{23} + b_{24}x_{24} + b_{25}x_{25} + b_{26}x_{26} + b_{27}x_{27}$$

	$F_{n\nabla}$											
b_i	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
b_0	0.175770	0.425175	0.883371	1.586455	1.899628	2.026007	2.058107	2.054187	2.069412	2.139490	2.357645	2.804328
b_2	-0.305797	-0.267412	-0.418518	-0.925061	-0.732979	-0.429611	-0.236105	-0.159138	-0.247707	-0.519239	-1.217875	-2.230391
b_7	0.320664	0.041130	0.000000	0.042296	-0.272631	-0.472295	-0.567934	-0.598107	-0.404230	0.067575	1.151239	2.370417
b_9	0.176390	0.241145	0.000000	-0.084784	-0.058048	0.161832	0.268277	0.368525	0.669915	1.201438	1.930230	2.947236
b_{10}	-0.168558	-0.934109	-2.238618	-1.883672	-1.591190	-1.353073	-0.947598	-0.708352	-0.347415	0.176509	1.221190	2.742371
b_{11}	-0.103548	-0.954868	-2.517245	-2.251547	-1.890948	-1.075607	-0.193149	0.622003	1.787335	3.090234	5.035156	6.999101
b_{14}	0.349273	-0.937388	-4.491429	-4.304975	-3.724361	-2.552354	-1.283559	-0.064981	1.761040	3.810824	7.157361	11.191026
b_{18}	0.326051	-0.038224	-1.623344	-1.807736	-1.671105	-1.377836	-0.998759	-0.527308	0.283465	1.353872	3.053972	5.423357
b_{19}	-1.040794	-0.806155	-0.703548	-1.350820	-0.537518	0.572009	1.628775	2.391011	3.030234	3.601990	3.710648	3.494198
b_{20}	-0.383581	-1.348226	-2.781868	-2.624578	-2.221716	-1.349497	-0.344831	0.462487	1.541927	2.711225	4.260309	5.970641
b_{23}	-0.078395	-0.557945	-2.635119	-2.260988	-1.924576	-1.229068	-0.401107	0.224152	1.045894	1.831750	3.596974	5.946926
b_{24}	-0.358077	-0.434711	-1.523450	-2.023459	-1.530763	-1.018361	-0.408611	-0.118446	0.089195	0.194697	0.643298	1.373248
b_{25}	0.794836	0.941649	1.417955	1.829837	1.260767	0.440716	-0.394656	-0.923245	-1.456637	-2.136660	-2.931875	-3.776398
b_{26}	-0.342870	-1.516333	-4.571710	-4.392841	-3.747926	-2.234404	-0.422111	1.016714	2.912691	4.895521	8.005211	11.584463
b_{27}	0.467308	0.865865	1.315406	1.042975	0.496254	-0.048087	-0.516796	-0.524839	-0.360649	-0.141175	-0.179117	-0.382556
R^2	0.9702	0.9656	0.9816	0.9858	0.9866	0.9881	0.9916	0.9938	0.9955	0.9965	0.9965	0.9948
F-test	83.6	72.1	169.2	178.4	188.7	212.8	305.3	414.3	564.7	734.0	741.9	496.4
Std. err.	0.0192	0.0813	0.1397	0.1386	0.1186	0.0935	0.0687	0.0569	0.0541	0.0573	0.0701	0.1038
Number of cases	51	51	51	51	51	51	51	51	51	51	51	51

Table 2 Dynamic trim polynomial terms, regression coefficients and important statistics

APPENDIX 2 - SPEED-DEPENDENT MODELS

$$\left(\frac{R_T}{\Delta}\right)_{100000} = \sum_{i=0}^{8}\alpha_i\varphi_{n\nabla i} + \left(\sum_{i=0}^{8}\alpha_{i+9}\varphi_{n\nabla i}\right)x_1 + \left(\sum_{i=0}^{8}\alpha_{i+18}\varphi_{n\nabla i}\right)x_2 + \left(\sum_{i=0}^{8}\alpha_{i+27}\varphi_{n\nabla i}\right)x_3 + \left(\sum_{i=0}^{8}\alpha_{i+36}\varphi_{n\nabla i}\right)x_5 + \left(\sum_{i=0}^{8}\alpha_{i+45}\varphi_{n\nabla i}\right)x_{12} + \left(\sum_{i=0}^{8}\alpha_{i+54}\varphi_{n\nabla i}\right)x_{13} +$$
$$+\left(\sum_{i=0}^{8}\alpha_{i+63}\varphi_{n\nabla i}\right)x_{16} + \left(\sum_{i=0}^{8}\alpha_{i+72}\varphi_{n\nabla i}\right)x_{19} + \left(\sum_{i=0}^{8}\alpha_{i+81}\varphi_{n\nabla i}\right)x_{20} + \left(\sum_{i=0}^{8}\alpha_{i+90}\varphi_{n\nabla i}\right)x_{22} + \left(\sum_{i=0}^{8}\alpha_{i+99}\varphi_{n\nabla i}\right)x_{23} + \left(\sum_{i=0}^{8}\alpha_{i+108}\varphi_{n\nabla i}\right)x_{26} + \left(\sum_{i=0}^{8}\alpha_{i+117}\varphi_{n\nabla i}\right)x_{27}$$

$\varphi_{n\nabla 0}=1$		$\varphi_{n\nabla 1}=\varphi_{n\nabla}$		$\varphi_{n\nabla 2}=\varphi_{n\nabla}^2$		$\varphi_{n\nabla 3}=\varphi_{n\nabla}^3$		$\varphi_{n\nabla 4}=\varphi_{n\nabla}^4$		$\varphi_{n\nabla 5}=\varphi_{n\nabla}^5$		$\varphi_{n\nabla 6}=\varphi_{n\nabla}^6$		$\varphi_{n\nabla 7}=\varphi_{n\nabla}^7$		$\varphi_{n\nabla 8}=\varphi_{n\nabla}^8$		R^2	F-test	Std. error	Number of cases
α_0	0.093013	α_1	0.078344	α_2	-0.009610	α_3	-0.198929	α_4	0.053610	α_5	0.640899	α_6	-0.468529	α_7	-0.553399	α_8	0.476649	0.9999	8740.0	0.00047	12
α_9	0.006121	α_{10}	0.036347	α_{11}	-0.019097	α_{12}	-0.364107	α_{13}	0.187922	α_{14}	0.881361	α_{15}	-0.805230	α_{16}	-0.683057	α_{17}	0.672619	0.9998	1962.1	0.00068	12
α_{18}	-0.043443	α_{19}	-0.021043	α_{20}	-0.001592	α_{21}	0.317451	α_{22}	-0.138387	α_{23}	-1.128385	α_{24}	0.051995	α_{25}	0.995728	α_{26}	-0.829093	0.9997	1136.1	0.00065	12
α_{27}	0.023486	α_{28}	0.057879	α_{29}	-0.023726	α_{30}	-0.348106	α_{31}	0.178199	α_{32}	0.737798	α_{33}	-0.707577	α_{34}	-0.548689	α_{35}	0.562535	0.9997	1268.3	0.00091	12
α_{36}	0.007789	α_{37}	-0.012854	α_{38}	0.048101	α_{39}	0.128945	α_{40}	0.017943	α_{41}	-0.038860	α_{42}	-0.182194	α_{43}	-0.067477	α_{44}	0.169689	0.9997	1138.5	0.00081	12
α_{45}	0.012387	α_{46}	0.000699	α_{47}	-0.041454	α_{48}	-0.078758	α_{49}	-0.411017	α_{50}	-0.020603	α_{51}	1.004690	α_{52}	0.100288	α_{53}	-0.614613	0.9989	337.9	0.00150	12
α_{54}	-0.045620	α_{55}	-0.043439	α_{56}	0.100568	α_{57}	-0.136304	α_{58}	0.117069	α_{59}	0.787725	α_{60}	-0.349993	α_{61}	-0.584287	α_{62}	0.212039	0.9980	184.2	0.00316	12
α_{63}	0.005396	α_{64}	0.006646	α_{65}	0.001613	α_{66}	0.027995	α_{67}	-0.038633	α_{68}	-0.088732	α_{69}	0.106999	α_{70}	0.065021	α_{71}	-0.077782	0.9979	175.6	0.00033	12
α_{72}	0.005845	α_{73}	0.015480	α_{74}	0.037527	α_{75}	-0.014682	α_{76}	-0.141940	α_{77}	-0.070899	α_{78}	0.174299	α_{79}	0.062046	α_{80}	-0.101799	0.9985	247.4	0.00095	12
α_{81}	0.002700	α_{82}	0.023412	α_{83}	-0.052987	α_{84}	-0.415022	α_{85}	0.269027	α_{86}	1.022733	α_{87}	-0.841008	α_{88}	-0.773952	α_{89}	0.683990	0.9994	588.2	0.00158	12
α_{90}	0.035937	α_{91}	0.044983	α_{92}	0.032879	α_{93}	0.147806	α_{94}	-0.993943	α_{95}	-1.042473	α_{96}	2.219790	α_{97}	0.948219	α_{98}	-1.455871	0.9987	289.4	0.00239	12
α_{99}	-0.007521	α_{100}	-0.010052	α_{101}	0.052718	α_{102}	-0.110748	α_{103}	-0.257833	α_{104}	0.301729	α_{105}	0.394981	α_{106}	-0.194182	α_{107}	-0.220723	0.9980	186.6	0.00130	12
α_{108}	-0.003859	α_{109}	-0.016928	α_{110}	0.006589	α_{111}	-0.297305	α_{112}	-0.143828	α_{113}	0.608868	α_{114}	0.059827	α_{115}	-0.591199	α_{116}	0.059515	0.9988	321.5	0.00249	12
α_{117}	-0.068471	α_{118}	-0.066821	α_{119}	0.061252	α_{120}	-0.300738	α_{121}	0.829897	α_{122}	1.379352	α_{123}	-1.668275	α_{124}	-1.031222	α_{125}	1.236290	0.9995	245.7	0.00389	12

Table 4 Resistance regression coefficients and their important statistics

$$(S)=c_0+c_1X_1+c_2X_2+c_3X_3+c_5X_5+c_8X_8+c_{10}X_{10}+c_{15}X_{15}+c_{18}X_{18}+c_{19}X_{19}+c_{22}X_{22}+c_{24}X_{24}+c_{26}X_{26}$$

c_0	6.699562
c_1	-2.538538
c_2	3.615313
c_3	0.513948
c_5	0.071497
c_8	1.172089
c_{10}	-0.427145
c_{15}	2.249989
c_{18}	-1.769779
c_{19}	-0.125797
c_{22}	-0.631931
c_{24}	1.501974
c_{26}	0.342678

R^2	0.9999
F-test	29467.7
Std. err.	0.0143
Number of cases	61

Table 3 Wetted surface polynomial terms, regression coefficients and important statistics

$$\tau^{o}=\sum_{i=0}^{8}\beta_i\varphi_{n\nabla i}+\left(\sum_{i=0}^{8}\beta_{i+9}\varphi_{n\nabla i}\right)X_2+\left(\sum_{i=0}^{8}\beta_{i+18}\varphi_{n\nabla i}\right)X_7+\left(\sum_{i=0}^{8}\beta_{i+27}\varphi_{n\nabla i}\right)X_9+\left(\sum_{i=0}^{8}\beta_{i+36}\varphi_{n\nabla i}\right)X_{10}+\left(\sum_{i=0}^{8}\beta_{i+45}\varphi_{n\nabla i}\right)X_{11}+\left(\sum_{i=0}^{8}\beta_{i+54}\varphi_{n\nabla i}\right)X_{14}+\left(\sum_{i=0}^{8}\beta_{i+63}\varphi_{n\nabla i}\right)X_{18}+$$
$$+\left(\sum_{i=0}^{8}\beta_{i+72}\varphi_{n\nabla i}\right)X_{19}+\left(\sum_{i=0}^{8}\beta_{i+81}\varphi_{n\nabla i}\right)X_{20}+\left(\sum_{i=0}^{8}\beta_{i+90}\varphi_{n\nabla i}\right)X_{23}+\left(\sum_{i=0}^{8}\beta_{i+99}\varphi_{n\nabla i}\right)X_{24}+\left(\sum_{i=0}^{8}\beta_{i+108}\varphi_{n\nabla i}\right)X_{25}+\left(\sum_{i=0}^{8}\beta_{i+117}\varphi_{n\nabla i}\right)X_{26}+\left(\sum_{i=0}^{8}\beta_{i+126}\varphi_{n\nabla i}\right)X_{27}$$

$\varphi_{n\nabla 0}=1$		$\varphi_{n\nabla 1}=\varphi_{n\nabla}$		$\varphi_{n\nabla 2}=\varphi_{n\nabla}^2$		$\varphi_{n\nabla 3}=\varphi_{n\nabla}^3$		$\varphi_{n\nabla 4}=\varphi_{n\nabla}^4$		$\varphi_{n\nabla 5}=\varphi_{n\nabla}^5$		$\varphi_{n\nabla 6}=\varphi_{n\nabla}^6$		$\varphi_{n\nabla 7}=\varphi_{n\nabla}^7$		$\varphi_{n\nabla 8}=\varphi_{n\nabla}^8$		R^2	F-test	Std. error	Number of cases
β_0	2.033088	β_1	0.127322	β_2	-1.698102	β_3	7.385583	β_4	-5.298216	β_5	-15.392440	β_6	17.085345	β_7	10.387316	β_8	-11.826407	0.9993	565.3	0.0394	12
β_9	-0.445368	β_{10}	2.163424	β_{11}	-2.043204	β_{12}	-13.114889	β_{13}	17.699598	β_{14}	25.430458	β_{15}	-40.013487	β_{16}	-17.534173	β_{17}	25.628880	0.9950	74.2	0.0605	12
β_{18}	-0.474875	β_{19}	-1.209746	β_{20}	2.488439	β_{21}	3.532062	β_{22}	-8.678590	β_{23}	-1.985603	β_{24}	19.995371	β_{25}	0.638182	β_{26}	-10.933303	0.9990	384.6	0.0508	12
β_{27}	0.143585	β_{28}	0.932906	β_{29}	-1.156282	β_{30}	-2.255990	β_{31}	11.399220	β_{32}	4.873652	β_{33}	-16.820768	β_{34}	-2.129468	β_{35}	7.680419	0.9997	1358.4	0.0206	12
β_{36}	-1.313384	β_{37}	1.397767	β_{38}	-0.172147	β_{39}	10.077473	β_{40}	-5.904343	β_{41}	-43.222770	β_{42}	42.104698	β_{43}	39.686211	β_{44}	-39.913842	0.9973	136.9	0.1402	12
β_{45}	-1.074402	β_{46}	4.285762	β_{47}	1.742878	β_{48}	7.116343	β_{49}	-1.468058	β_{50}	-38.316588	β_{51}	35.581547	β_{52}	38.278200	β_{53}	-38.148097	0.9994	607.5	0.1423	12
β_{54}	-2.538026	β_{55}	5.789688	β_{56}	1.867719	β_{57}	19.736459	β_{58}	-1.220753	β_{59}	-97.746012	β_{60}	78.168134	β_{61}	94.757031	β_{62}	-85.623977	0.9993	569.9	0.2349	12
β_{63}	-1.365006	β_{64}	1.504390	β_{65}	0.933899	β_{66}	9.069458	β_{67}	3.835651	β_{68}	-40.422098	β_{69}	23.518788	β_{70}	39.803665	β_{71}	-31.454296	0.9998	1555.6	0.0647	12
β_{72}	0.569334	β_{73}	7.844968	β_{74}	-2.930311	β_{75}	-28.830351	β_{76}	37.833001	β_{77}	61.195760	β_{78}	-84.867294	β_{79}	-43.279656	β_{80}	56.181719	0.9984	240.1	0.1527	12
β_{81}	-1.323953	β_{82}	5.036428	β_{83}	1.245051	β_{84}	1.417878	β_{85}	4.007709	β_{86}	-24.916260	β_{87}	17.637162	β_{88}	26.233226	β_{89}	-23.368297	0.9994	667.4	0.1268	12
β_{90}	-1.200749	β_{91}	3.403904	β_{92}	1.653146	β_{93}	13.498226	β_{94}	-13.727450	β_{95}	-88.020821	β_{96}	72.651723	β_{97}	66.400207	β_{98}	-68.913814	0.0985	246.6	0.1675	12
β_{99}	-0.977243	β_{100}	3.779033	β_{101}	-1.710287	β_{102}	-8.874362	β_{103}	12.493018	β_{104}	-1.835102	β_{105}	-6.622121	β_{106}	11.688065	β_{107}	-6.468723	0.9984	239.8	0.0738	12
β_{108}	0.419815	β_{109}	-5.817447	β_{110}	2.168412	β_{111}	17.290572	β_{112}	-24.321826	β_{113}	-29.826274	β_{114}	42.437218	β_{115}	17.202880	β_{116}	23.530578	0.9987	1194.7	0.0624	12
β_{117}	-2.187318	β_{118}	6.724710	β_{119}	3.483912	β_{120}	6.944942	β_{121}	-1.839095	β_{122}	-63.179692	β_{123}	59.375899	β_{124}	67.577259	β_{125}	-66.469013	0.9993	568.6	0.2494	12
β_{126}	-0.087033	β_{127}	-3.018830	β_{128}	3.801864	β_{129}	4.991400	β_{130}	-5.547206	β_{131}	3.758720	β_{132}	-5.683562	β_{133}	-7.913331	β_{134}	9.316142	0.9986	277.4	0.0453	12

Table 5 Dynamic trim regression coefficients and their important statistics

APPENDIX 3 - THE DEGREE OF AGREEMENT FOR THE DATA EMPLOYED IN THE DEVELOPMENT OF THE MODELS

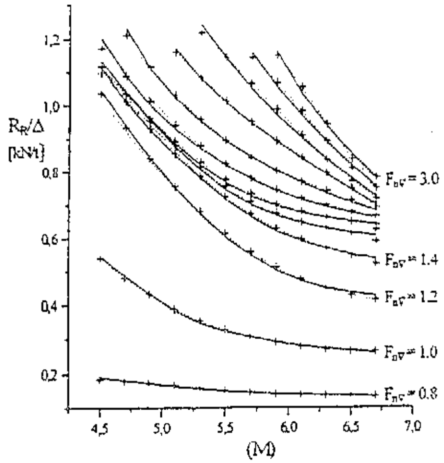

Fig. 3 $R_R/\Delta = f(F_{n\nabla},(M))$ for $L/B = 3.33$

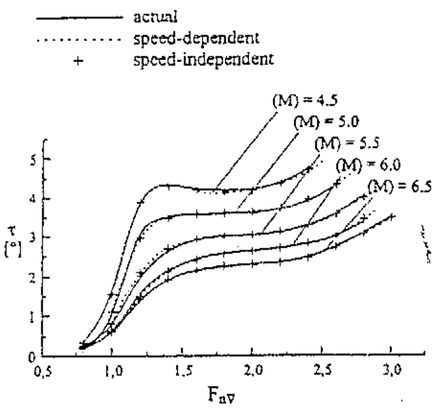

Fig. 4 $\tau = f(F_{n\nabla},(M))$ for $L/B = 3.33$

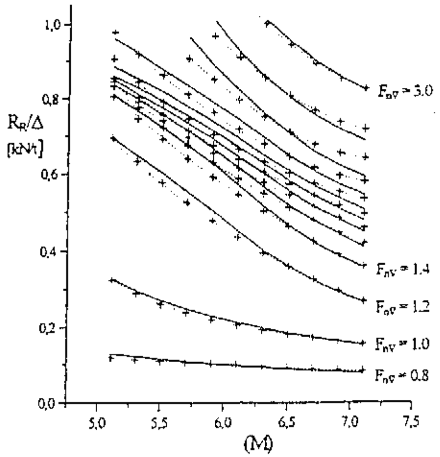

Fig. 5 $R_R/\Delta = f(F_{n\nabla},(M))$ for $L/B = 5.41$

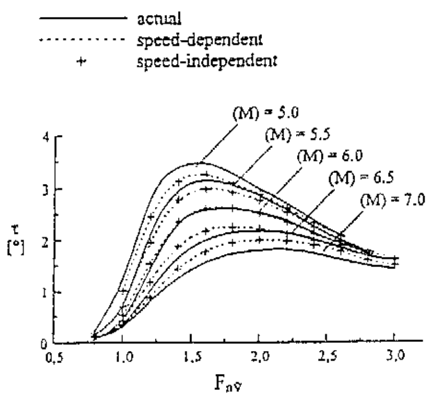

Fig. 6 $\tau = f(F_{n\nabla},(M))$ for $L/B = 5.41$

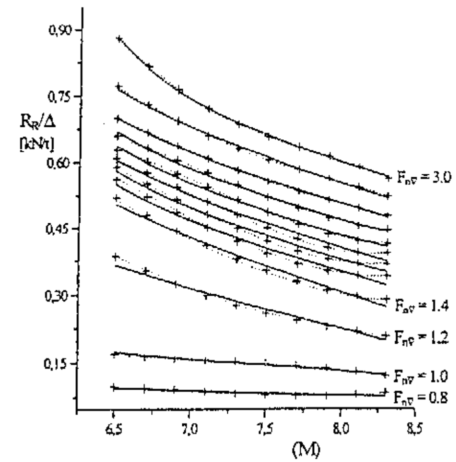

Fig. 7 $R_R/\Delta = f(F_{n\nabla},(M))$ for $L/B = 7.50$

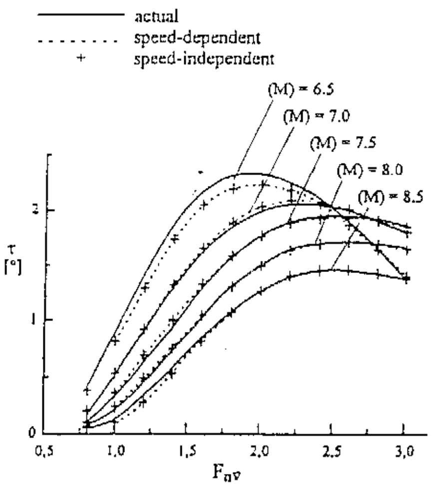

Fig. 8 $\tau = f(F_{n\nabla},(M))$ for $L/B = 7.50$

		Towing power					Dynamic trim						Wetted surface coefficient	
L/B		3.33	4.54	5.41	6.25	7.50	L/B	3.33	4.54	5.41	6.25	7.50		
Number of errors between 3 to 5 %	s-i	1	9	33	6	5	Number of errors between 0.2 to 0.3° s-i	-	1	10	3	-	Number of errors between 0.2 to 0.3 %	13
	s-d	1	10	32	4	5	s-d	-	1	10	4	-		
Number of errors above 5 %	s-i	-	1	2	-	1	Number of errors above 0.3° s-i	-	-	1	1	-	Number of errors above 0.3 %	6
	s-d	-	1	2	2	1	s-d	-	-	1	1	-		
Number of observations		123	152	119	164	120	Number of observations	108	132	108	156	108	Number of observations	61

Table 6 Discrepancies between calculated data and data given in Reference [1]
(s-i - speed-independent s-d - speed-dependent)