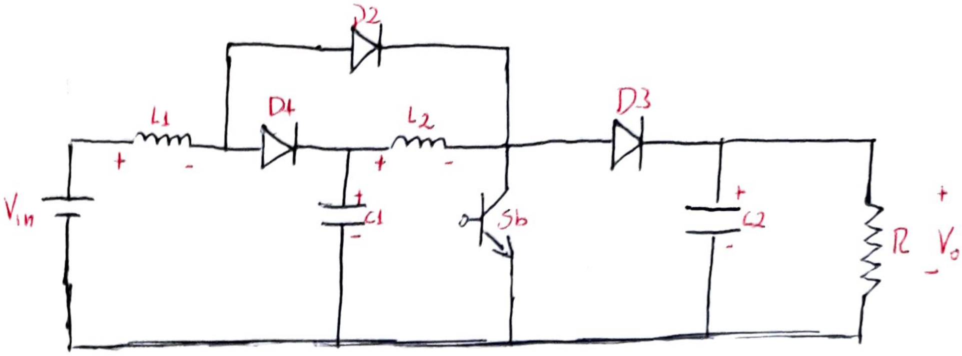
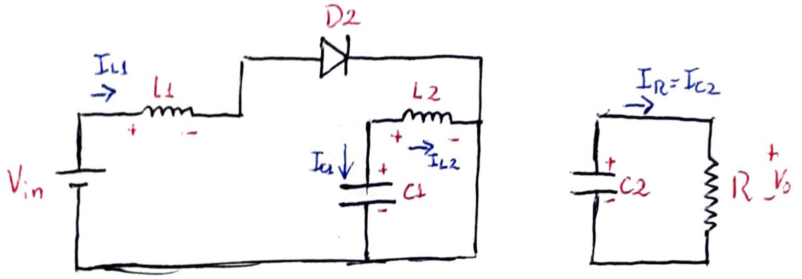


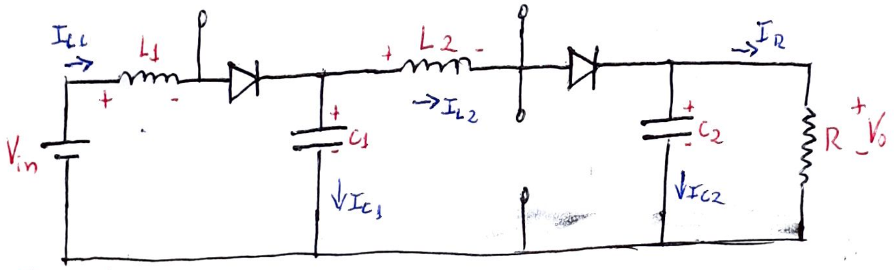
Chave ON



$V_{L1} = V_{in}$

$V_{L2} = V_{C1} - V_{in} + V_{L1} = V_{C1}$

Chave OFF



$V_{L1} = V_{in} - V_{C1}$

$V_{L2} = V_{C1} - V_o$

Encontrando D

Logo.

$$V_{L1} = D V_{in} + (1-D)(V_{in} - V_{C1}) = 0$$

$$V_{L2} = D V_{C1} + (1-D)(V_{C1} - V_o) = 0$$

P/ V_{L1}

$$D V_{in} = -(1-D)(V_{in} - V_{C1})$$

$$D V_{in} = (1-D) V_{C1} - (1-D) V_{in}$$

$$D V_{in} + (1-D) V_{in} = (1-D) V_{C1}$$

$$V_{in}(D + 1 - D) = (1-D) V_{C1}$$

$$V_{C1} = \frac{V_{in}}{1-D}$$

P/ V_{L2}

$$D V_{C1} = -(1-D)(V_{C1} - V_o)$$

$$D V_{C1} = (1-D) V_o - (1-D) V_{C1}$$

$$D V_{C1} + (1-D) V_{C1} = (1-D) V_o$$

$$V_{C1}(D + 1 - D) = (1-D) V_o$$

$$V_{C1} = (1-D) V_o$$

Igualando as expressões:

$$\frac{V_{in}}{1-D} = (1-D)V_o \rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{1}{(1-D)^2}}$$

Encontrando L_1, L_2, ΔI_{L1} e ΔI_{L2}

Os indutores são carregados quando a chave esta ON. Logo:

P/ V_{L1} temos

$$V_{L1} = L_1 \frac{dI_{L1}}{dt} = V_{in} = L_1 \frac{\Delta I_{L1}}{DT_{ch}} \Rightarrow \boxed{L_1 = \frac{V_{in} D}{\Delta I_{L1} \cdot f_{ch}} \Rightarrow \Delta I_{L1} = \frac{V_{in} D}{L_1 f_{ch}}}$$

P/ V_{L2} temos:

$$V_{L2} = L_2 \frac{dI_{L2}}{dt} = V_{C1} = \frac{V_{in}}{1-D}$$, fazendo

$$L_2 \frac{dI_{L2}}{dt} = \frac{V_{in}}{1-D} = L_2 \frac{\Delta I_{L2}}{DT_{ch}} \Rightarrow \boxed{L_2 = \frac{V_{in} D}{\Delta I_{L2} f_{ch} (1-D)} \Rightarrow \Delta I_{L2} = \frac{V_{in} D}{L_2 f_{ch} (1-D)}}$$

Encontrando I_{L1} e I_{L2}

I_{L1}:

$$P_{in} = P_o$$

$$V_{in} \cdot I_{L1} = V_o I_R$$

$$V_{in} I_{L1} = V_o \frac{V_o}{R}$$

$$I_{L1} = \frac{V_o^2 \cdot V_{in}}{V_{in}^2 R} = \frac{V_{in}}{R \cdot (1-D)^4}$$

$$\boxed{I_{L1} = \frac{V_{in}}{R \cdot (1-D)^4}}$$

I_{L2}:

$$I_{L2} = I_{C1} \cdot D + (I_{L1} - I_{C1})(1-D)$$

$$I_{L2} = I_{C1} D + I_{L1}(1-D) - I_{C1}(1-D)$$

$$I_{L2} = I_{C1}(D - 1 + D) + I_{L1}(1-D)$$

$$I_{L2} = I_{C1}(2D-1) + \frac{V_{in}}{R(1-D)^3}$$

Em regime permanente $I_{C1} = 0$

$$\boxed{I_{L2} = \frac{V_{in}}{R(1-D)^3}}$$

Logo $I_{L1\,máx/min}$ e $I_{L2\,máx/min}$ são:

$$\boxed{\begin{aligned} I_{L1\,máx} &= I_{L1} + \frac{\Delta I_{L1}}{2} \\ I_{L1\,min} &= I_{L1} - \frac{\Delta I_{L1}}{2} \end{aligned}}$$

$$I_{L1\,máx} = \frac{V_{in}}{R\cdot(1-D)^4} + \frac{V_{in} D}{2L_1 f_{ch}}$$

$$I_{L\,min} = \frac{V_{in}}{R(1-D)^4} - \frac{V_{in} D}{2 L_1 f_{ch}}$$

$$\boxed{\begin{aligned} I_{L2\,máx} &= I_{L2} + \frac{\Delta I_{L2}}{2} \\ I_{L2\,min} &= I_{L2} - \frac{\Delta I_{L2}}{2} \end{aligned}}$$

$$I_{L2\,máx} = \frac{V_{in}}{R(1-D)^3} + \frac{V_{in} D}{2L_2 f_{ch}(1-D)}$$

$$I_{L2\,min} = \frac{V_{in}}{R(1-D)^3} - \frac{V_{in} D}{2 L_2 f_{ch}(1-D)}$$

$$\boxed{I_{L1(RMS)} = \sqrt{I_{L1}^2 + \frac{\Delta I_{L1}^2}{12}}} \quad \boxed{I_{L2(RMS)} = \sqrt{I_{L2}^2 + \frac{\Delta I_{L2}^2}{12}}}$$

Encontrando $L_{1/2(crítico)}$

P/ $L_{1/2(crítico)}$ precisamos que $I_{L1/L2\,min} = 0$

$$I_{L1(min)} = \frac{V_{in}}{R(1-D)^4} - \frac{V_{in} D}{2 L_{1(crítico)} f_{ch}} = 0$$

$$\boxed{L_{1(crítico)} = \frac{D R (1-D)^4}{2 f_{ch} V_{in}}}$$

$$I_{L2(min)} = \frac{V_{in}}{R(1-D)^3} - \frac{V_{in} D}{2 L_{2(crítico)} f_{ch}(1-D)} = 0$$

$$\boxed{L_{2(crítico)} = \frac{D R (1-D)^2}{2 f_{ch}}}$$

No período em que temos a chave ON

$I_{C1} = -I_{L2}$

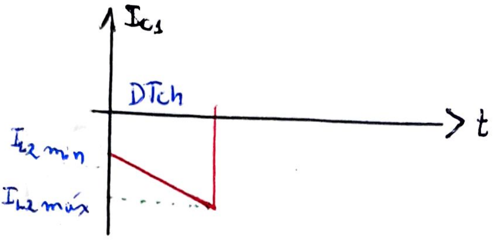
$I_{C2} = -I_R$

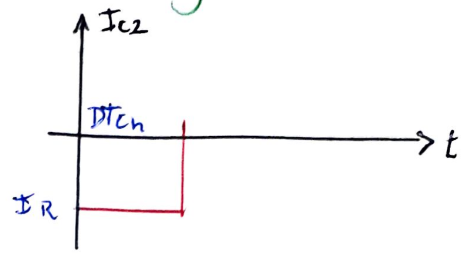
E p/ a chave OFF

$I_{C1} = I_{L1} - I_{L2}$

$I_{C2} = I_{L2} - I_R$

note que é mais fácil trabalhar com a região ON





logo, temos que:

P/ C_1

$$\Delta V_{C1} \cdot C_1 = \Delta Q$$

$$\Delta V_{C1} \cdot C_1 = \frac{\left(I_{L2(máx)} + I_{L2(min)}\right) D T_{ch}}{2}$$

$$= \frac{\left(I_{L2} + \frac{\Delta I_{L2}}{2} + I_{L2} - \frac{\Delta I_{L2}}{2}\right) D T_{ch}}{2}$$

$$= \frac{2 I_{L2} D T_{ch}}{2} = I_{L2} D T_{ch}$$

Substituindo I_{L2}, temos:

$$\Delta V_{C1} C_1 = \frac{V_{in}}{R(1-D)^3} D T_{ch}$$

$$\boxed{C_1 = \frac{V_{in} D}{R \cdot f_{ch} \, \Delta V_{C1} (1-D)^3}}$$

P/ C_2

$$\Delta V_{C2} C_2 = \Delta Q$$

Como $V_{C2} = V_o$

$$\Delta V_o C_2 = \Delta Q$$

$$\Delta V_o C_2 = I_R \cdot D T_{ch}$$

Substituindo $I_R = \frac{V_o}{R}$

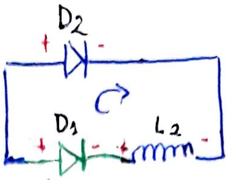
$$\Delta V_o C_2 = \frac{V_o}{R} D T_{ch}$$

$$\boxed{C_2 = \frac{V_o D}{R \, \Delta V_o \, f_{ch}}}$$

Diodo D1

Tensão

Chave ON



$$-\cancel{V_{D2}}^{\,0} + V_{L2} + V_{D1} = 0$$

$$V_{D1} = -V_{L2} = -V_{C1}$$

Substituindo

$$\boxed{V_{D1} = -\frac{V_{in}}{1-D}}$$

Corrente

Chave OFF

$$I_{D1} = I_{L1}$$

$$I_{D1_{máx}} = I_{L1_{máx}}$$

Substituindo.

$$\boxed{I_{D1_{máx}} = \frac{V_{in}}{R(1-D)^4} + \frac{V_{in} D}{2L_1 \cdot f_{ch}}}$$

Diodo D2

tensão

Chave OFF

$$V_{D2} = V_0 - V_{C1}$$; substituindo:

$$V_{D2} = V_0 - [(1-D)V_0] \Rightarrow V_{D2} = V_0(1-1+D) \rightarrow \boxed{V_{D2} = V_0 D}$$

Corrente

Chave ON

$$I_{D2} = I_{L1}$$

$$I_{D2_{máx}} = I_{L1_{máx}}$$

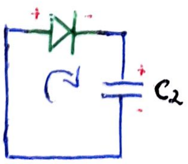
Substituindo.

$$\boxed{I_{D2_{máx}} = \frac{V_{in}}{R(1-D)^4} + \frac{V_{in} D}{2L_1 f_{ch}}}$$

Diodo D3

tensão

Chave ON



$$-V_{D3} - V_{C2} = 0$$

$$V_{D3} = -V_{C2} = -V_0$$

$$\boxed{V_{D3} = -V_0}$$

Corrente

Chave OFF

$$I_{D3} = I_{L2}$$

$$I_{D3_{máx}} = I_{L2_{máx}}$$

$$\boxed{I_{D3_{máx}} = \frac{V_{in}}{R(1-D)^3} + \frac{V_{in} D}{2L_2 f_{ch}(1-D)}}$$

Chave Sb

tensão

Chave OFF

$$\boxed{V_{Sb} = V_o}$$

Corrente

Chave ON

$$I_{Sb} = I_{L1} + I_{L2}$$

$$I_{Sb_{máx}} = I_{L1_{máx}} + I_{L2_{máx}}$$

$$I_{Sb_{máx}} = \frac{V_{in}}{R.(1-D)^4} + \frac{V_{in} D}{2L_1 f_{ch}} + \frac{V_{in}}{R(1-D)^3} + \frac{V_{in} D}{2L_2 f_{ch} (1-D)}$$

$$I_{Sb_{máx}} = \frac{V_{in}}{R(1-D)^3}\left(\frac{1}{1-D} + 1\right) + \frac{V_{in} D}{2 f_{ch}}\left(\frac{1}{L_1} + \frac{1}{L_2(1-D)}\right)$$

$$I_{Sb_{máx}} = \frac{V_{in}}{R(1-D)^3}\left(\frac{1+1-D}{1-D}\right) + \frac{V_{in} D}{2 f_{ch}}\left(\frac{L_2(1-D) + L_1}{L_1 L_2 (1-D)}\right)$$

$$\boxed{I_{Sb_{máx}} = \frac{V_{in}(2-D)}{R(1-D)^4} + \frac{V_{in} D}{2 f_{ch}}\left(\frac{L_1 + (1-D)L_2}{L_1 L_2 (1-D)}\right)}$$

PRojetando.

Especificações

$V_{in} = 5\,V$ $\qquad V_m = 1$ $\qquad \frac{\Delta I_{L1}}{I_{L1}} = 31\%$ $\qquad \frac{\Delta V_{C1}}{V_{C1}} = 1\%$

$V_o = 20\,V$ $\qquad H_v = 0{,}1$

$P_o = 8\,W$ $\qquad f_{ch} = 10\,KHz$ $\qquad \frac{\Delta I_{L2}}{I_{L2}} = 28\%$ $\qquad \frac{\Delta V_{C2}}{V_{C2}} = 2\%$

$T_s = 100\,ms$

Sendo $\frac{V_o}{V_{in}} = \frac{1}{(1-D)^2} \Rightarrow D = 1 - \sqrt{\frac{V_{in}}{V_o}}$

$$D = 1 - \sqrt{\frac{5}{20}} \Rightarrow \boxed{D = 0{,}5}$$

$$\Delta I_{L1} = 0{,}31\, I_{L1} = 0{,}31 \cdot \frac{V_{in}}{R \cdot (1-D)^4} \quad ; \quad R = \frac{V_o^2}{P_o} = \frac{20^2}{8} = 50$$

$$I_{L1} = \frac{5}{50 \cdot (1-0{,}5)^4} \Rightarrow \boxed{I_{L1} = 1{,}6\ A} \quad \boxed{\Delta I_{L1} = 0{,}496}$$

$\Delta I_{L2} = 0{,}28\, I_{L2}$

$$I_{L2} = \frac{V_{in}}{R(1-D)^3} = \frac{5}{50(1-0{,}5)^3} \Rightarrow \boxed{I_{L2} = 0{,}8\,A} \quad \boxed{\Delta I_{L2} = 0{,}224}$$

Assim, o valor das indutâncias são dados por:

$$L_1 = \frac{V_{in}\, D}{\Delta I_{L1}\, f_{ch}} = \frac{5 \cdot 0{,}5}{0{,}496 \cdot 10 \cdot 10^3} \Rightarrow \boxed{L_1 = 0{,}504\ mH}$$

$$L_2 = \frac{V_{in}\, D}{\Delta I_{L2} \cdot f_{ch}(1-D)} = \frac{5 \cdot 0{,}5}{0{,}224 \cdot 10 \cdot 10^3 \cdot 0{,}5} \Rightarrow \boxed{L_2 = 2{,}232\ mH}$$

Calculo das Capacitâncias

Capacitor C1

$$C_1 = \frac{V_{in}\, D}{R \cdot f_{ch} \cdot \Delta V_{c1} (1-D)^3} \quad , \quad \Delta V_{c1} = 0{,}01\, V_{c1}$$

$$\Delta V_{c1} = 0{,}01 \left(\frac{V_{in}}{1-D}\right)$$

$$C_1 = \frac{\cancel{V_{in}}\, D}{R \cdot f_{ch} \cdot 0{,}01 \frac{\cancel{V_{in}}}{\cancel{(1-D)}} (1-D)^{\cancel{3}\,2}} = \frac{D}{R\, f_{ch}\, 0{,}01\, (1-D)^2}$$

Substituindo os valores:

$$C_1 = \frac{0{,}5}{50 \cdot 10 \cdot 10^3 \cdot 0{,}01 (1-0{,}5)^2} \Rightarrow C_1 = 400 \cdot 10^{-6}$$

$$\boxed{C_1 = 400\,\mu F}$$

$$\boxed{\text{Obs: } V_{c1} = \frac{V_0}{2} = 10\,V}$$

Capacitor C2

$$C_2 = \frac{V_0\, D}{R\, \Delta V_0\, f_{ch}} \quad , \text{ como } V_0 = V_{c2} \qquad \frac{\Delta V_{c2}}{V_{c2}} = \frac{\Delta V_0}{V_0} = \frac{2}{100}$$

$$C_2 = \frac{50 \cdot 0{,}5}{50 \cdot 10 \cdot 10^3} = 50 \cdot 10^{-6}$$

$$\boxed{C_2 = 50\,\mu F}$$

$$\boxed{\text{Obs: } V_{c2} = V_0 = 20\,V}$$

Projetando os Diodos;

Diodo D1.

$$V_{D1} = \frac{-V_{in}}{1-D} \Rightarrow V_{D1} = \frac{-5}{1-0{,}5} \Rightarrow \boxed{V_{D1} = -10\,V}$$

$$I_{D1_{máx}} = \frac{V_{in}}{R(1-D)^4} + \frac{V_{in}\, D}{2 L_1 f_{ch}} \Rightarrow I_{D1_{máx}} = \frac{5}{50(1-0{,}5)^4} + \frac{5 \cdot 0{,}5}{2 \cdot 0{,}504 \cdot 10^{-3} \cdot 10 \cdot 10^3}$$

$$\boxed{I_{D1_{máx}} = 1{,}848\,A}$$

Diodo D2

$V_{D2} = V_o \cdot D$

$V_{D2} = 20 \cdot 0{,}5 \Rightarrow \boxed{V_{D2} = 10\,V}$

$$I_{D2_{máx}} = \frac{V_{in}}{R(1-D)^4} + \frac{V_{in}\,D}{2 L_1 \cdot f_{ch}} \Rightarrow I_{D2_{máx}} = \frac{5}{50(1-0{,}5)^4} + \frac{5 \cdot 0{,}5}{2 \cdot 0{,}504 \cdot 10^{-3} \cdot 10 \cdot 10^3}$$

$$\boxed{I_{D2_{máx}} = 1{,}848\ A}$$

Diodo D3

$V_{D3} = -V_o$

$\boxed{V_{D3} = -20\,V}$

$$I_{D3_{máx}} = \frac{V_{in}}{R(1-D)^3} + \frac{V_{in}\,D}{2 L_2\, f_{ch}\,(1-D)} \Rightarrow I_{D3} = \frac{5}{50(1-0{,}5)^3} + \frac{5 \cdot 0{,}5}{2 \cdot 2{,}232 \cdot 10^{-3} \cdot 10 \cdot 10^3 (1-0{,}5)}$$

$$\boxed{I_{D3_{máx}} = 0{,}912\ A}$$

Chave Sb

$V_{Sb} = V_o \Rightarrow \boxed{V_{Sb} = 20\,V}$

$$I_{Sb_{máx}} = \frac{V_{in}(2-D)}{R \cdot (1-D)^4} + \frac{V_{in}\,D}{2 f_{ch}} \left(\frac{L_1 + (1-D)\,L_2}{L_1 L_2 (1-D)} \right)$$

$$I_{Sb_{máx}} = \frac{5(2-0{,}5)}{50 \cdot (1-0{,}5)^4} + \frac{5 \cdot 0{,}5}{2 \cdot 10 \cdot 10^3} \left(\frac{0{,}504 \cdot 10^{-3} + (1-0{,}5) \cdot 2{,}232 \cdot 10^{-3}}{0{,}504 \cdot 10^{-3} \cdot 2{,}232 \cdot 10^{-3} \cdot (1-0{,}5)} \right)$$

Projetando o controlador

P/ aproximar o sistema de um sistema de segunda ordem, primeiro vamos retirar algumas informações:

$T_p = 3{,}731$ ms

$V_{o(máx)} = 29{,}489V$

$V_{o(inicial)} : 6{,}14V$

$V_{o(final)} : 20V$

P/ retirar a função transferencia foi usado

$D_{(inicial)} = 0{,}1$

$D_{(final)} = 0{,}5$

$$\%UP = \frac{29{,}489 - 20}{20} \cdot 100 \Rightarrow \%UP = 47{,}445$$

$$\zeta = \frac{-\ln\left(\frac{47{,}445}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{47{,}445}{100}\right)}} \Rightarrow \zeta\ 0{,}23091$$

$$\omega_n = \frac{\pi}{3{,}731 \cdot 10^{-3} \cdot \sqrt{1 - 0{,}23091^2}} \Rightarrow \omega_n = 865{,}413$$

$$k = \frac{20 - 6{,}14}{0{,}5 - 0{,}1} \Rightarrow k = 34{,}65$$

Sendo a função $G(S) = K \dfrac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$

$$G(S) = 34{,}65 \frac{748940{,}5145}{S^2 + 399{,}665\,S + 748940{,}5145}$$

PS

O ganho $K_{PID} = \dfrac{4 \cdot V_m}{T_s \cdot K \omega_n^2 \cdot H_v}$, sendo $T_s = 100\,ms$

$H_v = 1$

$V_m = 1$

$$K_{PID} = \frac{4 \cdot 1}{100 \cdot 10^{3} \cdot 34{,}65 \cdot 748940{,}5145 \cdot 1}$$

$$\underline{K_{PID} = 1{,}5413 \cdot 10^{-6}}$$

$$PID(s) = 1{,}5413 \cdot 10^{-6} \cdot \frac{s^2 + 399{,}6655 + 748940{,}5145}{s}$$

$\boxed{K_D = 1{,}5413 \cdot 10^{-6}}$ $\boxed{K_P = 6{,}1603 \cdot 10^{-4}}$ $\boxed{K_I = 1{,}1544}$

Obs: O projeto é feito para $H_v = 1$ para que assim seja mais genérico. A compensação do ganho de 0,1 é feita via software, para que assim tenhamos $H_v = 1$