The feedback gain vector is then

$$\mathbf{f}^T = \mathbf{e}_1^T \begin{bmatrix} \dfrac{1}{T^2} & -\dfrac{1}{2T} \\ -\dfrac{1}{T^2} & -\dfrac{3}{2T} \end{bmatrix} \begin{bmatrix} 1 + d_1 + d_0 & (2 + d_1)T \\ 0 & 1 + d_1 + d_0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1 + d_1 + d_0}{T^2} & \dfrac{3 + d_1 - d_0}{2T} \end{bmatrix}$$

It is interesting to note that the gains are varying inversely with the sampling interval.

8.9 REGULATION WITH NONZERO REFERENCE INPUTS

In this section we seek to control the system such that the output vector $\mathbf{y}_k$ tracks a set of reference variables $\mathbf{r}_k$ of the same dimension as $\mathbf{y}_k$. We shall also assume that the number of control efforts in vector $\mathbf{u}_k$ is the same as the number of outputs $\mathbf{y}_k$. The system is governed by

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{8.9.1}$$

with an output relation

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k \tag{8.9.2}$$

We shall seek to accomplish this task by modifying the control law to be of the form

$$\mathbf{u}_k = -\mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{r}_k \tag{8.9.3}$$

where $\mathbf{G}$ is a square matrix yet to be calculated. This situation is illustrated in Fig. 8.13. If the control effort of (8.9.3) is substituted into relation (8.9.1), the result is

$$\mathbf{x}_{k+1} = (\mathbf{A} - \mathbf{B}\mathbf{F})\mathbf{x}_k + \mathbf{B}\mathbf{G}\mathbf{r}_k \tag{8.9.4}$$

Let us now take the z-transform of (8.9.4) to give

$$\mathbf{X}(z) = (z\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{F})^{-1}\mathbf{B}\mathbf{G}\mathbf{R}(z) \tag{8.9.5}$$

Then the transform of the output vector $\mathbf{y}_k$ is, from relation (8.9.2),

$$\mathbf{Y}(z) = \mathbf{C}(z\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{F})^{-1}\mathbf{B}\mathbf{G}\mathbf{R}(z) \tag{8.9.6}$$

For lack of a better strategy, let us choose $\mathbf{G}$ such that the dc gain between $\mathbf{R}(z)$ and $\mathbf{Y}(z)$ is the identity matrix, or

$$\mathbf{C}(z\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{F})^{-1}\mathbf{B}\mathbf{G}\big|_{z=1} = \mathbf{I} \tag{8.9.7}$$

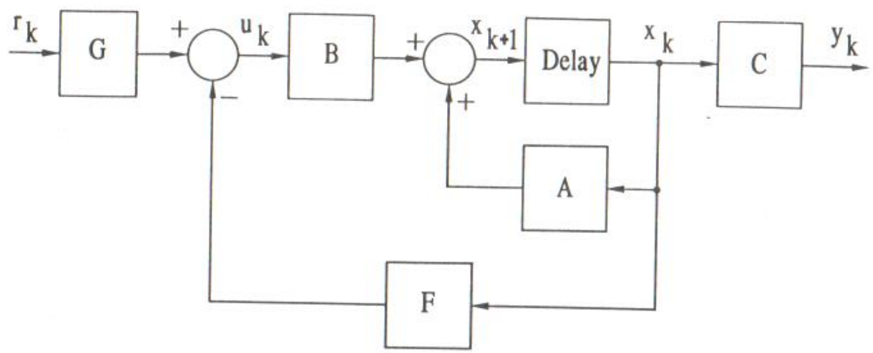

Figure 8.13. Complete state feedback system with reference input.

The gain matrix $\mathbf{G}$ is thus

$$\mathbf{G} = [\mathbf{C}(\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{F})^{-1}\mathbf{B}]^{-1} \tag{8.9.8}$$

The closed-loop poles are still placed by the selection of the elements of $\mathbf{F}$ as in the simple state feedback regulator.

Example 8.6. Let us design a state feedback control system to control the temperature $x_1(k)$ about some nonzero fixed point $\bar{y}$ and have the system poles located at $z = 0.5 \pm j0.2$. This is similar to the problem given in Example 8.2.

The system matrices are

$$\mathbf{A} = \begin{bmatrix} 0.6277 & 0.3597 \\ 0.0899 & 0.8526 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 0.0251 \\ 0.1150 \end{bmatrix}$$

and

$$\mathbf{c}^T = [1 \quad 0]$$

The pole locations will be given by the roots of $\det(z\mathbf{I} - \mathbf{A} + \mathbf{b}\mathbf{f}^T) = 0$, where the elements of $\mathbf{f}^T$ will be selected to place them as specified above. This was already done in Example 8.2, and the state feedback gain matrix was found to be

$$\mathbf{f}^T = [2.248 \quad 3.684]$$

Now all we need to do is to evaluate the reference input gain G as given by expression (8.9.8). The calculations of relation (8.9.8) are most easily carried out by using the digital computer, and for the numerical values just