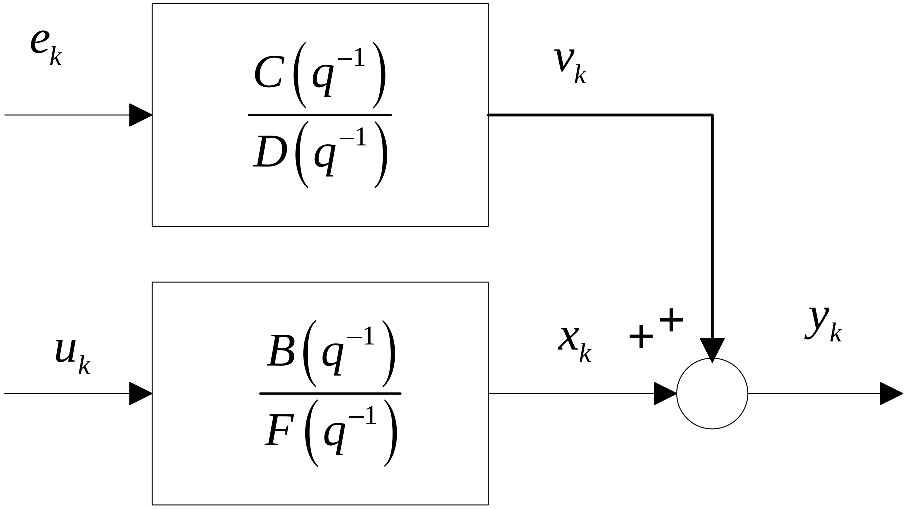
1. Modelos y Predictores

1.1. Principales modelos



Donde

$$B = b_0 z^{-n_k} + \cdots b_m z^{-n_k - n_b}$$

$$F = 1 + f_1 z^{-1} + \cdots + f_{n_f} z^{-n_f}$$

$$C = 1 + \cdots + c_{n_c} z^{-n_c}$$

$$D = 1 + \cdots + c_{n_d} z^{-n_d}$$

$$y_{k+1} = ay_k + bu_k + e_{k+1} + ce_k$$ [1.1]

Predictor convencional

$$\hat{y}_{k+1} = ay_k + bu_k$$ [1.2]

Nuevo Predictor

$$\hat{y}_{k+1} = ay_k + bu_k + ce_k$$ [1.3]

$$e_{k+1} = y_{k+1} - \hat{y}_{k+1}$$ [1.4]

$$\hat{y}_{k+1} = ay_k + bu_k + c \underbrace{y_k - \hat{y}_k}_{señal\ extra}$$ [1.5]

En forma polinómica,

$$C\hat{y}_k = C - A \; y_k + Bu_k$$ [1.6]

$$y_k = x_k + v_k \tag{1.7}$$

$$x_k = G_k u_k = \frac{B}{F} u_k = \frac{b_0 z^{-nk} + \cdots}{1 + f_1 z^{-1} + \cdots} u_k \tag{1.8}$$

$$x_k + f_1 x_{k-1} + \cdots + f_{nf} x_{k-nf} = b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \tag{1.9}$$

$$x_k = -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \tag{1.10}$$

$$v_k = H_k e_k = \frac{C}{D} e_k = \frac{1 + \cdots + c_{nc} z^{-nc}}{1 + \cdots + d_{nd} z^{-nd}} e_k \tag{1.11}$$

$$v_k + d_1 v_{k-1} + \cdots + d_{nd} v_{k-nd} = e_k + c_1 e_{k-1} + \cdots + c_{nc} e_{k-nc} \tag{1.12}$$

$$v_k = -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + e_k + c_1 e_{k-1} + \cdots + c_{nc} e_{k-nc}$$

Si se toma como predicción,

$$\hat{y}_k = f_1 y_{k-1} + \cdots b_1 u_{k-1-nk} + \cdots \quad [1.13]$$

la diferencia entre modelo y medición, $\hat{y}_k - y_k$, no sería ruido blanco que aseguraría la convergencia de los mínimos cuadrados

Lo ideal sería que

$$y_k - \hat{y}_k = e_k \quad [1.14]$$

1.1.1. Box – Jenkin

$y_k = Gu_k + He_k$

hay que conocer nb, nc, nd, nf y nk

1.1.2. modelo de error de salida (OE)

$nc = nd = 0$

$H = 1$

$w = e$

$y = Gu + e$

1.1.3. ARMAX

$F = D$ [1.15] $= A = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}$

$Ay = Bu + Ce$

1.1.4. ARX

$C = 1$

$Ay = Bu + e$

[1.16]

1.2. Predicción. Caso general

El objetivo de la predicción es hacer que $y_k - \hat{y}_k = e_k$

$$y_k = Gu_k + He_k$$

$$y_k = \frac{B}{F} u_k + \frac{C}{D} e_k \qquad [1.20]$$

$$\frac{D}{C} y_k = \frac{D}{C} \frac{B}{F} u_k + e_k \qquad [1.21]$$

$$y_k = \left[1 - \frac{D}{C}\right] y_k + \frac{D}{C} \frac{B}{F} u_k + e_k \qquad [1.22]$$

la predicción será

$$\hat{y}_k = \left[1 - \frac{D}{C}\right] y_k + \frac{D}{C} \frac{B}{F} u_k \qquad [1.23]$$

$$CF\hat{y}_k = \left[CF - DF\right] y_k + DBu_k \qquad [1.24]$$

$$\hat{y}_k = \; 1 - CF \;\; \hat{y} + CFy_k - DFy_k + DBu_k \qquad [1.25]$$

$$\hat{y}_k = \; 1 - CF \;\;\; \hat{y} - y \;\; + \;\; 1 - DF \;\; y_k + DBu_k \qquad [1.26]$$

1.2.1. Predicción en el modelo ARMAX

$$Ay = Bu + Ce \qquad [1.27]$$

la predicción será

$$\hat{y}_k = \left[1 - \frac{A}{C}\right] y_k + \frac{A}{C}\frac{B}{A} u_k \qquad [1.28]$$

$$C\hat{y}_k = \left[C - A\right] y_k + Bu_k \qquad [1.29]$$

$$\hat{y}_k = -c_1\hat{y}_{k-1} + \cdots + y_k + c_1 y_{k-1} + \cdots - y_k - a_1 y_{k-1} - \cdots + b_1 u_{k-1} \qquad [1.30]$$

$$\hat{y}_k = \left[1 - C\right] \underbrace{y_k - \hat{y}_k}_{\varepsilon_k} + \left[1 - A\right] y_k + Bu = \varphi\theta \qquad [1.31]$$

$$\begin{aligned} \varphi^T &= \left[y_{k-1} \cdots \varepsilon_k \cdots u_{k-1} \cdots \right] \\ \theta &= \left[a_1 \cdots c_1 \cdots b_{1\cdots} \right] \end{aligned} \qquad [1.32]$$

1.2.2. Predicción en el modelo Box-Jenkin

Modelo

$$y_k = \frac{B}{F} u_k + \frac{C}{D} e_k \qquad [1.33]$$

predicción

$$\hat{y}_k = \; 1 - CF \quad \hat{y}_k - y_k \; + \; 1 - DF \;\; y_k + DBu_k \qquad [1.34]$$

se define el error de predicción como

$$\varepsilon_k = y_k - \hat{y}_k = \frac{D}{C}\left[y_k - \frac{B}{F} u_k \right] = e_k \qquad [1.35]$$

Por otro lado,

$$y_k = x_k + v_k \qquad [1.36]$$

$$x_k = G_k u_k = \frac{B}{F} u_k = \frac{b_0 z^{-nk} + \cdots}{1 + f_1 z^{-1} + \cdots} u_k \qquad [1.37]$$

$$x_k + f_1 x_{k-1} + \cdots + f_{nf} x_{k-nf} = b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \quad [1.38]$$

$$x_k = -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \quad [1.39]$$

$$v_k = H_k e_k = \frac{C}{D} e_k = \frac{1 + \cdots + c_{nc} z^{-nc}}{1 + \cdots + d_{nd} z^{-nd}} e_k \quad [1.40]$$

$$v_k + d_1 v_{k-1} + \cdots + d_{nd} v_{k-nd} = e_k + c_1 e_{k-1} + \cdots + c_{nc} e_{k-nc} \quad [1.41]$$

$$v_k = -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + e_k + c_1 e_{k-1} + \cdots + c_{nc} e_{k-nc}$$

$$\begin{aligned} y_k &= x_k + v_k \\ &= -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \\ &\quad -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + e_k + c_1 e_{k-1} + \cdots + c_{nc} e_{k-nc} \end{aligned} \quad [1.42]$$

$$\begin{aligned} y_k &= -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \\ &\quad -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + \varepsilon_k + c_1 \varepsilon_{k-1} + \cdots + c_{nc} \varepsilon_{k-nc} \end{aligned} \quad [1.43]$$

$$\begin{aligned} \hat{y}_k &= -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk} \\ &\quad -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + \cancel{\varepsilon}_k + c_1 \varepsilon_{k-1} + \cdots + c_{nc} \varepsilon_{k-nc} \end{aligned}$$

Resumen

$$\hat{y}_k = {\varphi_k}^T \theta_k$$

donde

$$\begin{aligned} {\varphi_k}^T &= \begin{bmatrix} u_{k-1} & \cdots & -x_{k-1} & \cdots & \varepsilon_{k-1} & \cdots & -v_{k-1} & \cdots \end{bmatrix} \\ \theta_k^T &= \begin{bmatrix} b_1 & \cdots & f_1 & \cdots & c_1 & \cdots & d_1 & \cdots \end{bmatrix} \end{aligned}$$

Las variables auxiliares se calculan,

$$x_k = -f_1 x_{k-1} - \cdots - f_{nf} x_{k-nf} + b_0 u_{k-nk} + \cdots + b_{nb} u_{k-nb-nk}$$

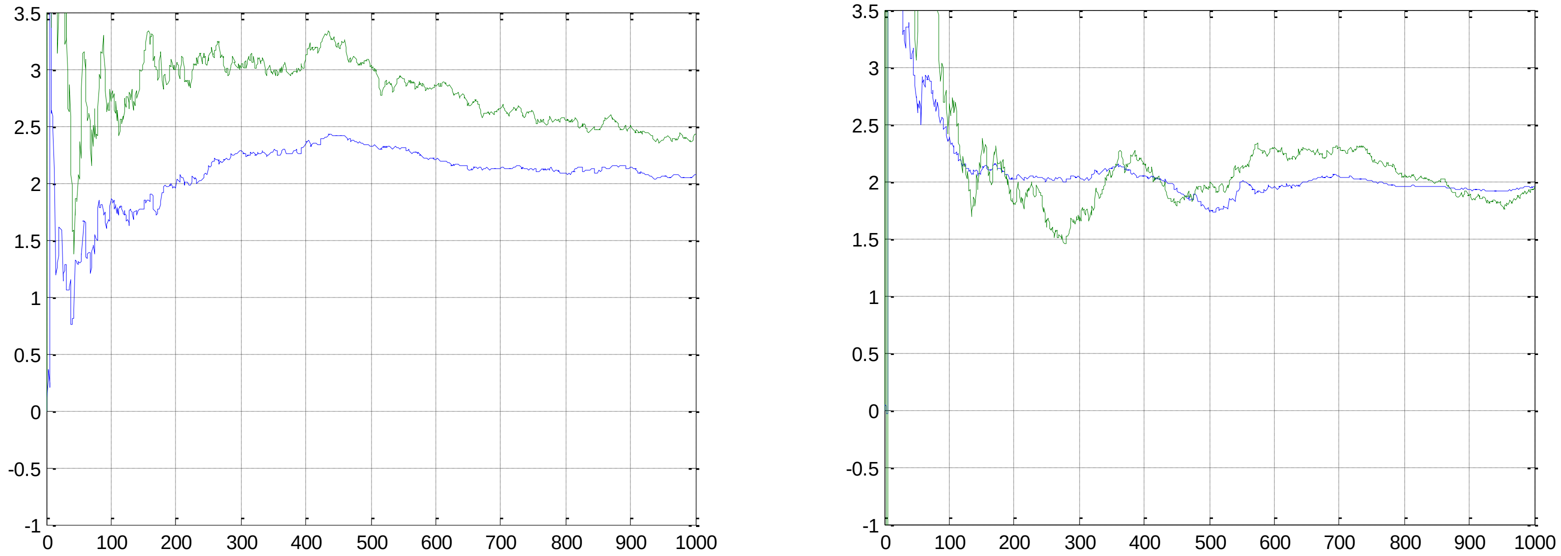
$$\varepsilon_k = y_k - \hat{y}_k \text{ o } \varepsilon_k = y_k - {\varphi_k}^T \theta_k$$

$$v_k = -d_1 v_{k-1} - \cdots - d_{nd} v_{k-nd} + \varepsilon_k + c_1 \varepsilon_{k-1} + \cdots + c_{nc} \varepsilon_{k-nc}$$

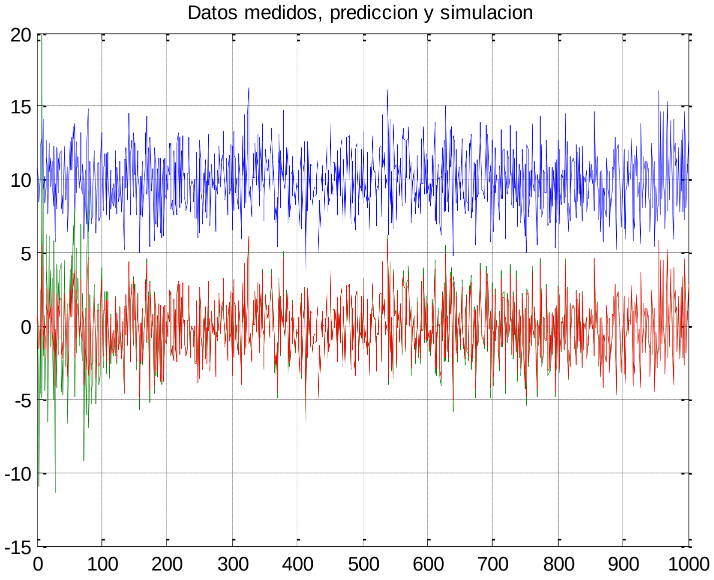
1.3. Ejemplo de Sesgo en la Estimación

1. Planta con nivel de continua

$$y_k = bu_k + c$$



dos realizaciones de la estimación (azul: con modelo de ruido)



simulación del modelo (no reproducen el nivel de continua)

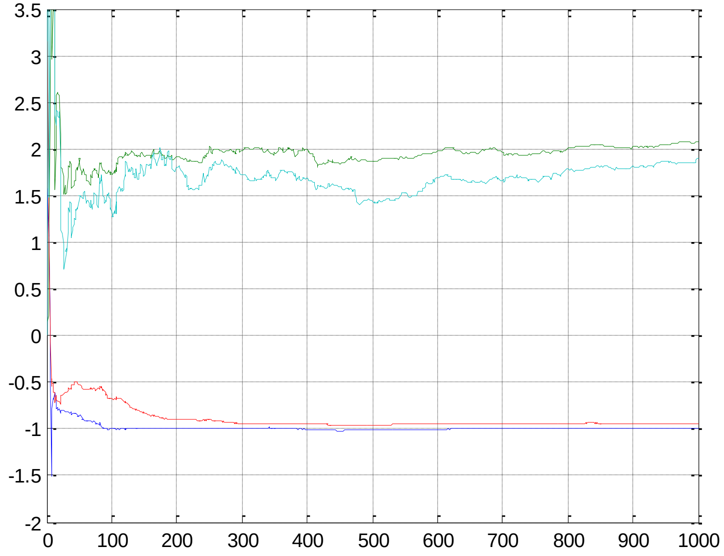
```
% Ejemplos para ver el sesgo
% Identificación de planta con nivel de continua
% Planta
n=10000;
A= [1];
B= [0 2];
C=1;
```

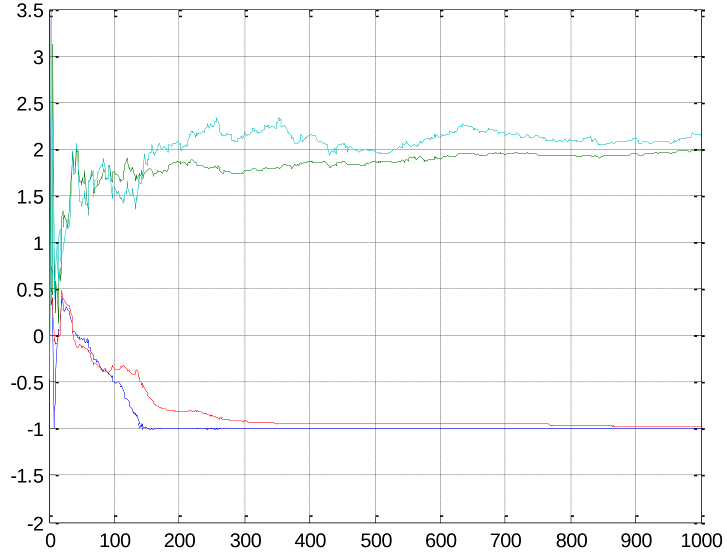
```
D=1;
F=1;
P = idpoly(A,B,C,D,F,0,1);
u= randn(n,1);
e= randn(n,1);
y=sim(P,u)+10;
figure(1)
plot(y);grid

na=0; % cantidad de polos
nb=1; % cantidad de ceros menos 1
nc=5; %
nk=1; % retardo
% minimos cuadrados recursivos
[th,yh] = rplr([y u],[na,nb,nc,0,0,nk],'ff',1);
[th1,yh] = rplr([y u],[na,nb,0,0,0,nk],'ff',1);
figure(2)
plot([th(:,1) th1]);grid
axis([0,n,-1, 3.5])

% simulación
```

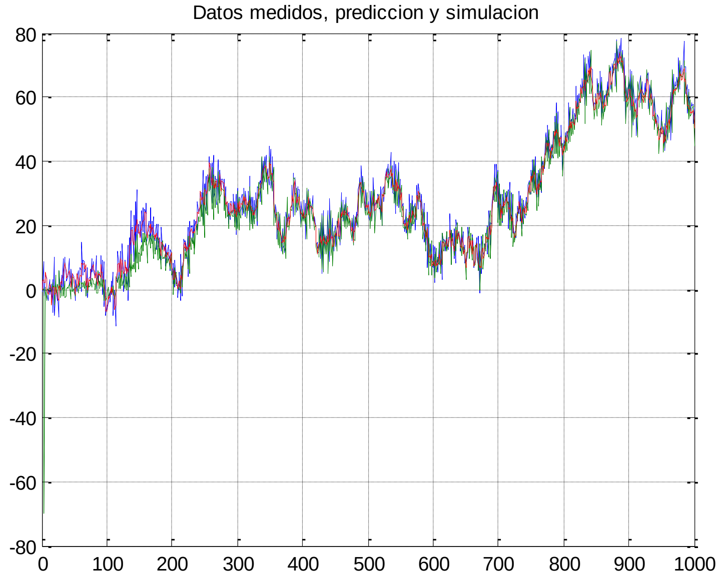
```
Am= [1 th(n,1:na)];
Bm= [0 th(n,na+1:na+nb)];
Cm= [1 th(n,na+nb+1:na+nb+nc)];
Dm=1;
Fm=1;
% Modelo obtenido
M = idpoly(Am,Bm,Cm,Dm,Fm,0,1);

% Comparacion. Se simula el modelo obtenido
ym=idsim(M,[u e]);

figure(3)
plot([y yh ym]);grid; title('Datos medidos, prediccion y simulacion')
%axis([0,160,0,10]);
```

2. Planta con ruido coloreado

$$Ay_k = Bu_k + Ce_k$$



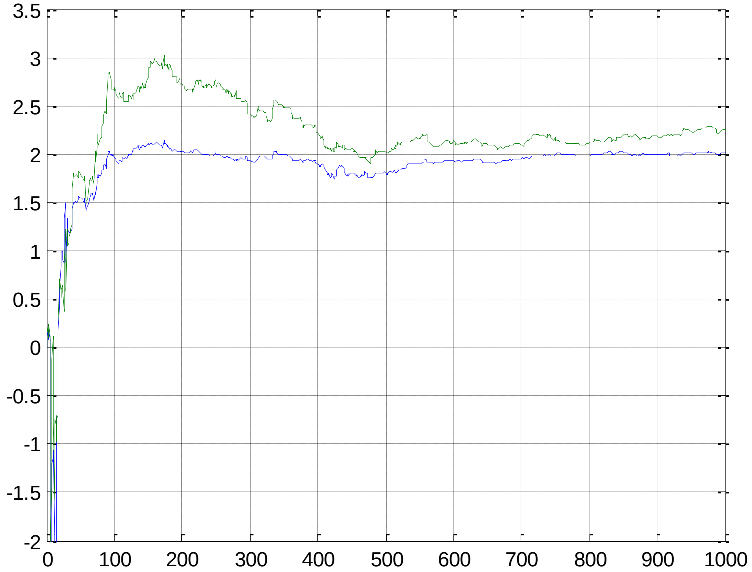


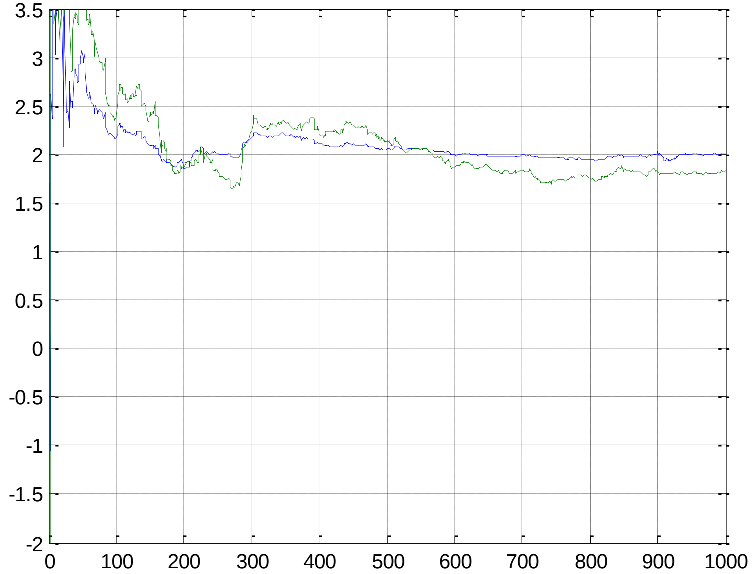


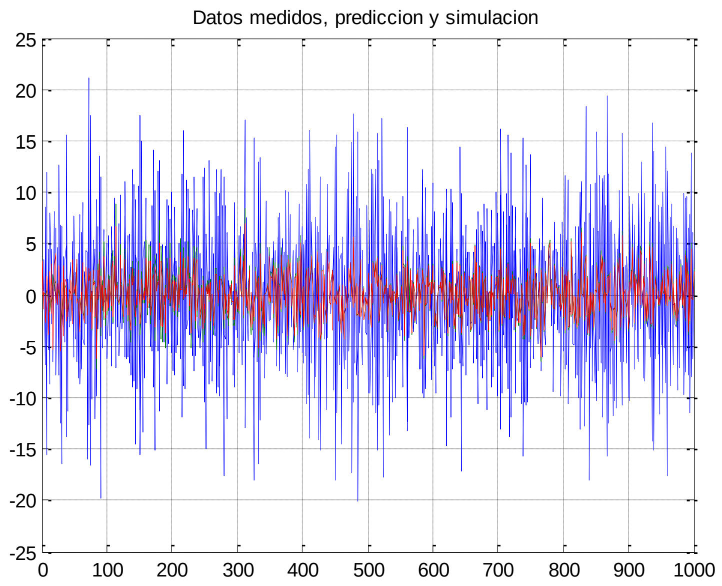
Datos medidos, prediccion y simulacion
80
60
40
20
0
-20
-40
-60
-80
0
100
200
300
400
500
600
700
800
900
1000

3. Planta con ruido coloreado

$$y_k = Bu_k + Ce_k$$








```
% Planta
n=1000;
A= [1];
B= [0 2];
C=poly([.9 .9 .9 .9]);
D=1;
F=1;
P = idpoly(A,B,C,D,F,1,1);
u= randn(n,1);
```

```
e= randn(n,1);
y=sim([u e],P);
figure(7)
plot([y]);grid
na=0; % cantidad de polos
nb=1; % cantidad de ceros menos 1
nc=4; %
nk=1; % retardo
% minimos cuadrados recursivos
[th,yh] = rplr([y u],[na,nb,nc,0,0,nk],'ff',1);
[th1,yh] = rplr([y u],[na,nb,0,0,0,nk],'ff',1);
%[th,zh] = rplr([z u],[16,16,0,0,0,1],'ff',1);
figure(8)
plot([th(:,1:na+nb) th1]);grid
axis([0,n,-2, 3.5])

% simulación
Am= [1 th(n,1:na)];
Bm= [0 th(n,na+1:na+nb)];
Cm= [1 th(n,na+nb+1:na+nb+nc)];
Dm=1;
```

```
Fm=1;
% Modelo obtenido
M = idpoly(Am,Bm,Cm,Dm,Fm,0,1);

% Comparacion. Se simula el modelo obtenido
ym=idsim([u e],M);

figure(9)
plot([y yh ym]);grid; title('Datos medidos, prediccion y simulacion')
%axis([0,160,0,10]);
```