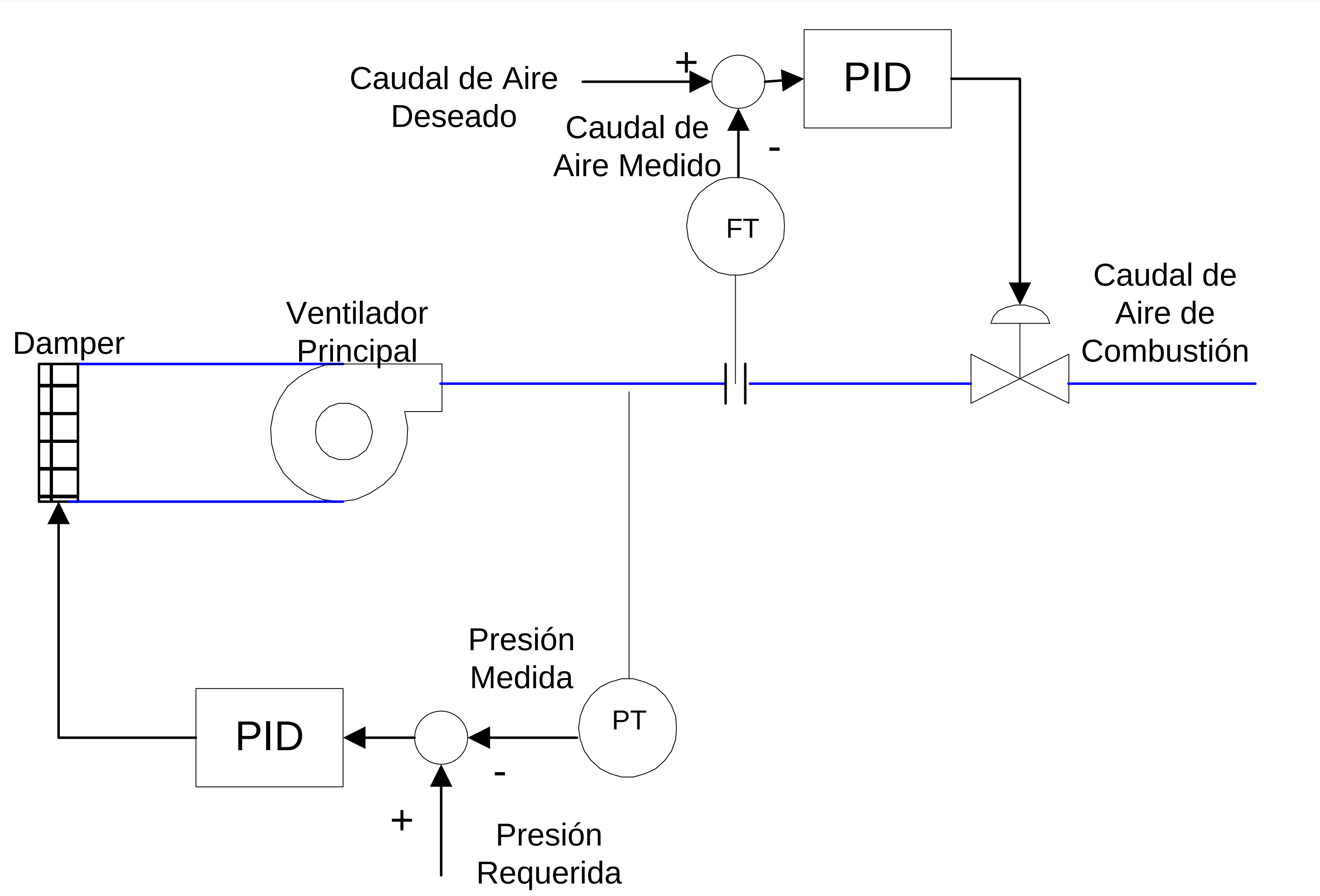
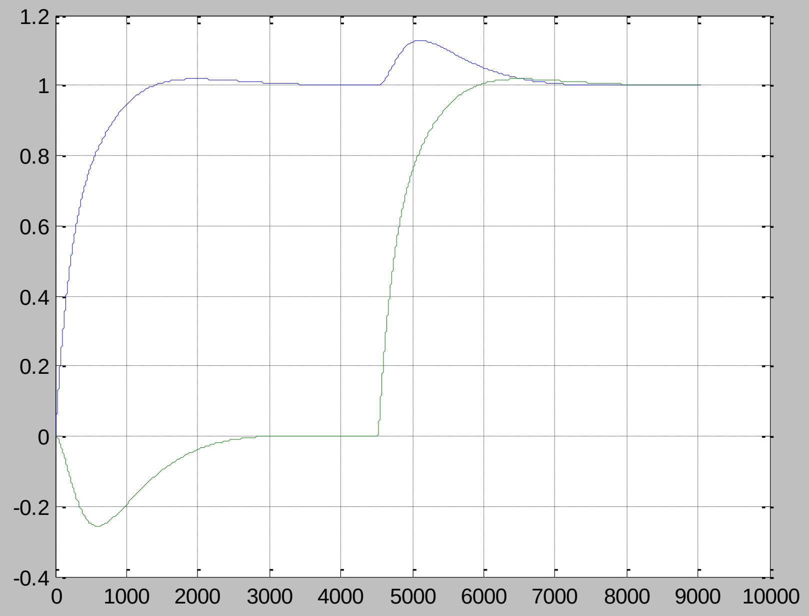
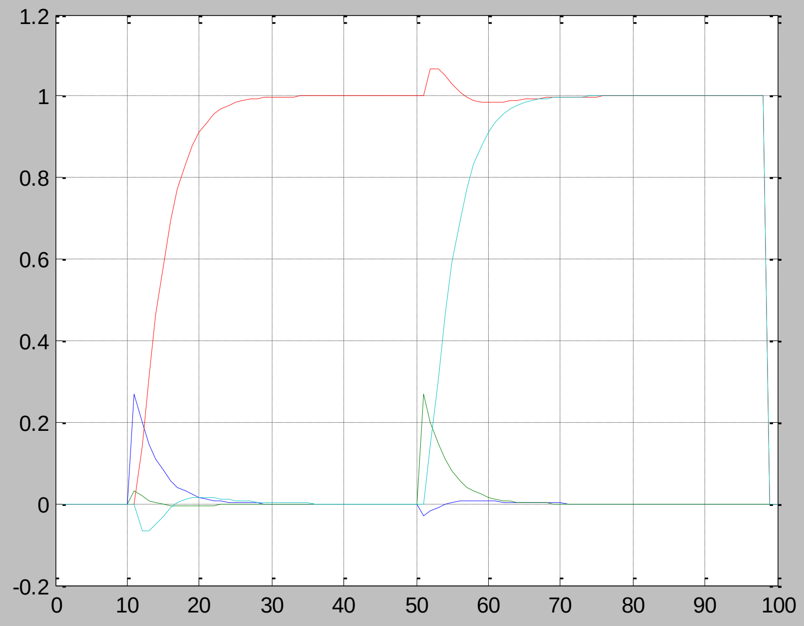
1. Control Por Matriz Dinámica

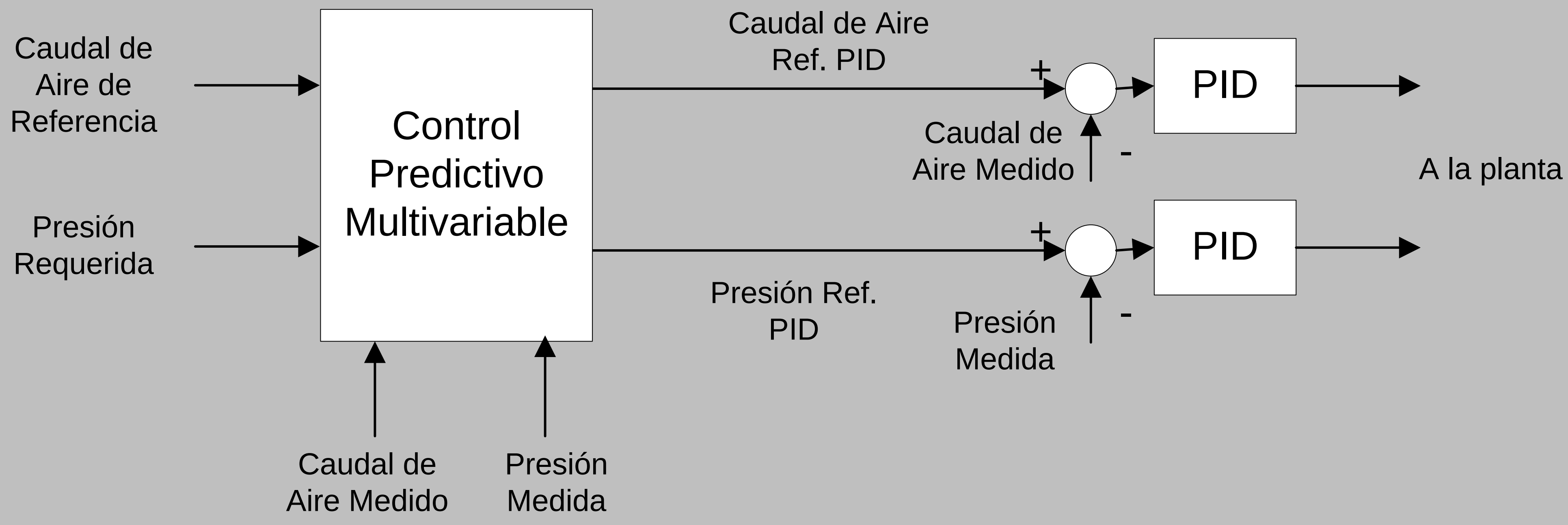
Ejemplo 1.1. *Control de Caudal y Presión.*







Control sin considerar el acoplamiento y Control Predictivo Multivariable


Caudal de
Aire de
Referencia
Presión
Requerida
Control
Predictivo
Multivariable
Caudal de
Aire Medido
Presión
Medida
Caudal de Aire
Ref. PID
Presión Ref.
PID
+
-
Caudal de
Aire Medido
+
-
Presión
Medida
PID
PID
A la planta

1.2. **Introducción**

Apto para sistemas multivariables

Se pueden manejar Restricciones fácilmente

Se basa en predicciones de las salidas futuras

Acciones de control basadas en predicciones y mediciones

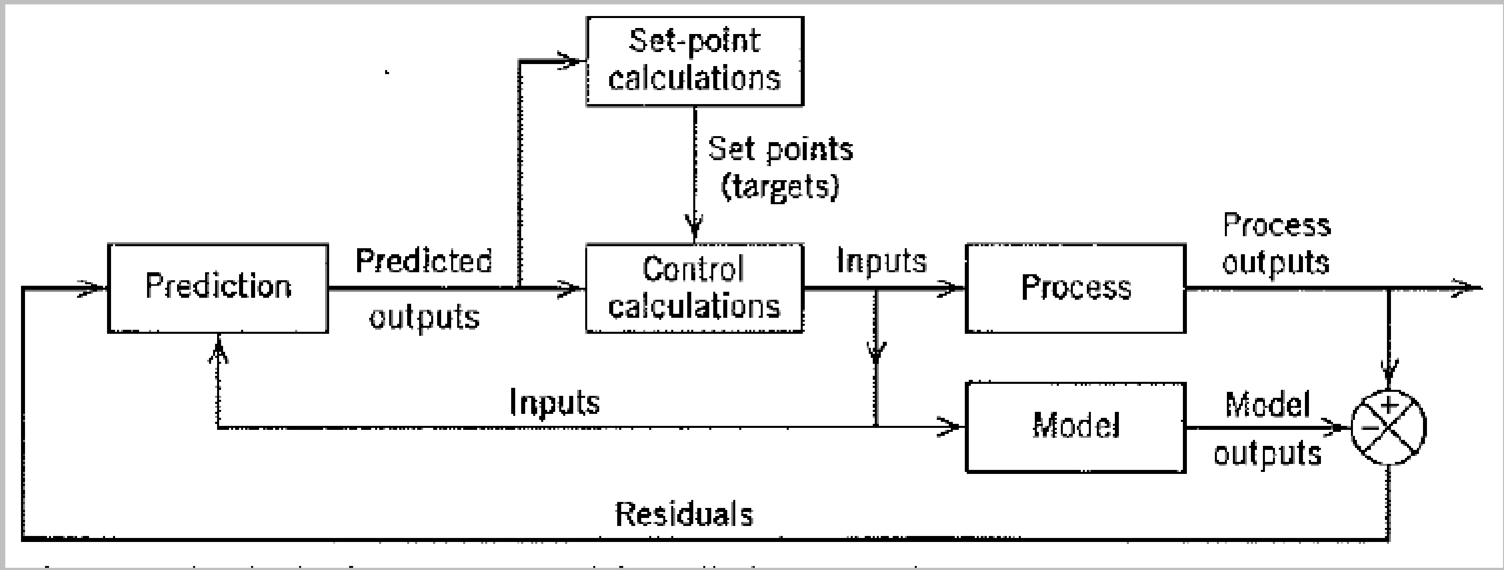
Se puede unir este control al cálculo óptimo de las referencias (programación lineal). RTO (Real Time Optimization)

Se necesita una buena precisión en el modelo para la predicción. Si no, puede ser peor que no usar nada.

Primeros esquemas:

DMC (*Dynamic Matrix Control):* Shell Oil y Adersa 1980.

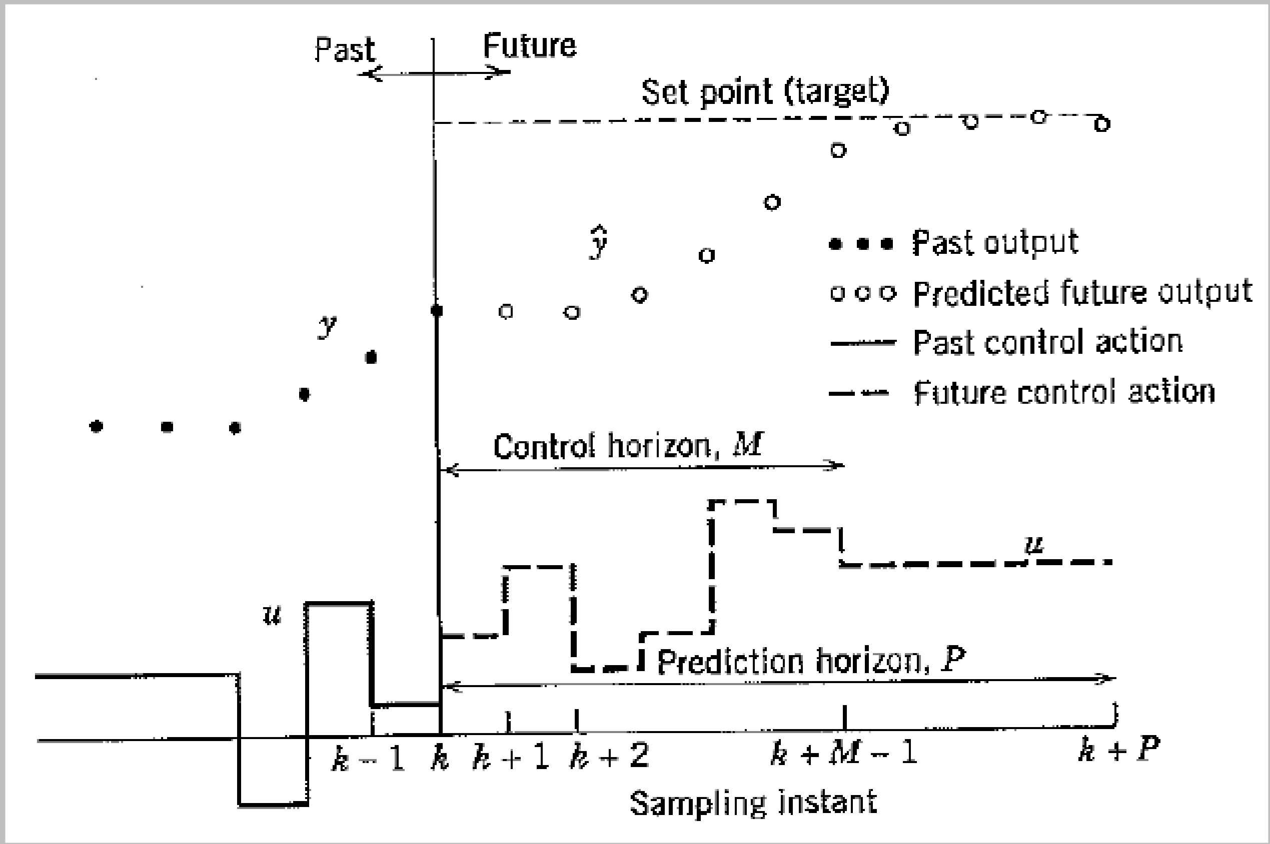
GPC (Generalized Predictive Control): Clarke 1987



Es un esquema similar al IMC o al Predictor de Smith

Pero más adecuado para sistemas multivariables y manejo de restricciones

La predicción se puede hacer con cualquier tipo de modelo (no lineal, difuso, redes neuronales)



Receding Horizon: (horizonte "que retrocede" – espejismo en la ruta). El horizonte se mueve a medida que avanza el control.

El modelo puede ser físico o empírico.

Puede ser basado en la respuesta al escalón discreta (DMC).

Se usa en sistemas estables.

Adecuado para sistemas con respuestas al escalón *raras* o no lineales.

Se necesita un gran número de muestras.

Se utiliza el modelo CARIMA

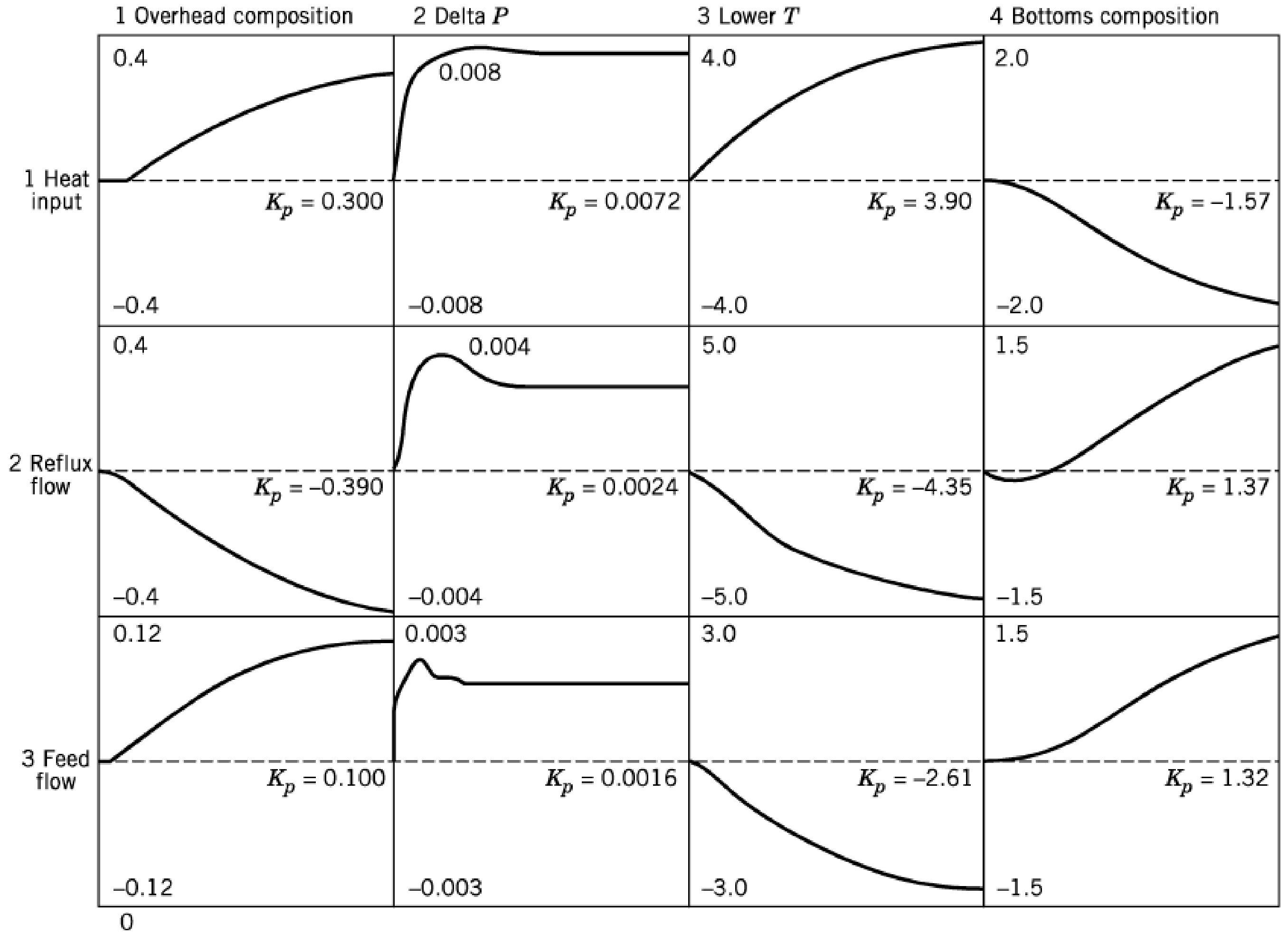


Figure 20.8 Individual step-response models for a distillation column with three inputs and four outputs. Each model represents the step response for 120 minutes (Hokanson and Gerstle, 1992).

Ejemplo de Control Predictivo

Un predictor simple es

$$y_{k+1} = ay_k + bu_k$$

ley de control

$$u_k = \frac{1}{b}\left(r_{k+1} - ay_k\right)$$

1.3. Predicción en Base a la Respuesta al Escalón

1.3.1. Ejemplo Sistema de Primer Orden

sistema original en modelo ARMA

$$y_{k+1} = ay_k + bu_k + n'_{k+1}$$

O lo que es lo mismo,

$$\left(1 - aq^{-1}\right) y_k = bu_{k-1} + n'_k$$

Utilizando el modelo CARIMA

$$\left(1 - aq^{-1}\right)\left(1 - q^{-1}\right) y_k = b\left(1 - q^{-1}\right) u_{k-1} + \left(1 - q^{-1}\right) n'_k$$

$$\left(1 - aq^{-1}\right)\left(1 - q^{-1}\right) y_k = b\Delta u_{k-1} + n_k$$

$$\left(1 - (1+a)q^{-1} + aq^{-2}\right) y_k = b\Delta u_{k-1} + n_k$$

La variable manipulada es ahora Δu (variaciones de u)

y n son variaciones en la perturbación

$$y_{k+1} = (1+a) y_k - ay_{k-1} + b\Delta u_k + n_{k+1}$$

Nota: Veamos la forma de la Respuesta al impulso y al escalón del sistema original

k	impulso	escalón
1	$g_0 = 0$	$s_0 = 0$
2	$g_1 = b$	$s_1 = b$
3	$g_2 = ab$	$s_2 = (a+1)b$
4	$g_3 = a^2 b$	$s_3 = (a^2 + a + 1)b$
5	$g_4 = a^3 b$	$s_3 = (a^3 + a^2 + a + 1)b$
6	$g_5 = a^4 b$	$s_3 = (a^4 + a^3 + a^2 + a + 1)b$

En el modelo ARMA, sin considerar la perturbación,

$$y_{k+1} = ay_k + bu_k$$

$$y_{k+1} = a\left(ay_{k-1} + bu_{k-1}\right) + bu_k = bu_k + abu_{k-1} + a^2 y_{k-1}$$

$$y_{k+1} = bu_k + abu_{k-1} + a^2\left(ay_{k-2} + bu_{k-2}\right) = bu_k + abu_{k-1} + a^2 bu_{k-2} + a^3 y_{k-2}$$

Reemplazando infinitas veces y agregando la perturbación,

$$y_{k+1} = \sum_{i=0}^{\infty} g_{i+1} u_{k-i} + n'_{k+1}$$

En el modelo CARIMA, sin considerar la perturbación,

$$y_{k+1} = b\Delta u_k + (1+a)\left(b\Delta u_{k-1} + (1+a)y_{k-1} - ay_{k-2}\right) - ay_{k-1}$$

$$= b\Delta u_k + (1+a)b\Delta u_{k-1} + \left((1+a)^2 - a\right)y_{k-1} - a(1+a)y_{k-2}$$

$$y_{k+1} = s_1\Delta u_k + s_2\Delta u_{k-1} + \left(1+a+a^2\right)y_{k-1} - a(1+a)y_{k-2}$$

$$= s_1\Delta u_k + s_2\Delta u_{k-1} + s_3\Delta u_{k-2} + \cdots$$

Reemplazando infinitas veces y agregando la perturbación,

$$y_{k+1} = \sum_{i=0}^{\infty} s_{i+1}\Delta u_{k-i} + n_{k+1}$$

Para predecir el valor de la salida en *k+1* se necesita conocer n_{k+1}, n_k

Se puede considerar que

$$n_{k+1} = n_k = y_k - s_1 \Delta u_{k-1} - s_2 \Delta u_{k-2} - s_3 \Delta u_{k-3} - s_4 \Delta u_{k-4} - \cdots$$

La predicción resultaría para cualquier horizonte h futuro

$$\hat{y}_{k+h} = s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots + n_k \tag{1.1}$$

$$\begin{aligned} \hat{y}_{k+h} &= s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots \\ &\quad + y_k - s_1 \Delta u_{k-1} - s_2 \Delta u_{k-2} - s_3 \Delta u_{k-3} - s_4 \Delta u_{k-4} - \cdots \end{aligned} \tag{1.2}$$

$$\begin{aligned} \hat{y}_{k+h} &= s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots \\ &\quad + y_k + \left(s_{h+1} - s_1\right) \Delta u_{k-1} + \left(s_{h+2} - s_2\right) \Delta u_{k-2} + \left(s_{h+3} - s_3\right) \Delta u_{k-3} + \cdots \end{aligned} \tag{1.3}$$

En forma compacta,

$$\hat{y}_{k+h} = \sum_{i=1}^{h} s_i \Delta u_{k+h-i} + y_k - \sum_{i=1}^{\infty} \left(s_{h+i} - s_i\right) \Delta u_{k-i} \tag{1.4}$$

Si la respuesta al escalón del sistema converge a un valor final, se cumple

$$s_{d+h+i} - s_i \to 0, \quad i \to \infty \tag{1.5}$$

Se podría suponer que la predicción se aproxima a

$$\hat{y}_{k+h} = \sum_{i=1}^{h} s_i \Delta u_{k+h-i} + y_k - \sum_{i=1}^{N} \left(s_{h+i} - s_i \right) \Delta u_{k-i} \tag{1.6}$$

Este método (DMC) solo es válido para este tipo de sistemas

Para predicciones desde *1* hasta h

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & s_1 \\ 0 & 0 & \cdots & s_1 & s_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & s_1 & \cdots & s_{h-2} & s_{h-1} \\ s_1 & s_2 & \cdots & s_{h-1} & s_h \end{bmatrix}_{h\times h} \begin{bmatrix} \Delta u_{k+h-1} \\ \Delta u_{k+h-2} \\ \vdots \\ \Delta u_{k+1} \\ \Delta u_k \end{bmatrix}_{h\times 1} +$$

$$+\begin{bmatrix} 1 & -(s_2 - s_1) & \cdots & -(s_N - s_{N-1}) & -(s_{N+1} - s_N) \\ 1 & -(s_3 - s_1) & \cdots & -(s_{N+1} - s_{N-1}) & -(s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & -(s_h - s_1) & \cdots & -(s_{h+N-2} - s_{N-1}) & -(s_{h+N-1} - s_N) \\ 1 & -(s_{h+1} - s_1) & \cdots & -(s_{h+N-1} - s_{N-1}) & -(s_{h+N} - s_N) \end{bmatrix}_{h\times(N+1)} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{(N+1)\times 1} \tag{1.7}$$

Otra simplificación es suponer que luego de N_u muestras, la acción de control permanece constante.

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & \cdots & s_1 \\ 0 & \cdots & s_2 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{h-1} \\ s_{h-N_u+1} & \cdots & s_h \end{bmatrix}_{h\times N_u} \begin{bmatrix} \Delta u_{k+Nu-1} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u\times 1} +$$

$$+\begin{bmatrix} 1 & (s_1 - s_2) & \cdots & (s_{N-1} - s_N) & (s_N - s_{N+1}) \\ 1 & (s_1 - s_3) & \cdots & (s_{N-1} - s_{N+1}) & (s_N - s_{N+2}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_1 - s_h) & \cdots & (s_{N-1} - s_{h+N-2}) & (s_N - s_{h+N-1}) \\ 1 & (s_1 - s_{h+1}) & \cdots & (s_{N-1} - s_{h+N-1}) & (s_N - s_{h+N}) \end{bmatrix}_{h\times(N+1)} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{(N+1)\times 1}$$

Más compacta

$$\hat{\mathbf{Y}}_{k+1} = \mathbf{F}_h \cdot \mathbf{\Delta U}_k + \mathbf{G}_h \cdot \mathbf{M}_k \tag{1.8}$$

donde

$$\hat{\mathbf{Y}}_{k+1} = \begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} \quad \mathbf{\Delta U}_k = \begin{bmatrix} \Delta u_{k+Nu-1} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u\times 1} \quad \mathbf{M}_k = \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{(N+1)\times 1}$$

$$\mathbf{F}_h = \begin{bmatrix} 0 & \cdots & s_1 \\ 0 & \cdots & s_2 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{h-1} \\ s_{h-N_u+1} & \cdots & s_h \end{bmatrix}_{h\times N_u} \quad \mathbf{G}_h = \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h - s_1) & \cdots & (s_{h+N-2} - s_{N-1}) & (s_{h+N-1} - s_N) \\ 1 & (s_{h+1} - s_1) & \cdots & (s_{h+N-1} - s_{N-1}) & (s_{h+N} - s_N) \end{bmatrix}_{h\times(N+1)}$$

1.4. Objetivo de Control.

Puede ser cualquiera basado en la predicción

Ver Matlab 203

EXAMPLE 20.2

Derive a predictive control law that is based on the following concept. A single control move, $\Delta u(k)$, is calculated so that the J-step-ahead prediction is equal to the set point, that is, $\hat{y}(k + J) = y_{sp}$ where integer J is a tuning parameter. This sampling instant, $k + J$, is referred to as a *coincidence point*. Assume that u is held constant after the single control move, so that $\Delta u(k + i) = 0$ for $i > 0$.

SOLUTION

In the proposed predictive control strategy, only a single prediction for J steps ahead is considered. Thus, we let $j = J$ in Eq. 20-12. Similarly, because we are only interested in calculating the current control move, $\Delta u(k)$, the future control moves in Eq. 20-12 are set equal to zero: $\Delta u(k + J - i) = 0$ for $i = 1, 2, \ldots, J - 1$. Thus, (20-12) reduces to

$$\hat{y}(k + J) = S_J \Delta u(k) + \hat{y}^o(k + J) \qquad (20\text{-}13)$$

Setting $\hat{y}(k + J) = y_{sp}$ and rearranging gives the desired predictive controller:

$$\Delta u(k) = \frac{y_{sp} - \hat{y}^o(k + J)}{S_J} \qquad (20\text{-}14)$$

The predicted unforced response $\hat{y}^o(k + J)$ can be calculated from Eq. 20-11 with $j = J$.

The control law in (20-14) is based on a single prediction that is made for J steps in the future. Note that the control law can be interpreted as the inverse of the predictive model in (20-13).

EXAMPLE 20.3

Apply the predictive control law of Example 20.2 to a fifth-order process:

$$\frac{Y(s)}{U(s)} = \frac{1}{(5s + 1)^5} \qquad (20\text{-}15)$$

Evaluate the effect of tuning parameter J on the set-point responses for values of $J = 3, 4, 6$, and 8 and $\Delta t = 5$ min.

SOLUTION

The y and u responses for a unit set-point change at $t = 0$ are shown in Figs. 20.4 and 20.5, respectively. As J increases, the y responses become more sluggish while the u responses become smoother. These trends occur because larger values of J allow the predictive controller more time before the J-step ahead prediction $\hat{y}(k + J)$ must equal the set point. Consequently, less strenuous control action is required. The Jth step-response coefficient S_J increases monotonically as J increases. Consequently, the input moves calculated from (20-14) tend to become smaller as S_J increases. (The u responses for $J = 4$ and 8 are omitted from Fig. 20.5.)

1.4.1. Objetivo de Control General.

Definir una referencia

$$\mathbf{R}_{k+1} = \begin{bmatrix} r_{k+1} \\ \vdots \\ r_{k+h} \end{bmatrix} \tag{1.9}$$

Definir un funcional

$$J = \left[\mathbf{R}_{k+1} - \hat{\mathbf{Y}}_{k+1}\right]^T \left[\mathbf{R}_{k+1} - \hat{\mathbf{Y}}_{k+1}\right] + \lambda \boldsymbol{\Delta}\mathbf{U}_k{}^T \boldsymbol{\Delta}\mathbf{U}_k \tag{1.10}$$

Minimizarlo respecto a la acción de control

$$\hat{\mathbf{Y}}_{k+1} = \mathbf{F}_h \bullet \boldsymbol{\Delta}\mathbf{U}_k + \mathbf{G}_h \bullet \mathbf{M}_k \tag{1.11}$$

$$J = \left[\mathbf{R}_{k+1} - \mathbf{F}_h \bullet \boldsymbol{\Delta}\mathbf{U}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right]^T \left[\mathbf{R}_{k+1} - \mathbf{F}_h \bullet \boldsymbol{\Delta}\mathbf{U}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right] + \lambda \boldsymbol{\Delta}\mathbf{U}_k{}^T \boldsymbol{\Delta}\mathbf{U}_k \tag{1.12}$$

$$\frac{\partial J}{\partial \boldsymbol{\Delta}\mathbf{U}} = -2\mathbf{F}_h{}^T \left[\mathbf{R}_{k+1} - \mathbf{F}_h \bullet \boldsymbol{\Delta}\mathbf{U}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right] + 2\lambda \boldsymbol{\Delta}\mathbf{U}_k \tag{1.13}$$

$$\boldsymbol{\Delta}\mathbf{U}_k{}^* = \left[\mathbf{F}_h{}^T \mathbf{F}_h + \lambda \mathbf{I}\right]^{-1} \mathbf{F}_h{}^T \left[\mathbf{R}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right] \tag{1.14}$$

el regulador resulta

$$\mathbf{\Delta U}_k^{\ *} = \mathbf{Q}_r \left[\mathbf{R}_{k+1} - \mathbf{E}_k \right] \qquad (1.15)$$

Donde

$$\mathbf{Q}_r = \left[\mathbf{F}_h^{\ T} \mathbf{F}_h + \lambda \mathbf{I} \right]^{-1} \mathbf{F}_h^{\ T} \quad , \quad \mathbf{E}_k = \mathbf{G}_h \bullet \mathbf{M}_k \qquad (1.16)$$

Solo se necesita la última fila de $\mathbf{\Delta U}^*$

Para $h=1,\ N_u=1$

$$\hat{y}_{k+1}=s_1\Delta u_k+\begin{bmatrix}1 & (s_2-s_1) & \cdots & (s_N-s_{N-1}) & (s_{N+1}-s_N)\end{bmatrix}\begin{bmatrix}y_k\\ \Delta u_{k-1}\\ \vdots\\ \Delta u_{k-N-1}\\ \Delta u_{k-N}\end{bmatrix}$$

$$\hat{\mathbf{Y}}_{k+1}=\mathbf{F}_h\bullet\mathbf{\Delta U}_k+\mathbf{G}_h\bullet\mathbf{M}_k$$

donde

$$\hat{\mathbf{Y}}_{k+1}=\hat{y}_{k+1}\quad \mathbf{\Delta U}_k=\Delta u_k\quad \mathbf{M}_k=\begin{bmatrix}y_k\\ \Delta u_{k-1}\\ \vdots\\ \Delta u_{k-N-1}\\ \Delta u_{k-N}\end{bmatrix}_{N\times 1}$$

$$\mathbf{F}_h=s_1\quad \mathbf{G}_h=\begin{bmatrix}1 & (s_2-s_1) & \cdots & (s_N-s_{N-1}) & (s_{N+1}-s_N)\end{bmatrix}$$

Controlador

$$\mathbf{R}_{k+1} = r_{k+1}$$

$$\Delta u_k = \frac{s_1}{s_1{}^2 + \lambda}\left[r_{k+1} - y_k - (s_2 - s_1)\Delta u_{k-1} - \cdots - (s_N - s_{N-1})\Delta u_{k-N-1} - (s_{N+1} - s_N)\Delta u_{k-N}\right]$$

Para $h=2, N_u=1$

$$\hat{y}_{k+1} = s_1 \Delta u_k + \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}$$

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \Delta u_k + \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \end{bmatrix}$$

$$\hat{\mathbf{Y}}_{k+1} = \mathbf{F}_h \bullet \mathbf{\Delta U}_k + \mathbf{G}_h \bullet \mathbf{M}_k$$

donde

$$\hat{\mathbf{Y}}_{k+1} = \begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \end{bmatrix} \quad \mathbf{\Delta U}_k = \Delta u_k \quad \mathbf{M}_k = \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N \times 1}$$

$$\mathbf{F}_h = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \quad \mathbf{G}_h = \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \end{bmatrix}$$

Controlador

$$\mathbf{R}_k = \begin{bmatrix} r_{k+1} \\ r_{k+2} \end{bmatrix}$$

$$\mathbf{Q}_r = \left[\mathbf{F}_h{}^T \mathbf{F}_h + \lambda \mathbf{I}\right]^{-1} \mathbf{F}_h{}^T = \frac{1}{s_1^2 + s_2^2 + \lambda} \begin{bmatrix} s_1 & s_2 \end{bmatrix}$$

$$\Delta u_k = \frac{1}{s_1{}^2 + s_2{}^2 + \lambda} \begin{bmatrix} s_1 & s_2 \end{bmatrix} \left[\begin{bmatrix} r_{k+1} \\ r_{k+2} \end{bmatrix} - \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix} \right]$$

1.4.2. Ejemplo Multivariable

sistema original (pensar en los cuatro tachitos)

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{21}(s) \\ G_{12}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{k_{11}}{\tau_{11}s+1} & \dfrac{k_{21}}{\tau_{21}s+1} \\ \dfrac{k_{12}}{\tau_{12}s+1} & \dfrac{k_{22}}{\tau_{22}s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y_1 = \frac{k_{11}}{\tau_{11}s+1}u_1 + \frac{k_{21}}{\tau_{21}s+1}u_2$$

$$y_2 = \frac{k_{12}}{\tau_{12}s+1}u_1 + \frac{k_{22}}{\tau_{22}s+1}u_2$$

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{h_1} s_{11i}\Delta u_{1k+h_1-i} + y_{1k} - \sum_{i=1}^{\infty}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_1} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{\infty}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{h_2} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{\infty}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_2} s_{22i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{\infty}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Si la respuesta al escalón del sistema converge a un valor final, se podría suponer que la predicción se aproxima a

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{h_1} s_{11i}\Delta u_{1k+h_1-i} + y_{1k} - \sum_{i=1}^{N_1}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_1} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{N_1}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{h_2} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{N_2}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_2} s_{22i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{N_2}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Otra simplificación es suponer que luego de N_u muestras, la acción de control permanece constante.

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{N_{1u}} s_{11i}\Delta u_{1k+N_{1u}-i} + y_{1k} - \sum_{i=1}^{N_1}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{N_{2u}} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{N_1}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{N_{1u}} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{N_2}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{N_{2u}} s_{12i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{N_2}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Para predicciones desde *1* hasta *h*

$$\begin{bmatrix} \hat{\mathbf{Y}}_1 \\ \hat{\mathbf{Y}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta}\mathbf{U}_1 \\ \boldsymbol{\Delta}\mathbf{U}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{\mathbf{12}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{M}_{11} \\ \mathbf{M}_{12} \\ \mathbf{M}_{21} \\ \mathbf{M}_{22} \end{bmatrix}$$

Siendo

$$\hat{\mathbf{Y}}_{1k} = \begin{bmatrix} \hat{y}_{1k+1} \\ \hat{y}_{1k+2} \\ \vdots \\ \hat{y}_{1k+h-1} \\ \hat{y}_{1k+h} \end{bmatrix}_{h\times 1} \quad \hat{\mathbf{Y}}_{2k} = \begin{bmatrix} \hat{y}_{2k+1} \\ \hat{y}_{2k+2} \\ \vdots \\ \hat{y}_{2k+h-1} \\ \hat{y}_{2k+h} \end{bmatrix}_{h\times 1} \quad \Delta U_{1k} = \begin{bmatrix} \Delta u_{1k+Nu} \\ \vdots \\ \Delta u_{1k} \end{bmatrix}_{N_u\times 1} \quad \Delta U_{2k} = \begin{bmatrix} \Delta u_{2k+Nu} \\ \vdots \\ \Delta u_{2k} \end{bmatrix}_{N_u\times 1}$$

$$M_{11k}=\begin{bmatrix} y_{1k} \\ \Delta u_{1k-1} \\ \vdots \\ \Delta u_{1k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1} \quad M_{22k}=\begin{bmatrix} y_{2k} \\ \Delta u_{2k-1} \\ \vdots \\ \Delta u_{2k-N-1} \\ \Delta u_{2k-N} \end{bmatrix}_{N\times 1} \quad M_{12k}=\begin{bmatrix} \Delta u_{2k-1} \\ \vdots \\ \Delta u_{2k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1} \quad M_{21k}=\begin{bmatrix} \Delta u_{1k-1} \\ \vdots \\ \Delta u_{1k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1}$$

(1.17)

$$F_{11}=\begin{bmatrix} 0 & \cdots & s_{111} \\ 0 & \cdots & s_{112} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{11h-1} \\ s_{111} & \cdots & s_{11h} \end{bmatrix}_{h\times N_u} \quad F_{12}=\begin{bmatrix} 0 & \cdots & s_{121} \\ 0 & \cdots & s_{122} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{12h-1} \\ s_{121} & \cdots & s_{12h} \end{bmatrix}_{h\times N_u} \quad F_{21}=\begin{bmatrix} 0 & \cdots & s_{211} \\ 0 & \cdots & s_{212} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{21h-1} \\ s_{211} & \cdots & s_{21h} \end{bmatrix}_{h\times N_u} \quad F_{22}=\begin{bmatrix} 0 & \cdots & s_{2} \\ 0 & \cdots & s_{2} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{22} \\ s_{221} & \cdots & s_{2} \end{bmatrix}$$

(1.18)

$$G_h=\begin{bmatrix} 1 & (s_2-s_1) & \cdots & (s_N-s_{N-1}) & (s_{N+1}-s_N) \\ 1 & (s_3-s_1) & \cdots & (s_{N+1}-s_{N-1}) & (s_{N+2}-s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h-s_1) & \cdots & (s_{h+N-2}-s_{N-1}) & (s_{h+N-1}-s_N) \\ 1 & (s_{h+1}-s_1) & \cdots & (s_{h+N-1}-s_{N-1}) & (s_{h+N}-s_N) \end{bmatrix}_{h\times N}$$

$$
\begin{bmatrix} \hat{y}_{1k+1} \\ \hat{y}_{1k+2} \\ \vdots \\ \hat{y}_{1k+h_1-1} \\ \hat{y}_{1k+h_1} \\ \hat{y}_{2k+1} \\ \hat{y}_{2k+2} \\ \vdots \\ \hat{y}_{2k+h_2-1} \\ \hat{y}_{2k+h_2} \end{bmatrix}_{(h_1+h_2)\times 1}
=
\begin{bmatrix}
0 & 0 & \cdots & 0 & s_{111} & 0 & 0 & \cdots & 0 & s_{211} \\
0 & 0 & \cdots & s_{111} & s_{112} & 0 & 0 & \cdots & s_{211} & s_{212} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & s_{111} & \cdots & s_{11h_1+N_{1u}-2} & s_{11h_1+N_{1u}-1} & & s_{211} & \cdots & s_{21h_1+N_{2u}-2} & s_{21h_1+N_{2u}-1} \\
s_{111} & s_{112} & \cdots & s_{11h_1+N_{1u}-1} & s_{11h_1+N_{1u}} & s_{211} & s_{212} & \cdots & s_{21h_1+N_{2u}-1} & s_{21h_1+N_{2u}}
\end{bmatrix}_{(h_1+h_2)\times(N_{1u}+N_{2u})}
\begin{bmatrix} \Delta u_{1k+N_{1u}} \\ \Delta u_{1k+N_{1u}-1} \\ \vdots \\ \Delta u_{1k+1} \\ \Delta u_{1k} \\ \Delta u_{2k+N_{2u}} \\ \Delta u_{2k+N_{2u}-1} \\ \vdots \\ \Delta u_{2k+1} \\ \Delta u_{2k} \end{bmatrix}_{(N_{1u}+N_{2u})\times 1}
+
$$

$$
+\begin{bmatrix}
1 & (s_{d+2}-s_1) & \cdots & (s_{d+N}-s_{N-1}) & (s_{d+N+1}-s_N) \\
1 & (s_{d+3}-s_1) & \cdots & (s_{d+N+1}-s_{N-1}) & (s_{d+N+2}-s_N) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
1 & (s_{d+h}-s_1) & \cdots & (s_{d+h+N-2}-s_{N-1}) & (s_{d+h+N-1}-s_N) \\
1 & (s_{d+h+1}-s_1) & \cdots & (s_{d+h+N-1}-s_{N-1}) & (s_{d+h+N}-s_N)
\end{bmatrix}_{h\times N}
\begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1}
$$

Más compacta

$$\hat{\mathbf{Y}}_k = \mathbf{F}_h \times \mathbf{\Delta U}_k + \mathbf{G}_h \times \mathbf{M}_k$$

1.5. Objetivo de Control.

Definir una referencia

$$R_k = \begin{bmatrix} r_{k+d+1} \\ \vdots \\ r_{k+d+h} \end{bmatrix}$$

Y minimizar el funcional

$$\hat{\mathbf{Y}}_k = \mathbf{F}_h \bullet \Delta\mathbf{U}_k + \mathbf{G}_h \bullet \mathbf{M}_k$$

$$J = \left[R_k - \hat{Y}_k\right]^T \left[R_k - \hat{Y}_k\right] + \lambda \Delta U_k{}^T \Delta U_k$$

$$J = \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k\right]^T \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k\right] + \lambda \Delta U_k{}^T \Delta U_k$$

$$\frac{\partial J}{\partial \Delta\mathbf{U}} = -2\mathbf{F}_h{}^T \left[\mathbf{R}_k - \mathbf{F}_h \bullet \Delta\mathbf{U}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right] + 2\lambda \Delta\mathbf{U}_k$$

$$\Delta\mathbf{U}_k{}^* = \left[\mathbf{F}_h{}^T \mathbf{F}_h + \lambda \mathbf{I}\right]^{-1} \mathbf{F}_h{}^T \left[\mathbf{R}_k - \mathbf{G}_h \bullet \mathbf{M}_k\right]$$

el regulador resulta

$$\Delta U_k{}^{*} = R_r\left[R_k - E_k\right]$$

Donde

$$\mathbf{R}_r = \left[\mathbf{F}_h{}^T\mathbf{F}_h + \lambda\mathbf{I}\right]^{-1}\mathbf{F}_h{}^T \quad , \quad \mathbf{E}_k = \mathbf{G}_h \times \mathbf{M}_k$$

sistema original

$$A\left(q^{-1}\right)y_k = q^{-3}B\left(q^{-1}\right)u_{k-1} + n'_k \qquad (1.19)$$

Modelo CARIMA

$$A\left(q^{-1}\right)\left(1-q^{-1}\right)y_k = q^{-3}B\left(q^{-1}\right)\left(1-q^{-1}\right)u_{k-1} + \left(1-q^{-1}\right)n'_k \qquad (1.20)$$

$$A\left(q^{-1}\right)\left(1-q^{-1}\right)y_k = q^{-3}B\left(q^{-1}\right)\Delta u_{k-1} + n_k \qquad (1.21)$$

$$\left(1+a_1q^{-1}+a_2q^{-2}\right)y_k = q^{-3}\left(b_0+b_1q^{-1}\right)u_{k-1} + n'_k \qquad (1.22)$$

Donde n es una perturbación

$$y_{k+4} = -a_1y_{k+3} - a_2y_{k+2} + b_0u_k + b_1u_{k-1} + n'_{k+4} \qquad (1.23)$$

ó

$$y_{k+4} = \left(1-a_1\right)y_{k+3} + \left(a_1-a_2\right)y_{k+2} + a_2y_{k+1} + b_0\Delta u_k + b_1\Delta u_{k-1} + n_{k+4} \qquad (1.24)$$

$$d=3 \quad N_1=4 \quad N_2=10 \quad N_u=2 \qquad (1.25)$$

$$\begin{aligned} y_{k+d+1} = {} & \left(1-a_1\right)\left[\left(1-a_1\right)y_{k+2} + \left(a_1-a_2\right)y_{k+1} + a_2y_k + b_0\Delta u_{k-1} + b_1\Delta u_{k-2}\right] + \\ & \left(a_1-a_2\right)y_{k+2} + a_2y_{k+1} + b_0\Delta u_k + b_1\Delta u_{k-1} + n_{k+d+1} \end{aligned} \qquad (1.26)$$

$$
\begin{aligned}
y_{k+d+1} &= b_0 \Delta u_k + \left[\left(1-a_1\right) b_0 + b_1 \right] \Delta u_{k-1} + \left(1-a_1\right) b_1 \Delta u_{k-2} + \\
&+ \left[\left(1-a_1\right)^2 + \left(a_1 - a_2\right) \right] y_{k+2} + \left[\left(1-a_1\right)\left(a_1 - a_2\right) + a_2 \right] y_{k+1} + \left(1-a_1\right) a_2 y_k + n_{k+d+1}
\end{aligned}
\tag{1.27}
$$

Respuesta al escalón

1	$s_{d-2} = 0$
2	$s_{d-1} = 0$
3	$s_{d} = 0$
4	$s_{d+1} = b_0$
5	$s_{d+2} = (1-a_1)b_0 + b_1$
6	$s_{d+3} = \left[1-a_2-(1-a_1)a_1\right]b_0 + (1-a_1)b_1$

$$\begin{aligned} y_{k+d+1} &= s_{d+1}\Delta u_k + s_{d+2}\Delta u_{k-1} + (1-a_1)b_1\Delta u_{k-2} + \left[(1-a_1)^2 + (a_1-a_2)\right]y_{k+2} + \\ &+ \left[(1-a_1)(a_1-a_2)+a_2\right]y_{k+1} + (1-a_1)a_2 y_k + n_{k+d+1} \end{aligned} \tag{1.28}$$

reemplazando varias veces resulta

$$y_{k+d+1} = s_{d+1}\Delta u_k + s_{d+2}\Delta u_{k-1} + s_{d+3}\Delta u_{k-2} + s_{d+4}\Delta u_{k-3} + \cdots + n_{k+d+1} \tag{1.29}$$

Para predecir el valor de la salida en $k+4$ se necesita conocer $n_{k+d+1}, n_{k+d} \cdots n_k$

Se puede considerar que

$$n_{k+d+1} = n_{k+d} = \cdots = n_k = y_k - s_{d+1}\Delta u_{k-d-1} - s_{d+2}\Delta u_{k-d-2} - s_{d+3}\Delta u_{k-d-3} - s_{d+4}\Delta u_{k-d-4} - \cdots \quad (1.30)$$

La predicción resultaría

$$\hat{y}_{k+d+h} = s_{d+1}\Delta u_{k+h-1} + s_{d+2}\Delta u_{k+h-2} + s_{d+3}\Delta u_{k+h-3} + s_{d+4}\Delta u_{k+h-4} + \cdots + n_k \quad (1.31)$$

$$\begin{aligned}\hat{y}_{k+d+h} = {} & s_{d+1}\Delta u_{k+h-1} + s_{d+2}\Delta u_{k+h-2} + s_{d+3}\Delta u_{k+h-3} + s_{d+4}\Delta u_{k+h-4} + \cdots \\ & + y_k - s_{d+1}\Delta u_{k-d-1} - s_{d+2}\Delta u_{k-d-2} - s_{d+3}\Delta u_{k-d-3} - s_{d+4}\Delta u_{k-d-4} - \cdots\end{aligned} \quad (1.32)$$

$$\begin{aligned}\hat{y}_{k+d+h} = {} & s_{d+1}\Delta u_{k+h-1} + s_{d+2}\Delta u_{k+h-2} + s_{d+3}\Delta u_{k+h-3} + s_{d+4}\Delta u_{k+h-4} + \cdots \\ & + s_{d+h}\Delta u_k + s_{d+h+1}\Delta u_{k-1} + \cdots + s_{2d+h+1}\Delta u_{k-d-1} + s_{2d+h+2}\Delta u_{k-d-2} + \cdots \\ & + y_k - s_{d+1}\Delta u_{k-d-1} - s_{d+2}\Delta u_{k-d-2} - s_{d+3}\Delta u_{k-d-3} - s_{d+4}\Delta u_{k-d-4} - \cdots\end{aligned} \quad (1.33)$$

$$\begin{aligned}\hat{y}_{k+d+h} = {} & s_{d+1}\Delta u_{k+h-1} + s_{d+2}\Delta u_{k+h-2} + s_{d+3}\Delta u_{k+h-3} + s_{d+4}\Delta u_{k+h-4} + \cdots + s_{d+h}\Delta u_k + \\ & + y_k + \left(s_{d+h+1} - s_1\right)\Delta u_{k-1} + \cdots + \left(s_{2d+h+1} - s_{d+1}\right)\Delta u_{k-d-1} + \left(s_{2d+h+2} - s_{d+2}\right)\Delta u_{k-d-2} + \cdots\end{aligned} \quad (1.34)$$

En forma compacta,

$$\hat{y}_{k+d+h} = \sum_{i=1}^{h} s_{d+i}\Delta u_{k+h-i} + y_k - \sum_{i=1}^{\infty}\left(s_{d+h+i} - s_i\right)\Delta u_{k-i} \quad (1.35)$$

Si la respuesta al escalón del sistema converge a un valor final, se cumple

$$s_{d+h+i} - s_i \to 0, \quad i \to \infty \quad (1.36)$$

Se podría suponer que la predicción se aproxima a

$$\hat{y}_{k+d+h} = \sum_{i=1}^{h} s_{d+i} \Delta u_{k+h-i} + y_k - \sum_{i=1}^{N} \left(s_{d+h+i} - s_i \right) \Delta u_{k-i} \qquad (1.37)$$

Este método (DMC) solo es válido para este tipo de sistemas

Para predicciones desde *1* hasta *h*

$$\begin{bmatrix} \hat{y}_{k+d+1} \\ \hat{y}_{k+d+2} \\ \vdots \\ \hat{y}_{k+d+h-1} \\ \hat{y}_{k+d+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & s_{d+1} \\ 0 & 0 & \cdots & s_{d+1} & s_{d+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & s_{d+1} & \cdots & s_{d+h-2} & s_{d+h-1} \\ s_{d+1} & s_{d+2} & \cdots & s_{d+h-1} & s_{d+h} \end{bmatrix}_{h\times h} \begin{bmatrix} \Delta u_{k+h} \\ \Delta u_{k+h-1} \\ \vdots \\ \Delta u_{k+1} \\ \Delta u_{k} \end{bmatrix}_{h\times 1} +$$

$$+\begin{bmatrix} 1 & (s_{d+2}-s_1) & \cdots & (s_{d+N}-s_{N-1}) & (s_{d+N+1}-s_N) \\ 1 & (s_{d+3}-s_1) & \cdots & (s_{d+N+1}-s_{N-1}) & (s_{d+N+2}-s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_{d+h}-s_1) & \cdots & (s_{d+h+N-2}-s_{N-1}) & (s_{d+h+N-1}-s_N) \\ 1 & (s_{d+h+1}-s_1) & \cdots & (s_{d+h+N-1}-s_{N-1}) & (s_{d+h+N}-s_N) \end{bmatrix}_{h\times N} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1} \quad (1.38)$$

Otra simplificación es suponer que luego de N_u muestras, la acción de control permanece constante.

$$\begin{bmatrix} \hat{y}_{k+d+1} \\ \hat{y}_{k+d+2} \\ \vdots \\ \hat{y}_{k+d+h-1} \\ \hat{y}_{k+d+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & \cdots & s_{d+1} \\ 0 & \cdots & s_{d+2} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{d+h-1} \\ s_{d+1} & \cdots & s_{d+h} \end{bmatrix}_{h\times N_u} \begin{bmatrix} \Delta u_{k+Nu} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u\times 1} +$$

$$+\begin{bmatrix} 1 & (s_{d+2}-s_1) & \cdots & (s_{d+N}-s_{N-1}) & (s_{d+N+1}-s_N) \\ 1 & (s_{d+3}-s_1) & \cdots & (s_{d+N+1}-s_{N-1}) & (s_{d+N+2}-s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_{d+h}-s_1) & \cdots & (s_{d+h+N-2}-s_{N-1}) & (s_{d+h+N-1}-s_N) \\ 1 & (s_{d+h+1}-s_1) & \cdots & (s_{d+h+N-1}-s_{N-1}) & (s_{d+h+N}-s_N) \end{bmatrix}_{h\times N} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1} \tag{1.39}$$

Más compacta

$$\hat{\mathbf{Y}}_k = \mathbf{F}_h \cdot \boldsymbol{\Delta}\mathbf{U}_k + \mathbf{G}_h \cdot \mathbf{M}_k \tag{1.40}$$

donde

$$\hat{\mathbf{Y}}_k = \begin{bmatrix} \hat{y}_{k+d+1} \\ \hat{y}_{k+d+2} \\ \vdots \\ \hat{y}_{k+d+h-1} \\ \hat{y}_{k+d+h} \end{bmatrix}_{h\times 1} \qquad \boldsymbol{\Delta}\mathbf{U}_k = \begin{bmatrix} \Delta u_{k+Nu} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u\times 1} \qquad \mathbf{M}_k = \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1} \tag{1.41}$$

$$\mathbf{F}_h = \begin{bmatrix} 0 & \cdots & s_{d+1} \\ 0 & \cdots & s_{d+2} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{d+h-1} \\ s_{d+1} & \cdots & s_{d+h} \end{bmatrix}_{h\times N_u} \qquad \mathbf{G}_h = \begin{bmatrix} 1 & (s_{d+2}-s_1) & \cdots & (s_{d+N}-s_{N-1}) & (s_{d+N+1}-s_N) \\ 1 & (s_{d+3}-s_1) & \cdots & (s_{d+N+1}-s_{N-1}) & (s_{d+N+2}-s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_{d+h}-s_1) & \cdots & (s_{d+h+N-2}-s_{N-1}) & (s_{d+h+N-1}-s_N) \\ 1 & (s_{d+h+1}-s_1) & \cdots & (s_{d+h+N-1}-s_{N-1}) & (s_{d+h+N}-s_N) \end{bmatrix}_{h\times N} \tag{1.42}$$

1.7. Objetivo de Control Generalizado.

Definir una referencia

$$\mathbf{R}_k = \begin{bmatrix} r_{k+d+1} \\ \vdots \\ r_{k+d+h} \end{bmatrix} \tag{1.43}$$

Y minimizar el funcional

$$\hat{\mathbf{Y}}_k = \mathbf{F}_h \cdot \Delta\mathbf{U}_k + \mathbf{G}_h \cdot \mathbf{M}_k \tag{1.44}$$

$$J = \left[\mathbf{R}_k - \hat{\mathbf{Y}}_k\right]^T \left[\mathbf{R}_k - \hat{\mathbf{Y}}_k\right] + \lambda \Delta\mathbf{U}_k{}^T \Delta\mathbf{U}_k \tag{1.45}$$

$$J = \left[\mathbf{R}_k - \mathbf{F}_h \cdot \Delta\mathbf{U}_k - \mathbf{G}_h \cdot \mathbf{M}_k\right]^T \left[\mathbf{R}_k - \mathbf{F}_h \cdot \Delta\mathbf{U}_k - \mathbf{G}_h \cdot \mathbf{M}_k\right] + \lambda \Delta\mathbf{U}_k{}^T \Delta\mathbf{U}_k \tag{1.46}$$

$$\frac{\partial J}{\partial \Delta\mathbf{U}} = -2\mathbf{F}_h{}^T \left[\mathbf{R}_k - \mathbf{F}_h \cdot \Delta\mathbf{U}_k - \mathbf{G}_h \cdot \mathbf{M}_k\right] + 2\lambda \Delta\mathbf{U}_k \tag{1.47}$$

$$\Delta\mathbf{U}_k{}^* = \left[\mathbf{F}_h{}^T \mathbf{F}_h + \lambda \mathbf{I}\right]^{-1} \mathbf{F}_h{}^T \left[\mathbf{R}_k - \mathbf{G}_h \cdot \mathbf{M}_k\right] \tag{1.48}$$

el regulador resulta

$$\mathbf{\Delta U}_k{}^* = \mathbf{K}_r\left[\mathbf{R}_k - \mathbf{E}_k\right] \tag{1.49}$$

Donde

$$\mathbf{K}_r = \left[\mathbf{F}_h{}^T\mathbf{F}_h + \lambda\mathbf{I}\right]^{-1}\mathbf{F}_h{}^T \quad , \quad \mathbf{E}_k = \mathbf{G}_h \cdot \mathbf{M}_k \tag{1.50}$$

Solo se necesita la última fila de $\mathbf{\Delta U}^*$

Pasos:

1- realizar la respuesta al escalón

2- elegir el período de muestreo: $T \cong 0{,}1\tau$ o $T \cong 0{,}5d$ el que dé más grande

3- elegir N, el *horizonte del modelo* o longitud de la respuesta al escalón y H, el horizonte de predicción, haciendo (es más o menos el tiempo de establecimiento)

$$N = H \cong 6\frac{\tau}{T}+1$$

4- elegir el horizonte de control, más grande que lo que tarda la respuesta al escalón en llegar al 60% del valor final, o

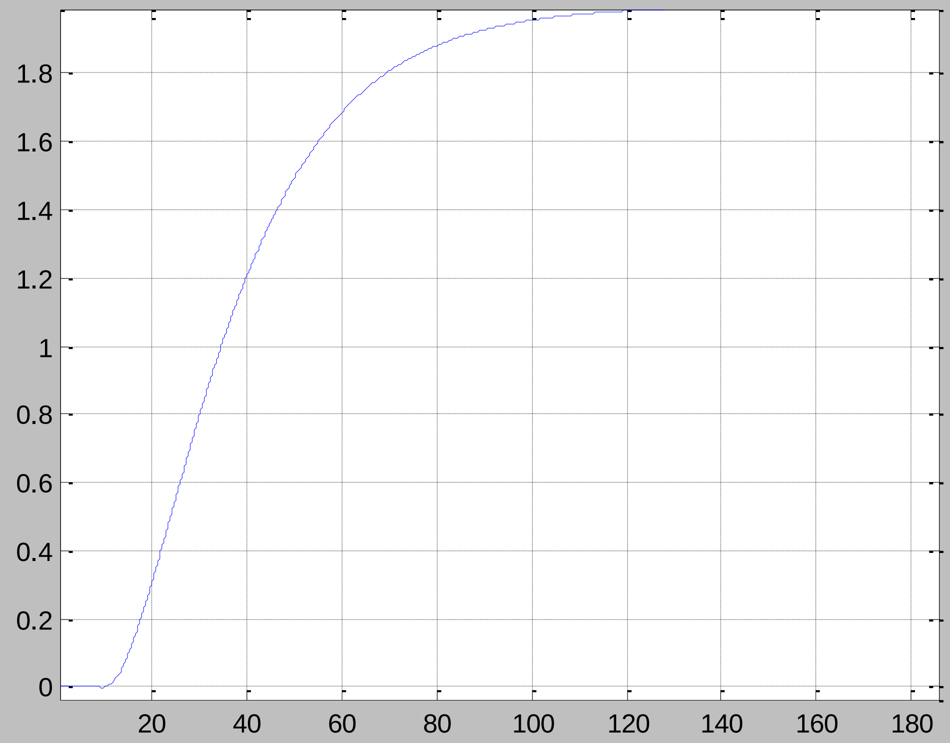
$$N_u \cong 2\frac{\tau}{T}+1$$

5- elegir el coeficiente $\lambda = \dfrac{N_u}{10}\left(\dfrac{3{,}5\tau}{T}+2-\dfrac{N_u-1}{2}\right)k_p$

1.7.1. Ejemplo Simple

sistema continuo original

$$G(s) = \frac{2}{(10s+1)(20s+1)} e^{-10s} \tag{1.51}$$

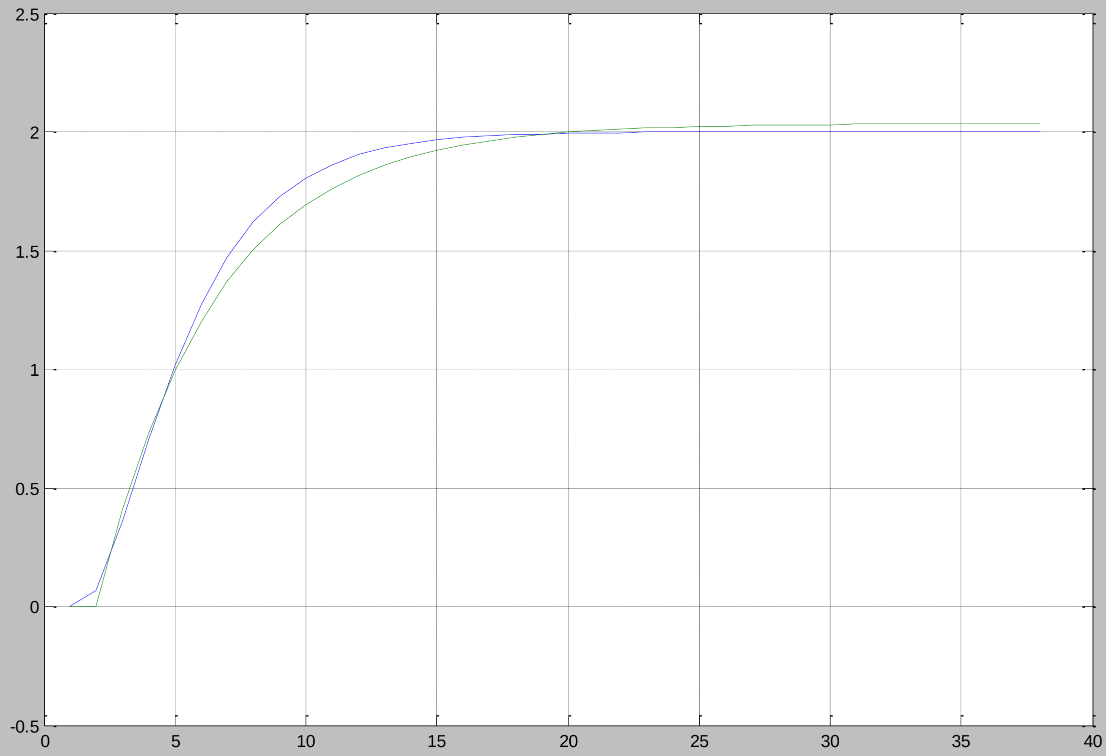


Se puede considerar un período de muestreo de

$$T = 7 \tag{1.52}$$

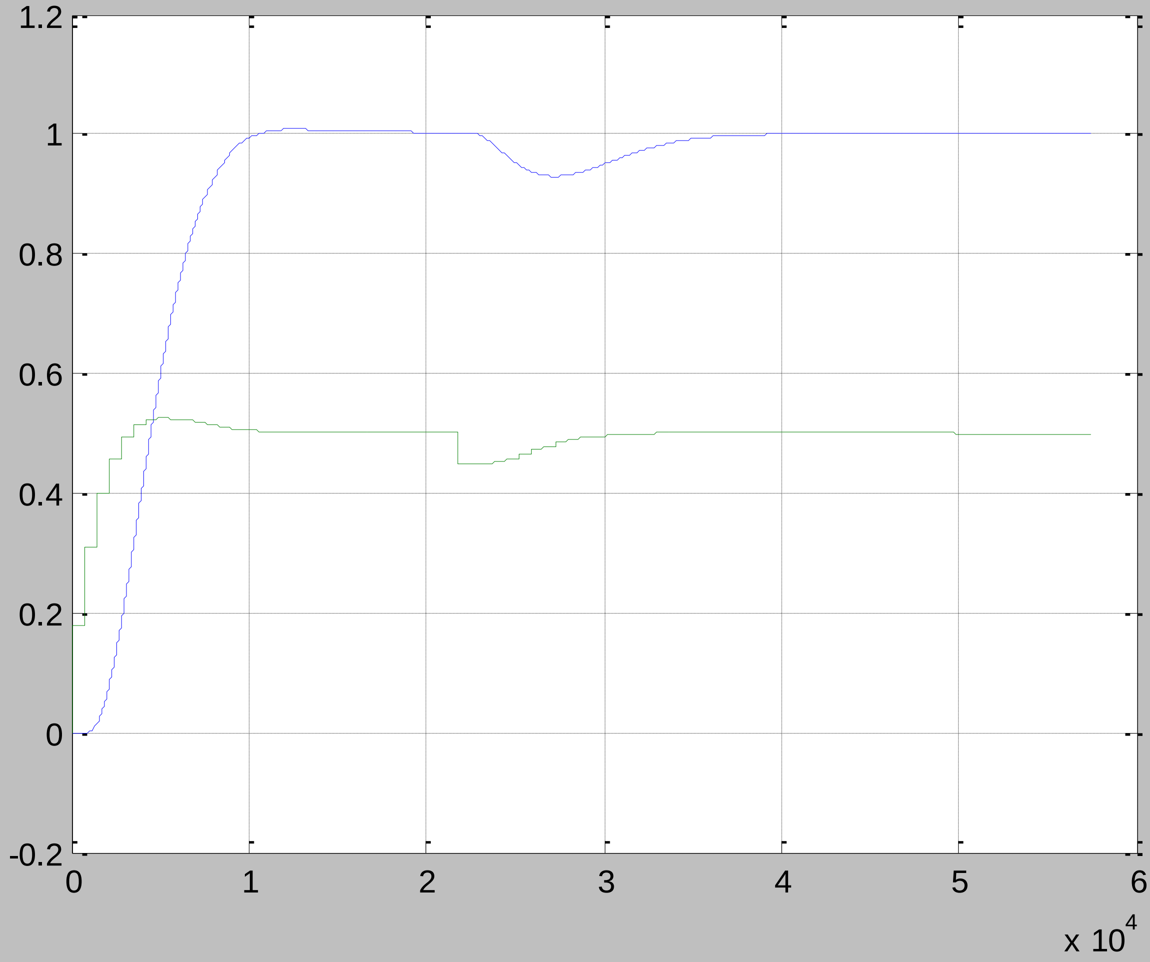
Una aproximación a un sistema de primer orden con retardo sería

$$y_{k+1} = 0{,}7996 y_k + 0{,}4076 u_{k-2} \tag{1.53}$$



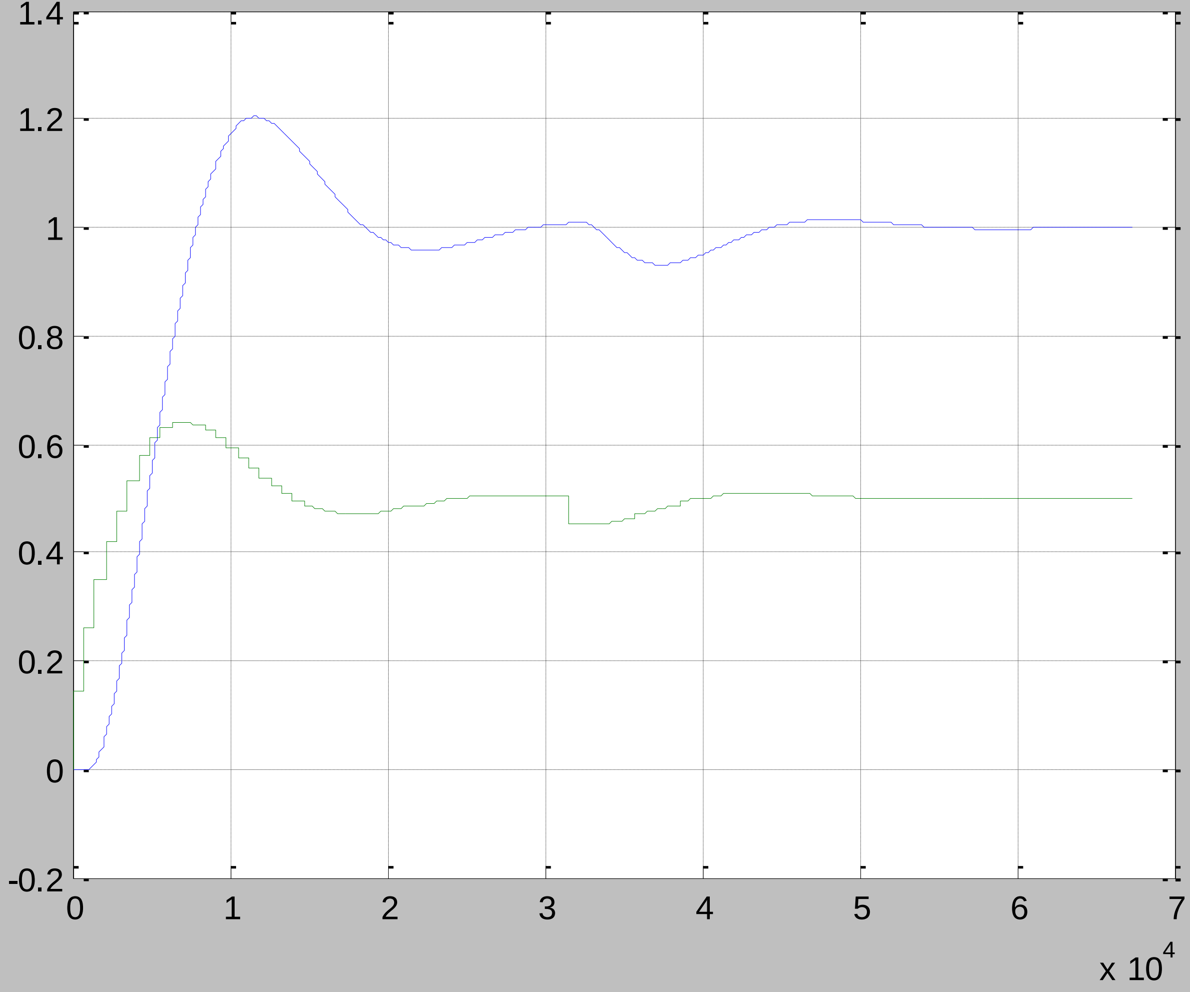
Según las reglas de selección se define

$$N = H = 19 \quad N_u = 5 \quad \lambda = 23,9 \tag{1.54}$$



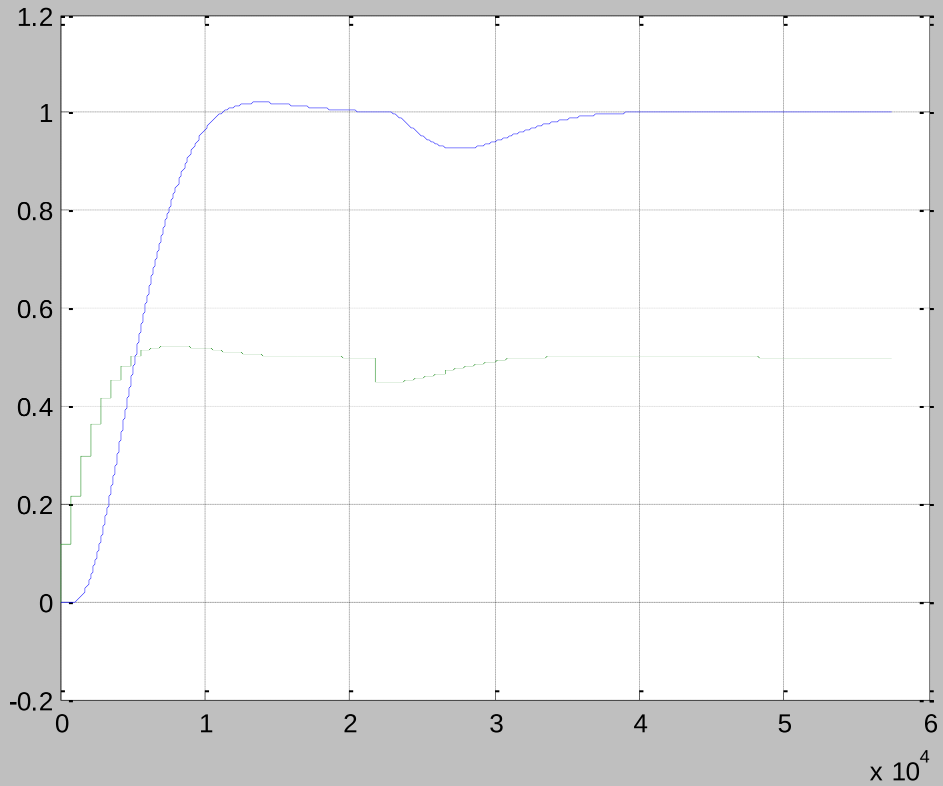
Influencia del Horizonte del Modelo

$$N = H = 5 \tag{1.55}$$



Influencia del Horizonte de Predicción

$$N = 19 \quad H = 3 \quad N_u = 4 \tag{1.56}$$



1.7.2. Ejemplo Sistema de Primer Orden

sistema original

$$\left(1-aq^{-1}\right)y_k = bu_{k-1} + n'_k$$

$$\left(1-aq^{-1}\right)\left(1-q^{-1}\right)y_k = b\left(1-q^{-1}\right)u_{k-1} + \left(1-q^{-1}\right)n'_k$$

$$\left(1-aq^{-1}\right)\left(1-q^{-1}\right)y_k = b\Delta u_{k-1} + n_k$$

$$\left(1-(1+a)q^{-1} + aq^{-2}\right)y_k = b\Delta u_{k-1} + n_k$$

Donde n es una perturbación

$$y_{k+1} = (1+a)y_k - ay_{k-1} + b\Delta u_k + n_{k+1}$$

Respuesta al escalón

1	$s_0 = 0$
2	$s_1 = b$
3	$s_2 =$
4	$s_3 =$
5	$s_4 =$

6	$s_5 =$

$$y_{k+1} = s_1 \Delta u_k + (1+a) y_k - a y_{k-1} + n_{k+1}$$

reemplazando varias veces resulta

$$y_{k+1} = s_1 \Delta u_k + s_2 \Delta u_{k-1} + s_3 \Delta u_{k-2} + s_4 \Delta u_{k-3} + \cdots + n_{k+1} \qquad (1.57)$$

Para predecir el valor de la salida en *k+1* se necesita conocer n_{k+1}, n_k

Se puede considerar que

$$n_{k+1} = n_k = y_k - s_1 \Delta u_{k-1} - s_2 \Delta u_{k-2} - s_3 \Delta u_{k-3} - s_4 \Delta u_{k-4} - \cdots$$

La predicción resultaría

$$\hat{y}_{k+h} = s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots + n_k \qquad (1.58)$$

$$\begin{aligned} \hat{y}_{k+h} &= s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots \\ &\quad + y_k - s_1 \Delta u_{k-1} - s_2 \Delta u_{k-2} - s_3 \Delta u_{k-3} - s_4 \Delta u_{k-4} - \cdots \end{aligned} \qquad (1.59)$$

$$\begin{aligned} \hat{y}_{k+h} &= s_1 \Delta u_{k+h-1} + s_2 \Delta u_{k+h-2} + s_3 \Delta u_{k+h-3} + s_4 \Delta u_{k+h-4} + \cdots \\ &\quad + y_k + \left(s_{h+1} - s_1\right) \Delta u_{k-1} + \left(s_{h+2} - s_2\right) \Delta u_{k-2} + \left(s_{h+3} - s_3\right) \Delta u_{k-3} + \cdots \end{aligned} \qquad (1.60)$$

En forma compacta,

$$\hat{y}_{k+h} = \sum_{i=1}^{h} s_i \Delta u_{k+h-i} + y_k - \sum_{i=1}^{\infty} \left(s_{h+i} - s_i \right) \Delta u_{k-i} \tag{1.61}$$

Si la respuesta al escalón del sistema converge a un valor final, se cumple

$$s_{d+h+i} - s_i \to 0, \quad i \to \infty \tag{1.62}$$

Se podría suponer que la predicción se aproxima a

$$\hat{y}_{k+h} = \sum_{i=1}^{h} s_i \Delta u_{k+h-i} + y_k - \sum_{i=1}^{N} \left(s_{h+i} - s_i \right) \Delta u_{k-i} \tag{1.63}$$

Este método (DMC) solo es válido para este tipo de sistemas

Para predicciones desde *1* hasta h

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & s_1 \\ 0 & 0 & \cdots & s_1 & s_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & s_1 & \cdots & s_{h-2} & s_{h-1} \\ s_1 & s_2 & \cdots & s_{h-1} & s_h \end{bmatrix}_{h\times h} \begin{bmatrix} \Delta u_{k+h} \\ \Delta u_{k+h-1} \\ \vdots \\ \Delta u_{k+1} \\ \Delta u_k \end{bmatrix}_{h\times 1} +$$

$$+ \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h - s_1) & \cdots & (s_{h+N-2} - s_{N-1}) & (s_{h+N-1} - s_N) \\ 1 & (s_{h+1} - s_1) & \cdots & (s_{h+N-1} - s_{N-1}) & (s_{h+N} - s_N) \end{bmatrix}_{h\times N} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1} \quad (1.64)$$

Otra simplificación es suponer que luego de N_u muestras, la acción de control permanece constante.

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} = \begin{bmatrix} 0 & \cdots & s_1 \\ 0 & \cdots & s_2 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{h-1} \\ s_1 & \cdots & s_h \end{bmatrix}_{h\times N_u} \begin{bmatrix} \Delta u_{k+Nu} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u\times 1} +$$

$$+\begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h - s_1) & \cdots & (s_{h+N-2} - s_{N-1}) & (s_{h+N-1} - s_N) \\ 1 & (s_{h+1} - s_1) & \cdots & (s_{h+N-1} - s_{N-1}) & (s_{h+N} - s_N) \end{bmatrix}_{h\times N} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1}$$

Más compacta

$$\hat{Y}_k = F_h \bullet \Delta U_k + G_h \bullet M_k \tag{1.65}$$

donde

$$\hat{Y}_k = \begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \\ \vdots \\ \hat{y}_{k+h-1} \\ \hat{y}_{k+h} \end{bmatrix}_{h\times 1} \qquad \Delta U_k = \begin{bmatrix} \Delta u_{k+Nu} \\ \vdots \\ \Delta u_k \end{bmatrix}_{N_u \times 1} \qquad M_k = \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1} \tag{1.66}$$

$$F_h = \begin{bmatrix} 0 & \cdots & s_1 \\ 0 & \cdots & s_2 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{h-1} \\ s_1 & \cdots & s_h \end{bmatrix}_{h\times N_u} \qquad G_h = \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h - s_1) & \cdots & (s_{h+N-2} - s_{N-1}) & (s_{h+N-1} - s_N) \\ 1 & (s_{h+1} - s_1) & \cdots & (s_{h+N-1} - s_{N-1}) & (s_{h+N} - s_N) \end{bmatrix}_{h\times N} \tag{1.67}$$

Objetivo de Control.

Definir una referencia

$$R_k = \begin{bmatrix} r_{k+1} \\ \vdots \\ r_{k+h} \end{bmatrix} \tag{1.68}$$

Y minimizar el funcional

$$\hat{Y}_k = F_h \bullet \Delta U_k + G_h \bullet M_k \tag{1.69}$$

$$J = \left[R_k - \hat{Y}_k \right]^T \left[R_k - \hat{Y}_k \right] + \lambda \Delta U_k{}^T \Delta U_k \tag{1.70}$$

$$J = \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right]^T \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right] + \lambda \Delta U_k{}^T \Delta U_k \tag{1.71}$$

$$\frac{\partial J}{\partial U} = -2 F_h{}^T \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right] + 2\lambda \Delta U_k \tag{1.72}$$

$$\Delta U_k{}^* = \left[F_h{}^T F_h + \lambda I \right]^{-1} F_h{}^T \left[R_k - G_h \bullet M_k \right] \tag{1.73}$$

el regulador resulta

$$\Delta U_k^{\ *} = R_r \left[R_k - E_k \right] \tag{1.74}$$

Donde

$$R_r = \left[F_h^{\ T} F_h + \lambda I \right]^{-1} F_h^{\ T} \quad , \quad E_k = G_h \bullet M_k \tag{1.75}$$

Solo se necesita la última fila de ΔU^*

Para $h=1,\ N_u=1$

$$\hat{y}_{k+1}=s_1\Delta u_k+\begin{bmatrix}1 & (s_2-s_1) & \cdots & (s_N-s_{N-1}) & (s_{N+1}-s_N)\end{bmatrix}\begin{bmatrix}y_k\\ \Delta u_{k-1}\\ \vdots\\ \Delta u_{k-N-1}\\ \Delta u_{k-N}\end{bmatrix}$$

$$\hat{Y}_k=F_h\bullet\Delta U_k+G_h\bullet M_k$$

donde

$$\hat{Y}_k=\hat{y}_{k+1}\quad \Delta U_k=\Delta u_k\quad M_k=\begin{bmatrix}y_k\\ \Delta u_{k-1}\\ \vdots\\ \Delta u_{k-N-1}\\ \Delta u_{k-N}\end{bmatrix}_{N\times 1}$$

$$F_h=s_1\quad G_h=\begin{bmatrix}1 & (s_2-s_1) & \cdots & (s_N-s_{N-1}) & (s_{N+1}-s_N)\end{bmatrix}$$

Controlador

$$R_k = r_{k+1}$$

$$\Delta u_k = \frac{s_1}{s_1{}^2 + \lambda}\left[r_{k+1} - y_k - \left(s_2 - s_1\right)\Delta u_{k-1} - \cdots - \left(s_N - s_{N-1}\right)\Delta u_{k-N-1} - \left(s_{N+1} - s_N\right)\Delta u_{k-N} \right]$$

Para $h=2, N_u=1$

$$\hat{y}_{k+1} = s_1\Delta u_k + \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}$$

$$\begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \Delta u_k + \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \end{bmatrix}$$

$$\hat{Y}_k = F_h \bullet \Delta U_k + G_h \bullet M_k$$

donde

$$\hat{Y}_k = \begin{bmatrix} \hat{y}_{k+1} \\ \hat{y}_{k+2} \end{bmatrix} \quad \Delta U_k = \Delta u_k \quad M_k = \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1}$$

$$F_h = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \quad G_h = \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \end{bmatrix}$$

Controlador

$$R_k = \begin{bmatrix} r_{k+1} \\ r_{k+2} \end{bmatrix}$$

$$\Delta u_k = \frac{s_1}{s_1{}^2 + \lambda}\left[r_{k+1} - y_k - (s_2 - s_1)\Delta u_{k-1} - \cdots - (s_N - s_{N-1})\Delta u_{k-N-1} - (s_{N+1} - s_N)\Delta u_{k-N} \right]$$

$$\Delta u_k = \frac{1}{s_1{}^2 + s_2{}^2 + \lambda}\begin{bmatrix} s_1 & s_2 \end{bmatrix}\left[\begin{bmatrix} r_{k+1} \\ r_{k+2} \end{bmatrix} - \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \end{bmatrix} \begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix} \right]$$

1.7.3. Ejemplo Multivariable

sistema original (pensar en los cuatro tachitos)

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{21}(s) \\ G_{12}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{k_{11}}{\tau_{11}s+1} & \dfrac{k_{21}}{\tau_{21}s+1} \\ \dfrac{k_{12}}{\tau_{12}s+1} & \dfrac{k_{22}}{\tau_{22}s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y_1 = \frac{k_{11}}{\tau_{11}s+1}u_1 + \frac{k_{21}}{\tau_{21}s+1}u_2$$

$$y_2 = \frac{k_{12}}{\tau_{12}s+1}u_1 + \frac{k_{22}}{\tau_{22}s+1}u_2$$

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{h_1} s_{11i}\Delta u_{1k+h_1-i} + y_{1k} - \sum_{i=1}^{\infty}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_1} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{\infty}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{h_2} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{\infty}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_2} s_{22i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{\infty}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Si la respuesta al escalón del sistema converge a un valor final, se podría suponer que la predicción se aproxima a

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{h_1} s_{11i}\Delta u_{1k+h_1-i} + y_{1k} - \sum_{i=1}^{N_1}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_1} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{N_1}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{h_2} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{N_2}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{h_2} s_{22i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{N_2}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Otra simplificación es suponer que luego de N_u muestras, la acción de control permanece constante.

$$\hat{y}_{1k+h_1} = \sum_{i=1}^{N_{1u}} s_{11i}\Delta u_{1k+N_{1u}-i} + y_{1k} - \sum_{i=1}^{N_1}\left(s_{11h_1+i} - s_{11i}\right)\Delta u_{1k-i} + \sum_{i=1}^{N_{2u}} s_{21i}\Delta u_{2k+h_1-i} - \sum_{i=1}^{N_1}\left(s_{21h_1+i} - s_{21i}\right)\Delta u_{2k-i}$$

$$\hat{y}_{2k+h_2} = \sum_{i=1}^{N_{1u}} s_{12i}\Delta u_{1k+h_2-i} + y_{2k} - \sum_{i=1}^{N_2}\left(s_{12h_2+i} - s_{12i}\right)\Delta u_{1k-i} + \sum_{i=1}^{N_{2u}} s_{12i}\Delta u_{2k+h_2-i} - \sum_{i=1}^{N_2}\left(s_{22h_2+i} - s_{22i}\right)\Delta u_{2k-i}$$

Para predicciones desde *1* hasta *h*

$$\begin{bmatrix} \hat{\mathbf{Y}}_1 \\ \hat{\mathbf{Y}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta U}_1 \\ \mathbf{\Delta U}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{\mathbf{12}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{M}_{11} \\ \mathbf{M}_{12} \\ \mathbf{M}_{21} \\ \mathbf{M}_{22} \end{bmatrix}$$

Siendo

$$\hat{Y}_{1k} = \begin{bmatrix} \hat{y}_{1k+1} \\ \hat{y}_{1k+2} \\ \vdots \\ \hat{y}_{1k+h-1} \\ \hat{y}_{1k+h} \end{bmatrix}_{h\times 1} \quad \hat{Y}_{2k} = \begin{bmatrix} \hat{y}_{2k+1} \\ \hat{y}_{2k+2} \\ \vdots \\ \hat{y}_{2k+h-1} \\ \hat{y}_{2k+h} \end{bmatrix}_{h\times 1} \quad \Delta U_{1k} = \begin{bmatrix} \Delta u_{1k+Nu} \\ \vdots \\ \Delta u_{1k} \end{bmatrix}_{N_u\times 1} \quad \Delta U_{2k} = \begin{bmatrix} \Delta u_{2k+Nu} \\ \vdots \\ \Delta u_{2k} \end{bmatrix}_{N_u\times 1}$$

$$
M_{11k} = \begin{bmatrix} y_{1k} \\ \Delta u_{1k-1} \\ \vdots \\ \Delta u_{1k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1} \quad M_{22k} = \begin{bmatrix} y_{2k} \\ \Delta u_{2k-1} \\ \vdots \\ \Delta u_{2k-N-1} \\ \Delta u_{2k-N} \end{bmatrix}_{N\times 1} \quad M_{12k} = \begin{bmatrix} \Delta u_{2k-1} \\ \vdots \\ \Delta u_{2k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1} \quad M_{21k} = \begin{bmatrix} \Delta u_{1k-1} \\ \vdots \\ \Delta u_{1k-N-1} \\ \Delta u_{1k-N} \end{bmatrix}_{N\times 1}
\tag{1.76}
$$

$$
F_{11} = \begin{bmatrix} 0 & \cdots & s_{111} \\ 0 & \cdots & s_{112} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{11h-1} \\ s_{111} & \cdots & s_{11h} \end{bmatrix}_{h\times N_u} \quad F_{12} = \begin{bmatrix} 0 & \cdots & s_{121} \\ 0 & \cdots & s_{122} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{12h-1} \\ s_{121} & \cdots & s_{12h} \end{bmatrix}_{h\times N_u} \quad F_{21} = \begin{bmatrix} 0 & \cdots & s_{211} \\ 0 & \cdots & s_{212} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{21h-1} \\ s_{211} & \cdots & s_{21h} \end{bmatrix}_{h\times N_u} \quad F_{22} = \begin{bmatrix} 0 & \cdots & s_{2} \\ 0 & \cdots & s_{2} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_{22} \\ s_{221} & \cdots & s_{2} \end{bmatrix}
\tag{1.77}
$$

$$
G_h = \begin{bmatrix} 1 & (s_2 - s_1) & \cdots & (s_N - s_{N-1}) & (s_{N+1} - s_N) \\ 1 & (s_3 - s_1) & \cdots & (s_{N+1} - s_{N-1}) & (s_{N+2} - s_N) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & (s_h - s_1) & \cdots & (s_{h+N-2} - s_{N-1}) & (s_{h+N-1} - s_N) \\ 1 & (s_{h+1} - s_1) & \cdots & (s_{h+N-1} - s_{N-1}) & (s_{h+N} - s_N) \end{bmatrix}_{h\times N}
$$

$$
\begin{bmatrix} \hat{y}_{1k+1} \\ \hat{y}_{1k+2} \\ \vdots \\ \hat{y}_{1k+h_1-1} \\ \hat{y}_{1k+h_1} \\ \hat{y}_{2k+1} \\ \hat{y}_{2k+2} \\ \vdots \\ \hat{y}_{2k+h_2-1} \\ \hat{y}_{2k+h_2} \end{bmatrix}_{(h_1+h_2)\times 1}
=
\begin{bmatrix}
0 & 0 & \cdots & 0 & s_{111} & 0 & 0 & \cdots & 0 & s_{211} \\
0 & 0 & \cdots & s_{111} & s_{112} & 0 & 0 & \cdots & s_{211} & s_{212} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & s_{111} & \cdots & s_{11h_1+N_{1u}-2} & s_{11h_1+N_{1u}-1} & & s_{211} & \cdots & s_{21h_1+N_{2u}-2} & s_{21h_1+N_{2u}-1} \\
s_{111} & s_{112} & \cdots & s_{11h_1+N_{1u}-1} & s_{11h_1+N_{1u}} & s_{211} & s_{212} & \cdots & s_{21h_1+N_{2u}-1} & s_{21h_1+N_{2u}}
\end{bmatrix}_{(h_1+h_2)\times(N_{1u}+N_{2u})}
\begin{bmatrix} \Delta u_{1k+N_{1u}} \\ \Delta u_{1k+N_{1u}-1} \\ \vdots \\ \Delta u_{1k+1} \\ \Delta u_{1k} \\ \Delta u_{2k+N_{2u}} \\ \Delta u_{2k+N_{2u}-1} \\ \vdots \\ \Delta u_{2k+1} \\ \Delta u_{2k} \end{bmatrix}_{(N_{1u}+N_{2u})\times 1}
+
$$

$$
+\begin{bmatrix}
1 & (s_{d+2}-s_1) & \cdots & (s_{d+N}-s_{N-1}) & (s_{d+N+1}-s_N) \\
1 & (s_{d+3}-s_1) & \cdots & (s_{d+N+1}-s_{N-1}) & (s_{d+N+2}-s_N) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
1 & (s_{d+h}-s_1) & \cdots & (s_{d+h+N-2}-s_{N-1}) & (s_{d+h+N-1}-s_N) \\
1 & (s_{d+h+1}-s_1) & \cdots & (s_{d+h+N-1}-s_{N-1}) & (s_{d+h+N}-s_N)
\end{bmatrix}_{h\times N}
\begin{bmatrix} y_k \\ \Delta u_{k-1} \\ \vdots \\ \Delta u_{k-N-1} \\ \Delta u_{k-N} \end{bmatrix}_{N\times 1}
$$

Más compacta

$$\hat{\mathbf{Y}}_k = \mathbf{F}_h \times \boldsymbol{\Delta}\mathbf{U}_k + \mathbf{G}_h \times \mathbf{M}_k$$

1.8. Objetivo de Control.

Definir una referencia

$$R_k = \begin{bmatrix} r_{k+d+1} \\ \vdots \\ r_{k+d+h} \end{bmatrix}$$

Y minimizar el funcional

$$\hat{Y}_k = F_h \bullet \Delta U_k + G_h \bullet M_k$$

$$J = \left[R_k - \hat{Y}_k \right]^T \left[R_k - \hat{Y}_k \right] + \lambda \Delta U_k{}^T \Delta U_k$$

$$J = \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right]^T \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right] + \lambda \Delta U_k{}^T \Delta U_k$$

$$\frac{\partial J}{\partial U} = -2 F_h{}^T \left[R_k - F_h \bullet \Delta U_k - G_h \bullet M_k \right] + 2\lambda \Delta U_k$$

$$\Delta U_k{}^* = \left[F_h{}^T F_h + \lambda I \right]^{-1} F_h{}^T \left[R_k - G_h \bullet M_k \right]$$

el regulador resulta

$$\Delta U_k^{\,*} = R_r \left[R_k - E_k \right]$$

Donde

$$\mathbf{R}_r = \left[\mathbf{F}_h^{\,T} \mathbf{F}_h + \lambda \mathbf{I} \right]^{-1} \mathbf{F}_h^{\,T} \quad , \quad \mathbf{E}_k = \mathbf{G}_h \times \mathbf{M}_k$$

1.9. Introducción de Restricciones

El funcional cuadrático, resultaba

$$J = \left[R - FB\cdot\Delta U - G_1\cdot M_a\right]^T \left[R - FB\cdot\Delta U - G_1\cdot M_a\right] + \lambda\Delta U^T\Delta U \quad (1.78)$$

se puede llevar a

$$J = \frac{1}{2}\Delta U^T H\Delta U + b^T\Delta U + f_0 \quad (1.79)$$

donde

$$H = 2\left(FB^T FB + \lambda I\right)$$

$$b^T = 2\left(R - G_1\cdot M_a\right)^T FB \quad (1.80)$$

$$f_0 = \left(R - G_1\cdot M_a\right)^T \left(R - G_1\cdot M_a\right)$$

el mínimo lineal es

$$\Delta U = -H^{-1}b \quad (1.81)$$

Puede haber restricciones en

- la actuación
- la velocidad de cambio de la actuación
- la salida
- etc.

Se puede construir el siguiente conjunto de restricciones:

$$\begin{array}{ccccc} U_{\inf} & \leq & U_k & \leq & U_{\sup} \\ DU_{\inf} & \leq & U_k - U_{k-1} & \leq & DU_{\sup} \\ Y_{\inf} & \leq & Y & \leq & Y_{\sup} \end{array} \tag{1.82}$$

o

$$\begin{array}{ccccc} \underline{1}U_{\inf} & \leq & T\Delta U_k + \underline{1}U_{k-1} & \leq & \underline{1}U_{\sup} \\ \underline{1}DU_{\inf} & \leq & \Delta U_k & \leq & \underline{1}DU_{\sup} \\ \underline{1}Y_{\inf} & \leq & FB \cdot \Delta U + G_1 \cdot M_a & \leq & \underline{1}Y_{\sup} \end{array} \tag{1.83}$$

con

$$\underline{\mathbf{1}} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \tag{1.84}$$

o

$$\mathbf{R} \cdot \mathbf{\Delta U} \leq \mathbf{C} \tag{1.85}$$

con

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_{nxn} \\ -\mathbf{I}_{nxn} \\ \mathbf{T} \\ -\mathbf{T} \\ \mathbf{FB} \\ -\mathbf{FB} \end{bmatrix} C = \begin{bmatrix} \underline{\mathbf{1}} \cdot \mathbf{DU}_{\sup} \\ -\underline{\mathbf{1}} \cdot \mathbf{DU}_{\inf} \\ \underline{\mathbf{1}} \cdot \mathbf{U}_{\sup} - \underline{\mathbf{1}} \cdot \mathbf{U}_{k-1} \\ -\underline{\mathbf{1}} \cdot \mathbf{U}_{\inf} + \underline{\mathbf{1}} \cdot \mathbf{U}_{k-1} \\ \underline{\mathbf{1}} \cdot \mathbf{Y}_{\sup} - G_1 \cdot M_a \\ -\underline{\mathbf{1}} \cdot \mathbf{Y}_{\inf} + G_1 \cdot M_a \end{bmatrix} \tag{1.86}$$

El problema es minimizar

$$J = \frac{1}{2}\Delta U^T H \Delta U + b^T \Delta U + f_0 \tag{1.87}$$

con la restricción

$$R\Delta U \leq C \tag{1.88}$$

en Matlab existe

du=quadprog(H,b,R,C)

1.10. Paquete MatLab MPC

Ver DMCEjercicio2011.m

1.11. Referencias

Seborg, D., Edgar, T., Mellichamp, D. *Process Dynamics and Control* – Wiley – 2nd Ed. 2004

Camacho, E.F., *Model Predictive Control in the Process Industry* – Springer – 1995

CStation:

Clarke, D.W., *Generalised Predictive Control with Input Constrains* – IEE Proceedings, vol 135, No 6 Nov 1988