Kinematic, Static, Dynamic and Control Analysis of a 6-DOF Serial Manipulator

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1 Introduction

This report presents the complete modeling and analysis of a 6-degree-of-freedom serial robotic manipulator. The study includes kinematic modeling using the Modified Denavit–Hartenberg (MDH) convention, Jacobian-based analysis, static and dynamic modeling, trajectory generation, and joint-space control.

All formulations follow the conventions presented in Craig [1].

2 Coordinate Frames and MDH Convention

The Modified Denavit–Hartenberg (Craig) convention is used throughout this work. The homogeneous transformation from frame $\{i-1\}$ to frame $\{i\}$ is defined as:

$$A_i = R_x(\alpha_{i-1})\,T_x(a_{i-1})\,R_z(\theta_i)\,T_z(d_i) \tag{1}$$

where:

- a_{i-1} is the distance from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ along $\hat{X}_{i-1}$
- α_{i-1} is the angle from $\hat{Z}_{i-1}$ to $\hat{Z}_i$ about $\hat{X}_{i-1}$
- d_i is the distance from $\hat{X}_{i-1}$ to $\hat{X}_i$ along $\hat{Z}_i$
- θ_i is the joint angle about $\hat{Z}_i$

3 MDH Parameter Table

The MDH parameters extracted from the CAD model are summarized in Table 1.

Table 1: Modified Denavit–Hartenberg Parameters

i	α_{i-1} [rad]	a_{i-1} [m]	d_i [m]
1	0	0	0.16099
2	$+\pi/2$	0	0
3	0	0.10596	0
4	$+\pi/2$	0.14652	0.13721
5	$-\pi/2$	0	0
6	$+\pi/2$	0	0
TCP	0	0	0.12539

4 Forward Kinematics

The forward kinematics from the base frame to the end-effector frame $\{6\}$ is given by:

$$T_0^6 = \prod_{i=1}^{6} A_i \tag{2}$$

The symbolic formulation was derived and later evaluated numerically in MATLAB. Orthonormality of the resulting rotation matrix was verified numerically.

5 Jacobian Analysis

The geometric Jacobian is defined as:

$$J(q) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \tag{3}$$

where:

$$J_{v,i} = \hat{z}_i \times (p_e - p_i) \tag{4}$$

$$J_{\omega,i} = \hat{z}_i \tag{5}$$

The Jacobian was validated using finite-difference approximation of the end-effector velocity.

6 Static Analysis

Static joint torques due to an external wrench applied at the end-effector were computed using:

$$\tau = J^T F \tag{6}$$

Additionally, gravity compensation torques were computed using the Jacobian-based formulation.

7 Dynamic Modeling

The Recursive Newton–Euler (RNE) algorithm was implemented to compute joint torques:

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) \tag{7}$$

Mass and inertia properties were extracted directly from the CAD model and converted to SI units.

8 Trajectory Generation

Joint-space trajectories were generated using:

- Cubic polynomials (position and velocity continuity)
- Quintic polynomials (position, velocity, and acceleration continuity)

9 Control

Two joint-space control strategies were implemented:

- Proportional-Derivative (PD) control
- Computed-torque control using inverse dynamics

10 Tool Frame Extension (Optional)

The physical robot includes a tool offset along the $\hat{Z}_6$ axis. This tool frame was not included in the numerical implementation but can be modeled as:

$$T_0^E = T_0^6 \, T_6^E \tag{8}$$

where T_6^E is a fixed homogeneous transform representing the TCP.

11 Conclusion

A complete and consistent modeling framework for a 6-DOF serial manipulator was developed and validated. The results demonstrate correct kinematic, static, and dynamic behavior aligned with standard robotics theory.

References

[1] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, 3rd ed. Pearson, 2005.