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12011 ECP011

Sistemas e Controle - Retorno 3 A

Nota - Cap 2

1-

$$a) u(t) \Rightarrow \mathcal{L}(u(t)) = \int_0^{\infty} e^{-st} \times 1 \times dt = \left. -\frac{1}{s} e^{-st} \right|_{t=0}^{\infty} = \frac{1}{s}$$

$$b) \mathcal{L}(tu(t)) = \mathcal{L}(t u(t)) = \int_0^{\infty} e^{-st} t dt = \frac{1}{s^2}$$

$$c) \mathcal{L}(\sin(\omega t) u(t)) = \frac{\omega}{s^2 + \omega^2}$$

2-

$$a) f(t) = e^{-at} \sin \omega t u(t) \Rightarrow \mathcal{L}(f(t)) = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$b) g(t) = e^{-at} \cos \omega t u(t) \Rightarrow \mathcal{L}(g(t)) = \frac{s+a}{(s+a)^2 + \omega^2}$$

$$8- y''' + 3y'' + 5y' + y = x''' + 4x'' + 6x' + 8x$$

Aplicando Laplace:

$$(s^3 + 3s^2 + 5s + 1)Y(s) = (s^3 + 4s^2 + 6s + 8)X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

$$X(s) = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}$$

9-

$$a) \frac{Y(s)}{F(s)} = \frac{7}{s^2 + 5s + 10} \Rightarrow (s^2 + 5s + 10)Y(s) = 7F(s)$$

$$\Downarrow$$
$$x'' + 5x' + 10x = 7f$$

$$11 - R(n) = \frac{n^4 + 2n^3 + 5n^2 + n + 1}{n^5 + 3n^4 + 2n^3 + 4n^2 + 5n + 2} \quad C(n) \rightarrow r(t) = 3t$$

$$(n^5 + 3n^4 + 2n^3 + 4n^2 + 5n + 2) \cdot C(n) = n^4 + 2n^3 + 5n^2 + n + 1 \quad | : C(n)$$

$$\frac{d^5 C}{dt^5} + 3 \frac{d^4 C}{dt^4} + 2 \frac{d^3 C}{dt^3} + 4 \frac{d^2 C}{dt^2} + 5 \frac{dC}{dt} + 2C = \frac{d^4 r}{dt^4} + 2 \frac{d^3 r}{dt^3} + 5 \frac{d^2 r}{dt^2} + \frac{dr}{dt} + r$$

substituindo $r(t) = t^3 \Rightarrow 18 \delta(t) + (36 - 90t + 9t^2 - 3t^3) u(t)$

Franklin - cap 3

3.2 - b) $3 + 7t + t^2 + \delta(t) = f(t)$

$$\mathcal{L}\{f(t)\} = \frac{3}{s} + \frac{7}{s} + \frac{2}{s^2} + 1$$

3.3 - c) $f(t) = t^2 + e^{-2t} \sin 3t$

$$\mathcal{L}\{f(t)\} = \frac{2}{s^3} + \frac{3}{(s+2)^2 + 9}$$

3.5 - a) $f(t) = \sin t \cdot \sin 3t \Rightarrow \frac{1}{2}(-\cos(t+3t) + \cos(t-3t))$

$$\mathcal{L}\{f(t)\} = -\frac{1}{2} \mathcal{L}\{\cos 4t\} + \frac{1}{2} \mathcal{L}\{\cos 2t\}$$

$$\mathcal{L}\{f(t)\} = \frac{1}{2} \frac{s}{s^2 + 16} + \frac{1}{2} \frac{s}{s^2 + 4} = \frac{6s}{(s^2 + 4)(s^2 + 16)}$$

$$3.7. a) \frac{2}{a(a+2)} = \frac{A}{a} + \frac{B}{a+2}$$

$$A(a+2) + Ba = 2, \quad a=0$$

$$2A = 2 \Rightarrow A = 1$$

$$a=2$$

$$-2B = 2, B = -1$$

$$\mathcal{L}^{-1}\left(\frac{2}{a(a+2)}\right) = \mathcal{L}^{-1}\left(\frac{1}{a}\right) - \mathcal{L}^{-1}\left(\frac{1}{a+2}\right)$$

$$\mathcal{L}^{-1}\left(\frac{2}{a(a+2)}\right) = 1 - e^{-2t} //$$

$$3.9. e) y'' + 2y' = e^t \quad y(0) = 1 \quad y'(0) = 2$$

$$\hat{a}^2 Y(s) - 2s - 2 + 2(aY(a) - 1) = \frac{1}{a-1}$$

$$Y(a) = \frac{a^2 + 3a + 2}{a(a-1)(a+2)} = \text{Por frações parciais}$$

$$Y(a) = \frac{1}{3(a-1)} + \frac{3}{2a} - \frac{5}{6(a+2)}$$

$$\mathcal{L}^{-1}(Y(a)) = \mathcal{L}^{-1}\left(\frac{1}{3(a-1)} + \frac{3}{2a} - \frac{5}{6(a+2)}\right)$$

$$y(t) = \frac{e^t}{3} + \frac{3}{2} - \frac{5e^{-2t}}{6} //$$