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ESE 118

HW1

1. Write truth tables for the following functions:

(a) $F = x(y + \bar{y} \cdot \bar{z})$

x	y	z	$\bar{y}$	$\bar{z}$	$\bar{y} \cdot \bar{z}$	$y + \bar{y} \cdot \bar{z}$	$F = x(y + \bar{y} \cdot \bar{z})$
0	0	0	1	1	1	1	0
0	0	1	1	0	0	0	0
0	1	0	0	1	0	1	0
0	1	1	0	0	0	1	0
1	0	0	1	1	1	1	1
1	0	1	1	0	0	0	0
1	1	0	0	1	0	1	1
1	1	1	0	0	0	1	1

- $\bar{y} \cdot \bar{z}$ is 1 only when $y=0$ and $z=0$
- $F=1$, only when $x=1$ and $y + \bar{y} \cdot \bar{z} = 1$

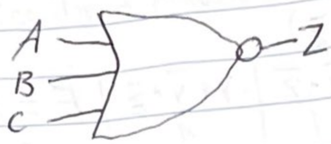
(b). $X = \overline{(b+\bar{c})} + (a \oplus b) + \bar{a} \cdot \bar{c}$

a	b	c	$\bar{c}$	$b+\bar{c}$	$\overline{(b+\bar{c})}$	$a \oplus b$	$\bar{a} \cdot \bar{c}$	X
0	0	0	1	1	0	0	1	1
0	0	1	0	0	1	0	0	1
0	1	0	1	1	0	1	1	1
0	1	1	0	1	0	1	0	1
1	0	0	1	1	0	1	0	1
1	0	1	0	0	1	1	0	1
1	1	0	1	1	0	0	0	0
1	1	1	0	1	0	0	0	0

- $a \oplus b$ is 1 when $a \neq b$

2. Write truth tables for the following two gates

(a)

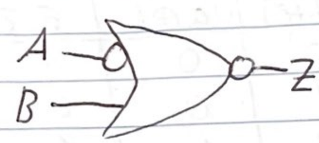


3-input NOR Gate

$Z = \overline{A+B+C}$

A	B	C	$A+B+C$	$Z = \overline{A+B+C}$
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

(b)

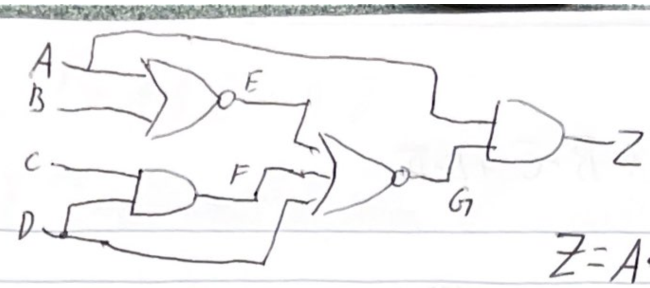


NOR Gate with one inverted input $(\bar{A})$

$Z = \overline{(\bar{A}+B)}$

A	B	$\bar{A}$	$\bar{A}+B$	$Z = \overline{(\bar{A}+B)}$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	1
1	1	0	1	0

3).



4 inputs

$Z = A \cdot G$

$Z = A \cdot \overline{(E+F+D)}$

$Z = A \cdot \overline{[(\overline{A+B}) + (C \cdot D) + D]}$

A	B	C	D	$E = (\overline{A+B})$	$F = (C \cdot D)$	$E+F+D$	$G = \overline{(E+F+D)}$	$Z = A \cdot G$
0	0	0	0	1	0	1	0	0
0	0	0	1	1	0	1	0	0
0	0	1	0	1	0	1	0	0
0	0	1	1	1	1	1	0	0
0	1	0	0	0	0	0	1	0
0	1	0	1	0	0	1	0	0
0	1	1	0	0	0	0	1	0
0	1	1	1	0	1	1	0	0
1	0	0	0	0	0	0	1	1
1	0	0	1	0	0	1	0	0
1	0	1	0	0	0	0	1	1
1	0	1	1	0	1	1	0	0
1	1	0	0	0	0	0	1	1
1	1	0	1	0	0	1	0	0
1	1	1	0	0	0	0	1	1
1	1	1	1	0	1	1	0	0

- When $A=0$, $Z=0$ regardless of B, C, D because of the AND Gate.

- $Z=1$ only when $A=1$ and $D=0$.
  - If $D=1$:
    The term $(C \cdot D) + D = 1$
  - If $D=0$:
    We get $\overline{[(\overline{A+B}) + 0 + 0]}$, Thus, $Z = 1 \cdot 1 = 1$

The truth table confirms that:

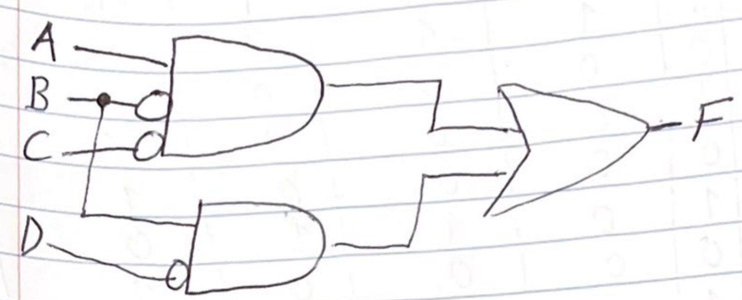
$$Z = A \cdot \overline{D}$$

Literal count: 2 literals $(A, \overline{D})$

4) Let $F = A \cdot \overline{B} \cdot \overline{C} + B \cdot \overline{D}$

(a). Implement F using AND, OR, and inverter gates.

- 4 inputs
- AND operations: $A \cdot \overline{B} \cdot \overline{C}$ and $B \cdot \overline{D}$
- OR operation: to combine the results of the AND operations



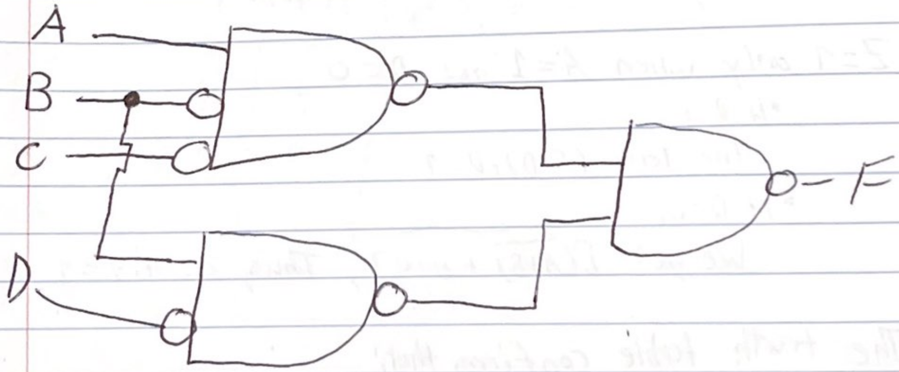
(b) Implement F using NAND gates and inverters only

- Step 1: Apply Double Complementation (T3)

$$F = \overline{\overline{[(A \cdot \overline{B} \cdot \overline{C} + B \cdot \overline{D})]}}$$

- Step 2: Apply DeMorgan's Theorem (T5(a))

$$\overline{(A \cdot \overline{B} \cdot \overline{C} + B \cdot \overline{D})} = \overline{(A \cdot \overline{B} \cdot \overline{C})} \cdot \overline{(B \cdot \overline{D})}$$

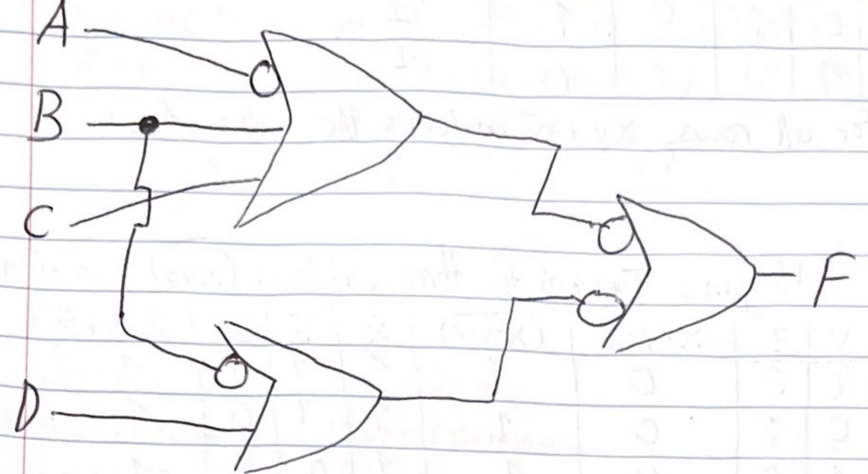


3 NAND Gates and 3 Inverters

$F = A \cdot \bar{B} \cdot \bar{C} + B \cdot \bar{D}$

4c). Implement F using OR gates and inverters only

Apply De Morgan's Law: $F = [\overline{(\bar{A}+B+C)} + \overline{(\bar{B}+D)}]$



- 3 OR Gates
- Inverters: 4 (2 input, 2 middle)

5). Use truth tables to prove the following theorems,

a). The Uniting Theorem T7(a): $xy + x\bar{y} = x$

x	y	$\bar{y}$	$x \cdot y$	$x \cdot \bar{y}$	$xy + x\bar{y}$
0	0	1	0	0	0
0	1	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1

- For all rows, $xy + x\bar{y}$ matches the value of x:

b). DeMorgan's Theorem for three variables: $\overline{(xyz)} = \bar{x} + \bar{y} + \bar{z}$

x	y	z	$x \cdot y \cdot z$	$\overline{(x \cdot y \cdot z)}$	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x} + \bar{y} + \bar{z}$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

- $\overline{(xyz)}$ and $\bar{x} + \bar{y} + \bar{z}$ match in every row.

6). Use Boolean Postulates and Theorems to simplify the following expressions to the given number of literals.

(a). Simplify to 1 literal: $xy + x\bar{y}$

Expression	Justification	Law/Postulate
$F = xy + x\bar{y}$	Original Expression	—
$F = x(y + \bar{y})$	Factor out x (common term)	Distributive Law (P4)
$F = x(1)$	$y + \bar{y} = 1$ (complements sum to 1)	Complement Theorem (P5(a))
$F = x$	$x \cdot 1 = x$ (identity multiplication)	Identity Theorem (P2(b))

$xy + x\bar{y} = x$ (1 literal)

(b). Simplify to 1 literal: $xyz + \bar{x}y + xy\bar{z}$

Expression	Justification	Law/Postulate
$xyz + \bar{x}y + xy\bar{z}$	Original Expression	—
$xyz + xy\bar{z} + \bar{x}y$	Rearrange terms	Commutative Law (P1)
$xy(z + \bar{z}) + \bar{x}y$	Factor out xy from first 2 terms	Distributive Law (P4)
$xy(1) + \bar{x}y$	$z + \bar{z} = 1$ (complements sum to 1)	Complement Theorem (P5(a))
$xy + \bar{x}y$	Simplify $xy \cdot 1 = xy$	Identity Theorem (P2(b))
$y(x + \bar{x})$	Factor out y	Distributive Law (P4)
$y(1)$	$x + \bar{x} = 1$ (complements sum to 1)	Complement Theorem (P5(a))
y	Simplify $y \cdot 1 = y$	Identity Theorem (P2(b))

$xyz + \bar{x}y + xy\bar{z} = y$ (1 literal) c). Simplify to 5 literals: $ab + ab\bar{c}d + abd\bar{e} + \bar{a}b\bar{c}e + \bar{a}\bar{b}\bar{c}e + a \cdot d \cdot \bar{d} \cdot e$

Expression	Justification	Law/Postulate
$ab + ab\bar{c}d + abd\bar{e} + \bar{a}b\bar{c}e + \bar{a}\bar{b}\bar{c}e + a \cdot d \cdot \bar{d} \cdot e$	Original Expression	
$ab + ab\bar{c}d + abd\bar{e} + \bar{a}b\bar{c}e + \bar{a}\bar{b}\bar{c}e + 0$	$d \cdot \bar{d} = 0$, annihilates term	Complement Theorem (P5(a))
$ab(1+\bar{c}d+d\bar{e}) + \bar{a}\bar{c}e(b+\bar{b})$	Factor ab and $\bar{a}\bar{c}e$	Distributive Law (P4)
$ab(1) + \bar{a}\bar{c}e(1)$	$1+x=1$ $b+\bar{b}=0$	Annulment Law T2 Complement Theorem
$ab + \bar{a}\bar{c}e$	simplify $ab \cdot 1 = ab$, $\bar{a}\bar{c}e \cdot 1 = \bar{a}\bar{c}e$	Identity Theorem P2(b)

$ab + \bar{a}\bar{c}e$ (5 literals)

d). Simplify to 1 literal: $\bar{w}x(\bar{z}+\bar{y}z)+x(w+\bar{w}yz)$

Expression	Justification	Law/Postulate
$\bar{w}x(\bar{z}+\bar{y}z)+x(w+\bar{w}yz)$	Original expression	
$\cancel{x[\bar{w}(\bar{z}+\bar{y}z)+w+\bar{w}yz]}$	~~Factor out x~~	~~Distributive Law (P4)~~
$\cancel{\bar{w}x\bar{z}+\bar{w}x\bar{y}z+xw+x\bar{w}yz}$	~~Distribute~~ terms	~~Distribution~~
$\cancel{\bar{w}x\bar{z}+xw+\bar{w}x\bar{y}z+x\bar{w}yz}$	~~Rearrange terms~~	~~Commutative Law~~
$x[\bar{w}(\bar{z}+\bar{y}z)+w+\bar{w}yz]$	Factor out x	Distributive Law (P4)
$x[\bar{w}\bar{z}+\bar{w}\bar{y}z+w+\bar{w}yz]$	Expand $\bar{w}(\bar{z}+\bar{y}z)$	Distributive Law (P4)
$x[\bar{w}(\bar{z}+\bar{y}z+yz)+w]$	Group $\bar{w}$-terms	Distributive Law (P4)
$x[\bar{w}(\bar{z}+z(\bar{y}+y))+w]$	Factor z from $\bar{y}z+yz$	Distributive Law
$x(\bar{w}(\bar{z}+z)+w]$	$\bar{y}+y=1$	Complement Theorem (P5(a))
$x[\bar{w}(1)+w]$	$\bar{z}+z=1$	Complement Thm
$x \cdot 1$	$\bar{w} \cdot w = 1$	Complement Thm.
x	$x \cdot 1 = x$	Identity Thm

$\bar{w}x(\bar{z}+\bar{y}z)+x(w+\bar{w}yz) = X$ (1 literal)