A) Complete problems 5a–5f on page 133. Problem 5 states "Prove that the following languages are not regular:"

a. $L = \{a^n b^l a^k : k \leq n + l\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^n b^l a^k$ such that $n + l = k = m$. Then, $w \in L$ and $xy = a^n b^l$. Next, we let $i = 0$, then $|xy^0| = n + l - |y| < k$. This contradicts the assumption that $w_i \in L$. □

b. $L = \{a^n b^l a^k : k \neq n + l\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^n b^l a^k$ such that $n + l = m$, then $xy = a^n b^l$ and let $k = m + |y|$. Then, $w = a^n b^l a^k \in L$. Next, we let $i = 2$, then $|xy^2| = m + |y| = k$. This contradicts the assumption that $w_i \in L$. □

c. $L = \{a^n b^l a^k : n = l \text{ or } l \neq k\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^n b^l a^k$ such that $n = k = l = m$, then $w \in L$ and $xy = a^m$. Next, we let $i = 2$, then $|xy^2| = 2m \neq l$, but $l = k$. This contradicts the assumption that $w_i \in L$. □

d. $L = \{a^n b^l : n \geq l\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^m b^m$, then $w \in L$ and $xy = a^m$. Next, we let $i = 0$, then $|xy^0| = m - |y| < l$. This contradicts the assumption that $w_i \in L$. □

e. $L = \{w : n_a(w) \neq n_b(w)\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^m b^k$, then $xy = a^m$, and let $k = m + |y|$. Then, $w \in L$. Next, we let $i = 2$, then $|xy^2| = m + |y| = k$. This contradicts the assumption that $w_i \in L$, since the number of a's ($|xy|$) is the same as the number of b's (k). □

f. $L = \{ww : w \in \{a, b\}^*\}$.

Proof. Suppose L is regular. Then, by the pumping lemma we have $w \in L$, and some $m \leq |w|$, $w = xyz$, $|xy| \leq m$, and $|y| \geq 1$, such that $w_i = xy^i z \in L$ for all $i \geq 0$. Let $w = a^m b a^m$, then $w \in L$ and $xy = a^m$. Next, we let $i = 2$, then $|xy^2| = 2m$, so $w_2 = a^{2m} b a^m \notin L$. This contradicts the assumption that $w_i \in L$. □