Optimal scaling for Metropolis-Hastings algorithms

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Abstract

A brief summary of the project goes here.

Contents

1 Introduction

Some general introduction to Monte Carlo methods and MCMC methods.
What are Monte Carlo and MCMC methods? What is the aim? What are the applications? A brief history? (Short comparison between numerical and stochastic methods? Advantages and disadvantages?)

1946, Los Alamos National Laboratory, New Mexico, U.S.A.. Mathematicians: John von Neumann, Stanislaw Ulam, Robert Richtmyer, Physicists: Enrico Fermi, Nick Metropolis, Edward Teller. Invention of first electronic computers as ENIAC (Electronic Numerical Integrator And Computer) made it possible to simulate (Monte Carlo Simulation).

Statement of the problem

Look at computational cost of MCMC methods as a function of dimension N.
Optimal Scaling for RWM and MALA.

Random Walk Metropolis algorithm

Look at Figure 1, the opitimal scaling of the acceptance probability and therefore the scaling of the proposal variance is illustrated by a Random Walk Metropolis-Hastings algorithm. A too small proposal variance and therefore a too high acceptance probability causes a very slow mixing of the Markov chain in the two modes of the target density. In the case of a too high proposal variance, the Markov chain stays very long in one state. Hence the effiecny of the algorithm is relatively low.

Own contributions

My contributions to the present topic.

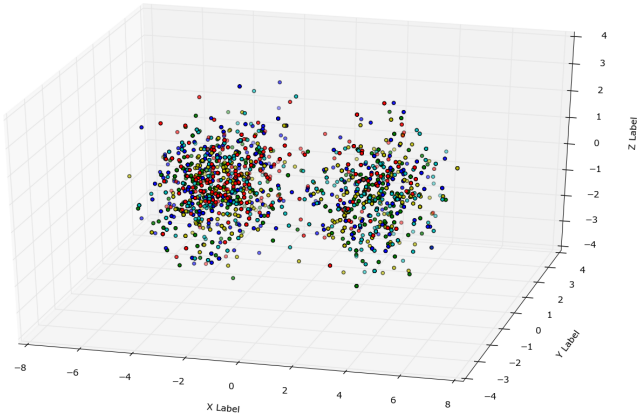
- First point.
- Second point.
- Last point.

Outline

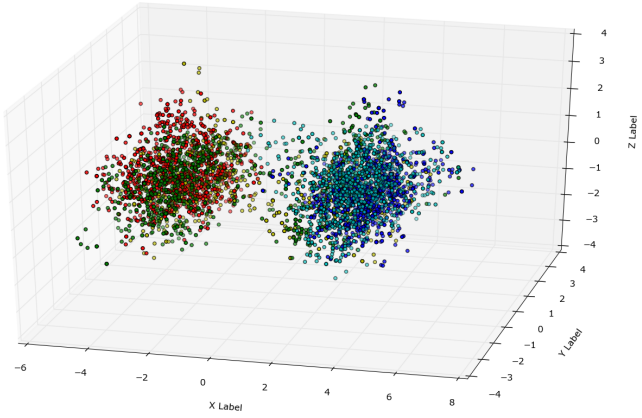
A brief outline of the structure of this work.
Give an overview of present results of optimal scaling: Roberts and Rosental, Bedard, Breyer and Piccioni and Scarlatti, Mattingly and Pillai and Stuart and Thiery

Acknowledgements

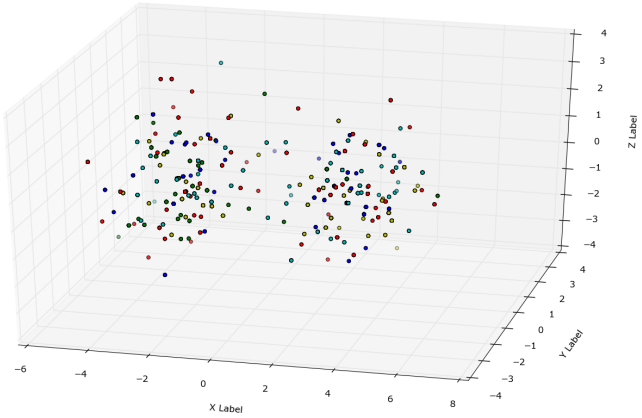
A list of persons, who deserve my acknowledgements: advisor, parents, friends.



(a) Optimal scaled acceptance probability with a good mixing between both modes.



(b) Too large acceptance probability with a bad mixing between both modes.



(c) Too low acceptance probability.

Figure 1: 5000 samples produced by a RWM algorithm of a multimodal non-product target density. Every 1000 consecutive samples are labeled in the same colour.

2 The Metropolis-Hastings algorithm

2.1 The MCMC Principle

2.2 The Metropolis-Hastings algorithm

2.2.1 Convergence Properties

2.2.2 The Random Walk Metropolis-Hastings algorithm

2.2.3 The Metropolis adjusted Langevin algorithm

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