Równania Różniczkowe i Różnicowe

Zadania, ćw 1

1. $y' = \frac{1}{1+x^2}$, $y(0) = 1$ (odp: $y(x) = 1 + \operatorname{arc\,tg} x$)
2. $y' = \frac{\cos x}{y}$, $y\left(\frac{\pi}{2}\right) = \sqrt{2}$ (odp: $y(x) = \sqrt{2 \sin x}$)
3. $y' = -\frac{1}{1-\frac{y}{x}} + \frac{1}{y-\frac{y^2}{x}}$ (odp: $x^2 + y^2 = C$)
4. $y' = \frac{x^2+y^2}{xy}$, $y(1) = 0$ (odp: $y = x\sqrt{2 \log |x|}$)
5. $y' = -\frac{x}{y}$, $y(0) = 1$ (odp: $x^2 + y^2 = 1$)
6. $y' = \frac{y-x}{y+x}$, $y(1) = 1$ (odp: $-\frac{y^2}{2x^2} - \frac{y}{2x} + \log\left(\frac{y+x}{x}\right) = \log |x| - 1 + \log 2$)
7. $y' = x + y - 1$, $y(0) = 2$ (odp: $y = 2e^x - x$)
8. $y' = e^{x+y}$, $y(0) = 0$ (odp: $y = -\log |2 - e^x|$)
9. $y' = \operatorname{tg}\left(\frac{y}{x}\right) + \frac{y}{x}$, $y(1) = \frac{\pi}{3}$ (odp: $y = x \operatorname{arc\,cos} \frac{x}{2}$)
10. $xy' = y + \sqrt{x^2 - y^2}$, $y(1) = 0$ (odp: $y = x \sin \log |x|$)
11. $y' = 2 - 3\frac{y+1}{x+y}$, $y(1) = 1 + \sqrt{3}$ (odp: $y^2 + 2xy - x^2 = 1$)