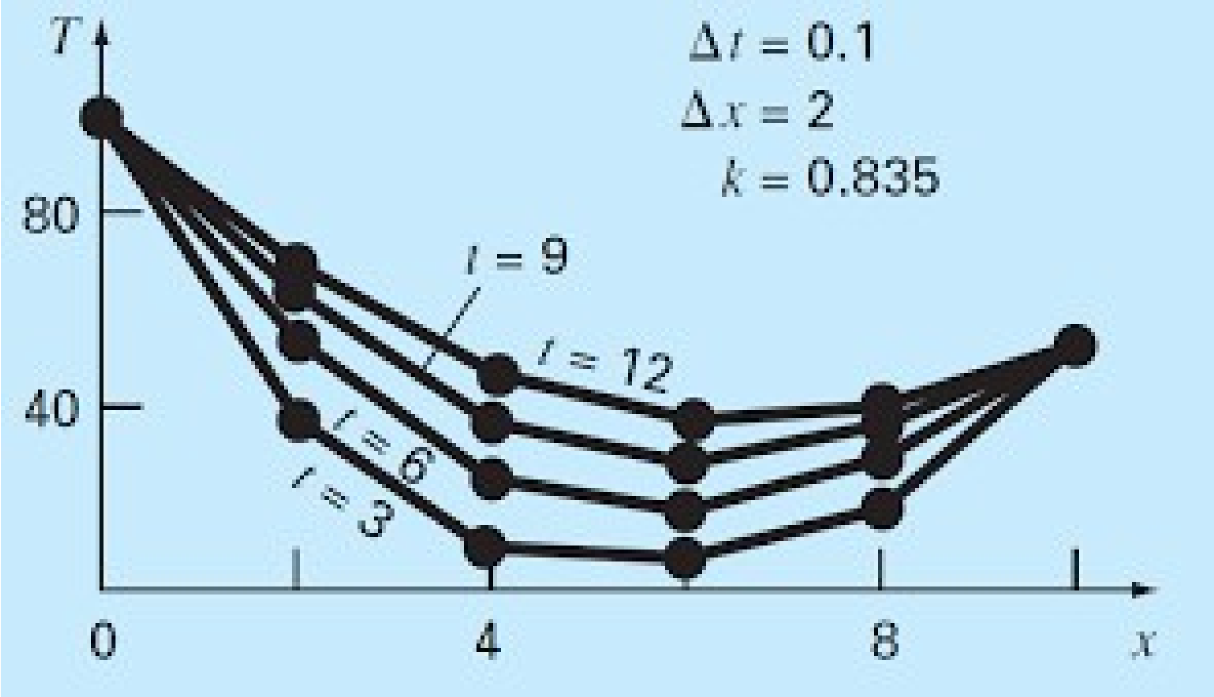
Parabólicas

$$\frac{\partial U}{dt} = \alpha \frac{\partial^2 U}{dt^2}$$



Parabólicas, explícito

$$\frac{\partial U}{dt}=\frac{T_i^{j+1}-T_i^j}{dt}$$

j indica tiempo
i indica espacio

$$\frac{\partial^2 U}{dx^2}=\frac{T_{i-1}^j-2T_i^j+T_{i+1}^j}{dx^2}$$

$$\frac{T_i^{j+1}-T_i^j}{dt}=\alpha\frac{T_{i-1}^j-2T_i^j+T_{i+1}^j}{dx^2}$$

$$T_i^{j+1}=T_i^j+\frac{dt\,\alpha}{dx^2}\left(T_{i-1}^j-2T_i^j+T_{i+1}^j\right)$$

Parabólicas, explícito

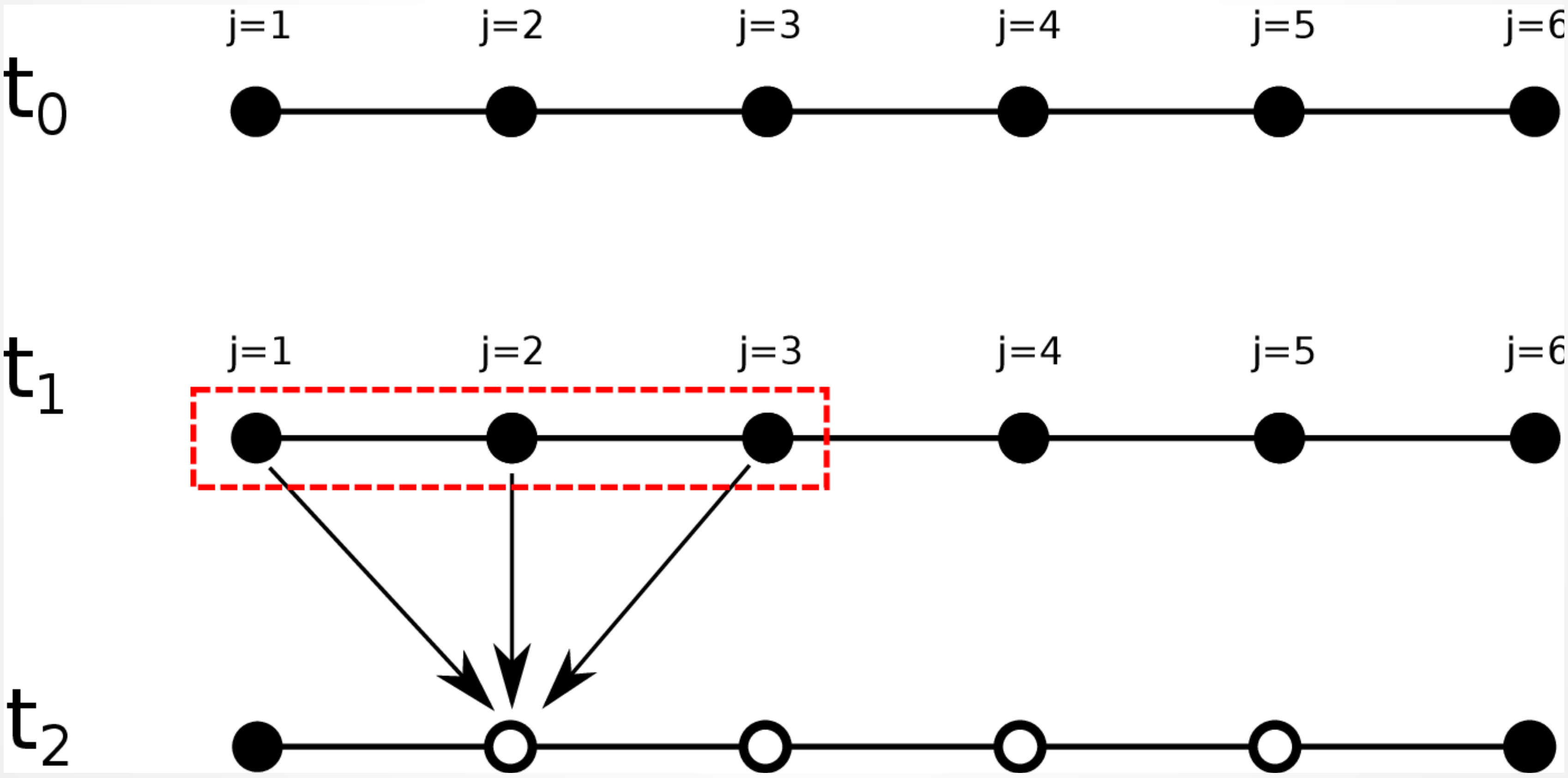
$$T_i^{j+1} = T_i^j + \frac{dt\,\alpha}{dx^2}\left(T_{i-1}^j - 2T_i^j + T_{i+1}^j\right)$$

r

$$T_i^{j+1} = r\,T_{i-1}^j + \left(1 - 2r\right)T_i^j + rT_{i+1}^j$$

Parabólicas, explícito

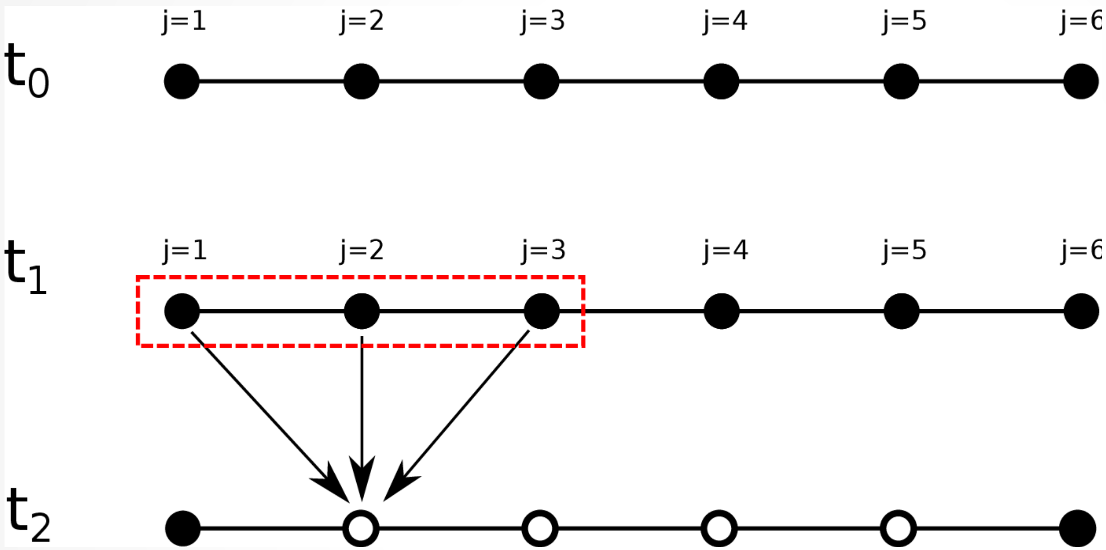
$$T_i^{j+1} = r\,T_{i-1}^{j} + (1-2r)\,T_i^{j} + r\,T_{i+1}^{j}$$



Parabólicas, explícito

$$T_i^{j+1} = r\,T_{i-1}^{j} + (1-2r)\,T_i^{j} + r\,T_{i+1}^{j}$$

Estable para r<1/2




```
tf=0
do while (tf.lt.30.)
  tant=t
  do i=2,nx-1
    t(i)=r*(tant(i-1)+tant(i+1))+(1-2*r)*tant(i)
  end do
  tf=tf+Dt
end do
```

Parabólicas, implícito

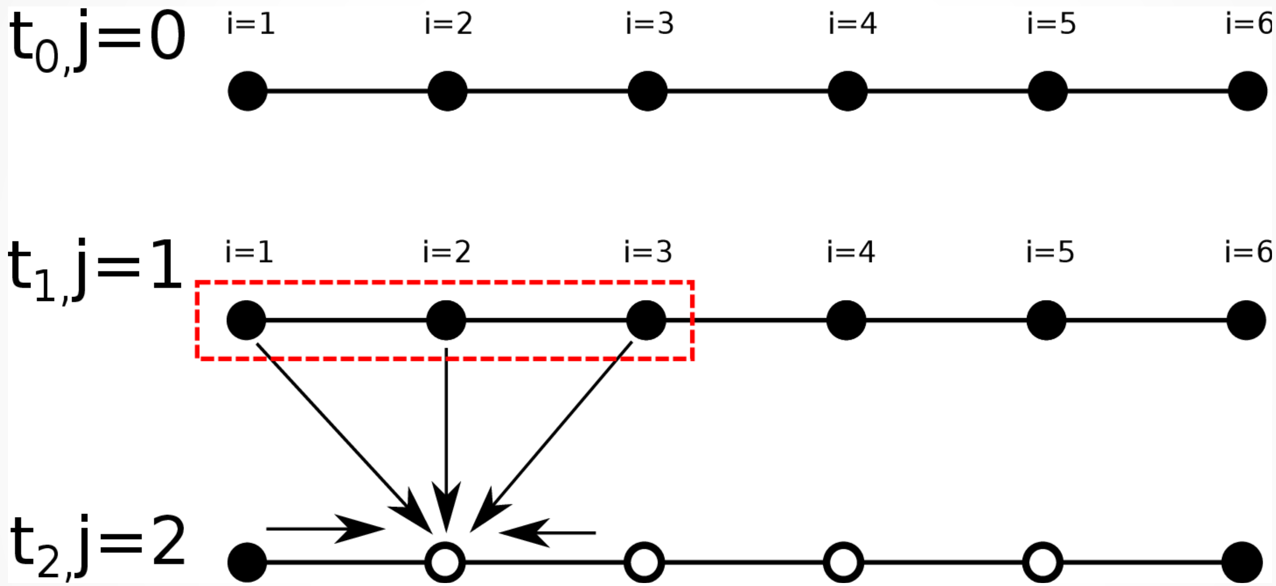
$$-rT_{i-1}^{j+1}+(2+2r)T_i^{j+1}-rT_{i+1}^{j+1}=r\,T_{i-1}^{j}+(2-2r)T_i^{j}+rT_{i+1}^{j}$$

-Estable para cualquier valor de r.

-Mas difícil de implementar (resuelvo un sistema de ecuaciones por cada paso de tiempo).

Parabólicas, implícito

$$-rT_{i-1}^{j+1}+(2+2r)T_i^{j+1}-rT_{i+1}^{j+1}=r\,T_{i-1}^{j}+(2-2r)\,T_i^{j}+rT_{i+1}^{j}$$



$$-u_{i-1}^{n+1}+\frac{2+2r}{r}u_i^{n+1}-u_{i+1}^{n+1}=u_{i-1}^{n}+\frac{2-2r}{r}u_i^{n}+u_{i+1}^{n}$$

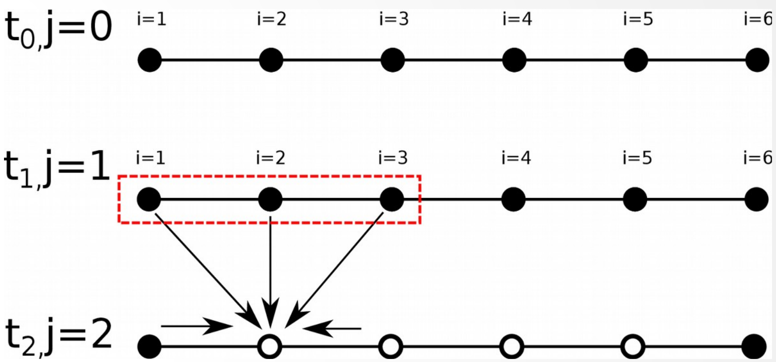
where $r = \alpha\Delta t/(\Delta x)^2$

On application of eq. at all grid points from i=1 to i=k+1 , the system of eqs. with boundary conditions u=A at x=0 and u=D at x=L can be expressed in the form of Ax = C

$$\begin{bmatrix} B(1) & -1 & 0 & 0 & \dots & 0 \\ -1 & B(2) & -1 & 0 & \dots & 0 \\ 0 & -1 & B(3) & -1 & \dots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \dots & -1 & B(k-1) \end{bmatrix} \begin{bmatrix} u_2^{n+1} \\ u_3^{n+1} \\ u_4^{n+1} \\ \vdots \\ u_k^{n+1} \end{bmatrix} = \begin{bmatrix} (C(1)+A)^n \\ C(2)^n \\ C(3)^n \\ \vdots \\ (C(k-1)+D)^n \end{bmatrix}$$

Parabólicas, implícito

$$-rT_{i-1}^{j+1}+(2+2r)T_i^{j+1}-rT_{i+1}^{j+1}=r\,T_{i-1}^{j}+(2-2r)\,T_i^{j}+rT_{i+1}^{j}$$




```
do while(time.lt.30)
  error=2*tol
  Tant=T !semilla

  do while (error>tol)
    Terr=T
    do i=2,nx-1
      C=r*Tant(i-1)+(2.-2.*r)*Tant(i)+r*Tant(i+1)
      T(i)=(1./(2.+2.*r))*(C+r*(T(i-1)+T(i+1)))
    Enddo
    error=maxval(abs(T-Terr))
  Enddo

  time=time+Dt
enddo
```

Parabólicas, borde adiabático

$$k\frac{\partial T}{\partial x} = q$$

$$\frac{T_{i+1}^{j} - T_{i}^{j}}{dx} = 0 \Rightarrow T_{i+1}^{j} = T_{i}^{j}$$

$$\frac{T_{i+1}^{j} - T_{i-1}^{j}}{2dx} = 0 \Rightarrow T_{i+1}^{j} = T_{i-1}^{j}$$

$$-rT_{i-1}^{j+1} + (2+2r)T_{i}^{j+1} - rT_{i+1}^{j+1} = rT_{i-1}^{j} + (2-2r)T_{i}^{j} + rT_{i+1}^{j}$$

$$2T_{i-1}^{j} - 4T_{i}^{j} + T_{i}^{j+1} + T_{i}^{j-1} = 0$$

r=1

Parabólicas, borde con convección

$$k\frac{\partial T}{\partial x} = q$$

$$k\frac{\partial T}{\partial x} = h(T - T_0)$$

$$-k\frac{T_{i+1}^{j} - T_{i}^{j}}{dx} = h\left(T_{i+1}^{j} - T_{0}\right)$$

Cambiar los subindices dependiendo de en que borde se encuetra.

El signo menos es para que el balance sea correcto

Hiperbólicas

$$\frac{\partial^2 u}{\partial t^2} = \frac{T\cdot g}{w}\cdot\frac{\partial u^2}{\partial x^2}$$

$$u_i^{j+1} = \frac{T\cdot g\cdot(\Delta t)^2}{w\cdot(\Delta x)^2}\cdot(u_{i+1}^j+u_{i-1}^j)-u_i^{j-1}+2\cdot\left(1-\frac{T\cdot g\cdot(\Delta t)^2}{w\cdot(\Delta x)^2}\right)\cdot u_i^j$$

$$\frac{T\cdot g\cdot(\Delta t)^2}{w\cdot(\Delta x)^2} = 1$$

$$u_i^{j+1} = u_{i+1}^j+u_{i-1}^j-u_i^{j-1}$$

Hiperbólicas

$$u_i^{j+1} = u_{i+1}^j + u_{i-1}^j - u_i^{j-1}$$

$$u_i^{-1} = u_i^1 - 2 \cdot v(x) \cdot \Delta t$$

MRU, solo en el primer paso