Dedução do eq. de ajuste do back propagation Lucas Kou Kinoshita.

. Para o vetor de entrada k com j saídas, a soma quadrática do erro dos m neurônios é dada por

$$\epsilon_k = \frac{1}{2}\sum_{j=1}^{m}(y_{kj} - \hat{y}_{kj})^2$$

$$= \frac{1}{2}\sum_{j=1}^{m}(y_{kj} - \overset{\text{saída}}{g}(\underbrace{\textstyle\sum w_i \overset{\text{intermediária}}{h_i}(x_k, z_i)}_{u_{kj}})))^2$$

A soma para todos os N amostras é dada como

$$\epsilon = \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{m}(y_{kj} - g(u_{kj}))^2$$

espera-se minimizar ϵ com a minimização dos ϵ_k.

Assim, para o elemento l do vetor de entrada.

$$\frac{\partial \epsilon_k}{\partial w_{ij}} = \frac{1}{2}\frac{\partial}{\partial w_{ij}}(y_j - g(u_{kj}))^2$$

$$\cancel{\frac{1}{2}}\cancel{2}\overbrace{(y_i - g(u_{kj}))}^{e_{kj}}{}^2 \frac{\partial}{\partial w_{ij}}(y_j - g(u_{kj}))$$

$$= e_{kj}(-1)\frac{\partial g(u_{kj})\,\partial u_{kj}}{\partial u_{kj}\cdot \partial w_{ij}}$$

$$\rightarrow h_i(x_k, z_i)$$

$$\frac{\partial \epsilon_k}{\partial w_{ij}} = -e_{kj}\, g'(u_{kj}) \frac{\partial \sum h_i(x_k, z_i)\, w_{ij}}{\partial w_{ij}}$$

$$\frac{\partial \epsilon_k}{\partial w_{ij}} = -e_{kj}\, g'(u_j)\, h_i(x_k, z_i) \qquad (I)$$

Δ

como $\Delta w_{ij} \propto -\dfrac{\partial \epsilon_k}{\partial w_{ij}}$

$$\Delta w_{ij} = \eta \underbrace{e_{kj}\, g'(u_{kj})}_{\delta_j} h_i(x_k, z_i)$$

e

$$w_{ij} = w_{ij} + \Delta w_{ij}$$

$$w_{ij} = w_{ij} + \eta\, \delta_j\, h_i(x, z_i) \qquad (III)$$

A derivada em relação ao peso genético

$$\frac{\partial \epsilon_k}{\partial w_{li}} = \frac{\partial}{\partial w_{li}} \frac{1}{2} \sum_{j=1}^{m} (y_{kj} - \hat{y}_{kj})^2$$

Para o termo I:

$$\frac{\partial G_{kr}}{\partial w_{li}} = -\underbrace{(y_{kr} - \hat{y}_{kr})}_{= e_{kr}} \frac{\partial \hat{y}_{kr}}{\partial w_{li}}$$

Lo erro do neurônio de saída r para a amostra k

$$\frac{\partial G_{kr}}{\partial w_{li}} = -e_{kr} \frac{\partial g(u_{kr})}{\partial u_{kr}} \cdot \frac{\partial u_{kr}}{\partial w_{li}}$$

$$\frac{\partial G_{kr}}{\partial w_{li}} = -e_{kr}\, g'(u_{kr}) \frac{\partial u_{kr}}{\partial w_{li}} \quad \frac{\partial (h_i(x_k, z_i) w_{ir})}{\partial w_{li}}$$

$$\frac{\partial G_{kr}}{\partial w_{li}} = -e_{kr}\, g'(u_{kr})\, w_{ir} \frac{\partial (h_i(u_{ki}))}{\partial w_{li}}$$

$$= e_{kr}\, g'(u_{kr})\, w_{ir} \cdot \frac{\partial (h_i(u_{ki}))}{u_{ki}} \cdot \frac{\partial (u_{ki})}{\partial w_{li}}$$

$$= e_{kr}\, g'(u_{kr})\, w_{ir} \cdot h_i'(u_{ki}) \frac{\partial (u_{ki})}{\partial w_{li}} \quad \frac{\partial (x_i w_{li})}{\partial w_{li}}$$

$$\frac{\partial G_{kr}}{\partial w_{li}} = -e_{kr}\, g'(u_{kr}) \cdot w_{ir}\, h_i'(u_{ki}) \cdot x_l \quad (\mathrm{II})$$

Retornando II em I

$$\frac{\partial \epsilon_k}{\partial w_{li}} = \sum_{j=1}^{m} \underbrace{-e_{kj}\, g'(u_{kj})}_{\delta_{kj}} \cdot \underbrace{w_{ij}\, h'_i(u_{ki}) \cdot x_l}_{cte}$$

$$\frac{\partial \epsilon_k}{\partial w_{li}} = -\left(\sum_{j=1}^{m} \delta_{kj}\, w_{ij}\right) h'_i(u_{ki}) \cdot x_l$$

$$w_{li} = w_{li} + \eta \underbrace{\left(\sum_{j=1}^{m} \delta_{kj}\, w_{ij}\right)}_{e_i} h'_i(u_{ki}) \cdot x_l$$

$$\boxed{w_{li} = w_{li} + \eta\, \delta_i\, x_l}$$

eq. do ajuste