first we note that $\{A_i^c\}_0^\infty$ are also independant since

$$P(A_i) = P(A_i \cap A_j) + P(A_i \cap A_j^c) = P(A_i)P(A_j) + P(A_i \cap A_j^c)$$

hence for all $i \neq j$

$$P(A_i \cap A_j^c) = P(A_i)P(A_j^c)$$

and by that same argument we can see that

$$P(A_i^c \cap A_j^c) = P(A_i^c)P(A_j^c)$$

moreover,

$$\begin{aligned}
P\Big(\bigcap_{k=0}^{\infty}\bigcup_{i=k}^{\infty} A_i\Big) &= \lim_{k\to\infty} P\Big(\bigcup_{i=k}^{\infty} A_i\Big) \\
&= \lim_{k\to\infty} 1 - P\bigg(\Big(\bigcup_{i=k}^{\infty} A_i\Big)^c\bigg) \\
&= \lim_{k\to\infty} 1 - P\Big(\bigcap_{i=k}^{\infty} A_i^c\Big) \\
&= \lim_{k\to\infty} 1 - \prod_{i=k}^{\infty} P(A_i^c)
\end{aligned}$$