Algebraic Topology: 3rd hw

Due on February 12, 2014 at 3:10pm

Section 1.2

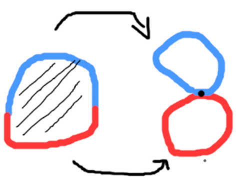
Problem 1

1.2.7 We can endow X with a cell structure by letting X^0 be a singleton and X^1 be homeo $S^1 \vee S^1$ by attaching two 1-cells in the obvious way, and then we can obtain X by attaching **two** 2-cells through a map that takes the upper semicircle of the boundary of the 2-cell to the first circle in $S^1 \vee S^1$ and the lower semicircle to the second. (see diagram below.) Now we can apply proposition 1.26.a to see that

$$\pi_1(X) \cong \pi_1(S^1 \vee S^1)/N \cong < a, b > /N \cong < a, ab > /N$$

such that N is the normal group generated by the loops that the characteristic maps of the 2-cells generate. But both generate the same loop that goes through both circles in $S^1 \vee S^1$ once. Therefore,

$$\pi_1(X) \cong \mathbb{Z}$$



1.2.9

1.2.17

1.2.18 (a) Let the quotient map of SX be q and put $A_i = q\big(\{0, 1/i\} \times I \cup \{1/n \mid n \neq i\} \times ([0, .25) \cup (.75, 1]\big)$ which are evidently open, and each deformation retracts to S^1, and they satisfy the assumptions of Van Kampen. Furthermore, $A_i \cap A_j$ for all $i \neq j$ is contractible. Hence,

$$\pi_1(SX) = \mathop{\Large *}^{\infty} \mathbb{Z}$$

Problem 2

a. Let $\mathbb{R}^*$ be the line with two origins and 0 and $0'$ are the two origins. Suppose f is a loop and let $F \colon I \times I \to \mathbb{R}^*$ be a homotopy between f and a loop that never traverses $0'$ defined as

$$F(s,t) = \begin{cases} f(s) & f(s) \neq 0' \\ 0' & f(s) = 0', t < 1/2 \\ 0 & f(s) = 0', t \geq 1/2 \end{cases}$$

note that $F^{-1}(\{0, 0'\}) = f^{-1}(\{0, 0'\}) \times I$ and $F^{-1}(S) = f^{-1}(S) \times I$ for any set $S \subset \mathbb{R}^*$ such that $S \cap \{0, 0'\} = \varnothing$. Now let O be an open subset of $\mathbb{R}^*$. O either contains both 0 and $0'$ or neither; if it didn't, $F^{-1}(O)$ is obviously open and if it did

$$F^{-1}(O) = F^{-1}(\{0, 0'\}) \cup F^{-1}(O \setminus \{0, 0'\}) = f^{-1}(\{0, 0'\}) \times I \cup f^{-1}(O \setminus \{0, 0'\}) \times I = f^{-1}(O) \times I$$

which is open hence F is continuous