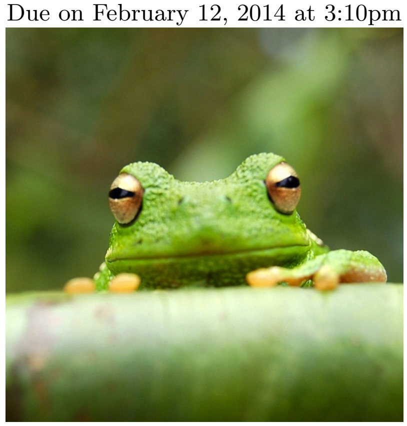
Folland Reading Group: 2nd week hw

Due on February 12, 2014 at 3:10pm



Chapter 1

Problem 1

a.
- if f is infinite on a set E with positive measure then

$$\int f \mathrm{d}\mu \geq \int N \chi_E \mathrm{d}\mu = N \cdot \mu(E)$$

for any integer N and simply letting $N \to +\infty$ gives us what we want.

- if f is simple then the theorem is trivial. For the general, due to p case there must exist an increasing sequence of simple functions $\{\phi_n\}_0^\infty$ that converge to f pointwise. Let $\mathcal{E}_n$ be the colleciton of sets where ϕ_n is positive, note that $\mathcal{E}_n$ is a finite collection of sets with finite measure. And it's obvious that f is positive on $\bigcup_{i=0}^{\infty} \bigcup_{E \in \mathcal{E}_n} E$ and hence we're done.

b. it's literally the same proof):¡.

Trivial Proof. f is measurable by proposition 2.11 and 2.12, and since for each x, $|f_n(x)| \to |f(x)|$ and $g_n(x) \to g(x)$ and since $|f_n(x)| \leq g_n(x)$ then $|f(x)| \leq g(x)$ then we have $|f| \leq g$ which means $f \in L^1$. By taking real and immaginary parts it suffices to assume f_n and f are real valued. And since $f_n - g_n \geq 0$ and $f_n + g_n \geq 0$ we can use Fatou's lemma:

$$\int g + \int f \leq \liminf \int (g_n - f_n) \leq \liminf \int g_n + \liminf \int f_n$$
$$\int g - \int f \leq \liminf \int (g_n - f_n) \leq \liminf \int g_n - \limsup \int f_n$$

Therefore, $\limsup \int f_n \leq \int f \leq \liminf \int f_n$ and the result follows. □

Problem 2

Problem 3

since simple functions are densa in $L^1(X)$ it suffices to prove that for all $\varepsilon > 0$ and all simple functions $\phi \in L^1(X)$ there exists a function $f \in C_c(X)$ such that $\int |f - \phi| < \varepsilon$. Let $\sum z_j \chi_{E_j}$ be the standard representation of ϕ