Measure Theory: 6th hw

Due on February 12, 2014 at 3:10pm

Chapter 7

Problem 1

1. It's just the topology generated by $f^{-1}(O)$ for all $f \in \mathcal{F}$ and O is open in Y_f
2. ($\Rightarrow$) fix some $x \in \mathfrak{X}$ and let O be a neighbourhood around $L(x)$ which means that $ev_x^{-1}(O)$ is a neighbourhood around L hence L_λ is eventually in $\mathrm{ev}_x^{-1}(O)$ which means $L_\lambda(x)$ is eventually in O and since x and O were arbitrary L_λ converges to L pointwise.

   ($\Leftarrow$) let O be a neighbourhood around L, it must then contain an open set of the form $\mathrm{ev}_x^{-1}(O)$ for some $x \in \mathfrak{X}$ and O open in $\mathbb{C}$ and contains $L(x)$

Problem 2

1. First, note that $0 \in A$ iff $\tau^0 A = A \in M$. Now suppose $0 \in A \iff \tau^n A \in M$ for all $n \leq N$ but then

   $$N + 1 \in A \iff N \in \tau A \iff \tau^N(\tau A) \in M \iff \tau^{N+1} A \in M$$

   hence the statement $\tau^n A \in M \iff n \in A$ is true for all $n \in \mathbb{N}$

   And since $\tau^n A \in M$ iff $A \in \tau^{-n}(M)$ hence $A \in \tau^{-n}(M) \iff n \in A$ from which

   $$a, a+n, \ldots, a+(k-1)n \in A \iff \tau^a A \in M \cap \tau^{-n}(M) \cap \cdots \tau^{-(k-1)n}(M)$$

   follows immediately hence A contains an arithmetic progression of size k iff there exists an $n \in \mathbb{N}$ such that $M \cap \tau^{-n}(M) \cap \cdots \tau^{-(k-1)n}(M)$ is non-empty.